Lecture 2:
Nearest Neighbor and Linear Classification

Data-driven approach:

1. Collect a dataset of images and labels
2. Use Machine Learning to train an image classifier
3. Evaluate the classifier on a withheld set of test images

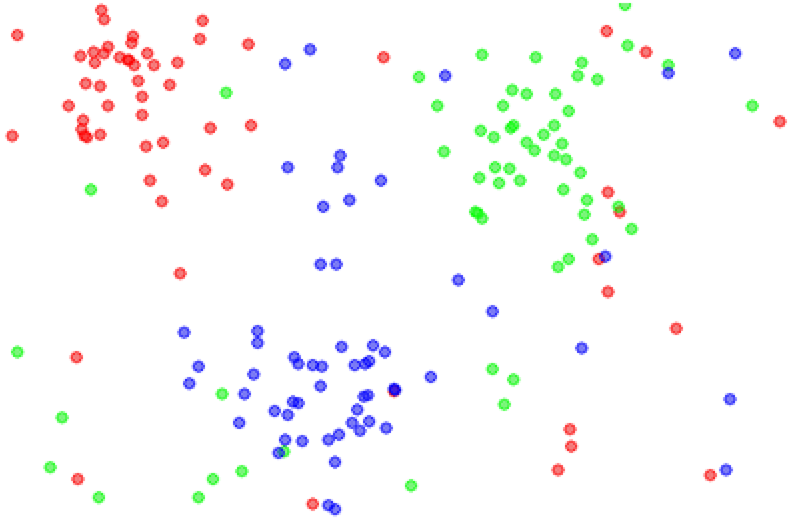
```python
def train(train_images, train_labels):
  # build a model for images -> labels...
  return model

def predict(model, test_images):
  # predict test_labels using the model...
  return test_labels
```

Example training set

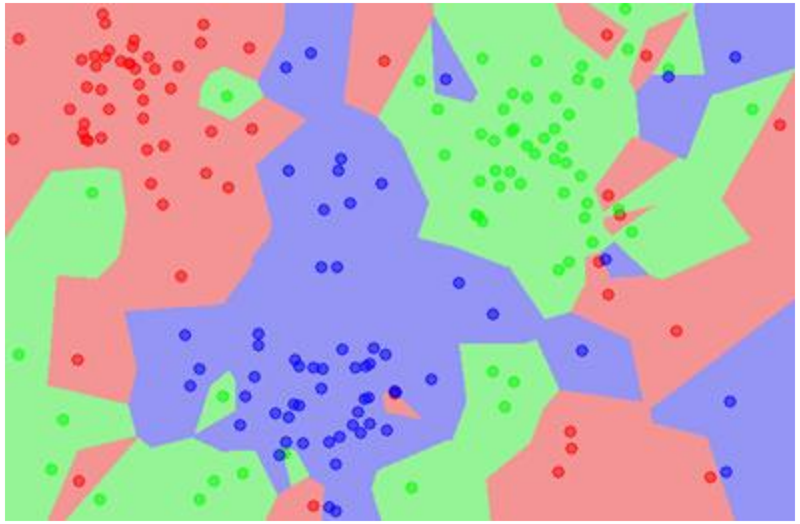
k-Nearest Neighbor

find the k nearest images, have them vote on the label

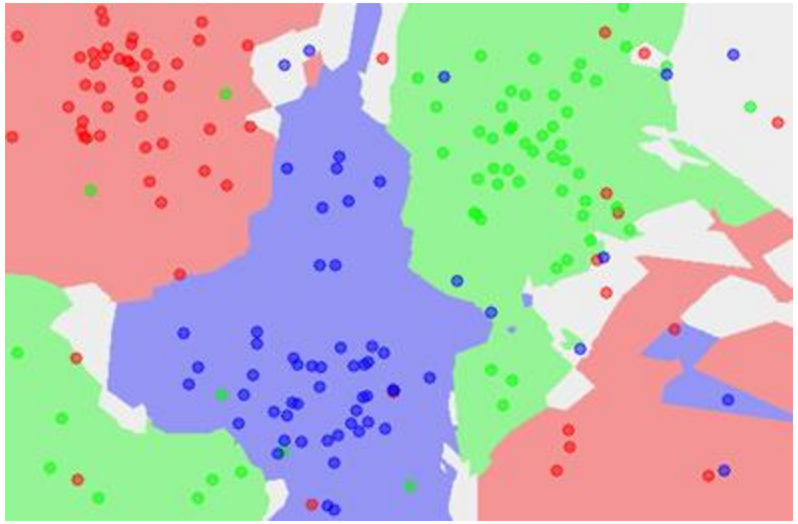
the data



NN classifier

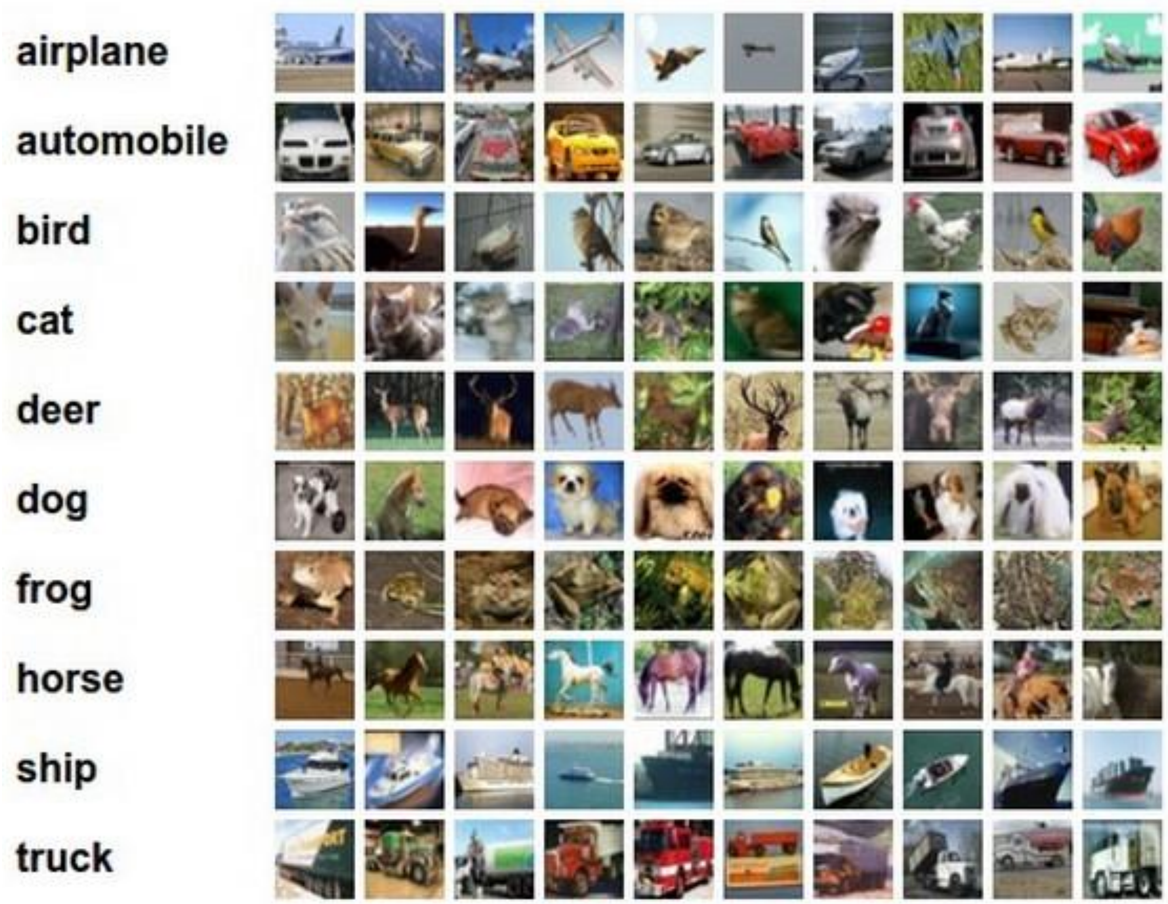


5-NN classifier



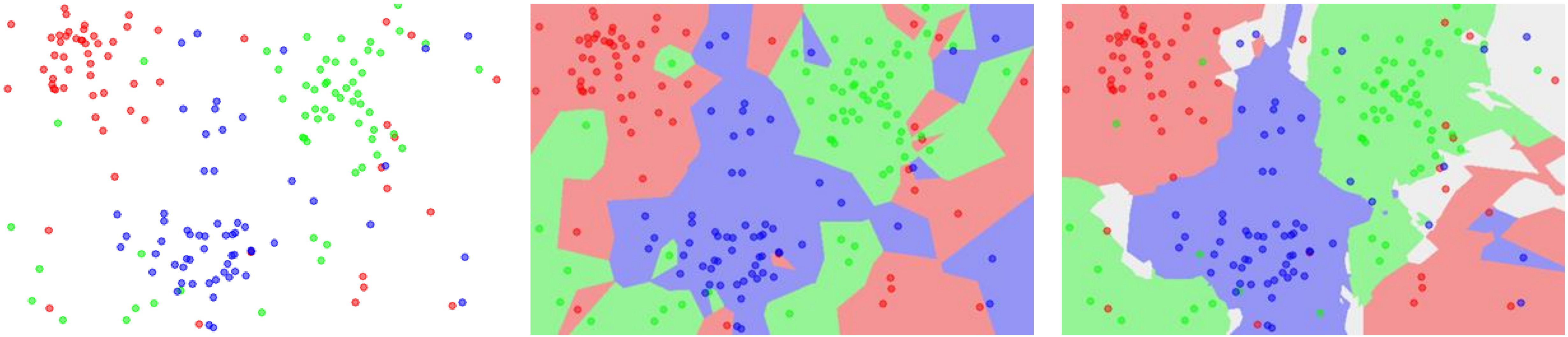
http://en.wikipedia.org/wiki/K-nearest_neighbors_algorithm

Example dataset: **CIFAR-10**
10 labels
50,000 training images
10,000 test images.

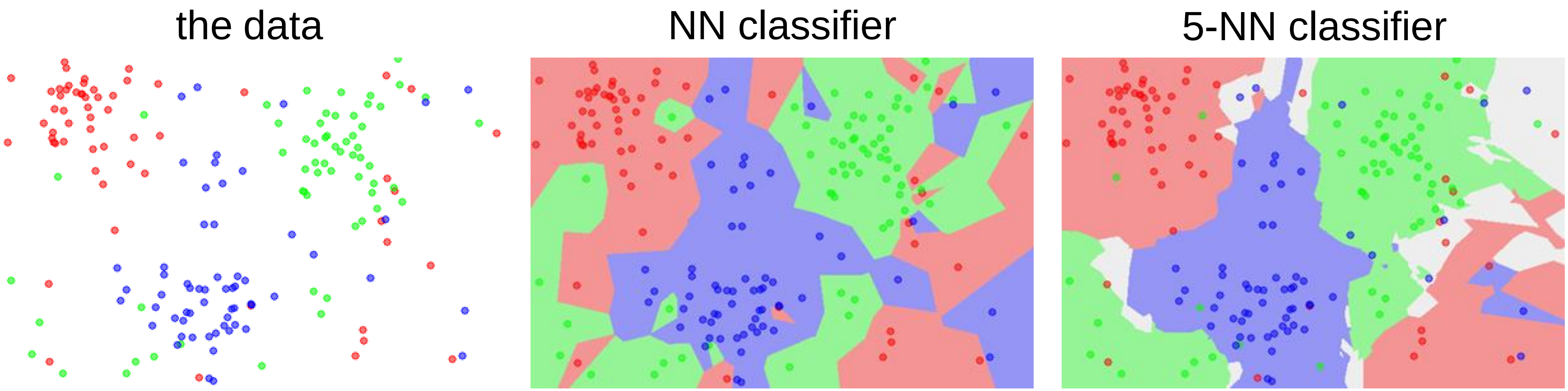


For every test image (first column), examples of nearest neighbors in rows

the data NN classifier 5-NN classifier



Q: what is the accuracy of the nearest neighbor classifier on the training data, when using the Euclidean distance?



Q2: what is the accuracy of the **k-**nearest neighbor classifier on the training data?

What is the best **distance** to use?
What is the best value of **k** to use?

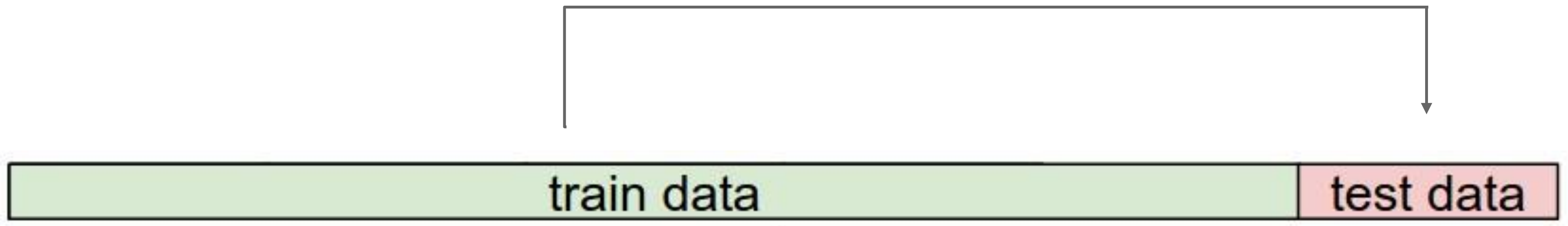
i.e. how do we set the **hyperparameters**?

What is the best **distance** to use?
What is the best value of **k** to use?

i.e. how do we set the **hyperparameters**?

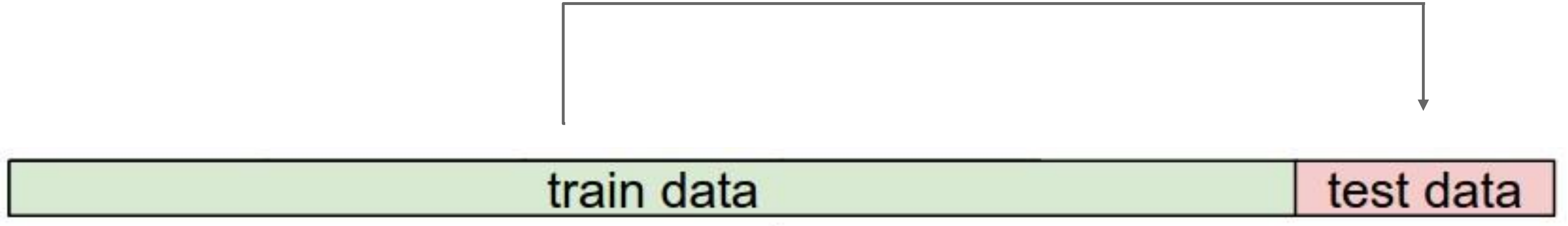
Very problem-dependent.
Must try them all out and see what works best.

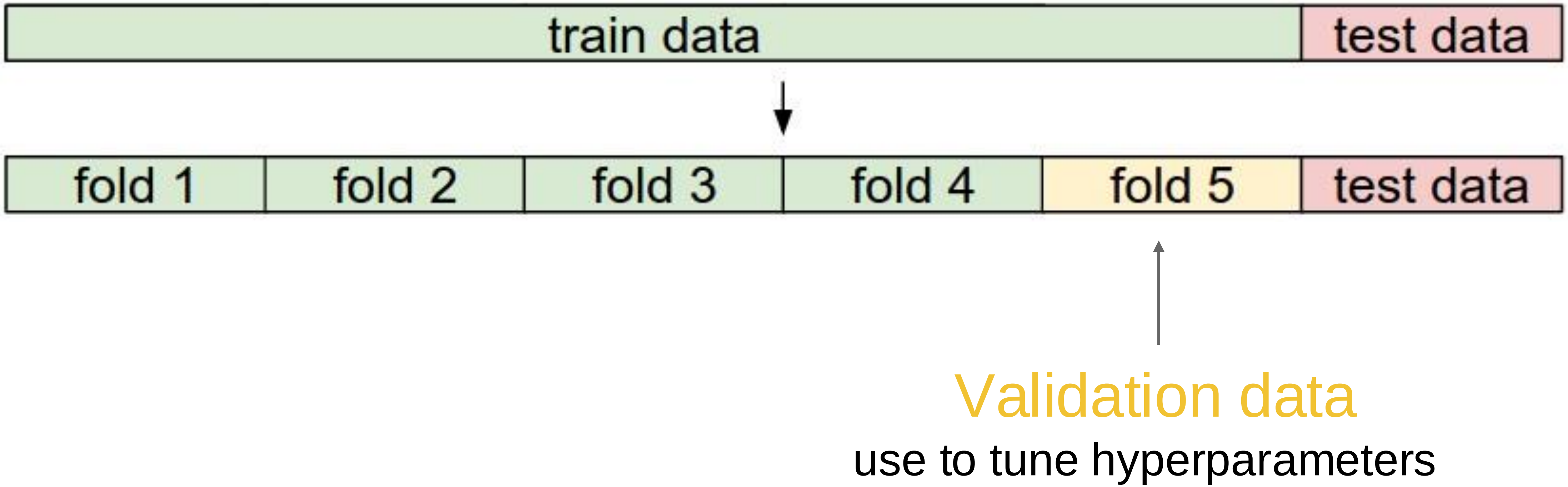
Try out what hyperparameters work best on test set.

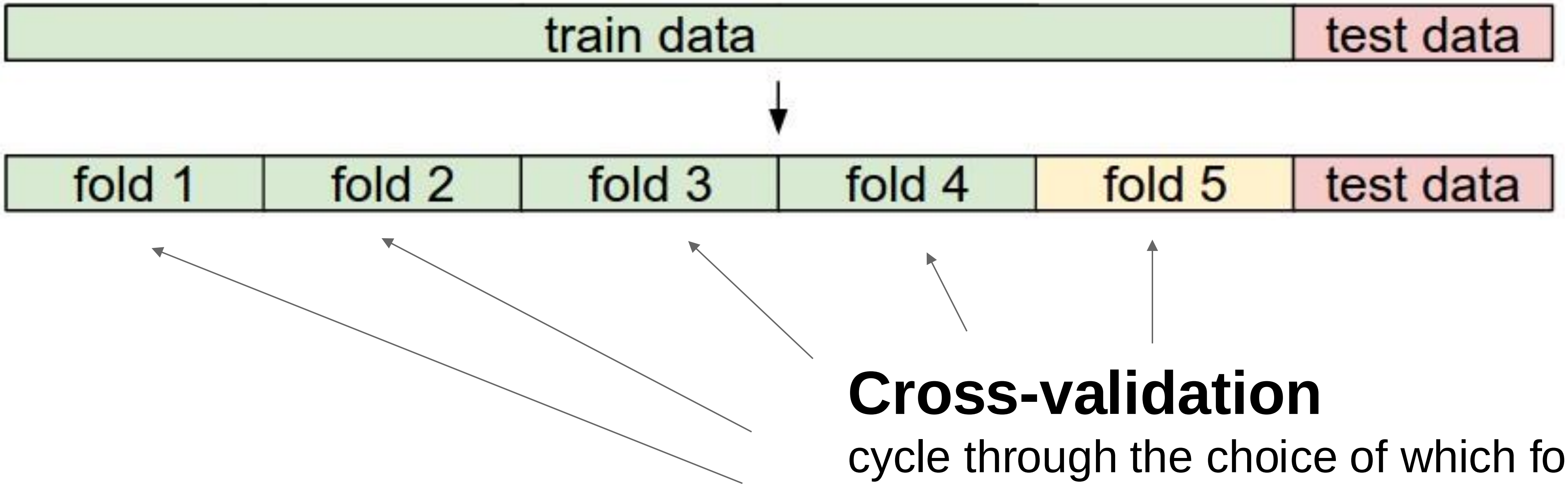


Trying out what hyperparameters work best on test set:

Very bad idea. The test set is a proxy for the generalization performance! Use only **VERY SPARINGLY,** at the end.

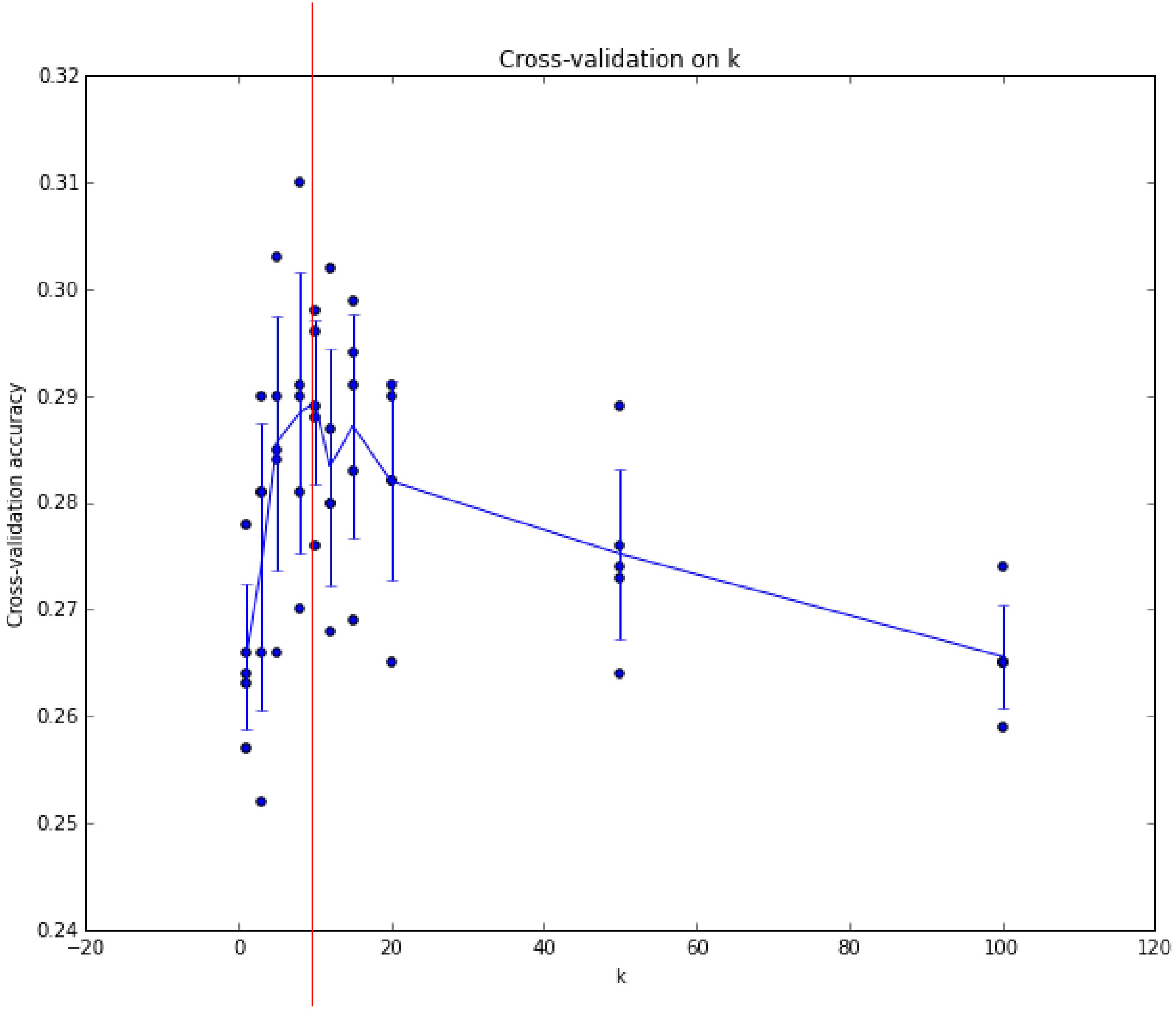



train data
test data
fold 1
fold 2
fold 3
fold 4
fold 5
test data
Validation data
use to tune hyperparameters



Cross-validation

cycle through the choice of which fold is the validation fold, average results.



Example of
5-fold cross-validation
for the value of **k.**

Each point: single
outcome.

The line goes
through the mean, bars
indicated standard
deviation

(Seems that k ~= 7 works best
for this data)

k-Nearest Neighbor on *raw* images is **never used.**

- terrible performance at test time
- distance metrics on level of whole images can be very unintuitive

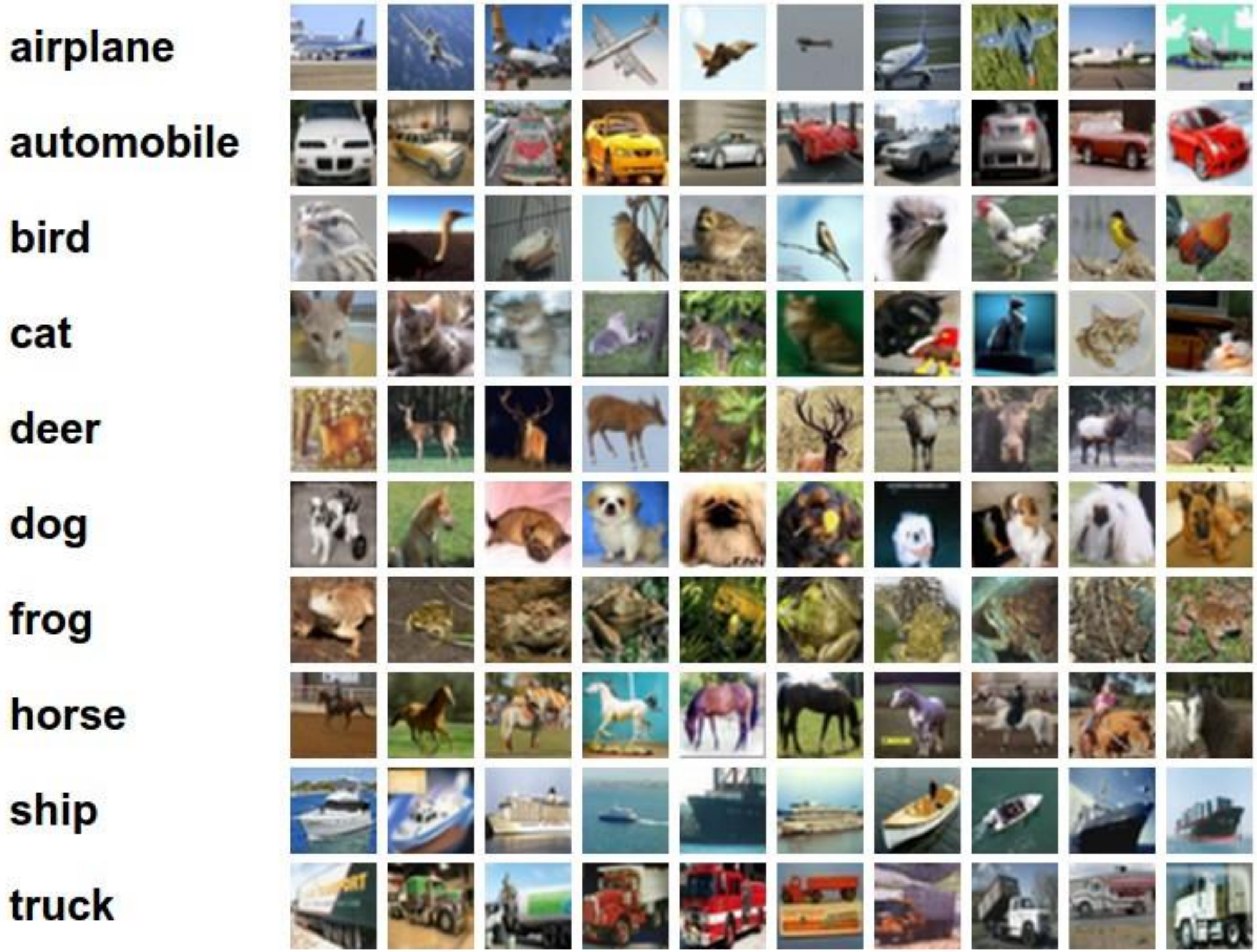
(all 3 images have same L2 distance to the one on the left)

Before moving on

- K-NN: the Rodney Dangerfield of classifiers

- Convergence of K-NN to the Bayes error rate.
- Universality of K-NN.

Linear Classification



Example dataset: **CIFAR-10**
10 labels
50,000 training images
each image is **32x32x3**
10,000 test images.

Parametric approach

[32x32x3]
array of numbers 0...1
(3072 numbers total)

image parameters

$f(\mathbf{x}, \mathbf{W})$ ⟶

10 numbers, indicating class scores

Parametric approach: **Linear classifier**

$$f(x, W) = Wx$$

[32x32x3]
array of numbers 0...1

10 numbers,
indicating class
scores

Parametric approach: **Linear classifier**

$$\boxed{f(x,W)} = \boxed{W}\boxed{x}$$

3072x1

10x1 **10x3072**

[32x32x3]
array of numbers 0...1

10 numbers, indicating class scores

parameters, or “weights”

Parametric approach: **Linear classifier**

$$\boxed{f(x,W)} = \boxed{W}\boxed{x} \quad \boxed{(+b)}$$

3072x1

10x1

10x1

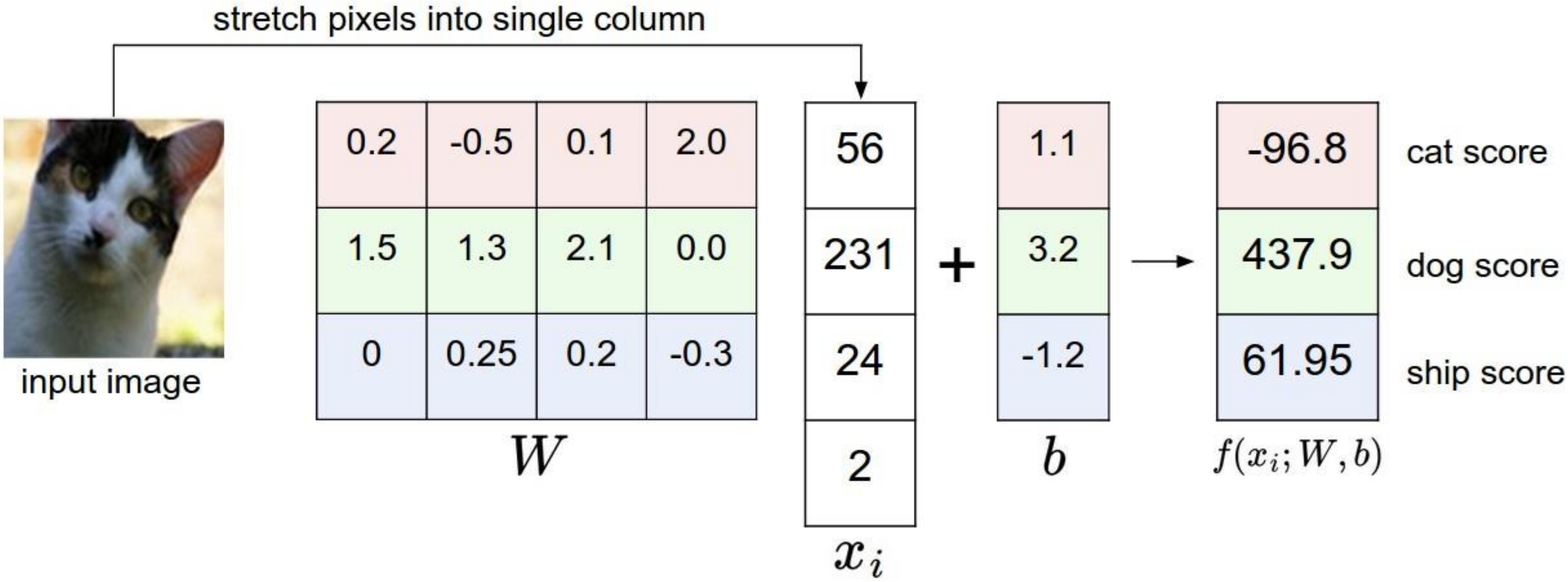
10x3072

[32x32x3]
array of numbers 0...1

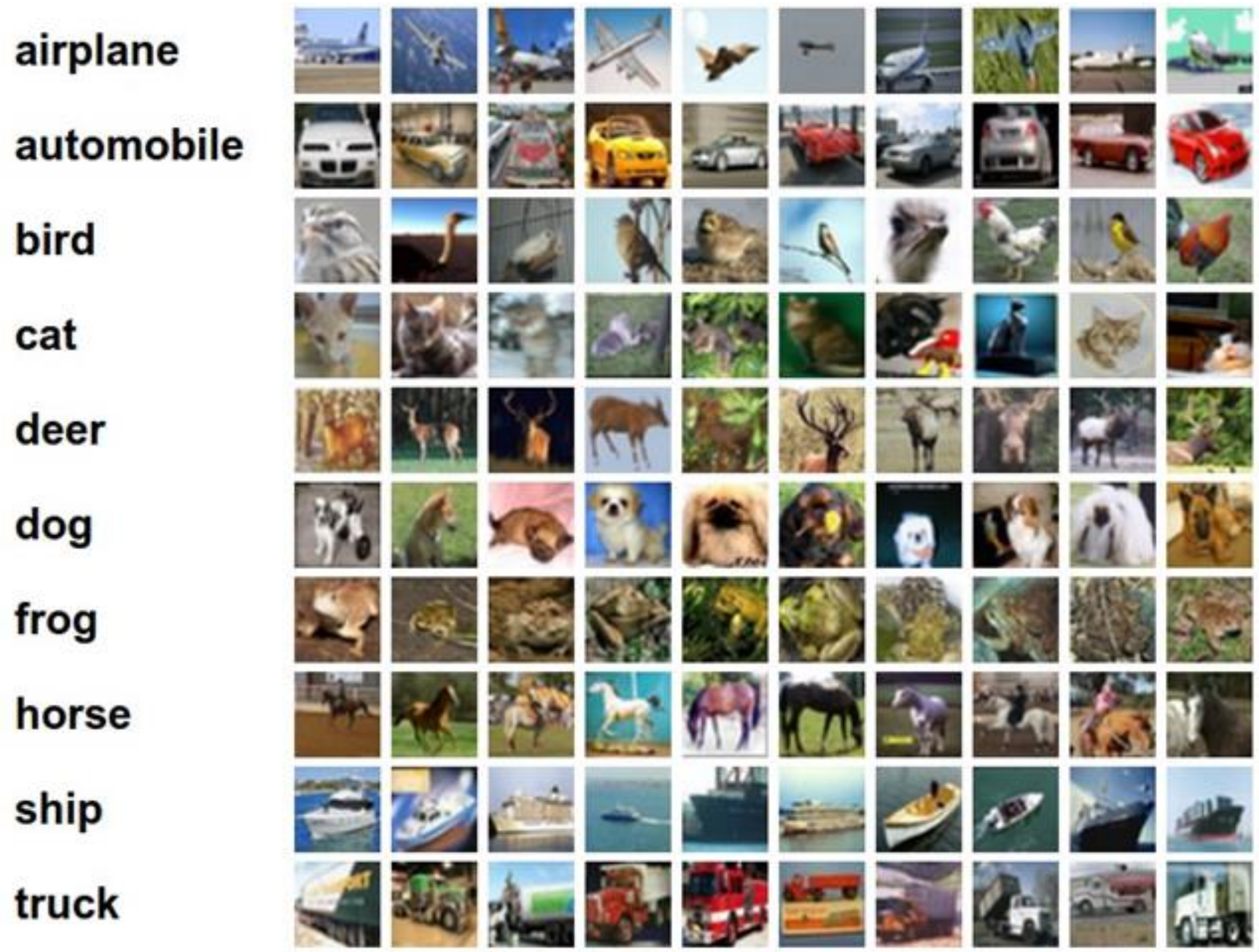
10 numbers, indicating class scores

parameters, or “weights”

Example with an image with 4 pixels, and 3 classes (cat/dog/ship)



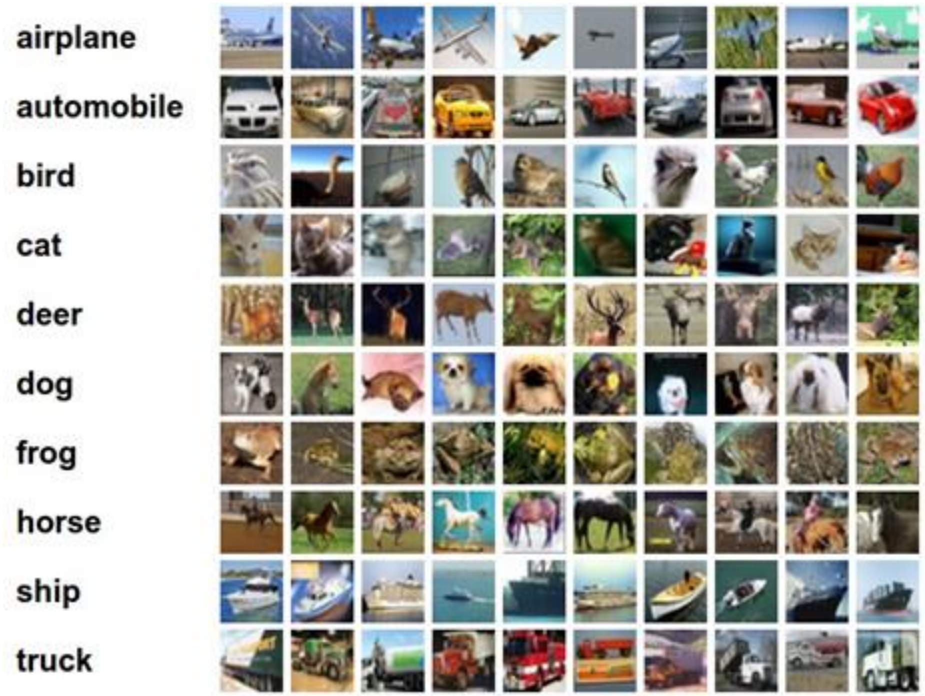
Interpreting a Linear Classifier



$$f(x_i, W, b) = Wx_i + b$$

Q: what does the linear classifier do, in English?

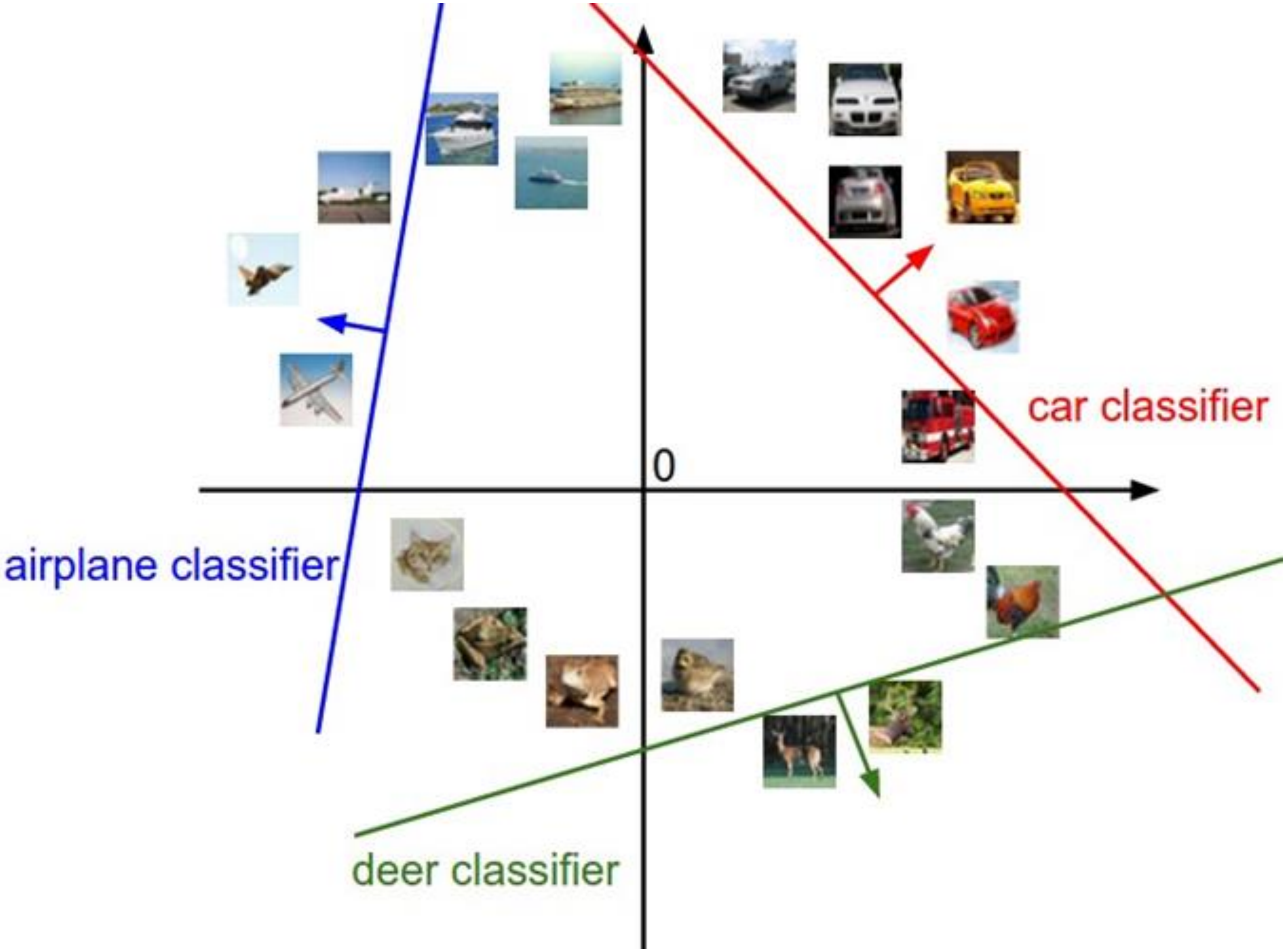
Interpreting a Linear Classifier (poll)



$$f(x_i, W, b) = Wx_i + b$$

Example trained weights of a linear classifier trained on CIFAR-10:

Interpreting a Linear Classifier



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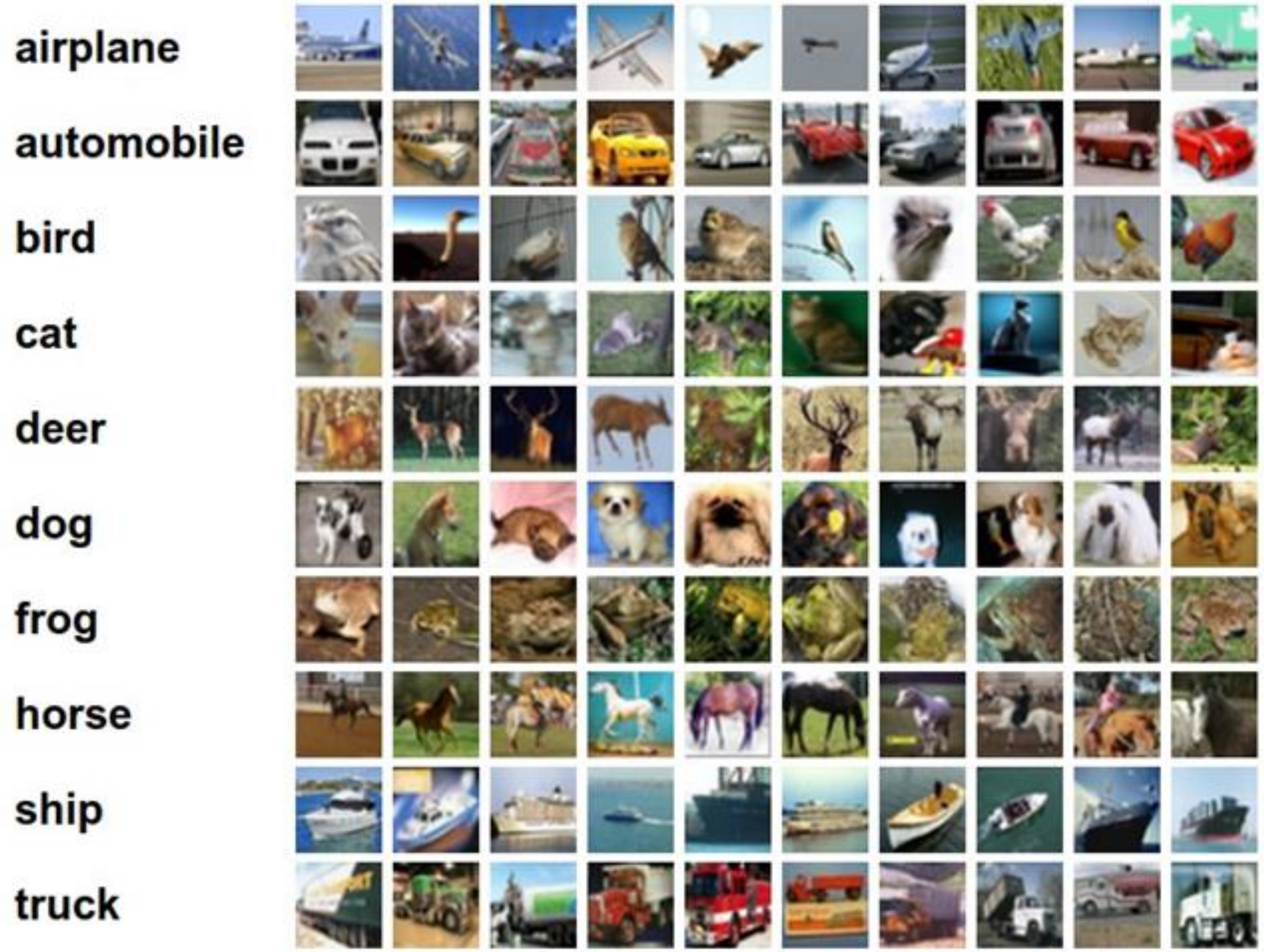
Interpreting a Linear Classifier



$$f(x_i, W, b) = Wx_i + b$$

Q2: what would be a very hard set of classes for a linear classifier to distinguish?

So far: We defined a (linear) **score function**: $f(x_i, W, b) = Wx_i + b$

really ***affine***

Example class scores for 3 images, with a random W:

airplane	-3.45	-0.51	3.42
automobile	-8.87	**6.04**	4.64
bird	0.09	5.31	2.65
cat	**2.9**	-4.22	5.1
deer	4.48	-4.19	2.64
dog	8.02	3.58	5.55
frog	3.78	4.49	**-4.34**
horse	1.06	-4.37	-1.5
ship	-0.36	-2.09	-4.79
truck	-0.72	-2.93	6.14

$$f(x, W) = Wx$$

Coming up:

- Loss function (quantifying what it means to have a “good” W)
- Optimization (start with random W and find a W that minimizes the loss)
- Neural nets! (tweak the functional form of f)

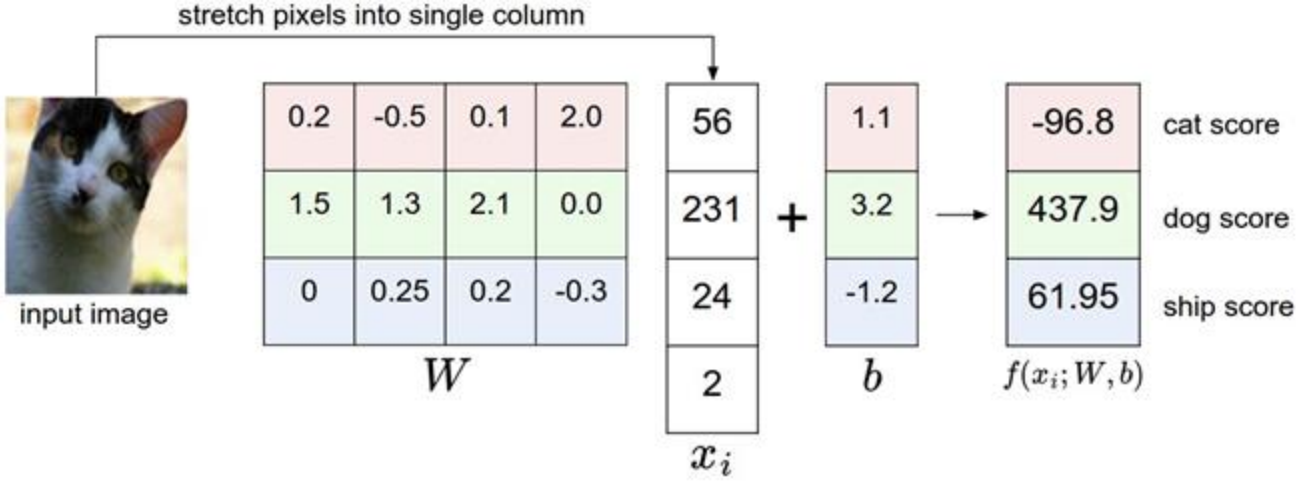
Summary so far... Linear classifier

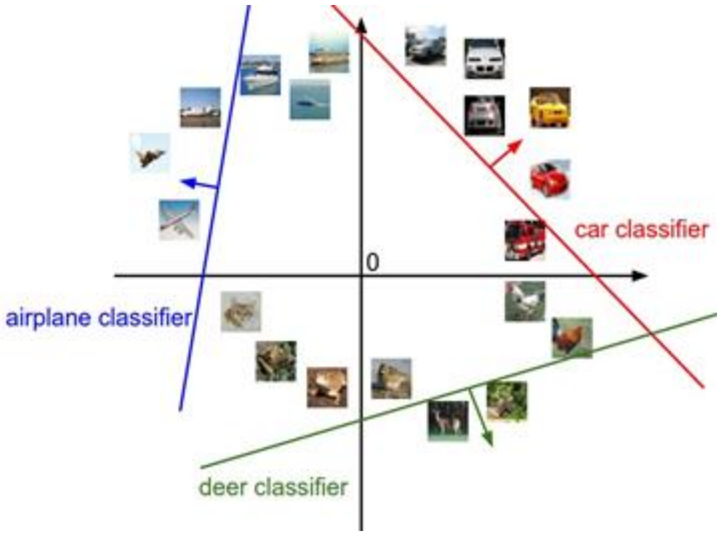
[32x32x3]
array of numbers 0...1
(3072 numbers total)

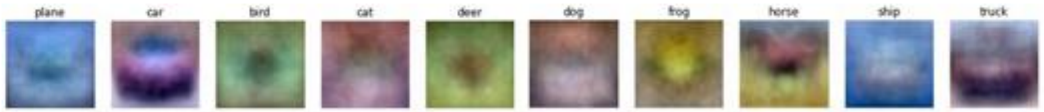
image parameters

$$f(\mathbf{x},\mathbf{W})$$

10 numbers, indicating class scores







Recall from last time… Going forward: Loss function/Optimization



airplane	-3.45	-0.51	3.42
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ship	-0.36	-2.09	-4.79
truck	-0.72	-2.93	6.14

TODO:

1. Define a **loss function** that quantifies our unhappiness with the scores across the training data.

1. Come up with a way of efficiently finding the parameters that minimize the loss function. **(optimization)**

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

  

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**

Suppose: 3 training examples, 3 classes.
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cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
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Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s_i = f(x_i, W)$

the SVM loss has the form:

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



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Losses:	2.9		

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= max(0, 5.1 - 3.2 + 1)
+max(0, -1.7 - 3.2 + 1)
= max(0, 2.9) + max(0, -3.9)
= 2.9 + 0
= 2.9

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

  

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
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Losses:	2.9	0	

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the SVM loss has the form:

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

= max(0, 1.3 - 4.9 + 1)
+max(0, 2.0 - 4.9 + 1)
= max(0, -2.6) + max(0, -1.9)
= 0 + 0
= 0

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	12.9

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the SVM loss has the form:

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

= max(0, 2.2 - (-3.1) + 1)
+max(0, 2.5 - (-3.1) + 1)
= max(0, 6.3) + max(0, 6.6)
= 6.3 + 6.6
= 12.9

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

  

cat	**3.2**	1.3	2.2
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Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s_i = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

and the full training loss is the mean over all examples in the training data:

$$L = \frac{1}{N} \sum_{i=1}^{N} L_i$$

L = (2.9 + 0 + 12.9)/3
= **5.3**

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x,W) = Wx$ are:

  

cat	**3.2**	1.3	2.2
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Multiclass SVM loss:

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where x_i is the image and
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and using the shorthand for the scores vector: $s_i = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q: what if the sum was instead over all classes? (including j = y_i)

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
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Losses:	2.9	0	12.9

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s_i = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q2: what if we used a mean instead of a sum here?

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

  

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Multiclass SVM loss:

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scores vector: $s_i = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q3: what if we used

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)^2$$

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

  

cat	**3.2**	1.3	2.2
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frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	12.9

Multiclass SVM loss:

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where y_i is the (integer) label,

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the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q4: what is the min/max possible loss?

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

  

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Losses:	2.9	0	12.9

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s_i = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q5: usually at initialization W are small numbers, so all s ~= 0. What is the loss?

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

  

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Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

			
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Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

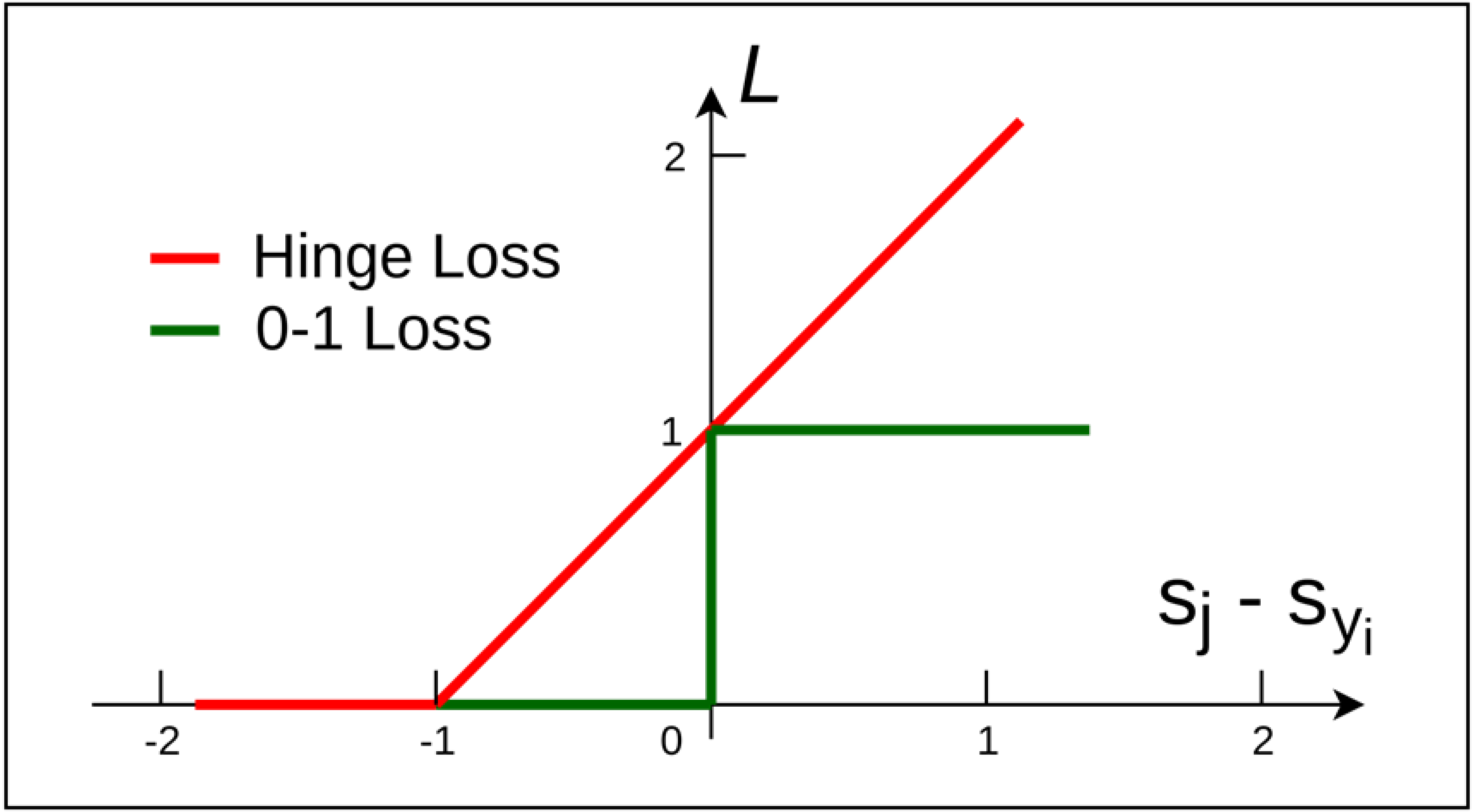
the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

and the full training loss is the mean over all examples in the training data:

$$L = \frac{1}{N} \sum_{i=1}^{N} L_i$$

L = (2.9 + 0 + 10.9)/3
= **5.3**



Hinge Loss
0-1 Loss
L
2
1
-2
-1
0
1
2
$s_j - s_{y_i}$

Multiclass SVM Loss:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	12.9

1. What happens to loss if car scores change a bit?
2. What is the min/max possible loss?
3. At initialization W is small so all s ≈ 0. What is the loss?
4. What if the sum was over all classes? (including j = y_i)
5. What if we used mean instead of sum?
6. What if we used squared SVM loss?

Example numpy code:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

```
def L_i_vectorized(x, y, W):
  scores = W.dot(x)
  margins = np.maximum(0, scores - scores[y] + 1)
  margins[y] = 0
  loss_i = np.sum(margins)
  return loss_i
```

Coding tip: Keep notes on dimensions:

```
N = X.shape[0]
D = X.shape[1]
C = W.shape[1]

scores=X.dot(W)                  # (N,D)*(D,C)=(N,C)
```