

Solution 1:

a) First, sort the table:

| ID | Actual Class | Score | Predicted Class |
|----|--------------|-------|-----------------|
| 6 | 0 | 0.63 | 1 |
| 7 | 1 | 0.62 | 1 |
| 10 | 0 | 0.57 | 1 |
| 4 | 1 | 0.38 | 0 |
| 1 | 0 | 0.33 | 0 |
| 8 | 1 | 0.33 | 0 |
| 2 | 0 | 0.27 | 0 |
| 5 | 1 | 0.17 | 0 |
| 9 | 0 | 0.15 | 0 |
| 3 | 1 | 0.11 | 0 |

| | Actual Class - 0 | Actual Class - 1 |
|----------------|------------------|------------------|
| Prediction - 0 | 3 | 4 |
| Prediction - 1 | 2 | 1 |

so we get

| FN | FP | TN | TP |
|----|----|----|----|
| 4 | 2 | 3 | 1 |

b)

$$\text{Precision} = \frac{\text{TP}}{\text{TP} + \text{FP}} = \frac{1}{3}$$

$$\text{Sensitivity} = \frac{\text{TP}}{\text{TP} + \text{FN}} = \frac{1}{5}$$

$$\text{Accuracy} = \frac{\text{TP} + \text{TN}}{\text{TP} + \text{TN} + \text{FP} + \text{FN}} = \frac{4}{10}$$

$$\text{Specificity} = \frac{\text{TN}}{\text{TN} + \text{FP}} = \frac{3}{5}$$

$$\text{Error Rate} = \frac{\text{FP} + \text{FN}}{\text{TP} + \text{TN} + \text{FP} + \text{FN}} = \frac{6}{10}$$

$$\text{F-measure} = \frac{2 \cdot \text{Precision} \cdot \text{Sensitivity}}{\text{Precision} + \text{Sensitivity}} = 0.25$$

$$\text{Negative Predictive Value} = \frac{\text{TN}}{\text{TN} + \text{FN}} = \frac{3}{7}$$

c) First we sort the results by the score:

| | true_labels | scores |
|----|-------------|--------|
| 6 | 0 | 0.63 |
| 7 | 1 | 0.62 |
| 10 | 0 | 0.57 |
| 4 | 1 | 0.38 |
| 1 | 0 | 0.33 |
| 8 | 1 | 0.33 |
| 2 | 0 | 0.27 |
| 5 | 1 | 0.17 |
| 9 | 0 | 0.15 |
| 3 | 1 | 0.10 |

Here we see that $\frac{1}{n_+} = \frac{1}{5} = 0.2$ and $\frac{1}{n_-} = \frac{1}{5} = 0.2$. Now we follow the algorithm as described in the lecture slides:

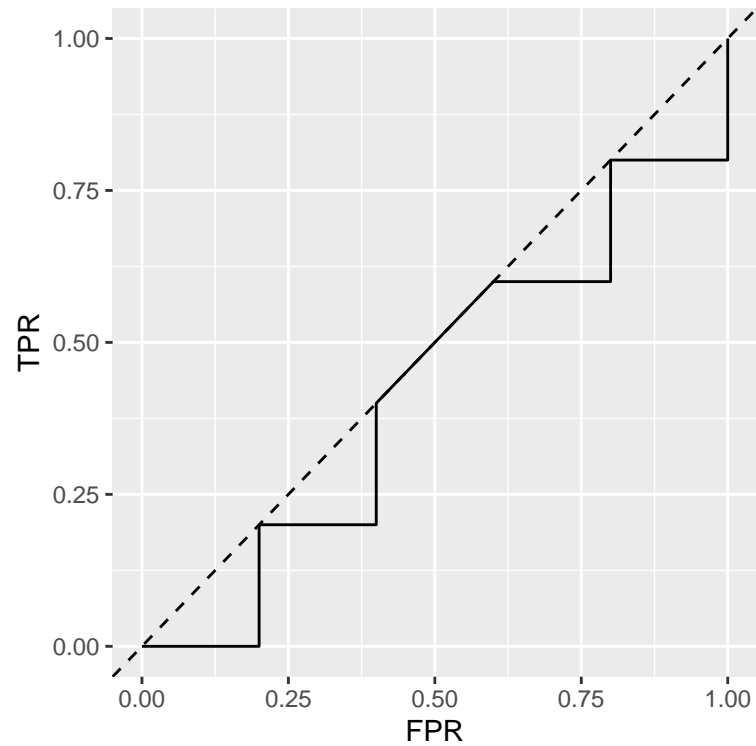
- (i) Set $\alpha = 1$, so we start in $(0, 0)$; we predict everything as 1.
- (ii) Set threshold $\tau = 0.625$ yields TPR $0 + \frac{1}{n_+} = 0.2$ and FPR $0 + \frac{1}{n_-} = 0.2$. (Obs. 6 is "0")
- (iii) Set threshold $\tau = 0.6$ yields TPR $0 + \frac{1}{n_+} = 0.2$ and FPR 0.2 . (Obs. 7 is "1")
- (iv) Set threshold $\tau = 0.5$ yields TPR $0.2 + \frac{1}{n_+} = 0.4$ and FPR $0.2 + \frac{1}{n_-} = 0.4$. (Obs. 10 is "0")
- (v) Set threshold $\tau = 0.35$ yields TPR $0.2 + \frac{1}{n_+} = 0.4$ and FPR 0.4 . (Obs. 4 is "1")
- (vi) Set threshold $\tau = 0.3$ yields TPR $0.4 + \frac{1}{n_+} = 0.6$ and FPR $0.4 + \frac{1}{n_-} = 0.6$. (Obs. 1/8 is "0"/"1")
- (vii) Set threshold $\tau = 0.2$ yields TPR 0.6 and FPR $0.6 + \frac{1}{n_-} = 0.8$. (Obs. 2 is "0")
- (viii) Set threshold $\tau = 0.16$ yields TPR $0.6 + \frac{1}{n_+} = 0.8$ and FPR 0.8 . (Obs. 5 is "1")
- (ix) Set threshold $\tau = 0.14$ yields TPR 0.8 and FPR $0.8 + \frac{1}{n_-} = 1$. (Obs. 9 is "0")
- (x) Set threshold $\tau = 0.09$ yields TPR $0.8 + \frac{1}{n_+} = 1$ and FPR 1 . (Obs. 3 is "1")

Therefore we get the polygonal path consisting of the ordered list of vertices

$(0, 0), (0.2, 0), (0.2, 0.2), (0.4, 0.2), (0.4, 0.4), (0.6, 0.6), (0.8, 0.6), (0.8, 0.8), (1, 0.8), (1, 1)$.

```
library(ggplot2)
roc_data <- data.frame(TPR = c(0, 0, 0.2, 0.2, 0.4, 0.6, 0.6, 0.8, 0.8, 1),
                      FPR = c(0, 0.2, 0.2, 0.4, 0.4, 0.6, 0.8, 0.8, 1, 1))

ggplot(roc_data, aes(x = FPR, y = TPR)) + geom_line() +
  geom_abline(slope = 1, intercept = 0, linetype = 'dashed')
```



We see that the resulting ROC lies below the line from the origin with a slope of 1, which represents a random classifier, i.e., the scoring algorithm performs worse than a random classifier. If this happens while evaluating the training data, the labels of the scoring algorithm should be inverted.

d) We can compute the AUC (*area under the curve*) by looking at the ROC, s.t.

$$AUC = 0.5 - 4 \cdot (0.2 \cdot 0.2 \cdot 0.5) = 0.42.$$