Bayesian inference and the Hierarchical Gaussian Filter

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Translational Neuromodeling Unit

Outline

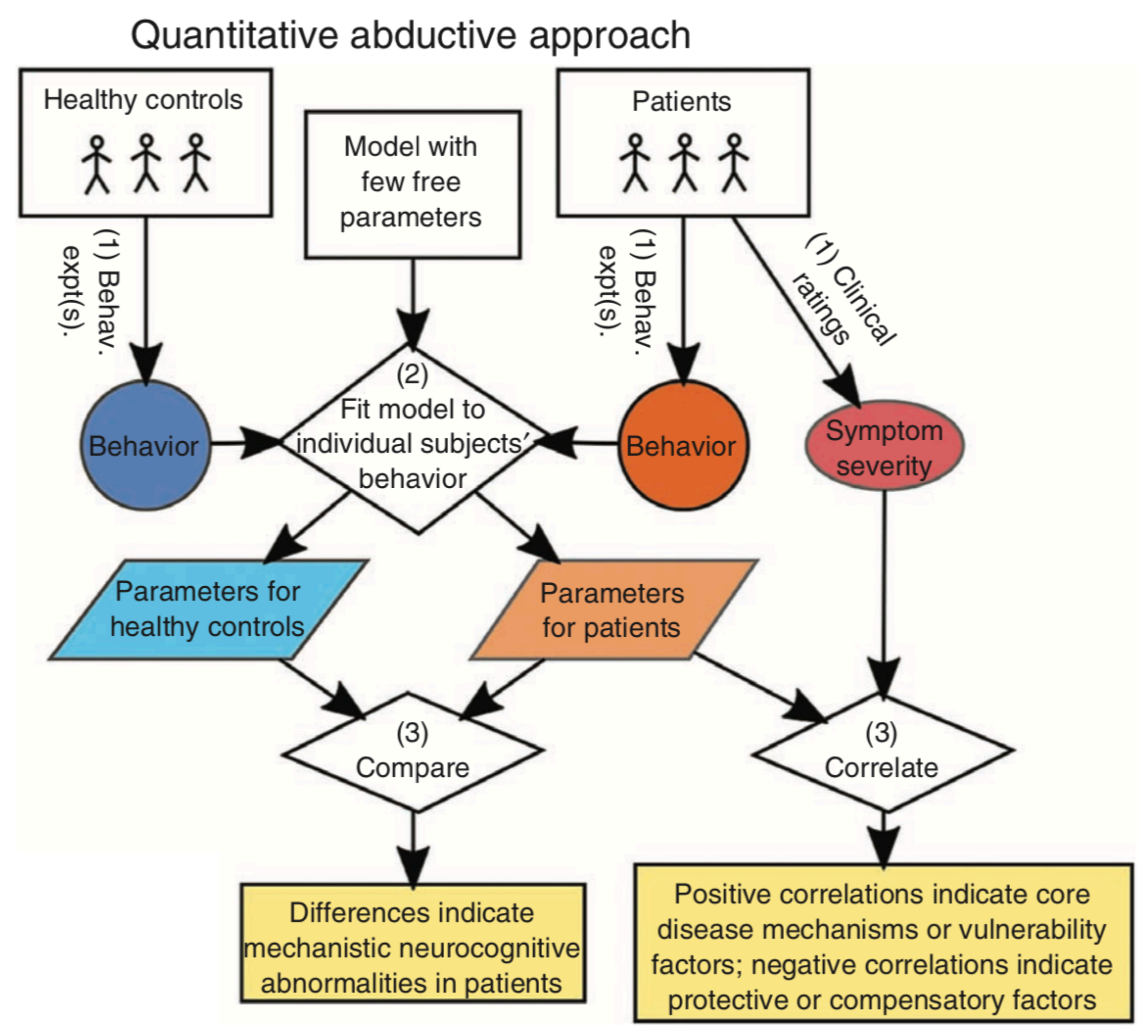
- Presentation (08.15 - 09.15)
    - Introduction
    - Meta-Bayesian modelling
    - The Hierarchical Gaussian Filter
    - Applications
- Exercises (09.15 - 09.45)
- Coffee break (09.45 - 10.15)
- Exercises (10.15 - 11.45)

The computational psychiatry approach

- The goals: nosology, biomarkers (computational phenotypes) and treatments
    - Dimensional / mathematical definition of diseases more differentiated and precise
    - Biomarkers can be parameter estimates of models
    - Better treatment selection / new treatments
- Central to this approach: “computational assays”. Examples of candidate mechanisms:
    - Ppl with autism over-estimate the volatility of the sensory environment
    - Ppl with delusions have attractor-like belief updating
    - Ppl with hallucinations have stronger perceptual priors
- Models are essential: they are precise, mathematical representations of mechanisms

The role of the HGF

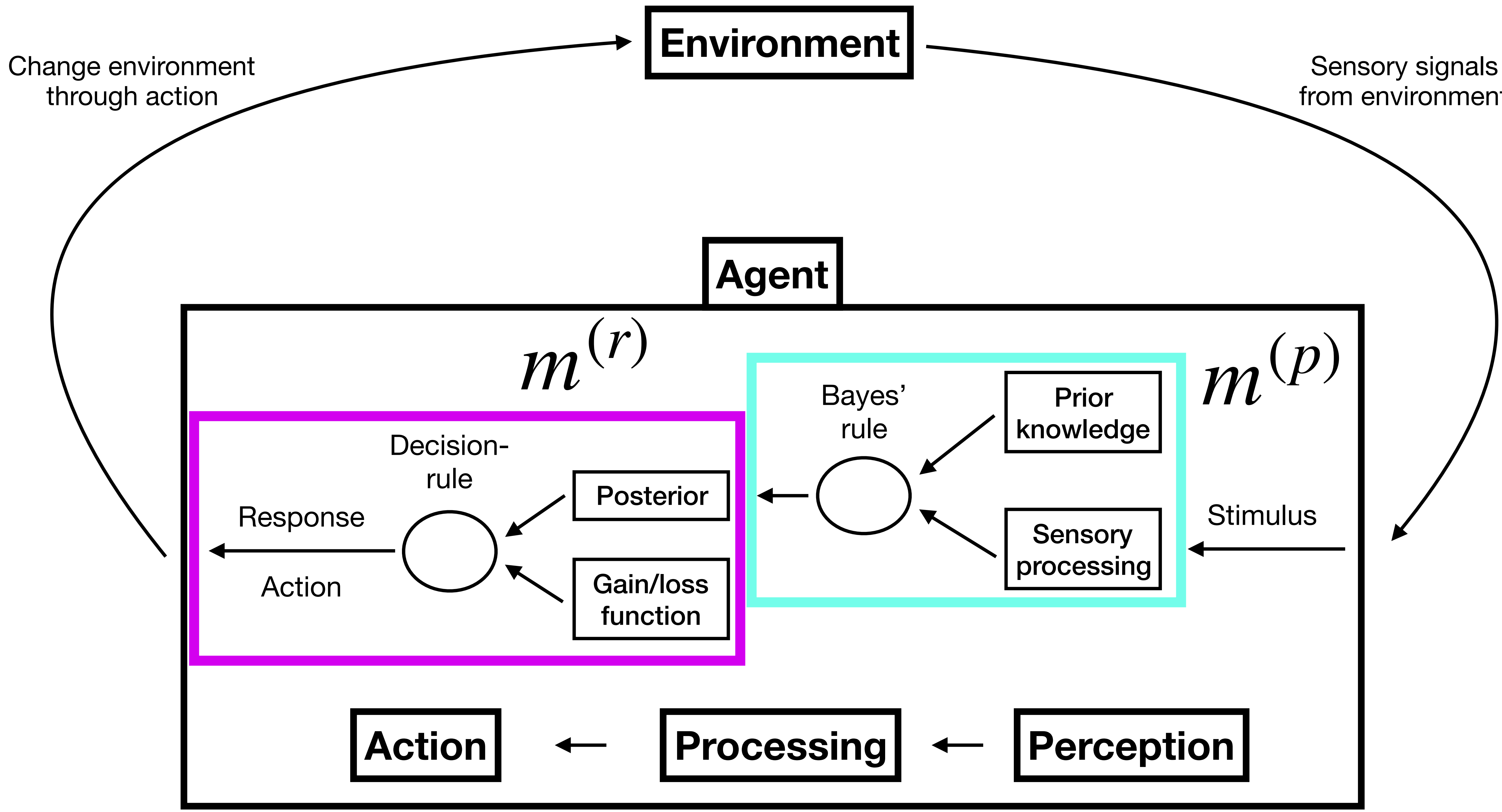
- The HGF provides opportunities to define biomarkers / “computational phenotypes” for hierarchical inference processes:
    - Individual-specific model: makes predictions for a single individual
    - Belief updating: the balance of top-down (prior) and bottom-up information (data / likelihood)
    - Separation of different types of uncertainty



(Maia & Frank, 2011))

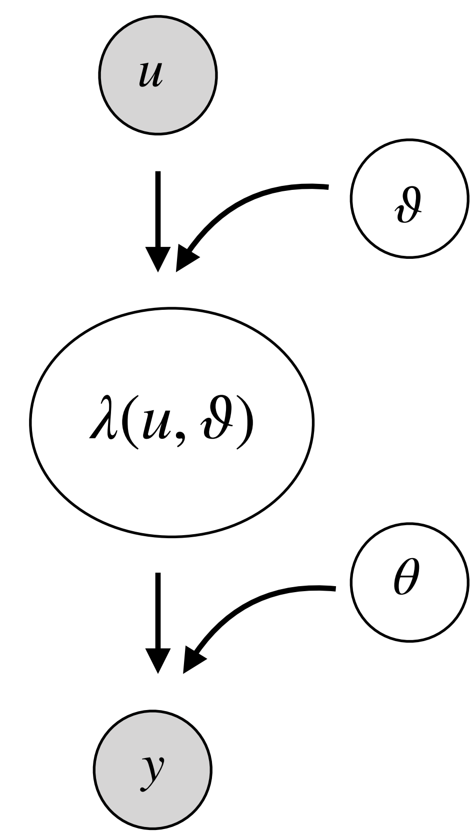
The meta-Bayesian modelling approach

Modelling the inference process



Meta-Bayesian modelling

- This is our generative model for the participants behaviour:
  - Assume:
    - (parameterised) perceptual model $x \overset{\vartheta}{\to} u$
    - (parameterised) response model $\lambda \overset{\theta}{\to} y$
  - Invert perceptual model to obtain inference process
  - Infer about parameters given behaviour and generative model for participants behaviour
- The outer generative model includes the generative model that we assume the participant to hold, therefore: Our assumptions should be weak / general



Example: Gambling task

- Two slot machines:

  For T trials, subjects can choose to play either machine to obtain a reward

- Generative process of task:
  At each time t only one of the machines will give a reward:

$$u^{(t)} \sim Ber(x^{(t)})$$

Subject's response in t-th trial:

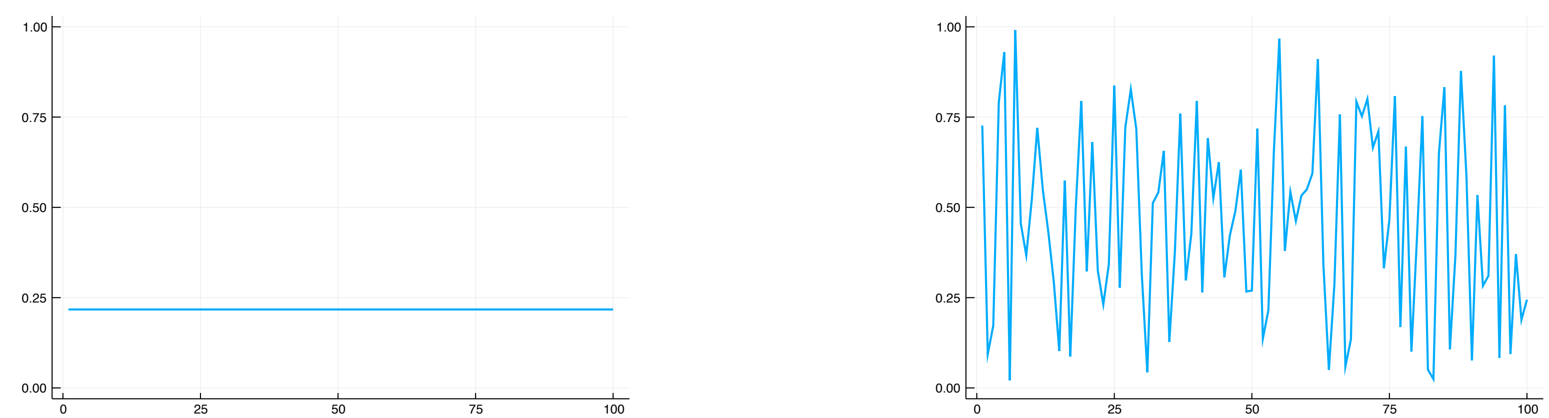
$$y^{(t)} \in \{1,2\}$$

Subject's reward in t-th trial:

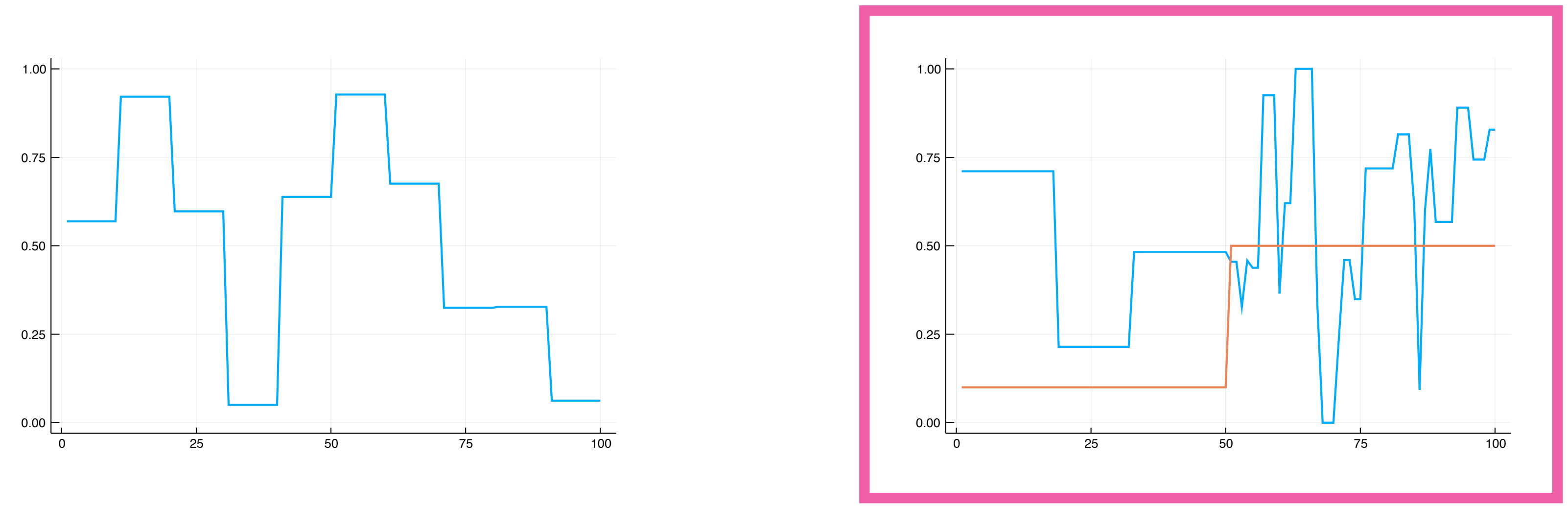
$$r^{(t)} = \begin{cases} 1, & if\ u^{(t)} = y^{(t)} \\ 0, & if\ else \end{cases}$$

Generative process of gambling task

$x^{(t)}$



$x^{(t)}$



Types of uncertainty:

- **Expected and irreducible**
- **Unexpected and reducible**
- **Unexpected and irreducible**

Generative models for the gambling task

- Different assumptions about the hidden variable $x^{(t)}$ lead to different models / predictions for behaviour

- If we assume a learner who's generative model has a constant $x^{(t)} = x,\ t = 1,2,\ldots,T$, one possible generative model is:

$$m^{(p)}: \begin{cases} p\left(u^{(t)} \,|\, x, m^{(p)}\right) = Ber(x) \quad \forall t = 1,\ldots,T \\ p\left(x \,|\, m^{(p)}\right) = Beta\,(1,1) \end{cases}$$

Derive inference process

We assume this perceptual model:

$$m^{(p)}: \begin{cases} p\left(u^{(t)} | x, m^{(p)}\right) = Ber(x) \quad \forall t = 1,\ldots,T \\ p\left(x | m^{(p)}\right) = Beta\,(1,1) \end{cases}$$

Which has this posterior:

$$\pi\left(x | u^{(1)}, \ldots, u^{(T)}\right) = Beta\left(a + \sum_{t=1}^{T} u^{(t)}; b + T - \sum_{t=1}^{T} u^{(t)}\right)$$

This gives the following sequence of parameters:

$$(a^{(t)}, b^{(t)}) = (a^{(t-1)} + u^{(t)}, b^{(t-1)} + 1 - u^{(t)})$$

And these expectations: $\hat{x}^{(t)} = \dfrac{a^{(t)}}{a^{(t)} + b^{(t)}}$

Choose response model

The responses are binary and our perceptual model gives us estimates for the latent probability for each choice.

$$m^{(r)} : \begin{cases} p\left(y^{(t)} | x, m^{(r)}\right) = Categorical(g(\theta, \vartheta)) \quad \forall t = 1, \ldots, T \\ g(\theta, \lambda(\vartheta)) = softmax\left(\left[\left(\dfrac{a^{(t)}}{a^{(t)} + b^{(t)}}\right), \left(\dfrac{b^{(t)}}{a^{(t)} + b^{(t)}}\right)\right], \theta\right) \\ p\left(\vartheta, \theta | m^{(r)}\right) \propto 1 \end{cases}$$

The θ in the above equation in the “decision temperature” (high temperature = random responding).

Generative model for behaviour in gambling task

$u^{(t)}, t = 1,\ldots,T$ **fixed**

initialise $(a^{(0)}, b^{(0)}) = (0,0)$

for t **in** $1,2,\ldots,T,$ **do**

Inference process

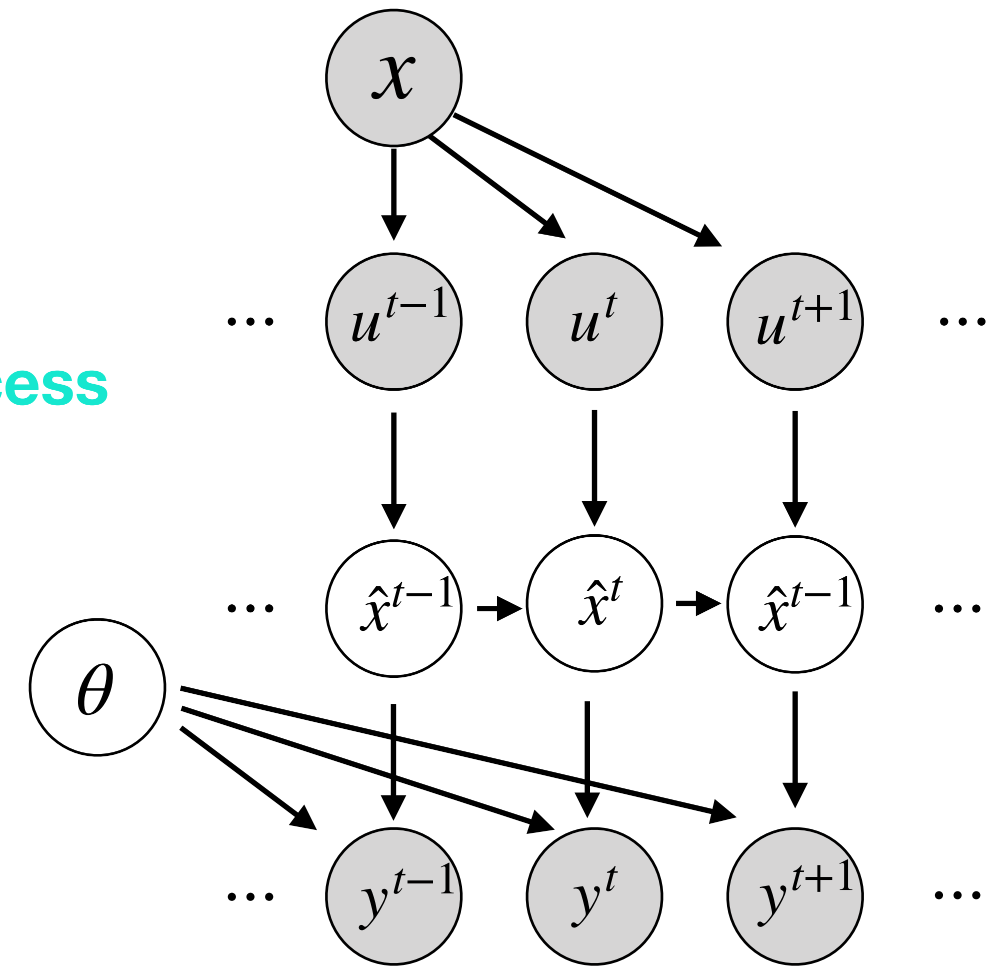
$$(a^{(t)}, b^{(t)}) = (a^{(t-1)} + u^{(t)}, b^{(t-1)} + 1 - u^{(t)})$$

$$\hat{x}^{(t)} = \frac{a^{(t)}}{a^{(t)} + b^{(t)}}$$

$$y^{(t)} \sim softmax\left(\left[\hat{x}^{(t)}, (1 - \hat{x}^{(t)})\right], \theta\right)$$

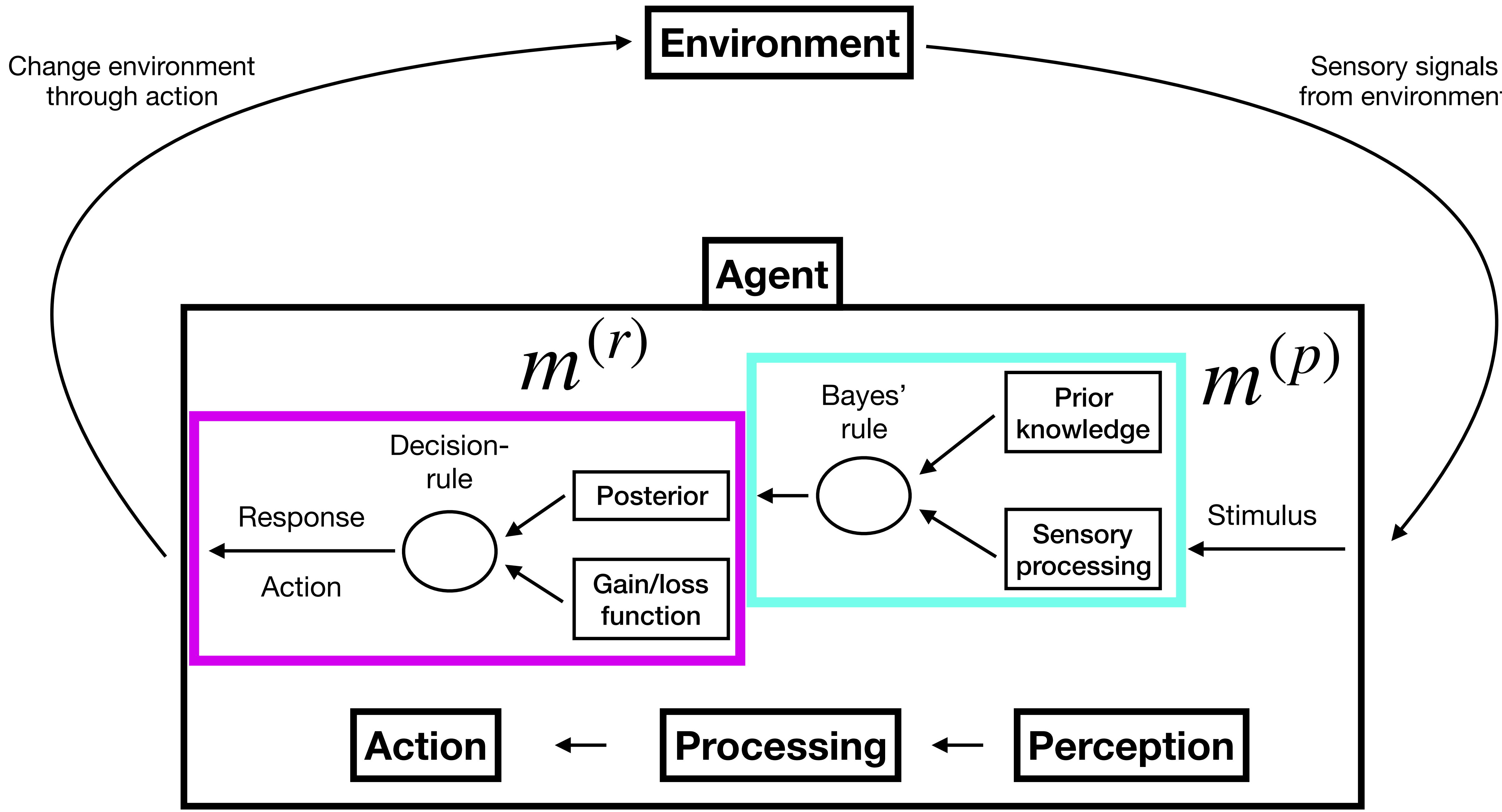
end

Response model

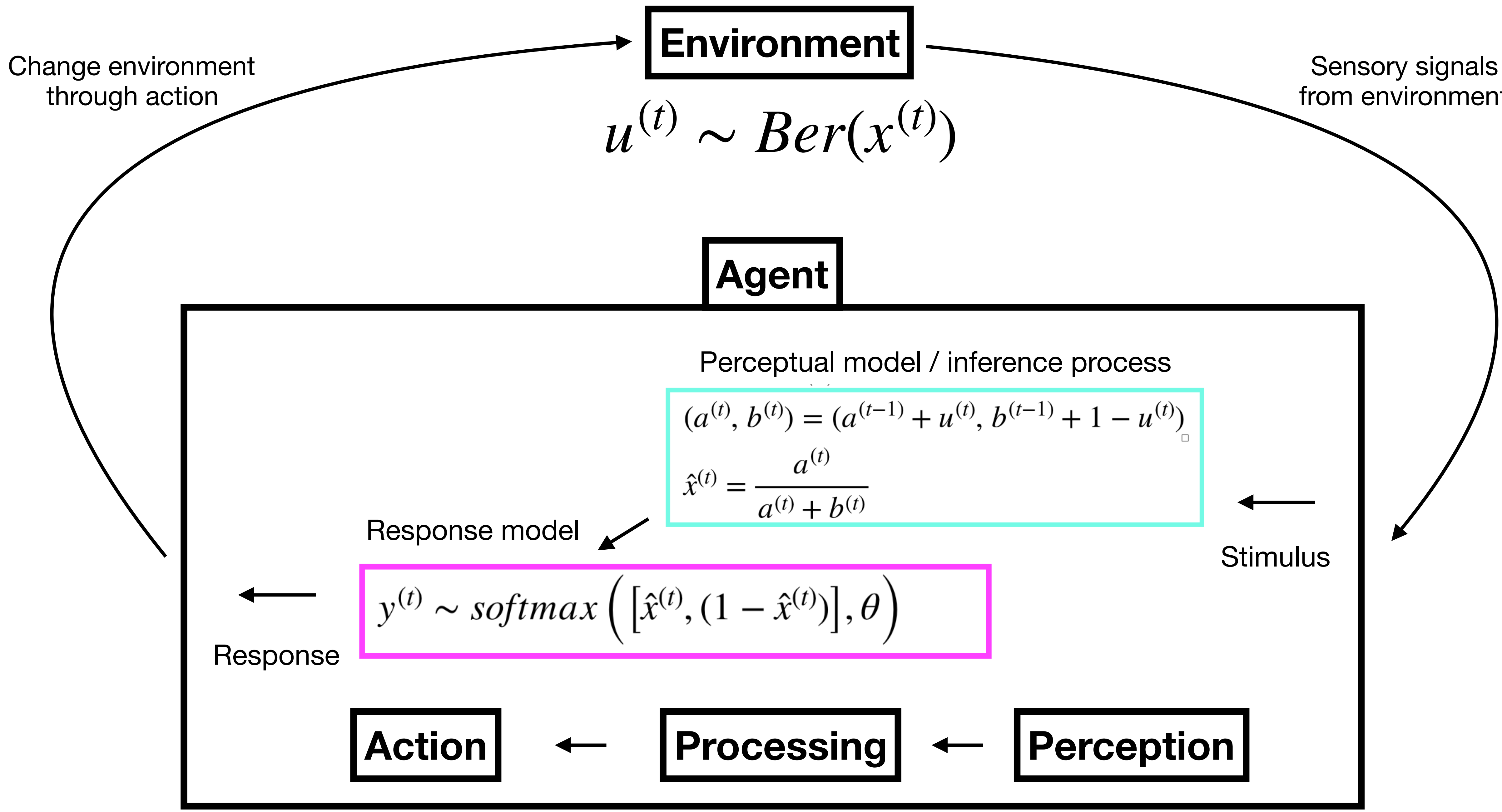


Theta: random responding / exploration

Modelling the inference process



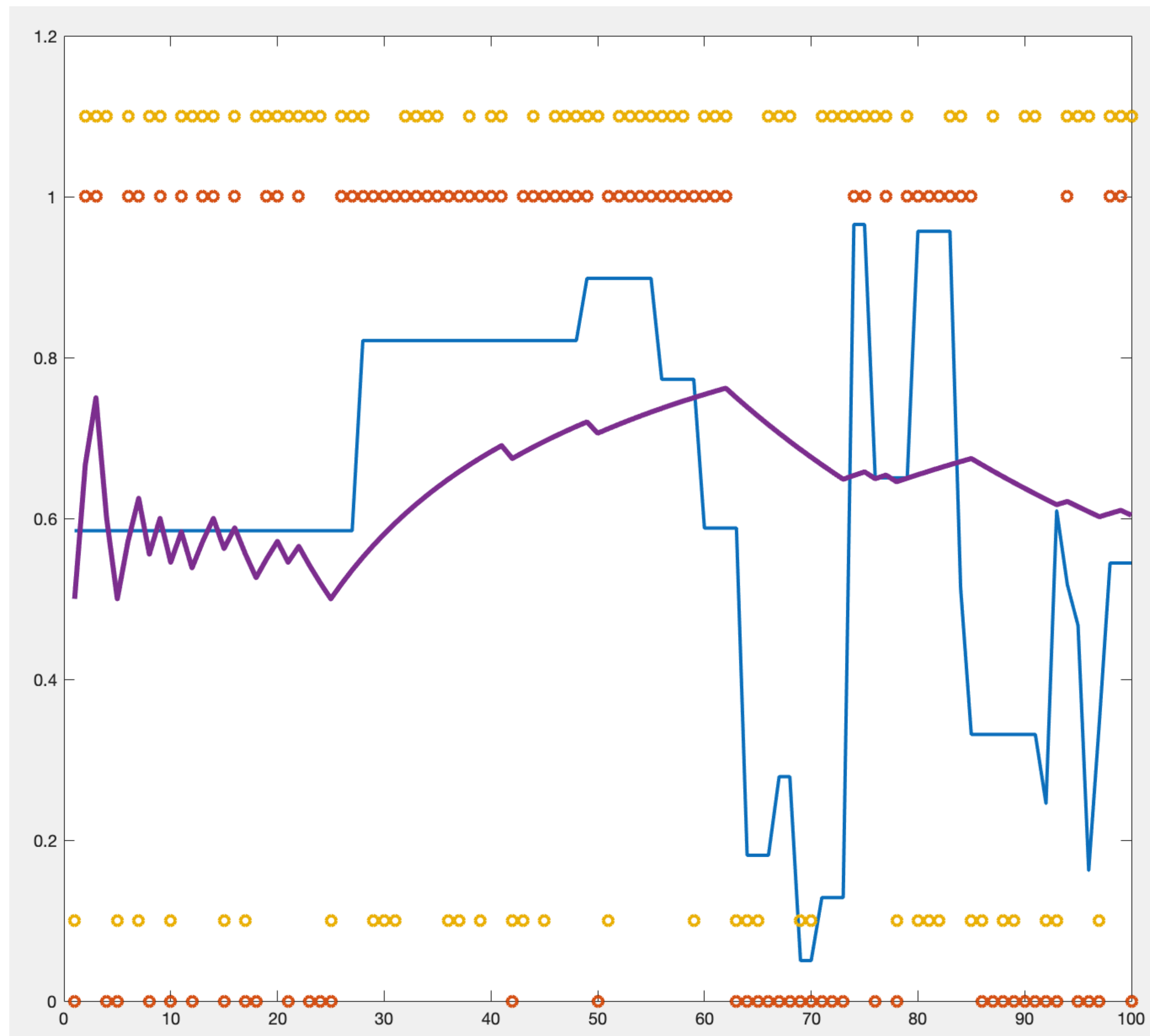
Example inference process



Behavioural predictions for our experiment

Need changing learning rate

Need changing rate of change of learning rate

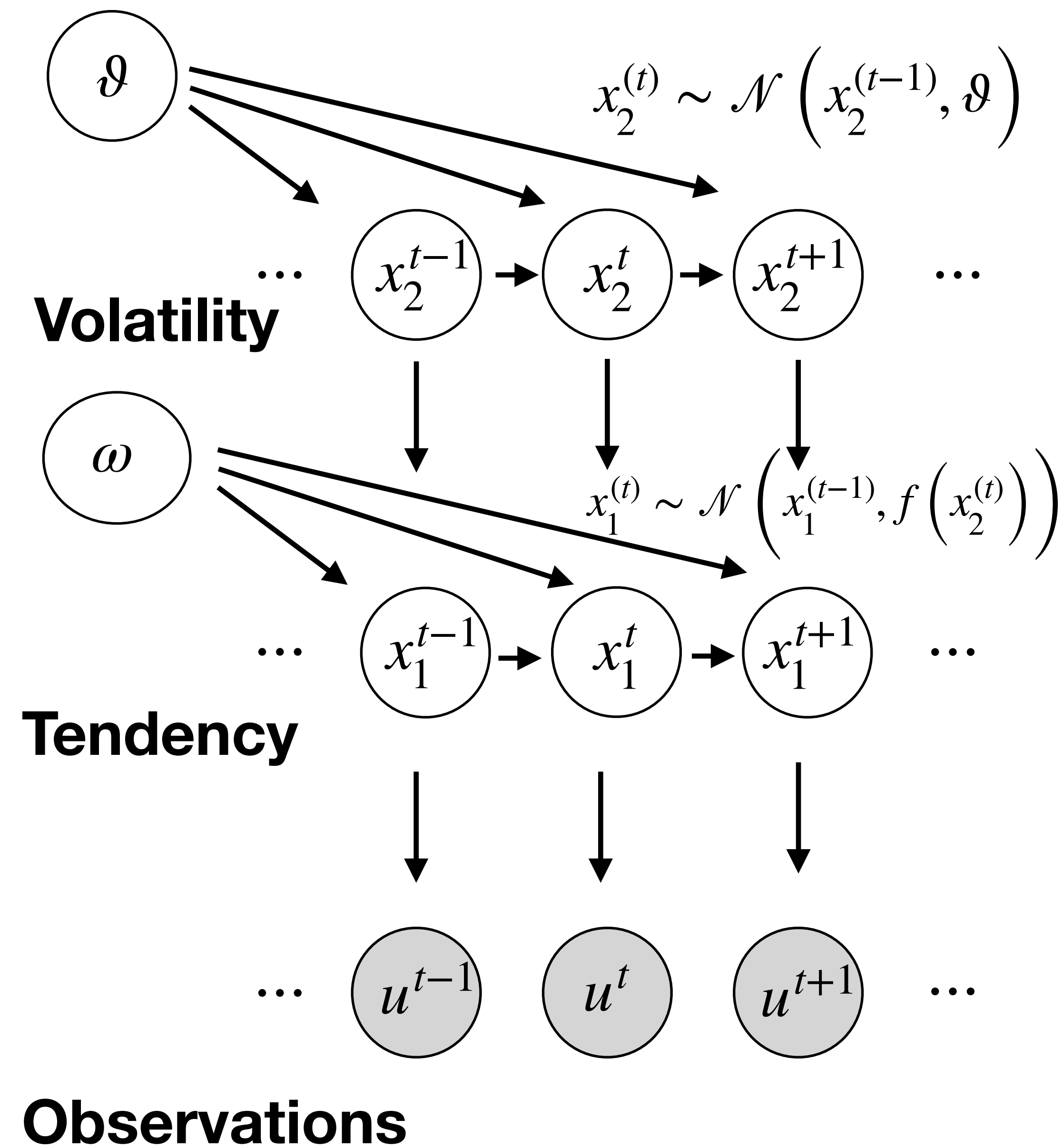


The hierarchical Gaussian filter (HGF)

HGF: Generative model at the base of inference process

- The HGF is defined through specific choices for the inference process:
    - Generative model: hierarchy of random walks
    - update equations derived through minimising perceptual free energy
- The HGF dynamically updates its learning rate with every observation

$$x_k^{(t)} \sim \mathcal{N}\left(x_k^{(t-1)}, \exp\left(\kappa_k x_{k+1}^{(t)} + \omega_k\right)\right)$$



Inference process of HGF

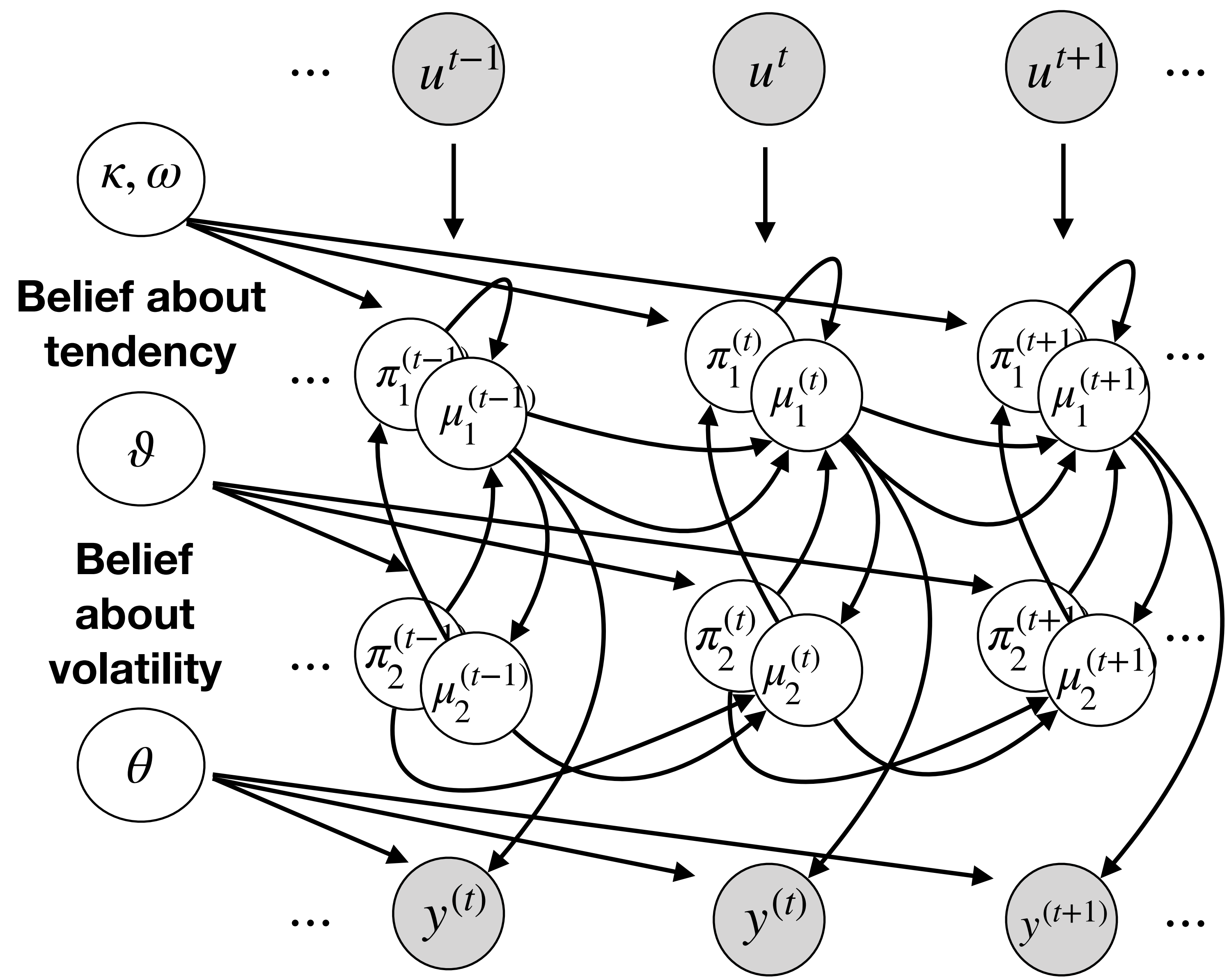
$u^{(t)}, \ t = 1, \ldots, T$ **fixed**

for t **in** $1, 2, \ldots, T,$ **do**

$\lambda^{(t)} = f(\lambda^{(t-1)}, u^{(t)}, \vartheta)$

$y^{(t)} = g(\lambda^{(t)}, \theta)$

end



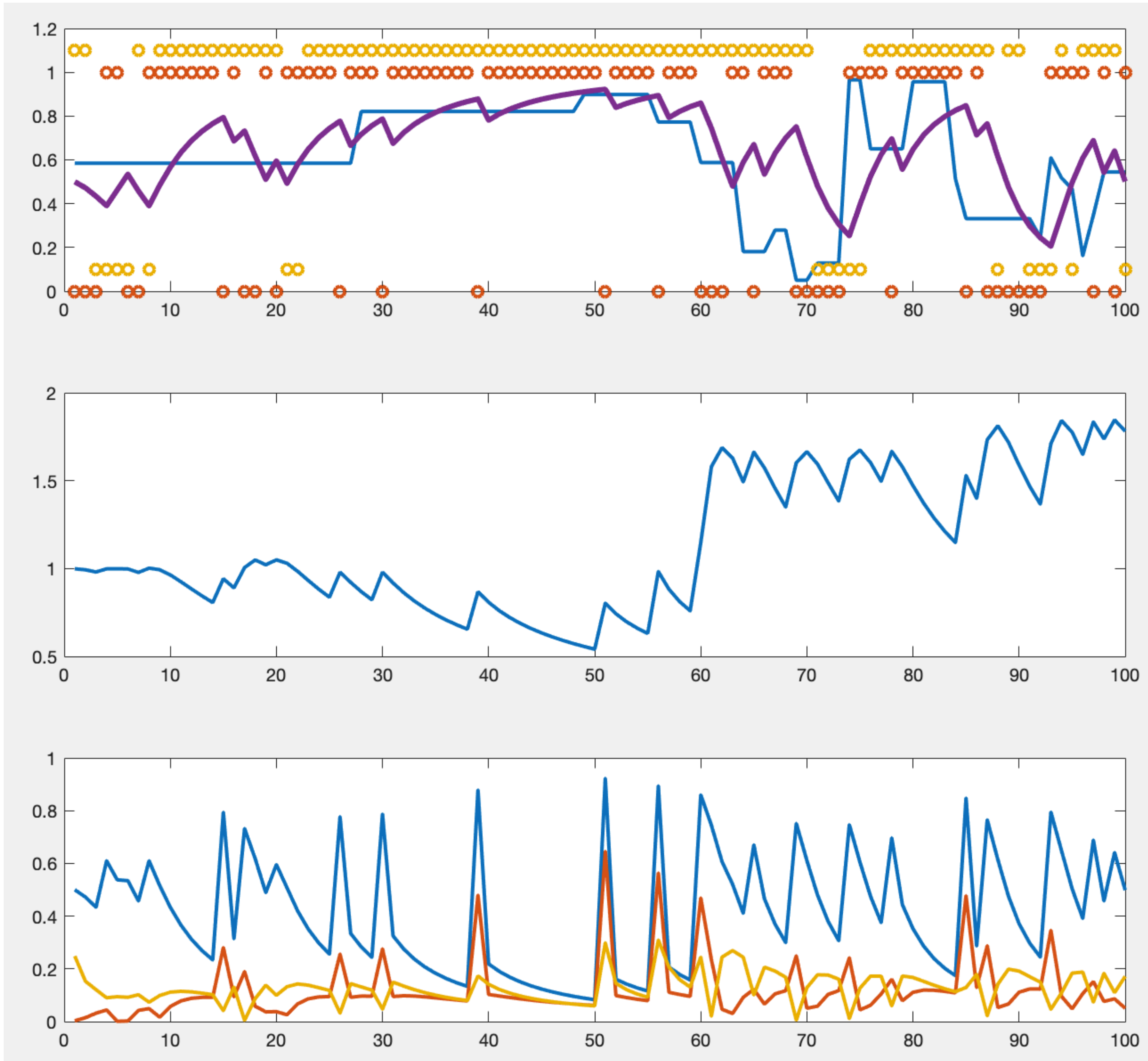
General pattern: $$\Delta\mu_i \propto \frac{\hat{\pi}_{i-1}}{\pi_i}\delta_{i-1}$$

HGF predictions

$x^{(t)}$ **and** $\hat{\mu}_1^{(t)}$

Belief about volatility, $\hat{\mu}_3^{(t)}$

Prediction errors, $\delta_{1:3}^{(t)}$



Precision weights and types of uncertainty

The learners observations are generated by:

$$u^{(t)} \sim \mathrm{Ber}\left(x_1^{(t)}\right)$$

which leads to these updates for the belief about the latent process:

$$\mu_2^{(t)} = \mu_2^{(t-1)} + \frac{1}{\pi_2^{(t)}}\delta_1^{(t)} \qquad \hat{\mu}_1^{(t)} = s\left(\mu_2^{(t)}\right)$$

The precision weight can be decomposed into factors corresponding to different kinds of uncertainty:

$$\frac{1}{\pi_2^{(t)}} = \frac{1}{\sigma_2^{(t-1)}\exp(\kappa\mu_3^{(t-1)} + \omega)} + \frac{1}{\hat{\mu}_1^{(t)}(1-\hat{\mu}_1^{(t)})}$$

Estimation uncertainty **Estimated volatility of the environment** **Irreducible uncertainty about the outcome**

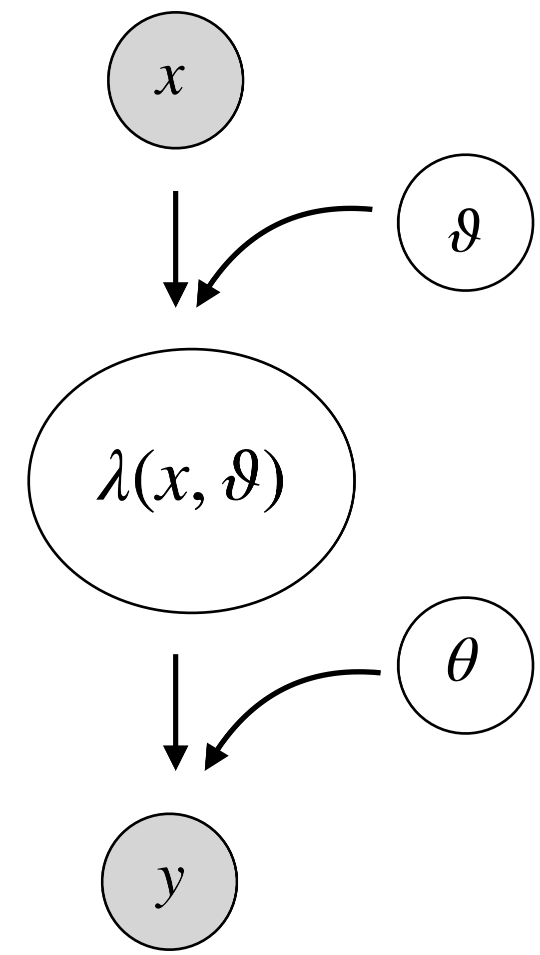
Parameter inference

After data collection we want to do inference based on the following posterior distribution:

$$p(\vartheta, \theta \,|\, \underline{u}, \underline{y}, \lambda) \propto p(\underline{y} \,|\, \vartheta, \theta, \underline{u}, \lambda) p(\vartheta, \theta)$$

Alternatively: Maximum-likelihood estimation:

$$(\hat{\vartheta}_{ML}, \hat{\theta}_{ML}) = \underset{(\vartheta, \theta)}{\arg\max}\; p(\underline{y} \,|\, \vartheta, \theta, \underline{u}, \lambda)$$

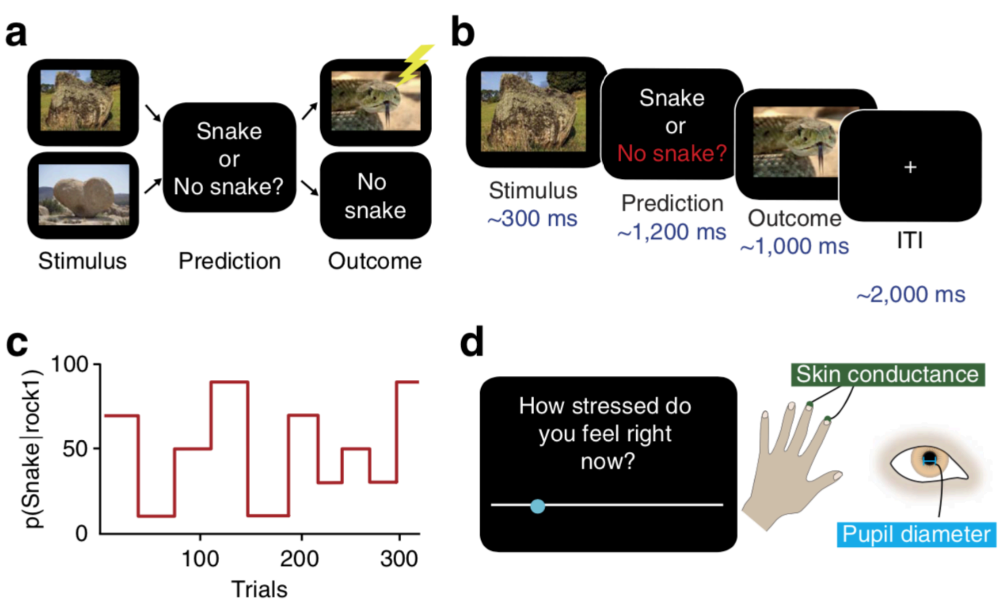


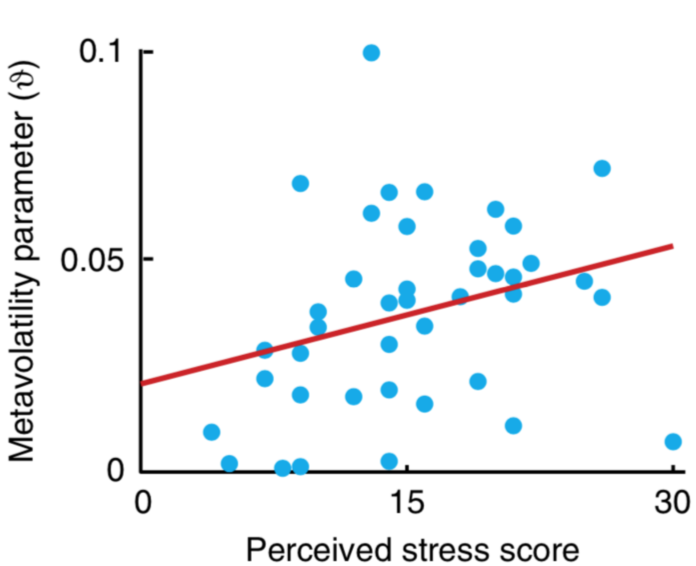
Implicit generative model

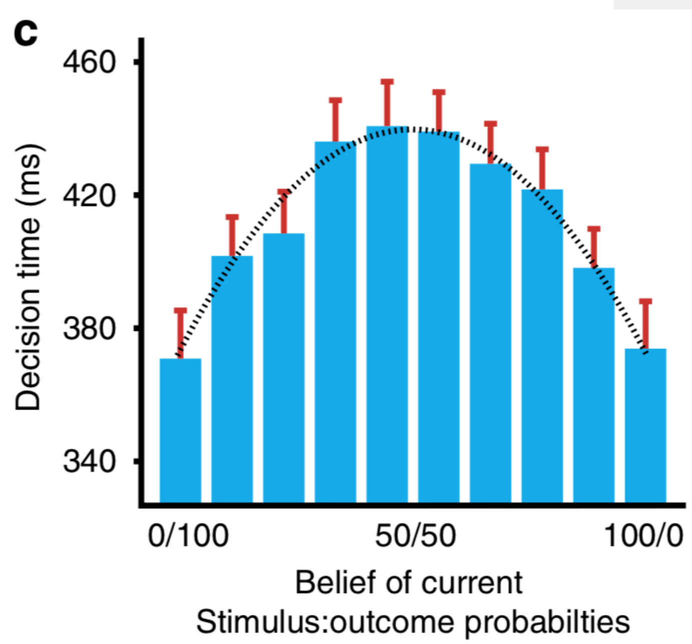
Important: Do simulations to check parameter recovery

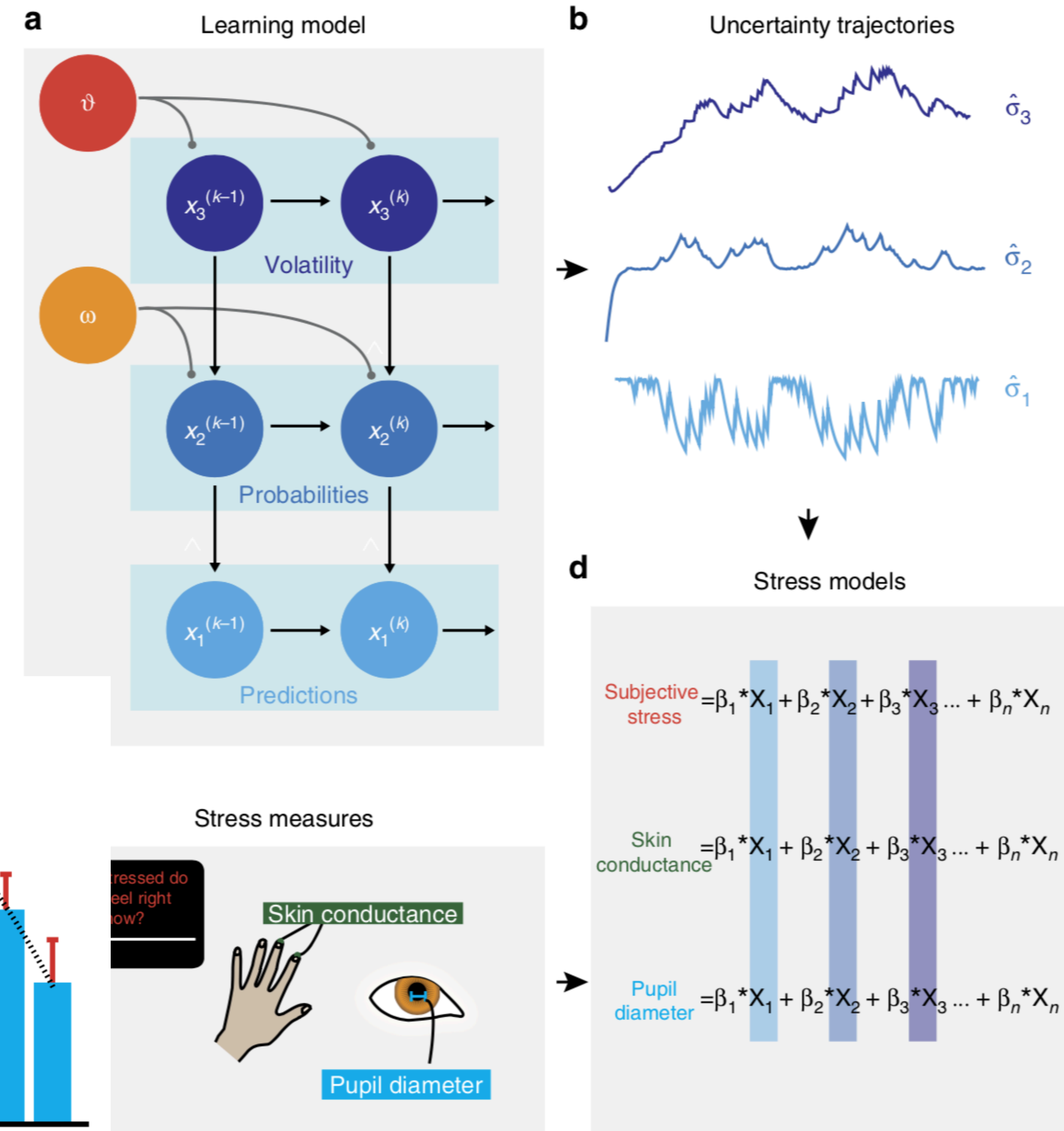
Applications

Irreducible uncertainty, volatility and stress



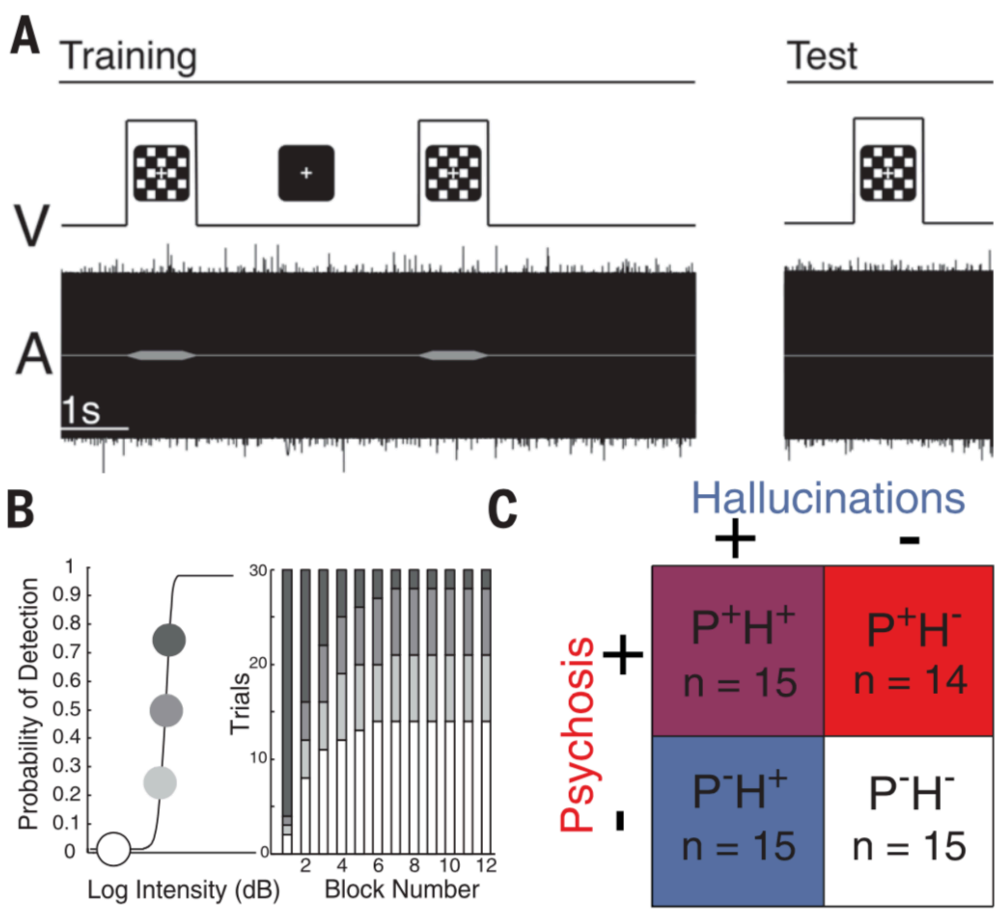


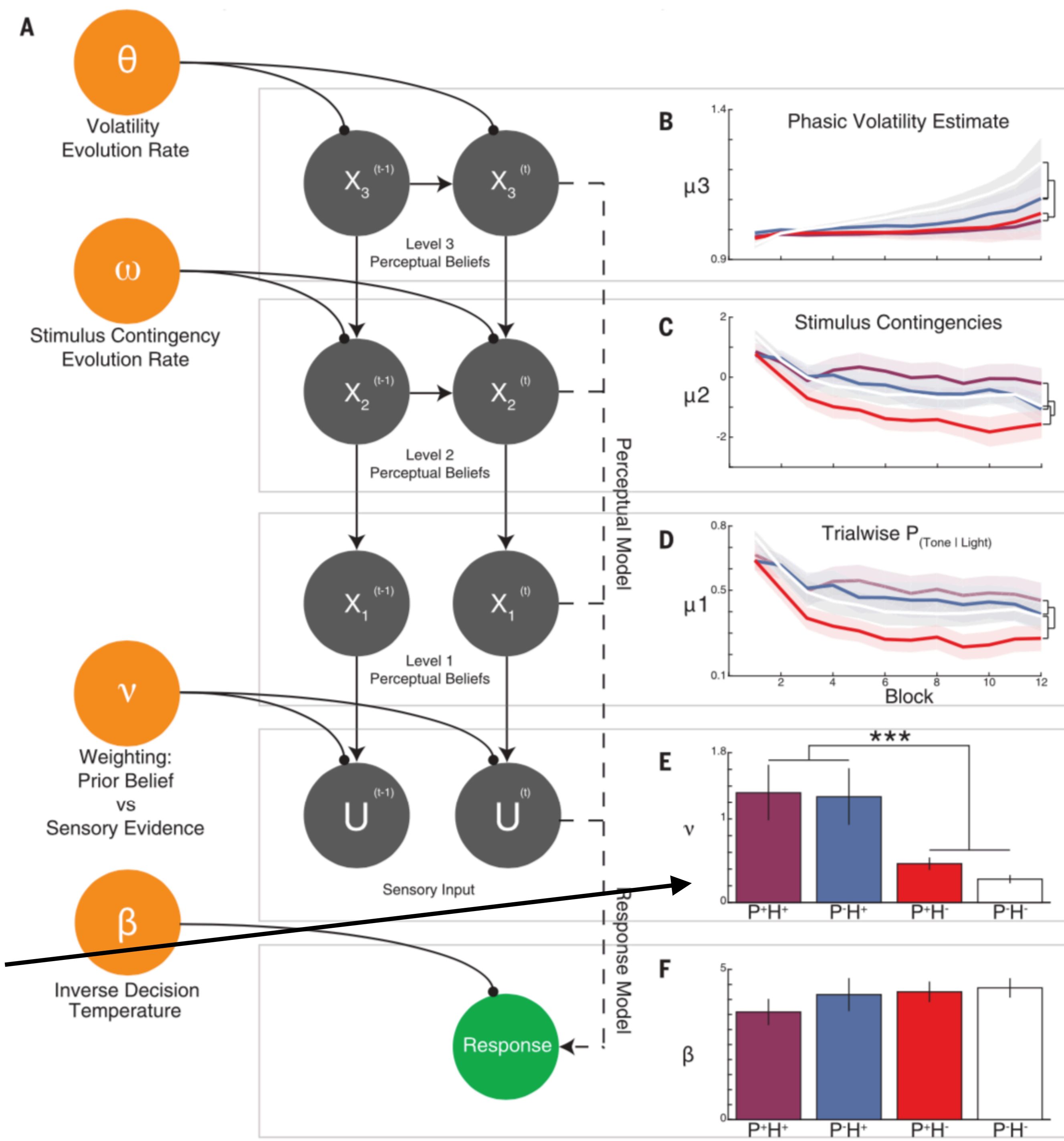




de Berker et al. (2015): “Computations of uncertainty mediate acute stress responses in humans”

Conditioned hallucinations





Subjects with hallucinations show higher estimates for weights on prior beliefs

Powers et al. (2017): "Pavlovian conditioning-induced hallucinations result from overweighting of perceptual priors"

References / literature

- Theory
    - Mathys et al. (2011): “A Bayesian foundation for individual learning under uncertainty”
    - Mathys et al. (2014): “Uncertainty in perception and the Hierarchical Gaussian Filter”
    - Daunizeau et al. (2010): “Observing the Observer (I): Meta-Bayesian Models of Learning and Decision-Making”
- Applications
    - Iglesias et al. (2013): “Hierarchical Prediction Errors in Midbrain and Basal Forebrain during Sensory Learning”
    - de Berker et al. (2015): “Computations of uncertainty mediate acute stress responses in humans”
    - Powers et al. (2017): “Pavlovian conditioning–induced hallucinations result from overweighting of perceptual priors”