University of Zurich[UZH]

Translational Neuromodeling Unit

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Predictive Coding

Computational Psychiatry Course
03.09.2019

Lilian A. Weber
Translational Neuromodeling Unit
University of Zurich & ETH Zurich

I. The idea: an algorithmic motif

From redundancy reduction in signal processing to approximate Bayesian inference in the visual cortex: *Rao & Ballard, 1999*

Predictive coding algorithm(s)

Linear predictive coding

O'Shaughnessy 1988 *IEEE Potentials*

Spatial decorrelation in the retina

Srinivasan et al. 1982 *Proc R Soc London Ser B*,
Hosoya et al. 2005 *Nature*

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Brain and Cognition

journal homepage: www.elsevier.com/locate/b&c

Special Invited Review

A review of predictive coding algorithms

M.W. Spratling *

(a)

Kersten 1987 *J Opt Soc Am A*

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Spatial decorrelation in the retina

Srinivasan et al. 1982 *Proc R Soc London Ser B*, Hosoya et al. 2005 *Nature*

Predictive coding in visual cortex

Rao & Ballard 1999 *Nat Neurosci*

Free-energy formulation of predictive coding

Friston 2005 *Phil Trans Royal Soc B*

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Special Invited Review

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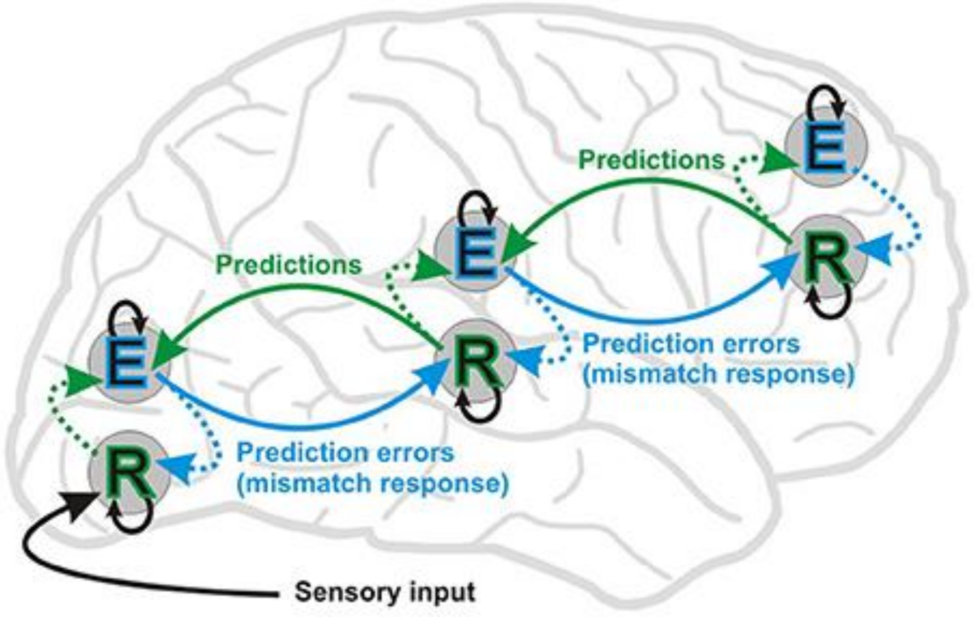


Stefanics et al. 2014 *Front Hum Neurosci*

III. Predictive coding as approximate inference

Firing rates/ Pixel values

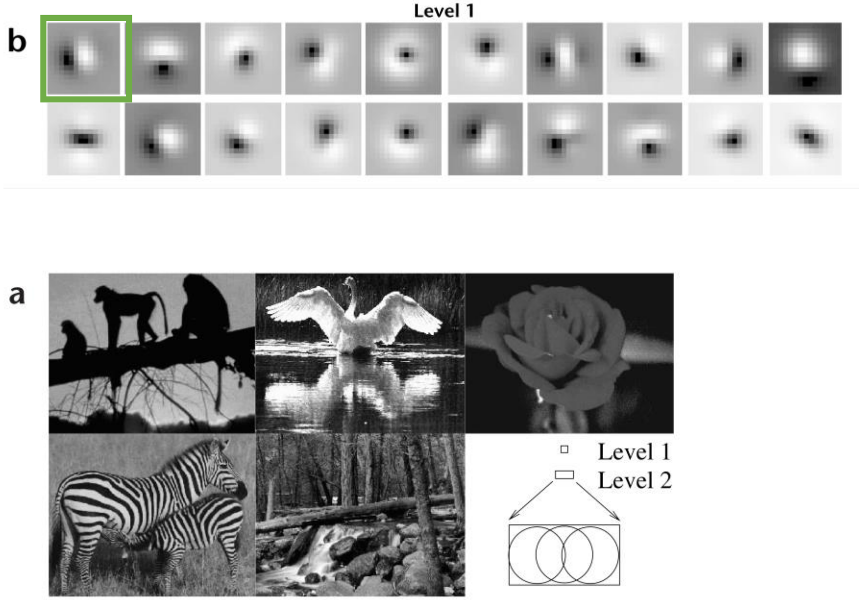
Model (possible patterns)

noise

Inferred causes (which patterns are present)

$$\boxed{\mathbf{I}} = f(\boxed{U}\boxed{\mathbf{r}}) + \boxed{\mathbf{n}}$$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ \vdots \\ i_K \end{bmatrix} = f\left(\begin{bmatrix} u_{1,1} & u_{1,2} & \cdots & u_{1,N} \\ u_{2,1} & u_{2,2} & \cdots & u_{1,N} \\ u_{3,1} & u_{3,2} & \cdots & u_{3,N} \\ \vdots & \vdots & \ddots & \vdots \\ u_{K,1} & u_{K,2} & \cdots & u_{K,N} \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_N \end{bmatrix} \right) + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ \vdots \\ n_K \end{bmatrix}$$



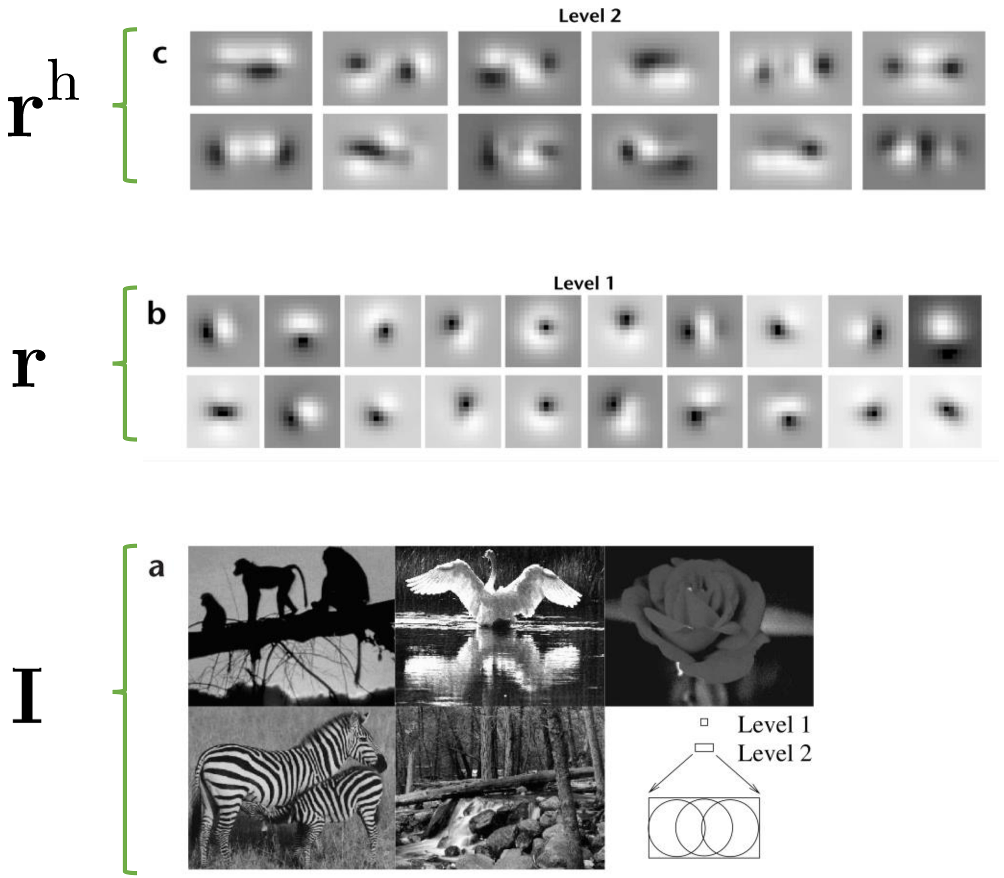
Rao & Ballard 1999 *Nat Neurosci*

III. Predictive coding as approximate inference

Introducing hierarchy

$$\mathbf{r} = \mathbf{r}^{\mathrm{td}} + \mathbf{n}^{\mathrm{td}}$$
$$= f(U^{\mathrm{h}}\mathbf{r}^{\mathrm{h}}) + \mathbf{n}^{\mathrm{td}}$$

$$\mathbf{I} = f(U\mathbf{r}) + \mathbf{n}$$



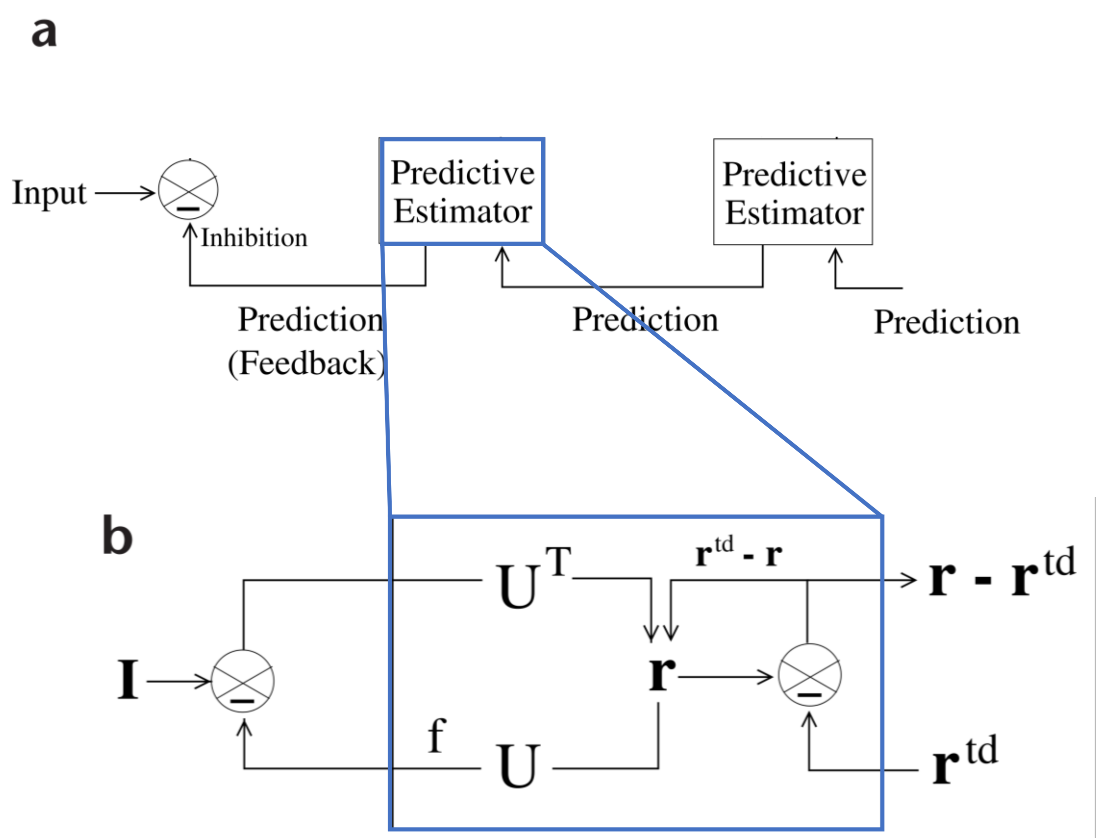
Rao & Ballard 1999 *Nat Neurosci*

III. Predictive coding as approximate inference

Implementation

$$\mathbf{r} = \mathbf{r}^{\mathrm{td}} + \mathbf{n}^{\mathrm{td}}$$
$$= f(U^{\mathrm{h}}\mathbf{r}^{\mathrm{h}}) + \mathbf{n}^{\mathrm{td}}$$
$$\mathbf{I} = f(U\mathbf{r}) + \mathbf{n}$$

Question: How to find *r* and *U*?



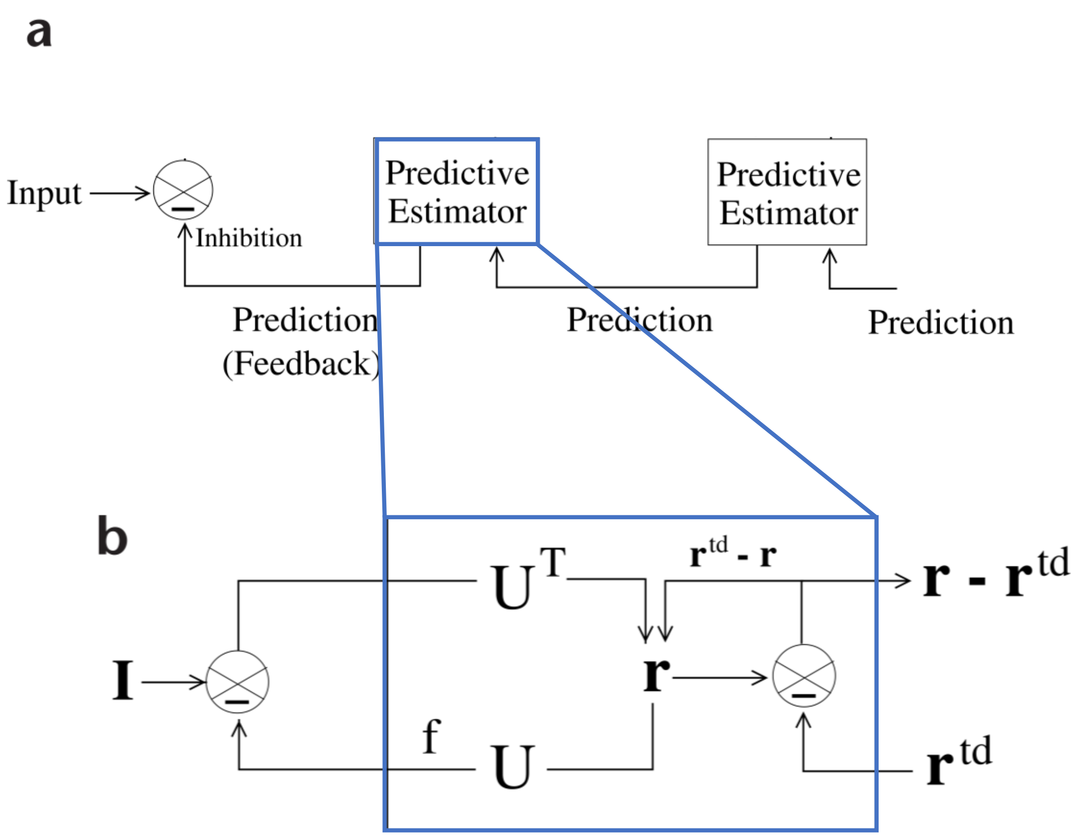
Rao & Ballard 1999 *Nat Neurosci*

Predictive coding as approximate inference

Optimization: Finding causes and associations

$$E = \underbrace{E_1}_{} - \log p(\mathbf{r}) - \log p(U)$$

$$E_1 = \underbrace{\frac{1}{\sigma^2}(\mathbf{I} - f(U\mathbf{r}))^\intercal(\mathbf{I} - f(U\mathbf{r})) + \frac{1}{\sigma_{\mathrm{td}}^2}(\mathbf{r} - \mathbf{r}^{\mathrm{td}})^\intercal(\mathbf{r} - \mathbf{r}^{\mathrm{td}})}_{\text{(precision-weighted) sum of squared prediction errors}}$$



Predictive coding as approximate inference

Inference: Estimating the causes based on inputs and expectations

Learning: Improving the model to reflect the statistics of the environment

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = -\frac{k_1}{2}\frac{\partial E}{\partial \mathbf{r}}$$

Inputs

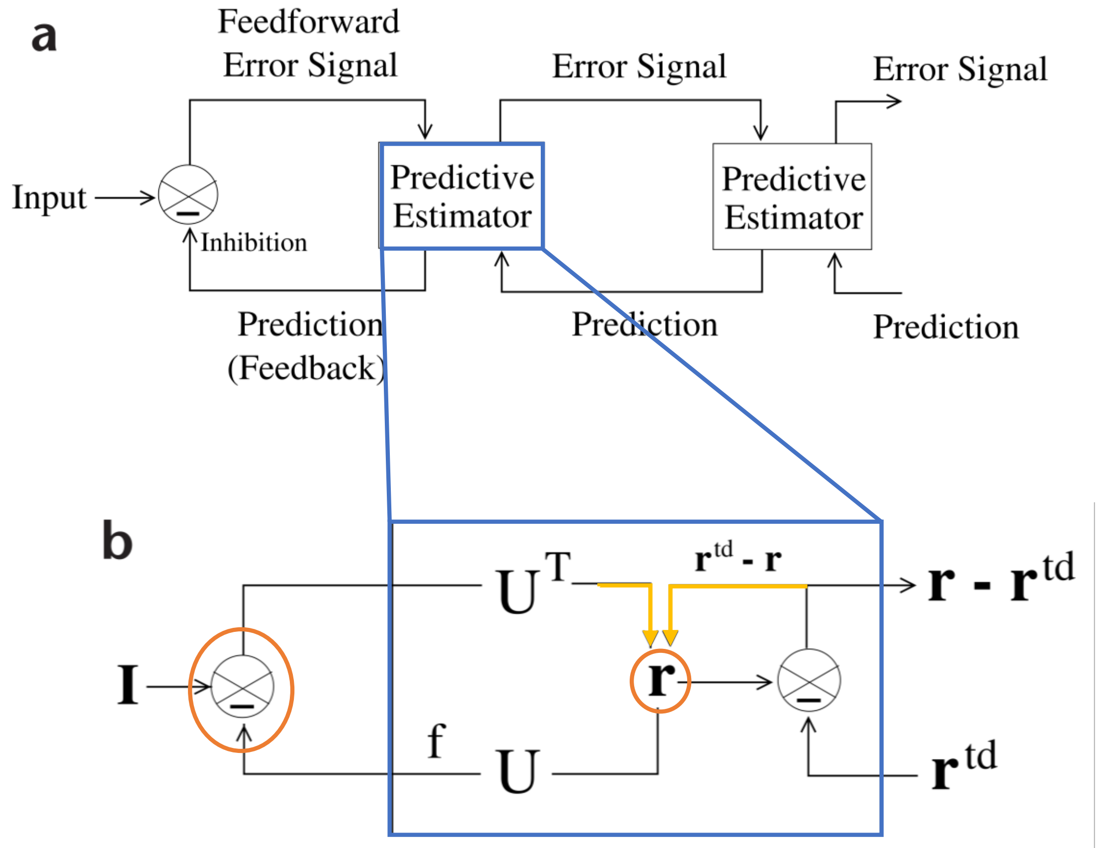
Expectations

$$= \frac{k_1}{\sigma^2} U^\top \frac{\partial f}{\partial U\mathbf{r}}^\top (\mathbf{I} - f(U\mathbf{r})) + \frac{k_1}{\sigma_{\mathrm{td}}^2}(\mathbf{r}^{\mathrm{td}} - \mathbf{r}) - k_1 \alpha \mathbf{r}$$

precision-weighting

$$\frac{\mathrm{d}U}{\mathrm{d}t} = -\frac{k_2}{2}\frac{\partial E}{\partial U} = \frac{k_2}{\sigma^2}\frac{\partial f}{\partial U\mathbf{r}}^\top (\mathbf{I} - f(U\mathbf{r}))\mathbf{r} - \frac{k_2}{2}\lambda U$$

Hebbian learning



Rao & Ballard 1999 *Nat Neurosci*

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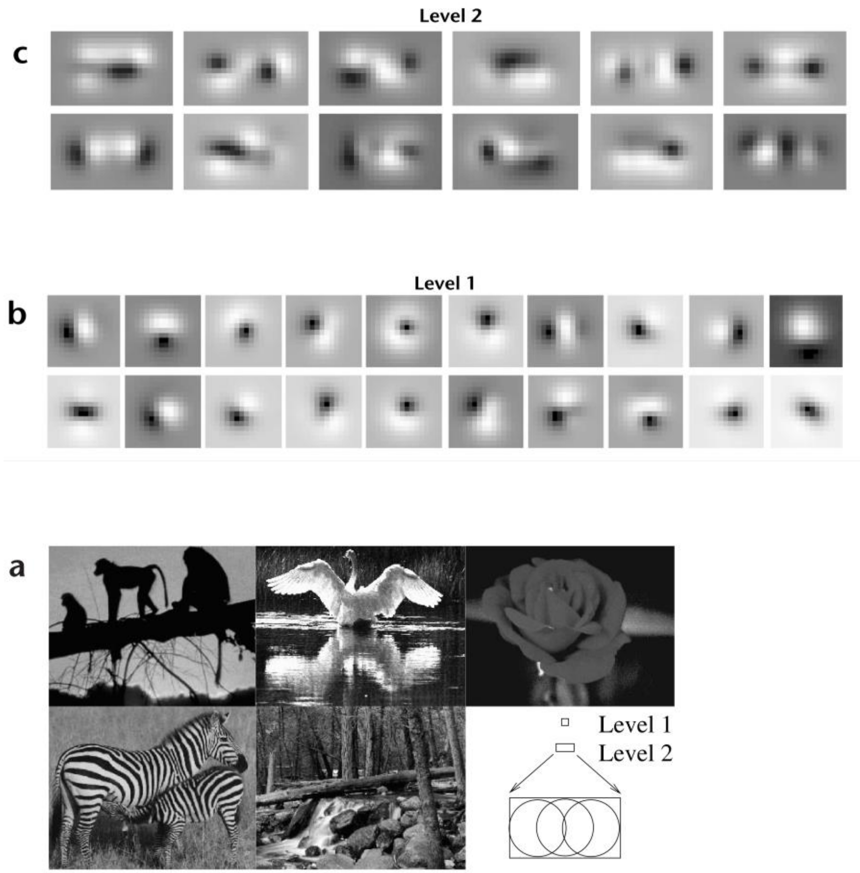
precision-weighting

$$\frac{dU}{dt} = -\frac{k_2}{2}\frac{\partial E}{\partial U} = \frac{k_2}{\sigma^2}\frac{\partial f}{\partial U\mathbf{r}}^\top (\mathbf{I} - f(U\mathbf{r}))\mathbf{r} - \frac{k_2}{2}\lambda U$$

Hebbian learning



Rao & Ballard 1999 *Nat Neurosci*

Predictive coding as approximate inference

Inference: Estimating the causes based on inputs and expectations

Learning: Improving the model to reflect the statistics of the environment

$$\frac{d\mathbf{r}}{dt} = -\frac{k_1}{2}\frac{\partial E}{\partial \mathbf{r}}$$

Inputs   Expectations

$$= \frac{k_1}{\sigma^2} U^\top \frac{\partial f}{\partial U\mathbf{r}}^\top (\mathbf{I} - f(U\mathbf{r})) + \frac{k_1}{\sigma_{td}^2}(\mathbf{r}^{td} - \mathbf{r}) - k_1 \alpha \mathbf{r}$$

precision-weighting

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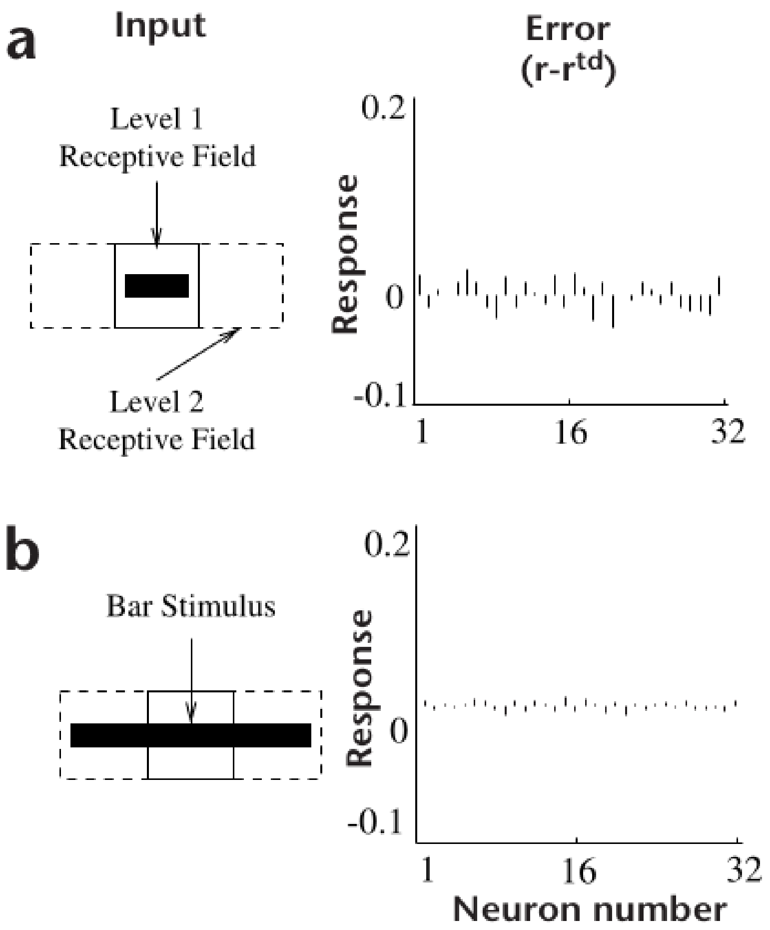
Hebbian learning

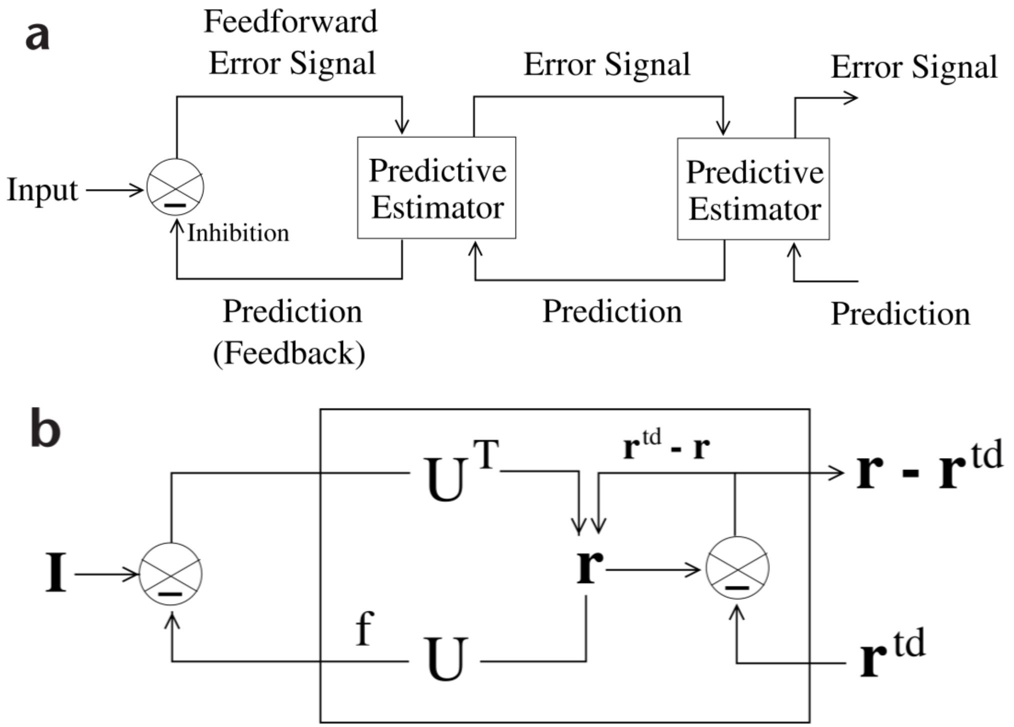


Rao & Ballard 1999 *Nat Neurosci*

Predictive coding as approximate inference

Summary

- A hierarchy of causes (predictive estimators and PE units)
- Recurrent message passing: PEs are signaled upwards (bottom-up) and predictions are signaled top-down
- Inference: finding the most appropriate causes to describe the data
- Learning: finding the most appropriate model to describe the data
- Both involve minimizing precision-weighted prediction errors
- (This looks like Bayesian inference for finding the most likely causes of sensory inputs (MAP estimates) and like Hebbian learning, respectively)

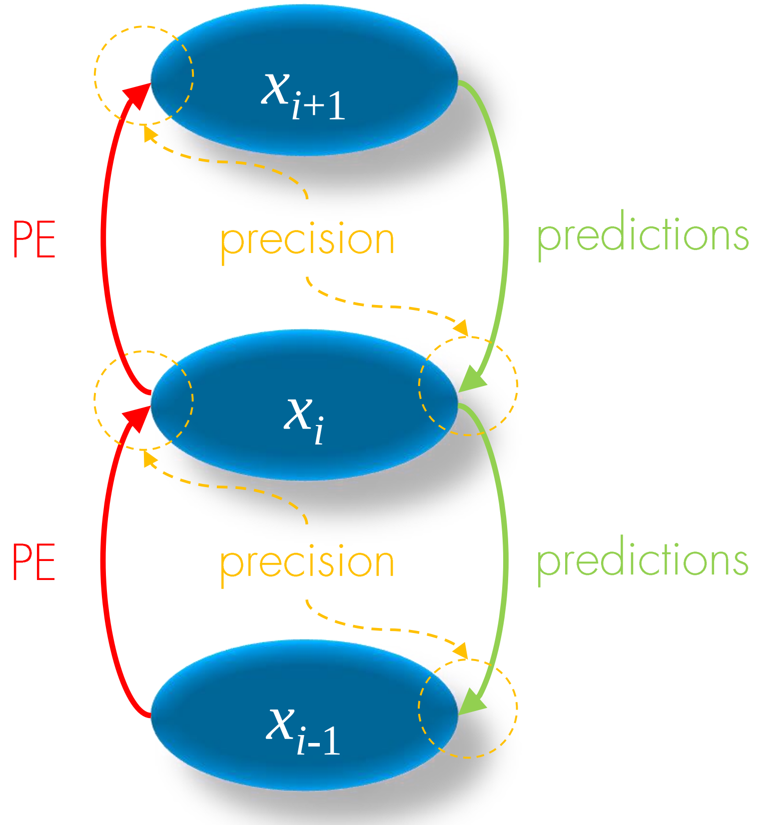


Rao & Ballard 1999 *Nat Neurosci*

The main ingredients of predictive coding (from a computational point of view)

- A hierarchical generative model of sensory inputs
- The estimates of the causes generate predictions of sensory input
- The estimates of causes are updated in response to prediction errors (mismatches)
- The relative influence of PEs and predictions is determined by their relative precision (certainty)
- These computations underlie both perception (inference) and learning (model update)

$$\Delta \text{belief} \sim \text{precision} \times \text{PE}$$



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Adapted from Stephan et al. 2016 *Brain*

II. Implementing (and testing) PC

Mapping the ingredients of predictive coding onto neuroanatomy and neurophysiology

Mapping predictive coding onto the brain

- A hierarchical generative model of sensory inputs
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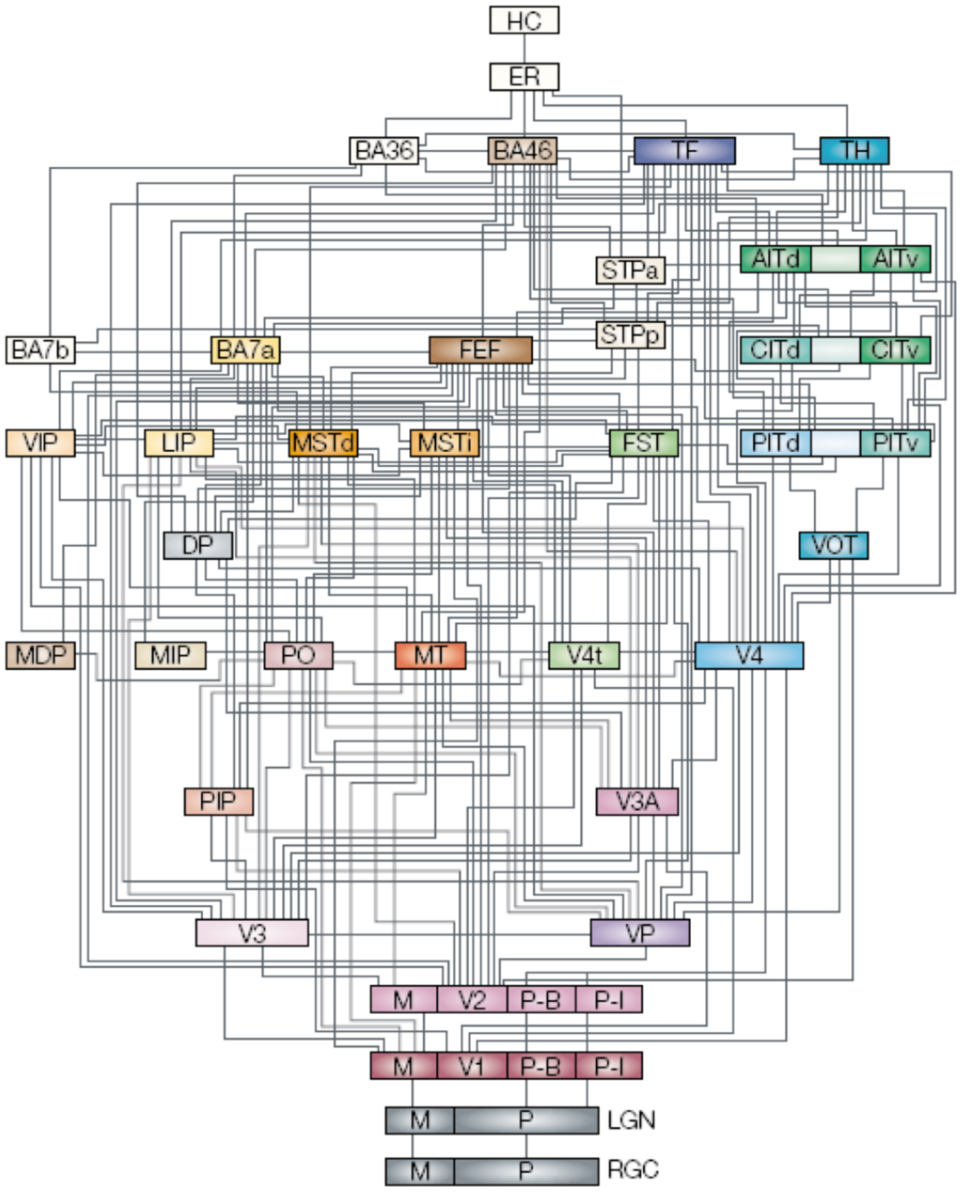

Felleman & Van Essen 1991 *Cereb Cortex*

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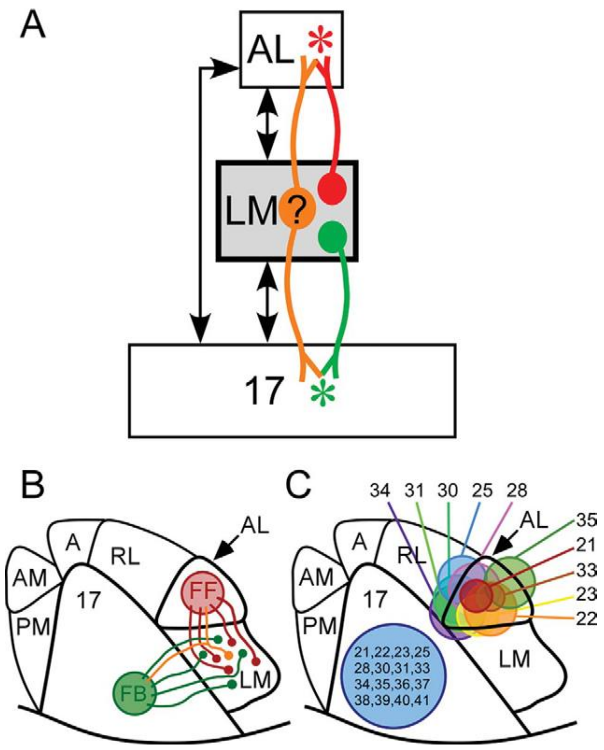
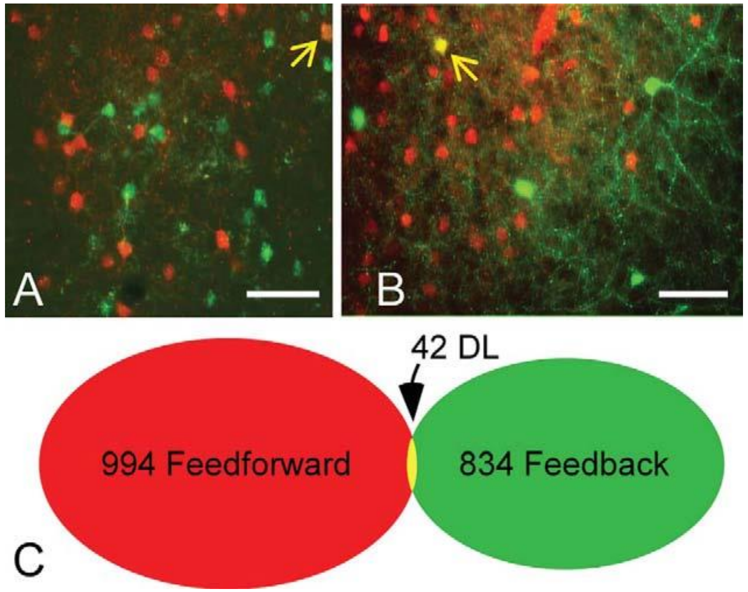
1. The source populations of forward and backward pathways should be completely separate, given their functional distinction.





Berezovskii et al. 2011 *J Comp Neurol*

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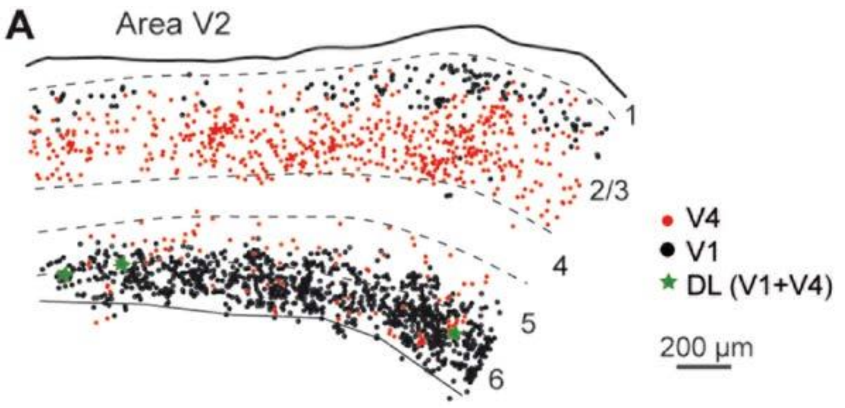

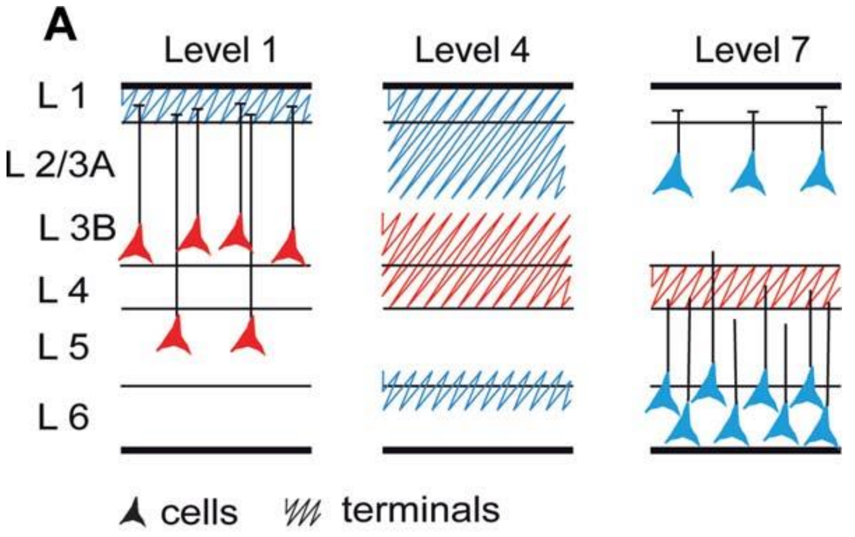

Markov et al. 2014 *J Comp Neurol*

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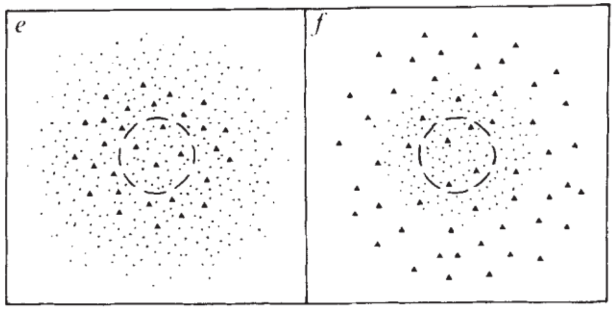
2. Backward connections encode predictions and expected precision, so they should be more divergent.



Zeki & Shipp 1988 *Nature*

3. Causes interact non-linearly to generate data, so backward connections should be more modulatory.

Self et al. 2012 *PNAS*
Olsen et al. 2012 *Nature*
Zilles et al. 2004 *J Anat*

4. Predictions are generated more slowly than prediction errors, suggesting a spectral asymmetry.

Roopun 2006 *PNAS*
Roopun et al. 2008 *Front. Cell. Neurosci*

Buffalo et al. 2011 *PNAS*
Bosman et al. 2012 *Neuron*
Bastos et al. 2015 *NeuroImage*

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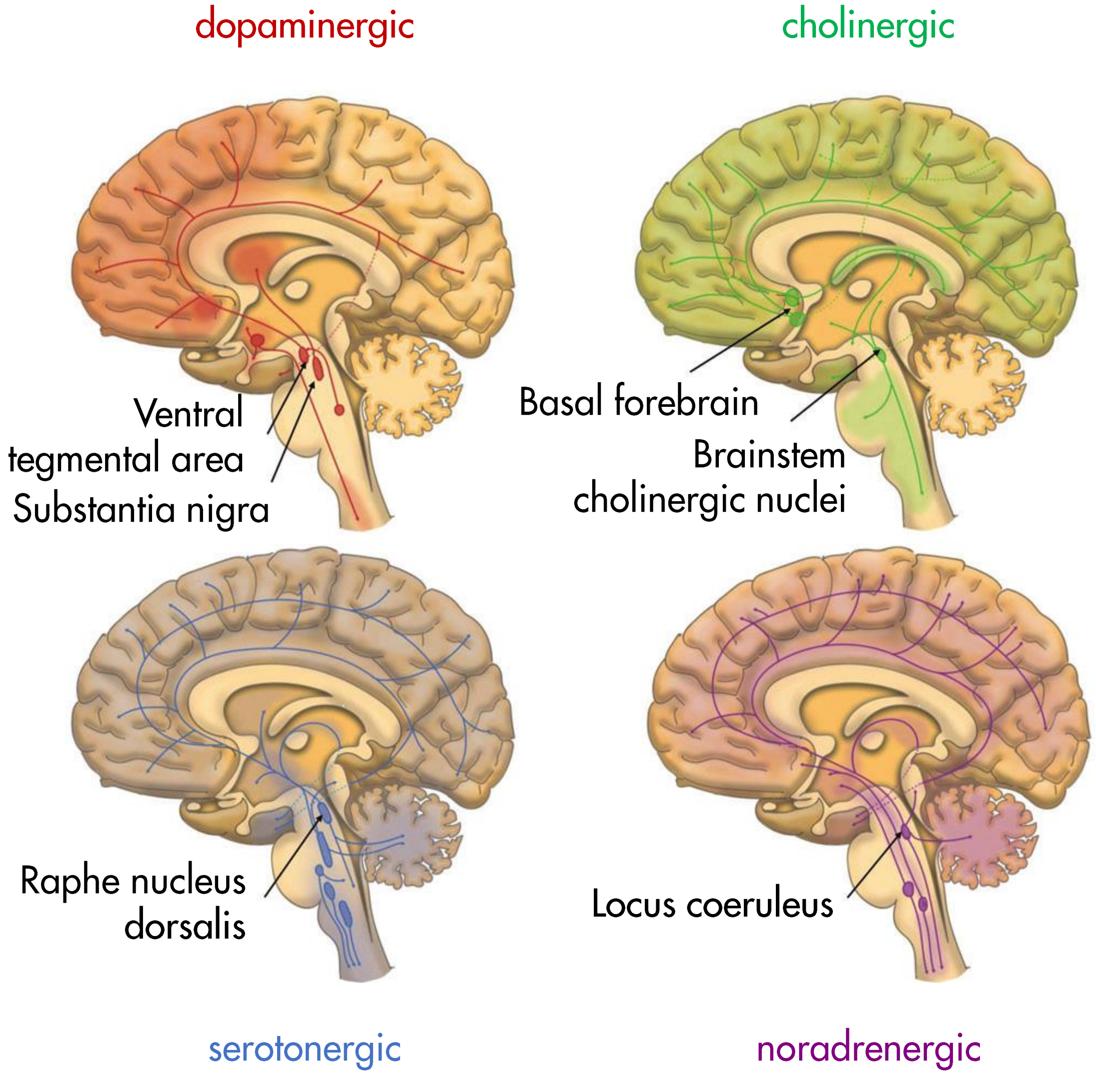

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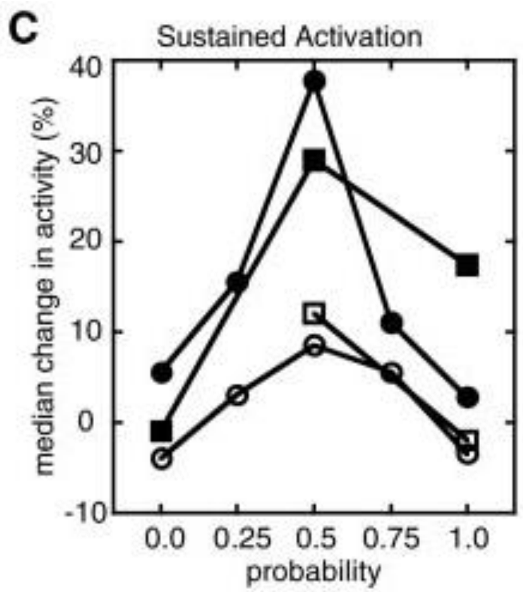
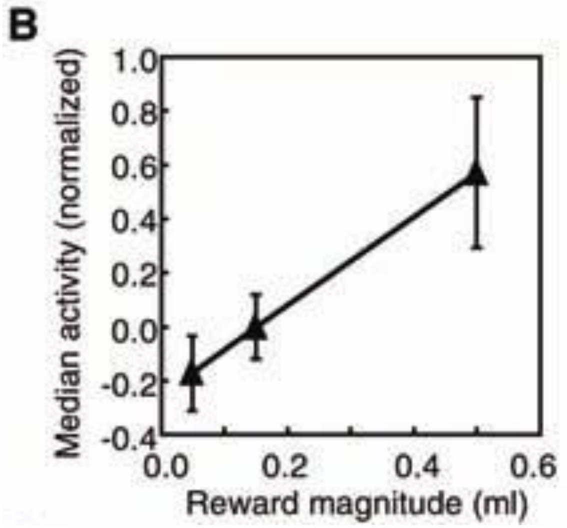
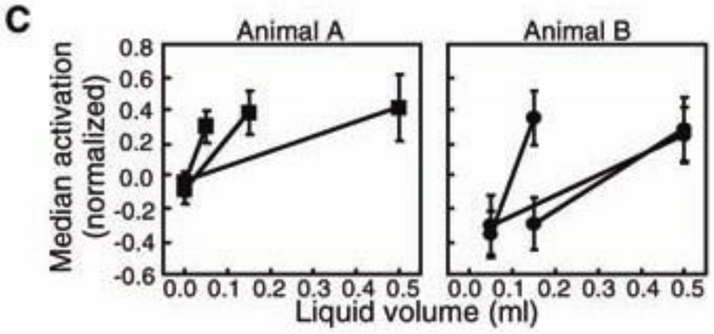


Fiorillo et al. 2003 *Science*





see also:
Fiorillo et al. 2005 *Behav Brain Func*
Bunzeck et al. 2010 *Hum. Brain. Map.*

Tobler et al. 2005 *Science*

Achieved via cholinergic modulation?
Naudé et al. 2018 *bioRχiv*

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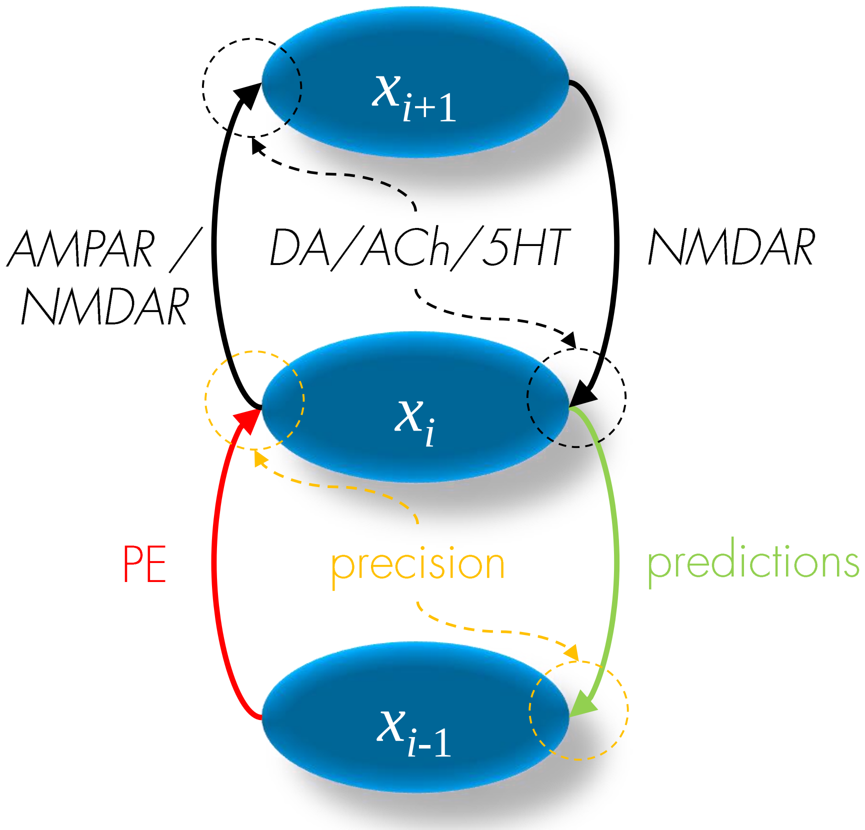


Stephan et al. 2016 *Brain*

frontiers in Computational Neuroscience

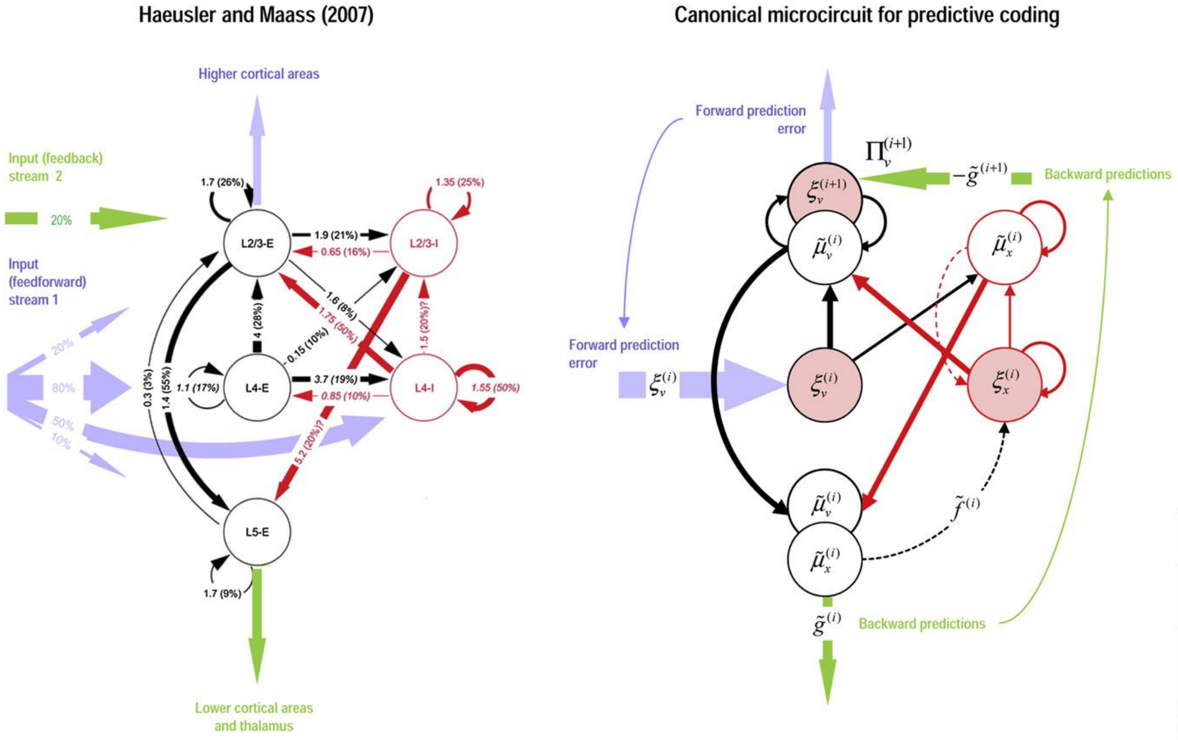
OPINION
published: 08 September 2015
doi: 10.3389/fncom.2015.00111

CrossMark

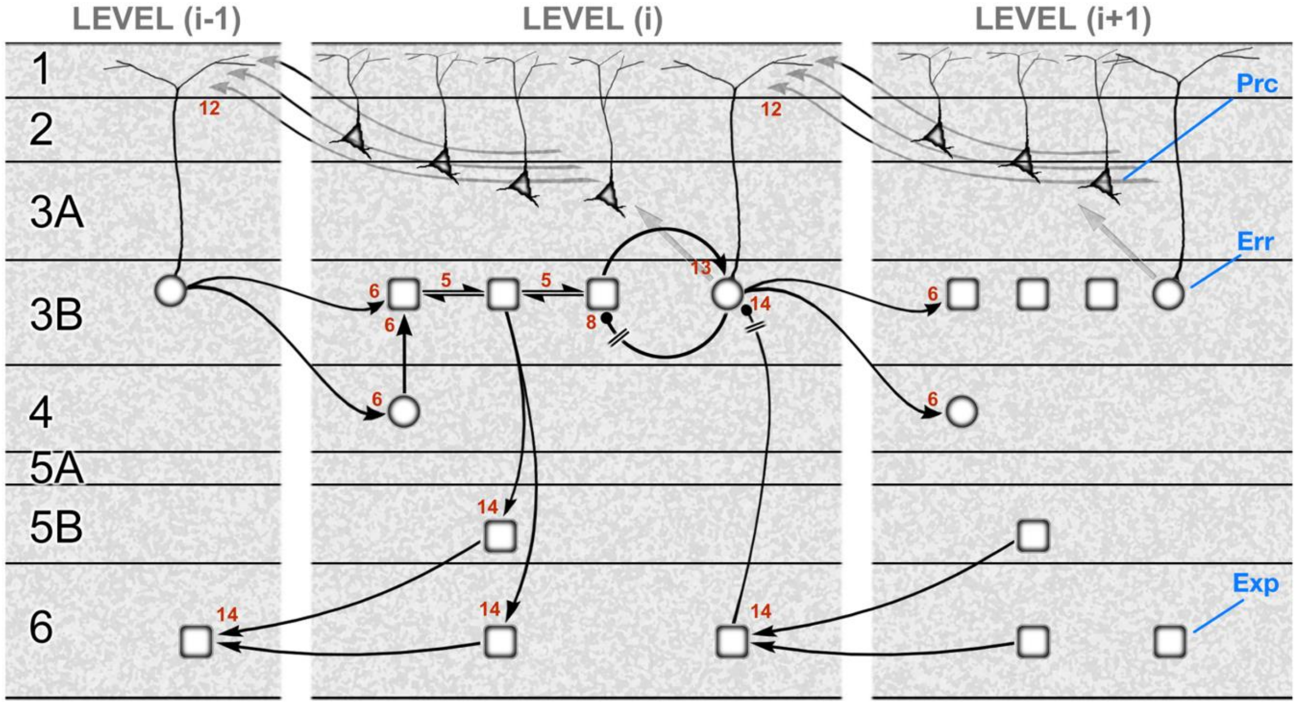
Is predictive coding theory articulated enough to be testable?

*Naoki Kogo * and Chris Trengove*

Mapping predictive coding onto the brain … *advanced*



Bastos et al. 2012 *Neuron*



Shipp 2016 *Frontiers in Psychology*

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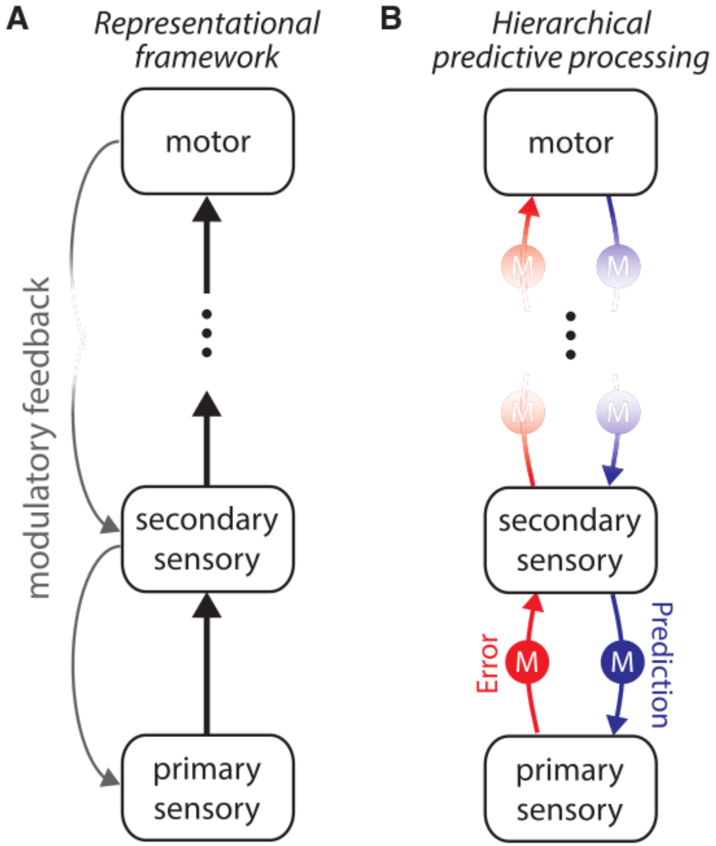
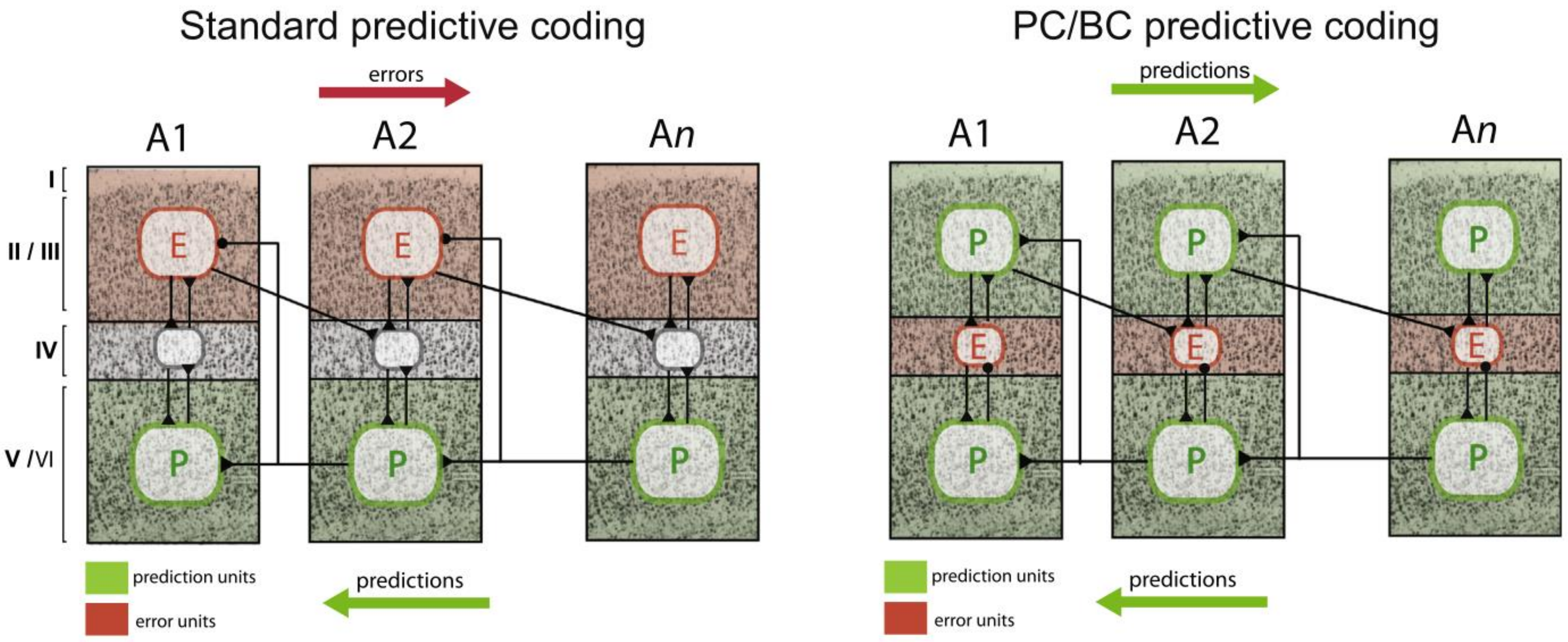
Testing the *relevant* predictions

- locomotion is sufficient to drive activity in mouse V1, even in the complete absence of visual input **(Keller et al., 2012, Saleem et al., 2013)**
- layer 2/3 of sensory cortices signals a mismatch between predicted and actual sensory feedback **(Eliades and Wang, 2008, Keller et al., 2012)**
- this results from a comparison of an excitatory motor-related input and an inhibitory visual input **(Attinger et al., 2017, Zmarz and Keller, 2016)**
- A24b/M2 provides a strong and dense projection to V1, which conveys motor-related signals that depend on the mouse's visuomotor experience. This projection fulfills all the criteria necessary to be interpreted as a prediction of visual flow given a motor output **(Leinweber et al., 2017)**



Keller & Mrsic-Flogel 2018 *Neuron*

Testing the *relevant* predictions



Heilbron & Chait 2018 *Neurosci*

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III. The computational goal

From representing probability distributions (and uncertainty) and the Laplace assumption to predictions on cortical infrastructure and plasticity: *Friston, 2005*

Available online at www.sciencedirect.com

ScienceDirect

Current Opinion in Neurobiology

With or without you: predictive coding and Bayesian inference in the brain

Laurence Aitchison[1] and Máté Lengyel[1,2]

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What does the free energy principle tell us about the brain?

Samuel J. Gershman

arXiv:1901.07945v1

Predictive coding as approximate inference: Rao and Ballard

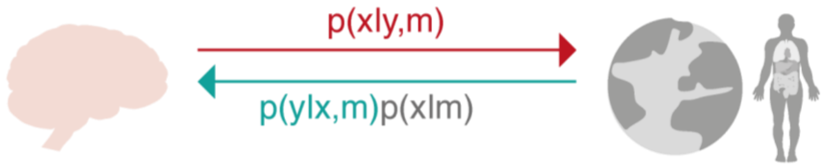
Optimization: Finding causes and associations

$$E = E_1 - \log p(\mathbf{r}) - \log p(U)$$

$$E_1 = \underbrace{\frac{1}{\sigma^2}(\mathbf{I} - f(U\mathbf{r}))^\intercal(\mathbf{I} - f(U\mathbf{r})) + \frac{1}{\sigma^2_{\text{td}}}(\mathbf{r} - \mathbf{r}^{\text{td}})^\intercal(\mathbf{r} - \mathbf{r}^{\text{td}})}_{-\log p(\mathbf{I}|\mathbf{r},U) + C}$$

$$\begin{aligned} E &= -\log p(\mathbf{I}|\mathbf{r},U) - \log p(\mathbf{r}) - \log p(U) \\ &= -\log \boxed{(p(\mathbf{I}|\mathbf{r},U)\, p(\mathbf{r})\, p(U))} \end{aligned}$$

Optimization looks like finding the most likely cause (MAP estimate) in Bayesian inference.



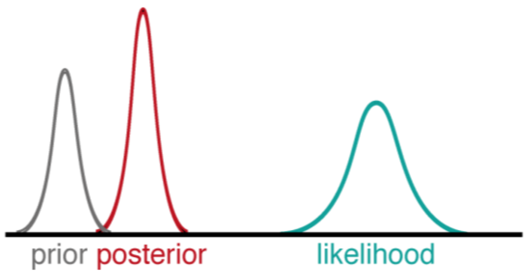
Bayes' Rule

posterior~ likelihood · prior

p(xIy,m)~ p(yIx,m)p(xIm)

likelihood: p(yIx,m)
prior: p(xIm)
posterior: p(xIy,m)

x = state of world & body
y = sensory data
m = model



Petzschner et al. 2017 *Biol Psychiatry*

The Bayesian Brain

Bayesian Inference

Analytical solutions

Approximate Inference

MAP estimates

Variational Bayes

Sampling (MCMC)

Gradient descent on neg. log joint

Gradient descent on free energy

Rao & Ballard 1999 Nat Neurosci

'original' PC

Friston 2005 Phil Trans Royal Soc B

'Fristonian' PC

Bogacz 2017 *J Math Psychol*

Point estimate (mode of posterior)

Full posterior distribution

$$p(x|y) = \frac{p(x)p(y|x)}{p(y)} \qquad p(y) = \int p(x)p(y|x)\,\mathrm{d}x$$

Representing uncertainty

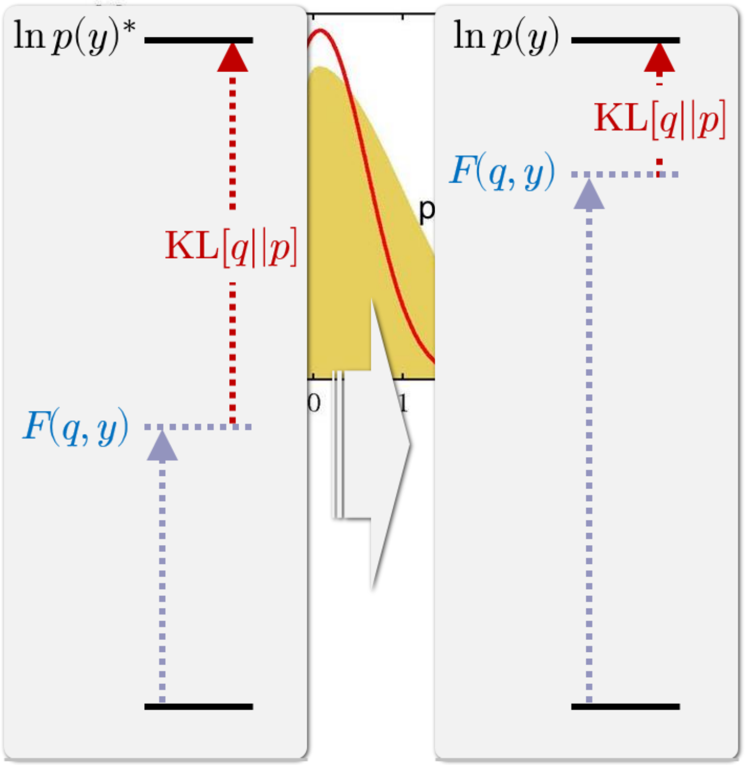
So far, we've only computed the MAP.
To be fully Bayesian: care about your uncertainty!

$q(x)$: our best guess about $p(x|y)$

$$\mathrm{KL}(q(x), p(x|y)) = \ln p(y) - \int q(x) \ln \frac{p(y,x)}{q(x)}\,\mathrm{d}x$$
$$= \ln p(y) - F$$

$$\ln p(y|m) = \mathrm{KL}(q(x), p(x|y,m)) + F(q(x), p(x,y|m))$$

$$F = \int q(x) \ln \frac{p(y,x)}{q(x)}\,\mathrm{d}x$$



Bishop, 2008; Friston et al. 2007 *NeuroImage*

‘Fristonian’ PC

Predictive Coding can be viewed as the implementation of a particular scheme of approximate Bayesian inference in the brain.

The free energy formulation of predictive coding

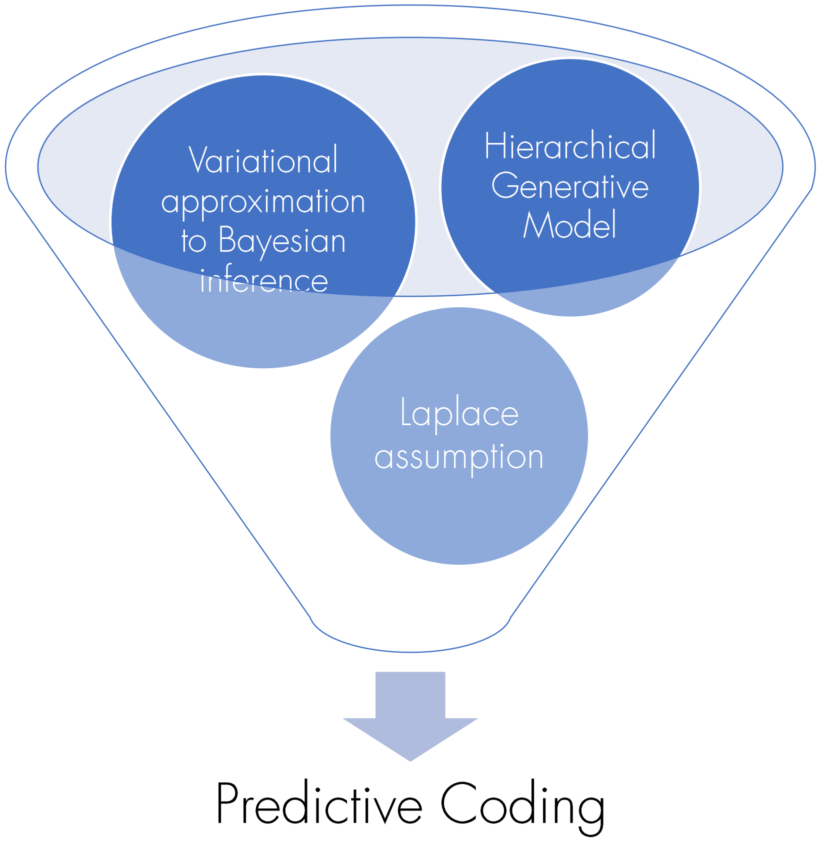
$$F = \int q(x) \ln \frac{p(y,x)}{q(x)} \, \mathrm{d}x$$

Inference: maximizing F to find the approximate posterior

Learning: maximizing F to improve the model and thereby reduce average surprise over time.

Additional assumption: *q(x)* is a Gaussian.
We only need to represent the mean and the variance.

→ *precision-weighted prediction errors*



IV. PC in CP

Stratifying psychiatric diseases using hierarchical Bayesian models

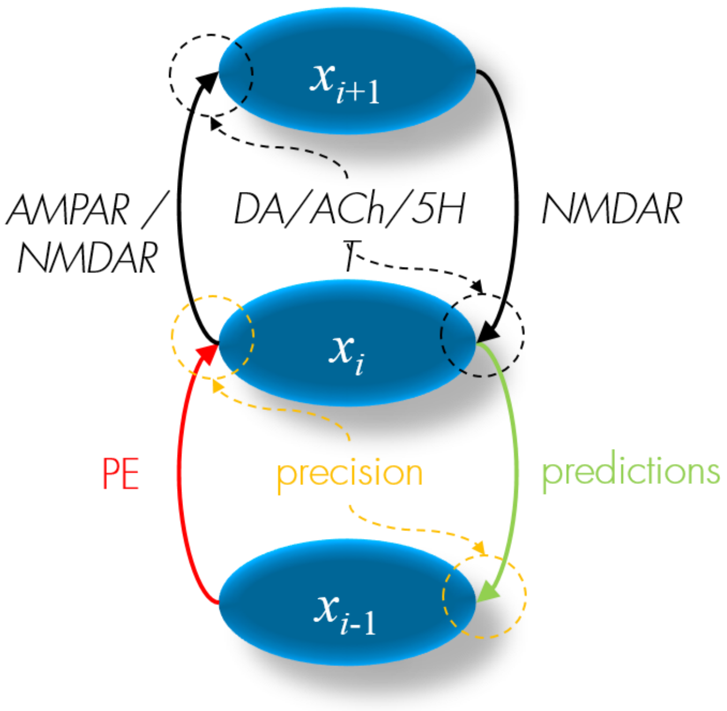
Hierarchical Bayesian Inference in Computational Psychiatry

Possible primary disruption at:

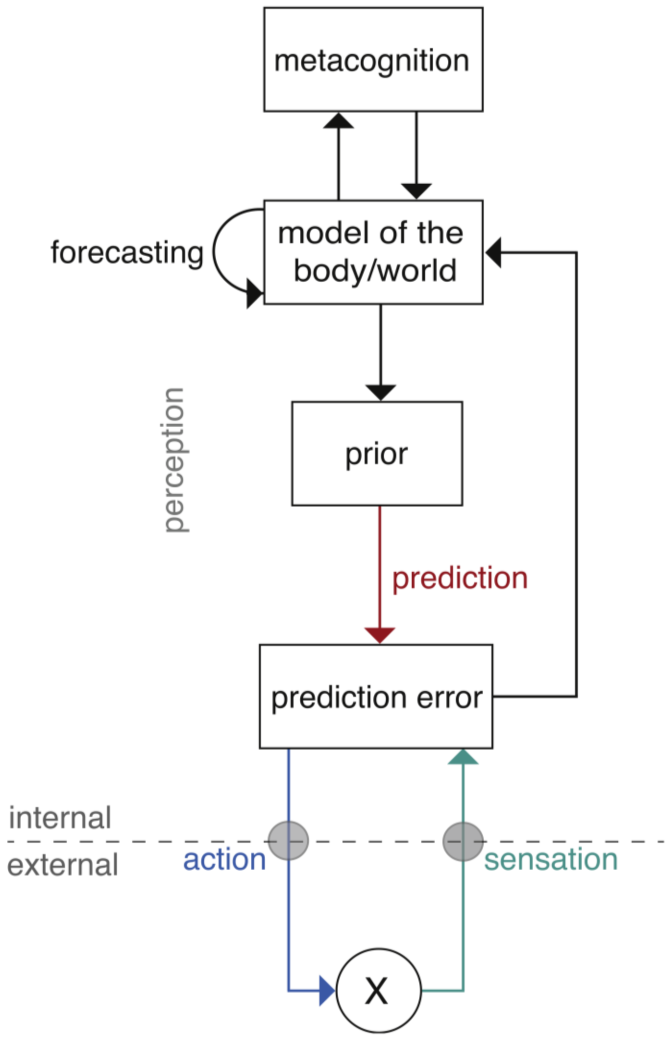
1. Sensory inputs (sensations)
2. Inference (perception)
3. Forecasting
4. Control (action)
5. Metacognition

At any of these, possible disturbance of:

- Prediction error (PE) computation
- Predictions/Expectations
- Estimation of their precision.



Stephan et al. 2016 *Brain*



Petzschner et al. 2017 *Biol Psychiatry*

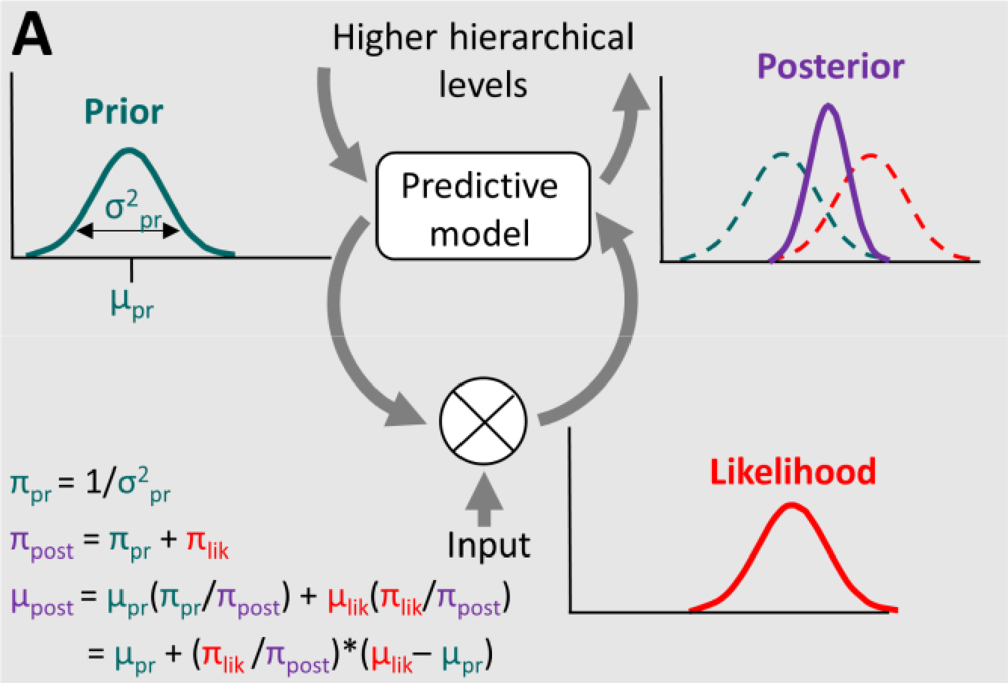
Precision: The right balance

Schizophrenia/Psychosis

Sterzer et al. 2018 *Biol Psychiatry*; Adams et al. 2013 *Front Psychiatry*; Corlett et al. 2011 *NPP*; Stephan et al. 2006 *Biol Psychiatry*; 2009; Powers et al. 2017 *Science*

Autism spectrum disorder

Pellicano & Burr 2012 *TiCS*; Van de Cruys et al. 2014 *Psychol Rev*; Lawson et al. 2014 *Front Hum Neurosci*; Lawson et al. 2017 *Nat Neurosci*



Sterzer et al. 2018 *Biol Psychiatry*

A role for interoception

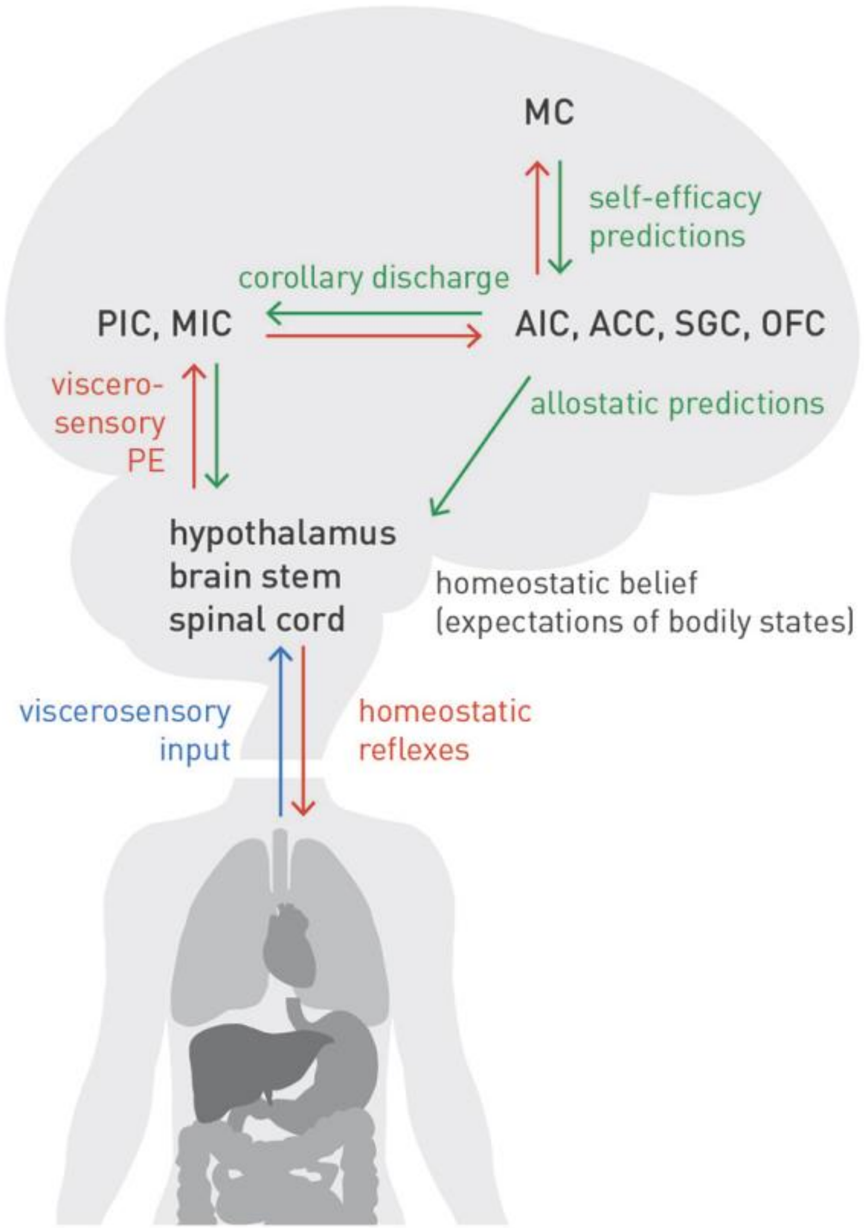
Seth 2013 *TiCS*

Barrett & Simmons 2015 *Nature Rev Neurosci*

Seth & Friston 2016 *Phil Trans Royal Soc B*

Stephan et al. 2016 *Front Hum Neurosci*

Petzschner et al. 2017 *Biol Psychiatry*



University of Zurich[UZH]
ETH Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich
Translational Neuromodeling Unit

news & views

PREDICTIVE CODING

Does predictive coding have a future?

In the 20th century we thought the brain extracted knowledge from sensations. The 21st century witnessed a 'strange inversion', in which the brain became an organ of inference, actively constructing explanations for what's going on 'out there', beyond its sensory epithelia. One paper played a key role in this paradigm shift.

Karl Friston

V. THE FUTURE OF PC

Tests and extensions

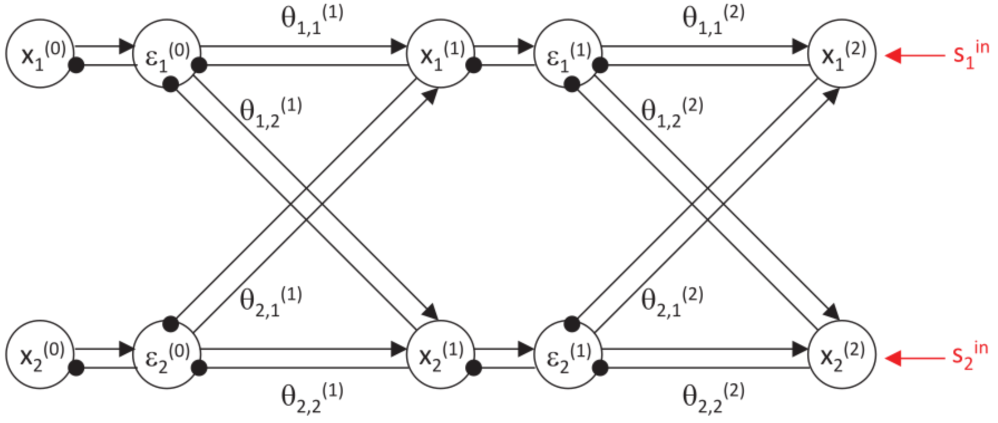
I. Testing further

- Sources of modulatory (precision) signals
  - Neuromodulators and how they interact
- Separate error units and prediction (representation) units
  - Laminar resolution, especially in the auditory domain (vision: Kok et al. 2016 *Curr Biol*)
  - High temporal resolution (predictions can precede a stimulus, PEs can only follow)
- Computationally explicit analyses ('model-based')
  - to test crucial theoretical distinctions, e.g. PEs vs. precision-weighted PEs
  - e.g., Iglesias et al. 2013 *Neuron*; Sedley et al. 2016 *eLife*; Bell et al. 2016 *Curr Biol*

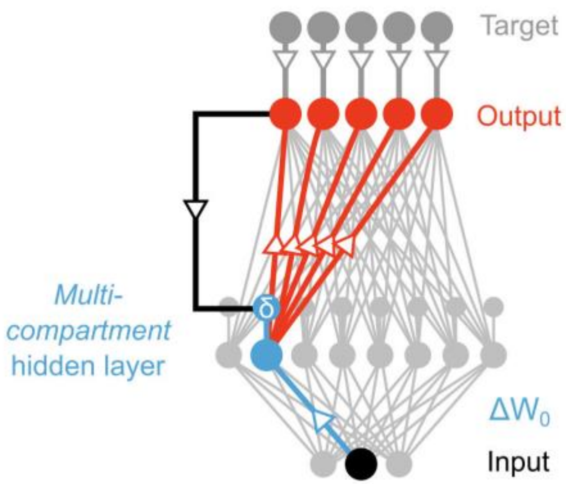
II. PC and ANNs

- A predictive coding network can perform supervised learning autonomously using simple Hebbian learning and approximates the backpropagation algorithm (Whittington & Bogacz 2017 *Neural Computation*)

- A single pyramidal neuron can implement predictive coding via its apical dendrites (Urbanczik & Senn 2014 *Neuron*; Sacramento et al. 2018 *NIPS*) which can help to solve the credit assignment problem… (Guerguiev et al. 2017 *eLife*)



B Segregated dendrites solution



Thank you

Translational Neuromodeling Unit

Rick Adams
Rafal Bogacz