



a

b

C

d

 \mathbf{e}

Inference

A

B

spm_MDP_VB_X.m

C

D

 \mathbf{E}

Deep temporal models

Model fitting

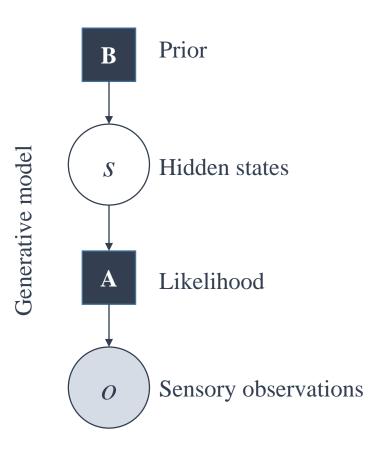
 $\mathcal{L}(\theta, u, o) = \ln P(u/\theta, o)$

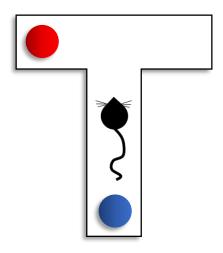
Precision

γ

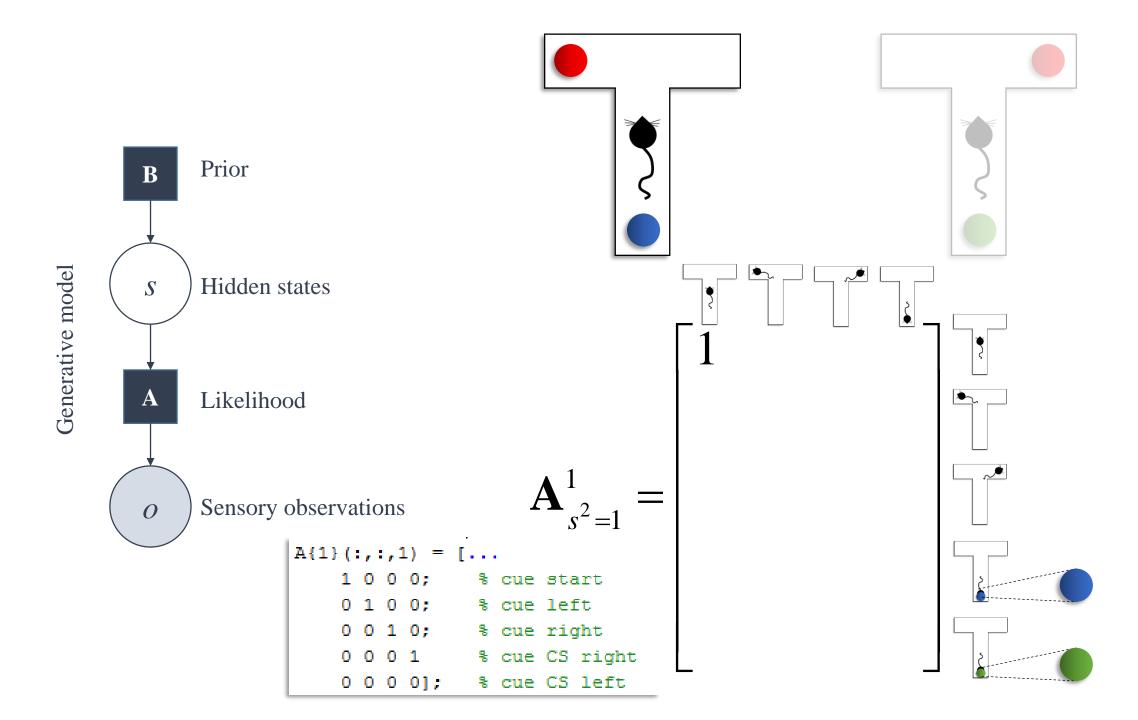
Mixed models $\partial_{\mu} \mathbf{\epsilon} \cdot \mathbf{\Pi} \mathbf{\epsilon}$

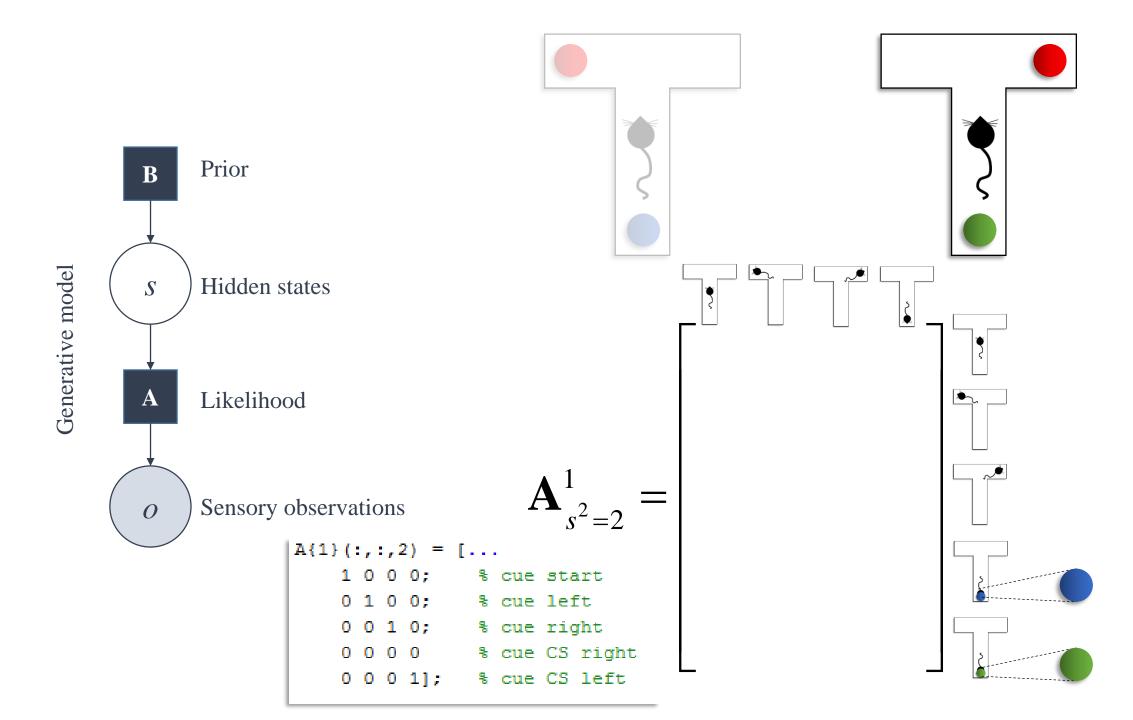
 $P(s \mid o) \propto P(o \mid s)P(s)$

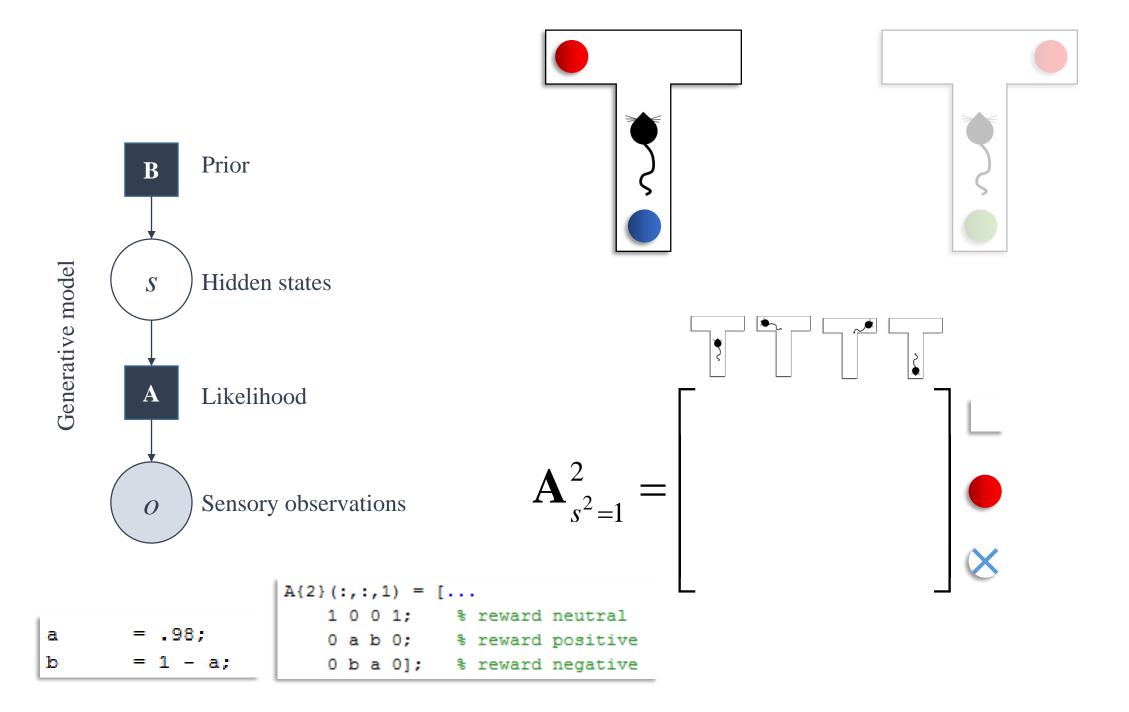


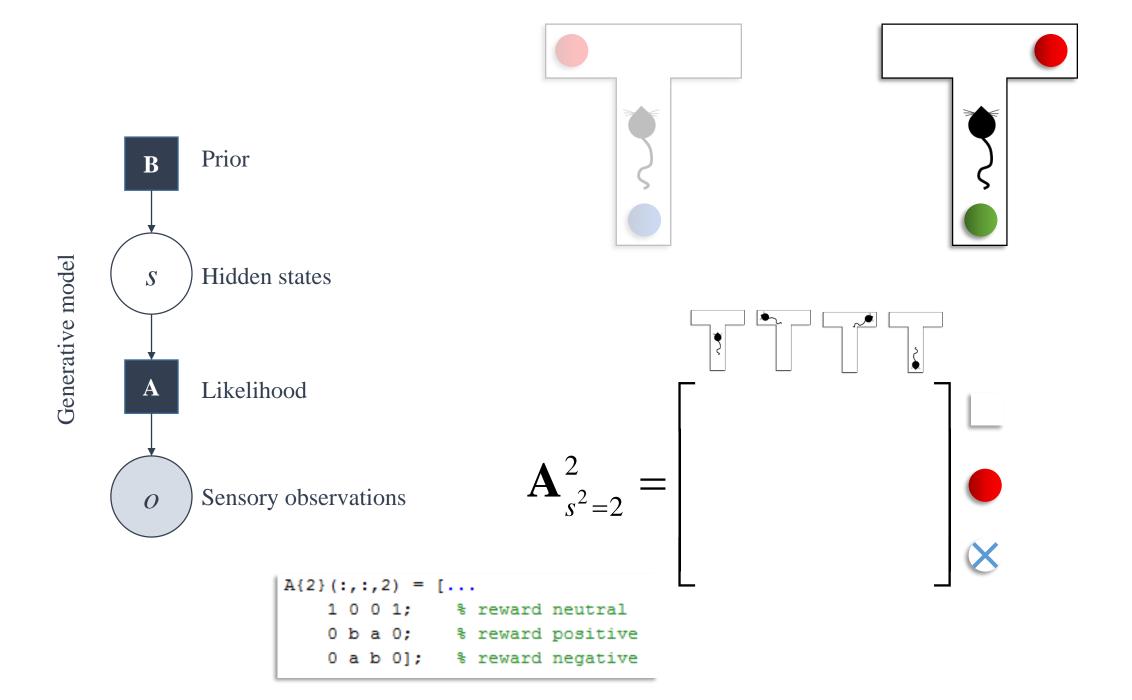


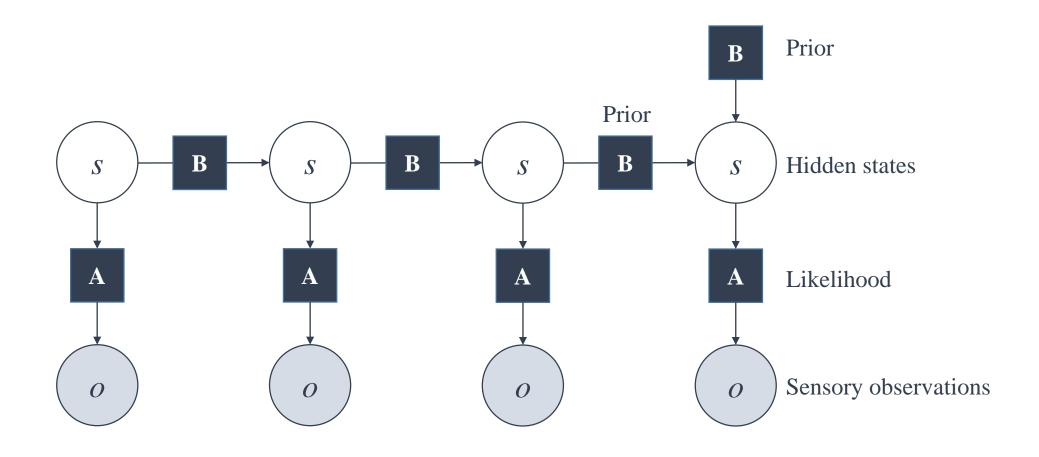
Navigating a T-maze



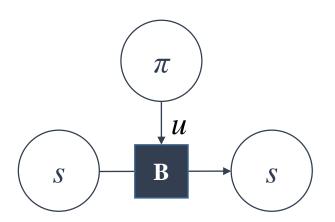


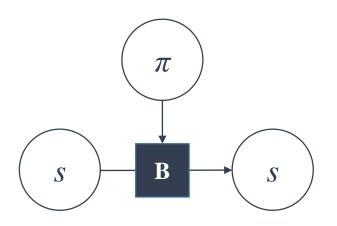




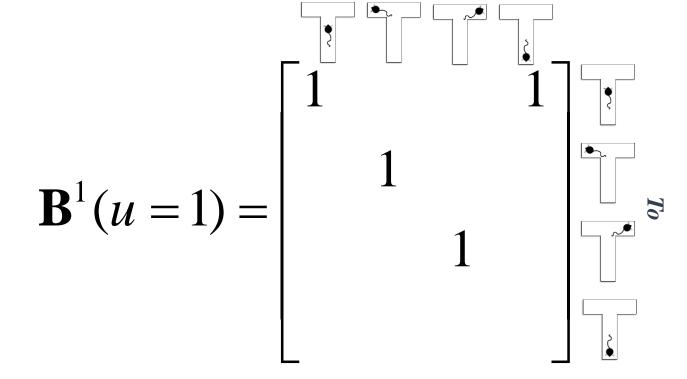


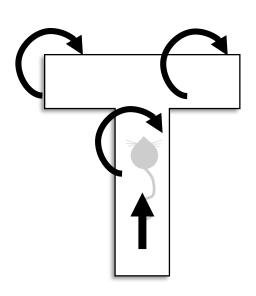
Time

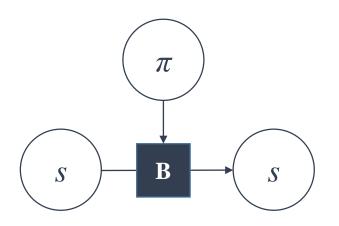




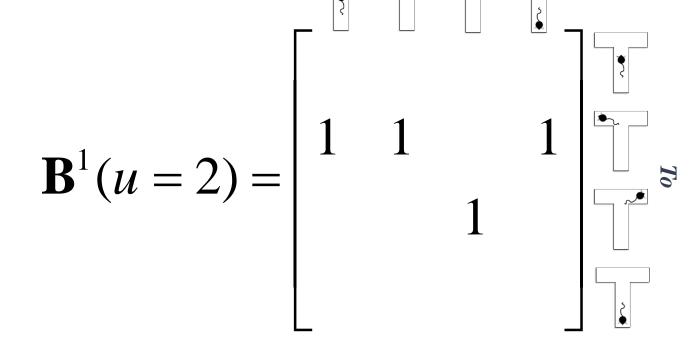
From

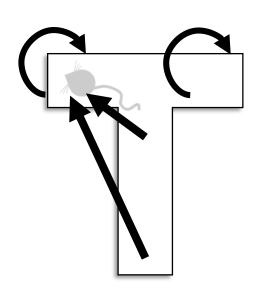


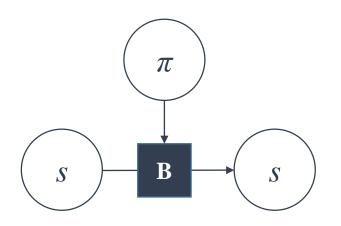




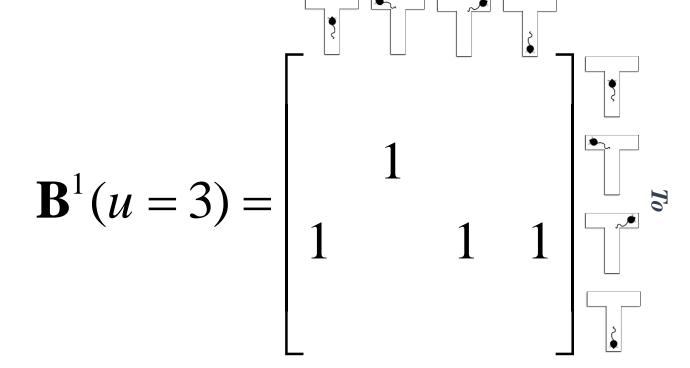
From

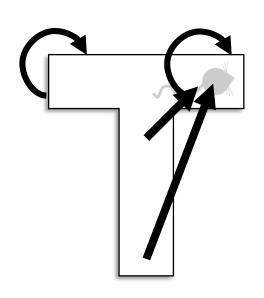


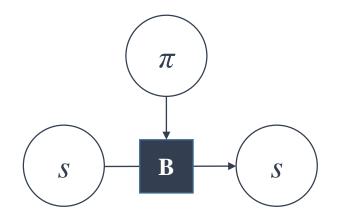


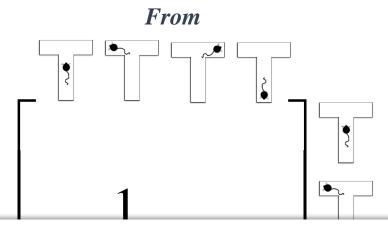


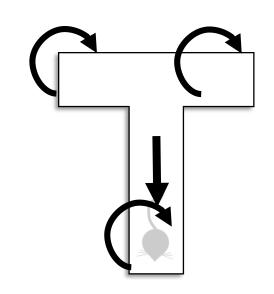
From



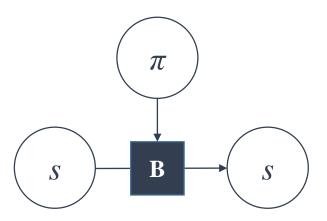


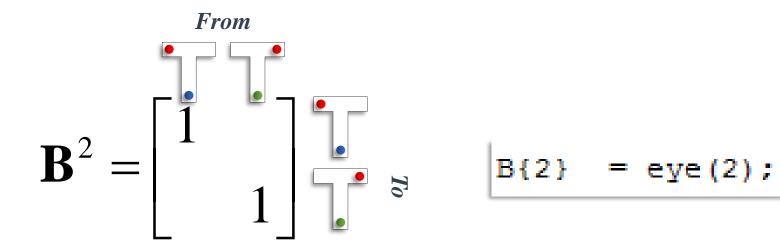






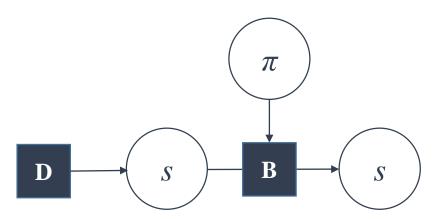
```
B\{1\}(:,:,1) = [1 \ 0 \ 0 \ 1; \ 0 \ 1 \ 0 \ 0; 0 \ 0 \ 1 \ 0; 0 \ 0 \ 0];
B\{1\}(:,:,2) = [0 \ 0 \ 0; 1 \ 1 \ 0 \ 1; 0 \ 0 \ 1 \ 0; 0 \ 0 \ 0];
B\{1\}(:,:,3) = [0 \ 0 \ 0; 0 \ 1 \ 0; 1 \ 0 \ 1 \ 1; 0 \ 0 \ 0];
B\{1\}(:,:,4) = [0 \ 0 \ 0; 0 \ 1 \ 0; 0 \ 0 \ 1 \ 0; 1 \ 0 \ 0];
```





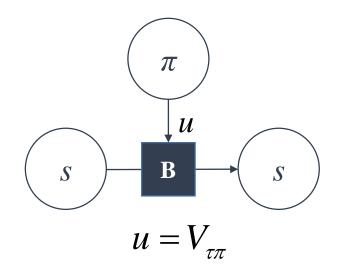
$$\mathbf{C}^{1} = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{C}^2 = \begin{bmatrix} 0 & 0 & 0 \\ c & c & c \\ -c & -c & -c \end{bmatrix}$$



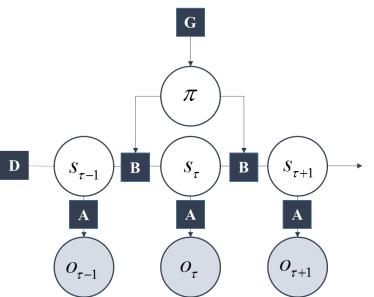
$$\mathbf{D}^1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{D}^2 = \frac{1}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



$$\mathbf{E} = \frac{1}{10} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}^T$$

E = ones(10,1)/10;



Generative model

$$P(\tilde{o}, \tilde{s}, \pi) = P(s_1)P(\pi) \prod_{\tau} P(s_{\tau+1} \mid s_{\tau}, \pi)P(o_{\tau} \mid s_{\tau})$$

$$P(o_{\tau} | s_{\tau}) = Cat(\mathbf{A})$$

$$P(s_{\tau} \mid s_{\tau-1}, \pi) = Cat(\mathbf{B}_{\pi\tau})$$

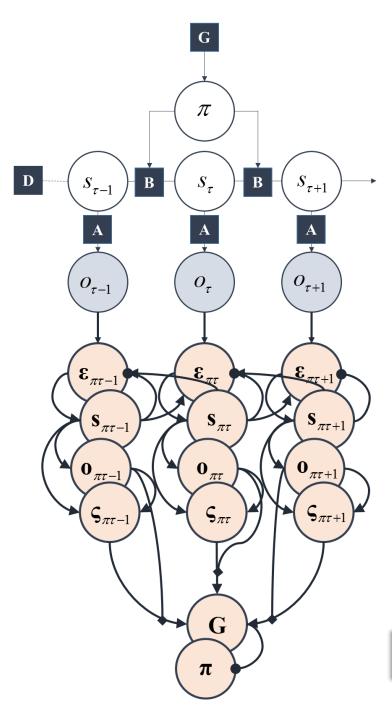
$$P(o_{\tau}) = Cat(\mathbf{C})$$

$$P(s_1) = Cat(\mathbf{D})$$

$$P(\pi) = \sigma(-\mathbf{G})$$

Can omit and instead specify mdp.T

```
mdp.E = E;
mdp.V = V;
mdp.A = A;
mdp.B = B;
mdp.C = C;
mdp.D = D;
mdp.S = [1 1]';
% true initial state
```



Generative model

$$P(\tilde{o}, \tilde{s}, \pi) = P(s_1)P(\pi) \prod_{\tau} P(s_{\tau+1} \mid s_{\tau}, \pi) P(o_{\tau} \mid s_{\tau})$$

$$P(o_{\tau} | s_{\tau}) = Cat(\mathbf{A})$$

$$P(s_{\tau} \mid s_{\tau-1}, \pi) = Cat(\mathbf{B}_{\pi\tau})$$

$$P(o_{\tau}) = Cat(\mathbf{C})$$

$$P(s_1) = Cat(\mathbf{D})$$

$$P(\pi) = \sigma(-\mathbf{G})$$

Bayesian message passing

$$\mathbf{s}_{\tau} = \mathbf{\pi} \cdot \mathbf{s}_{\pi \tau}$$

$$\mathbf{s}_{\pi\tau} = \sigma(\mathbf{v}_{\pi\tau}); \ \dot{\mathbf{v}}_{\pi\tau} = \mathbf{\varepsilon}_{\pi\tau}$$

$$\boldsymbol{\varepsilon}_{\pi\tau} = \ln \mathbf{A} \cdot o_{\tau} + \frac{1}{2} \ln(\mathbf{B}_{\pi\tau} \mathbf{s}_{\pi\tau}) + \frac{1}{2} \ln(\mathbf{B}_{\pi\tau+1}^{\dagger} \mathbf{s}_{\pi\tau+1}) - \ln \mathbf{s}_{\pi\tau}$$

$$\mathbf{o}_{\pi\tau} = \mathbf{A}\mathbf{s}_{\pi\tau}$$

$$\varsigma_{\pi\tau} = \ln \mathbf{o}_{\pi\tau} - \ln \mathbf{C} + \mathbf{H} \cdot \mathbf{s}_{\pi\tau}$$

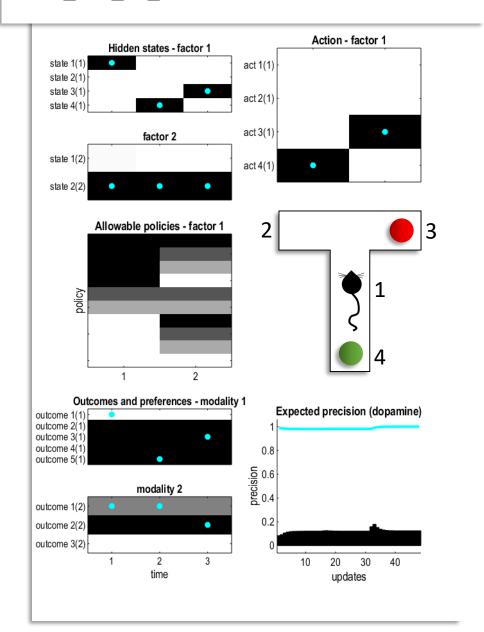
$$\mathbf{H} = -diag(\mathbf{A} \cdot \ln \mathbf{A})$$

$$\mathbf{G}_{\pi} = \mathbf{o}_{\pi\tau} \cdot \boldsymbol{\varsigma}_{\pi\tau}$$

$$\pi = \sigma(-\mathbf{G})$$

 $MDP = spm_MDP_VB_X(mdp);$

spm_figure('GetWin','Figure 1'); clf
spm_MDP_VB_trial(MDP)



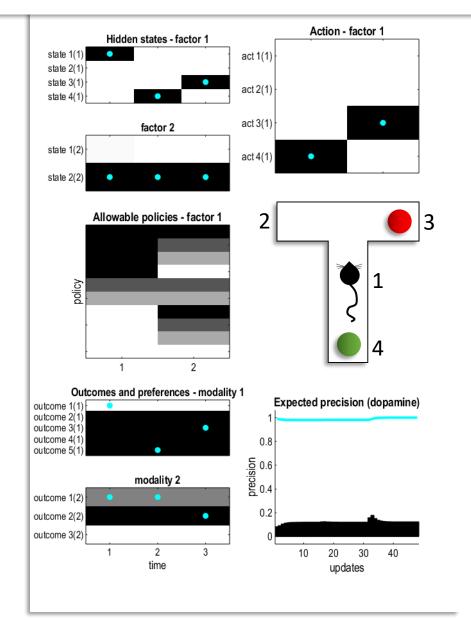
$$P(\pi \mid \mathbf{\gamma}, \mathbf{E}) = \sigma(\mathbf{I} - \mathbf{\gamma} \cdot \mathbf{G})$$

$$\mathbf{G}_{\pi} = -D_{KL}[P(o \mid s)Q(s \mid \pi) || Q(o \mid \pi)Q(s \mid \pi)]$$
Information gain
$$-E[\ln P(o)]$$

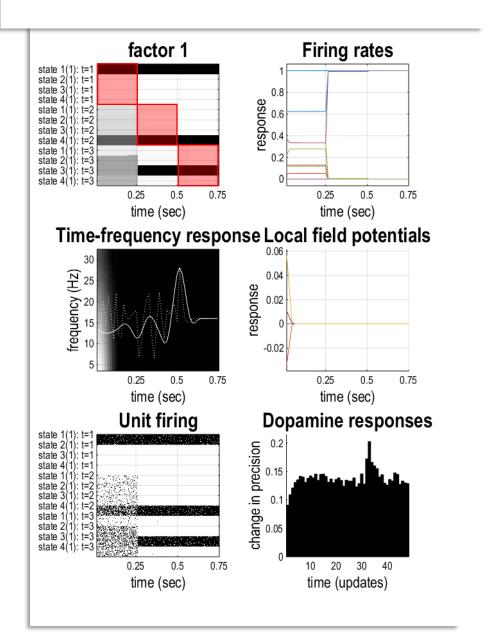
Preferences

$$\sigma(\mathbf{x}) \triangleq \frac{e^{\mathbf{x}}}{\sum_{i} e^{\mathbf{x}_{i}}}$$

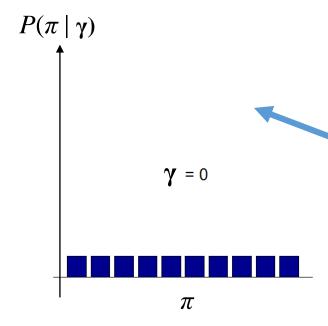
spm_figure('GetWin','Figure 1'); clf
spm MDP VB trial(MDP)



spm_figure('GetWin','Figure 2'); clf
spm MDP VB LFP(MDP)

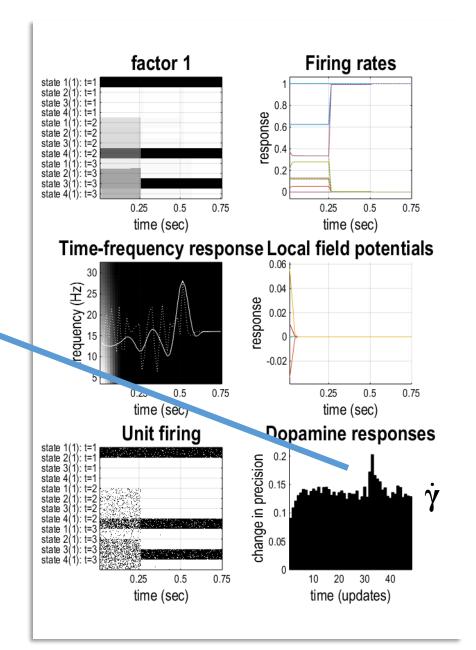


Precision



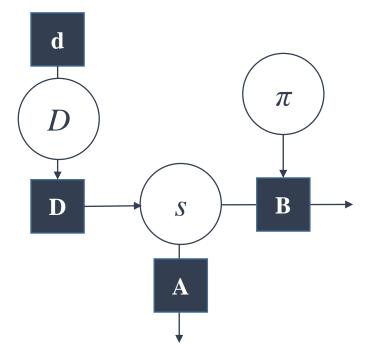
$$P(\gamma) = \Gamma(1,\beta)$$

 $P(\gamma) = \Gamma(1,\beta)$ mdp.beta



Neurotransmitter	Precision	Evidence
Acetylcholine	Likelihood	 Presence of presynaptic receptors on thalamocortical afferents (Sahin, Bowen et al. 1992, Lavine, Reuben et al. 1997) Modulation of gain of visually evoked responses (Gil, Connors et al. 1997, Disney, Aoki et al. 2007) Changes in effective connectivity with pharmacological manipulations (Moran, Campo et al. 2013) Modelling of behavioral responses under pharmacological manipulation (Vossel, Bauer et al. 2014, Marshall, Mathys et al. 2016)
Noradrenaline	Transitions	 Maintenance of persistent prefrontal (delay-period) activity (requiring precise transition probabilities) depends upon noradrenaline (Arnsten and Li 2005, Zhang, Cordeiro Matos et al. 2013) Pupillary responses to surprising (i.e. imprecise) sequences (Lavín, San Martín et al. 2013, Liao, Yoneya et al. 2016, Vincent et al, 2019) Modelling of behavioral responses under pharmacological manipulation (Marshall, Mathys et al. 2016)
Dopamine	Policies	 Expressed post-synaptically on striatal medium spiny neurons (Freund, Powell et al. 1984, Yager, Garcia et al. 2015) Computational fMRI reveals midbrain activity with changes in precision (Schwartenbeck, FitzGerald et al. 2015) Modelling of behavioral responses under pharmacological manipulation (Marshall, Mathys et al. 2016)
Serotonin	Preferences or interoceptive likelihood	 Receptors expressed on layer V pyramidal cells (Aghajanian and Marek 1999, Lambe, Goldman-Rakic et al. 2000, Elliott, Tanaka et al. 2018) in medial prefrontal cortex Medial prefrontal cortical regions heavily implicated in interoceptive processing and autonomic regulation (Marek, Strobel et al. 2013, Mukherjee, Sabharwal et al. 2016)

a, b, c, d, e



$$P(s_1 = i)$$

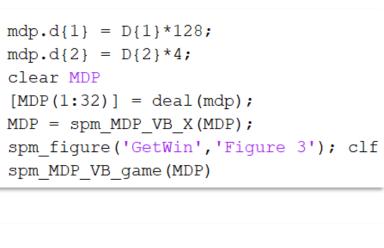
$$P(s_1 = i)$$

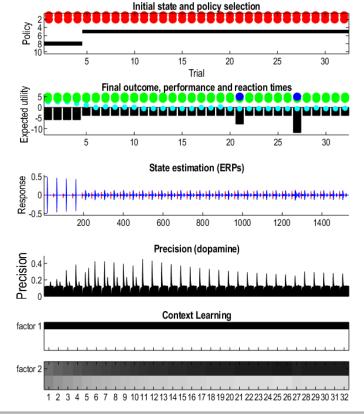
Learning

$$\nabla_{E[\ln D]} F = 0 \Leftrightarrow \mathbf{d} = d + \mathbf{s}_1$$

$$E[D_i] = \frac{\mathbf{d}_i}{\sum_{i} \mathbf{d}_i}$$

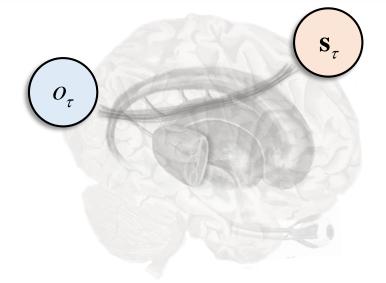


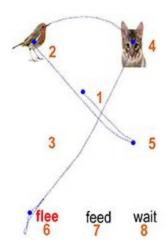




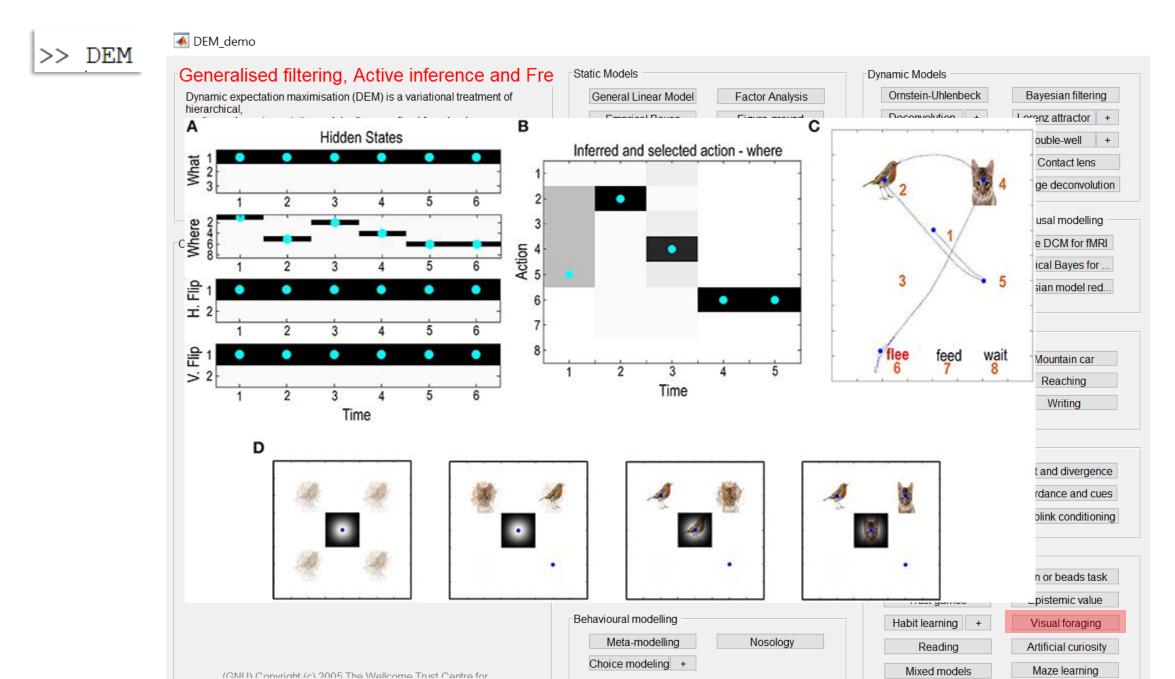
$$P(\pi) = \sigma(-\mathbf{G})$$

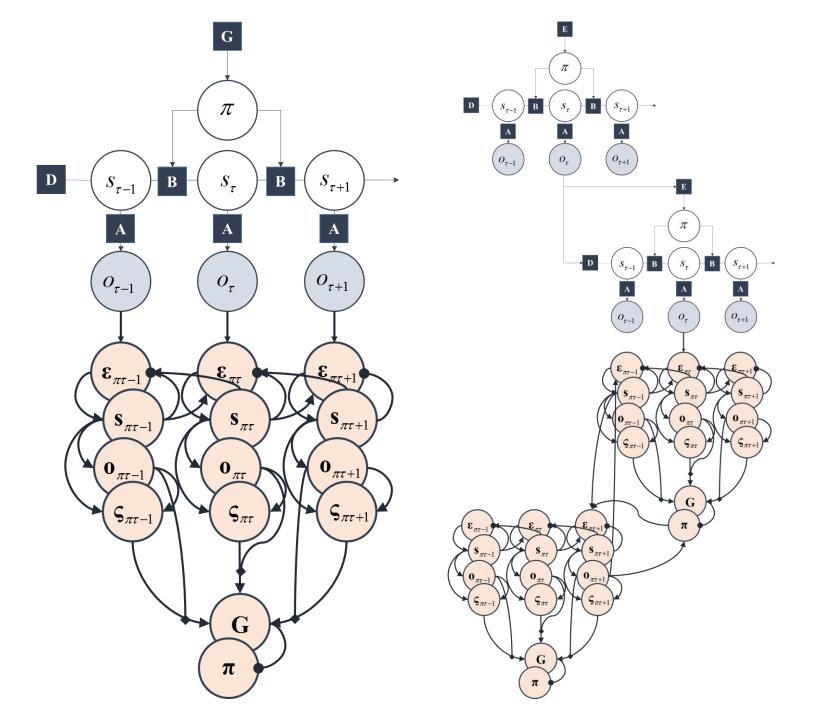
$$= \sigma \Big(E_{\tilde{Q}}[\ln P(\tilde{o}) + D_{KL}[Q(A \mid \tilde{o}) \parallel Q(A)]] \Big)$$

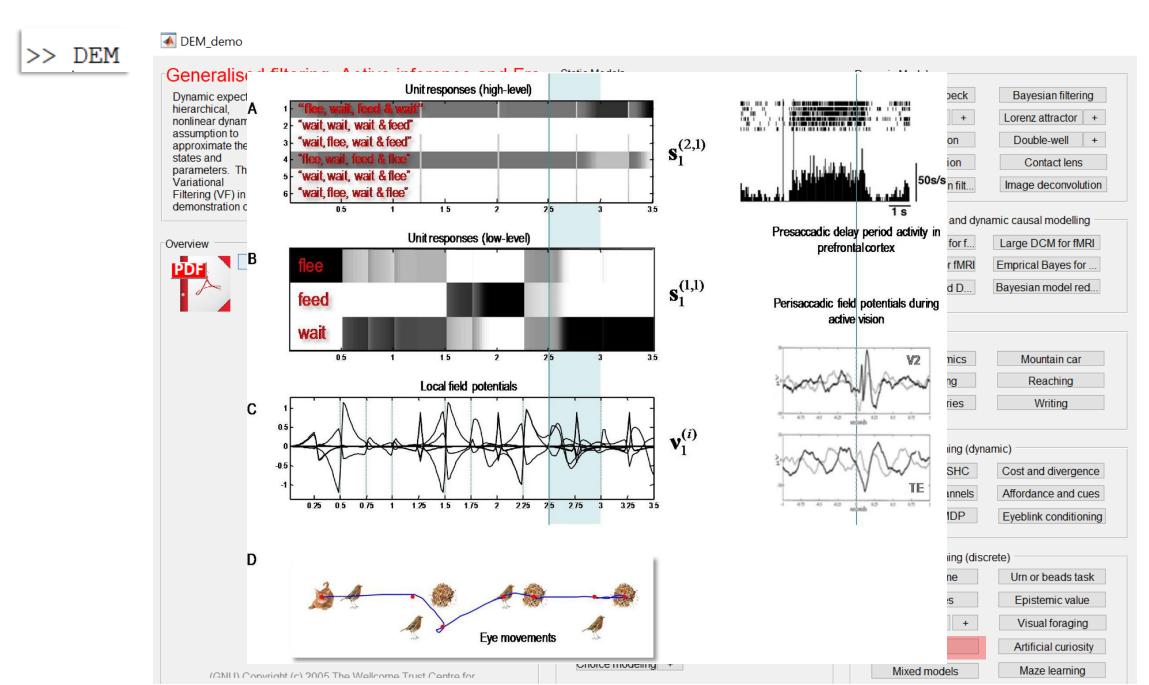


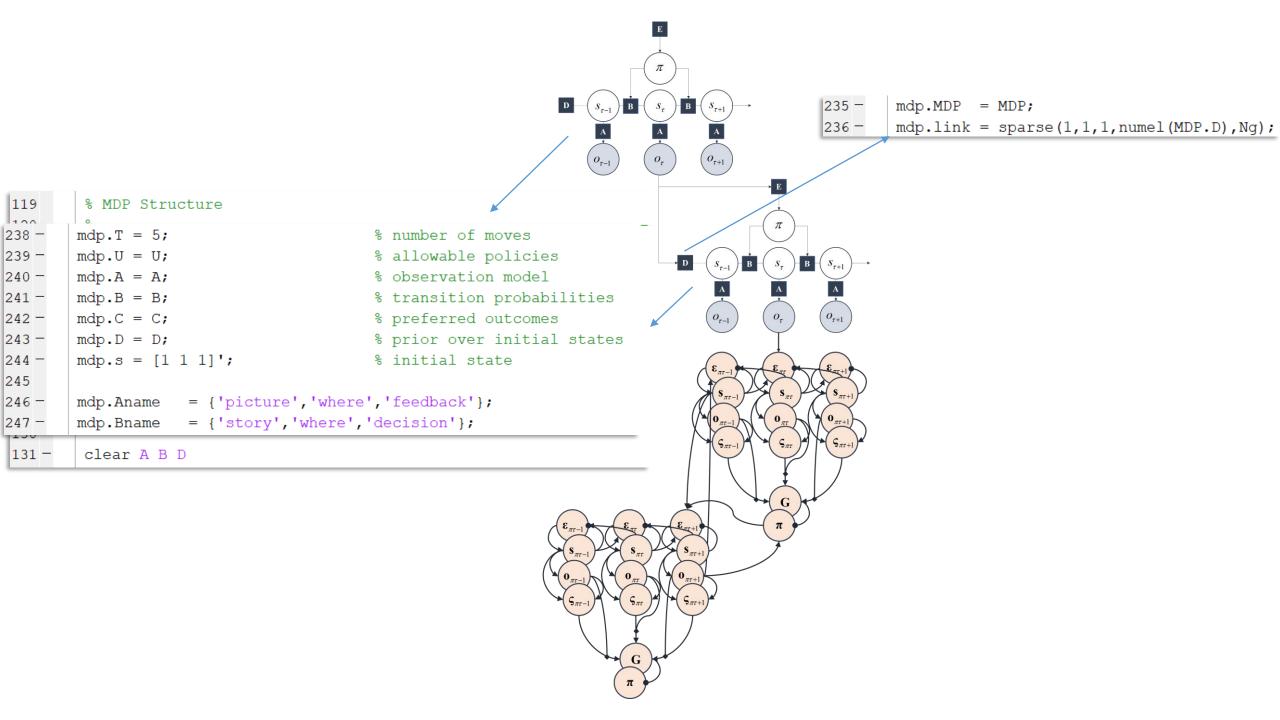


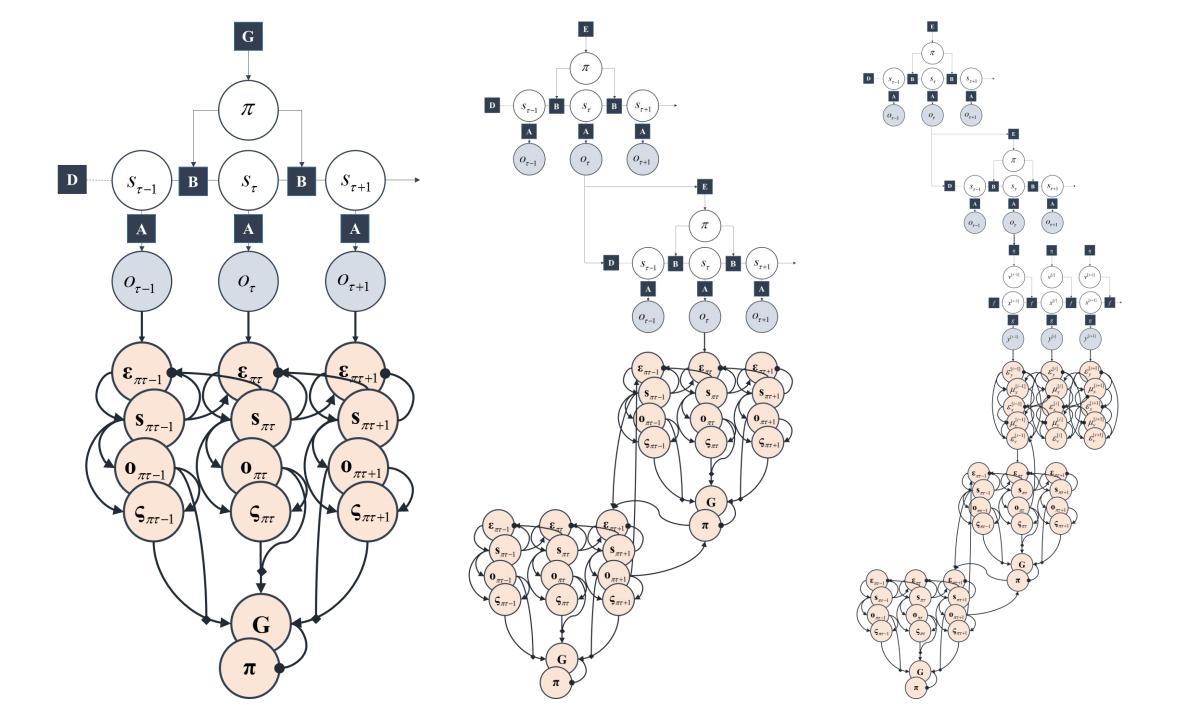
Visual foraging and hierarchical inference

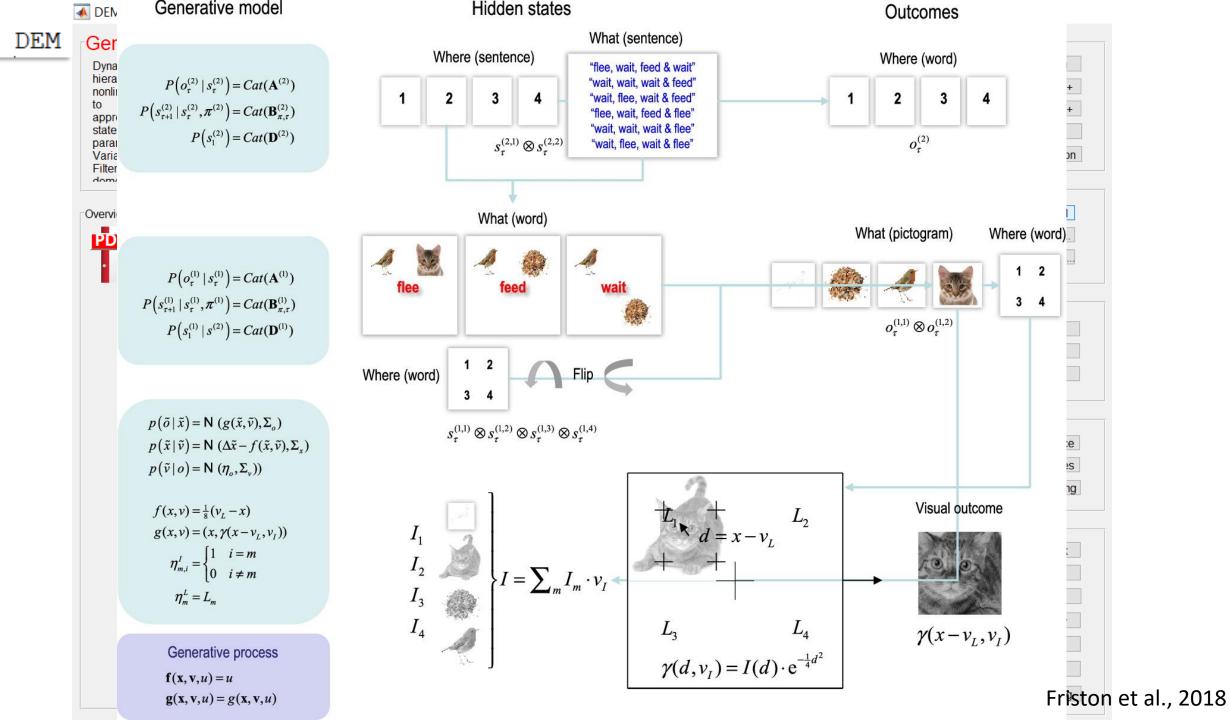














>> DEM

Generalised filtering, Active inference and Free

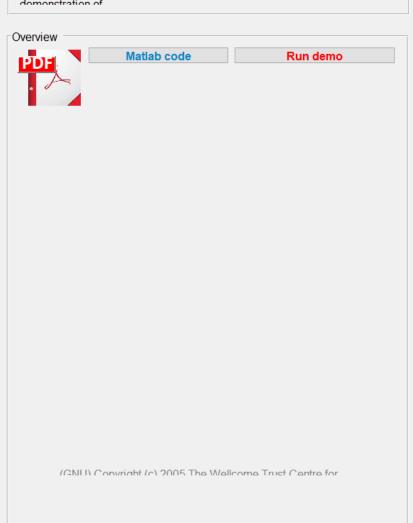
Dynamic expectation maximisation (DEM) is a variational treatment of hierarchical.

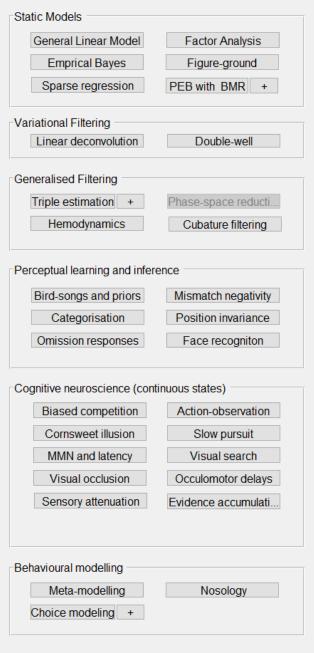
nonlinear dynamic or static models. It uses a fixed-form Laplace assumption to

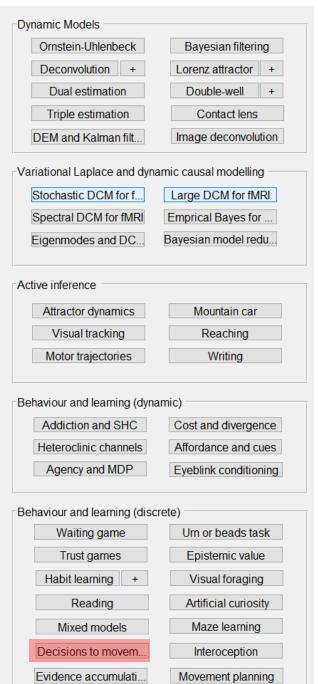
approximate the conditional, variational or ensemble density of unknown states and

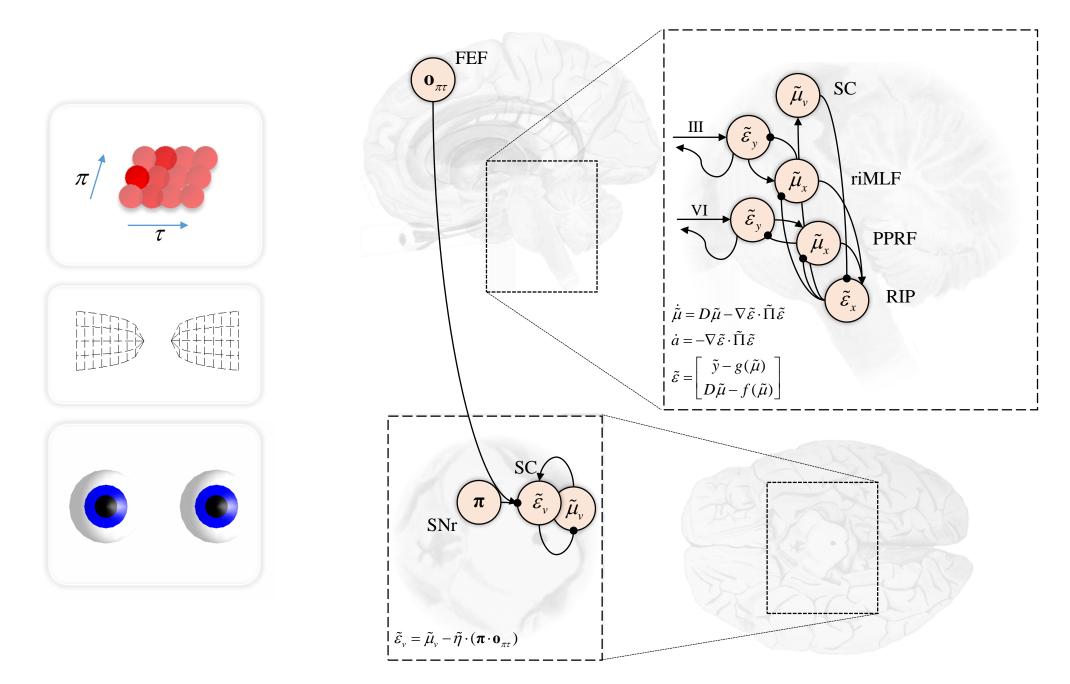
parameters. This is an approximation to the density that would obtain from Variational

Filtering (VF) in generalized coordinates of motion. Start with the

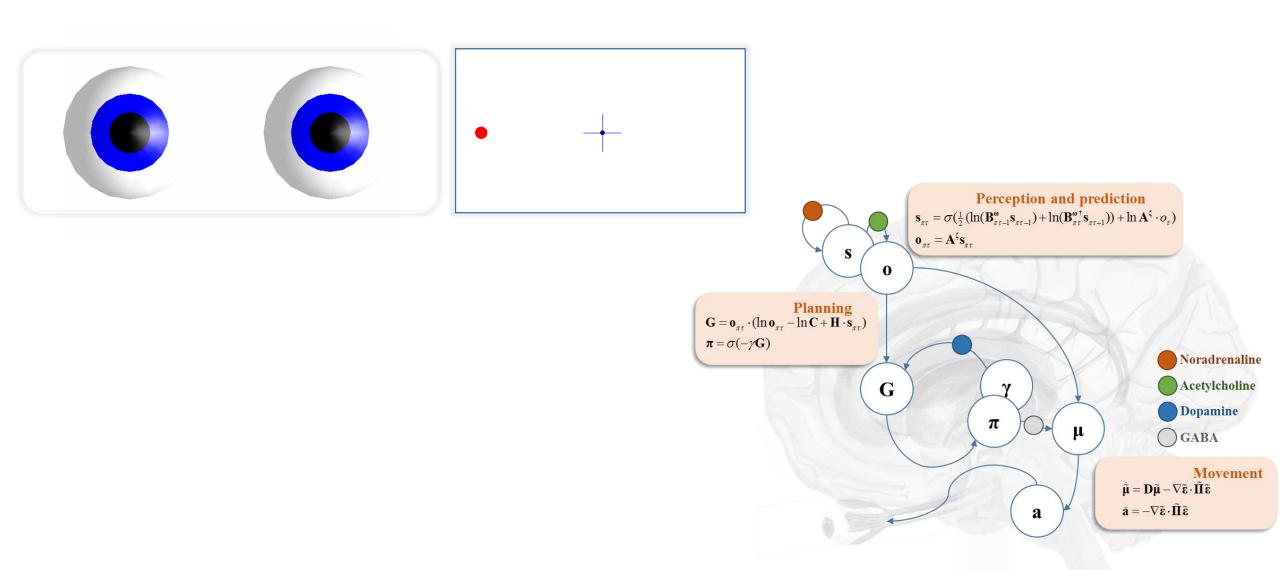


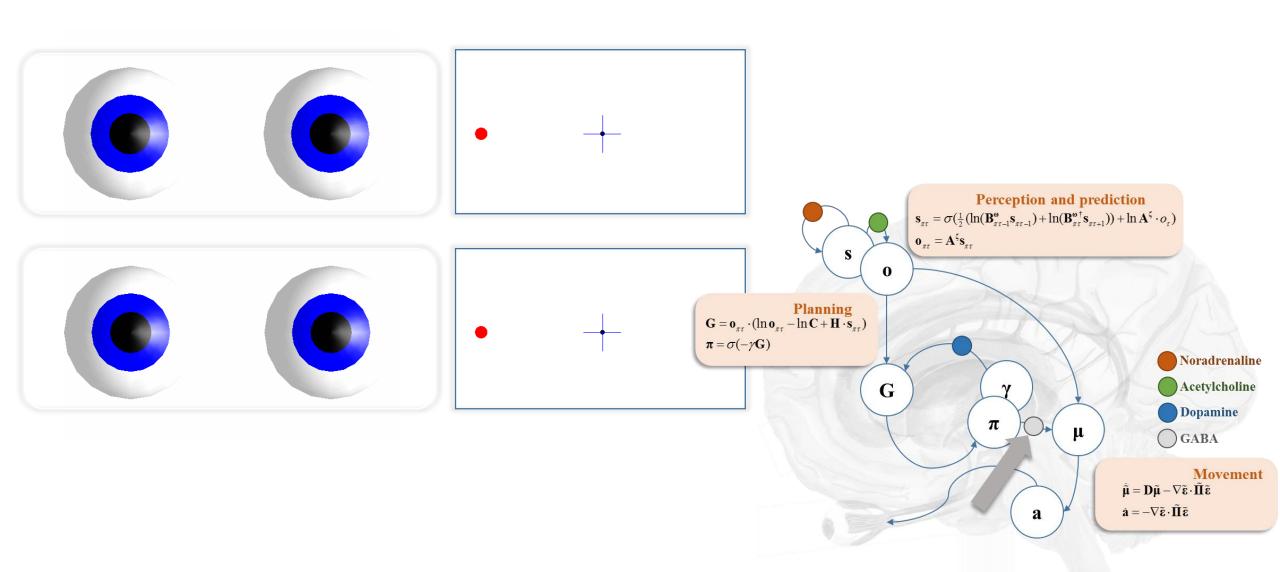




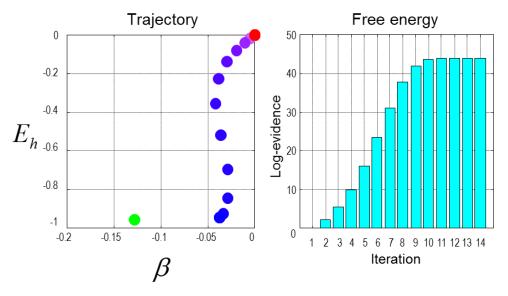


Parr and Friston 2018, Friston and Parr 2019



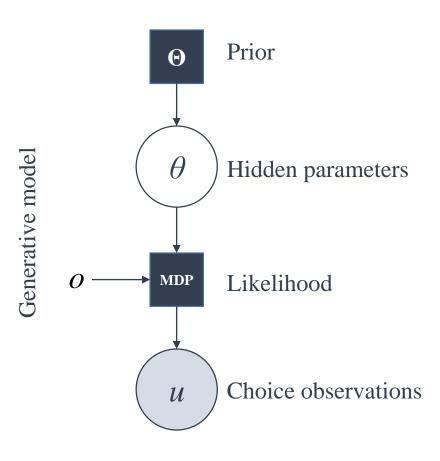


Parr and Friston 2019



Model fitting

 $P(\theta \mid o, u) \propto P(u \mid \theta, o) P(\theta)$





>> DEM

Generalised filtering, Active inference and Free

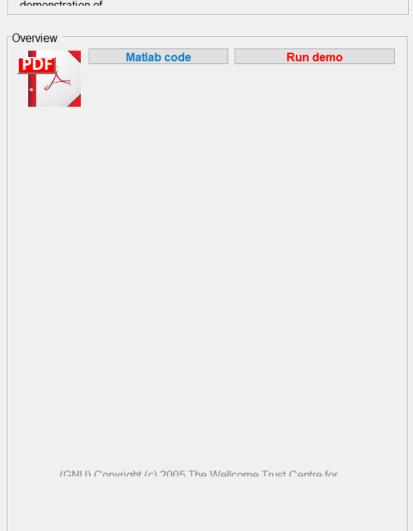
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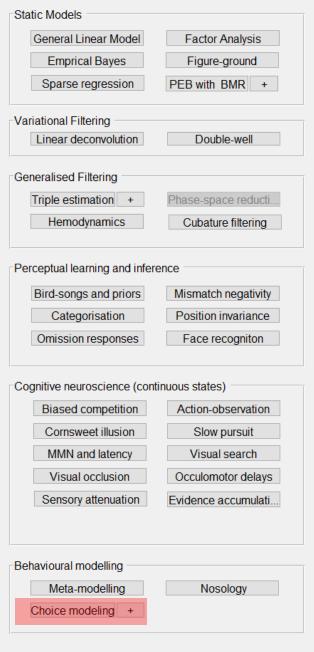
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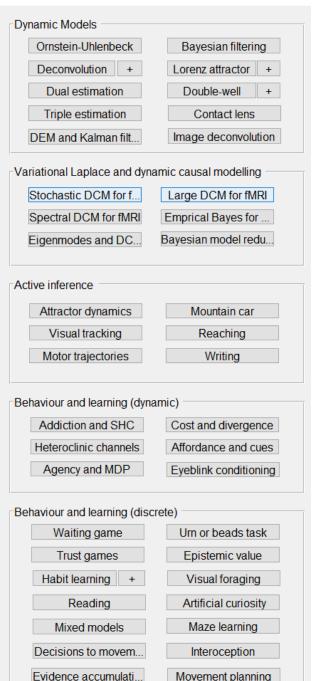
approximate the conditional, variational or ensemble density of unknown states and

parameters. This is an approximation to the density that would obtain from Variational

Filtering (VF) in generalized coordinates of motion. Start with the







Model

priors
$$p(\ln c) = \mathcal{N}(\eta, \Sigma)$$

generative model spm_MDP_gen(ln c)

likelihood

$$\mathcal{L}(\theta, u, o) = \ln P(u / \ln c, o)$$
$$\boldsymbol{\pi} = \sigma(\ln \mathbf{E} - \mathbf{F} - \gamma \cdot \mathbf{G})$$
$$P(u_{\tau} | \ln c, \tilde{o}) = \sigma(\alpha \cdot \boldsymbol{\pi})$$

$$\mathbf{C}^2 = \begin{bmatrix} 0 & 0 & 0 \\ c & c & c \\ -c & -c & -c \end{bmatrix}$$

Observed behaviour

$$U = \tilde{o}$$

$$Y = \tilde{u}$$

$$M.pE = \eta$$

$$M.pC = \Sigma$$

$$M.G = @spm_MDP_gen$$

$$M.L = @spm_MDP_L$$

 $[Ep,Cp,F] = spm_nlsi_Newton(M,U,Y);$

Variational Laplace

$$q(\ln c) = \mathcal{N}(\mu, \Pi^{-1})$$

$$F \approx \ln P(\tilde{u} \mid \tilde{o})$$



Novel Tools and Methods

Computational Phenotyping in Psychiatry: A Worked Example

Philipp Schwartenbeck, 1,2,3,4 and Karl Friston 1

DOI:http://dx.doi.org/10.1523/ENEURO.0049-16.2016

¹The Wellcome Trust Centre for Neuroimaging, UCL, London WC1N 3BG, UK, ²Centre for Cognitive Neuroscience, University of Salzburg, 5020 Salzburg, Austria, ³Neuroscience Institute, Christian-Doppler-Klinik, Paracelsus Medical University Salzburg, A-5020 Salzburg, Austria, and ⁴Max Planck UCL Centre for Computational Psychiatry and Ageing Research, London WC1B 5EH, UK



RESEARCH ARTICLE

Human visual exploration reduces uncertainty about the sensed world

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1 Wellcome Trust Centre for Neuroimaging, Institute of Neurology, University College London, London, United Kingdom, 2 Institute of Cognitive Neuroscience, University College London, London, United Kingdom, 3 Division of Psychiatry, University College London, London, United Kingdom, 4 Scuola Internazionale Superiore di Studi Avanzati (SISSA), Trieste, Italy, 5 Translational Neuromodeling Unit (TNU), Institute for Biomedical Engineering, University of Zurich and ETH Zurich, Zurich, Switzerland, 6 Max Planck UCL Centre for Computational Psychiatry and Ageing Research, London, United Kingdom



a

b

C

d

 \mathbf{e}

Inference

A

B

spm_MDP_VB_X.m

C

D

 \mathbf{E}

Deep temporal models

Model fitting

 $\mathcal{L}(\theta, u, o) = \ln P(u/\theta, o)$

Precision

γ

Mixed models $\partial_{\mu} \mathbf{\epsilon} \cdot \mathbf{\Pi} \mathbf{\epsilon}$

Thanks

Karl Friston Geraint Rees

Berk Mirza **David Benrimoh** Dimitrije Markovic **Ensor Palacios** Hayriye Cagnan Jakub Limanowski Jakob Hohwy Jelle Bruineberg Michael Kirchoff Peter Vincent Philipp Schwartenbeck **Rick Adams** Stefan Kiebel Tim Sandhu Takuya Isomura





