The drift-diffusion model of decision making

Ariel Zylberberg
Columbia University

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Drift-diffusion model

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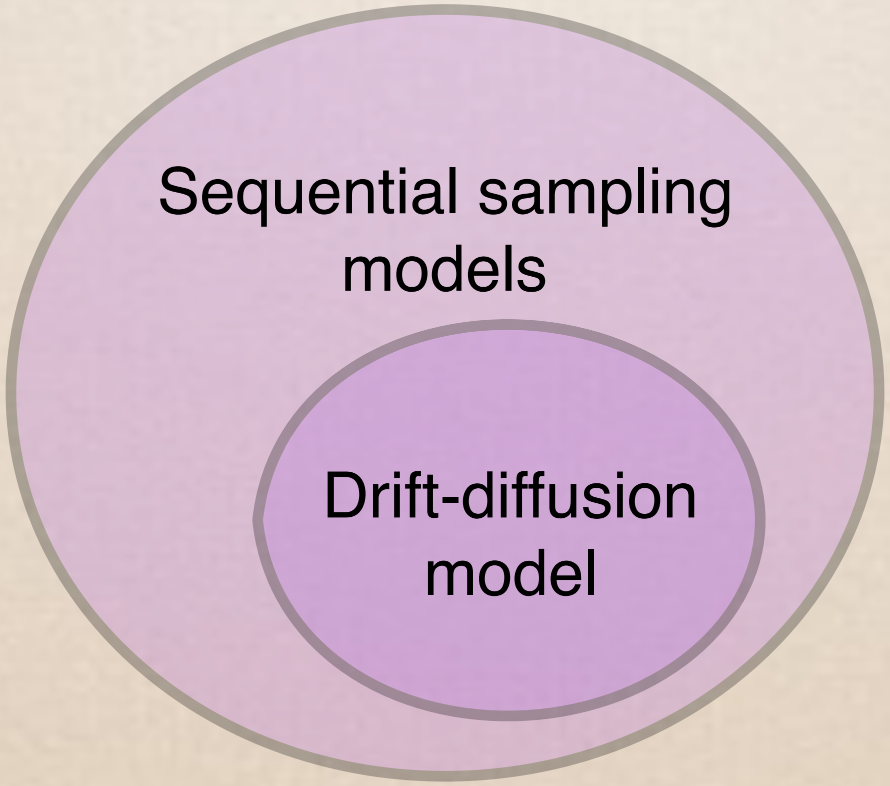

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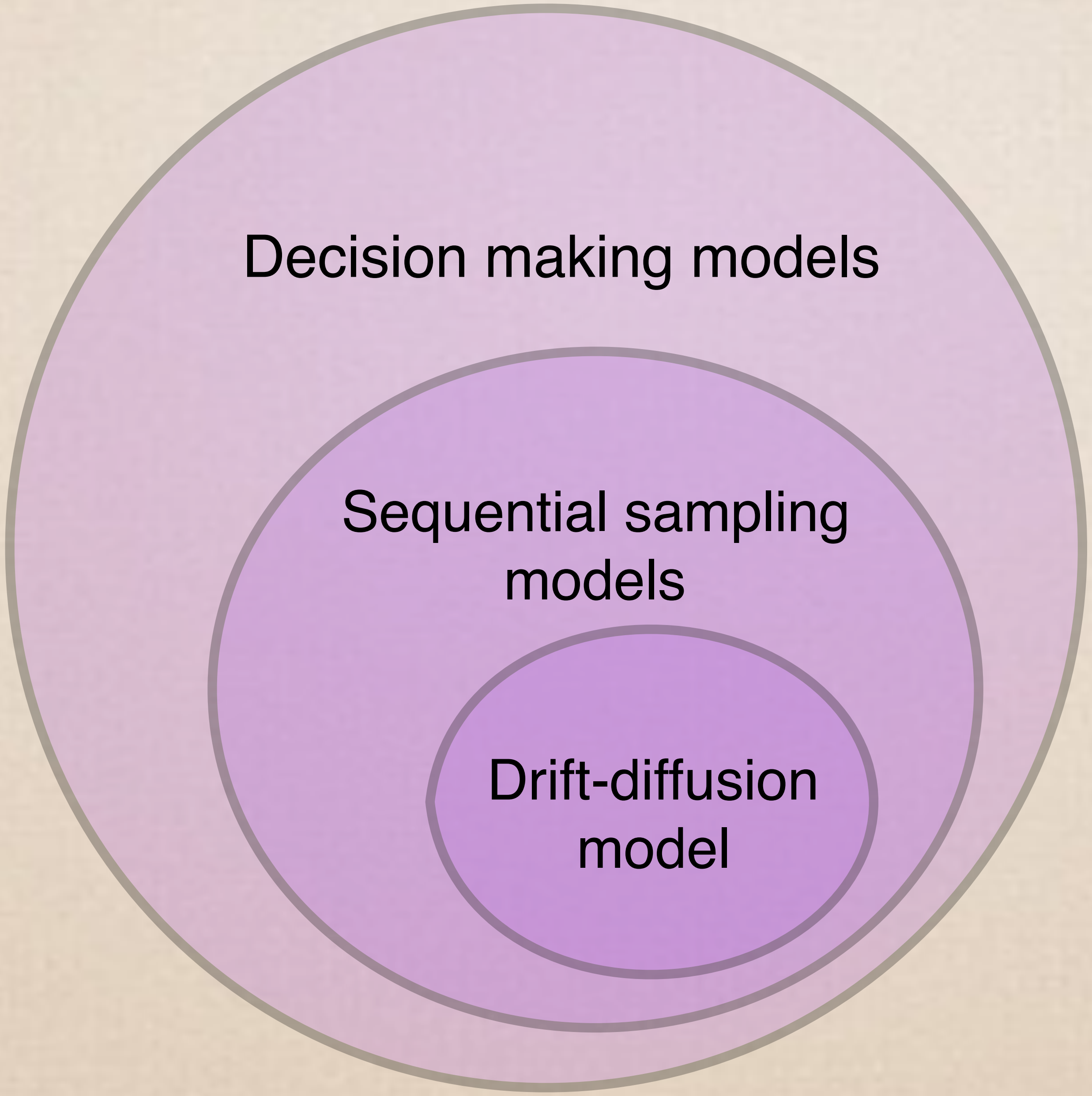

Speed, accuracy and difficulty

Speed, accuracy and difficulty

Speed, accuracy and difficulty

The garbage plate, Rochester, NY

Speed, accuracy and difficulty

The garbage plate, Rochester, NY

Speed, accuracy and difficulty

The garbage plate, Rochester, NY

Speed, accuracy and difficulty

The garbage plate, Rochester, NY

» Easy

Speed, accuracy and difficulty

The garbage plate, Rochester, NY

- Easy
- Fast

Speed, accuracy and difficulty

The garbage plate, Rochester, NY

- Easy
- Fast
- Consistent / accurate

Speed, accuracy and difficulty

The garbage plate, Rochester, NY

Speed, accuracy and difficulty

The garbage plate, Rochester, NY

Speed, accuracy and difficulty

The garbage plate, Rochester, NY

- » More difficult

Speed, accuracy and difficulty

The garbage plate, Rochester, NY

- More difficult
- Slower

Speed, accuracy and difficulty

The garbage plate, Rochester, NY

- » More difficult
- » Slower
- » Less consistent / accurate

Typical decision-making tasks

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Which snack do you prefer?

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Which snack do you prefer?

Word or non-word?

piggle

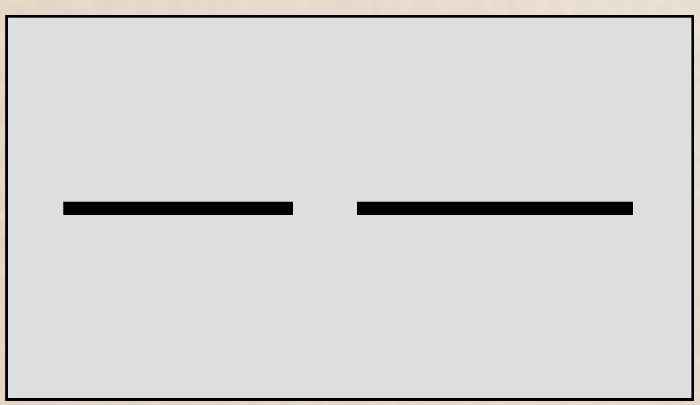
Typical decision-making tasks

Which snack do you prefer?

Word or non-word?

piggle

Which line is longer?



Typical decision-making tasks

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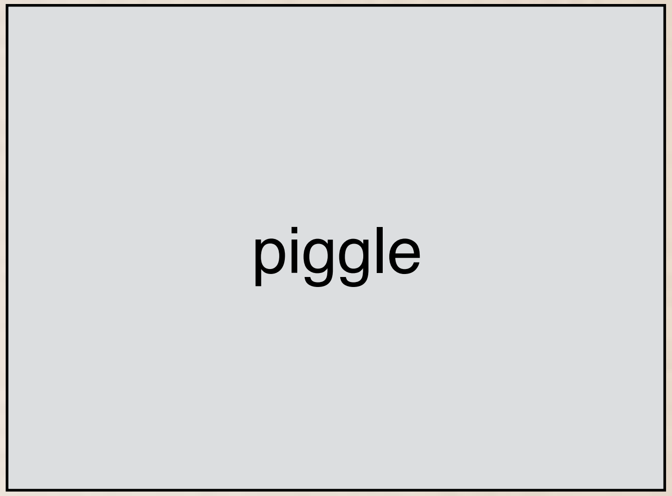
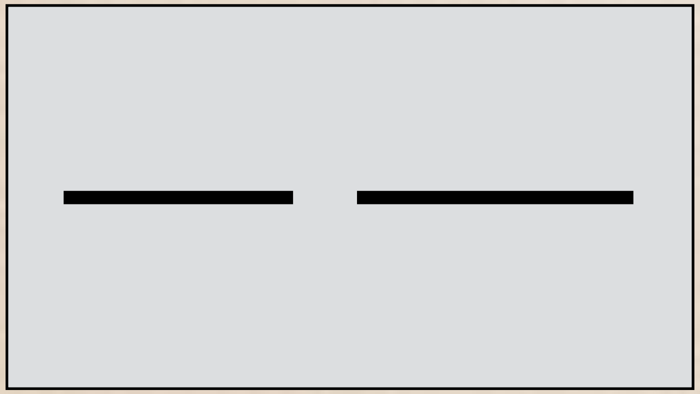
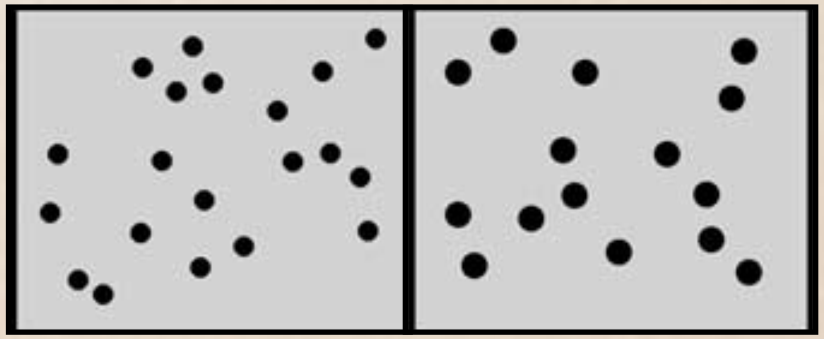


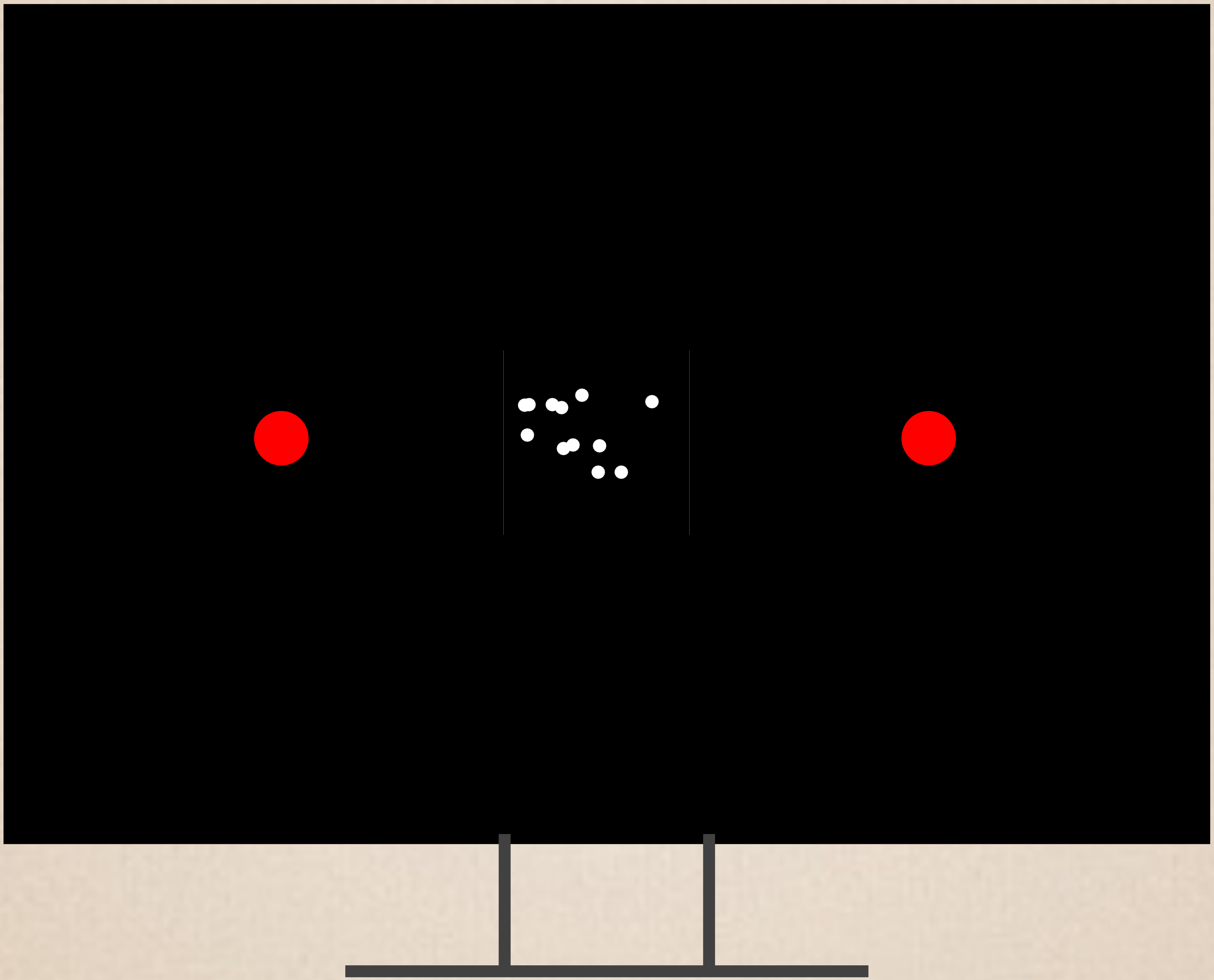
Which line is longer?



Which display has more dots?



Random dot motion discrimination task



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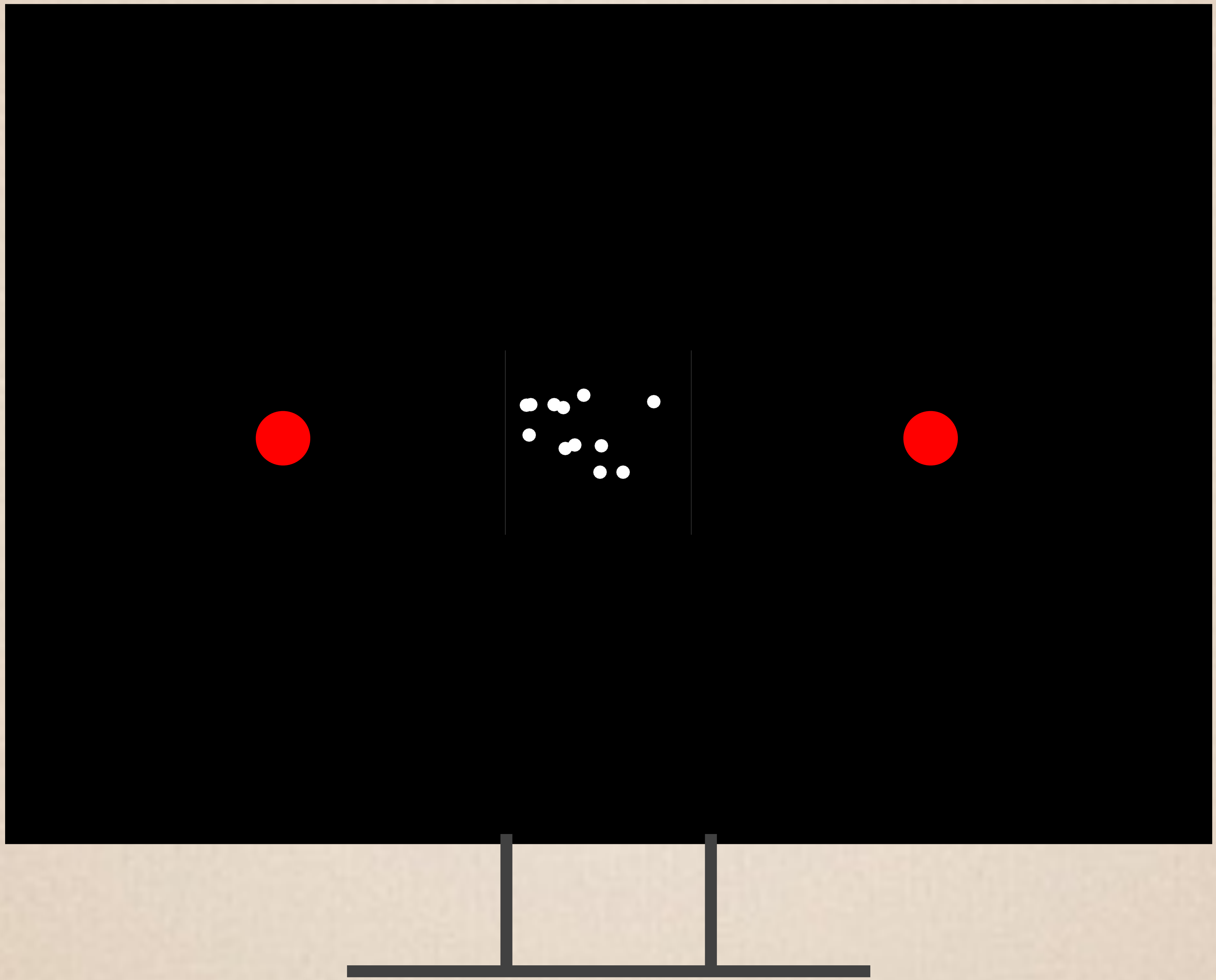

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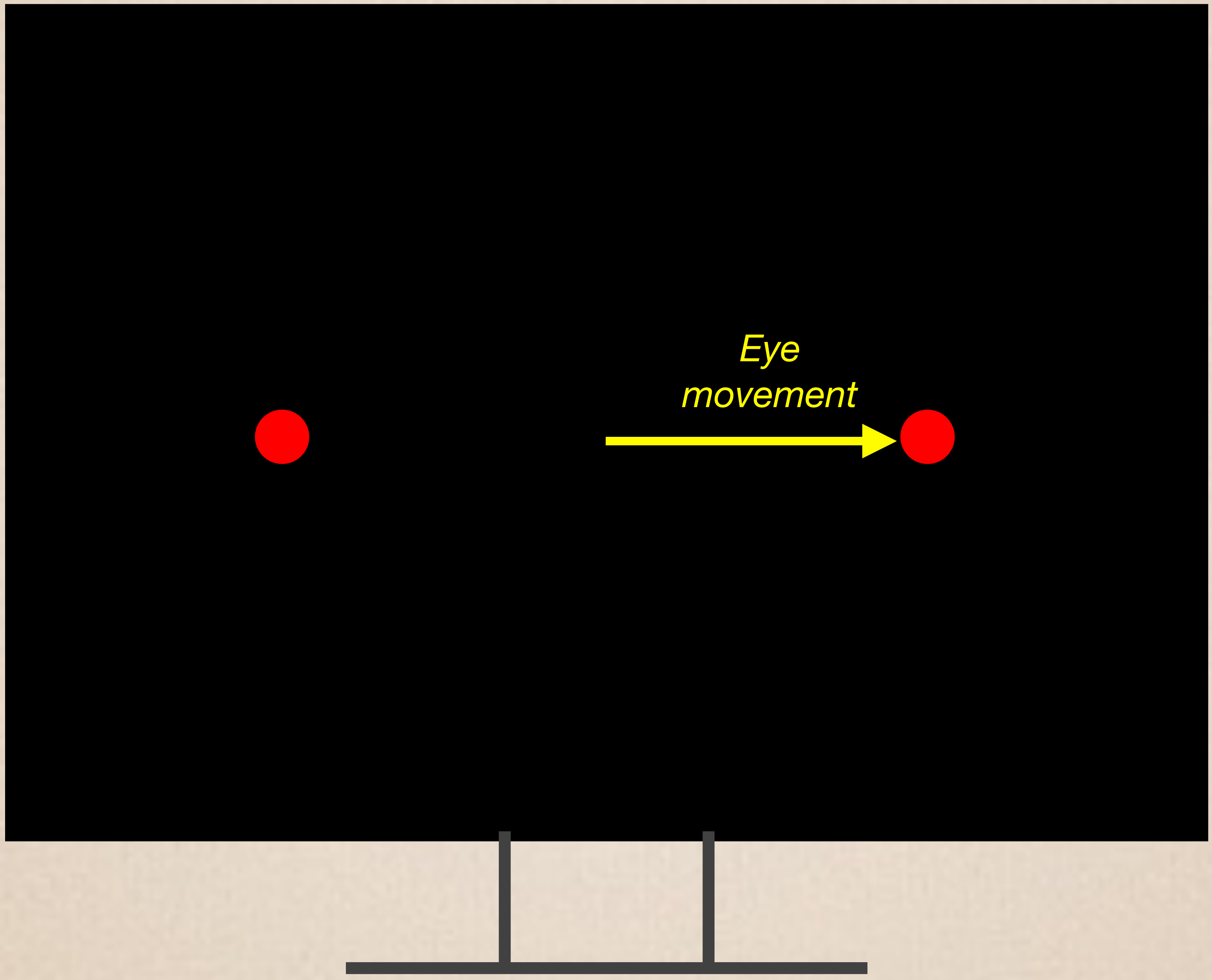
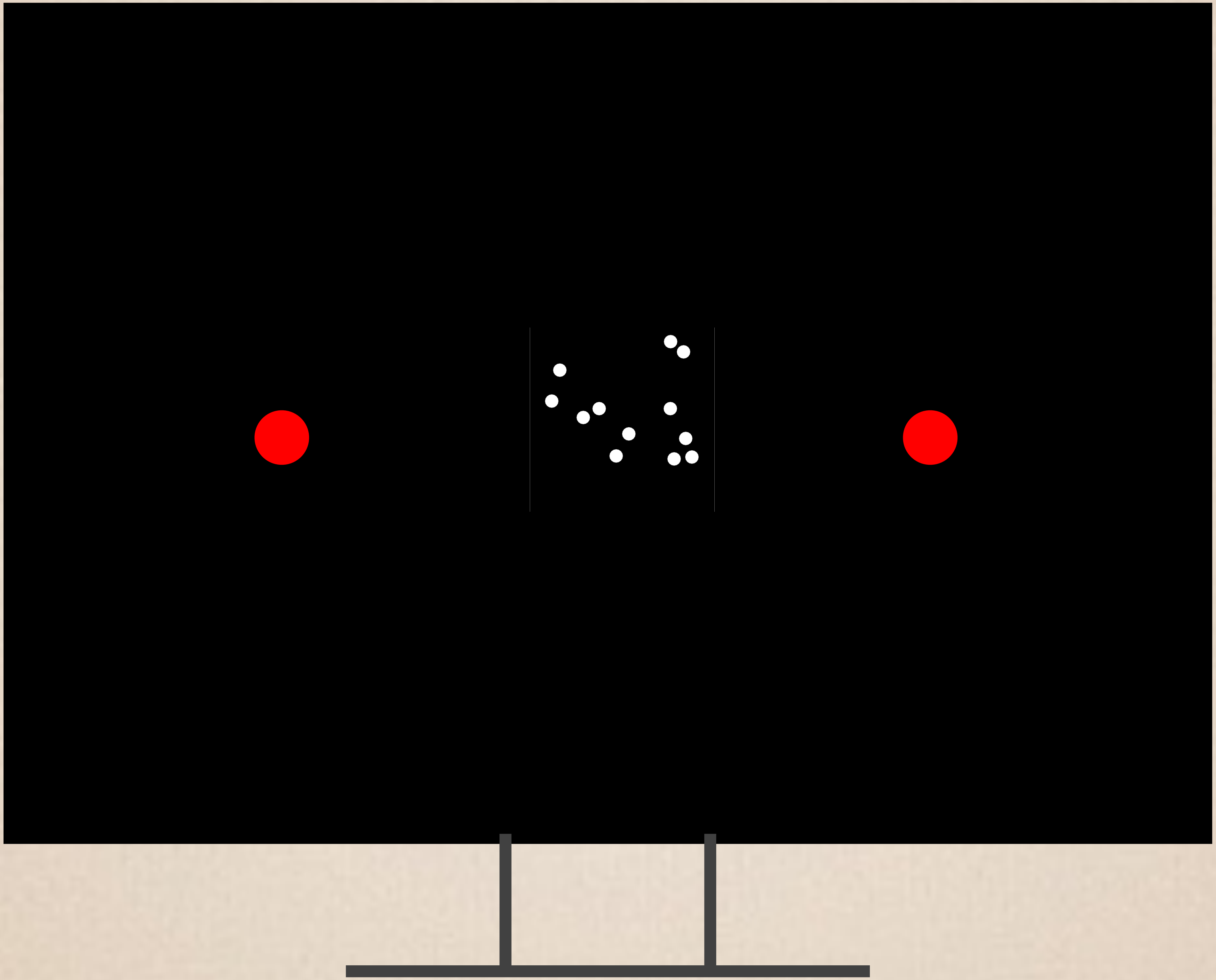


Random dot motion discrimination task

(More difficult)



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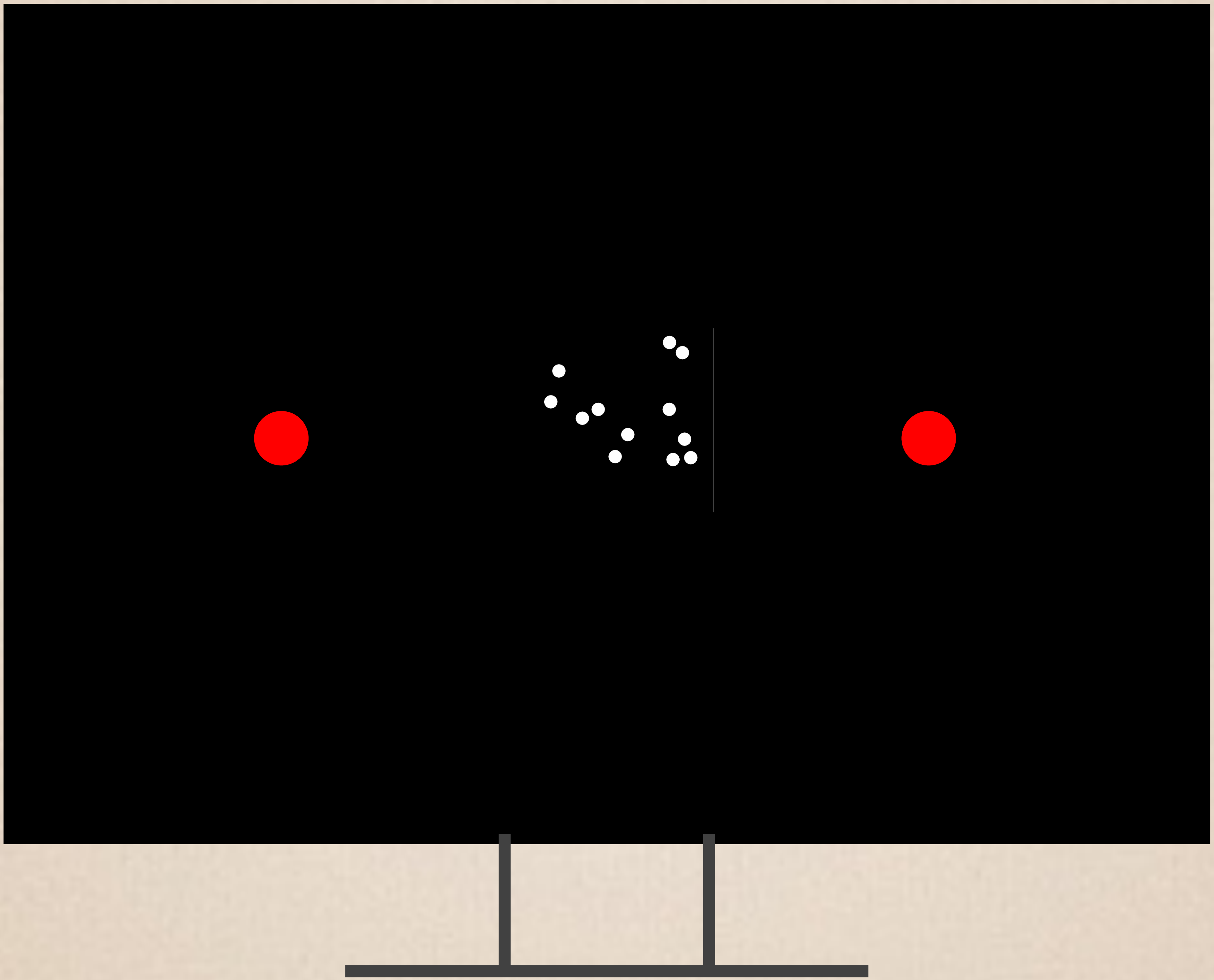

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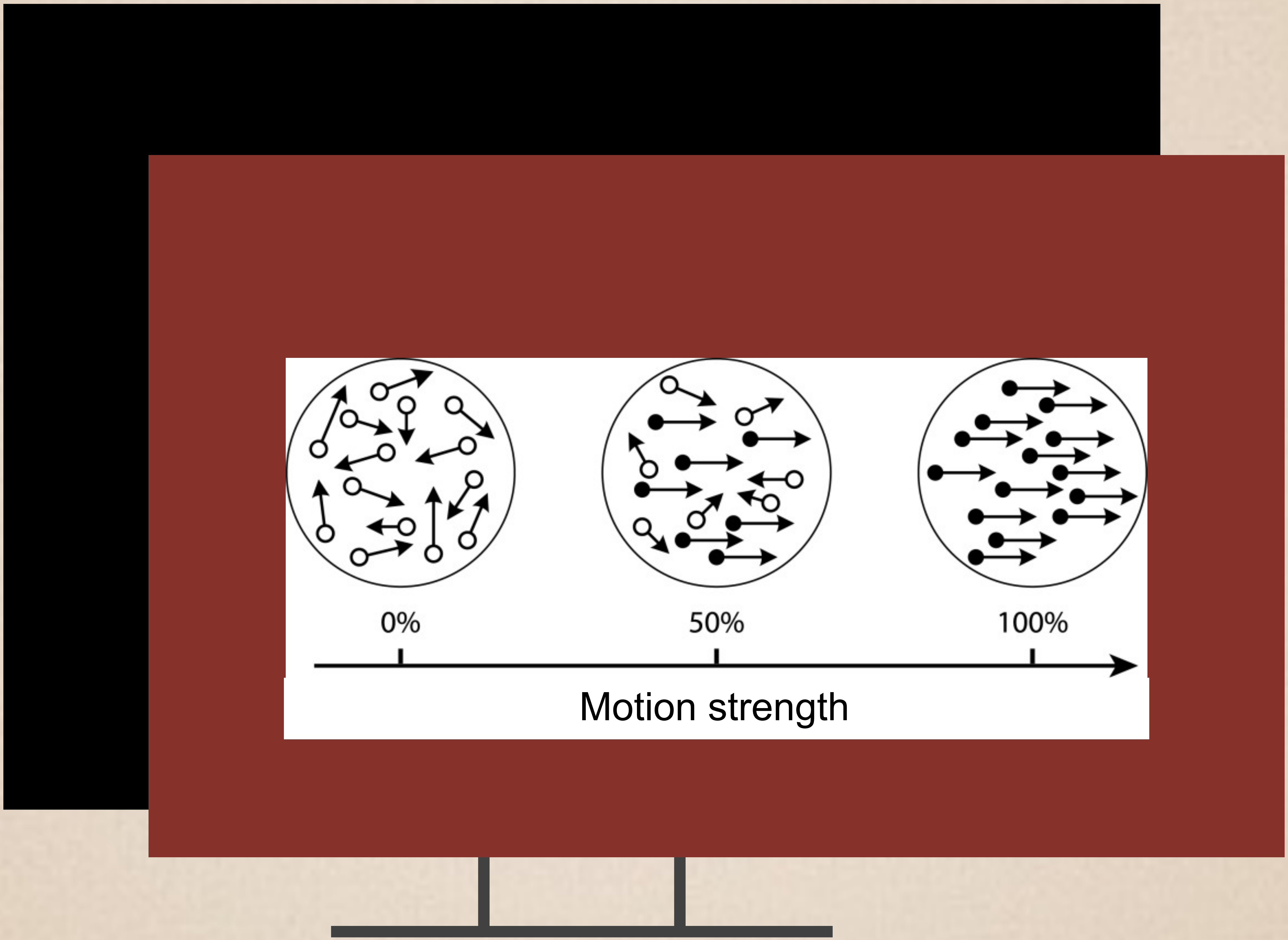

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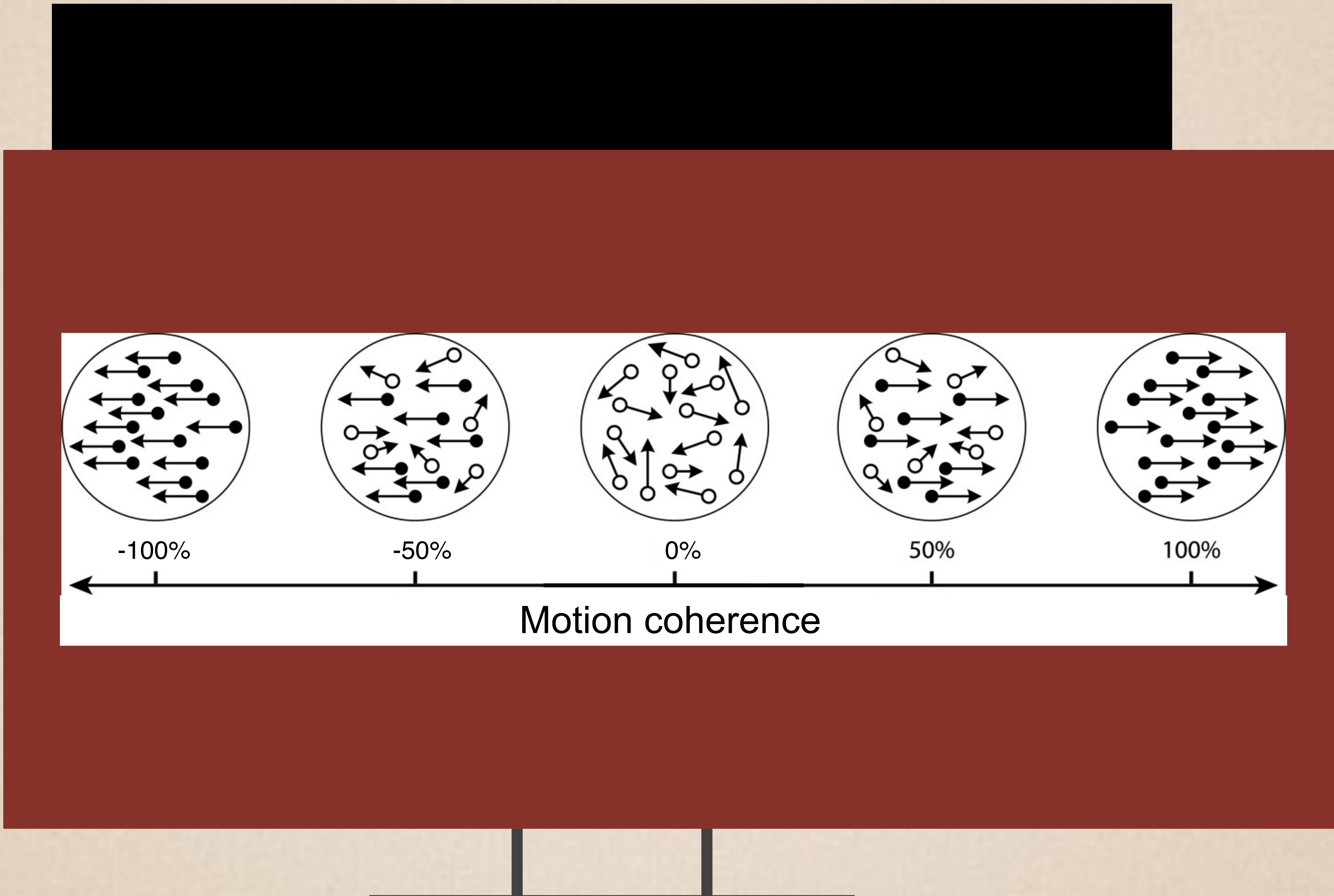
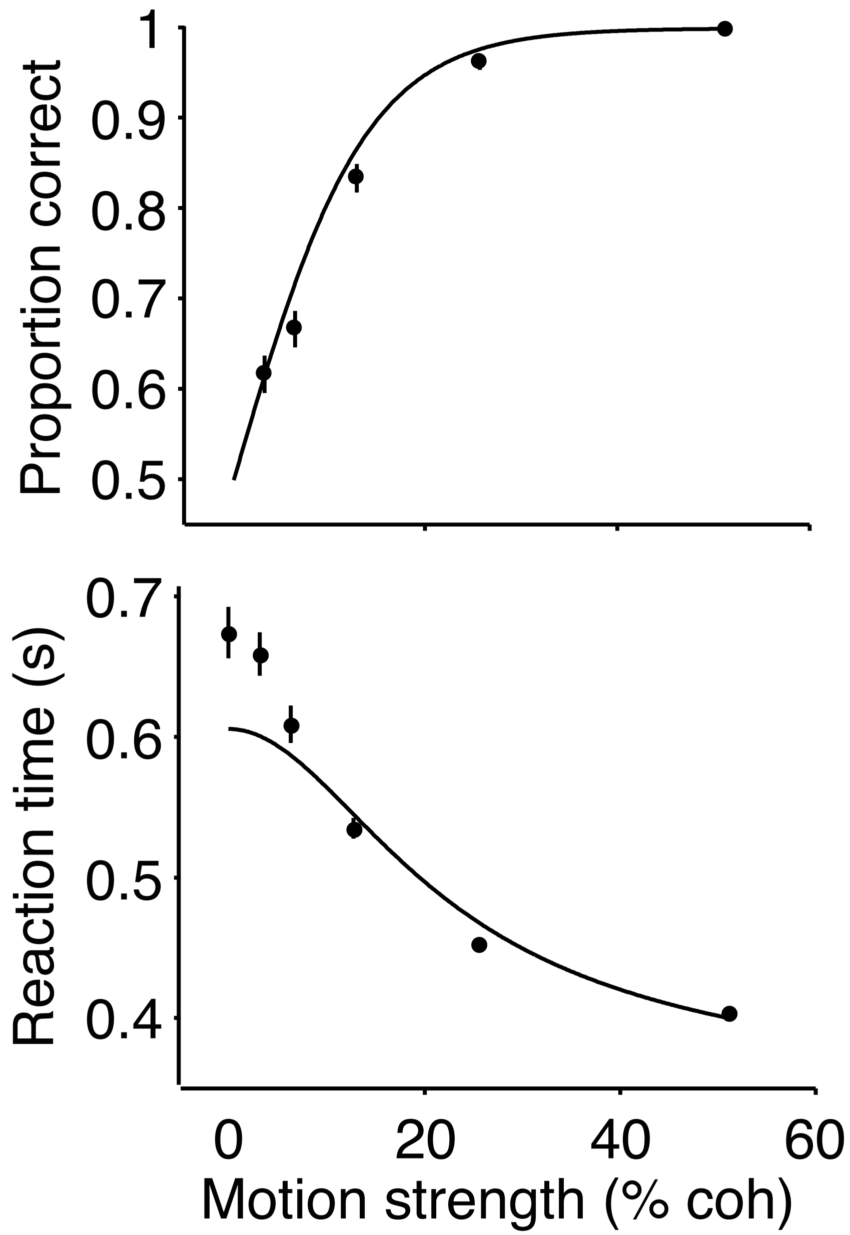


Speed-accuracy tradeoff



Hanks et al. eLife, 2014

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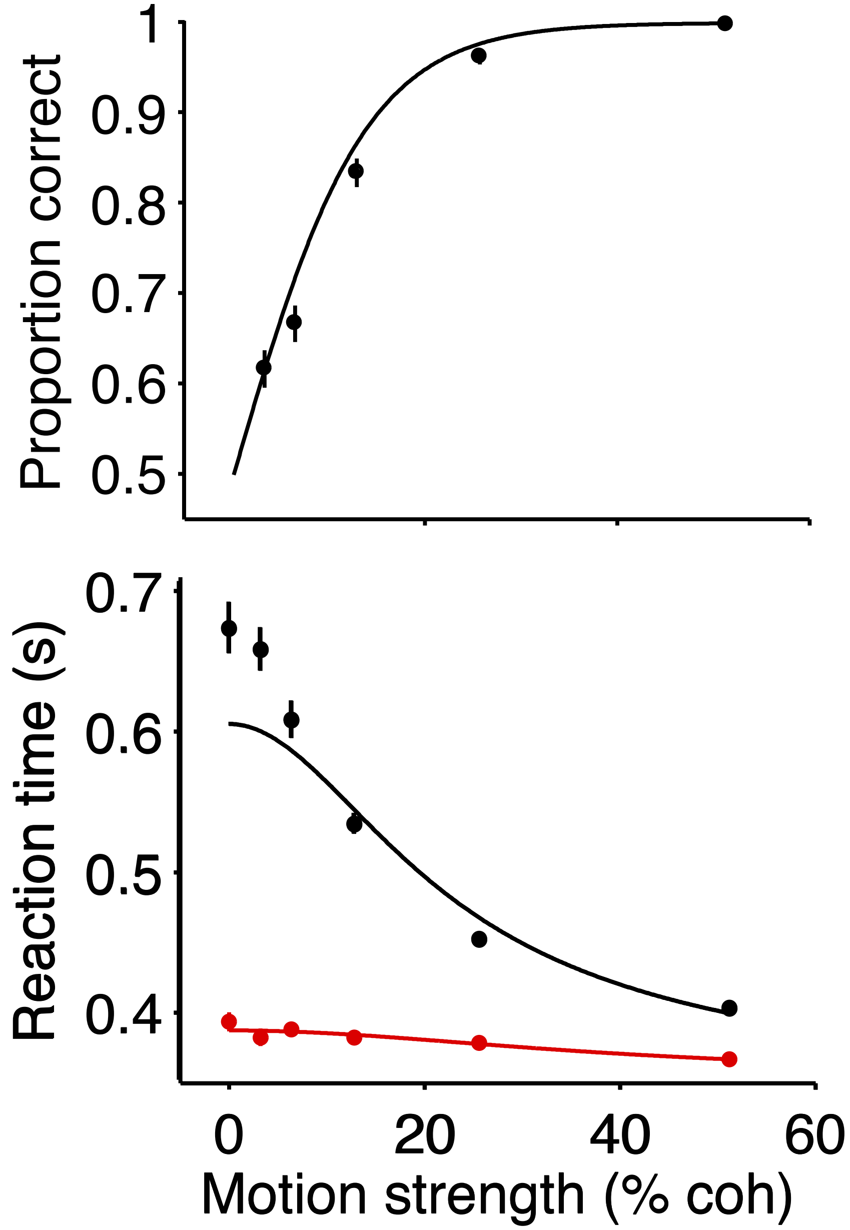

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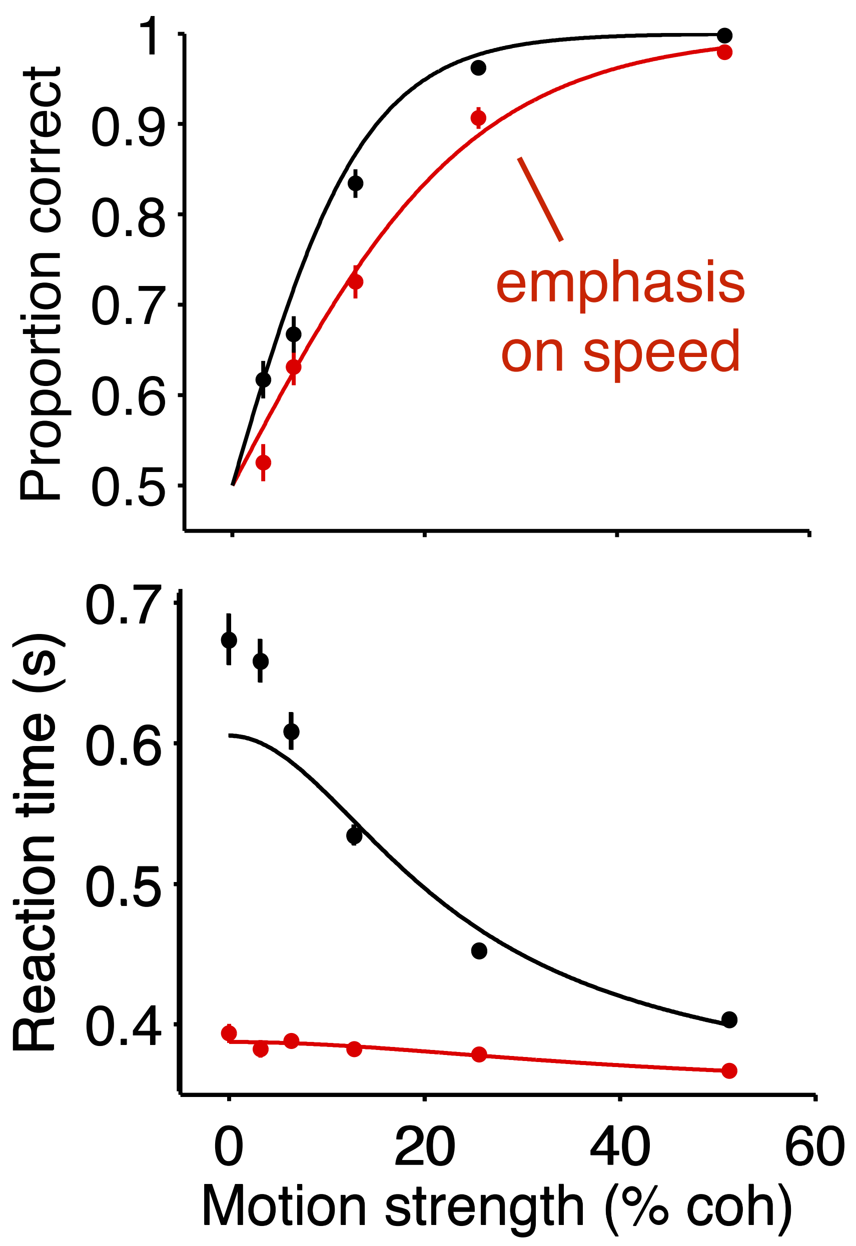

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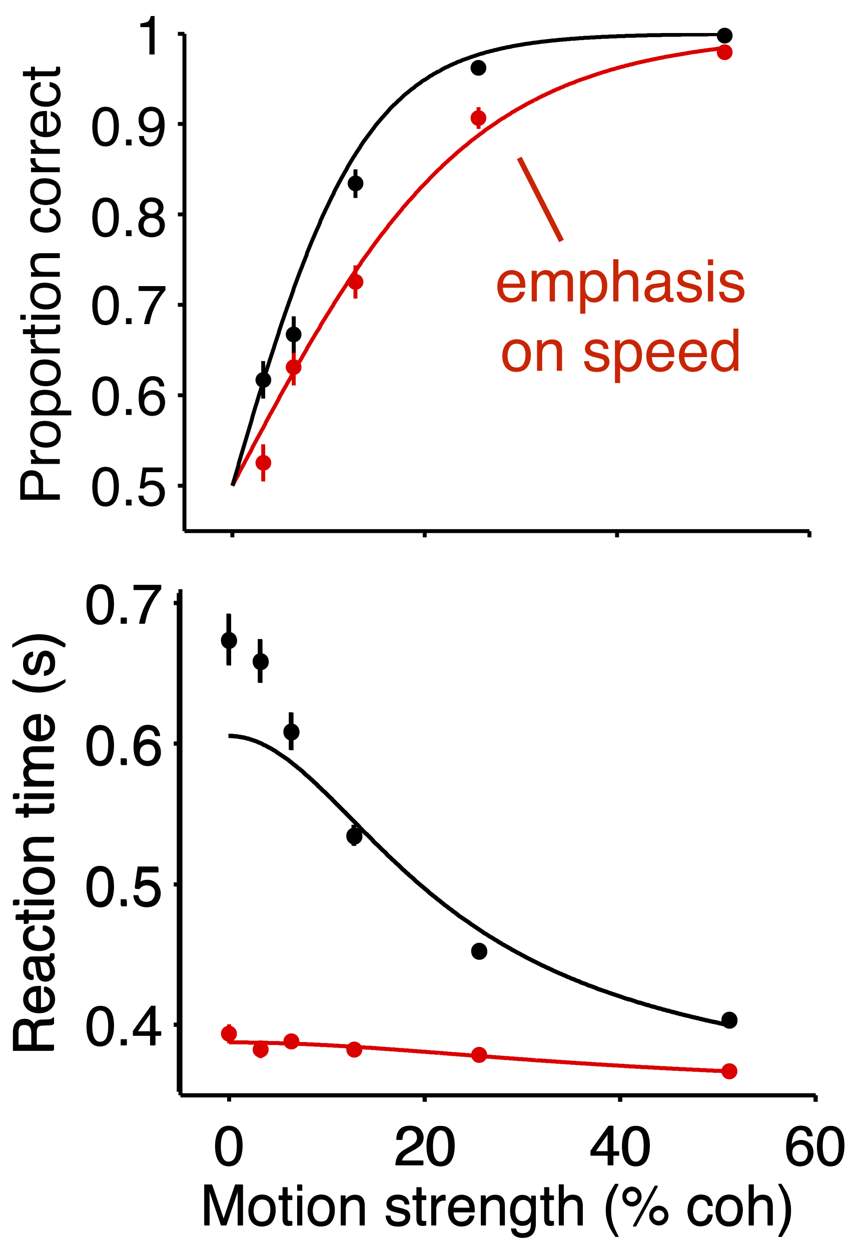

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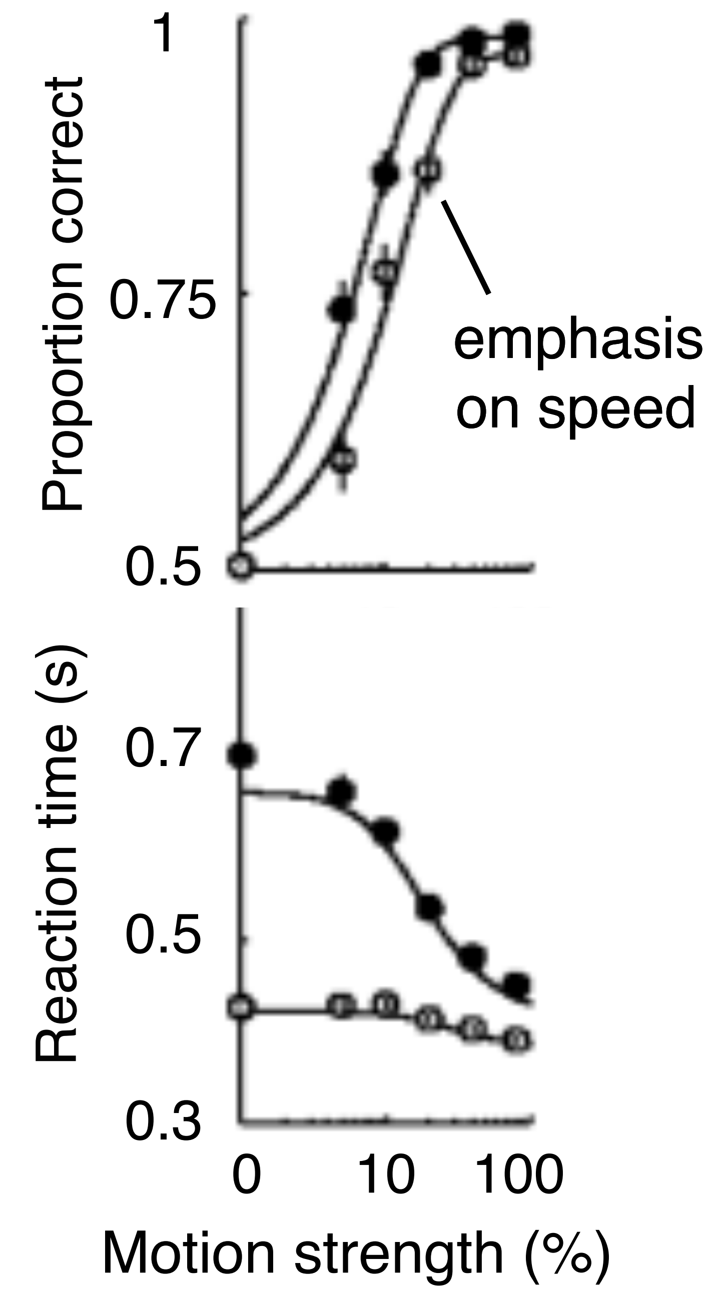
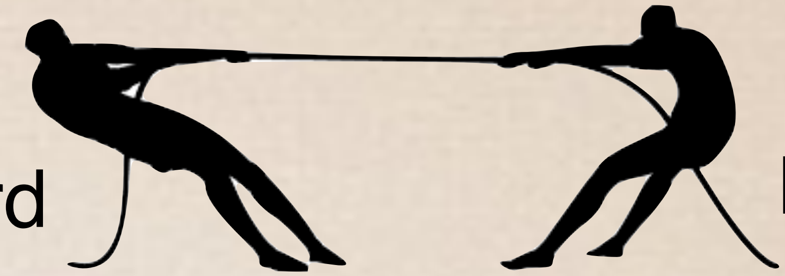


Mulder et al.
Att Percept Psych
2013

Opponent process theory of discrimination

Cartwright and Festinger (1943), “A quantitative theory of decision”.



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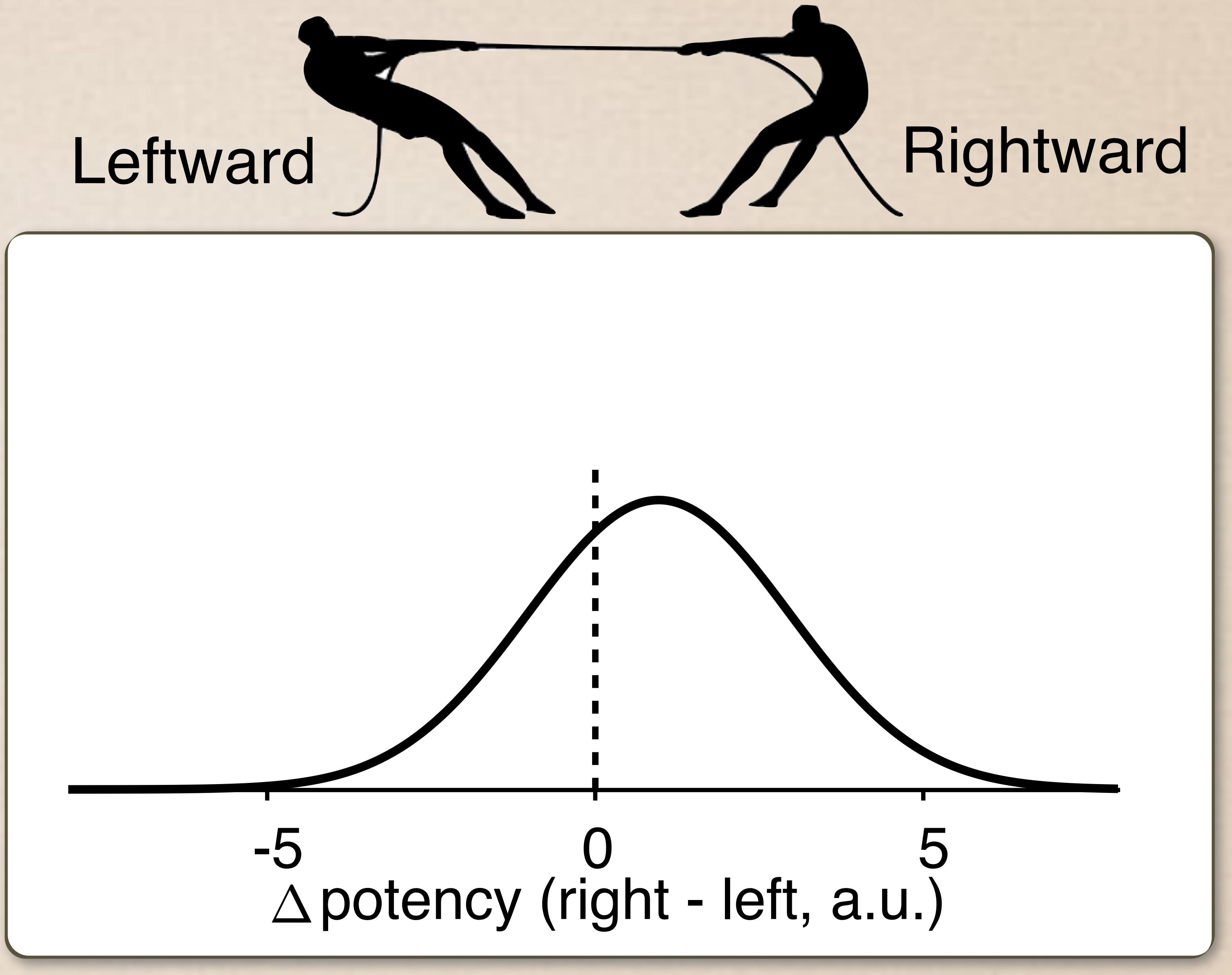

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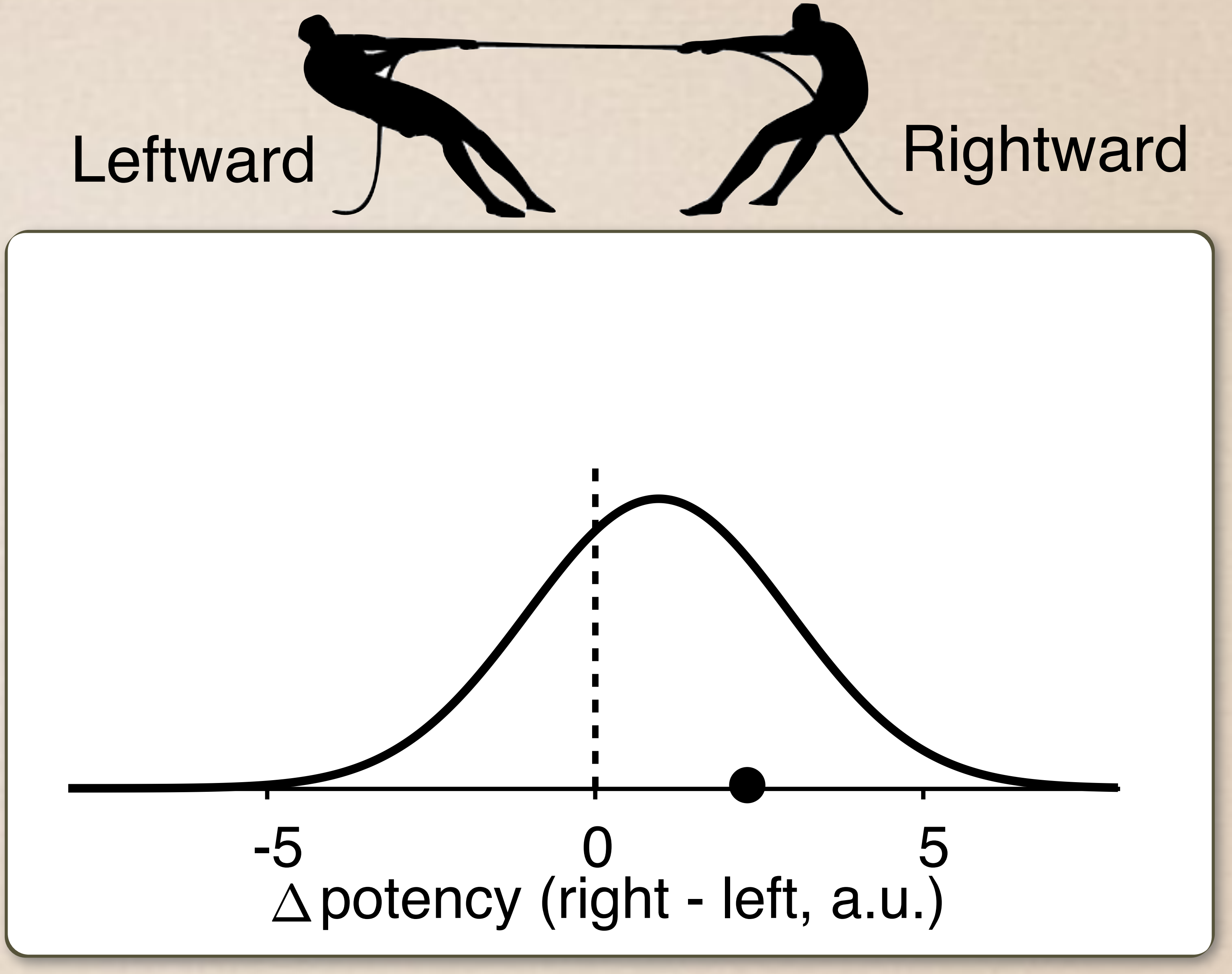

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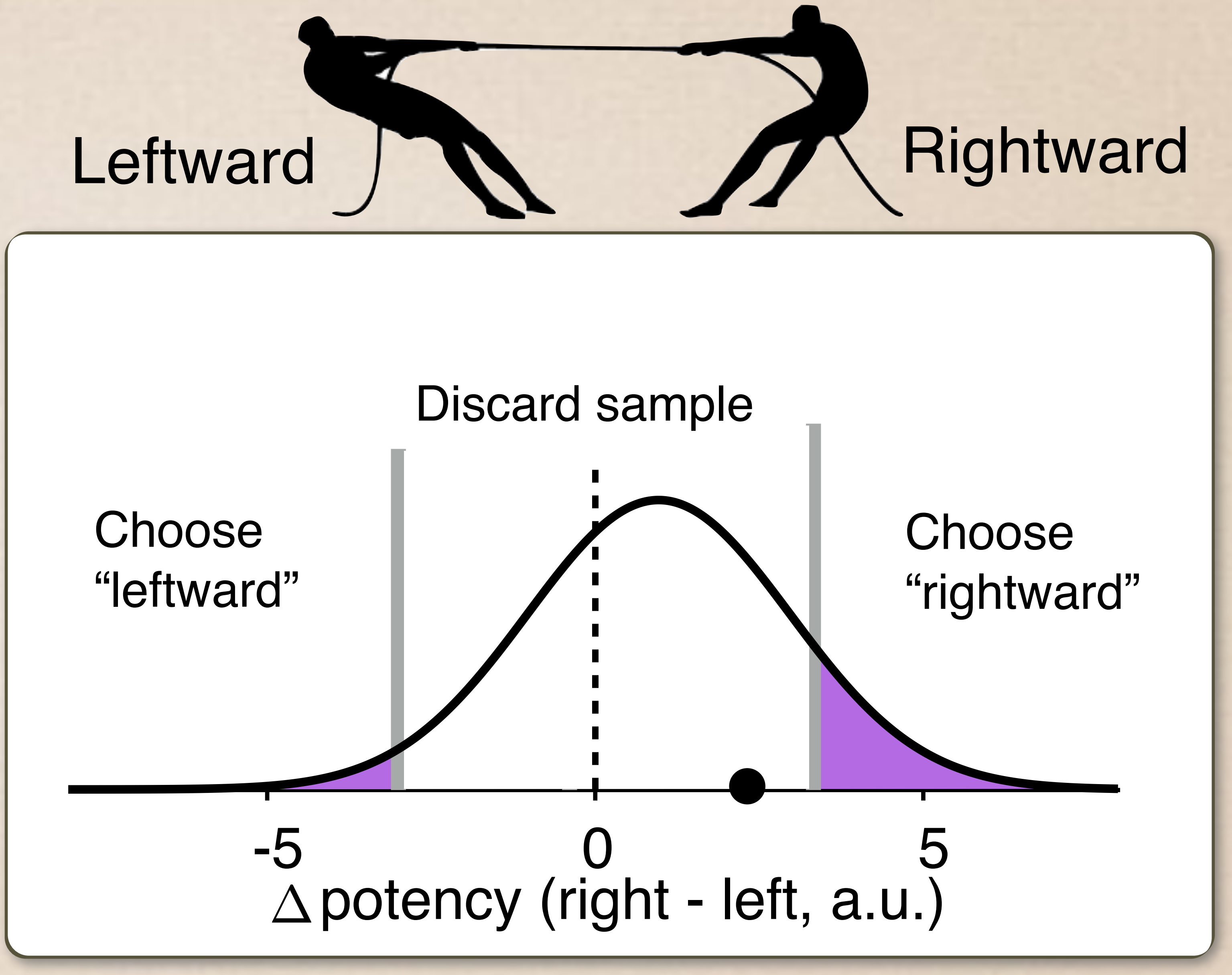

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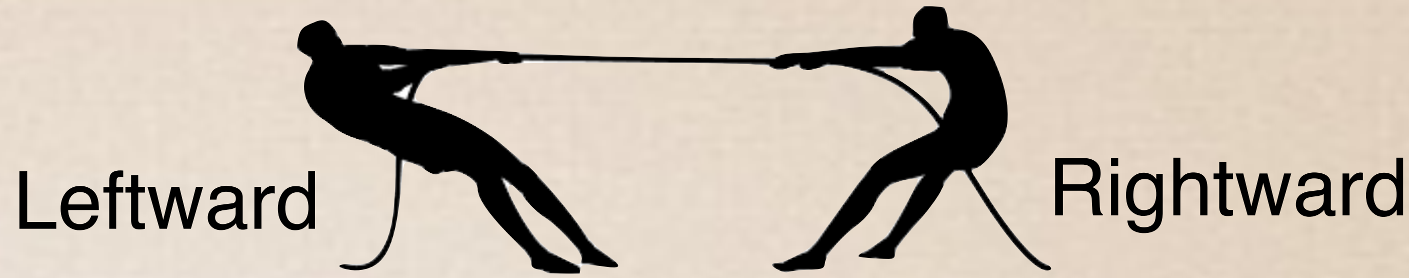

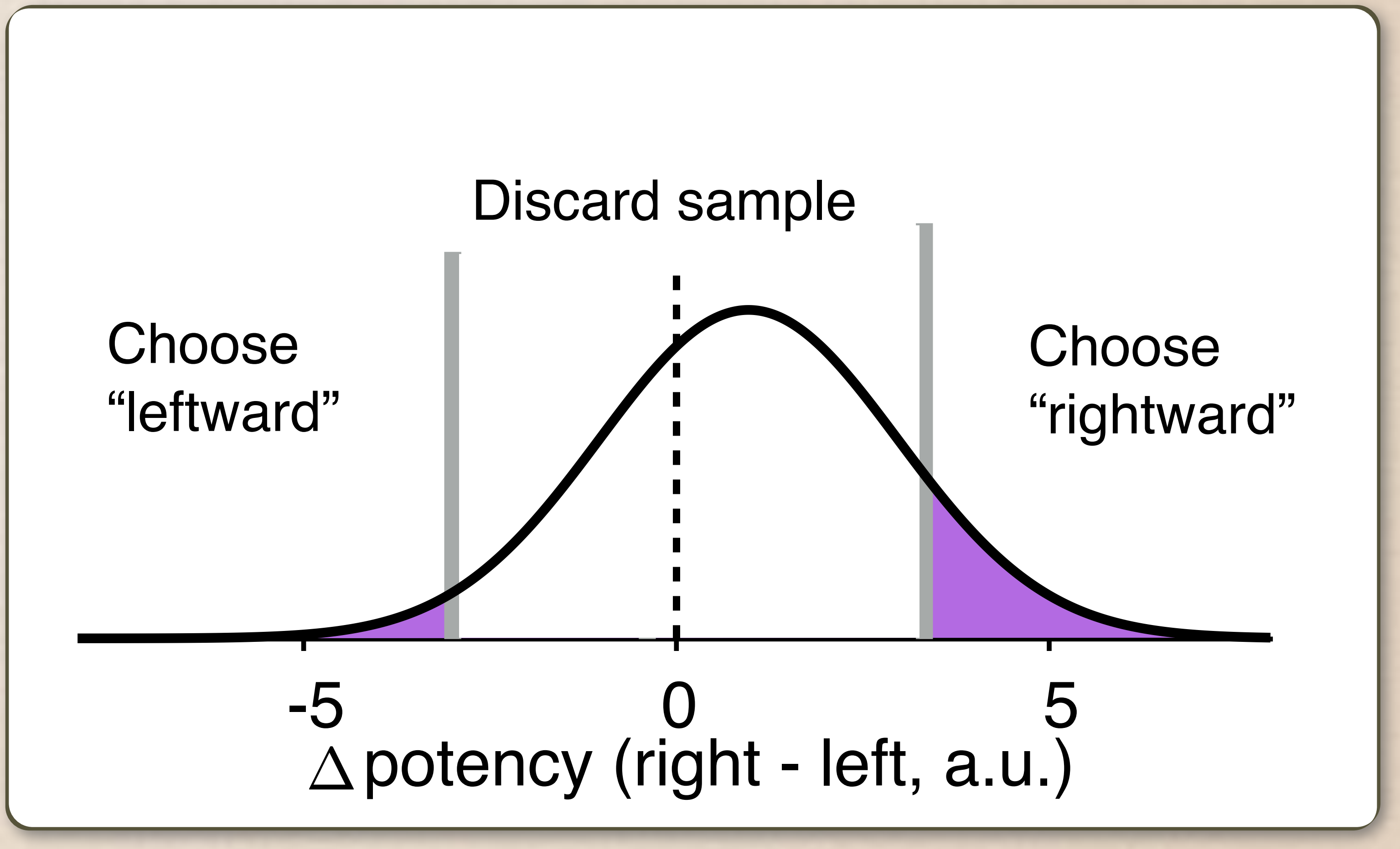

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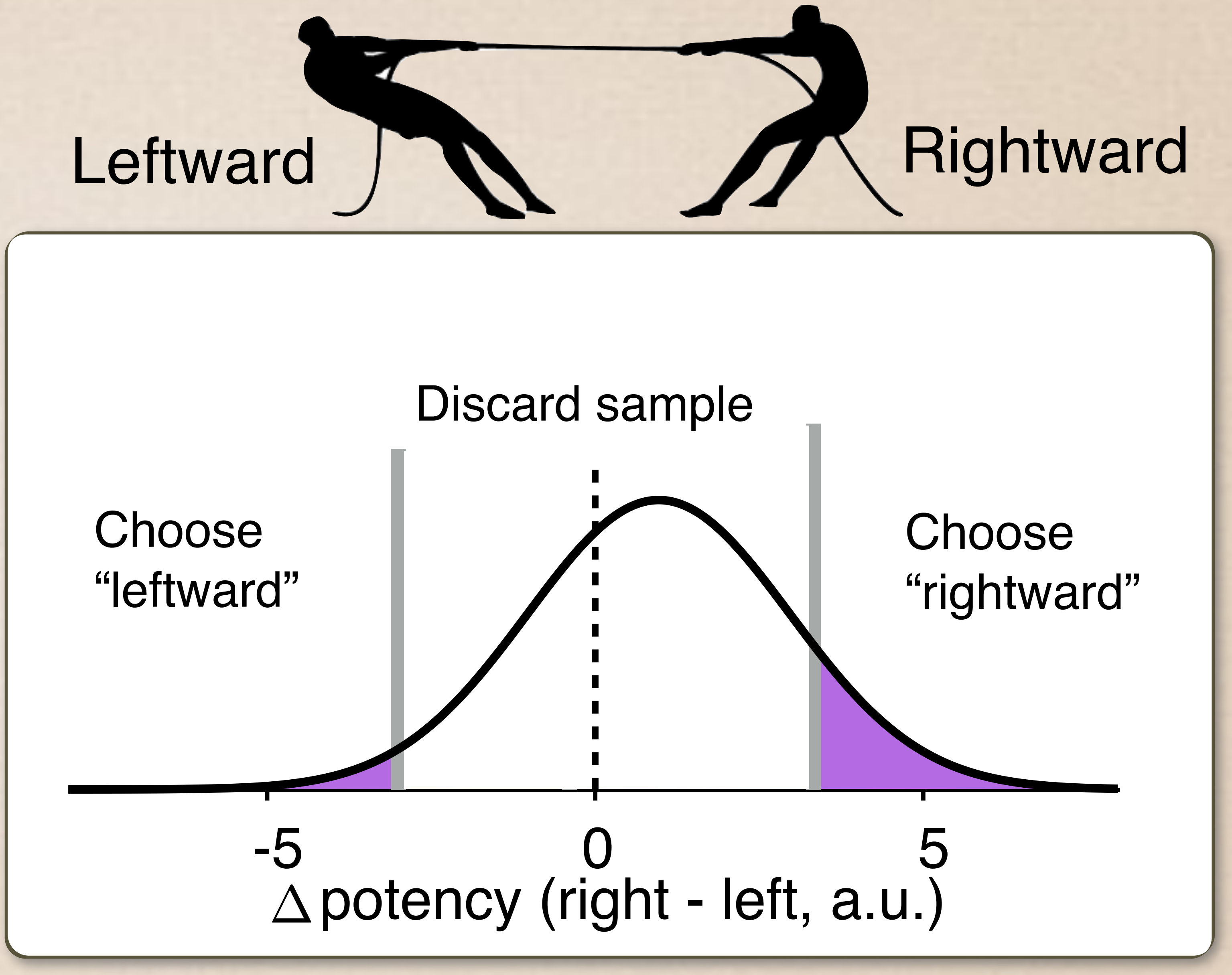

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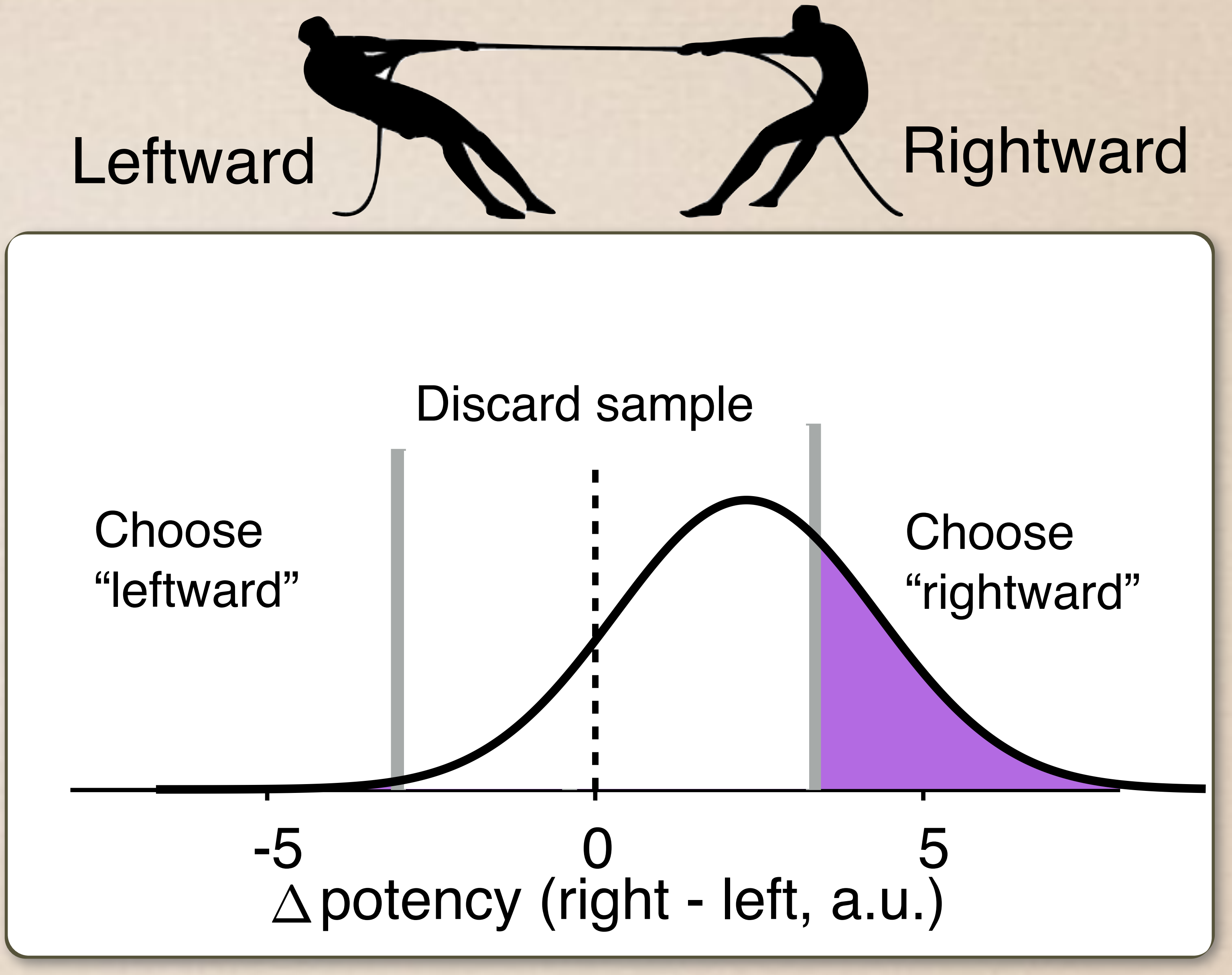

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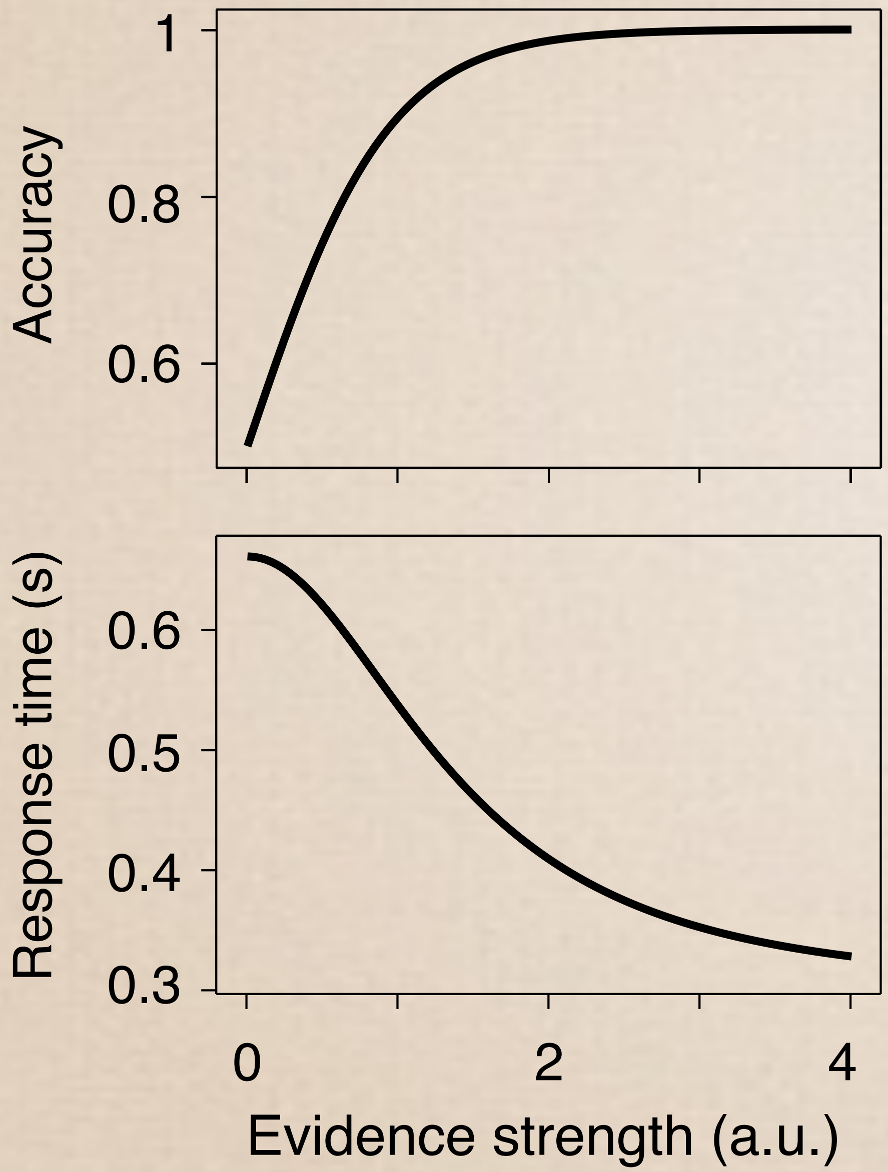

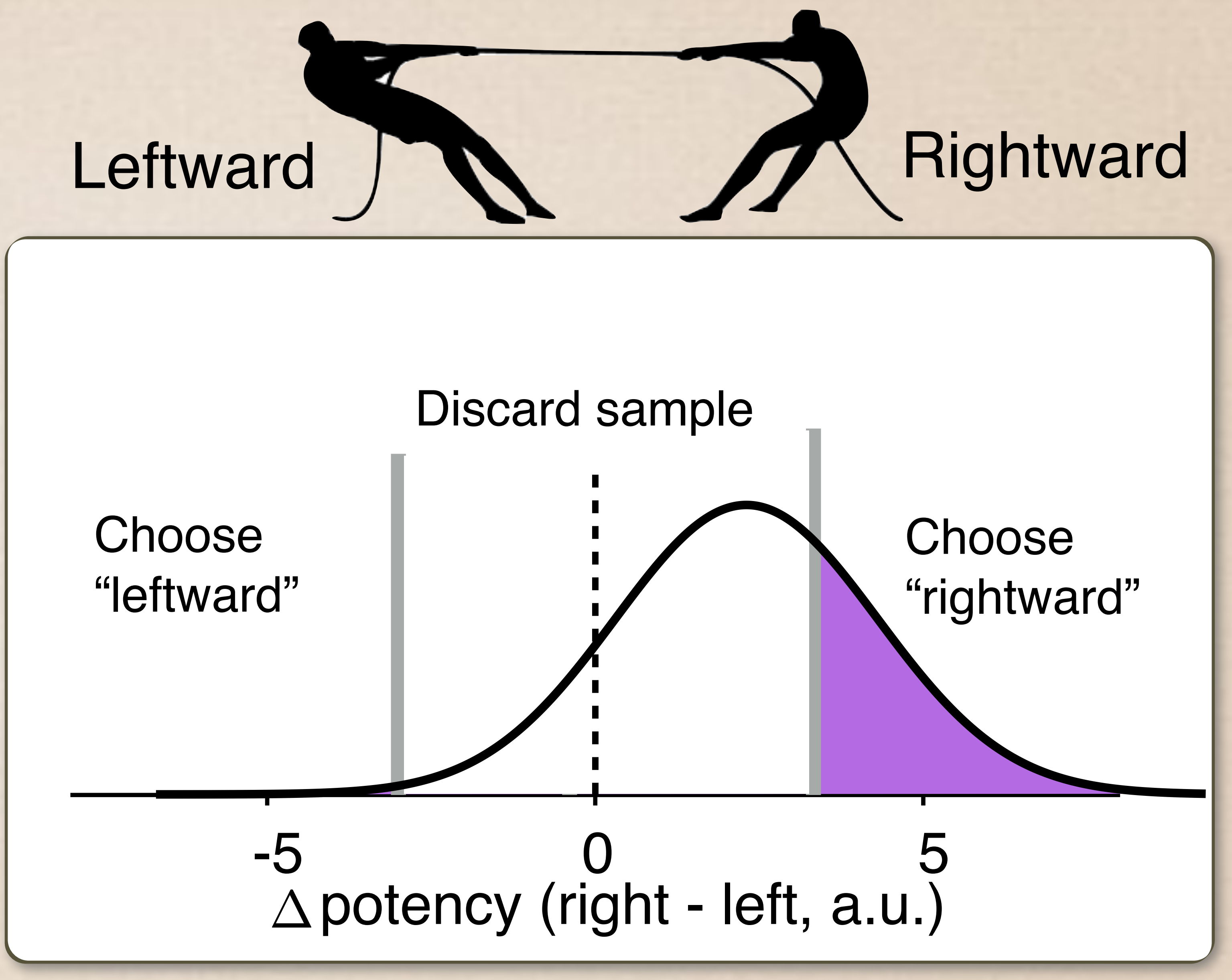

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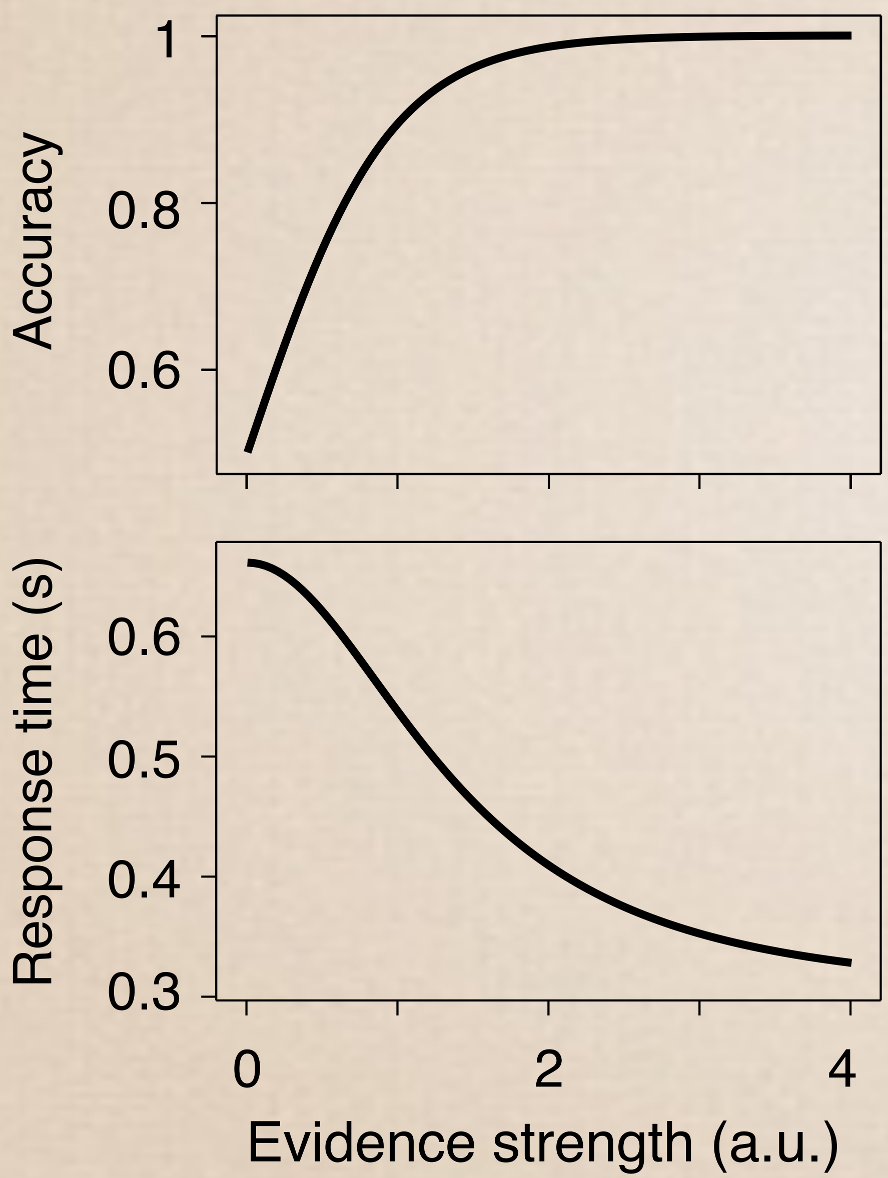

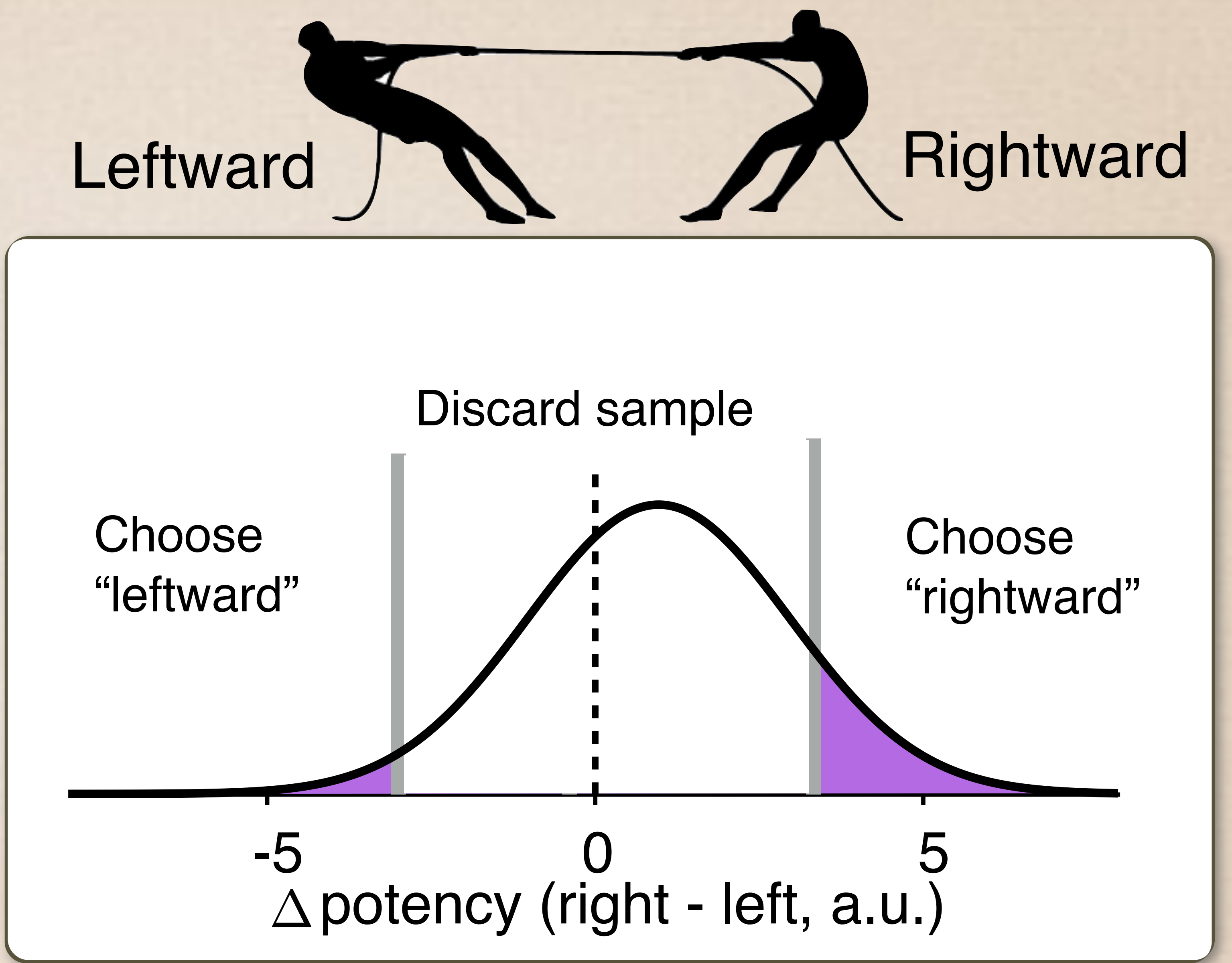

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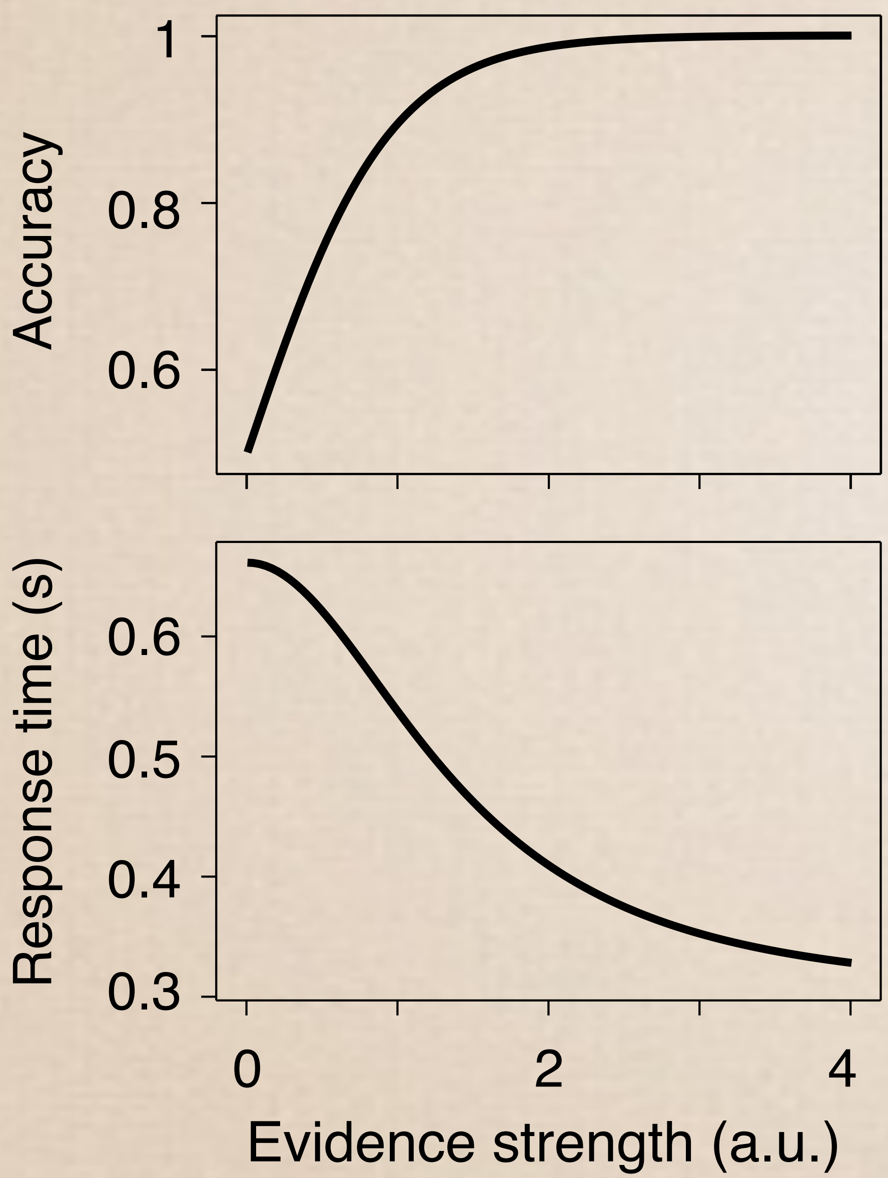

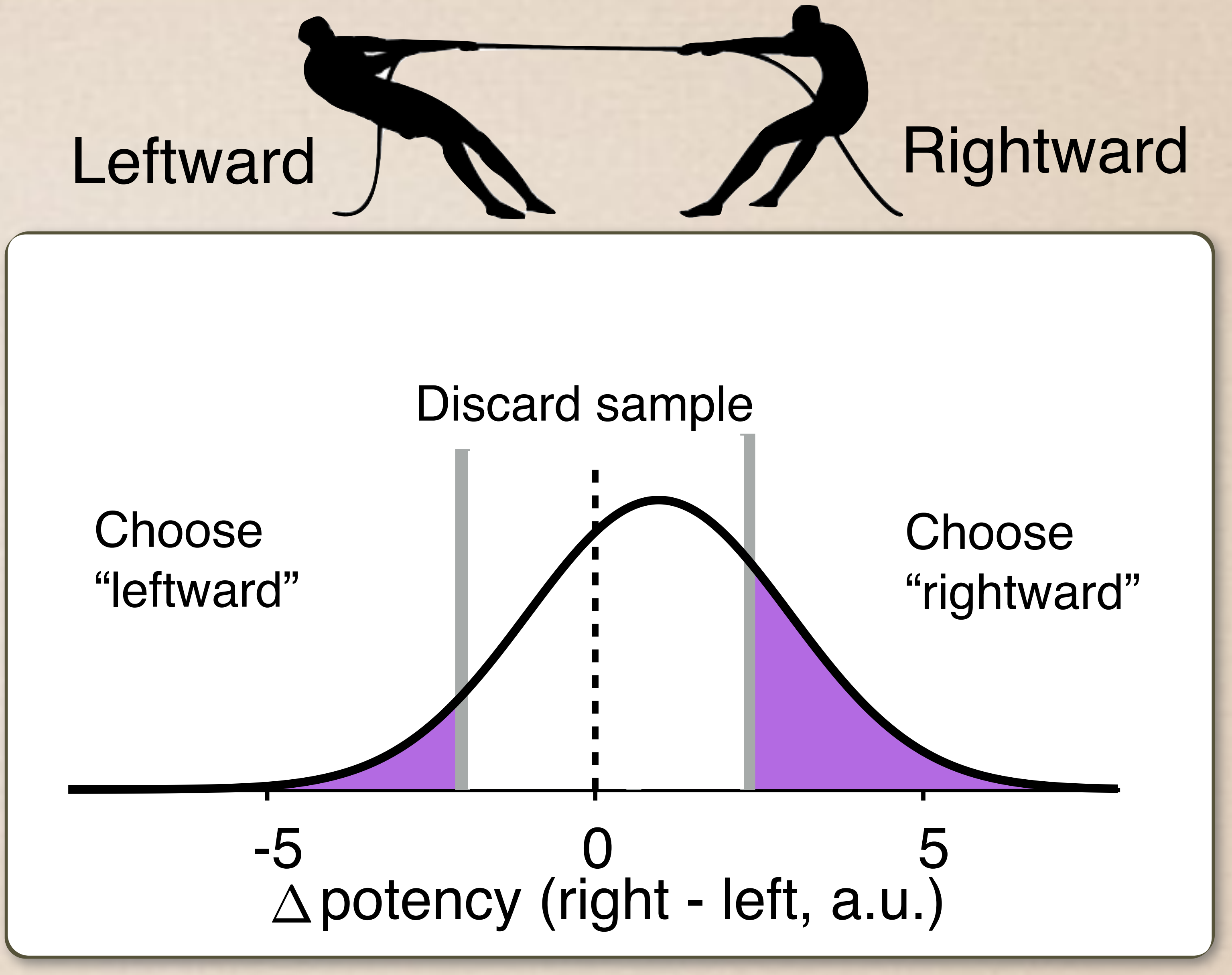

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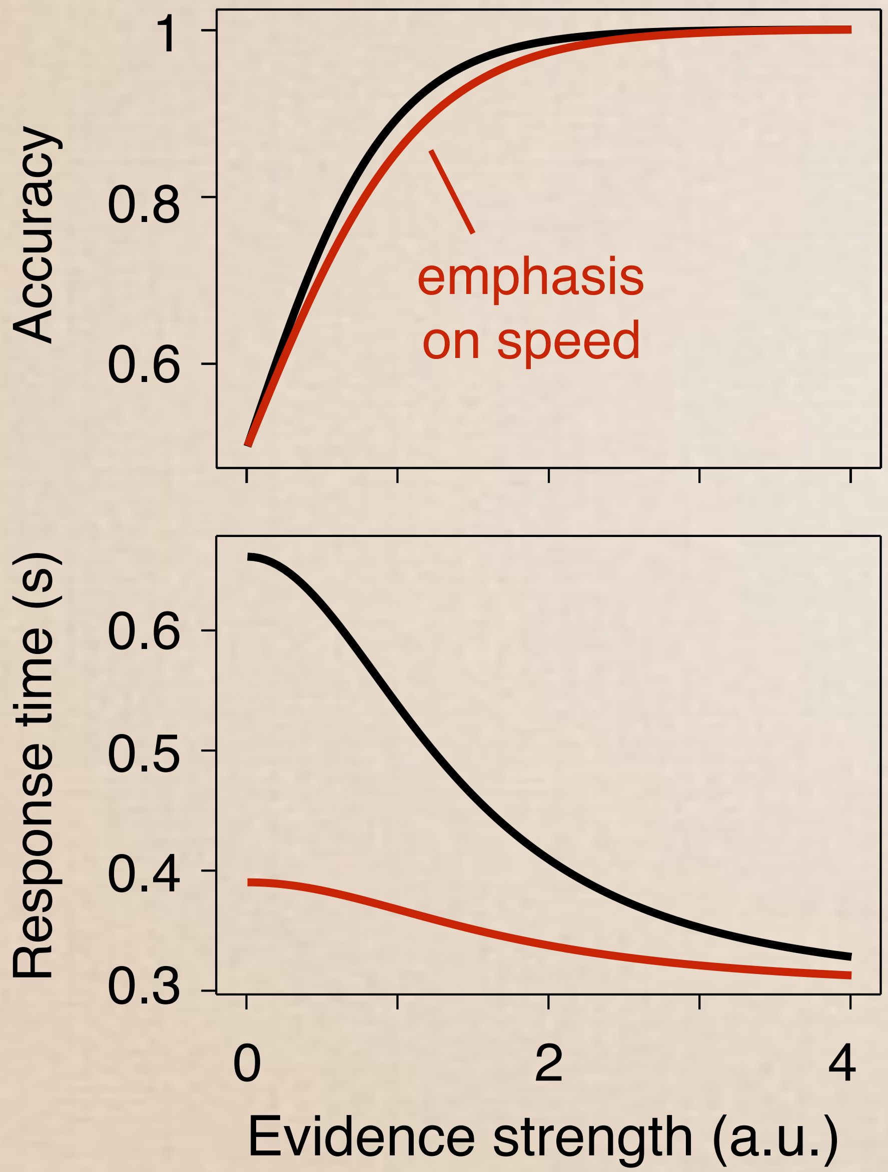

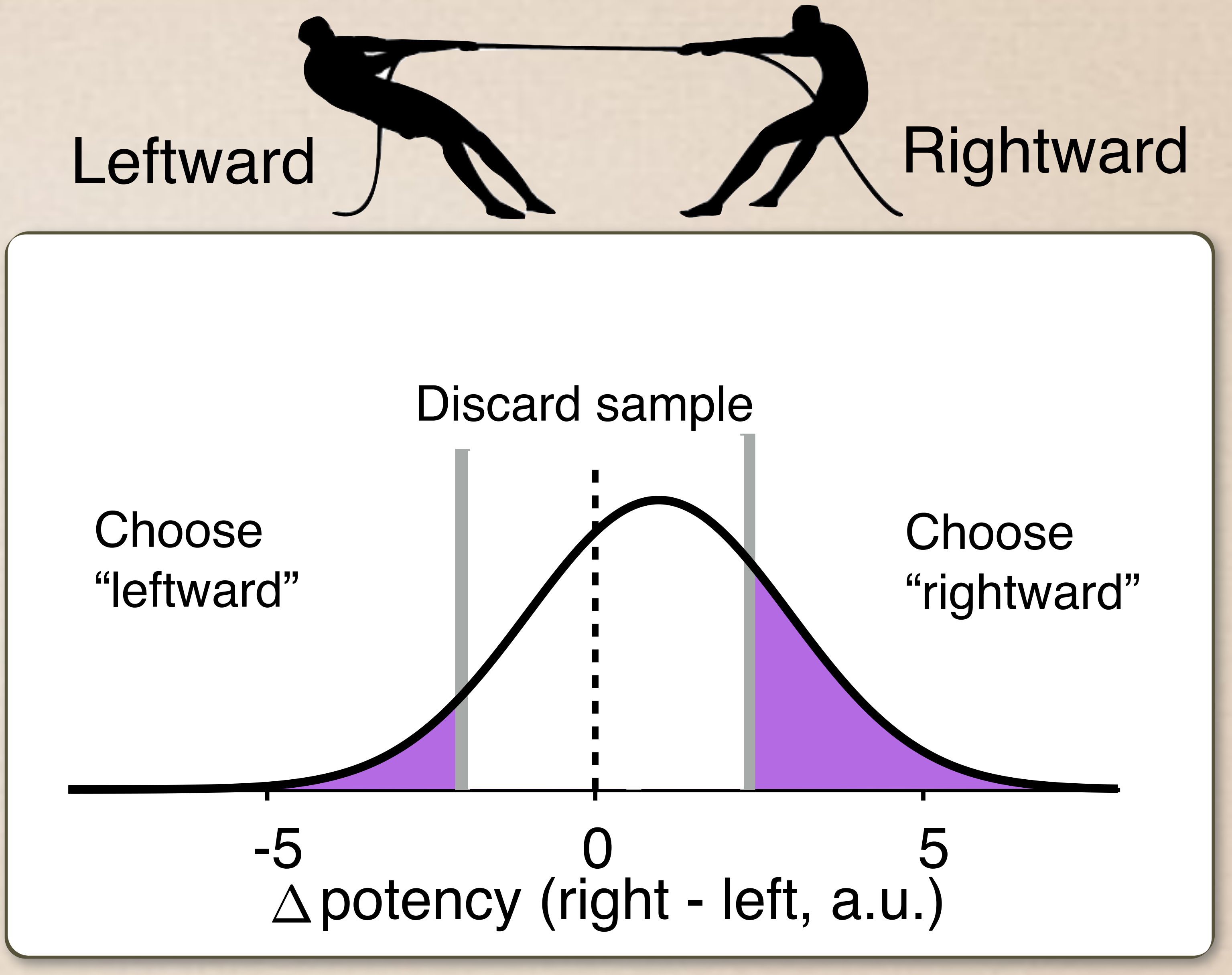

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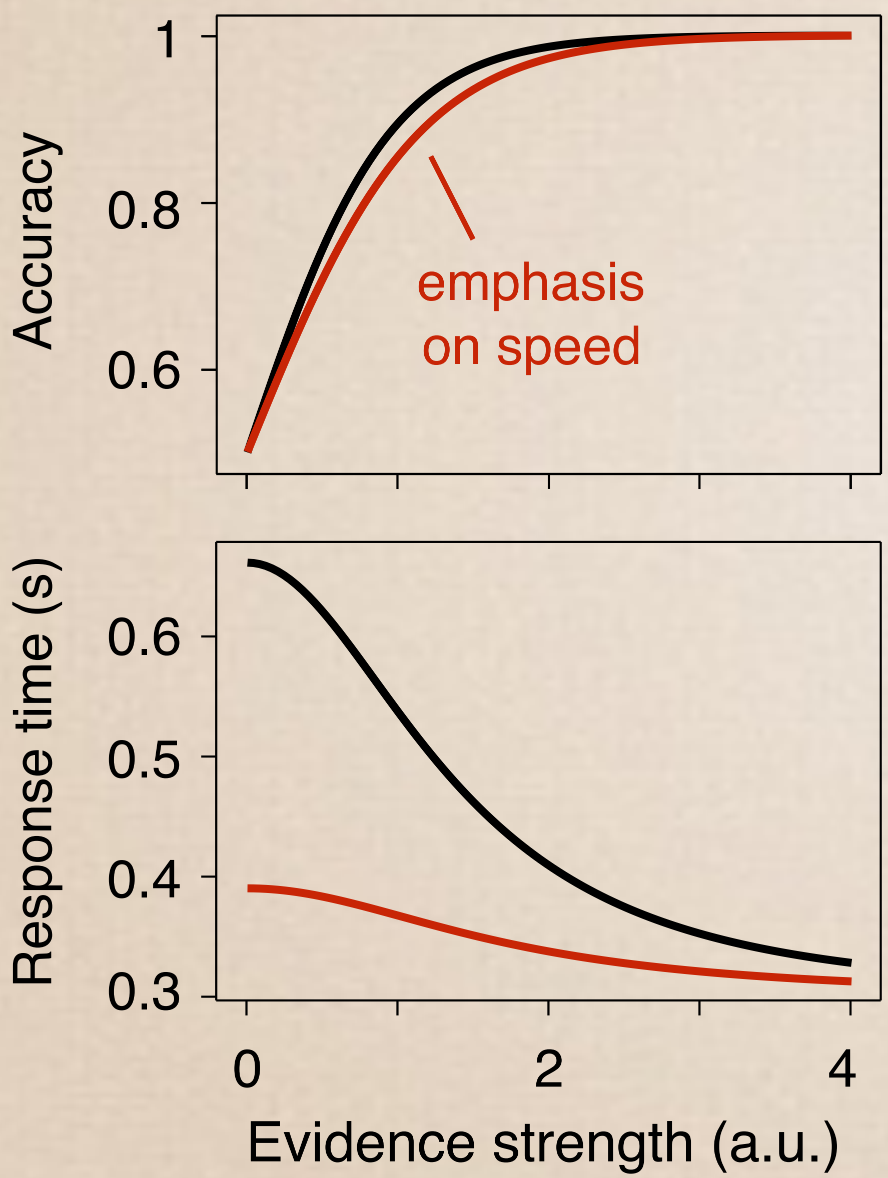

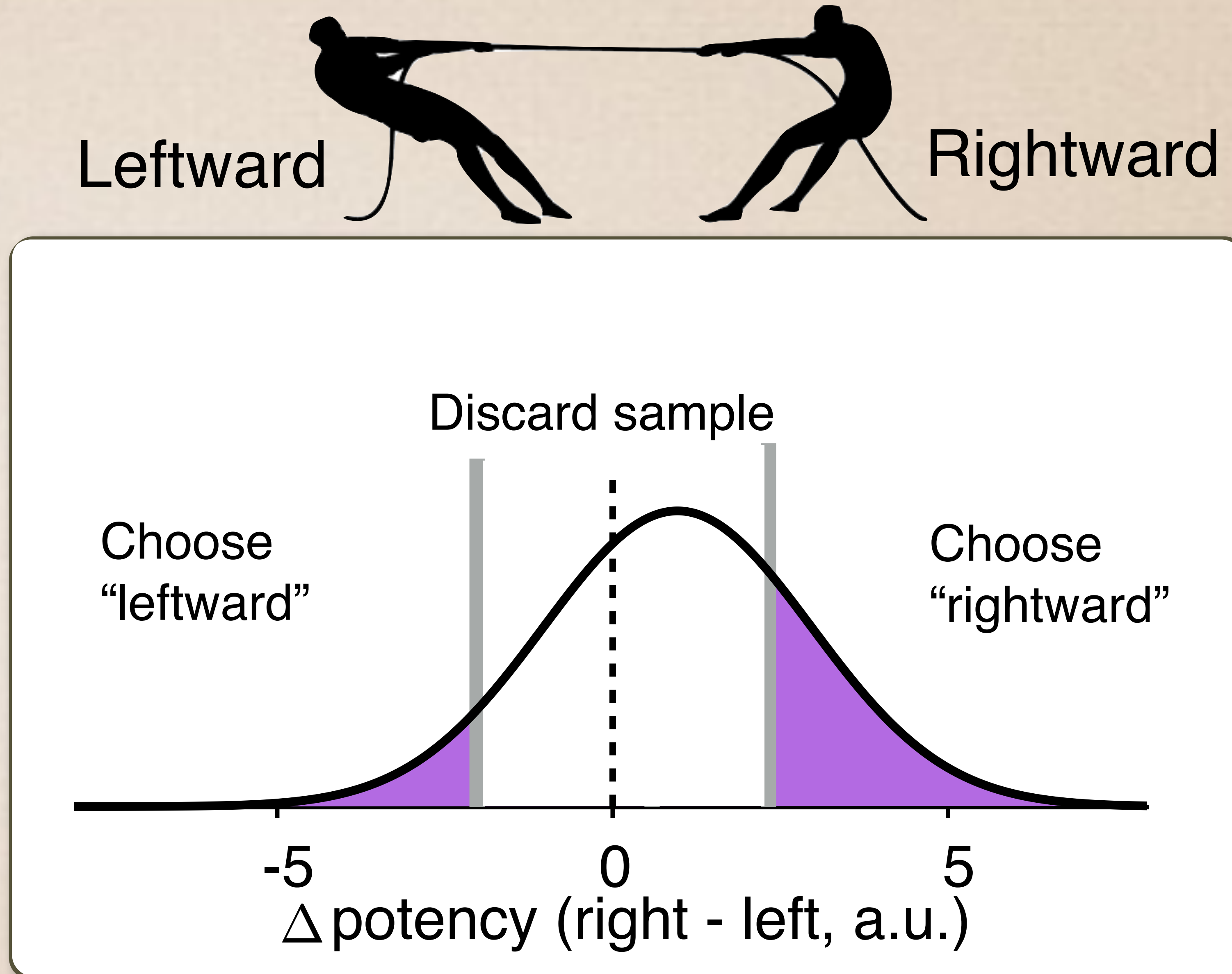

» Decisions may take a long time if high levels of accuracy are required.

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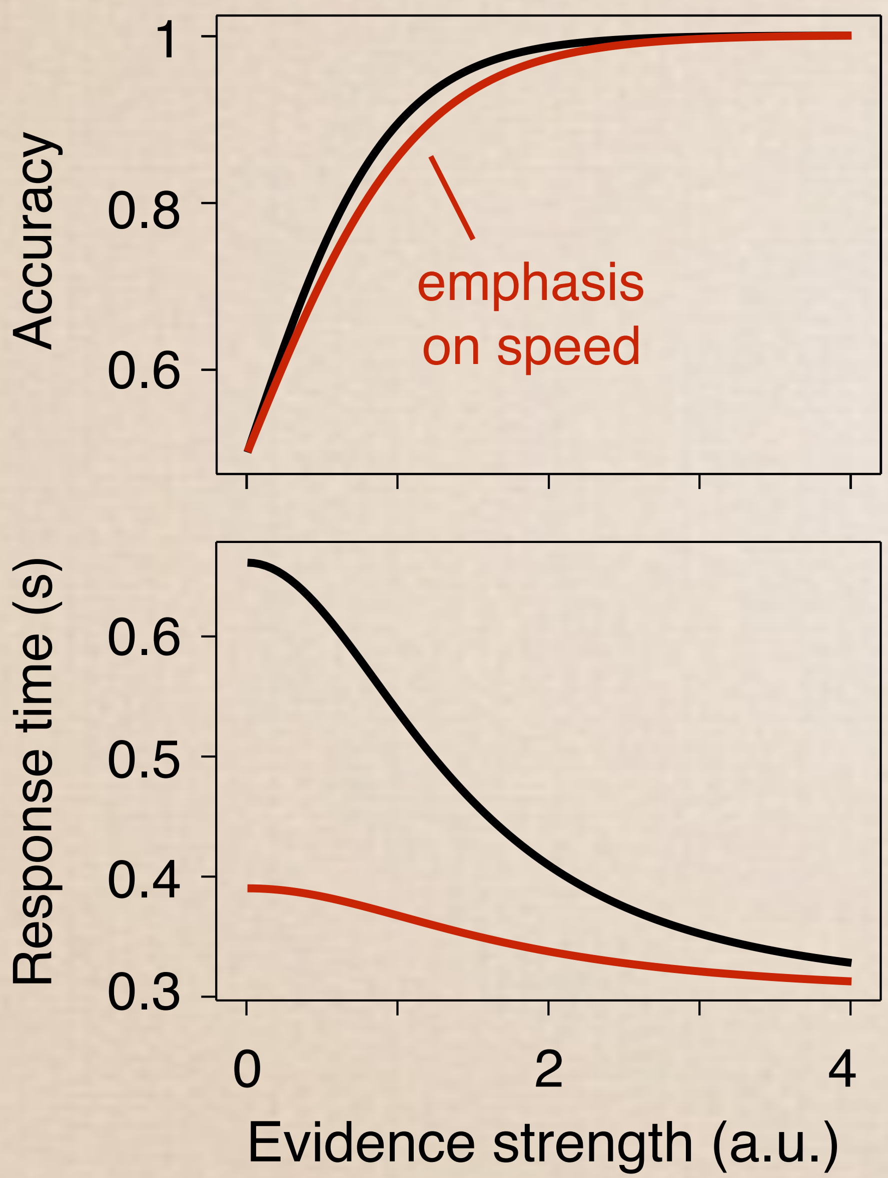

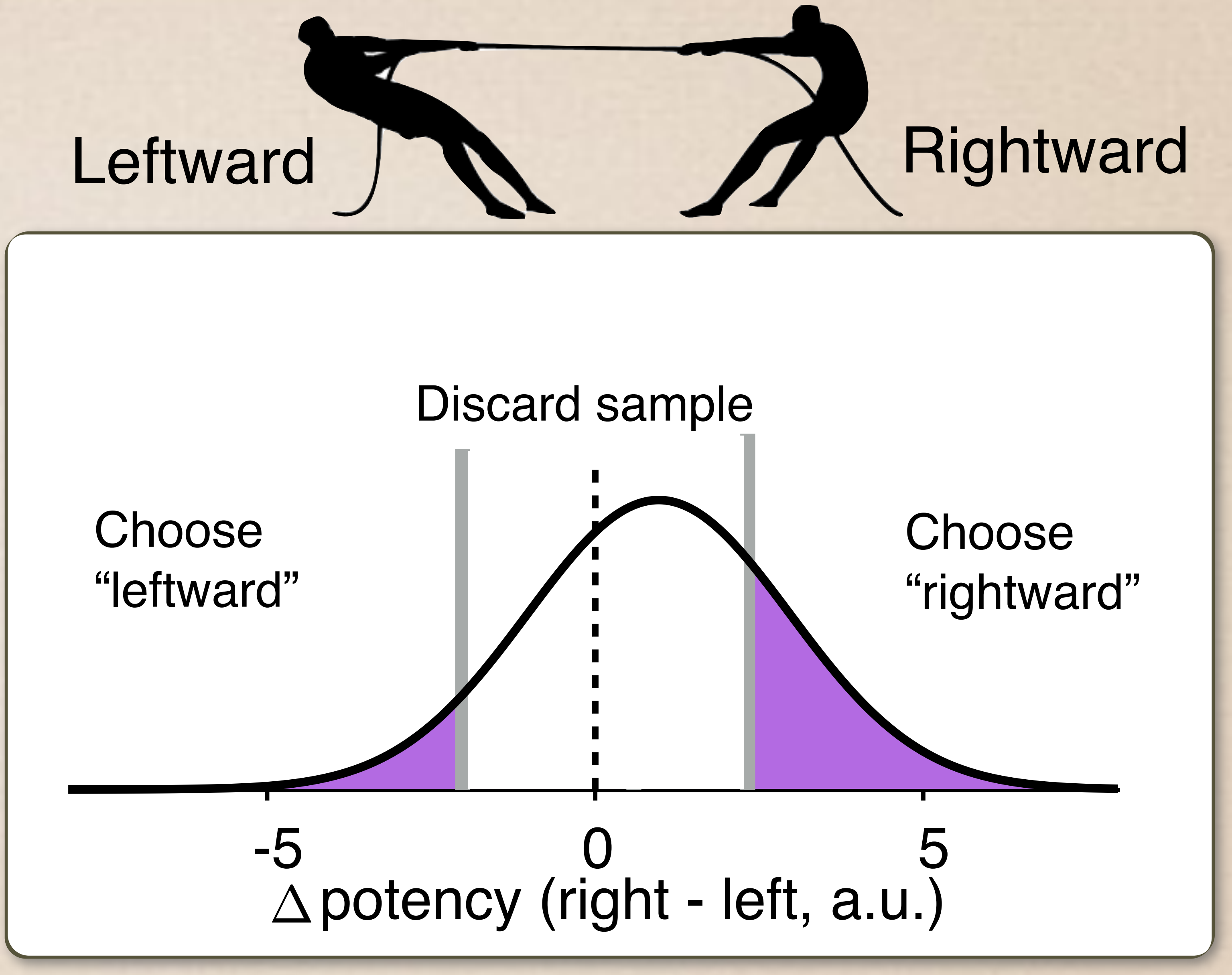

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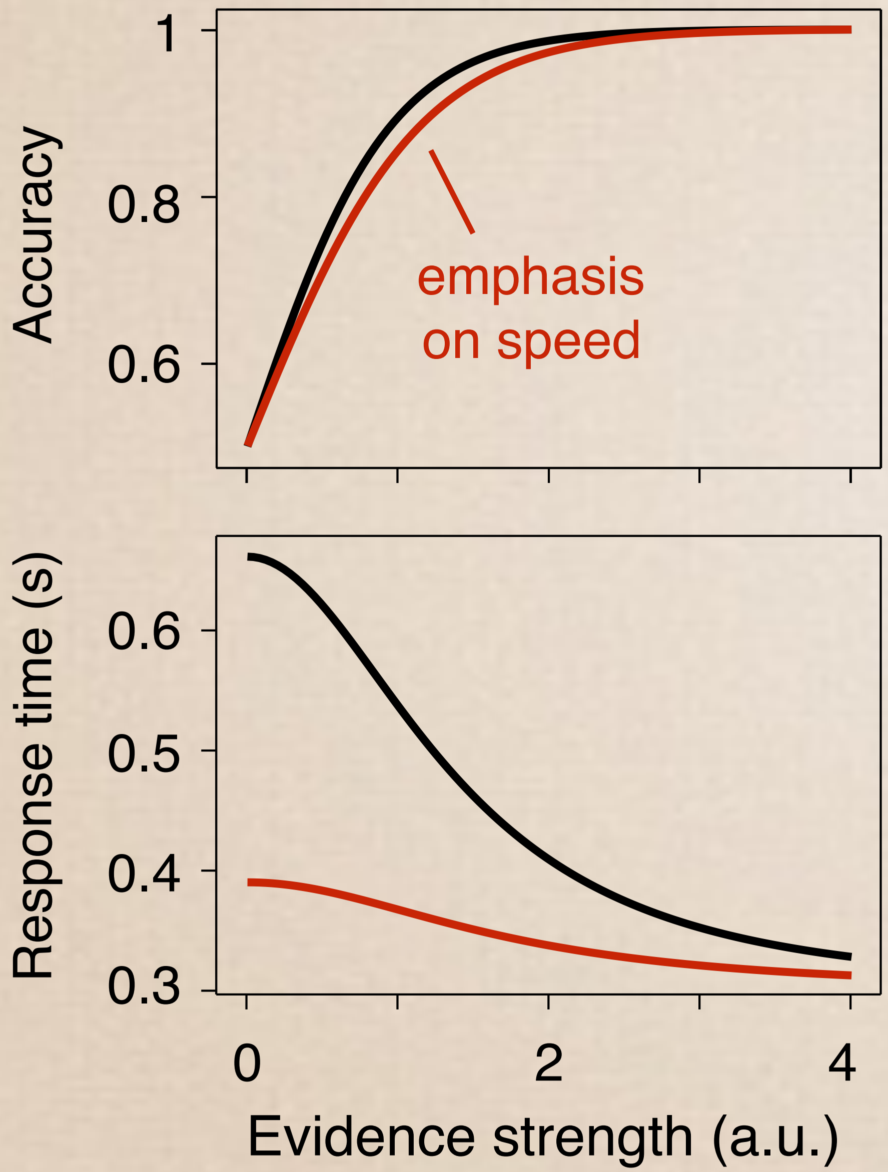

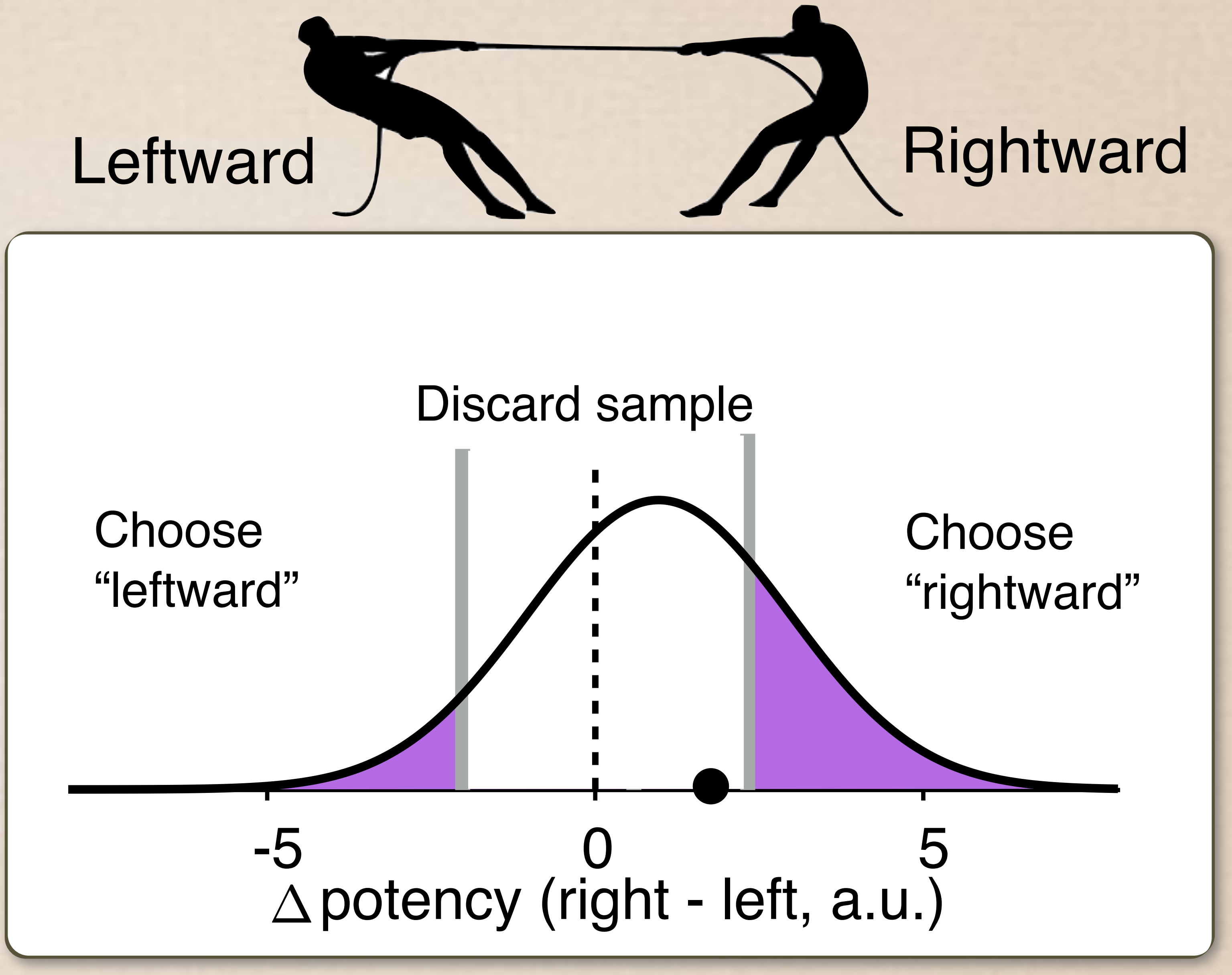

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Sequential probability ratio test (SPRT)

Wald & Wolfowitz (1948), A. Turing (194?)

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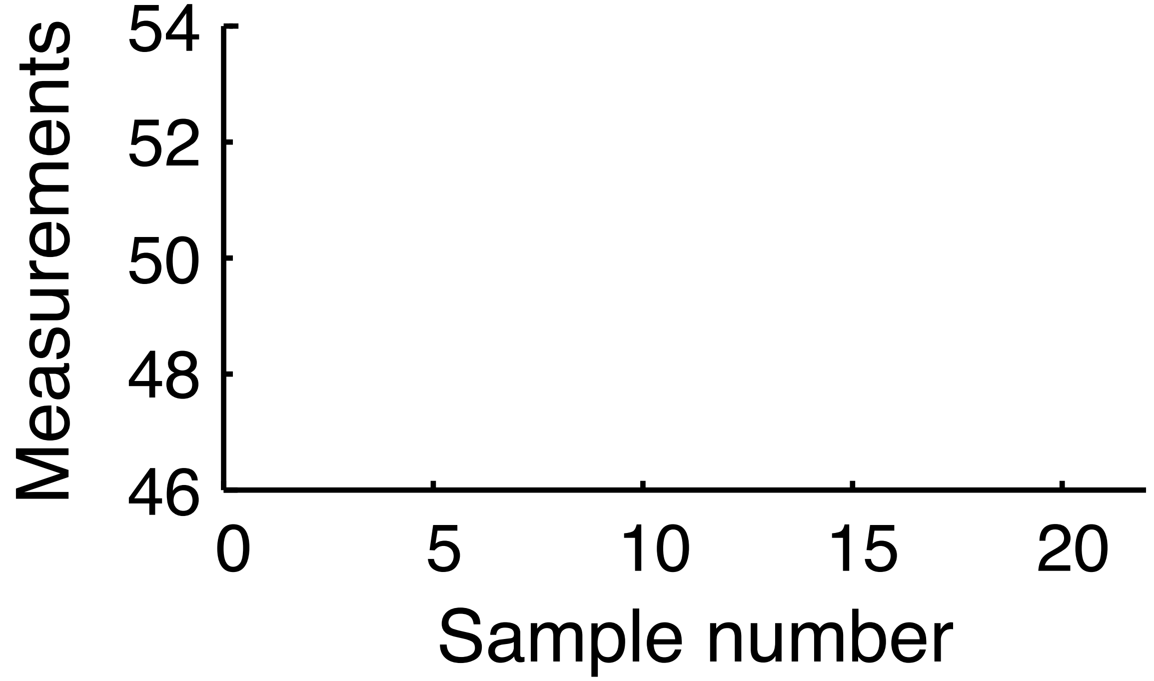

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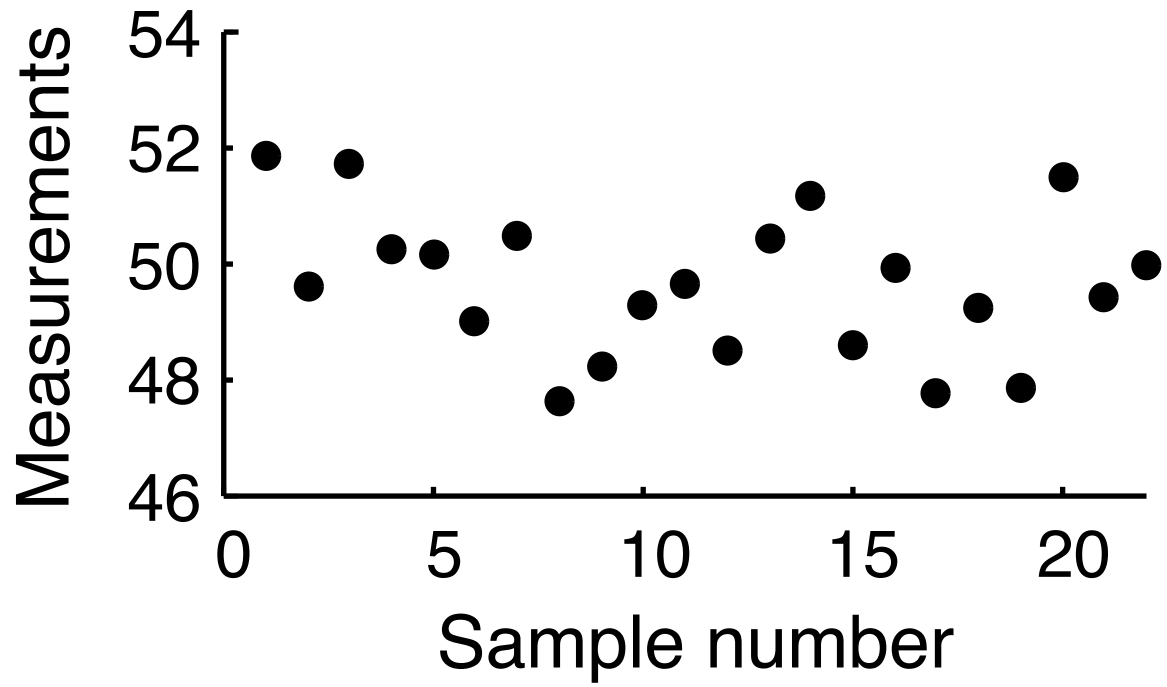

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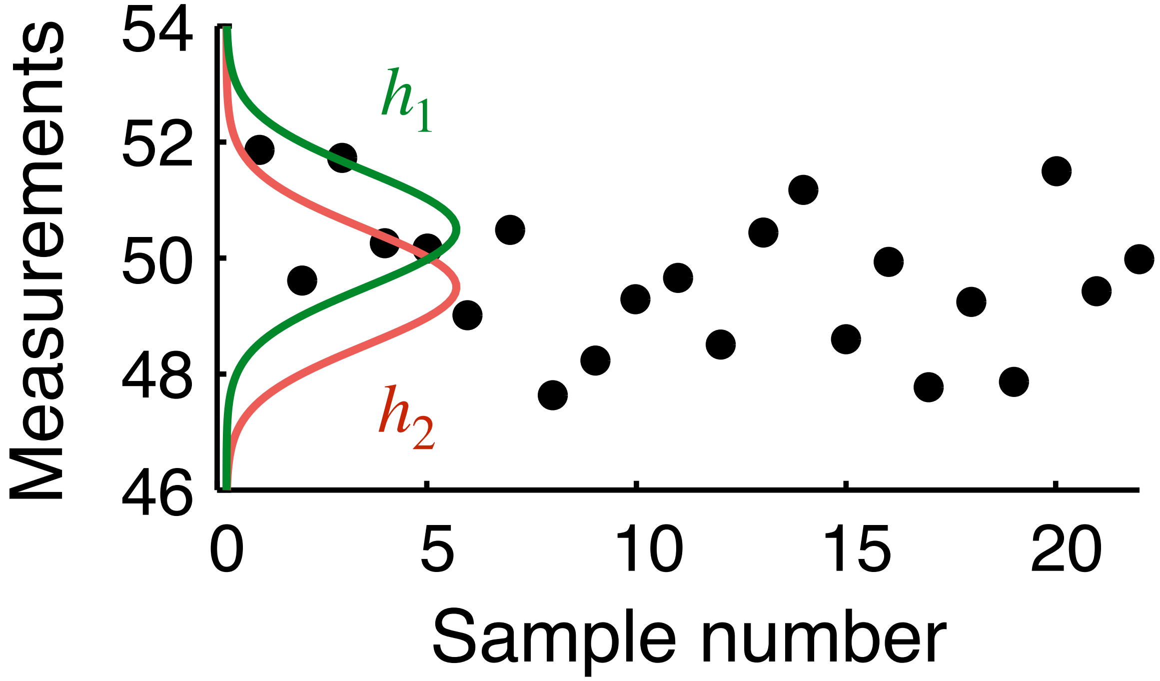

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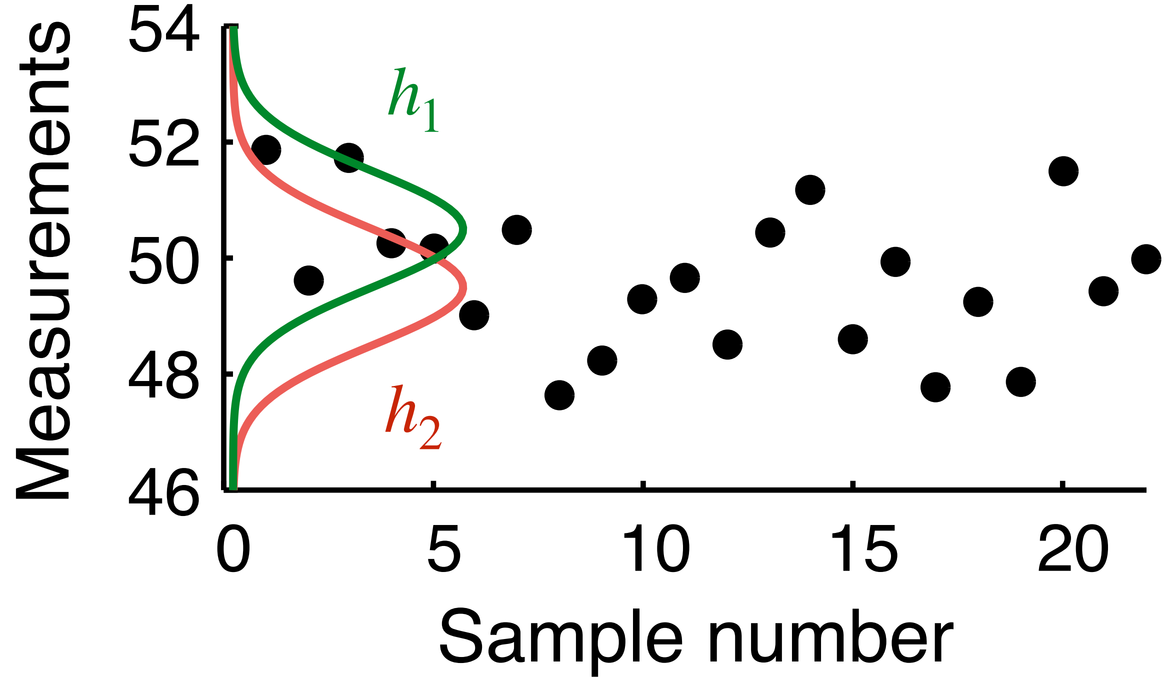

Weight of evidence e in favor of h_1 over h_2 =

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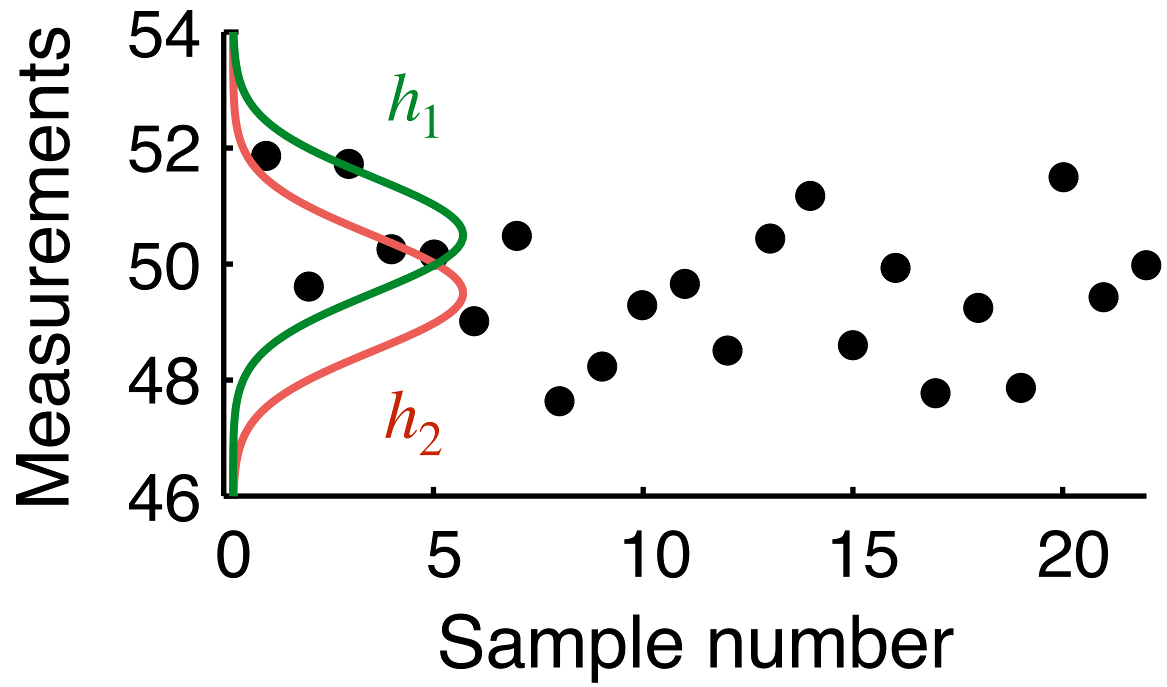

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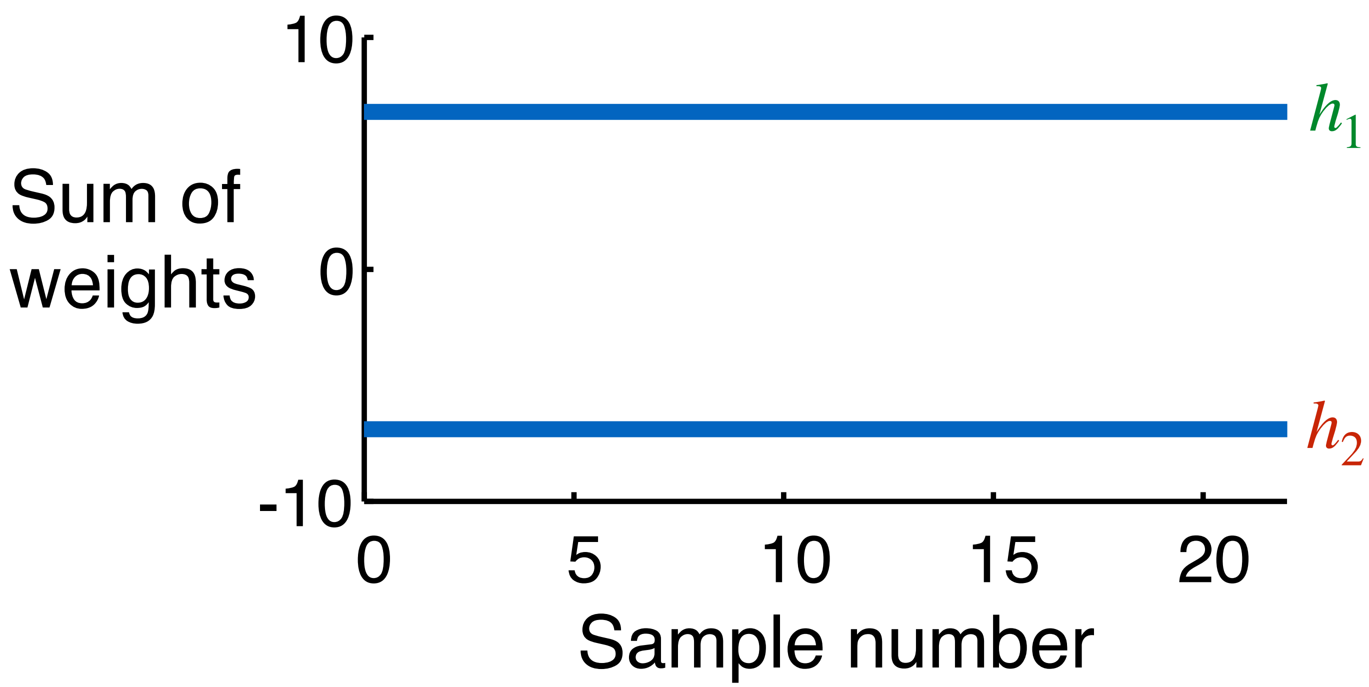

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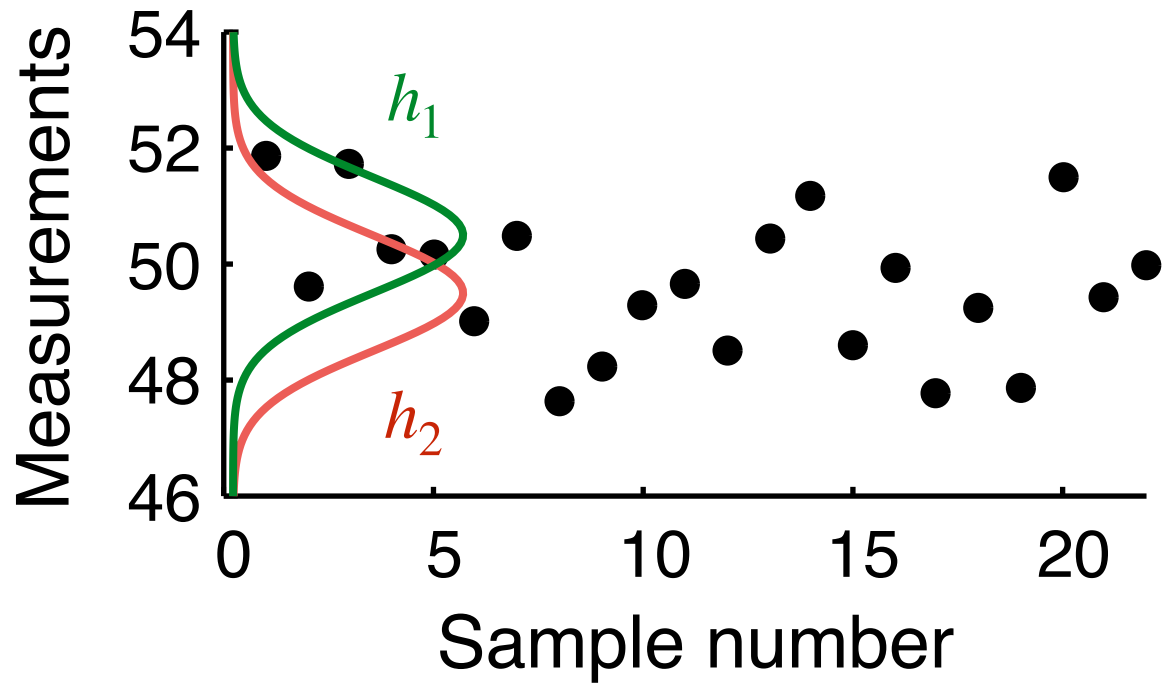

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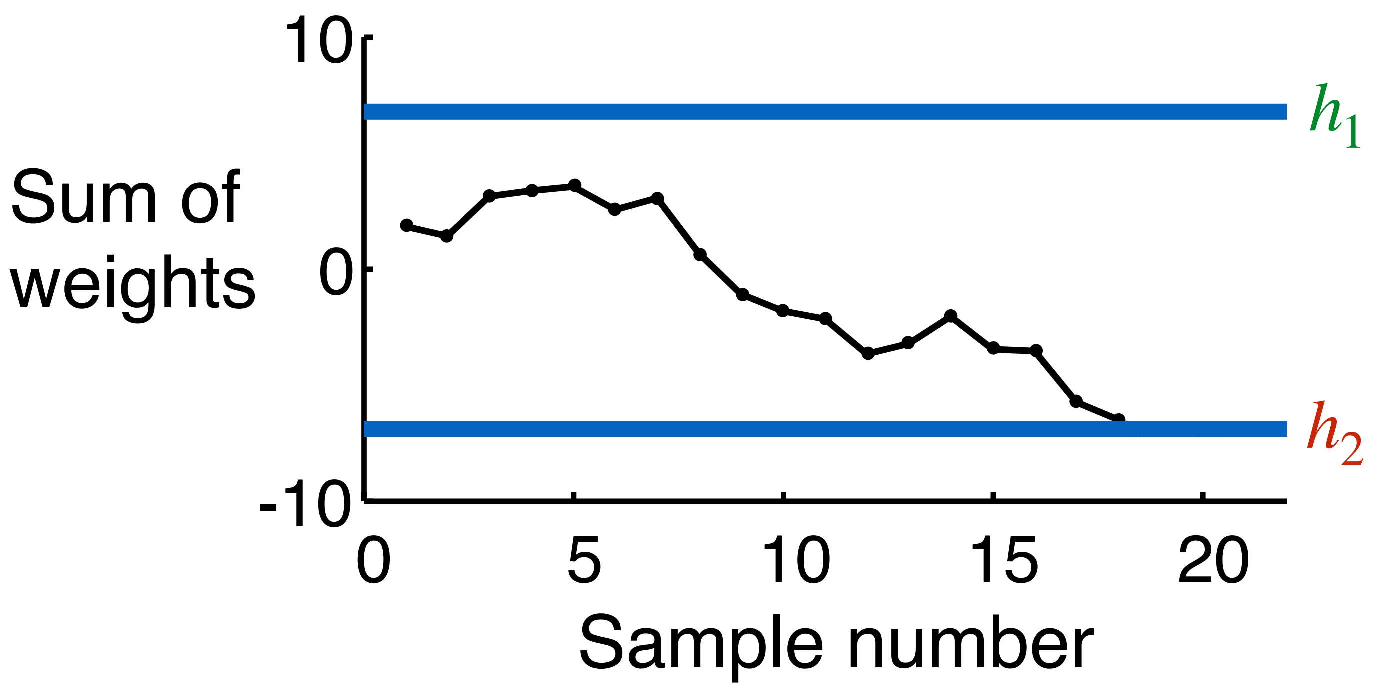

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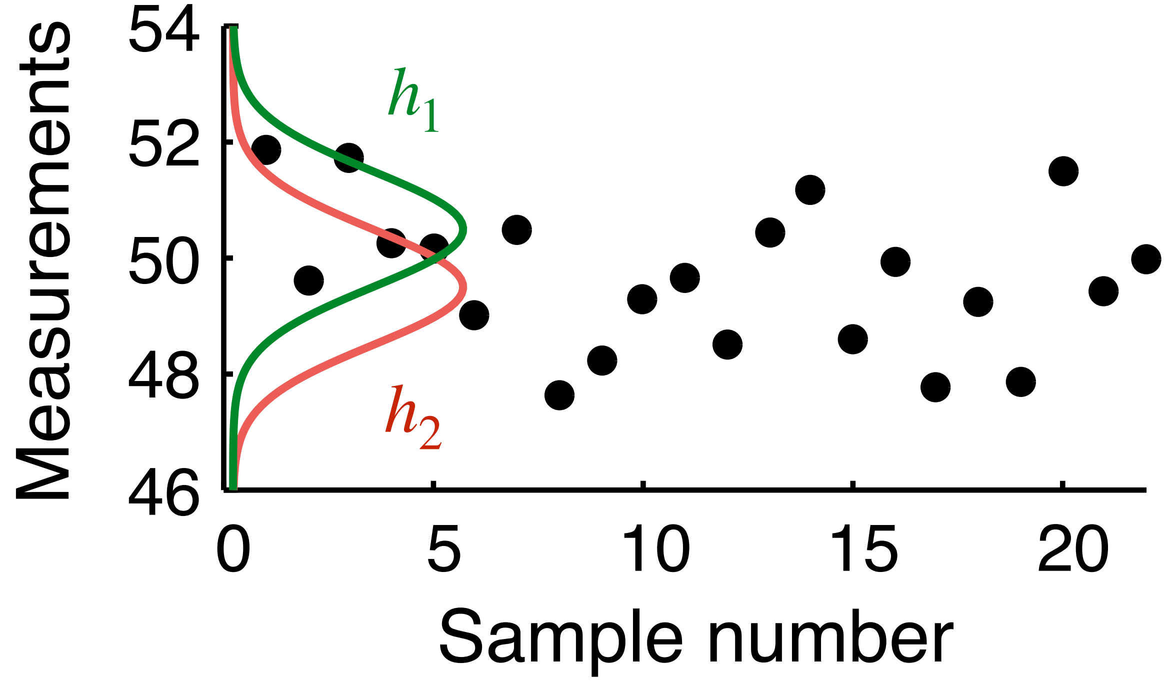

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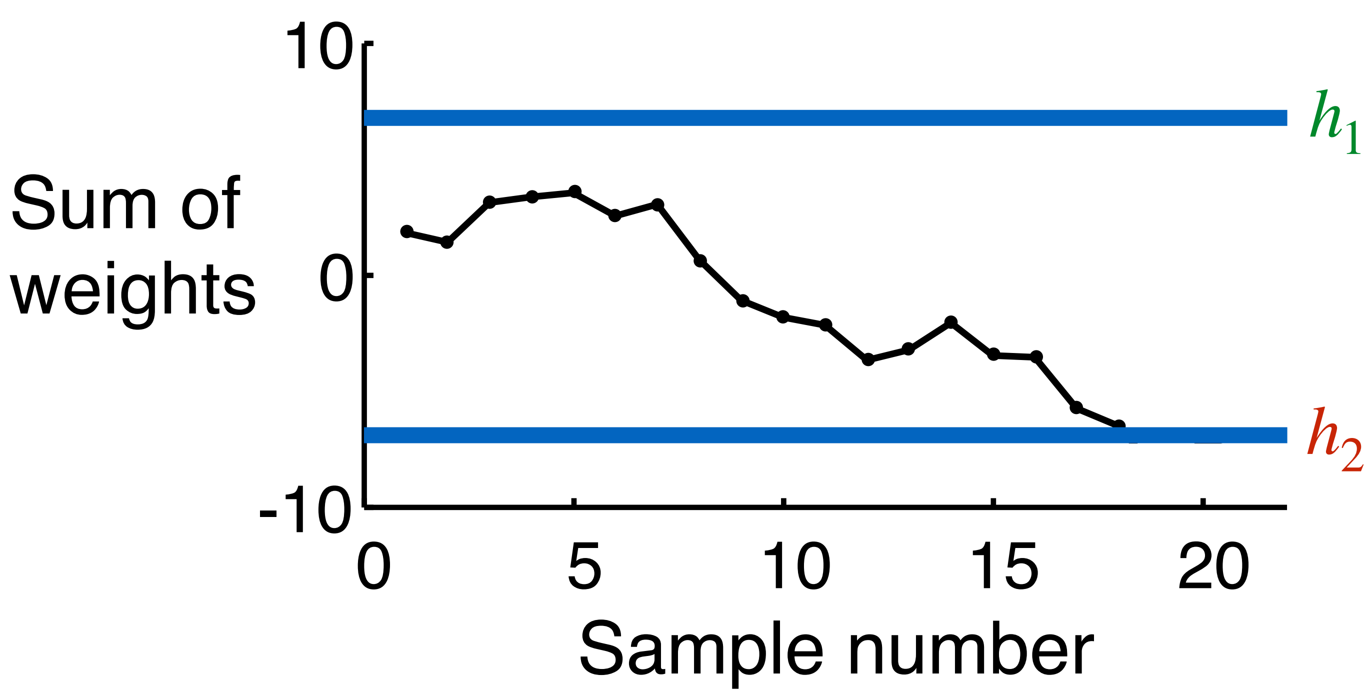

$$B \approx \log\left(\frac{\mathrm{acc}}{1-\mathrm{acc}}\right)$$

bound height

required accuracy

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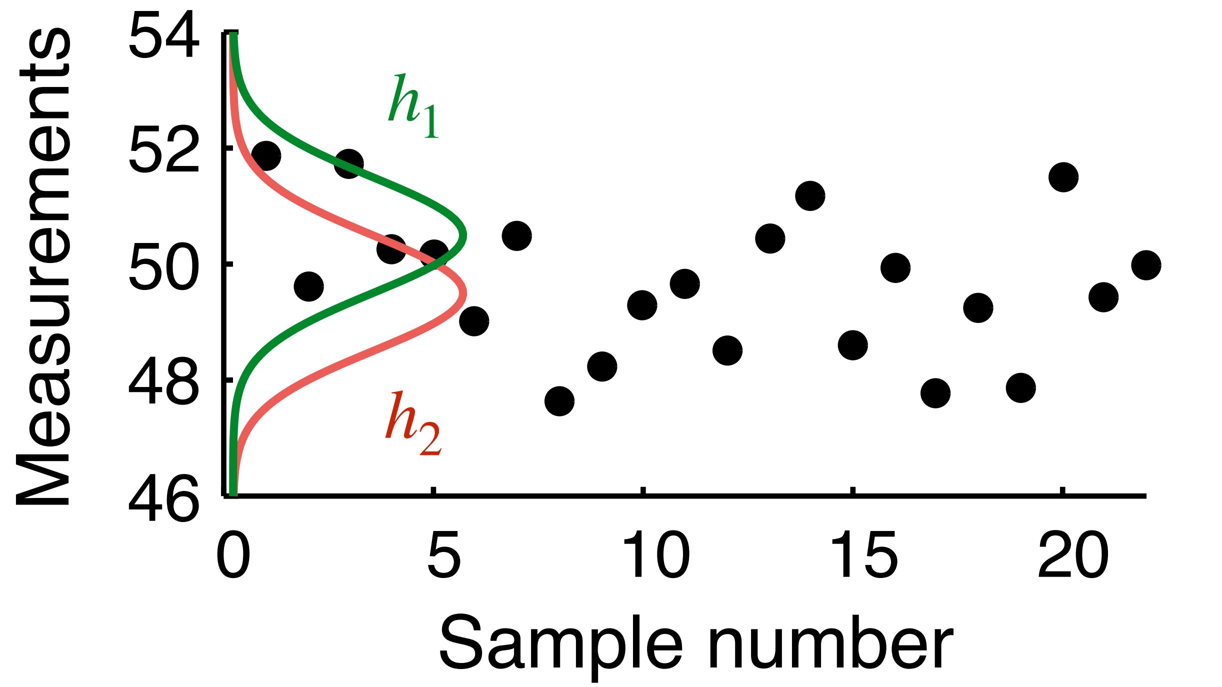

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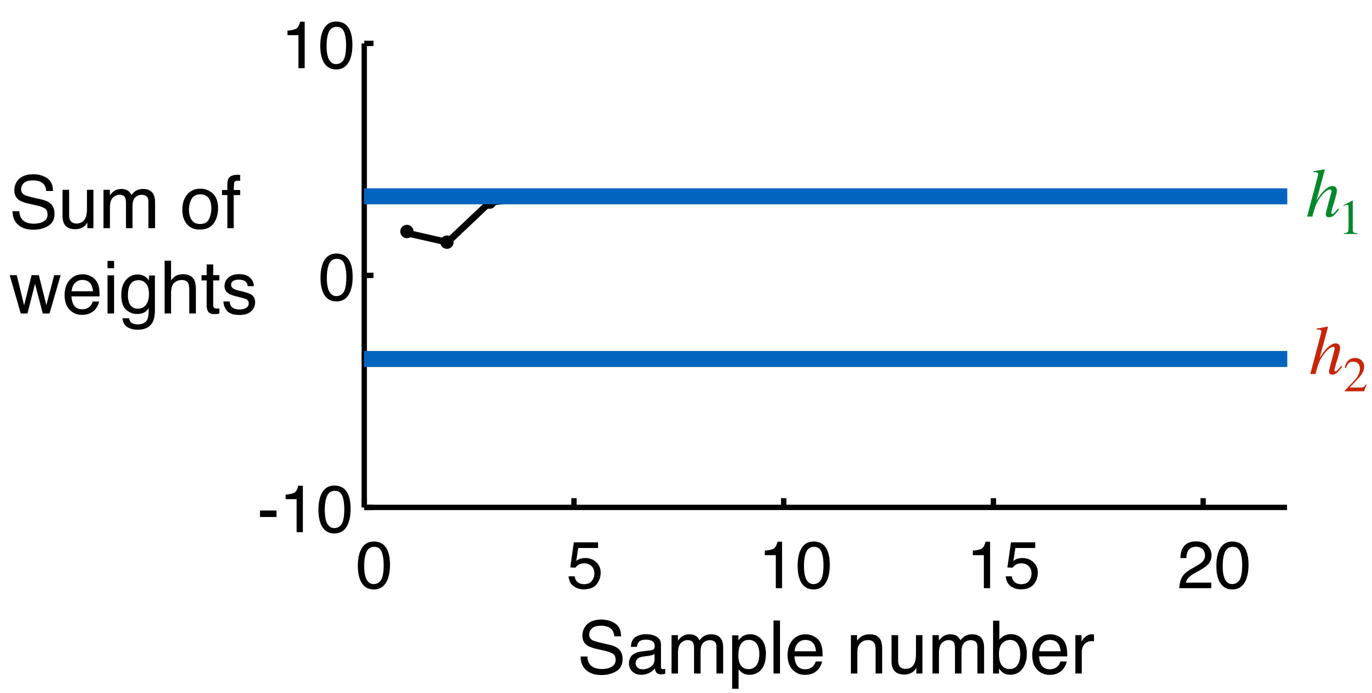

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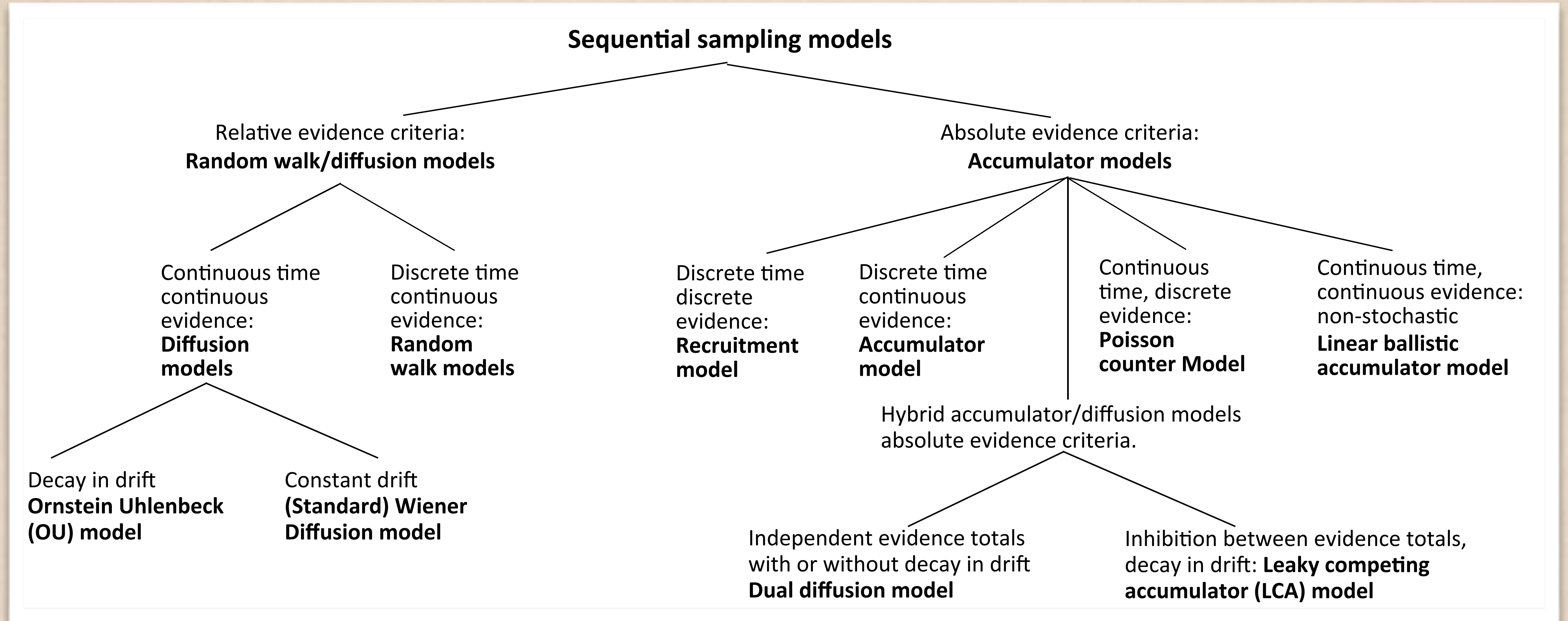
Many variants of evidence accumulation models for binary decisions



Ratcliff et al. TiCS 2016

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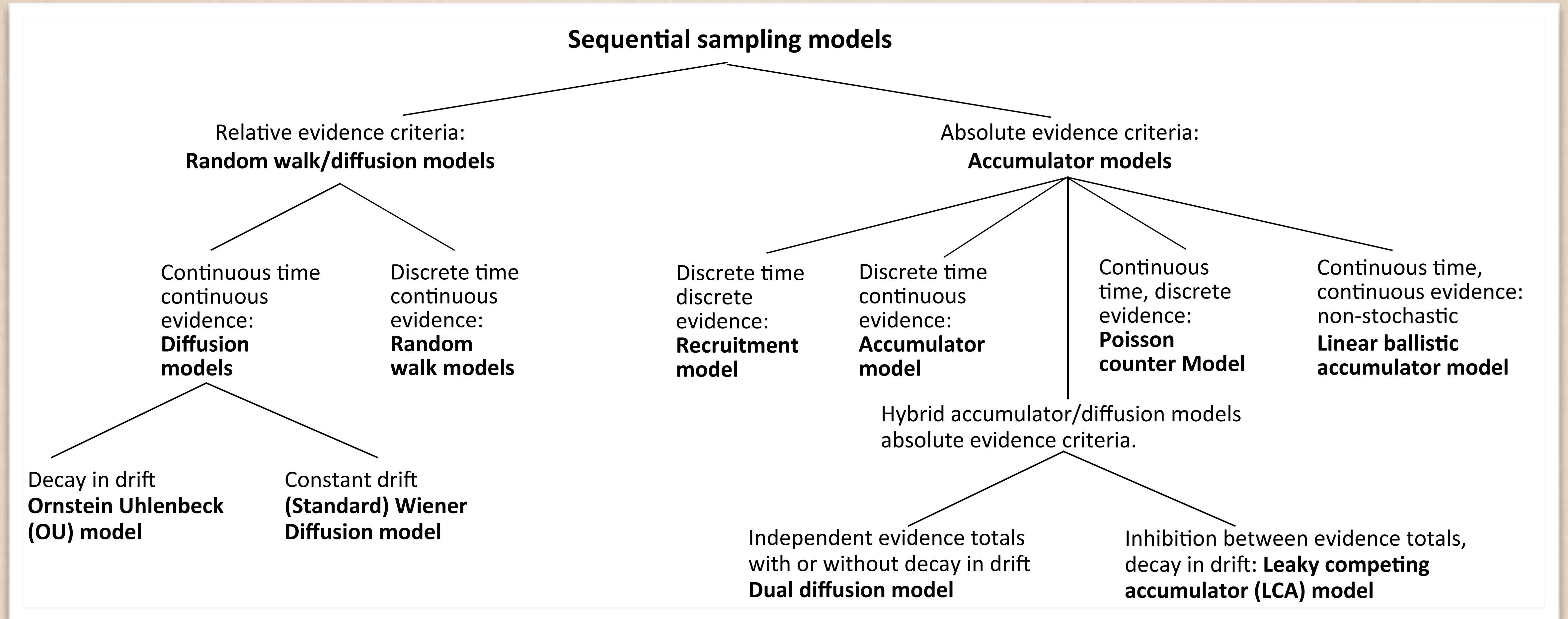

Ratcliff et al. TiCS 2016

» Time (continuous or discrete)

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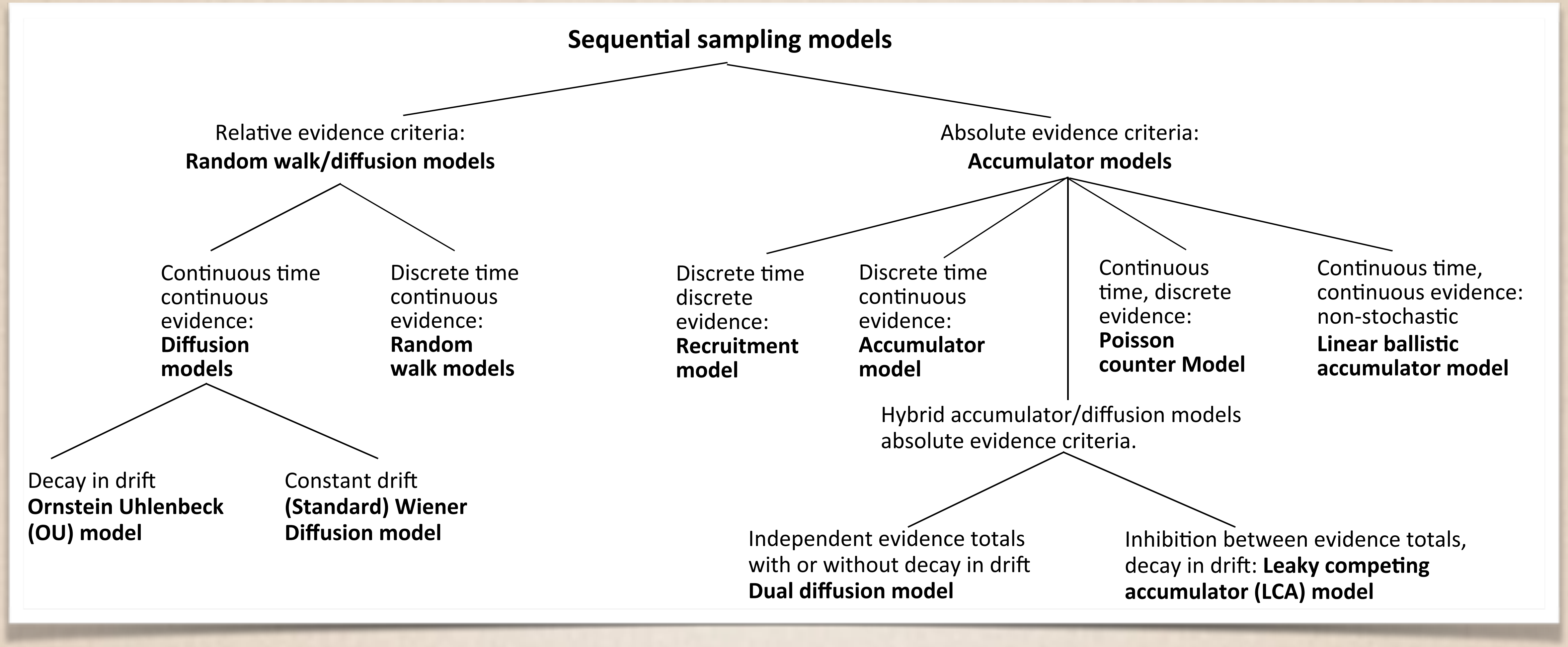

Ratcliff et al. TiCS 2016

- Time (continuous or discrete)
- Momentary evidence (continuous or discrete)

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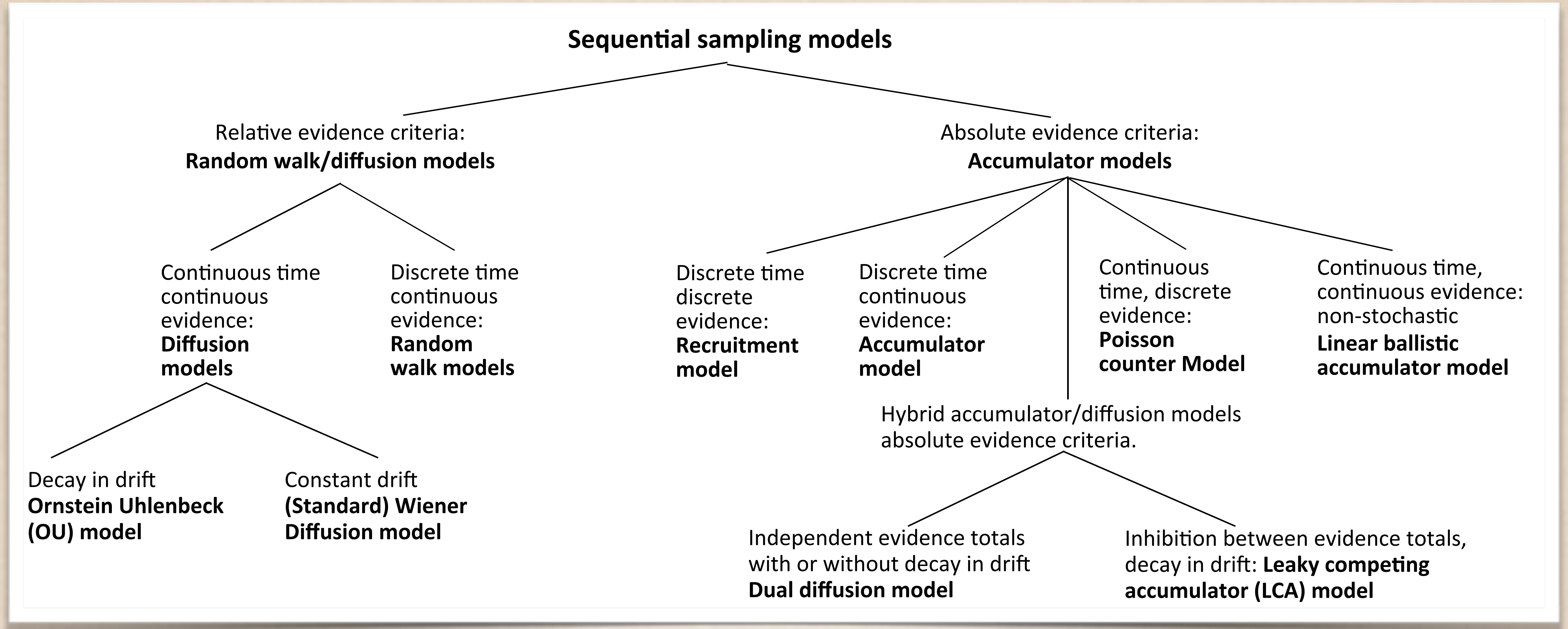

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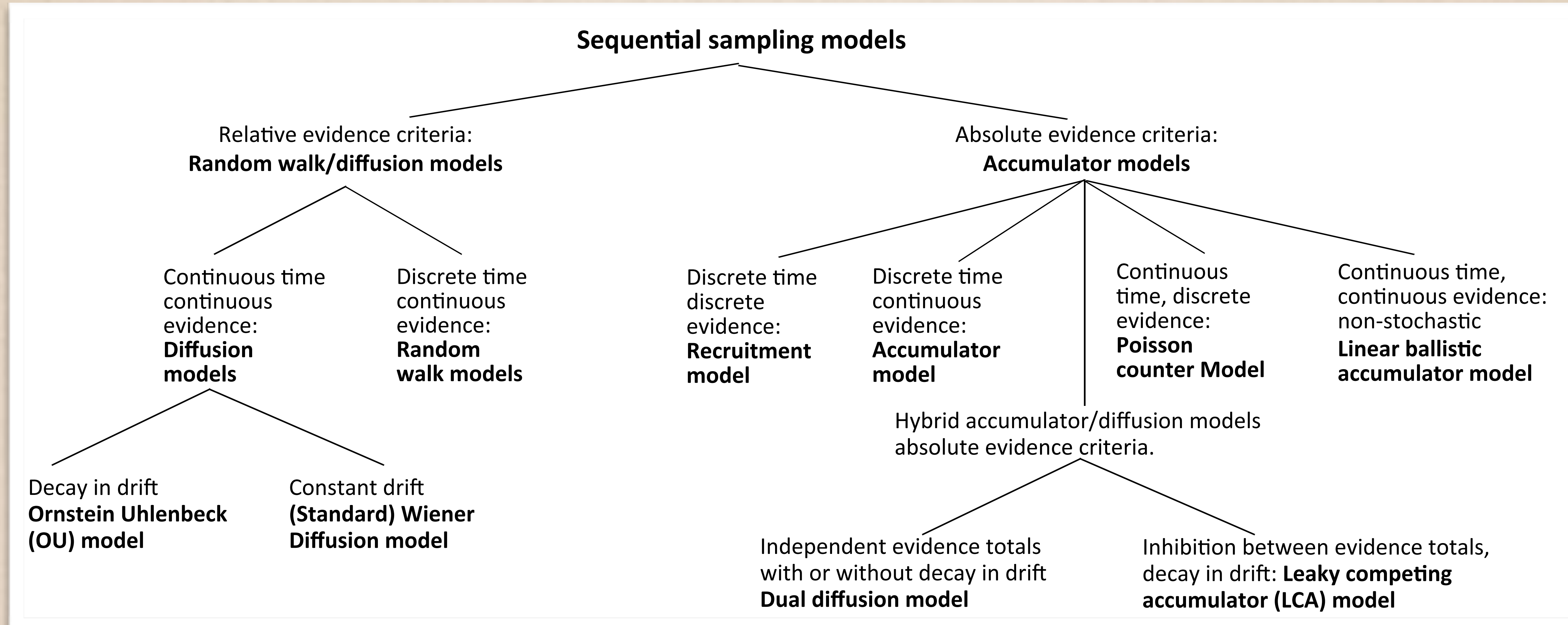

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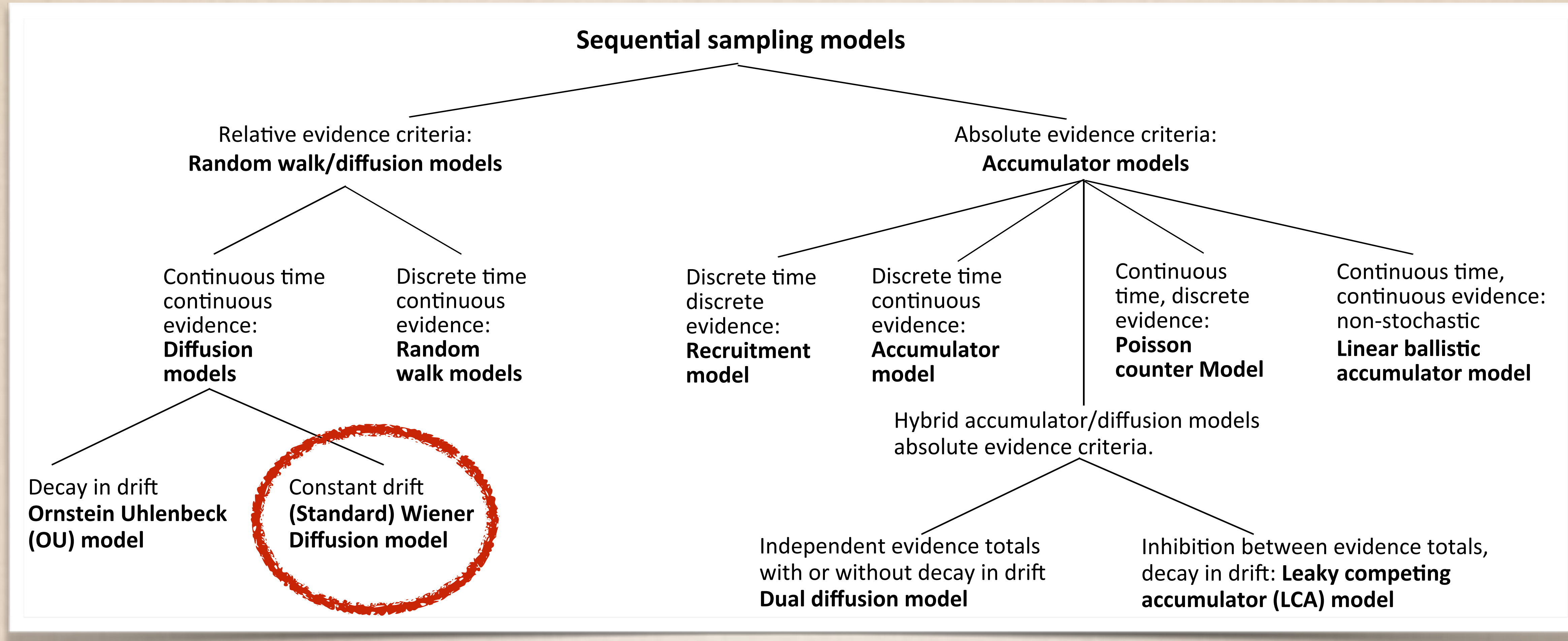

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Drift diffusion model

Roger Ratcliff, 1978

Drift diffusion model

- Decision based on the accumulation of samples of evidence arriving sequentially over time

Roger Ratcliff, 1978

Drift diffusion model

» Decision based on the accumulation of samples of evidence arriving sequentially over time

decision variable

$$x_{i+1} = x_i + e_i$$

evidence

i: iteration

Roger Ratcliff, 1978

Drift diffusion model

» Decision based on the accumulation of samples of evidence arriving sequentially over time

decision variable

$$x_{i+1} = x_i + e_i$$

i: iteration

evidence

» Evidence samples e_i are normally distributed

time step

$$e_i \sim \mathcal{N}(\mu\Delta t, \sigma^2\Delta t)$$

drift coefficient

diffusion coefficient

Roger Ratcliff, 1978

Drift diffusion model

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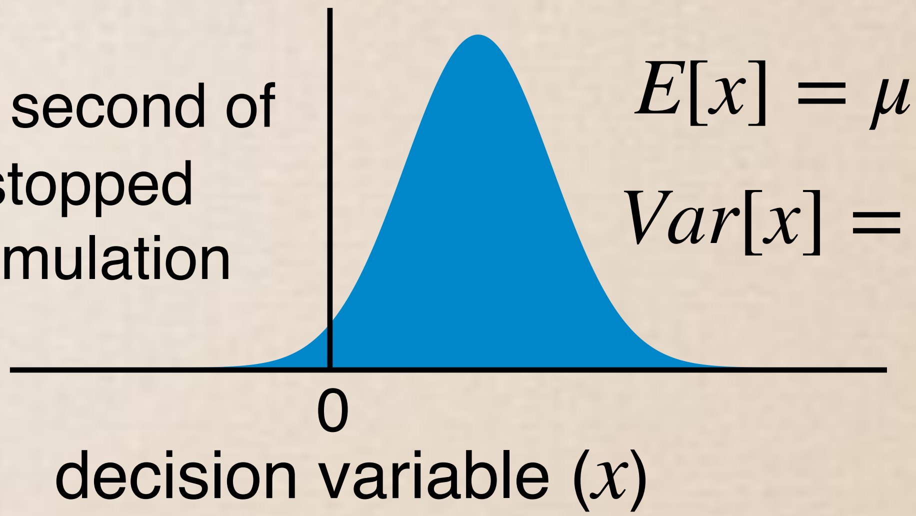
After t second of unstopped accumulation



$$E[x] = \mu t$$

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Roger Ratcliff, 1978

Drift diffusion model

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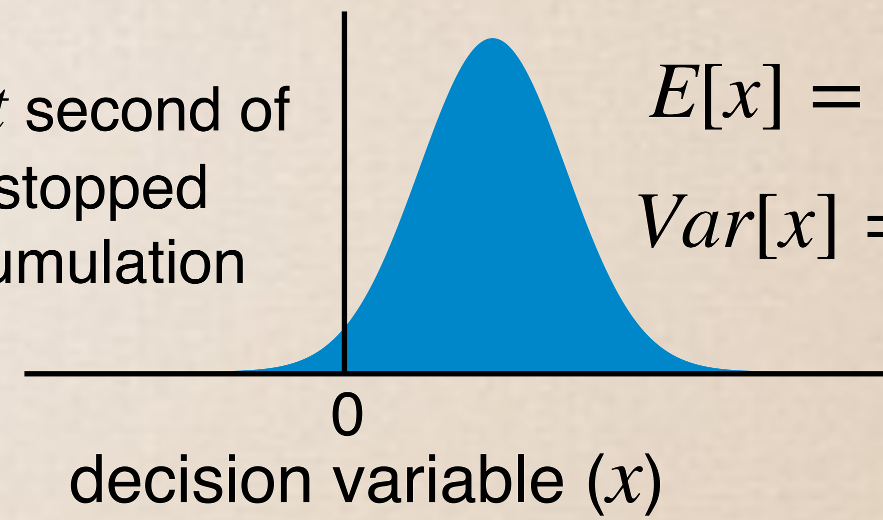


$E[x] = \mu t$

$Var[x] = \sigma^2 t$

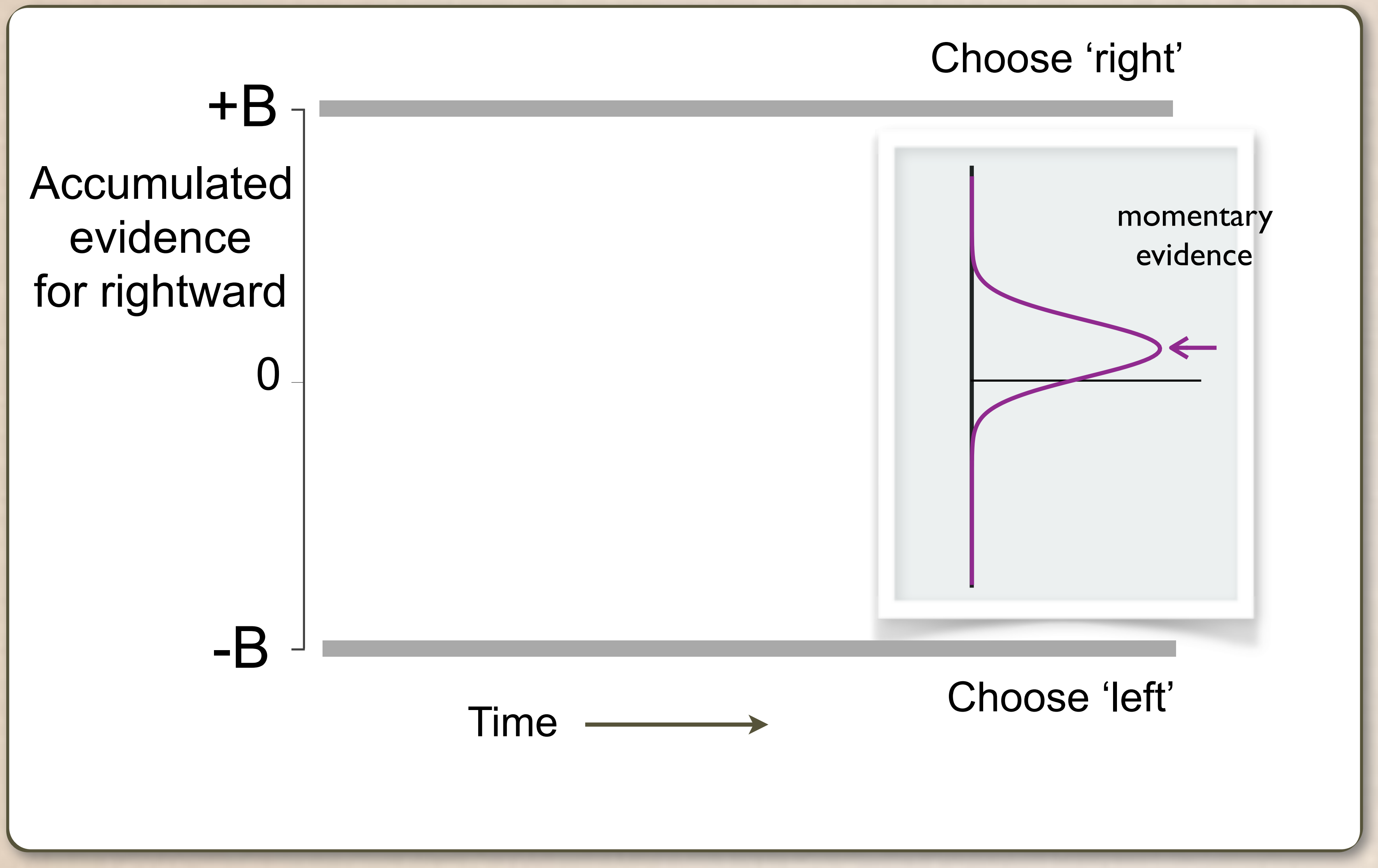
» In the limit $\Delta t \to 0$, the decision process is described by a Wiener process with drift:

decision variable

$$\frac{dx(t)}{dt} = \underbrace{\mu}_{\text{signal}} + \underbrace{\sigma W(t)}_{\text{noise}}$$

Roger Ratcliff, 1978

Drift diffusion model



Sample paths:

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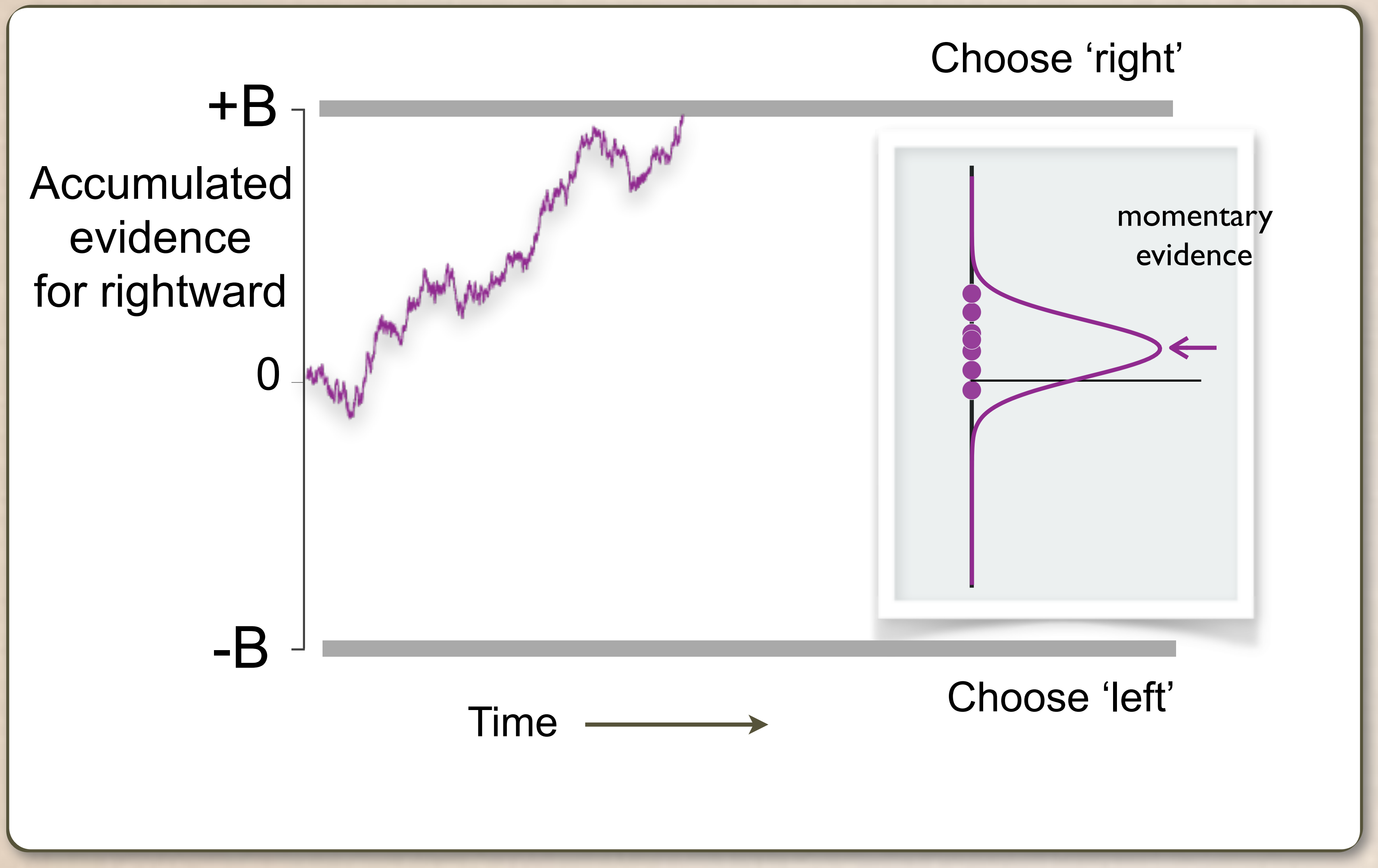
$$x_{i+1} = x_i + e_i$$

$$x_0 = 0$$

Drift diffusion model



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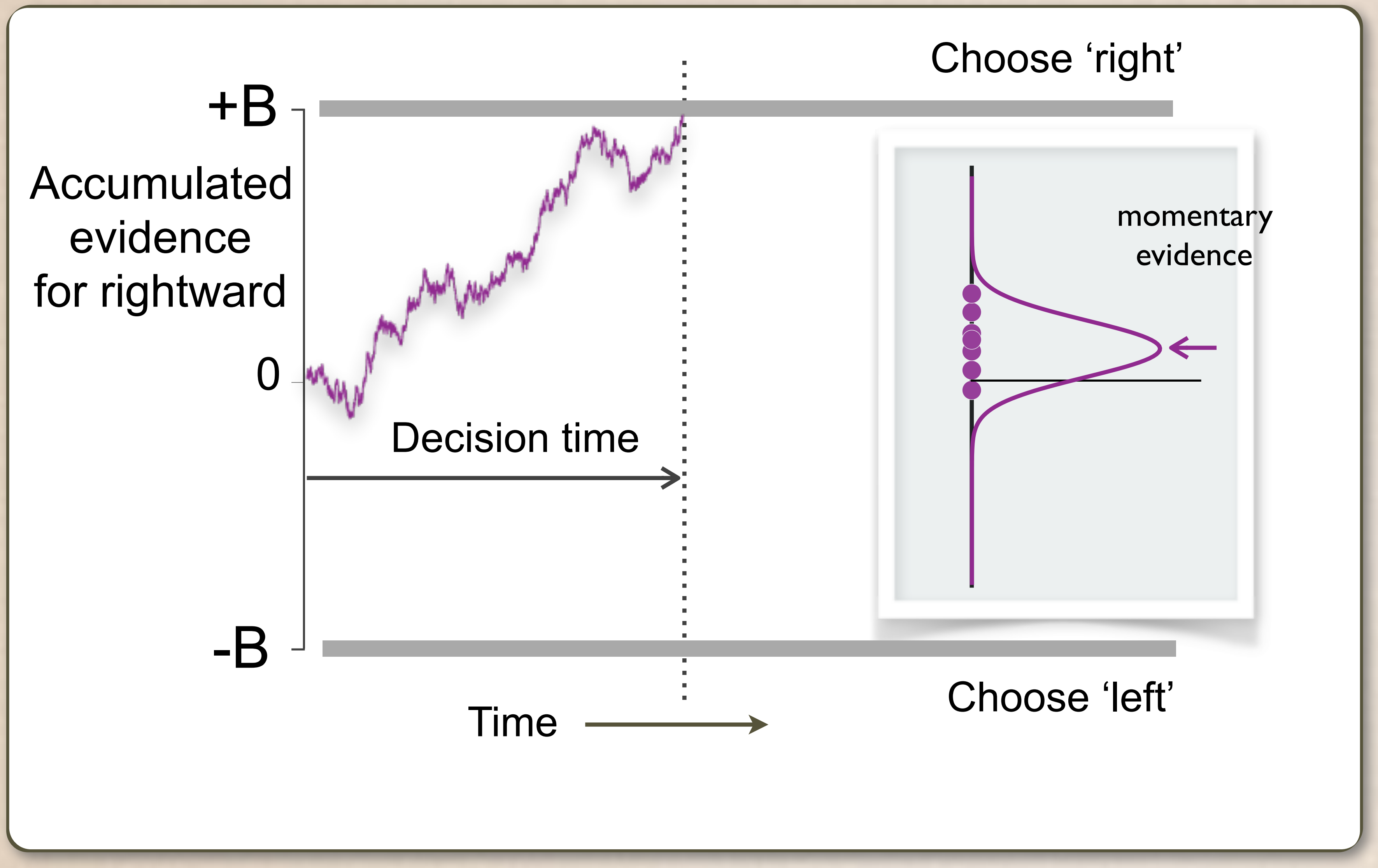

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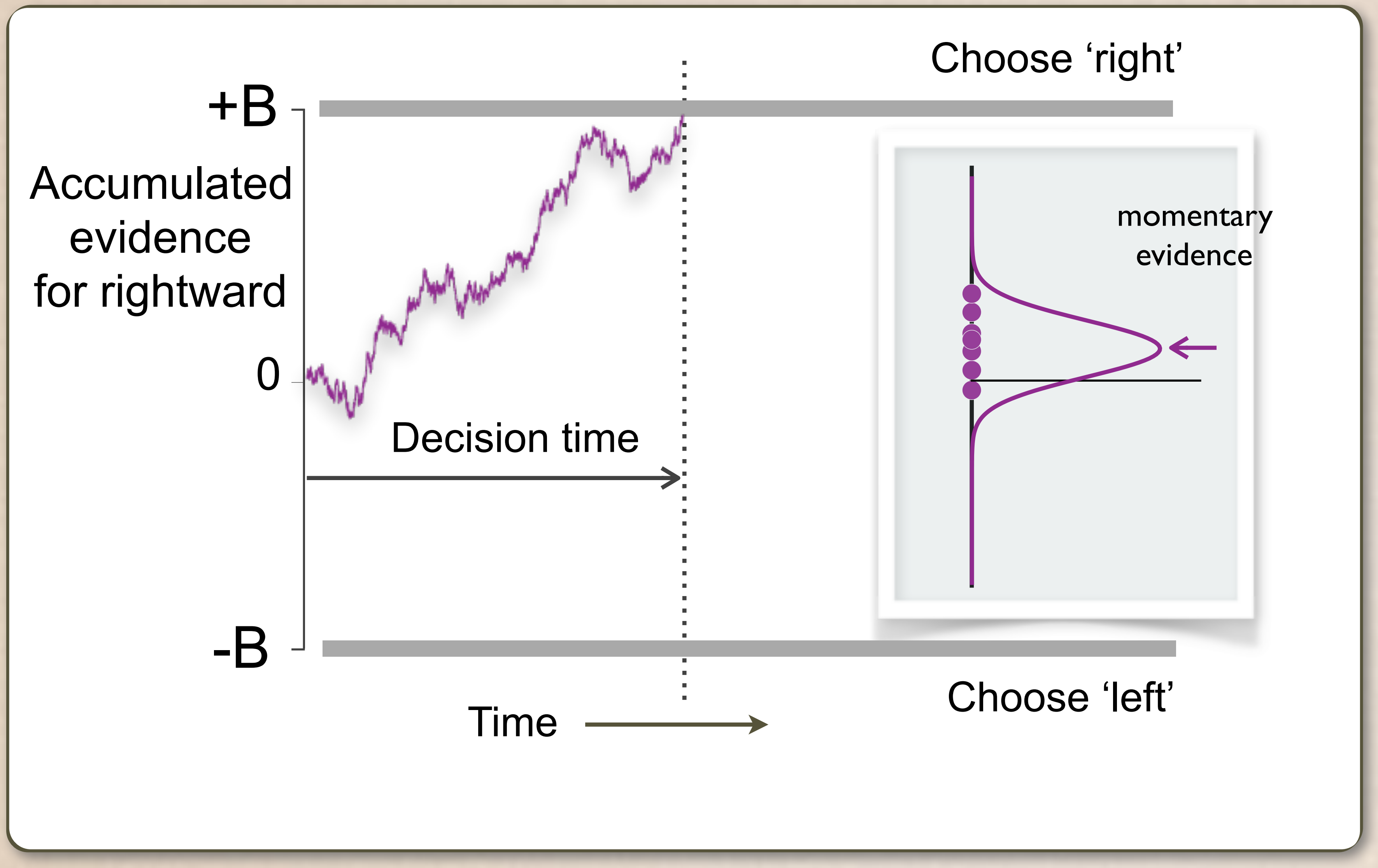

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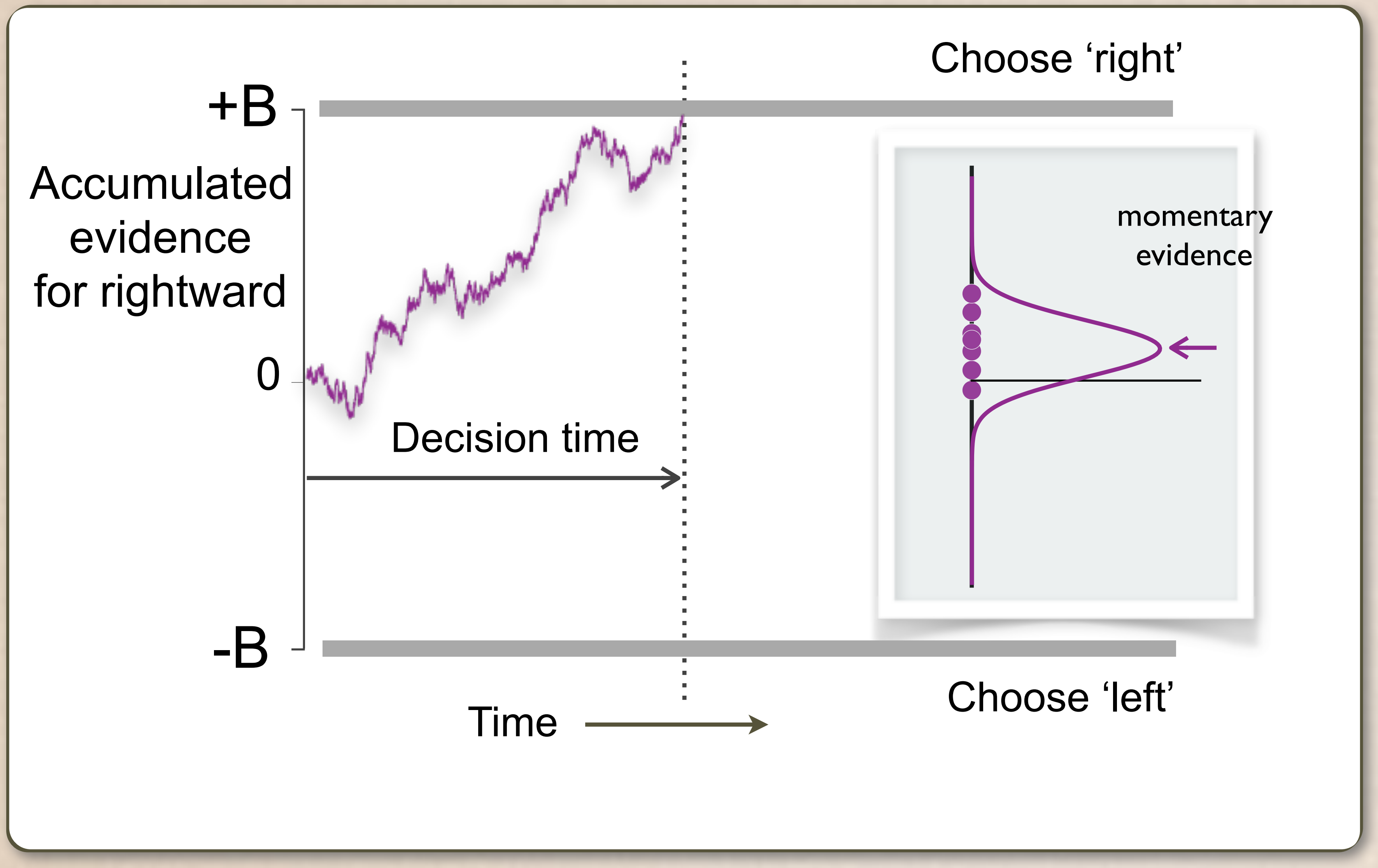
Motion task:

$\mu = \kappa \times$ Motion coherence

Drift diffusion model



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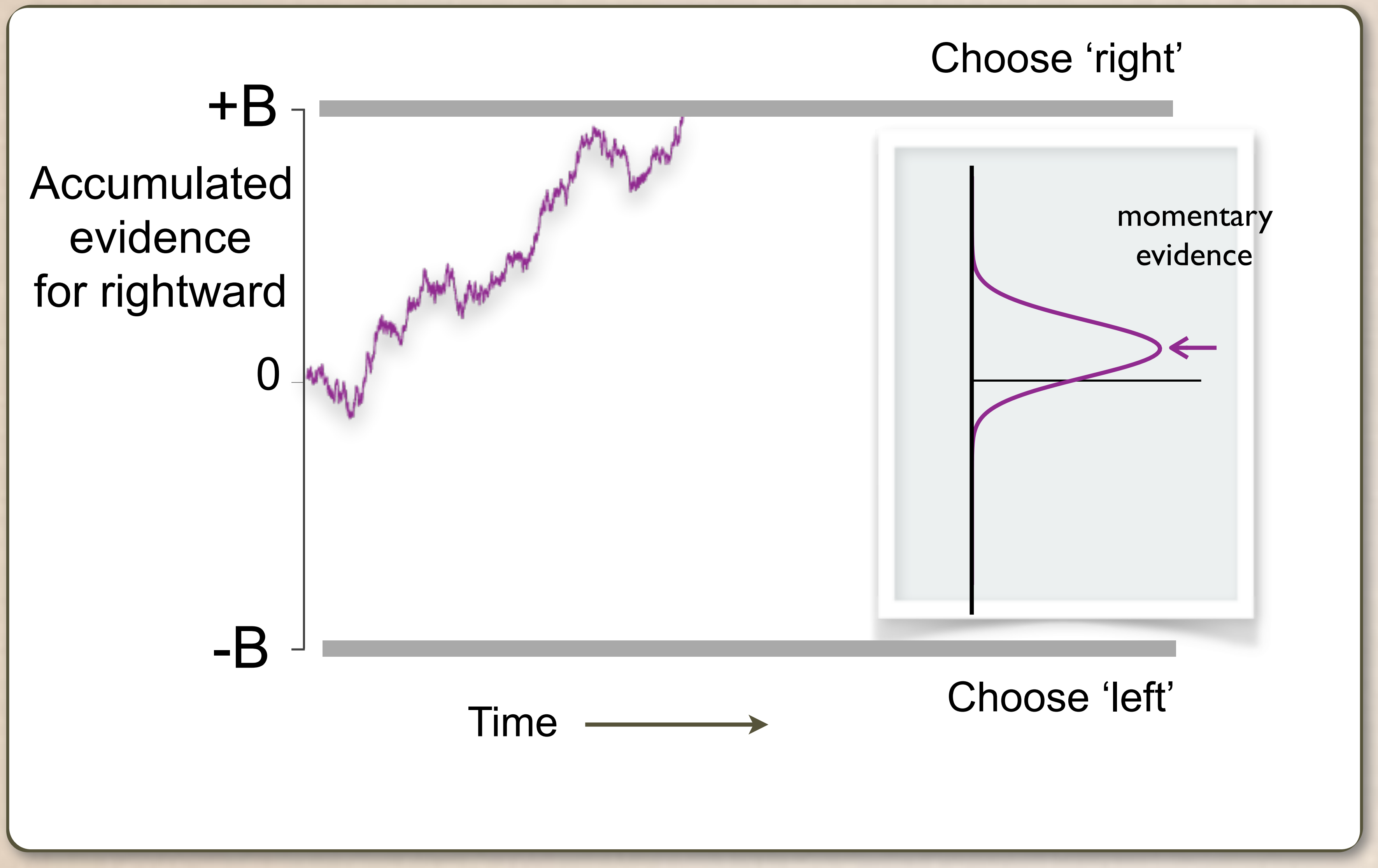
$\mu = \kappa \times$ Motion coherence

$|\mu|$ increases with motion strength

Drift diffusion model



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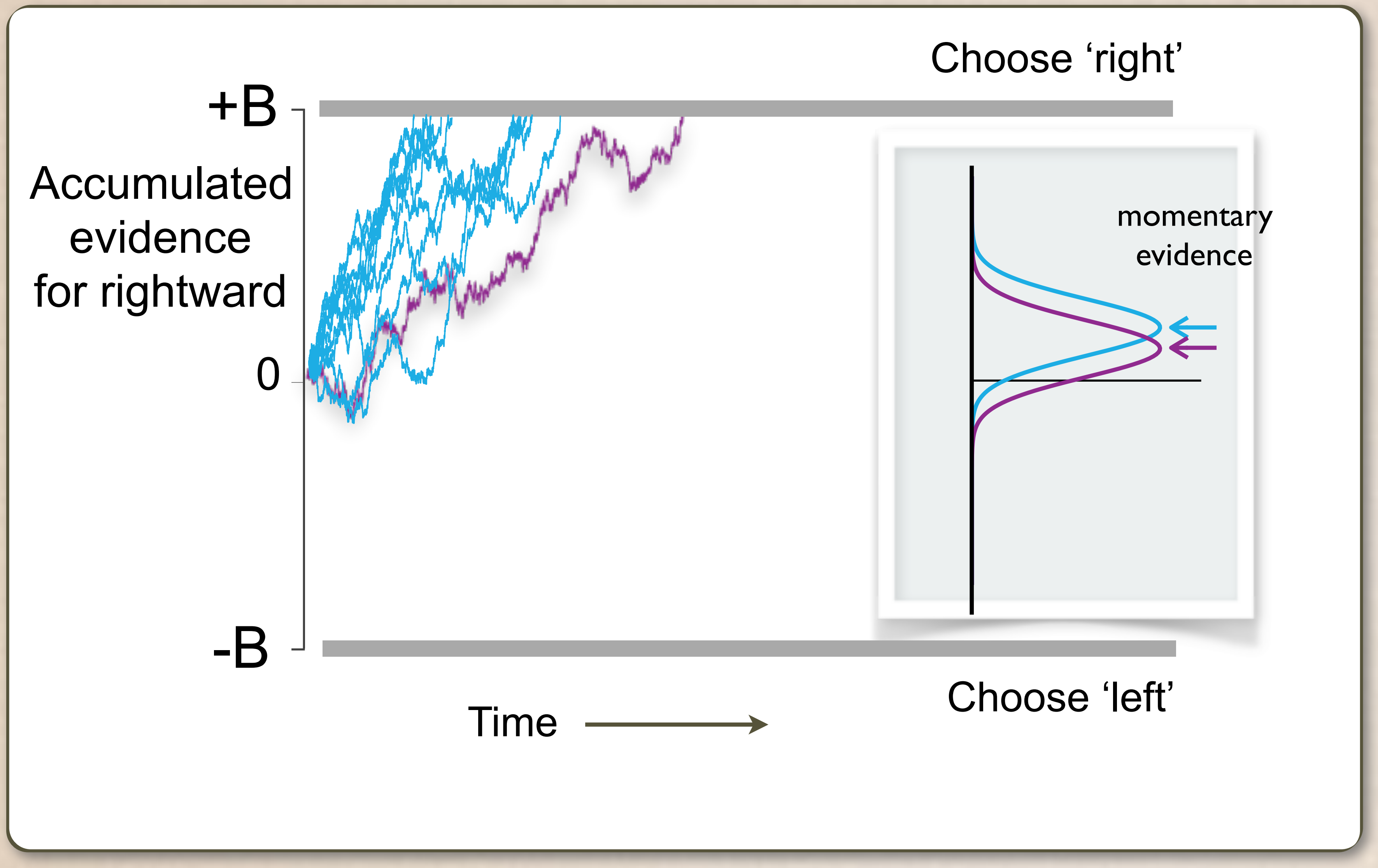

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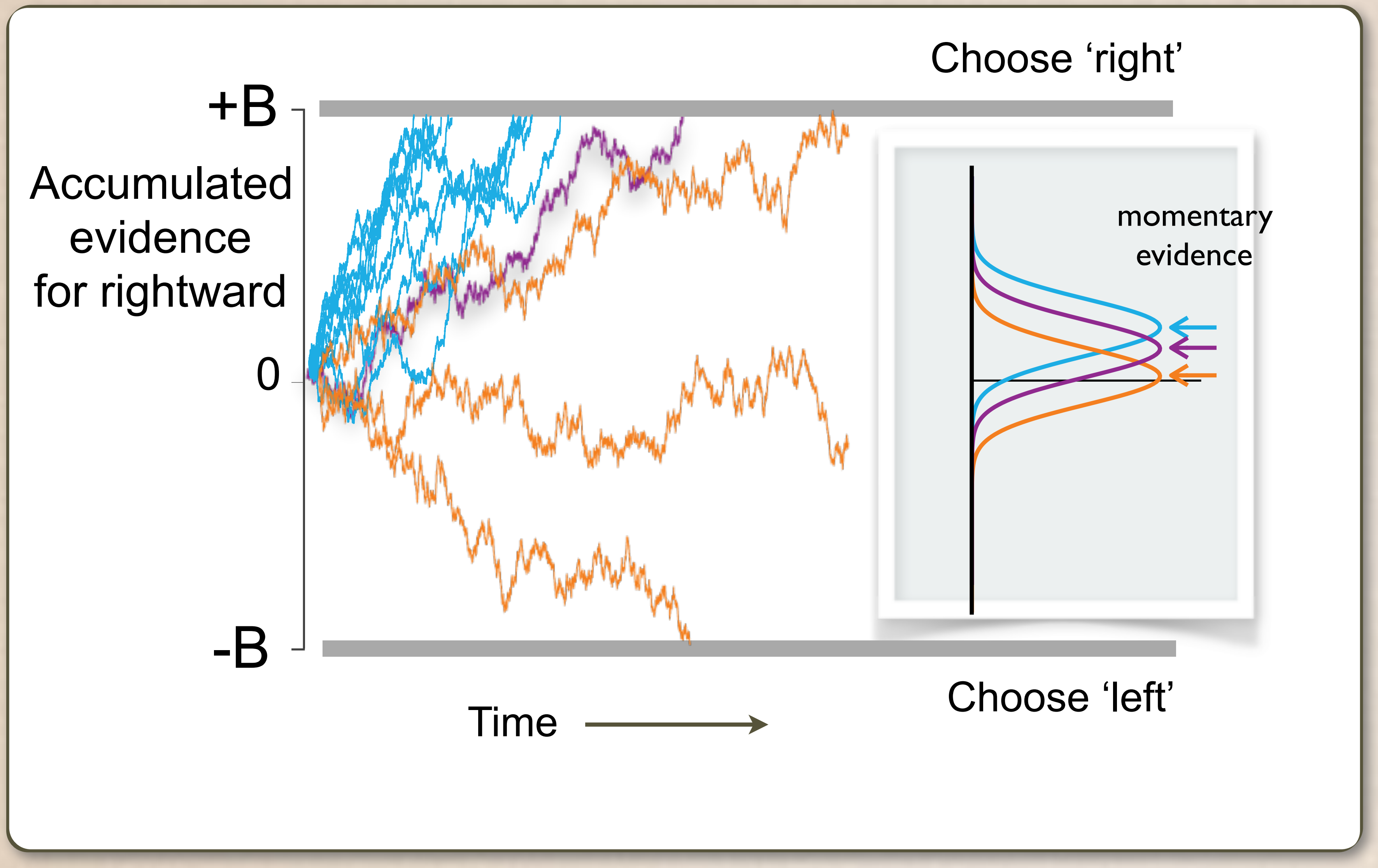

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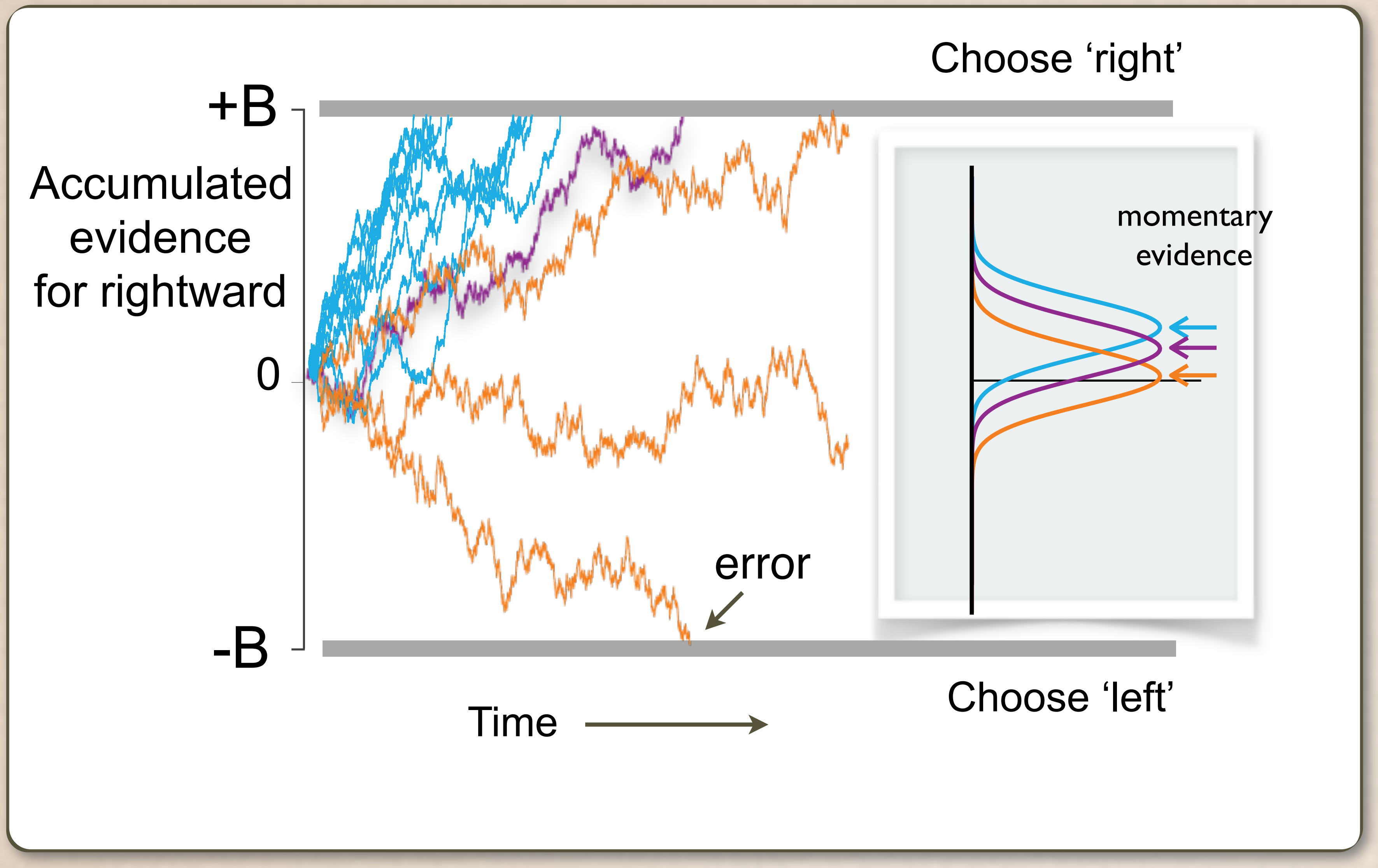

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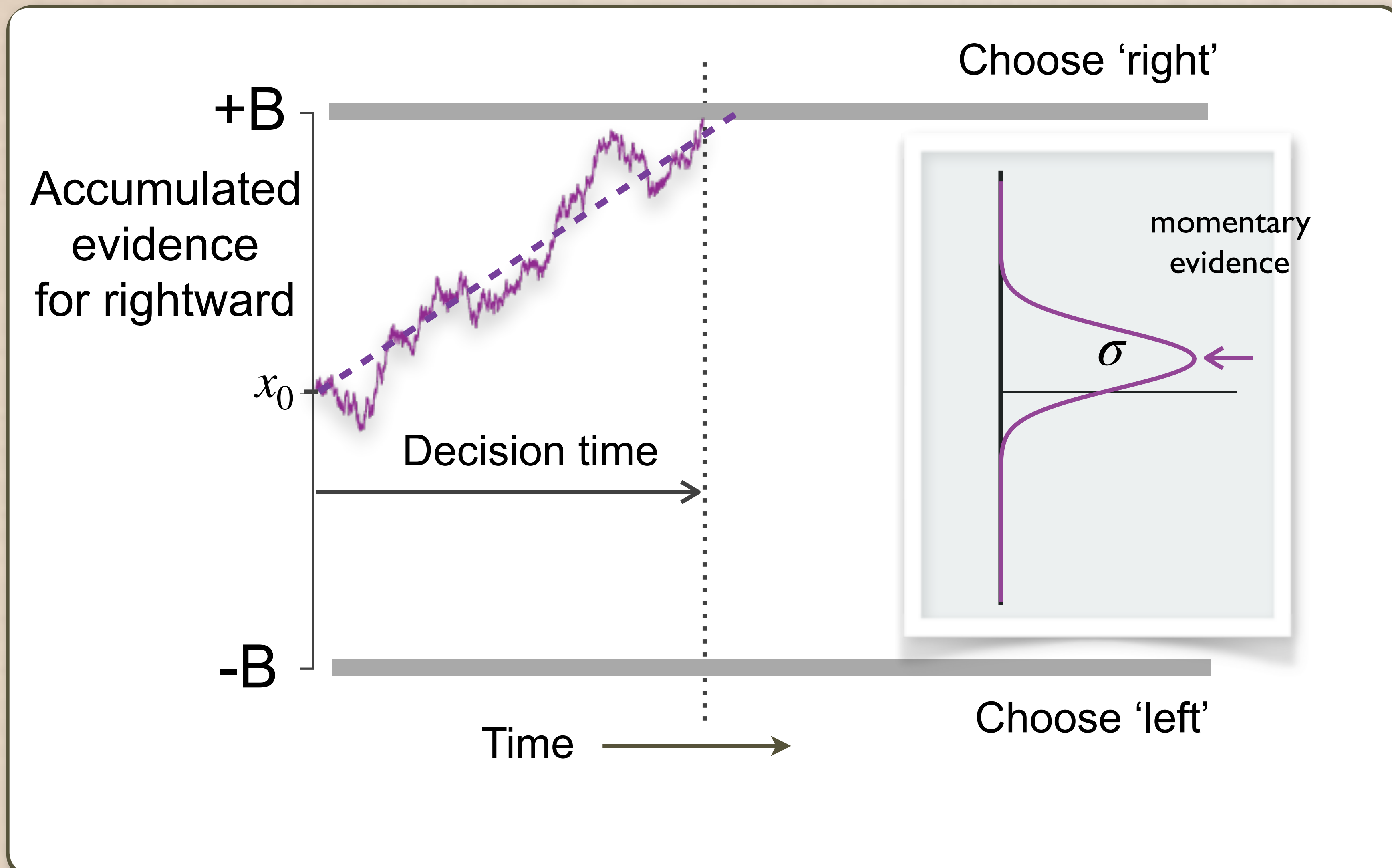
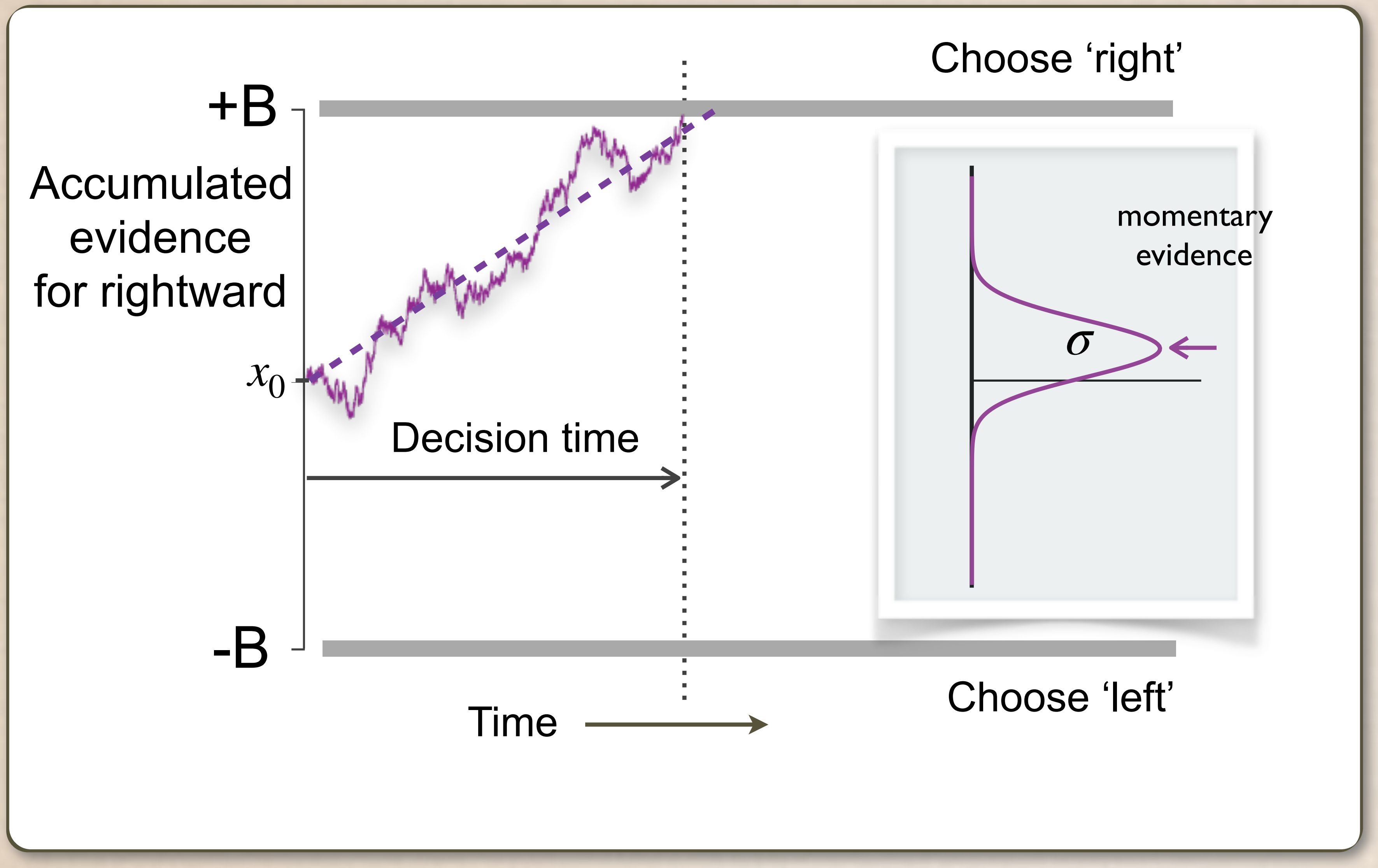


Parameters in the 'simple' version:

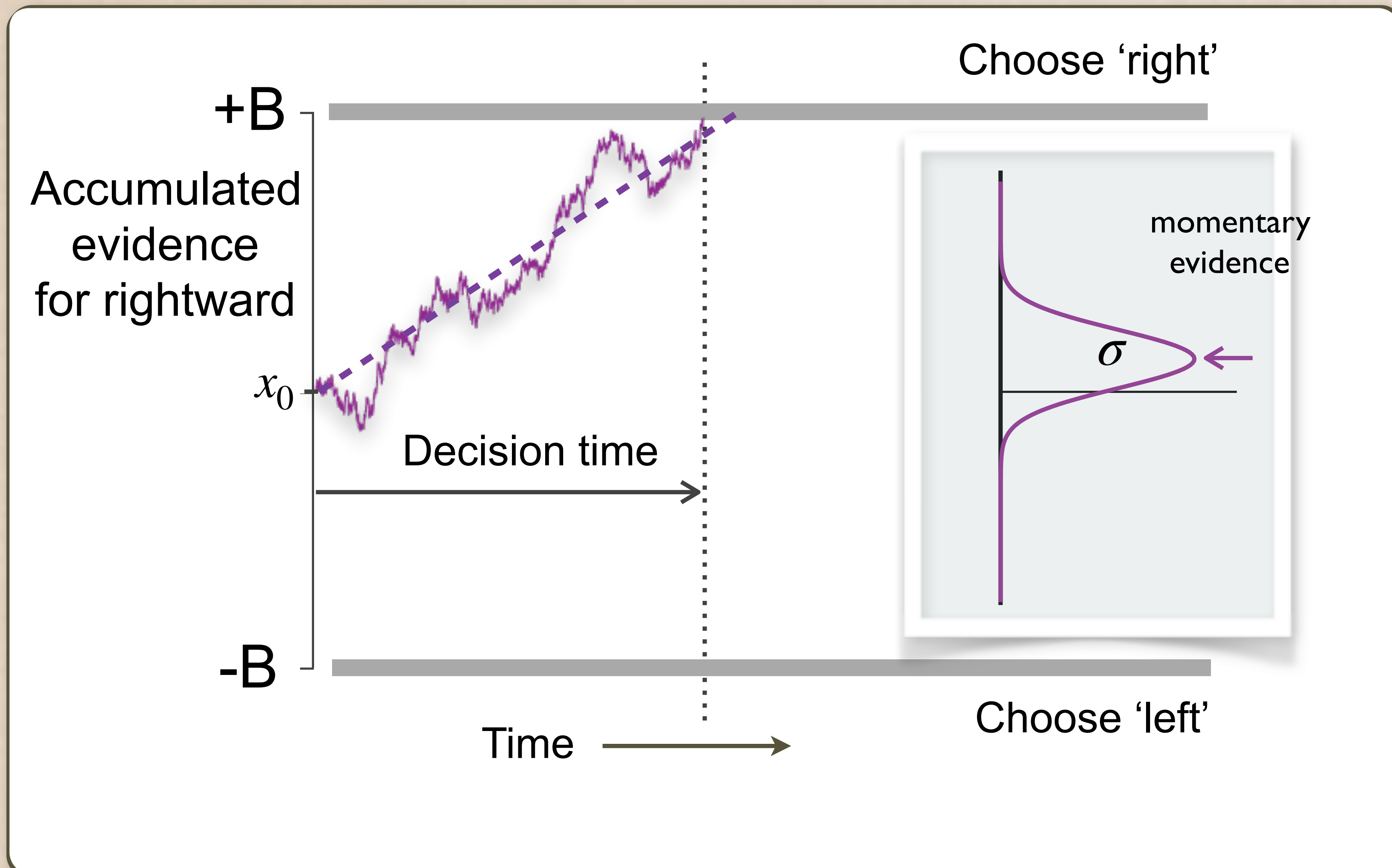
Drift diffusion model



Parameters in the 'simple' version:

- Drift rate, μ

Drift diffusion model



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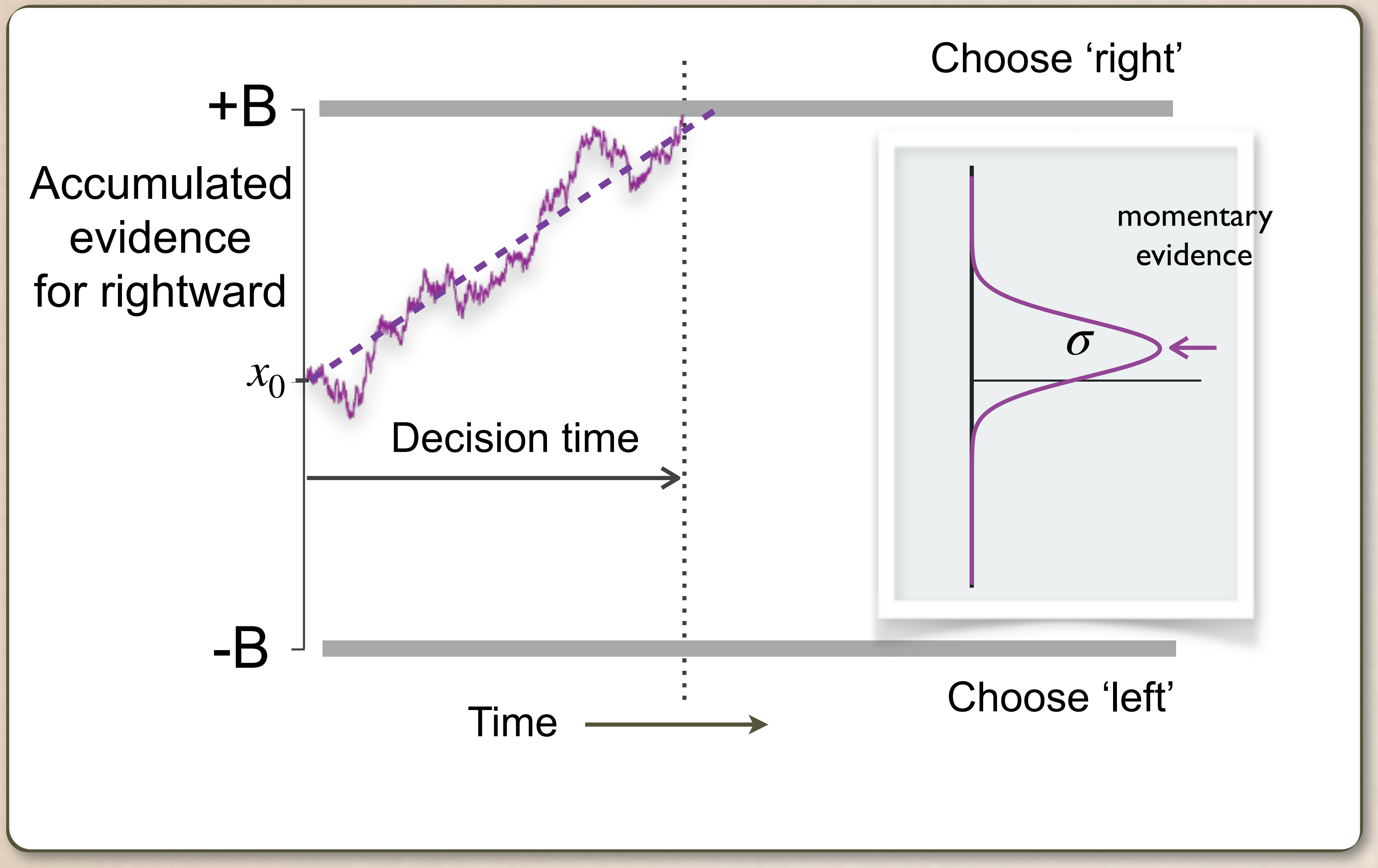
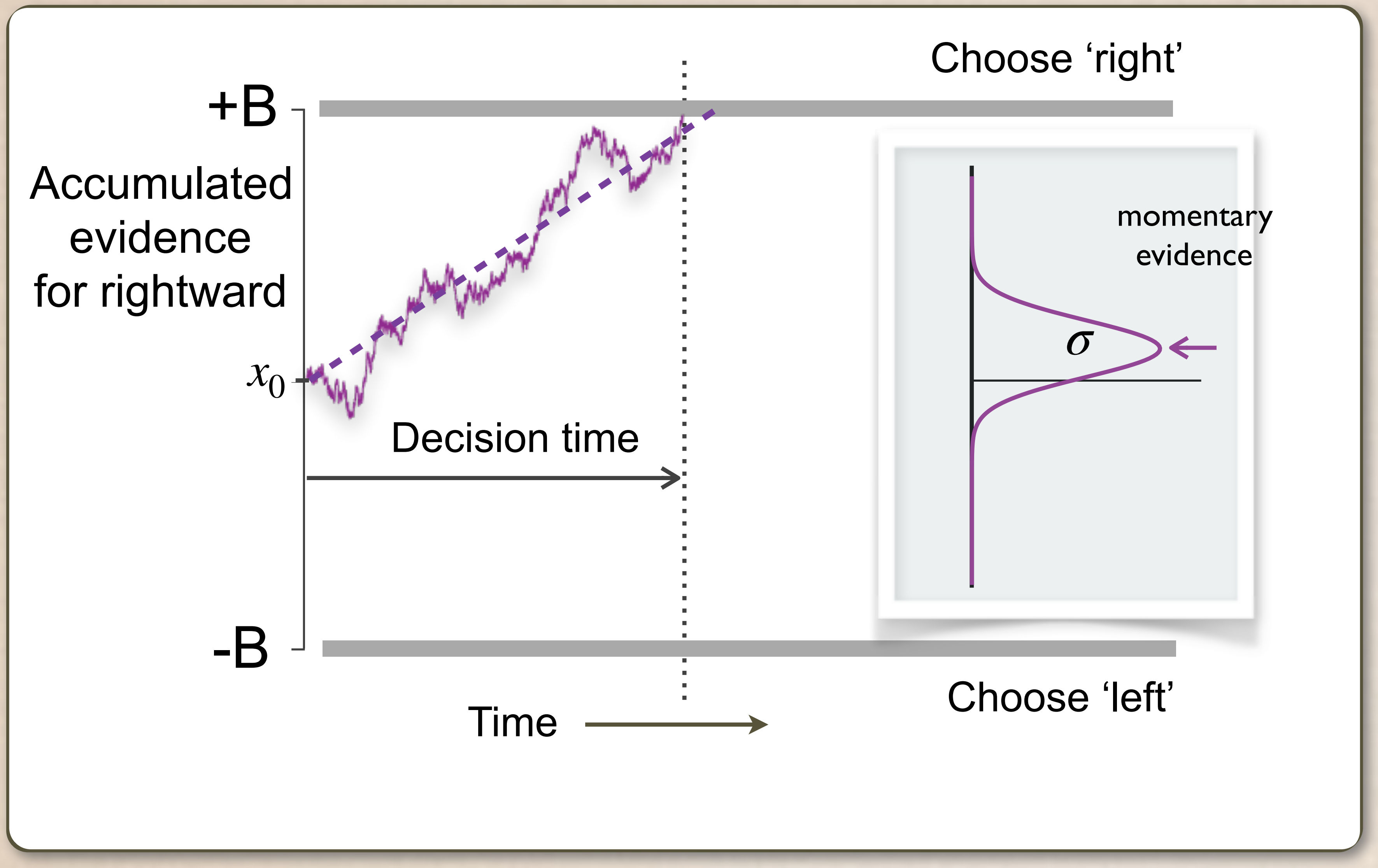


Parameters in the 'simple' version:

- Drift rate, μ
- Bound height, B

Drift diffusion model



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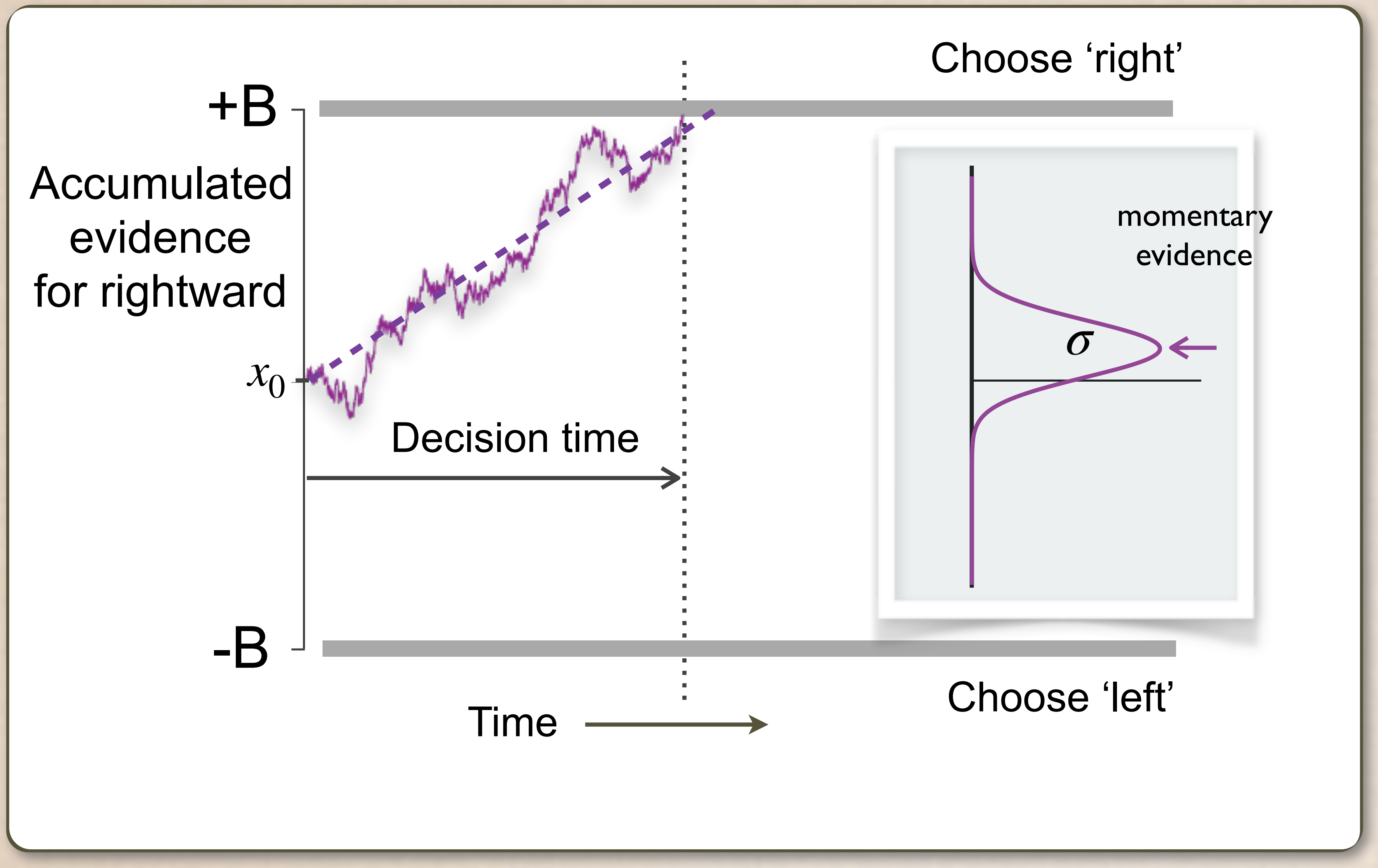

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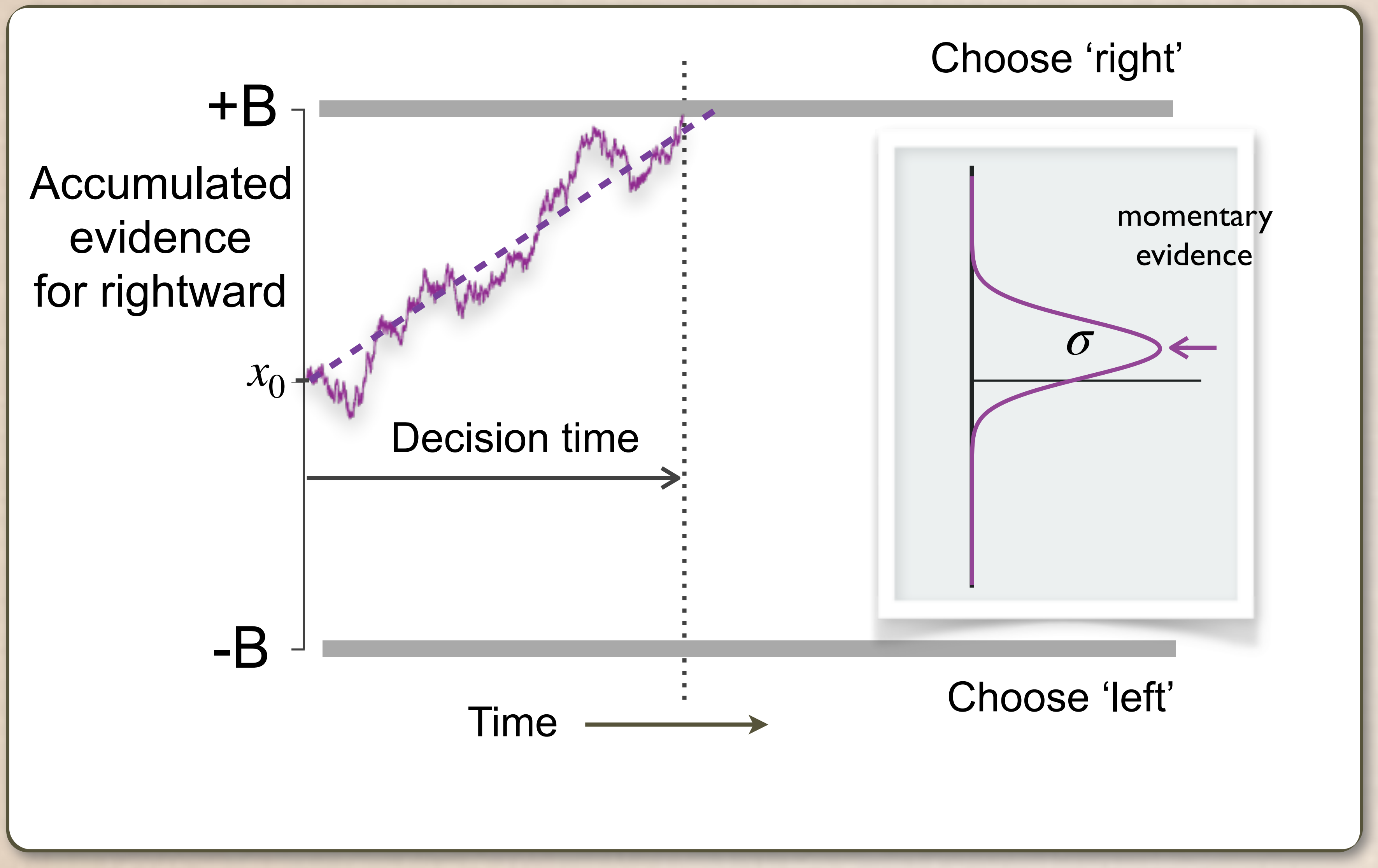

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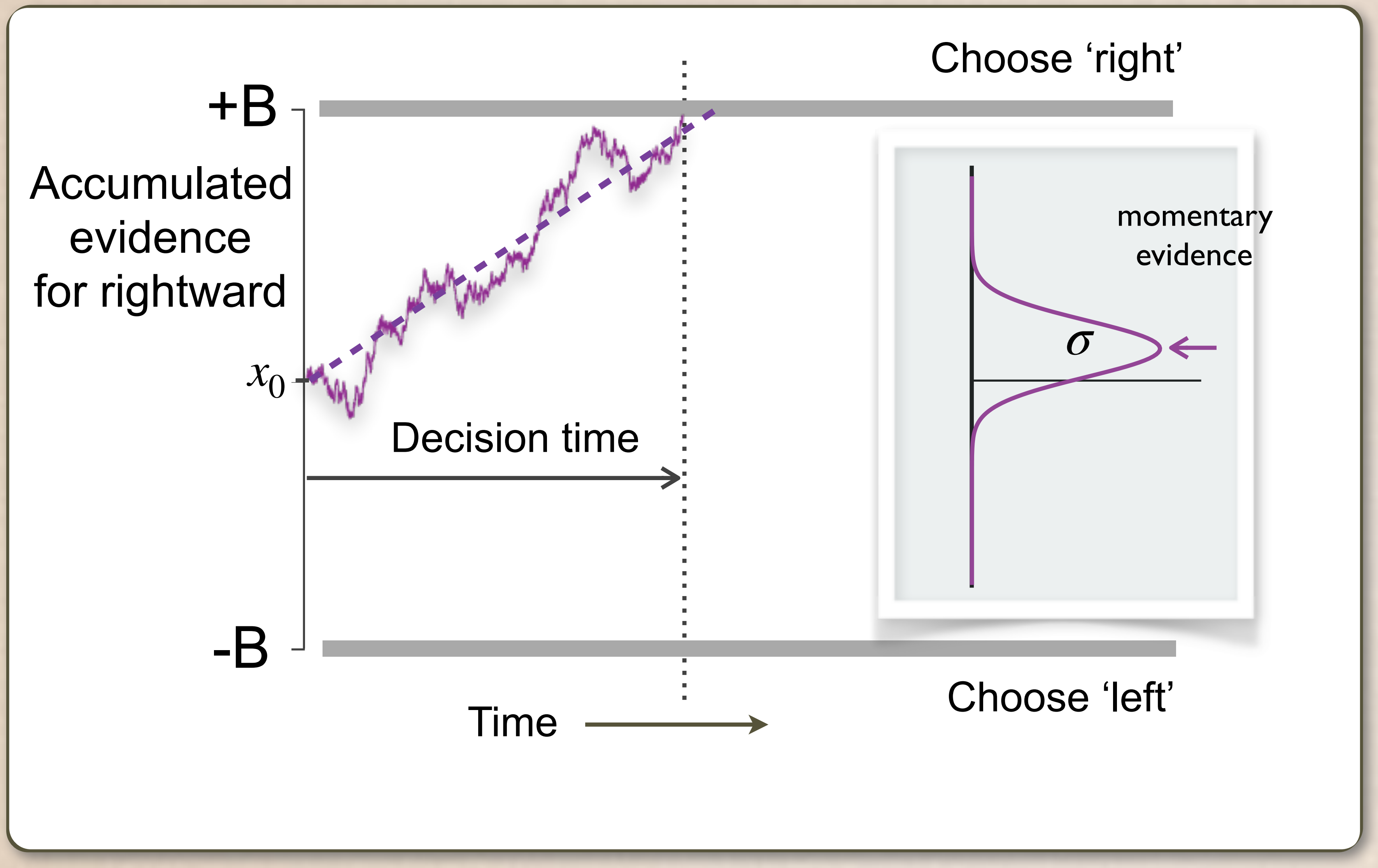
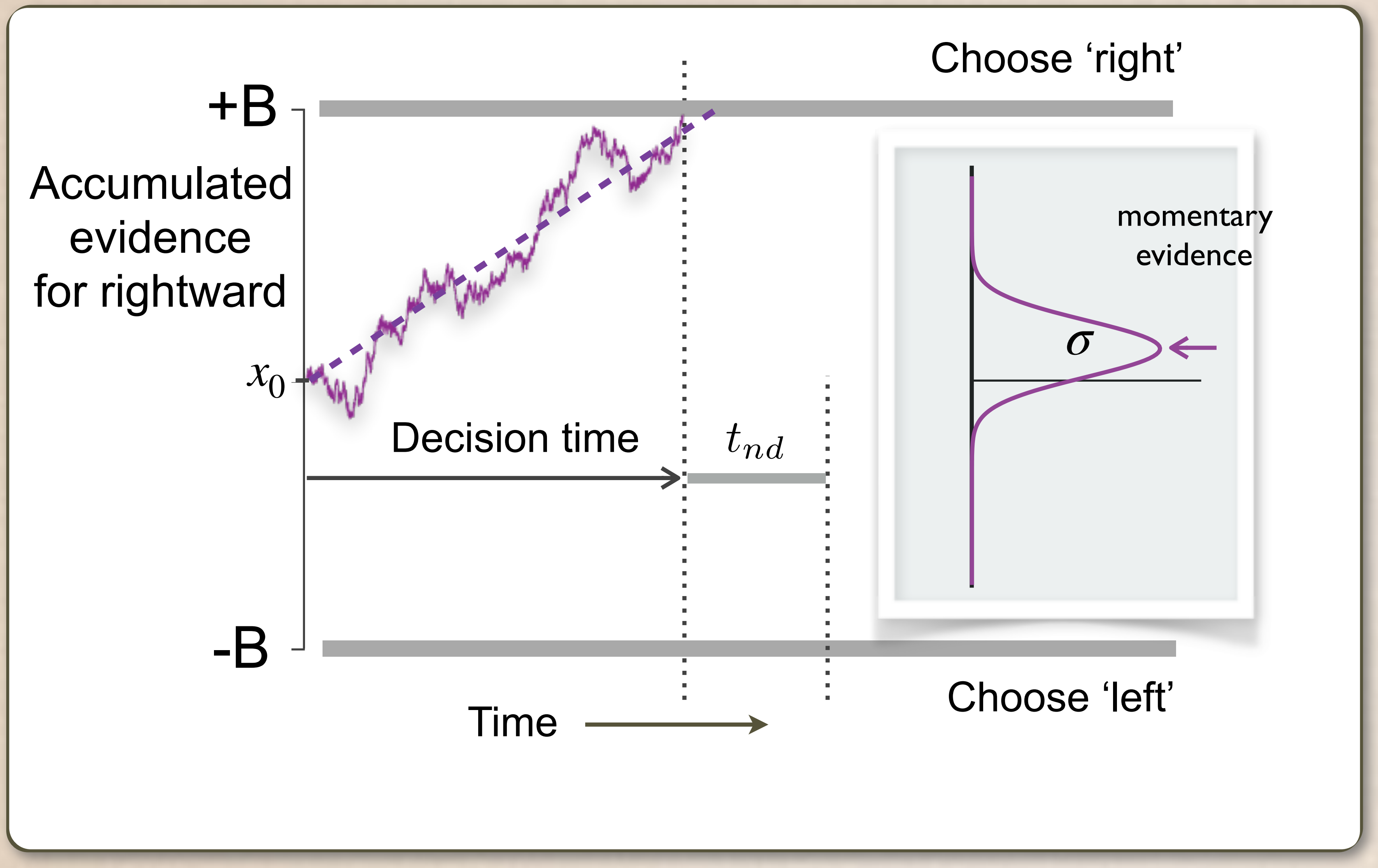


Parameters in the 'simple' version:

- Drift rate, μ
- Bound height, B
- Starting point, x_0
- Non-decision time, t_{nd}

Drift diffusion model



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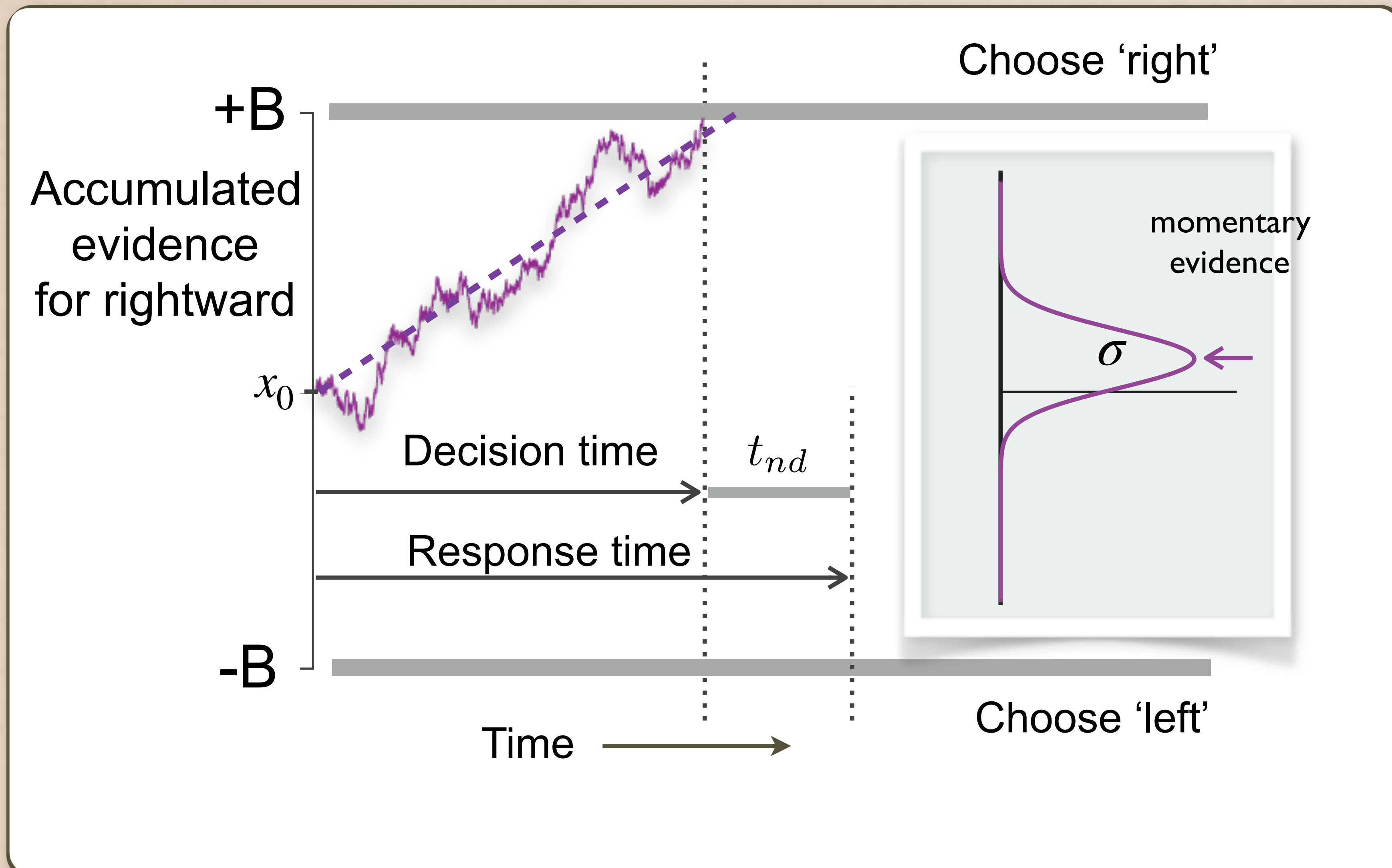

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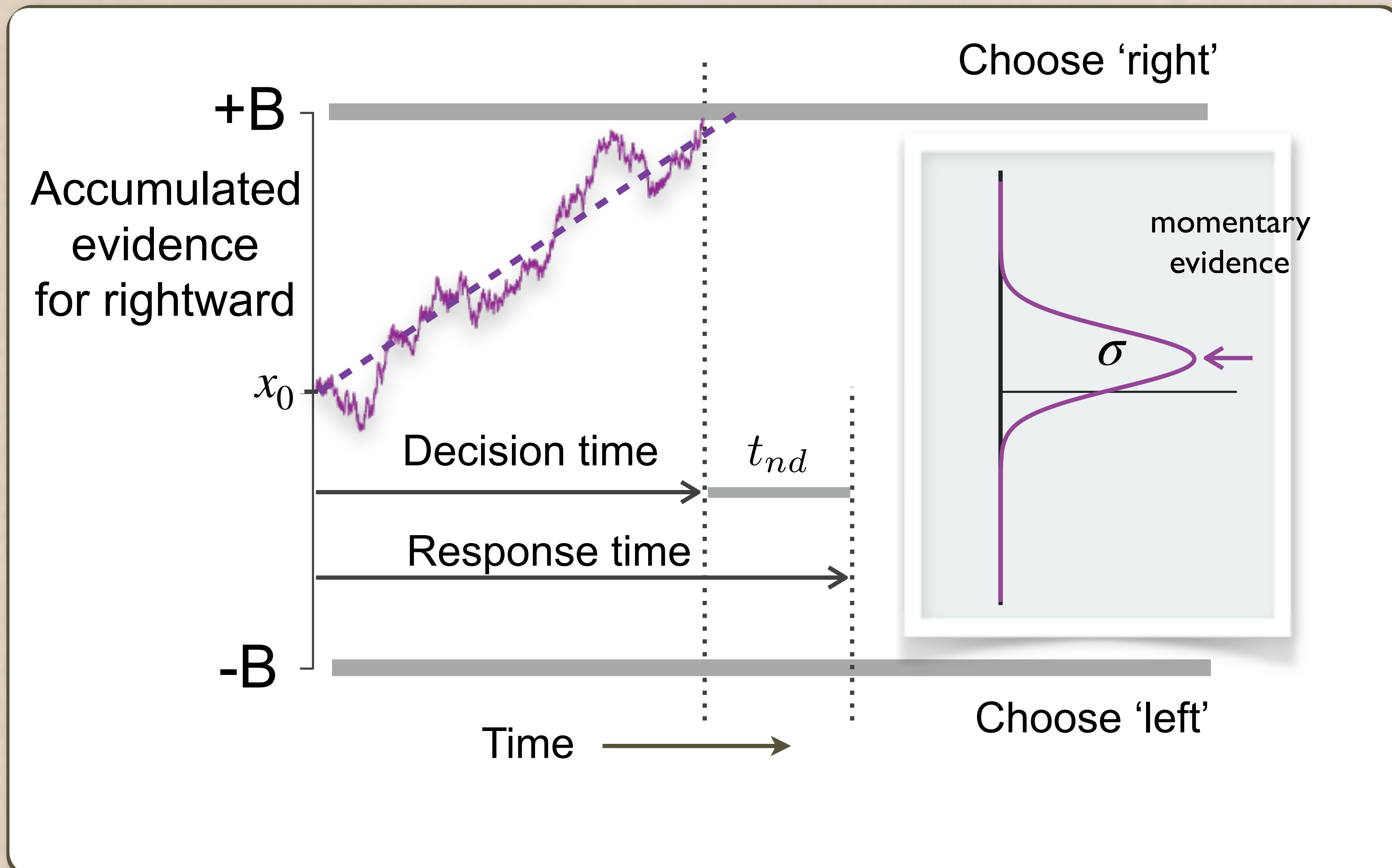
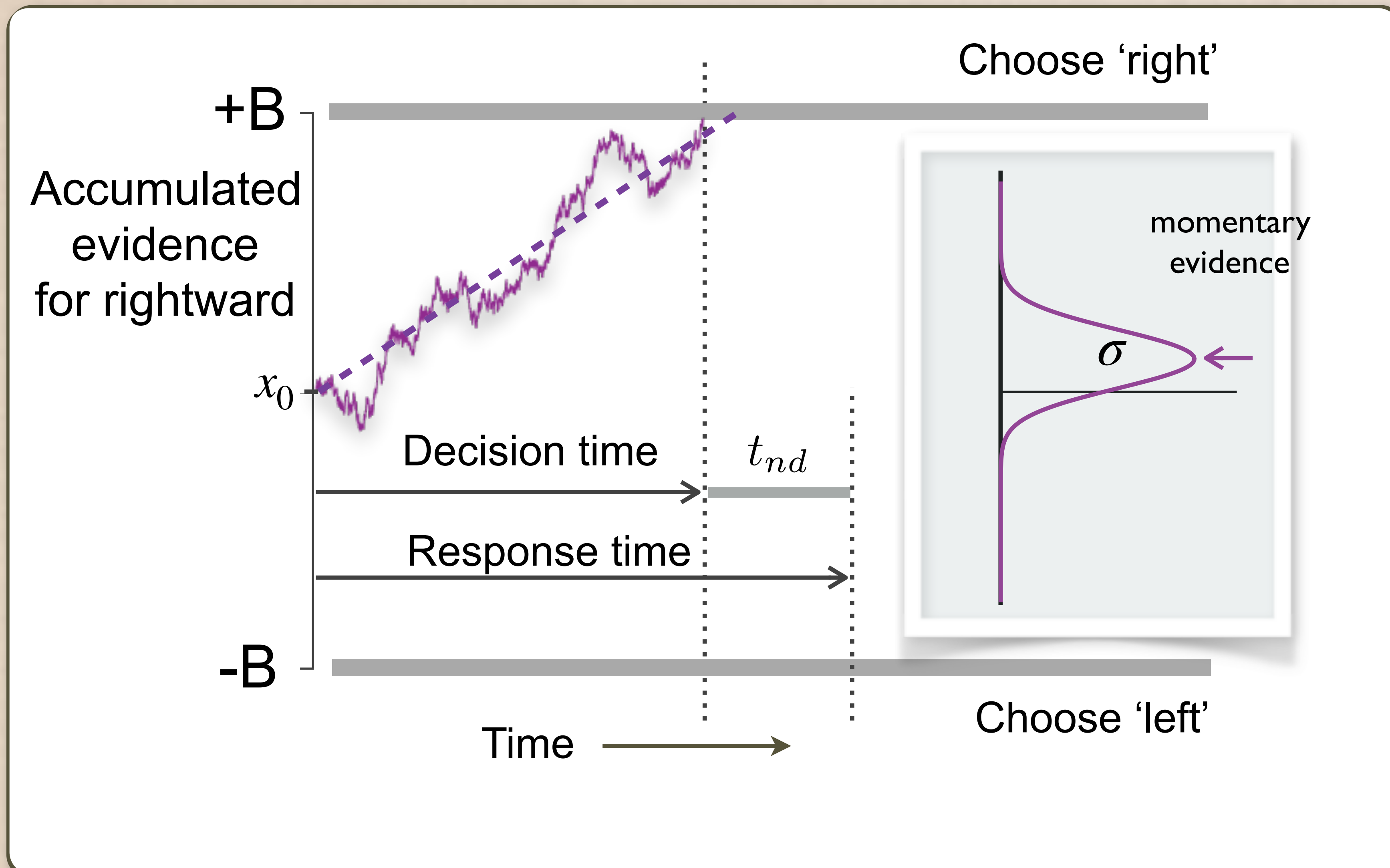
Drift diffusion model



Parameters in the 'simple' version:

- Drift rate, μ
- Bound height, B
- Starting point, x_0
- Non-decision time, t_{nd}
- Diffusion coefficient, σ

Drift diffusion model



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- Bound height, B
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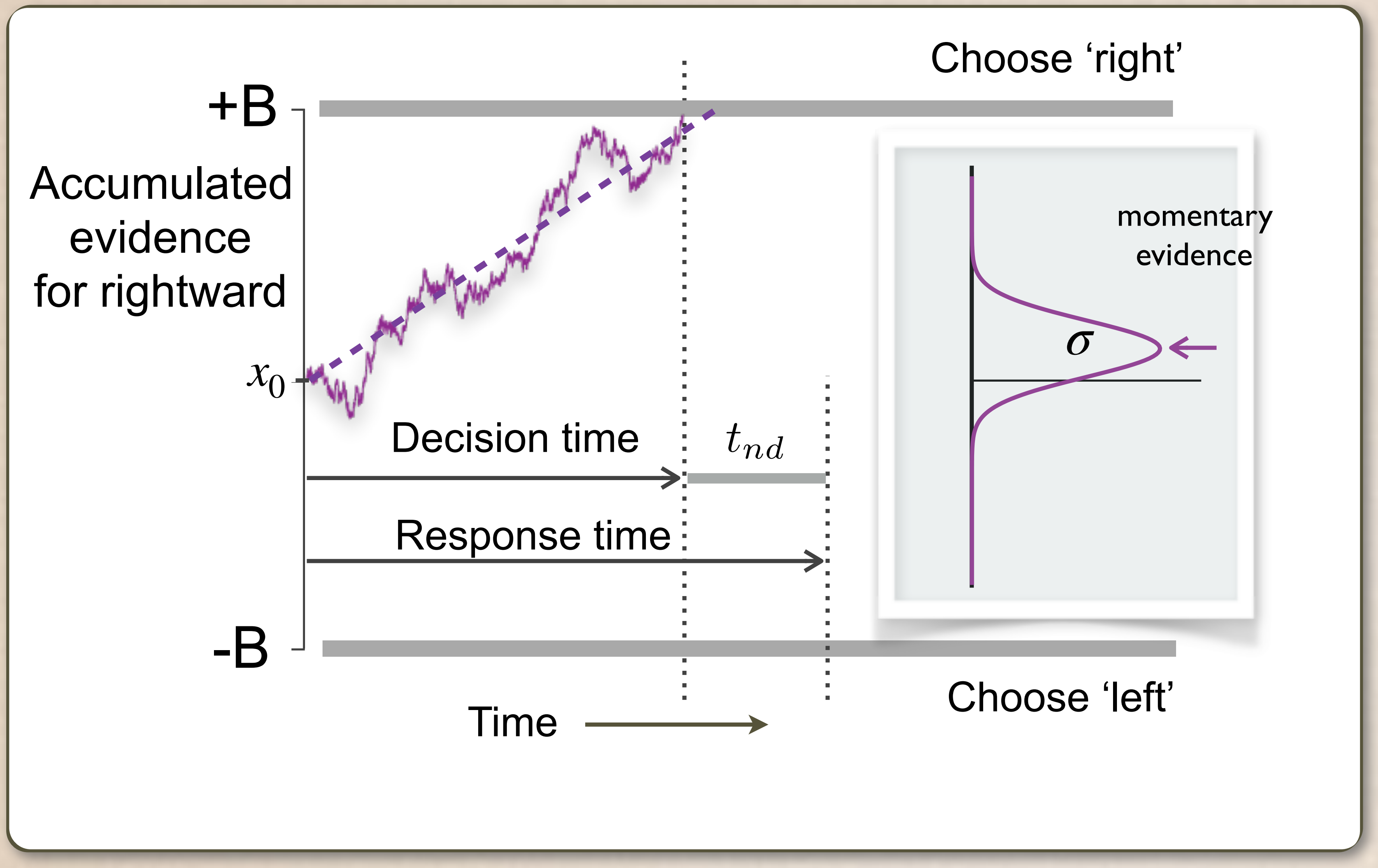
Drift diffusion model

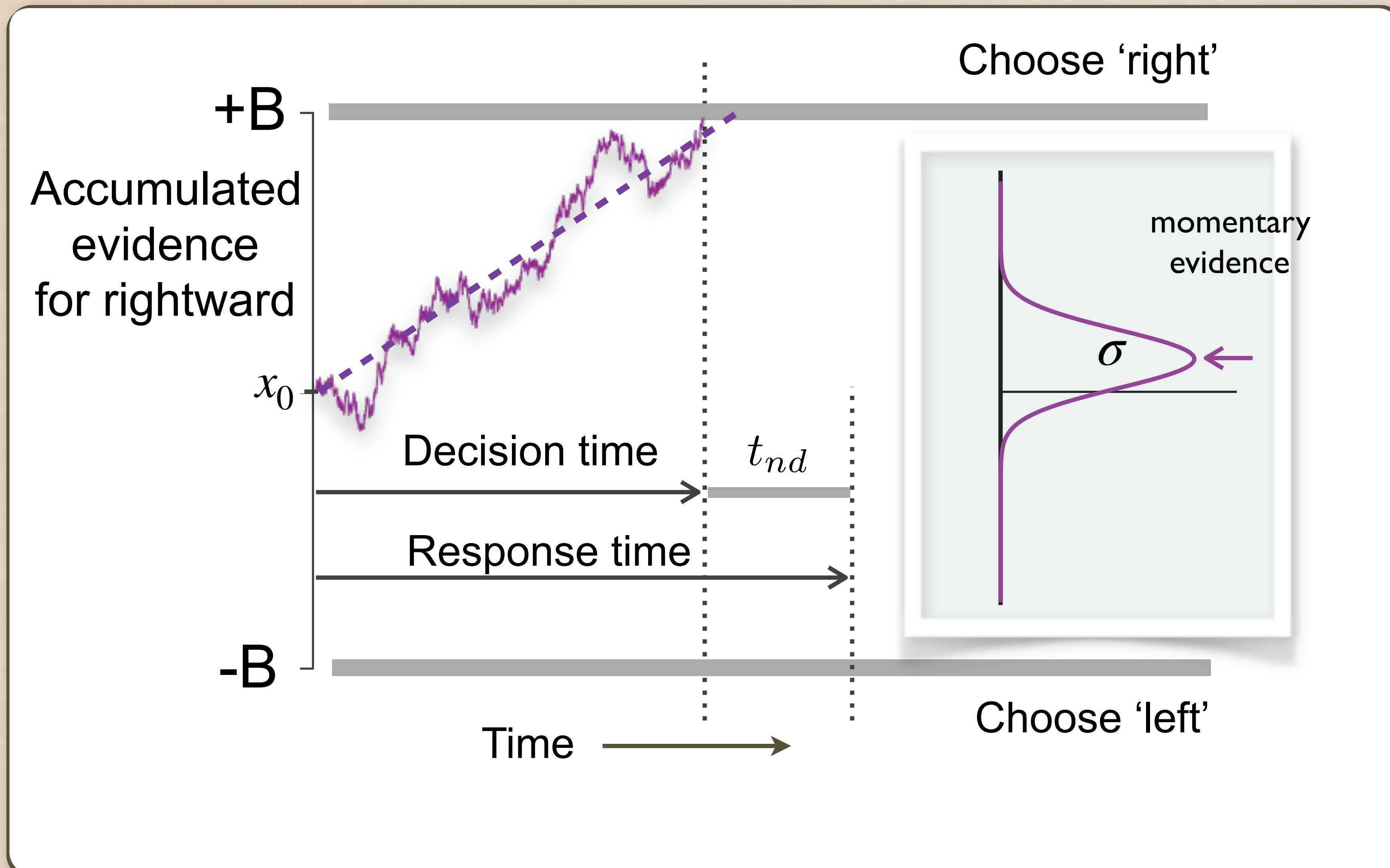


Parameters in the 'simple' version:

- Drift rate, μ
- Bound height, B
- Starting point, x_0
- Non-decision time, t_{nd}
- ~~Diffusion coefficient, σ~~

$$\sigma = 1$$

Drift diffusion model



Parameters in the 'simple' version:

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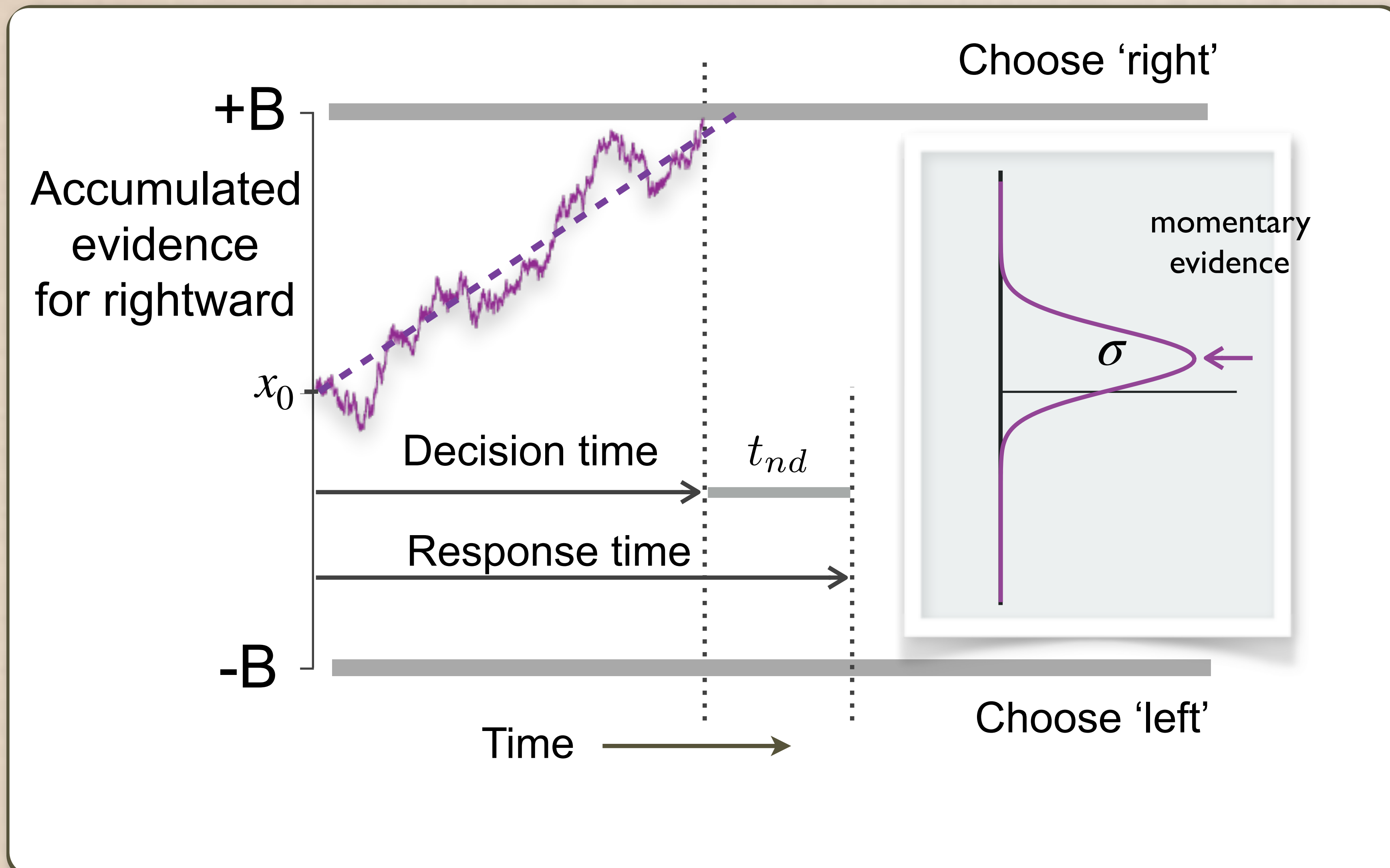
- Bound height, B
- Starting point, x_0
- Non-decision time, t_{nd}
- ~~Diffusion coefficient, σ~~

$$\sigma = 1 \times \mathbf{2}$$

Drift diffusion model

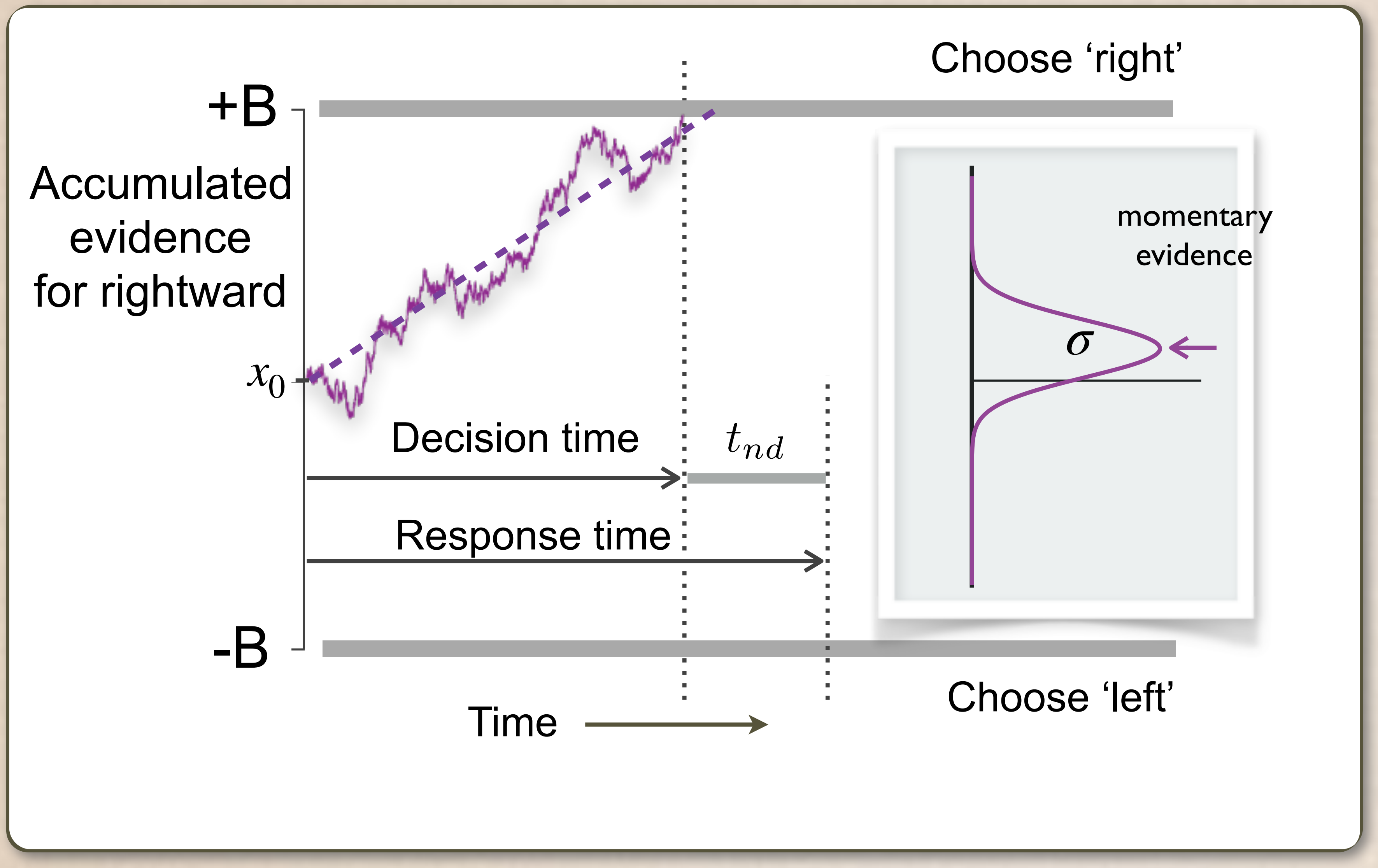


Parameters in the 'simple' version:

- Drift rate, μ $\times$ **2**
- Bound height, B $\times$ **2**
- Starting point, x_0 $\times$ **2**
- Non-decision time, t_{nd}
- ~~Diffusion coefficient, σ~~

$\sigma = 1$ $\times$ **2**

Drift diffusion model



Parameters in the 'simple' version:

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- Bound height, B
- Starting point, x_0
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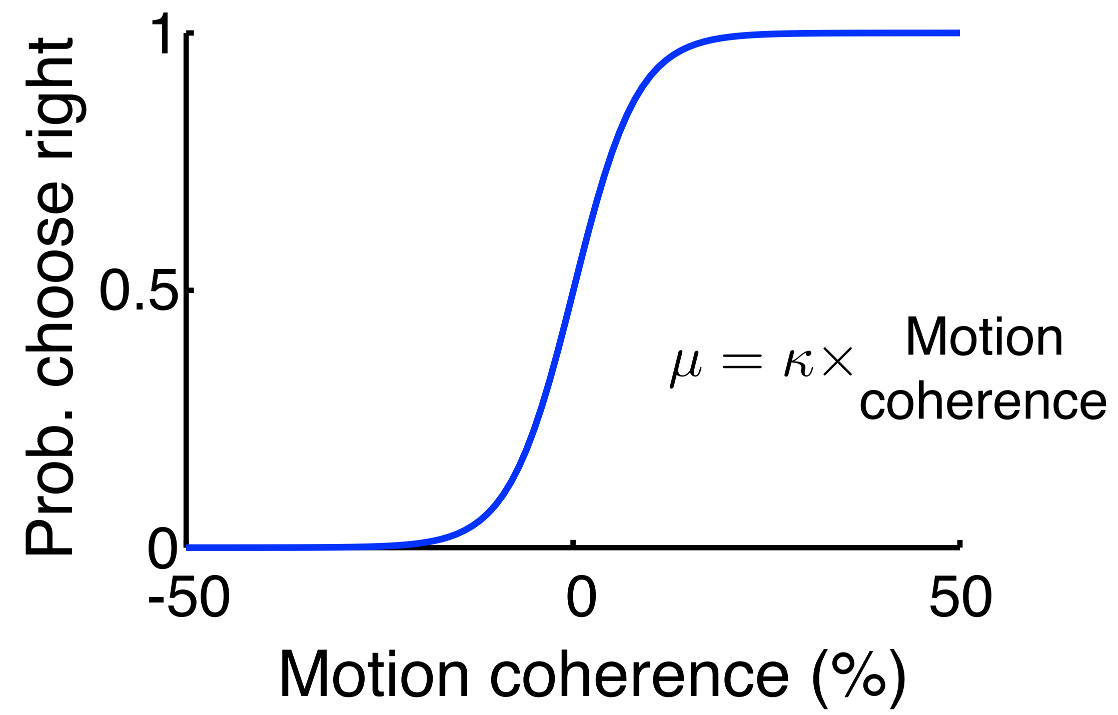
Analytic solution to the choice and response times

Probability of terminating at the upper bound:

$$p_+ = \frac{1}{1 + e^{-2\mu B}}$$



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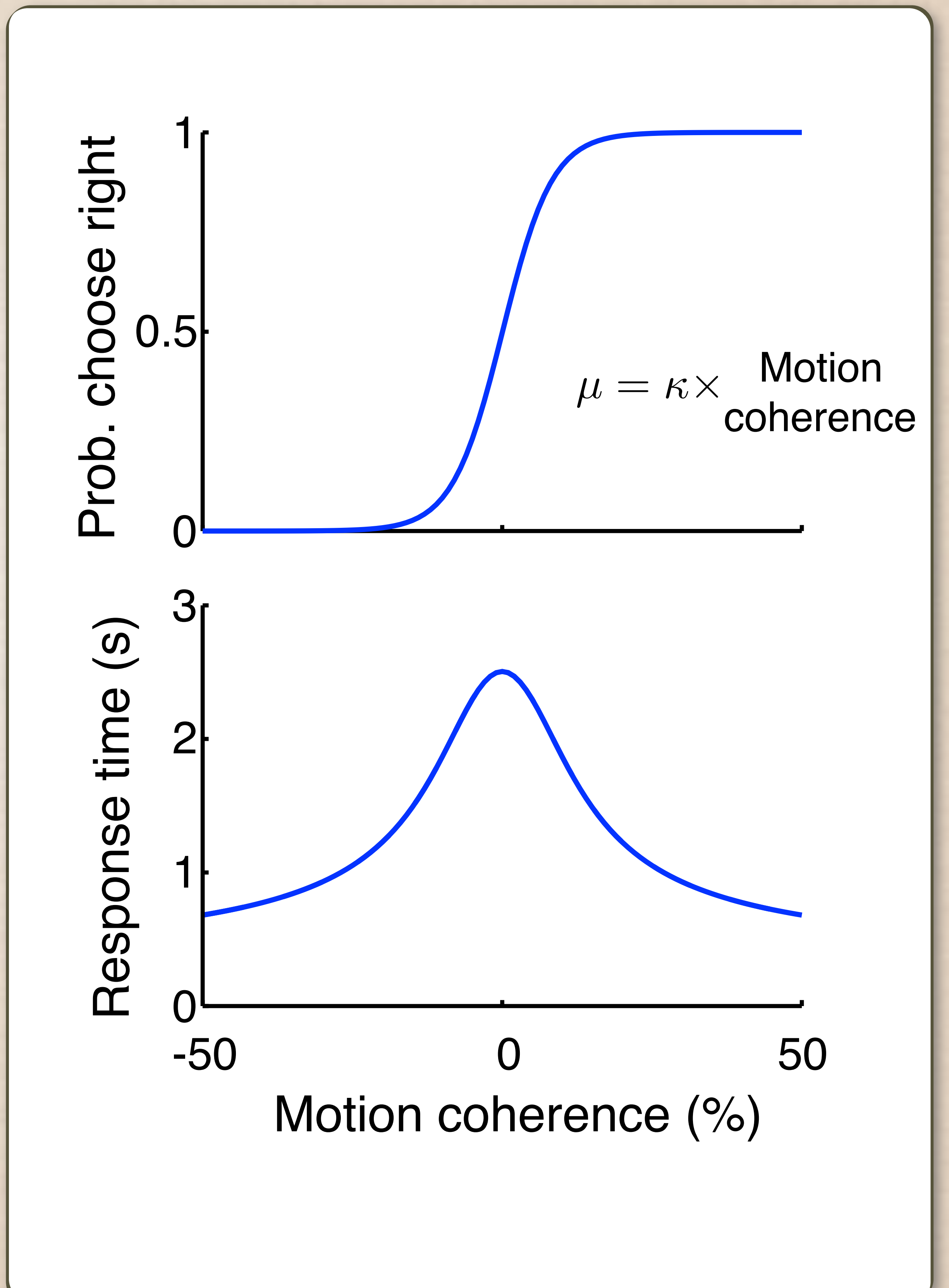

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Average response time:

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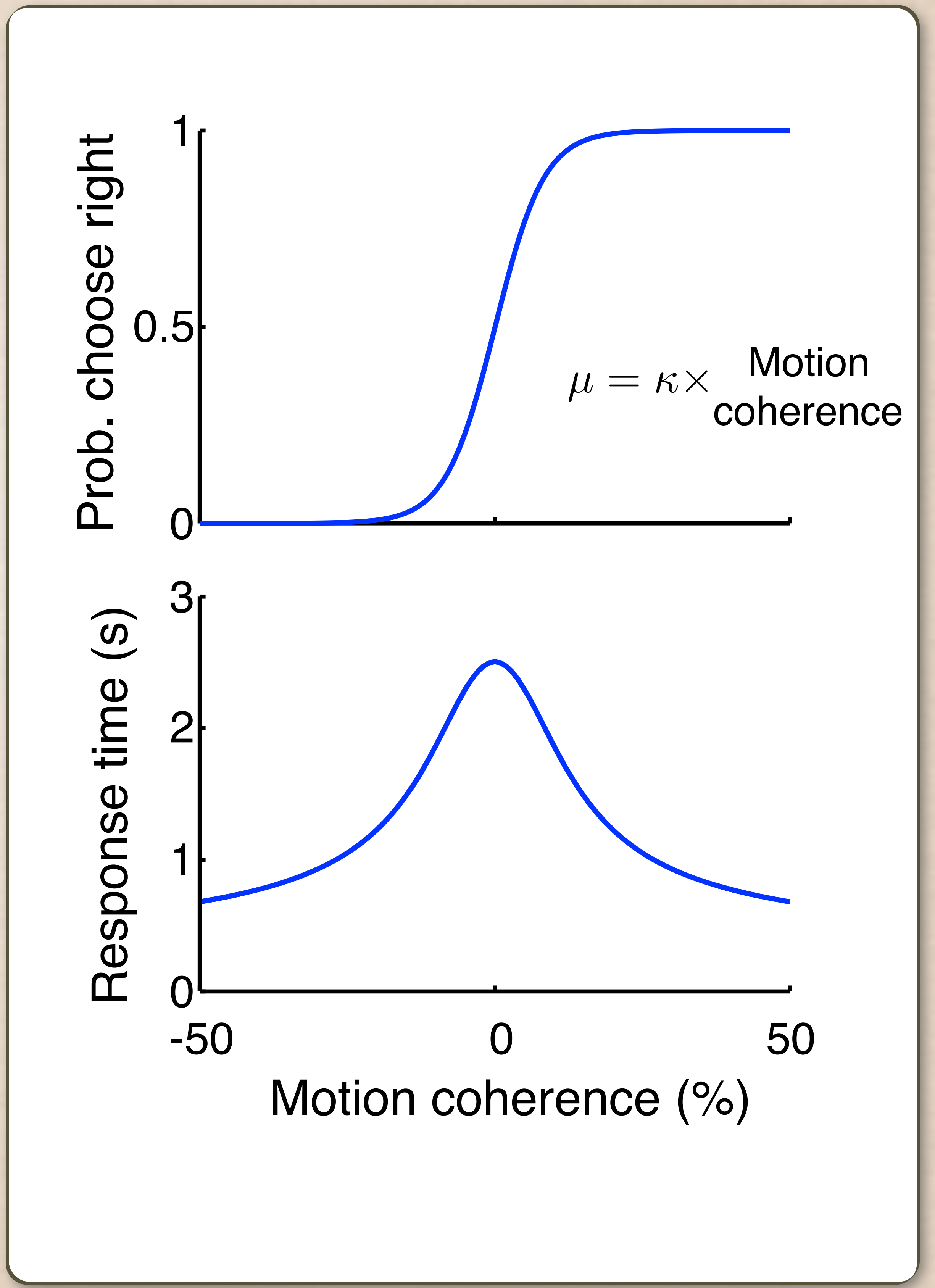

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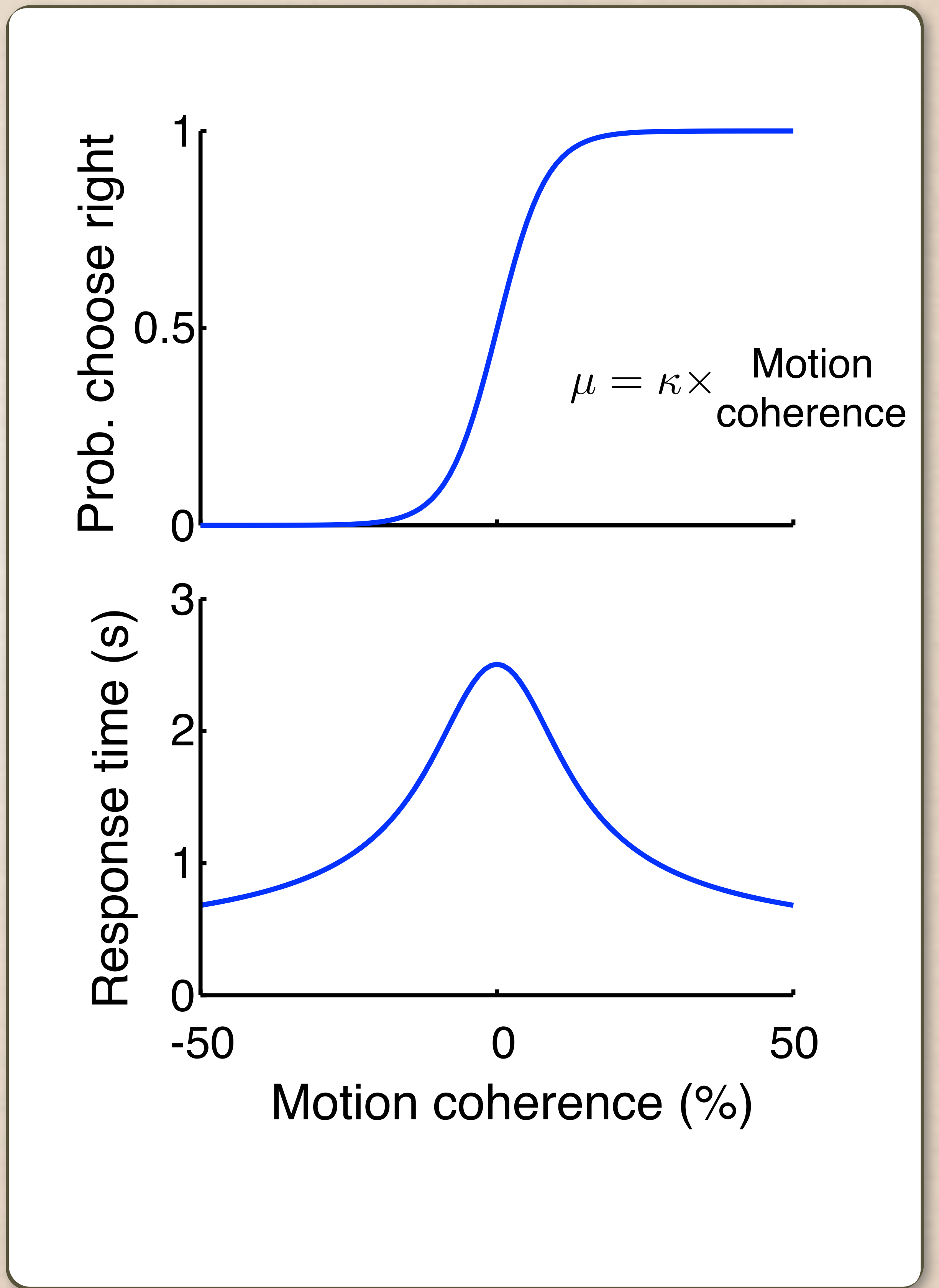

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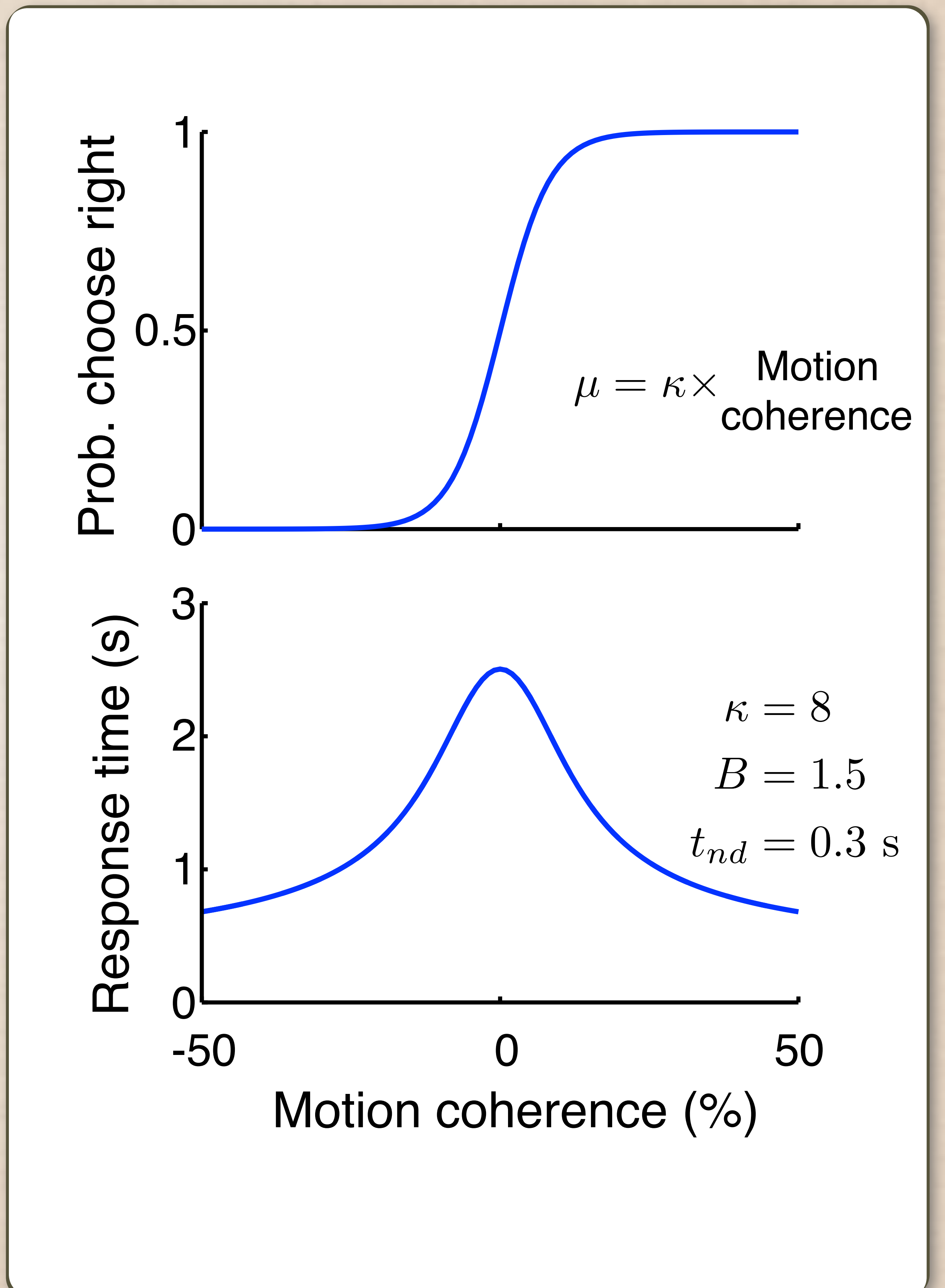
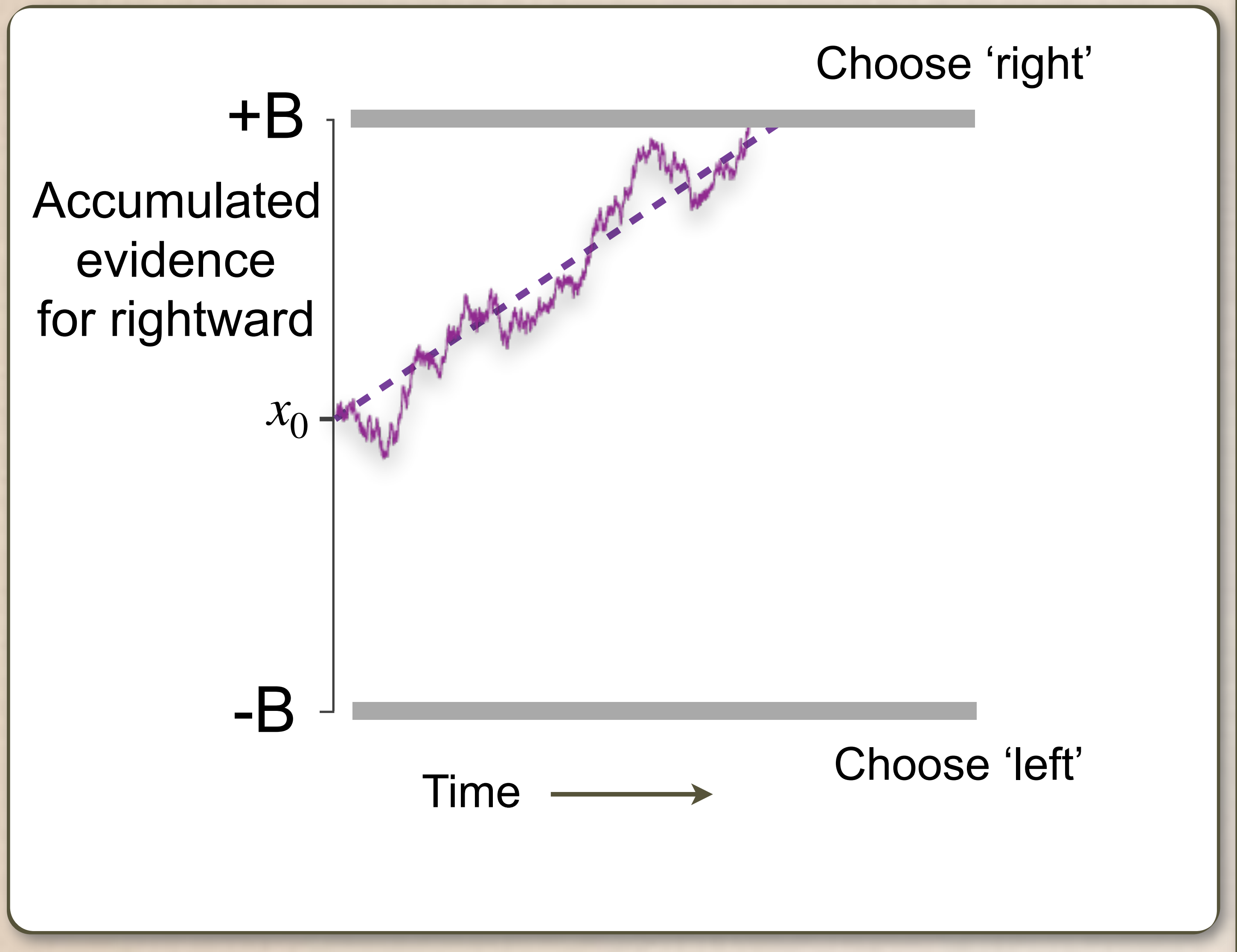
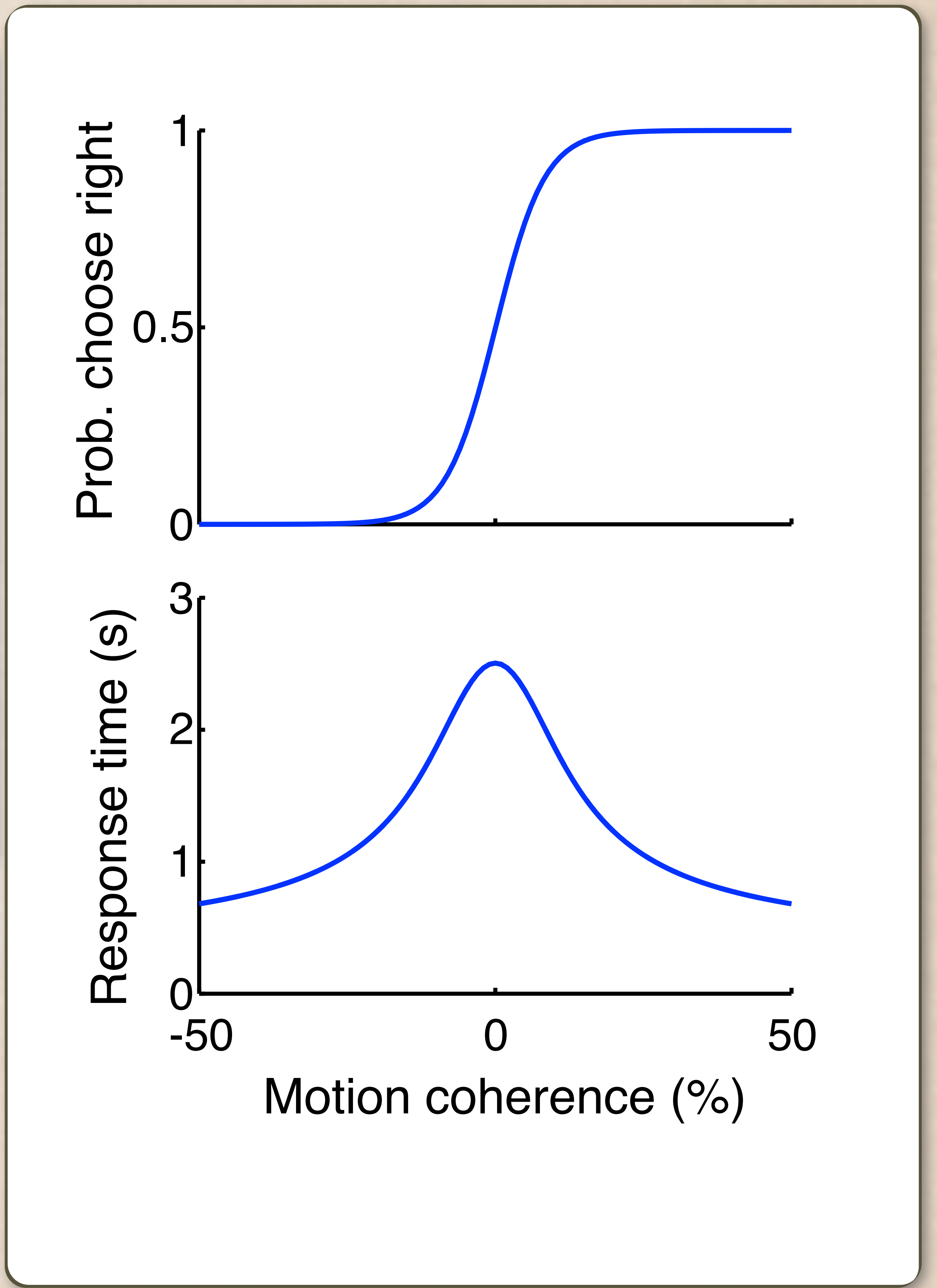


Change in bound height





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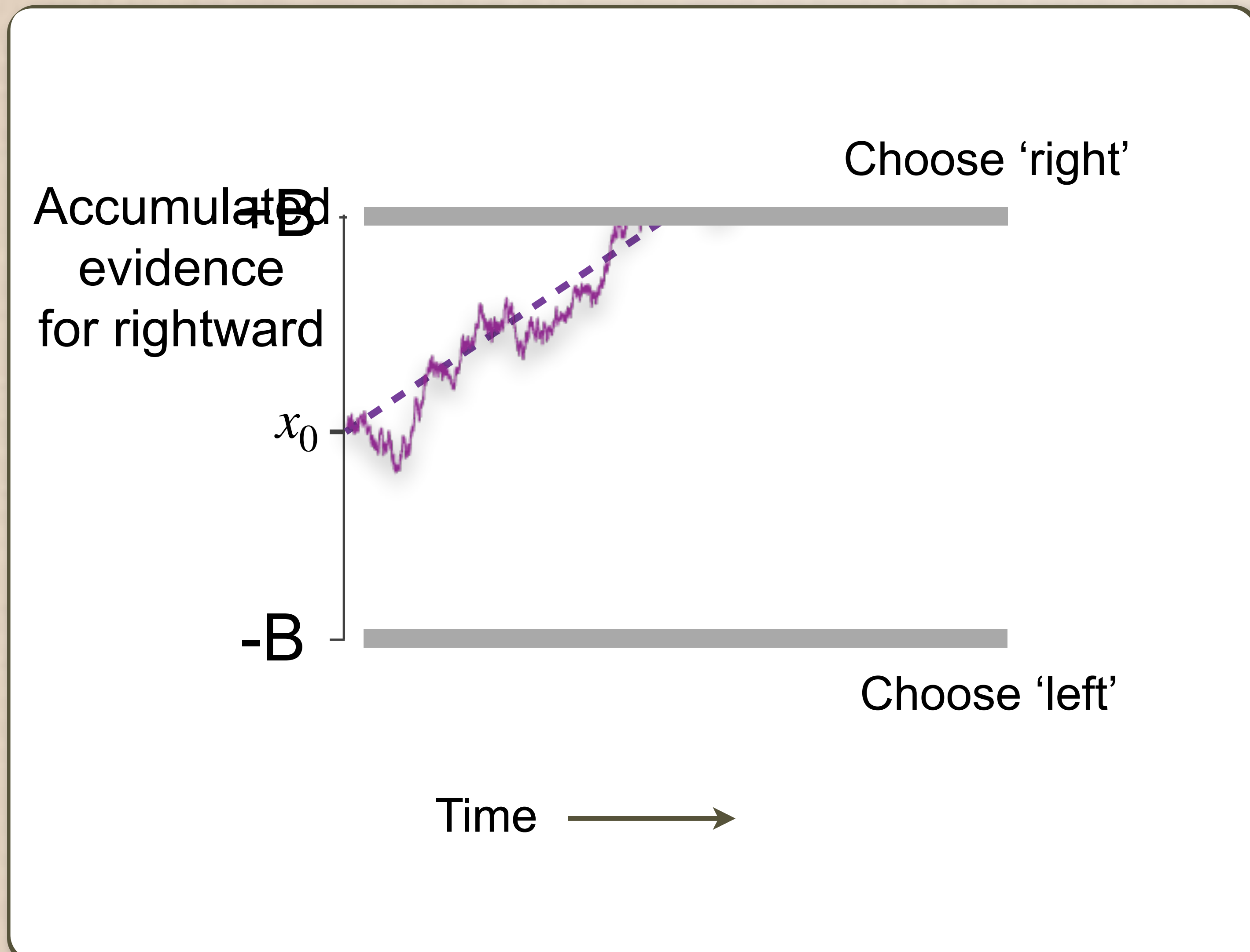

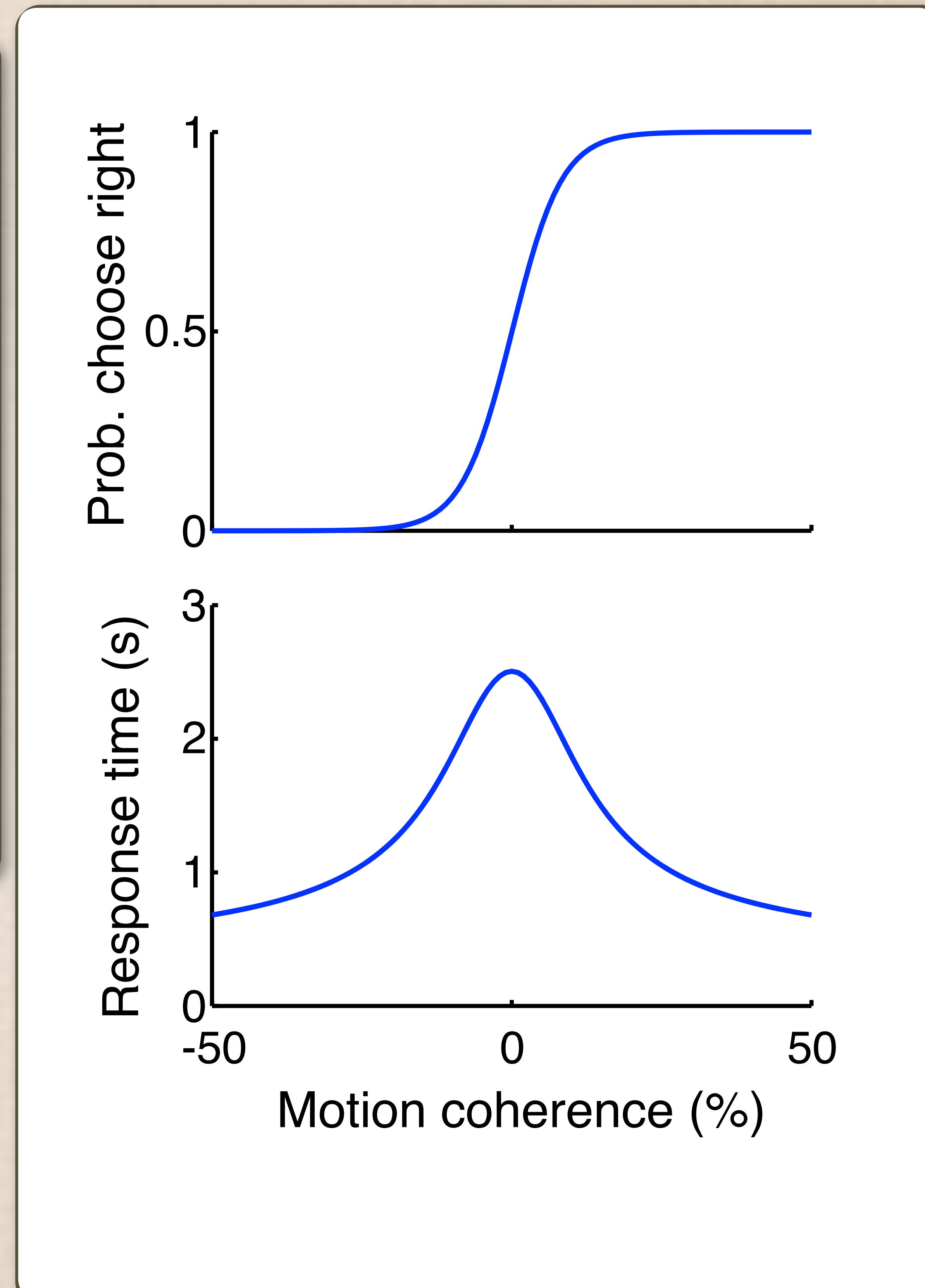

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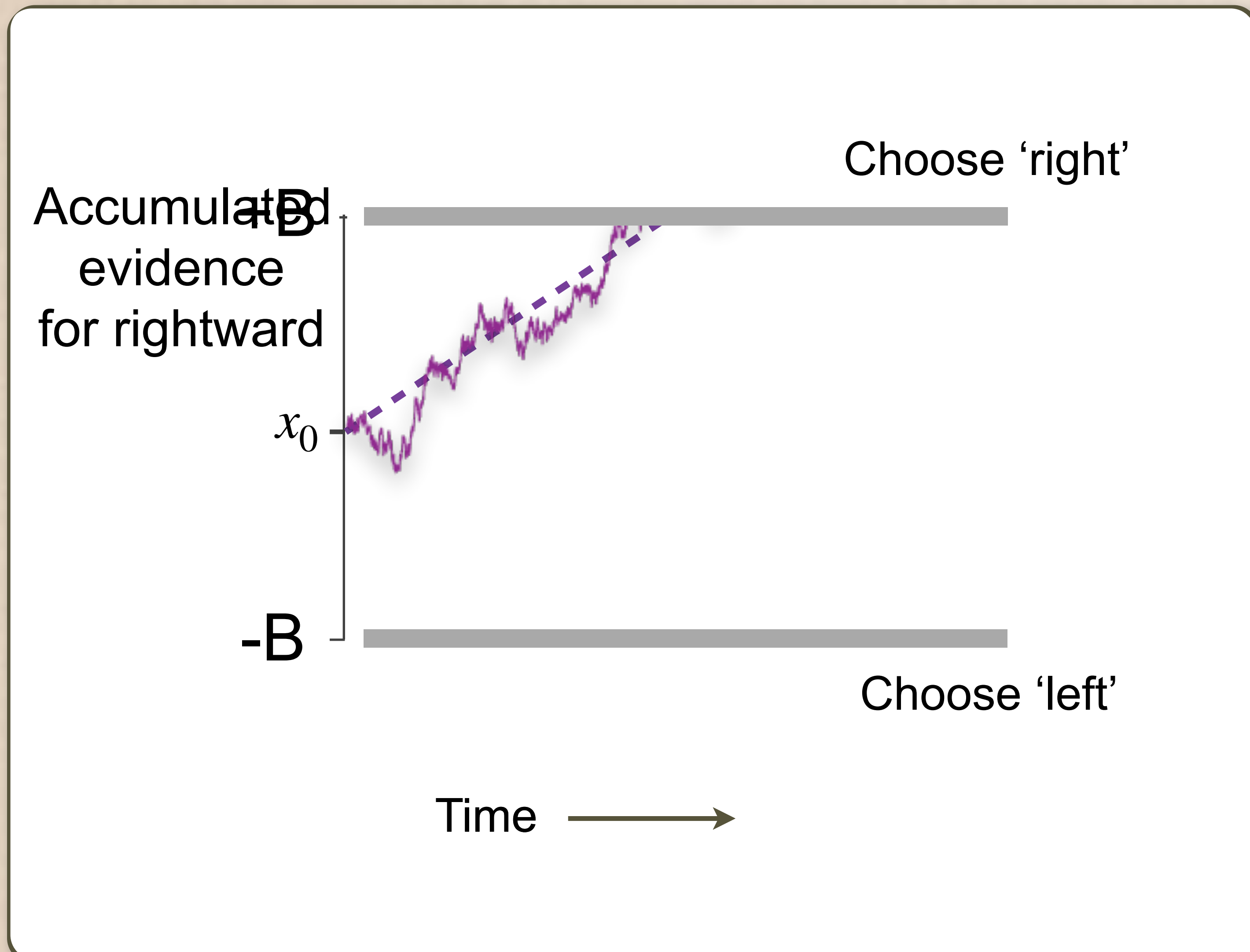

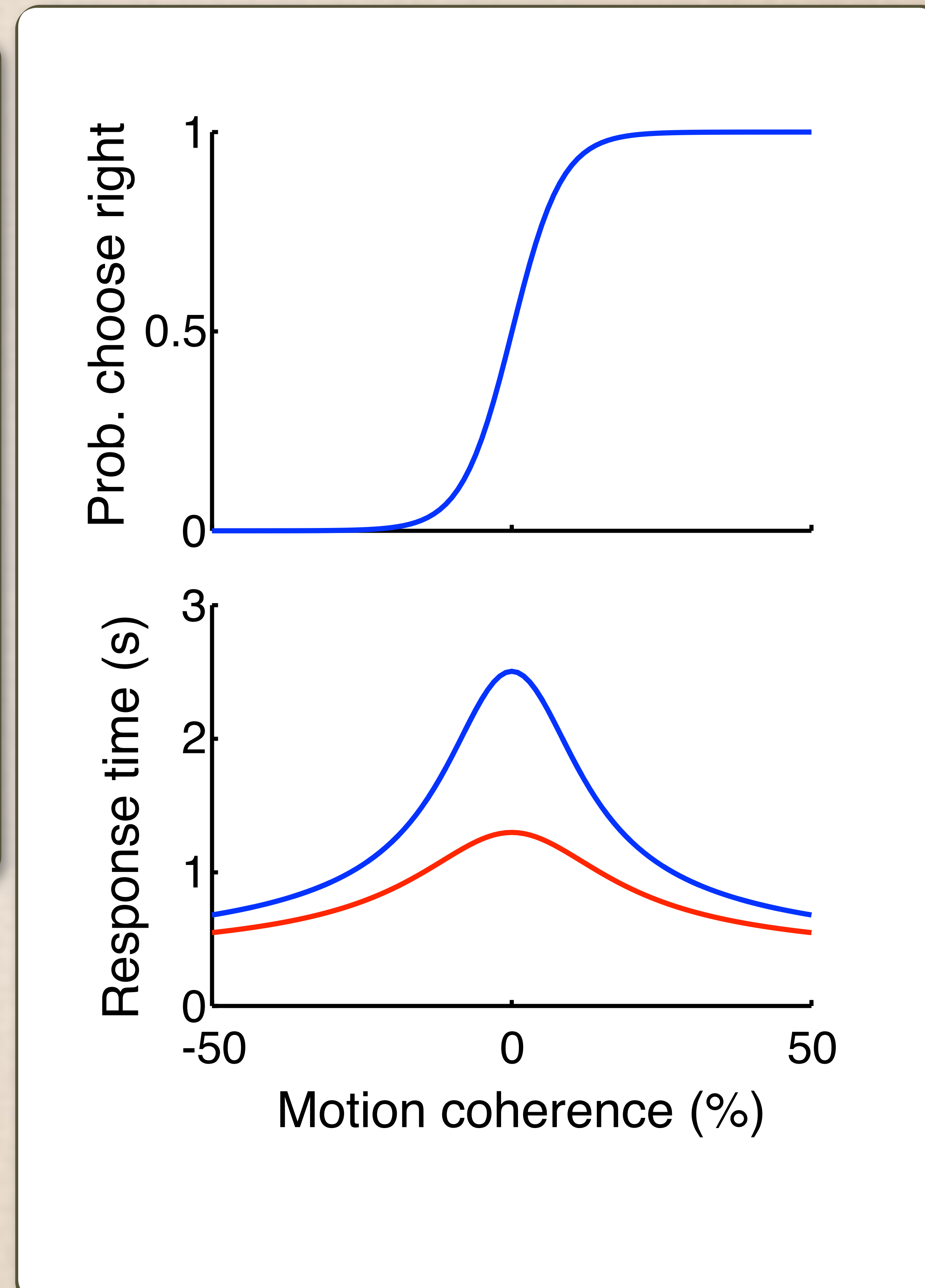

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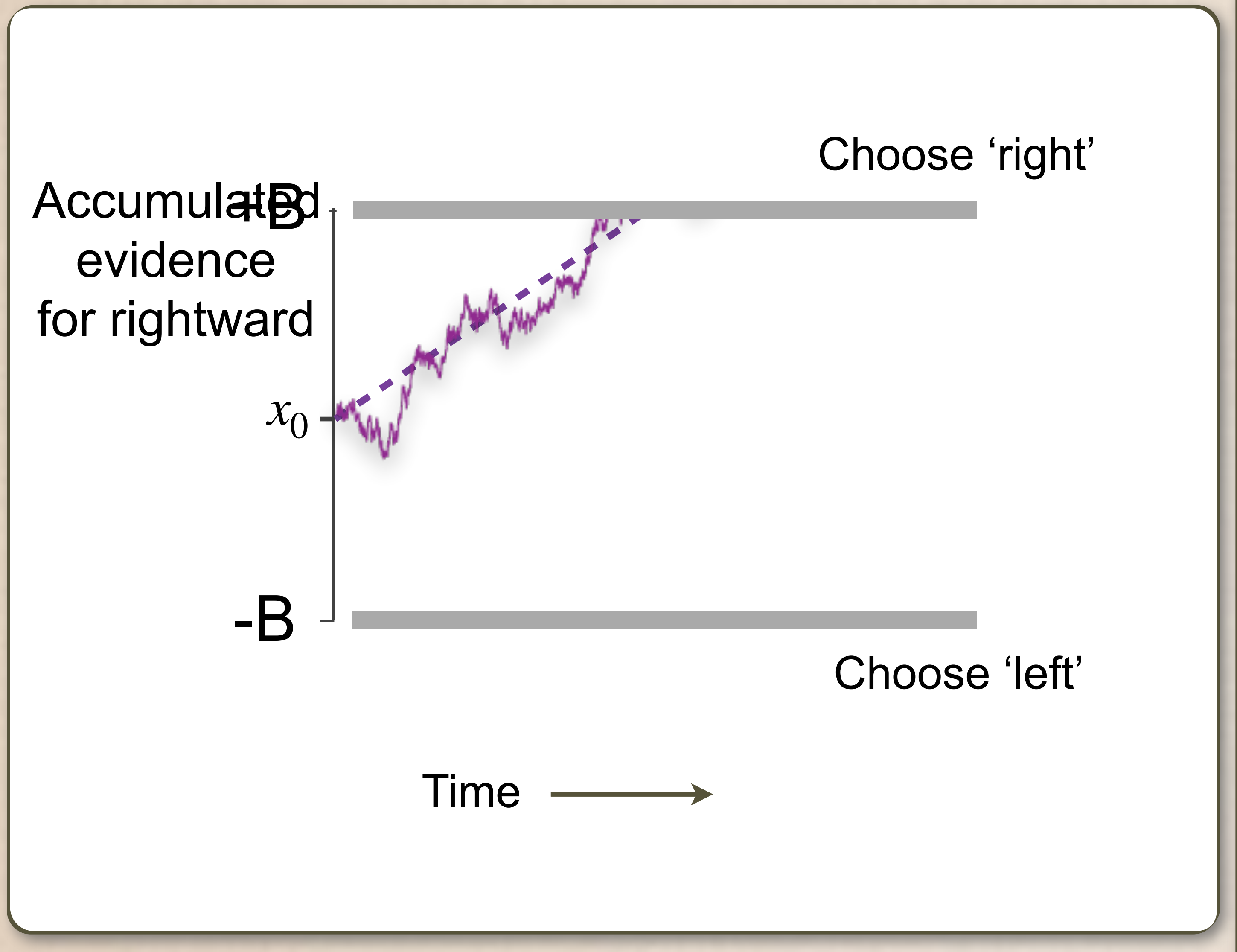
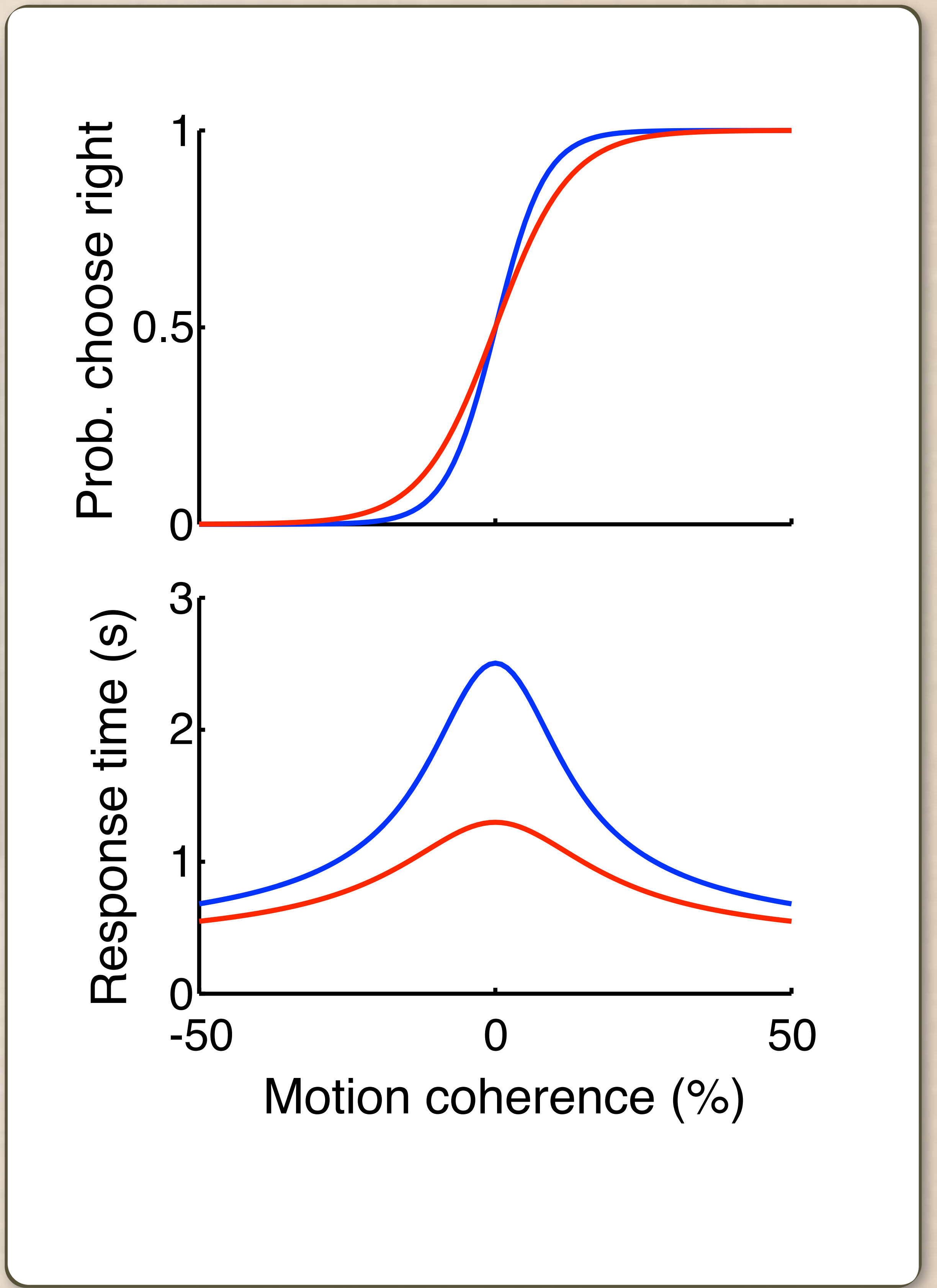
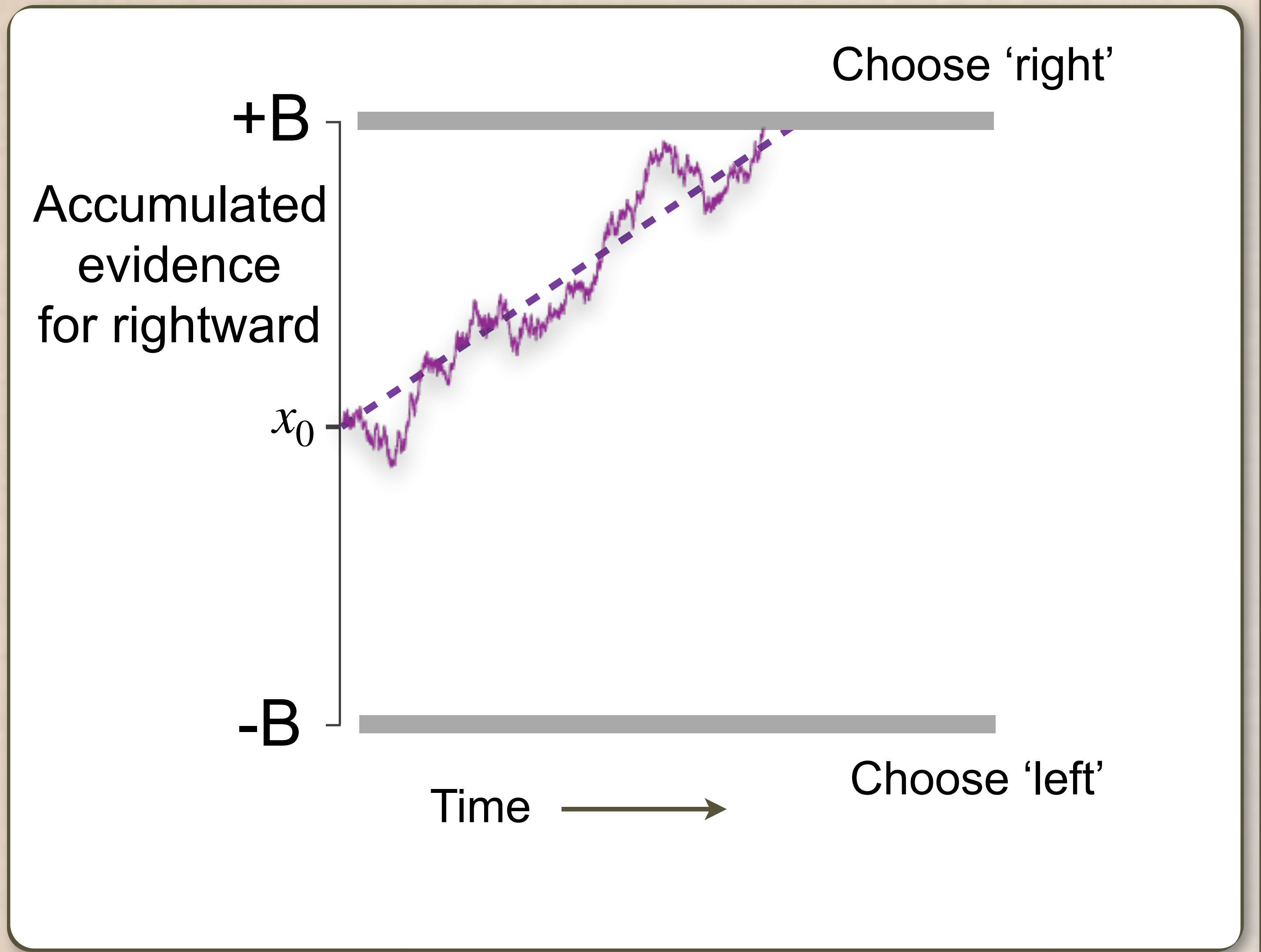


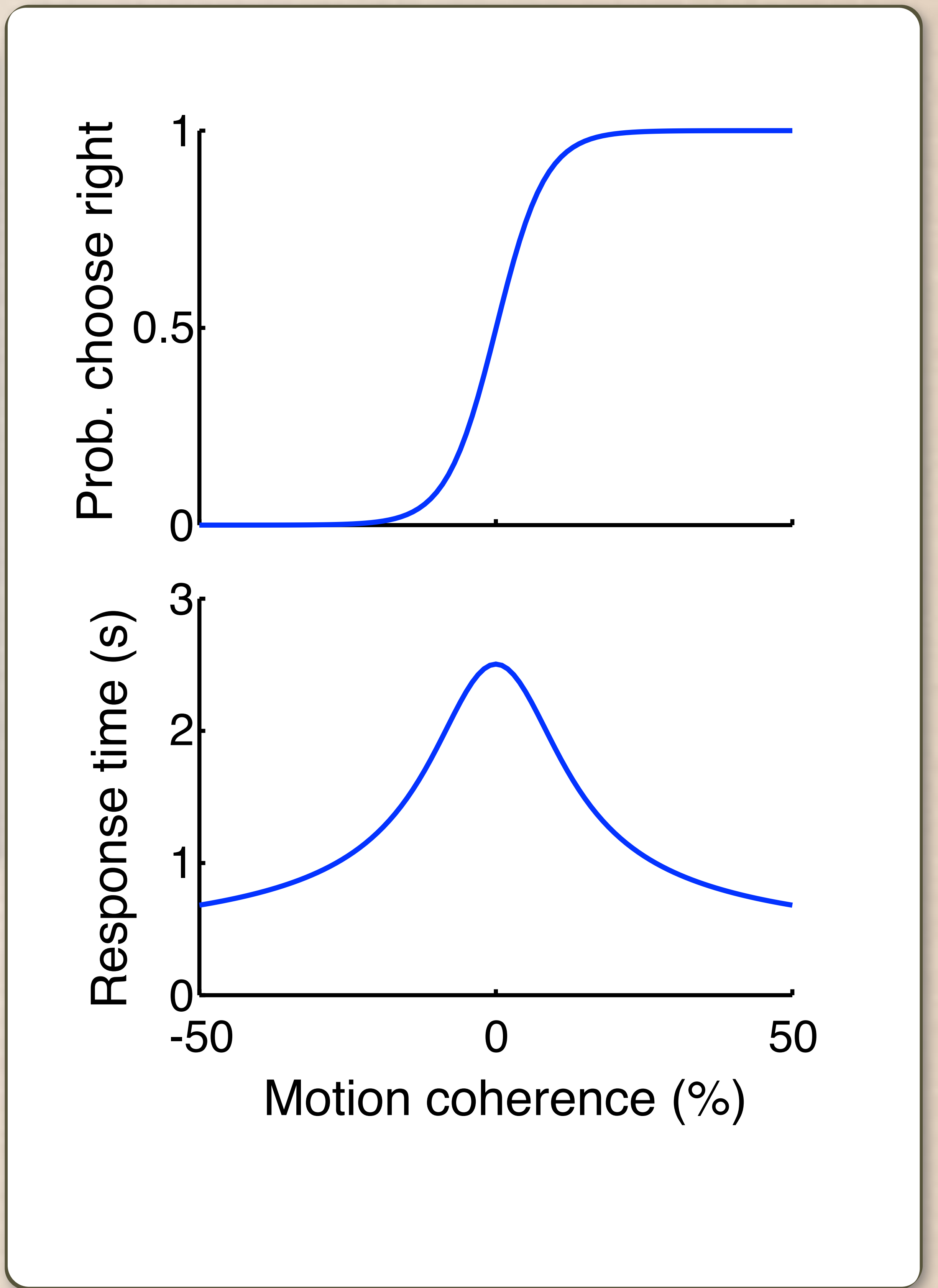


Change in drift rate

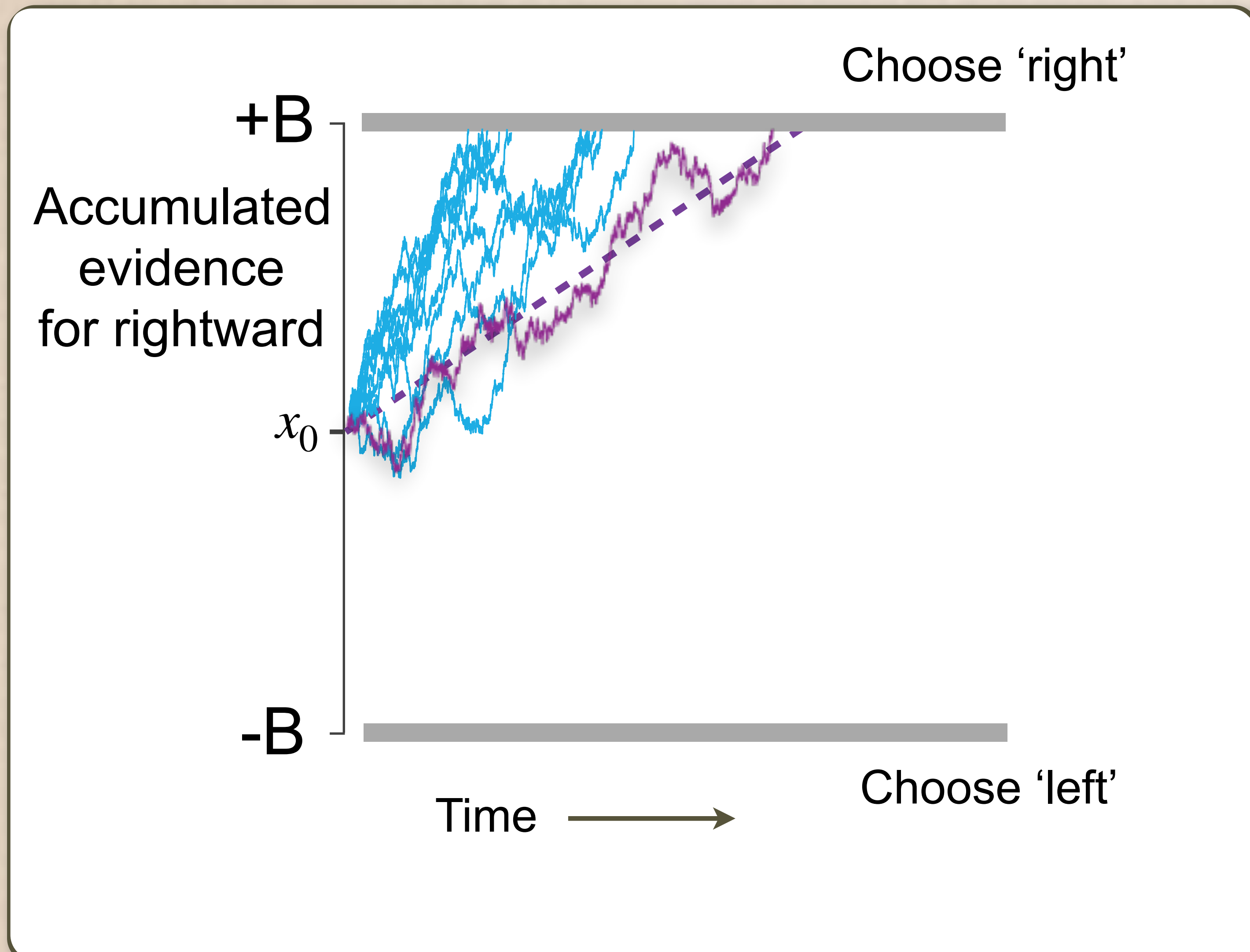


$\mu = \kappa \times$ Motion coherence

signal-to-noise



Change in drift rate



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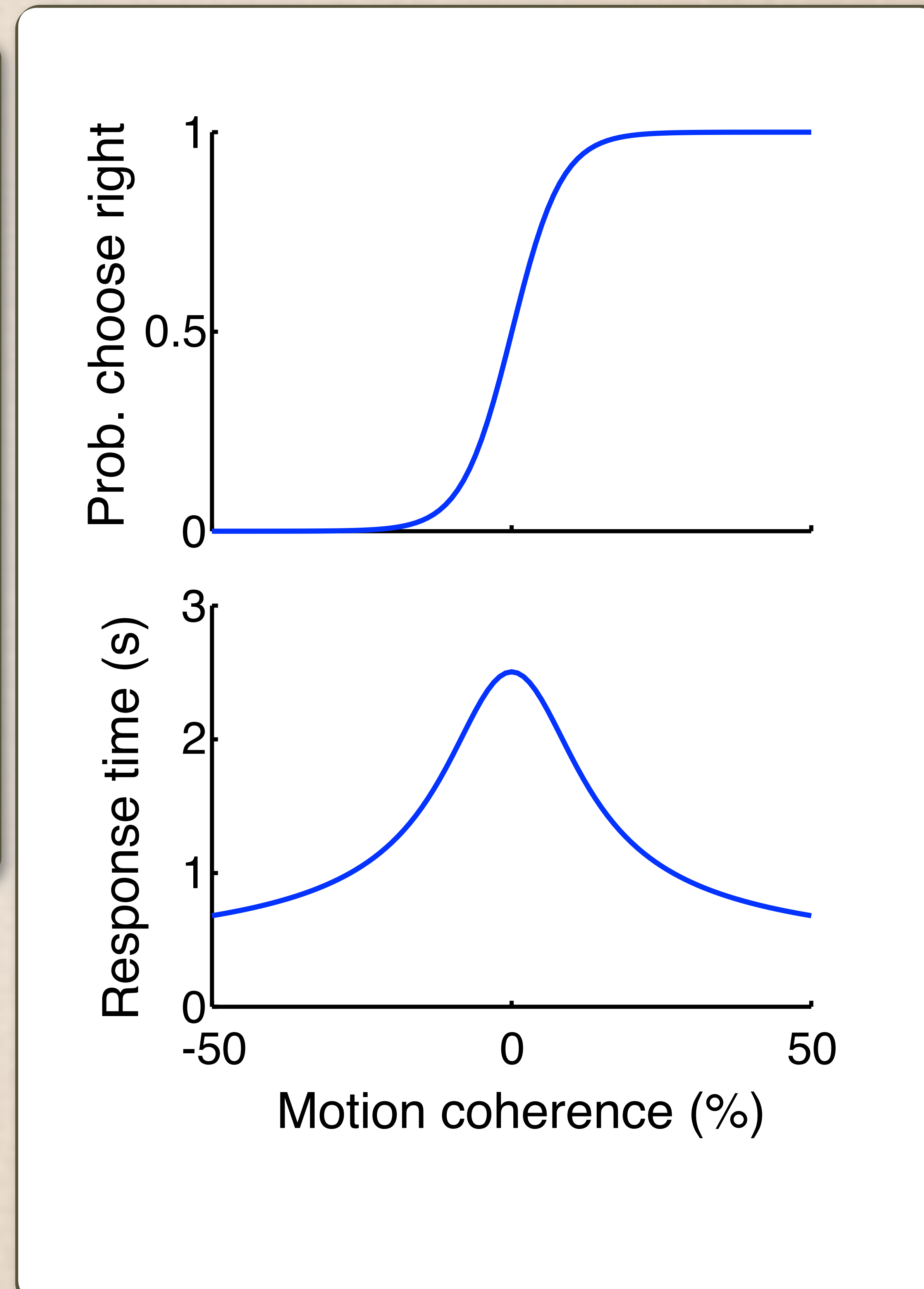

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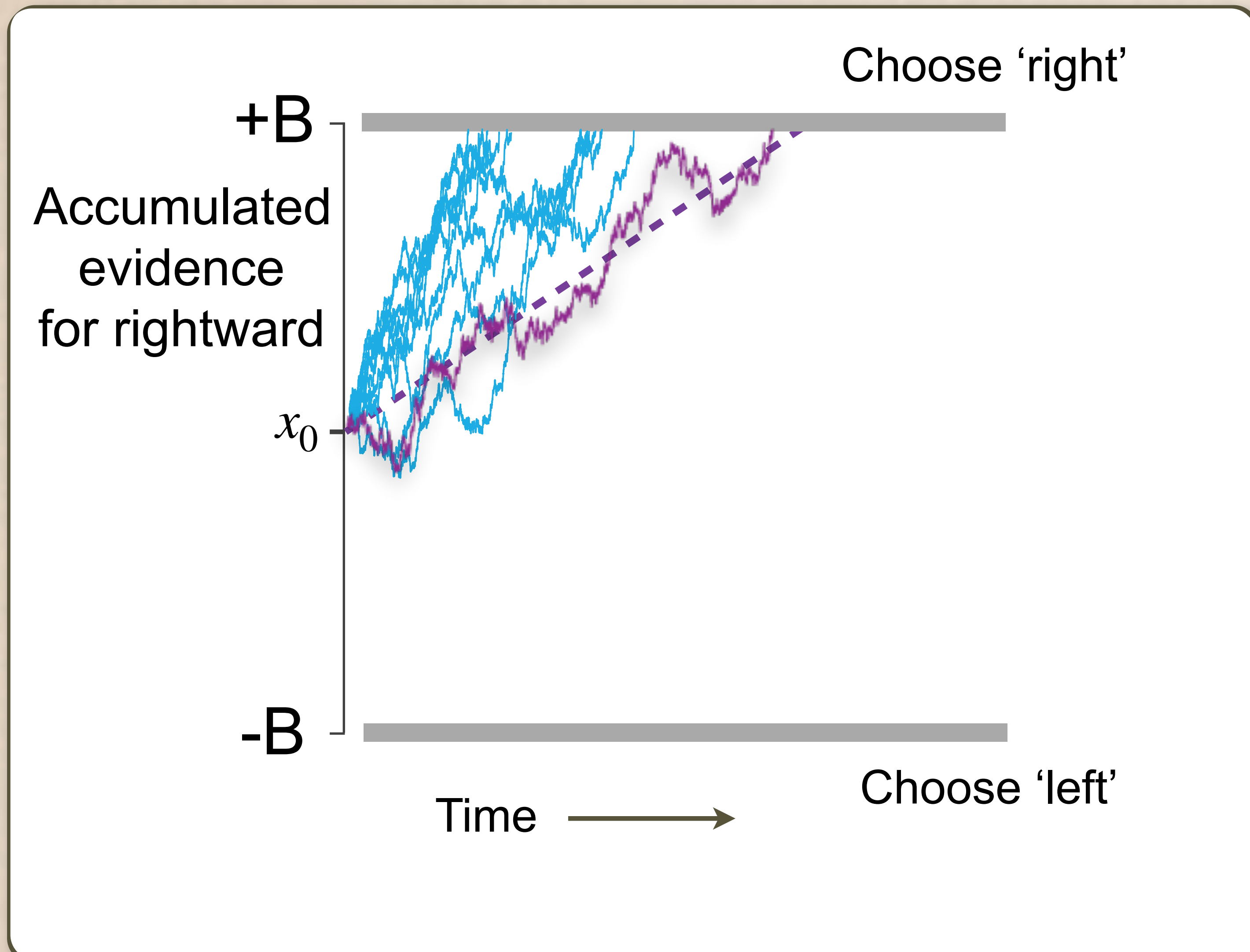

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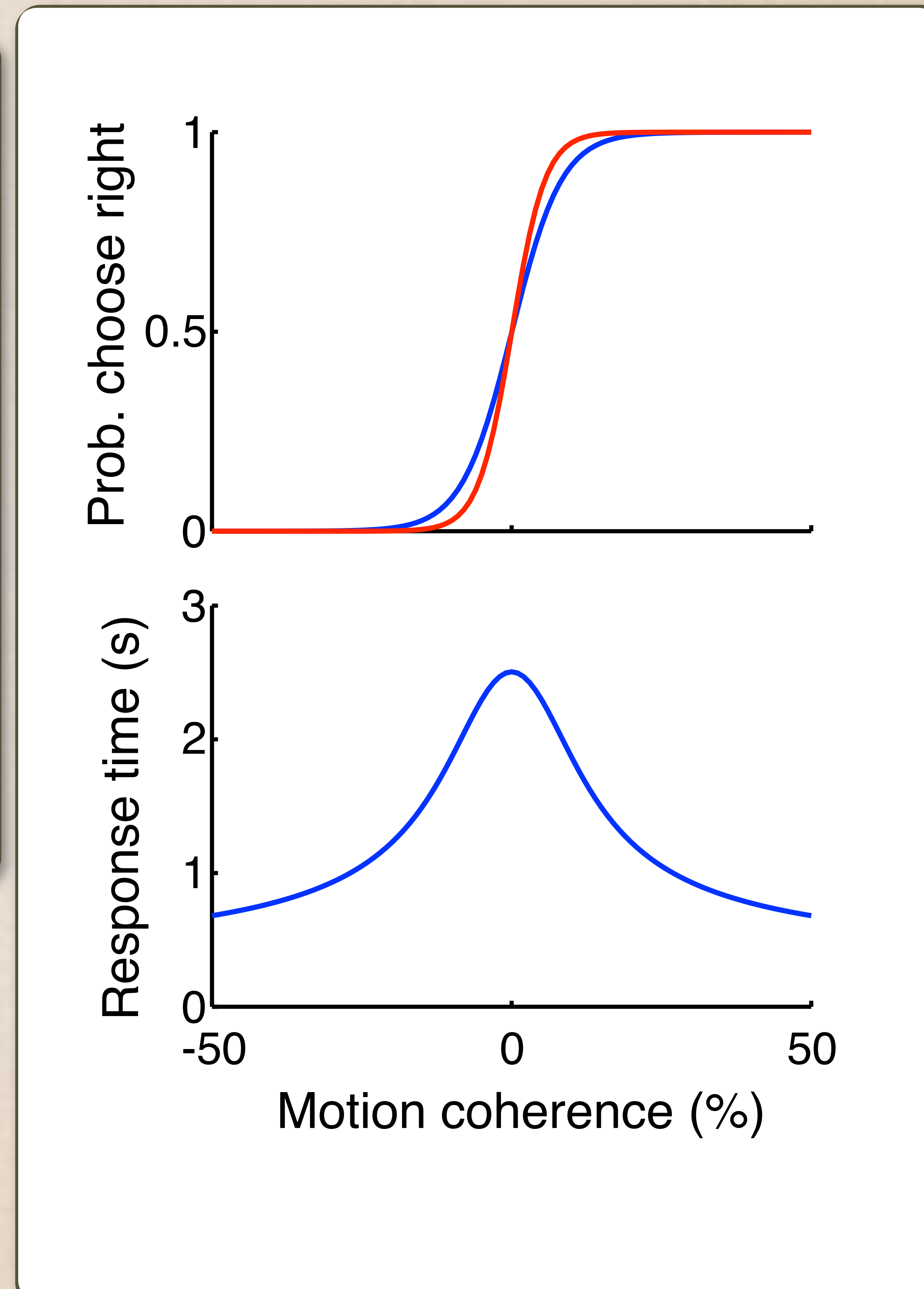

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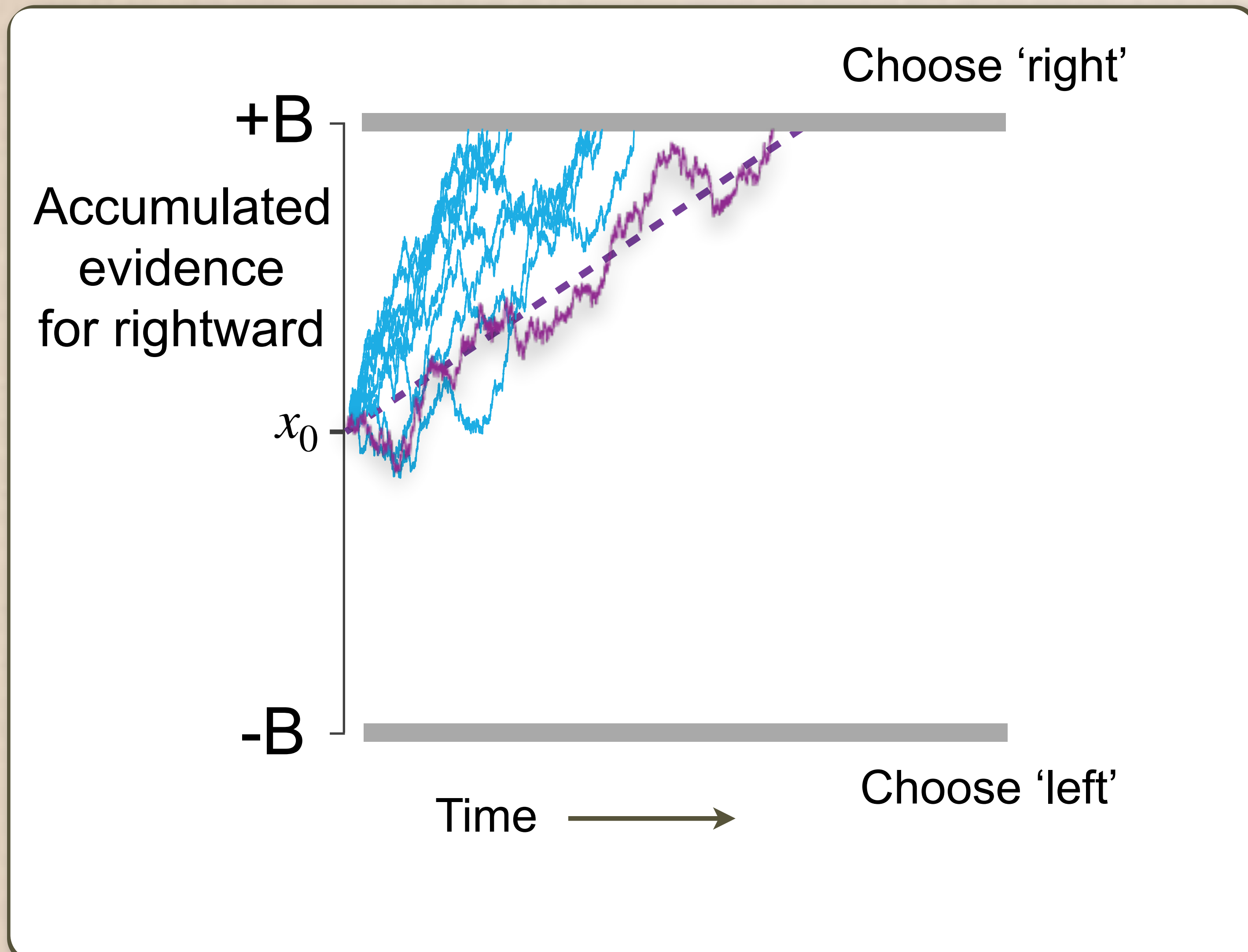

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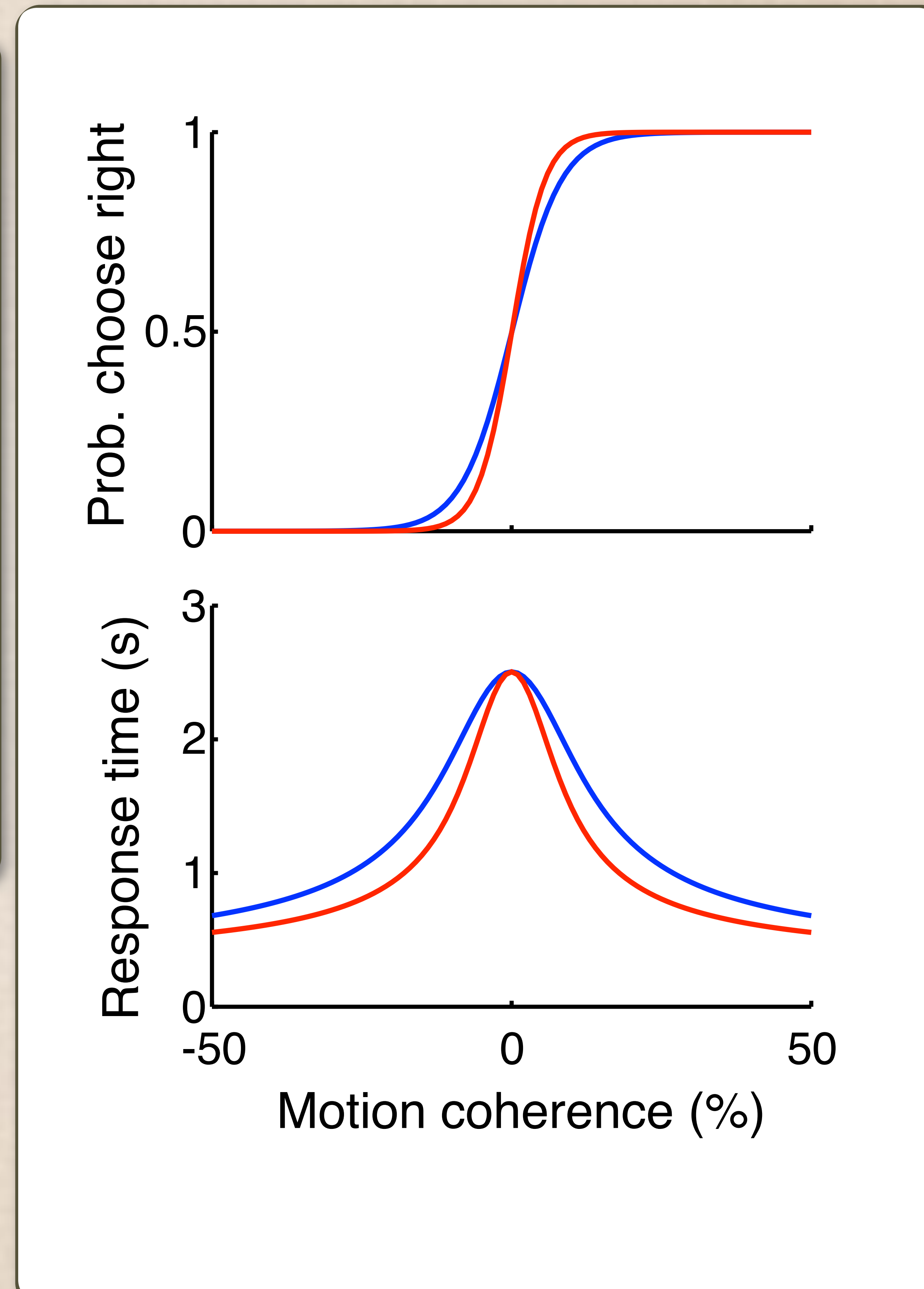
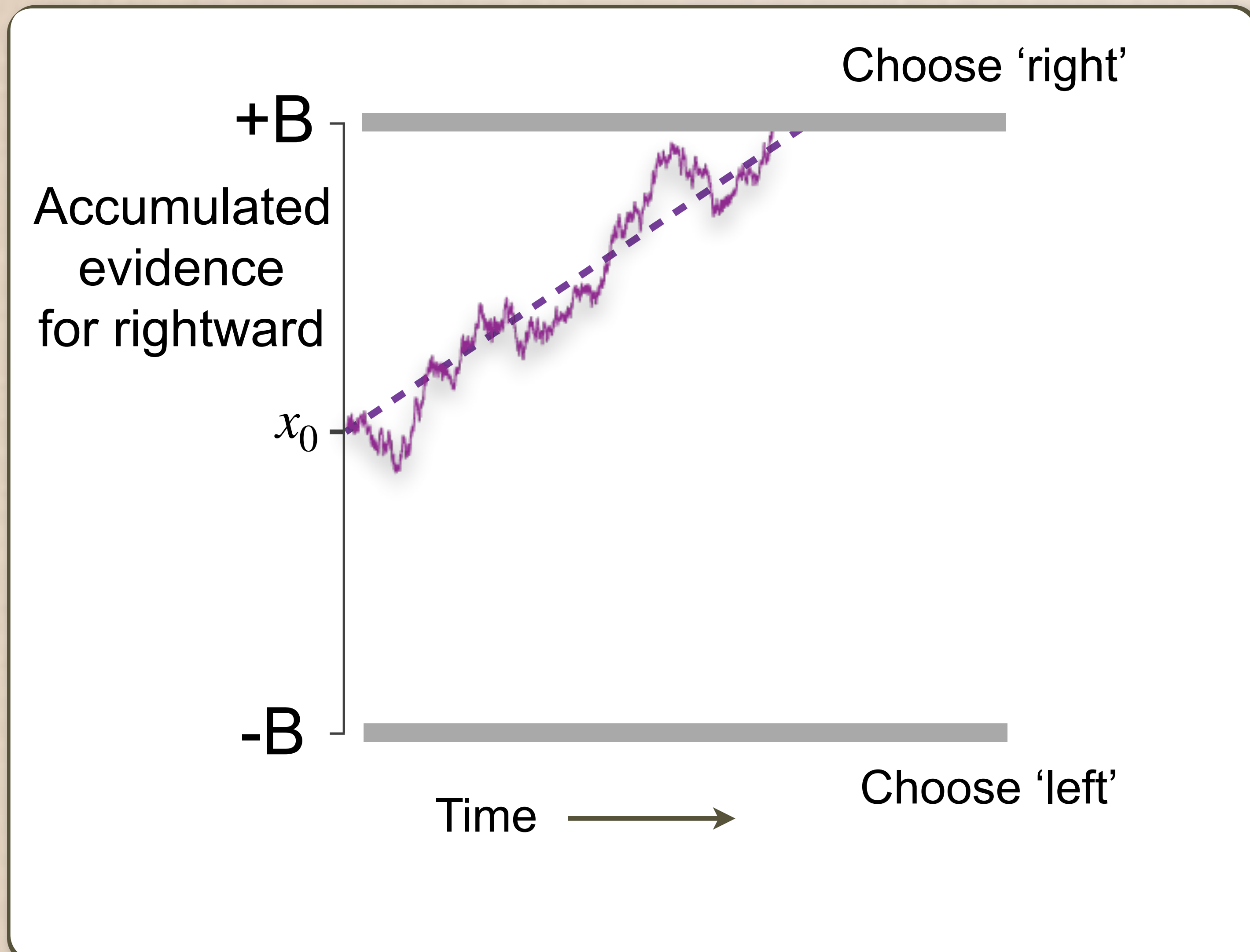
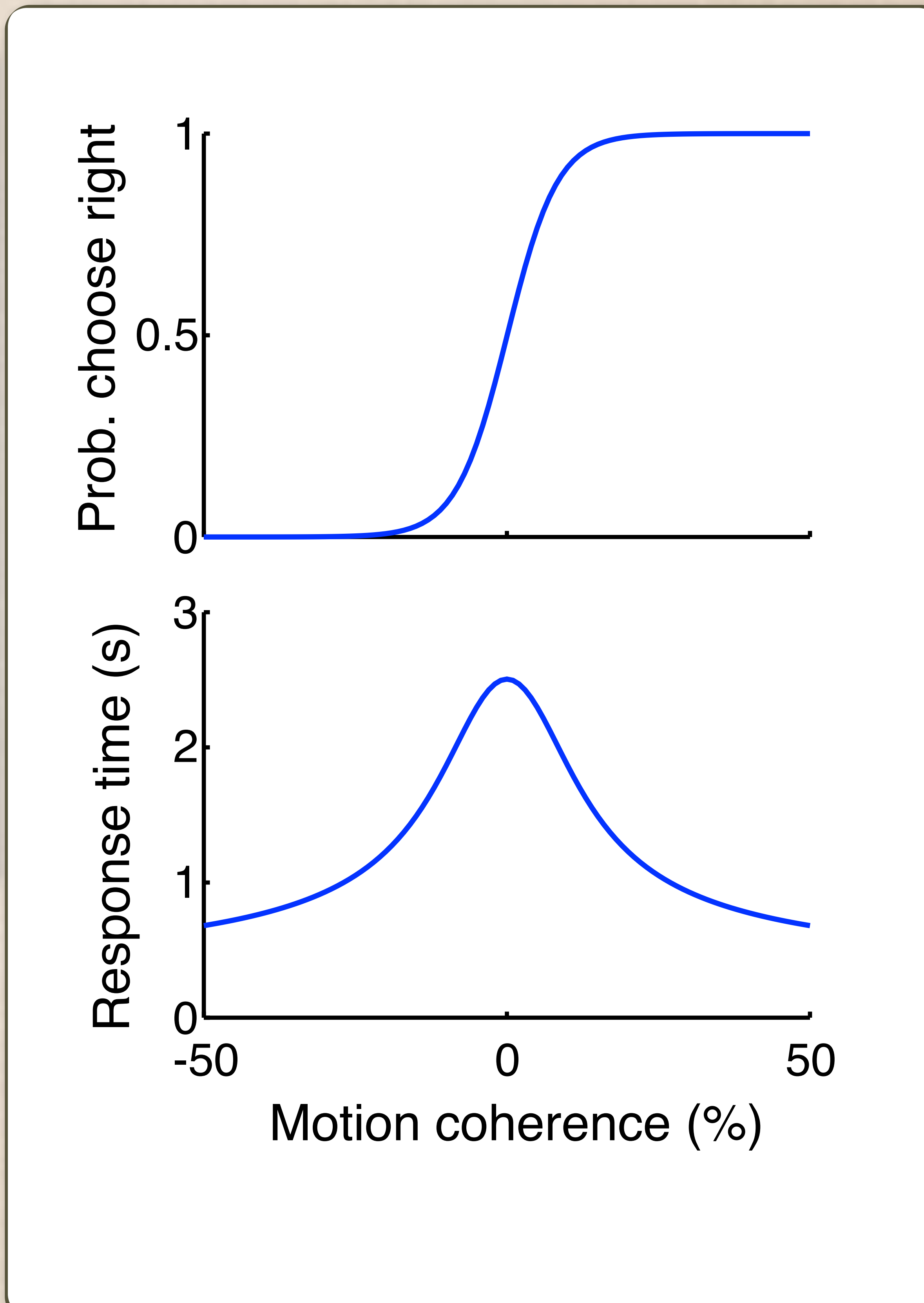
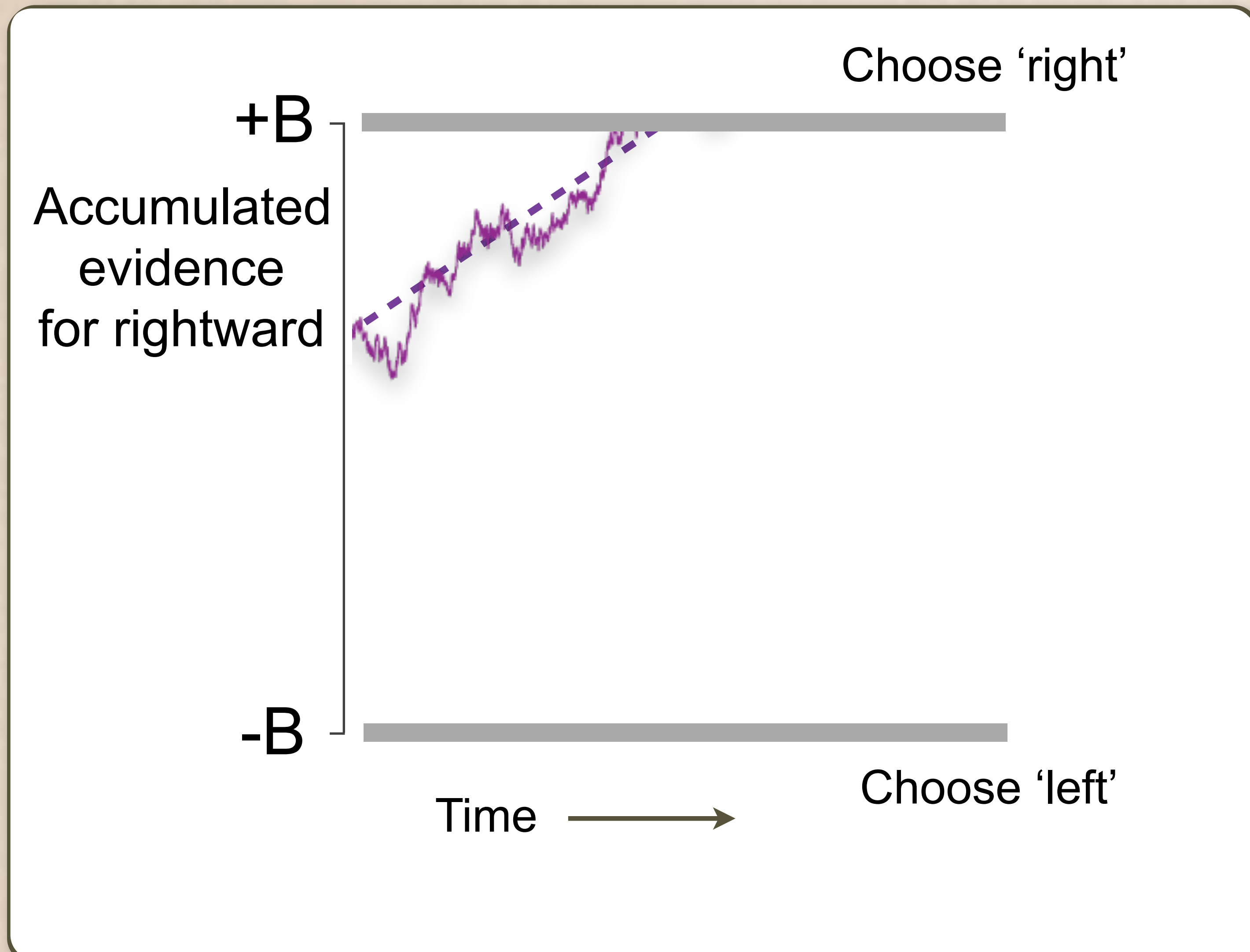


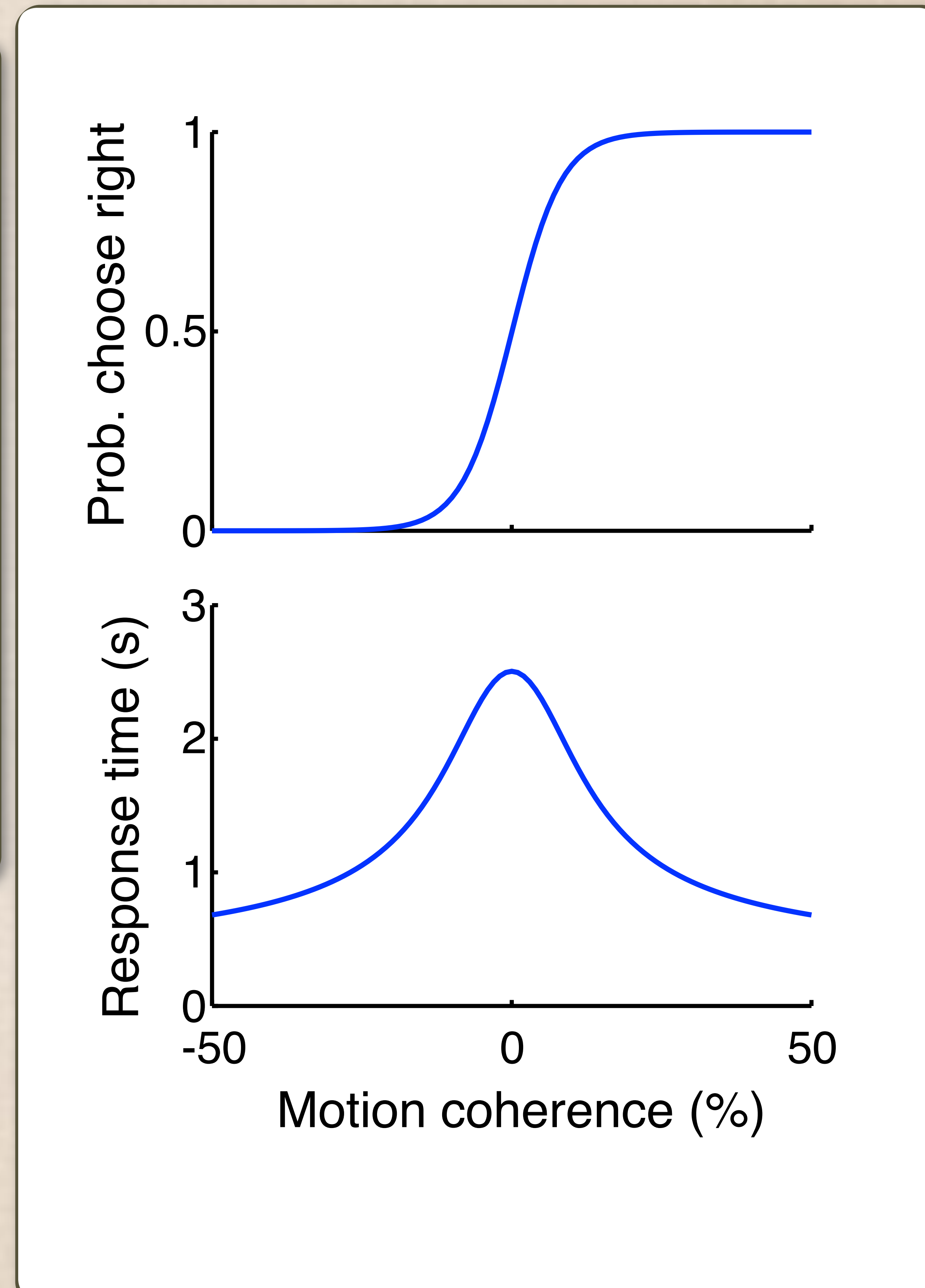
Change in starting point





Change in starting point





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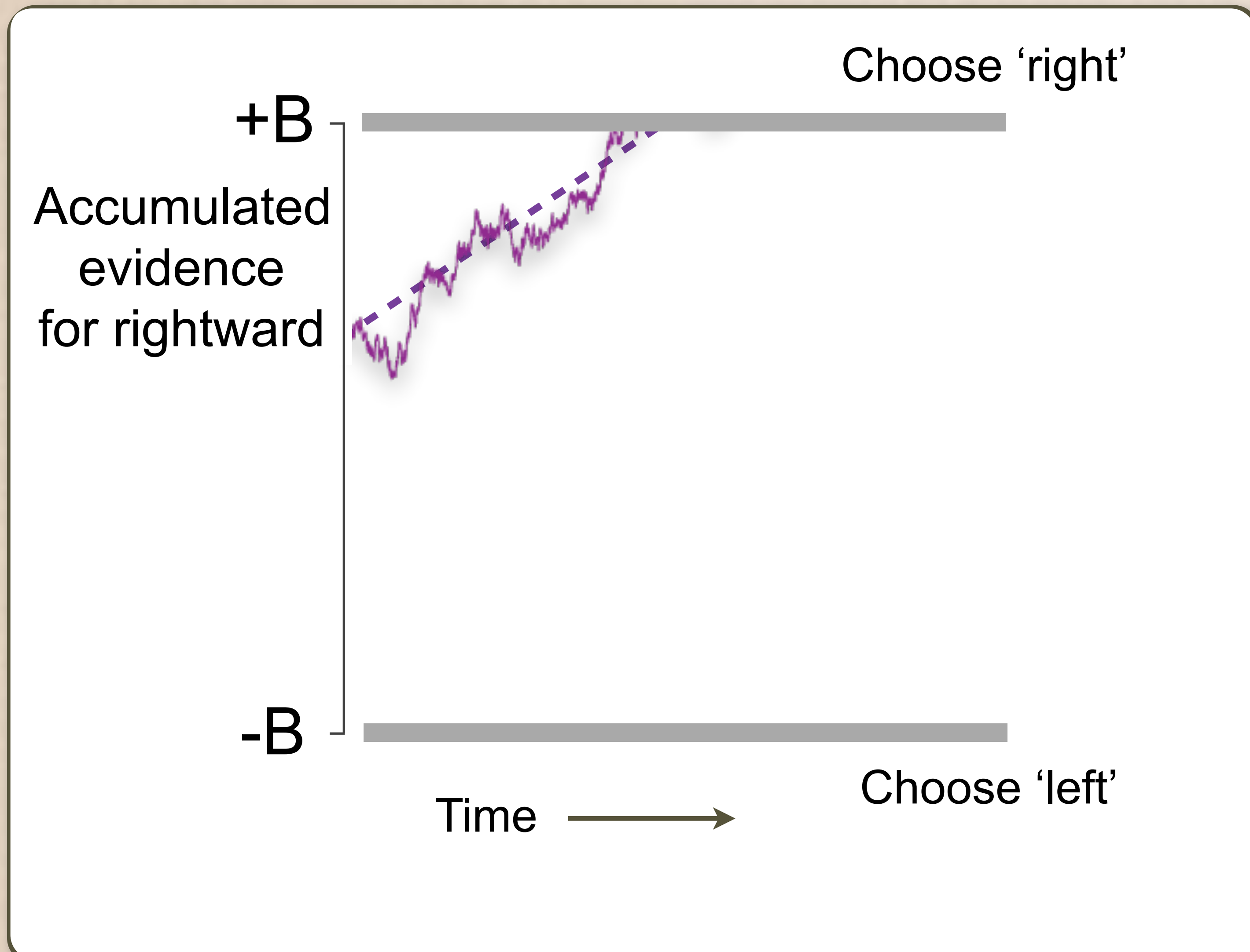
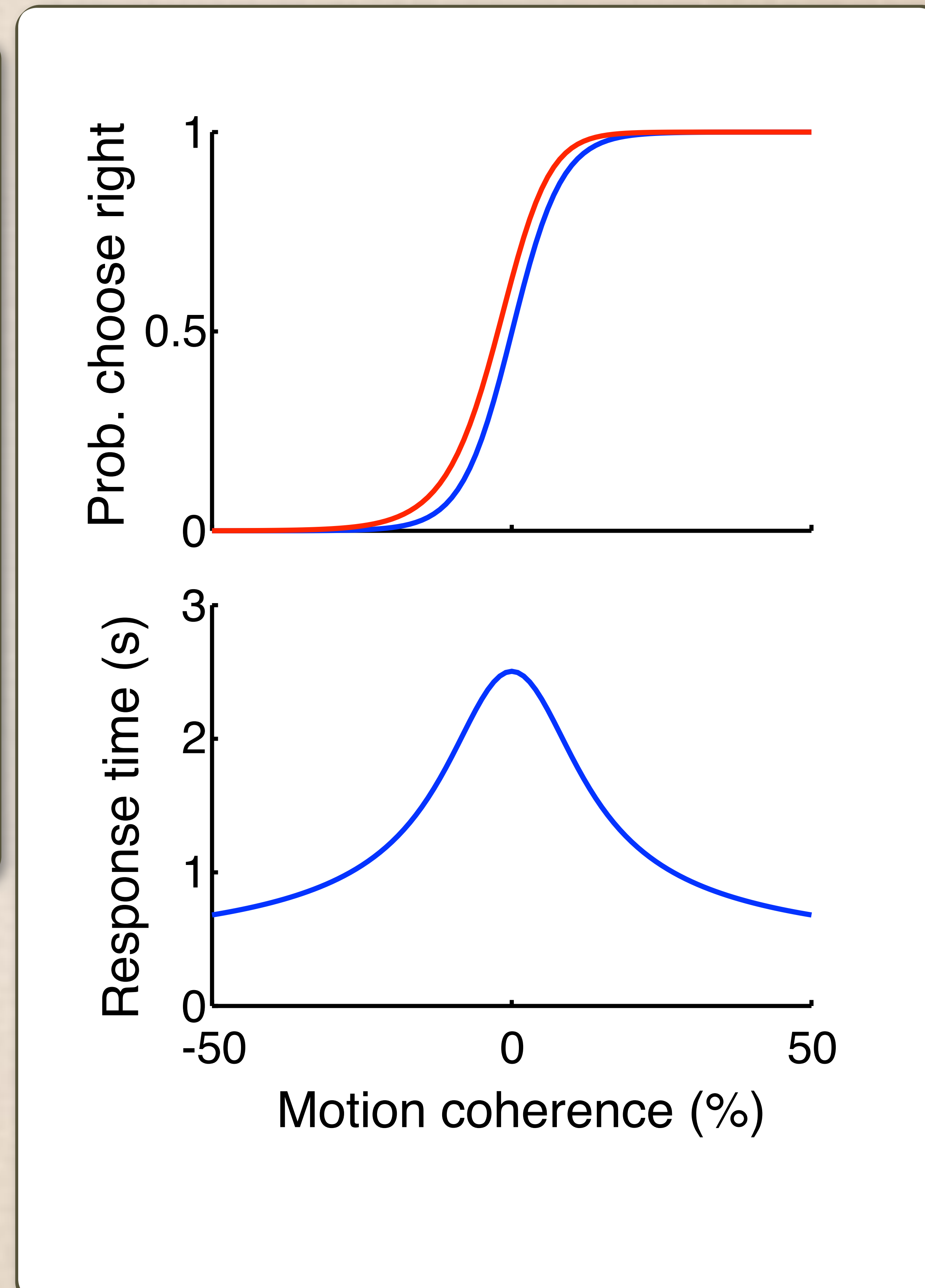
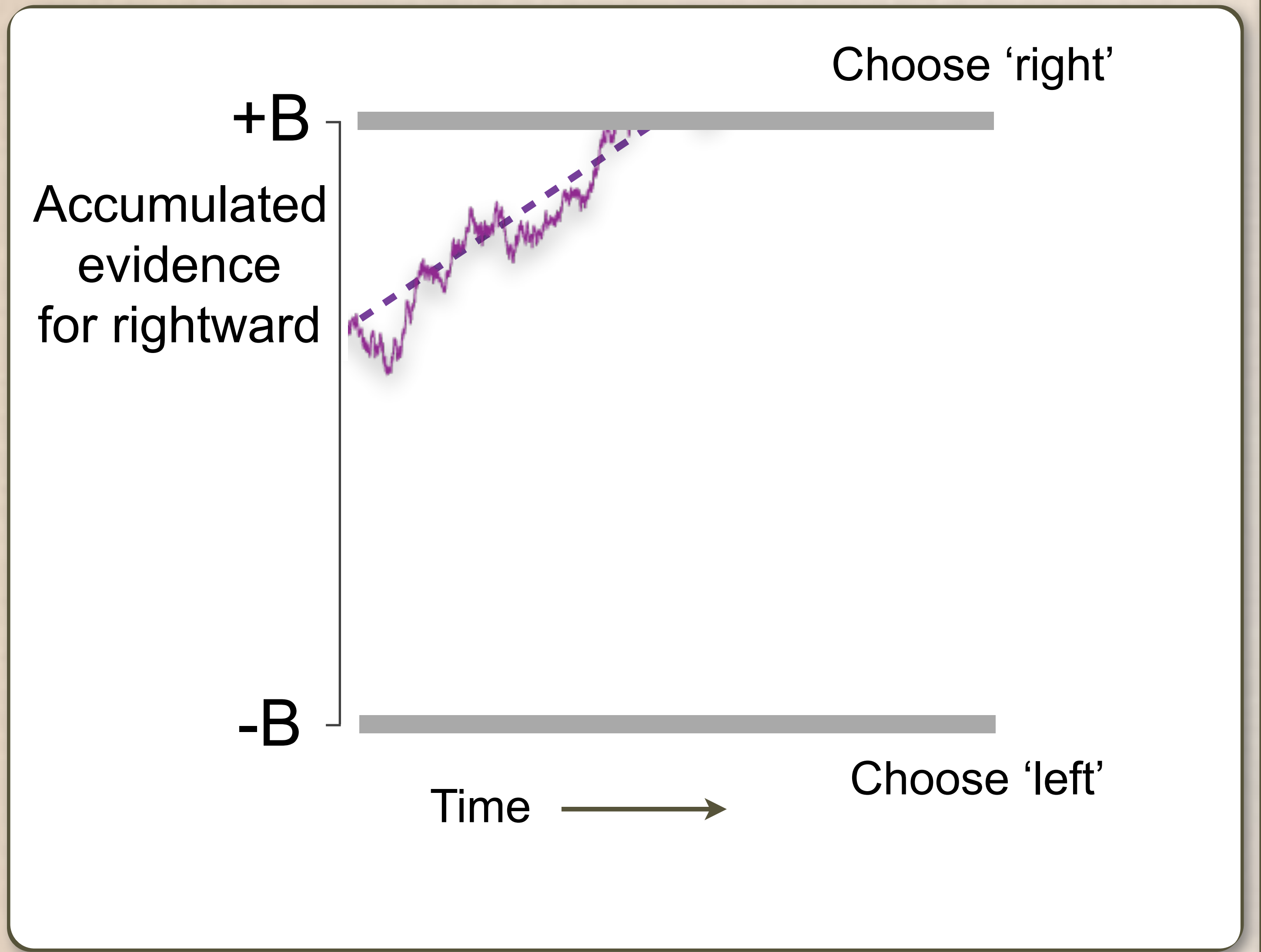
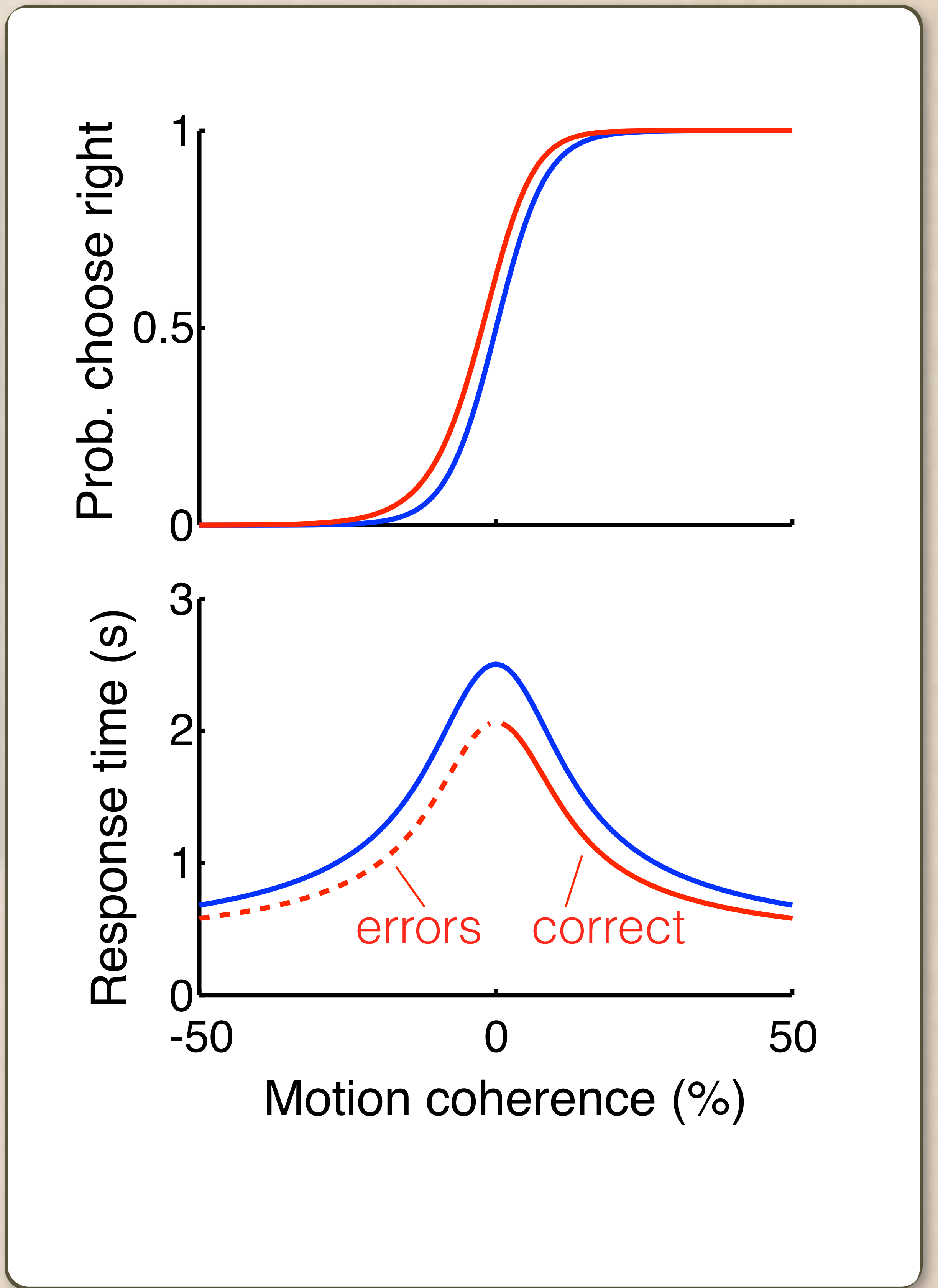


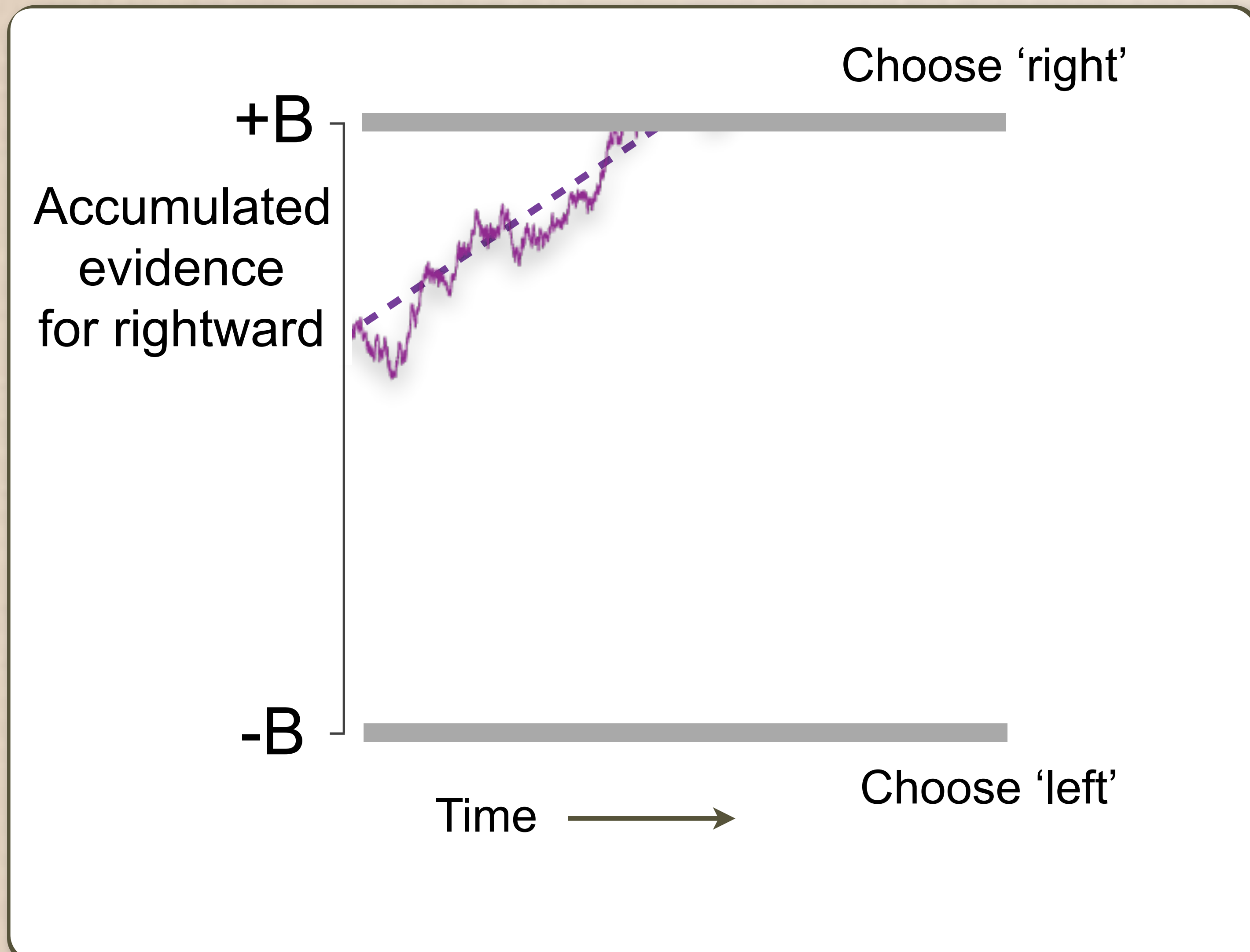


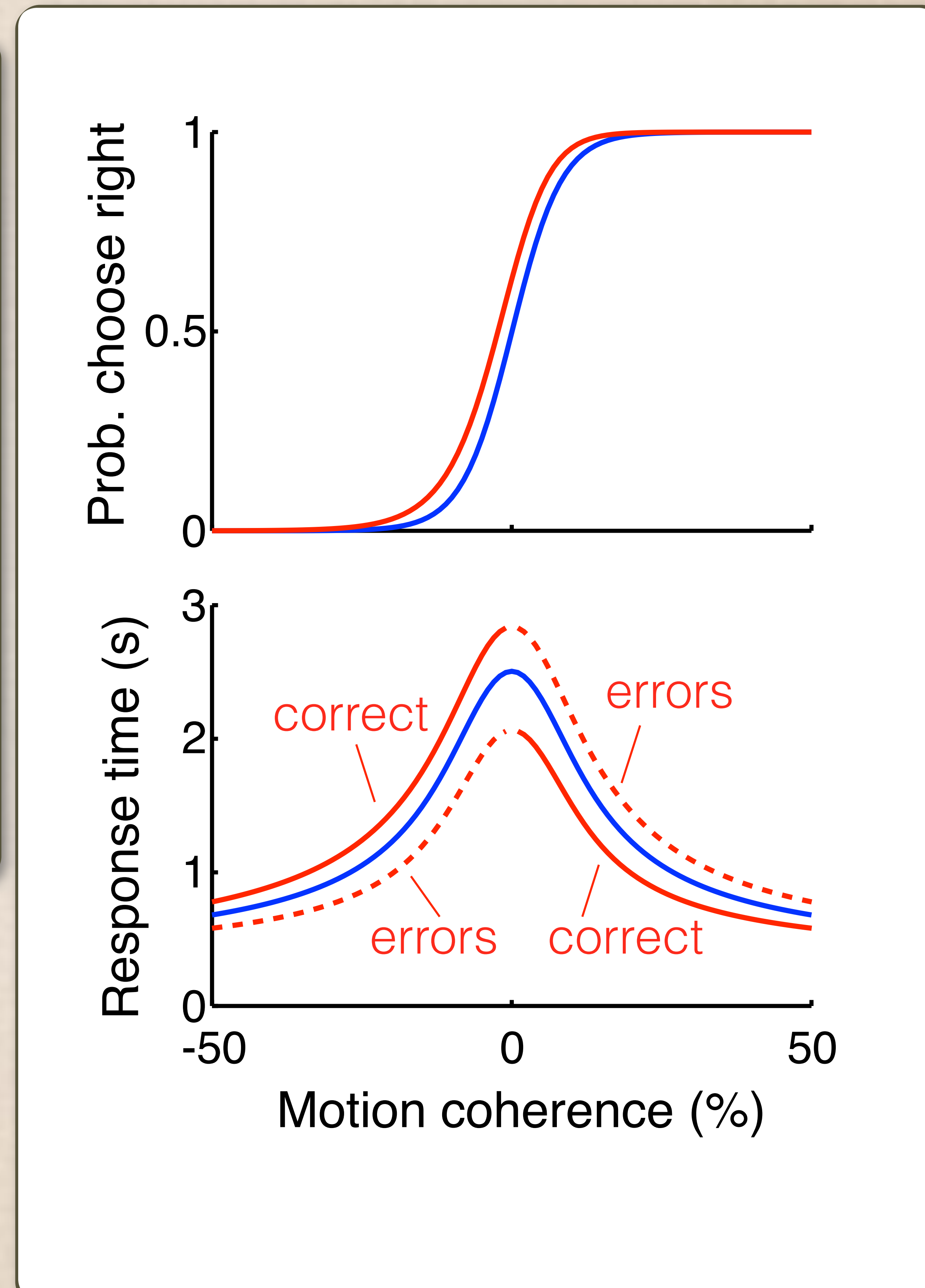
Change in starting point



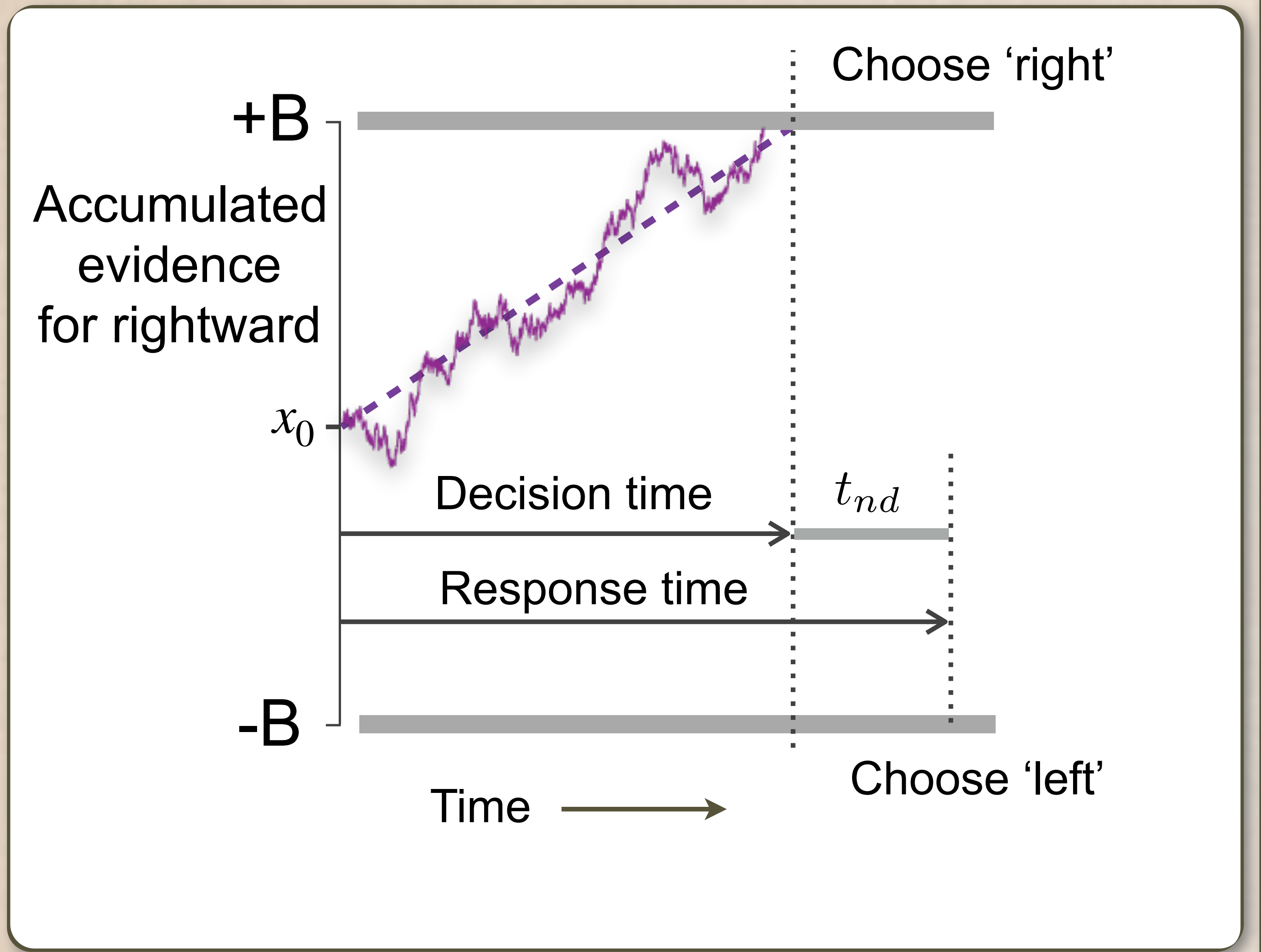


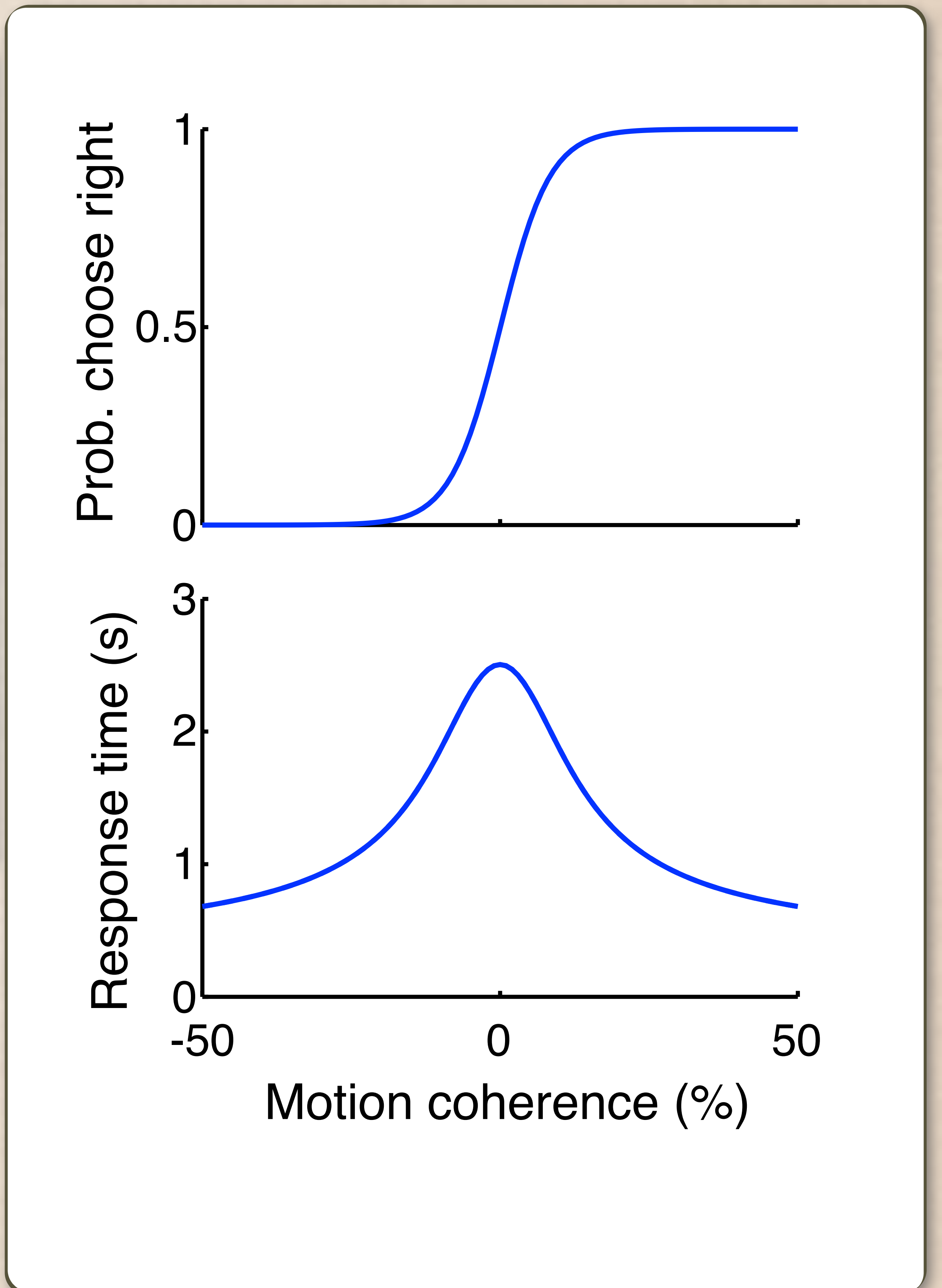
Change in starting point



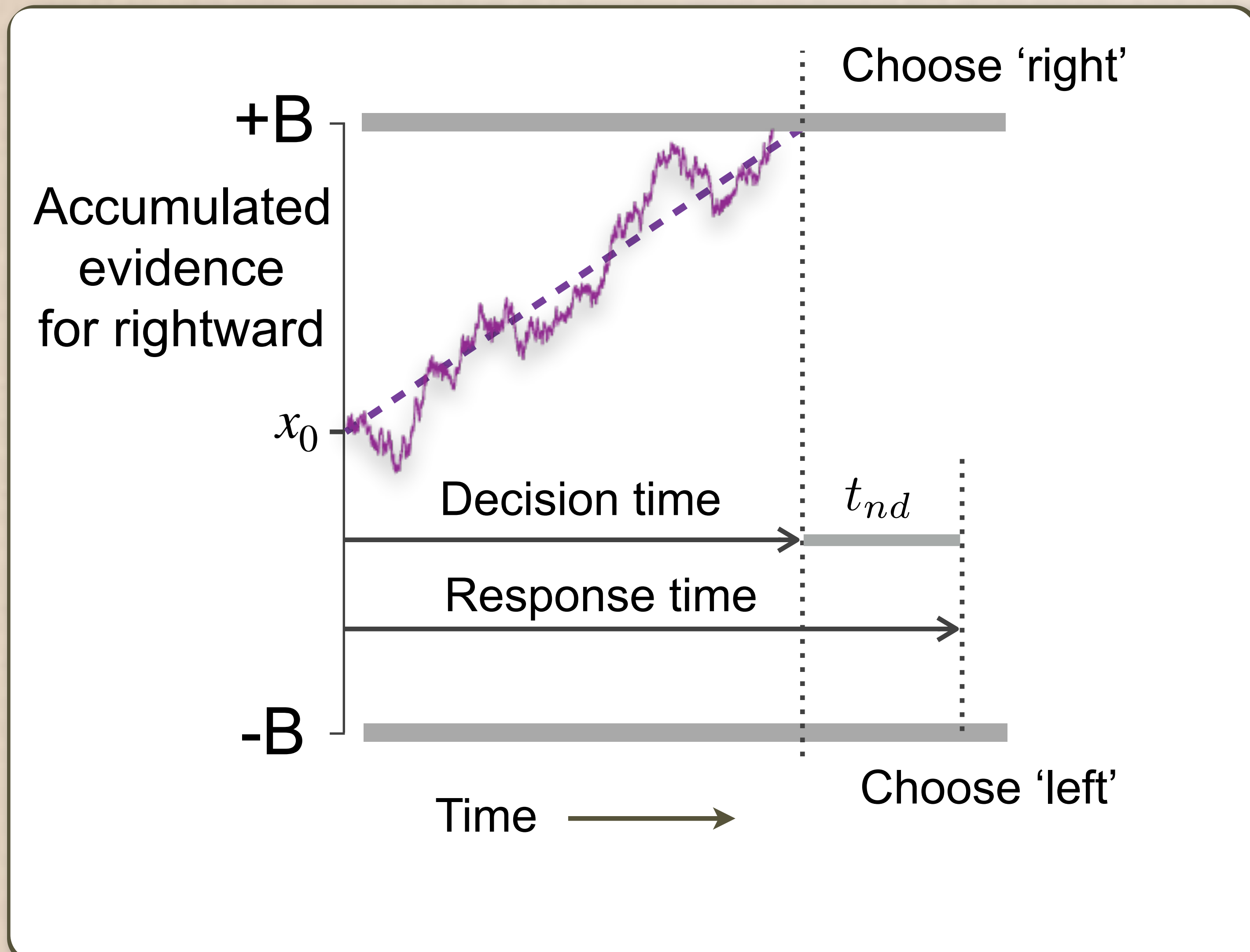


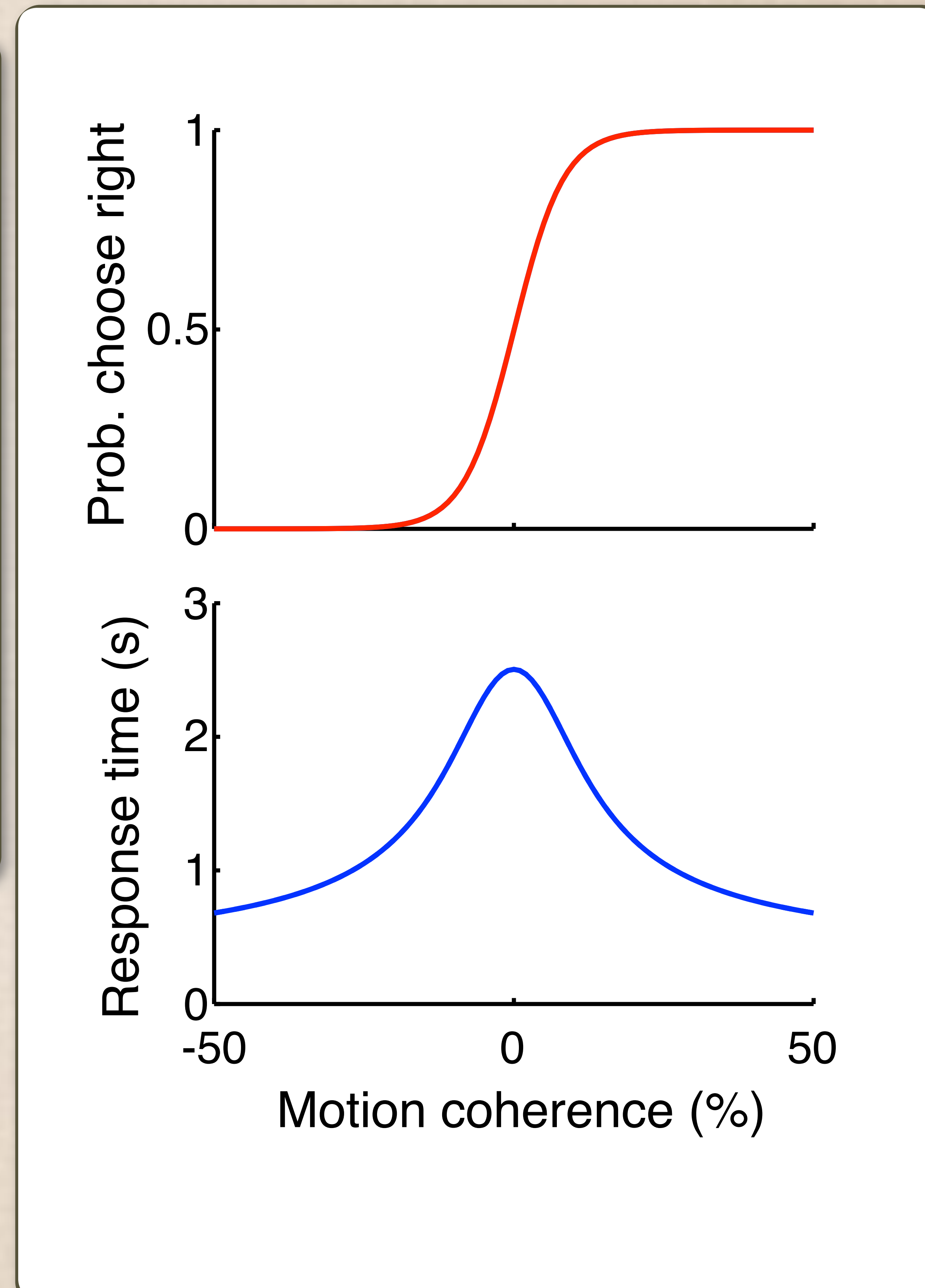
Change in non-decision time



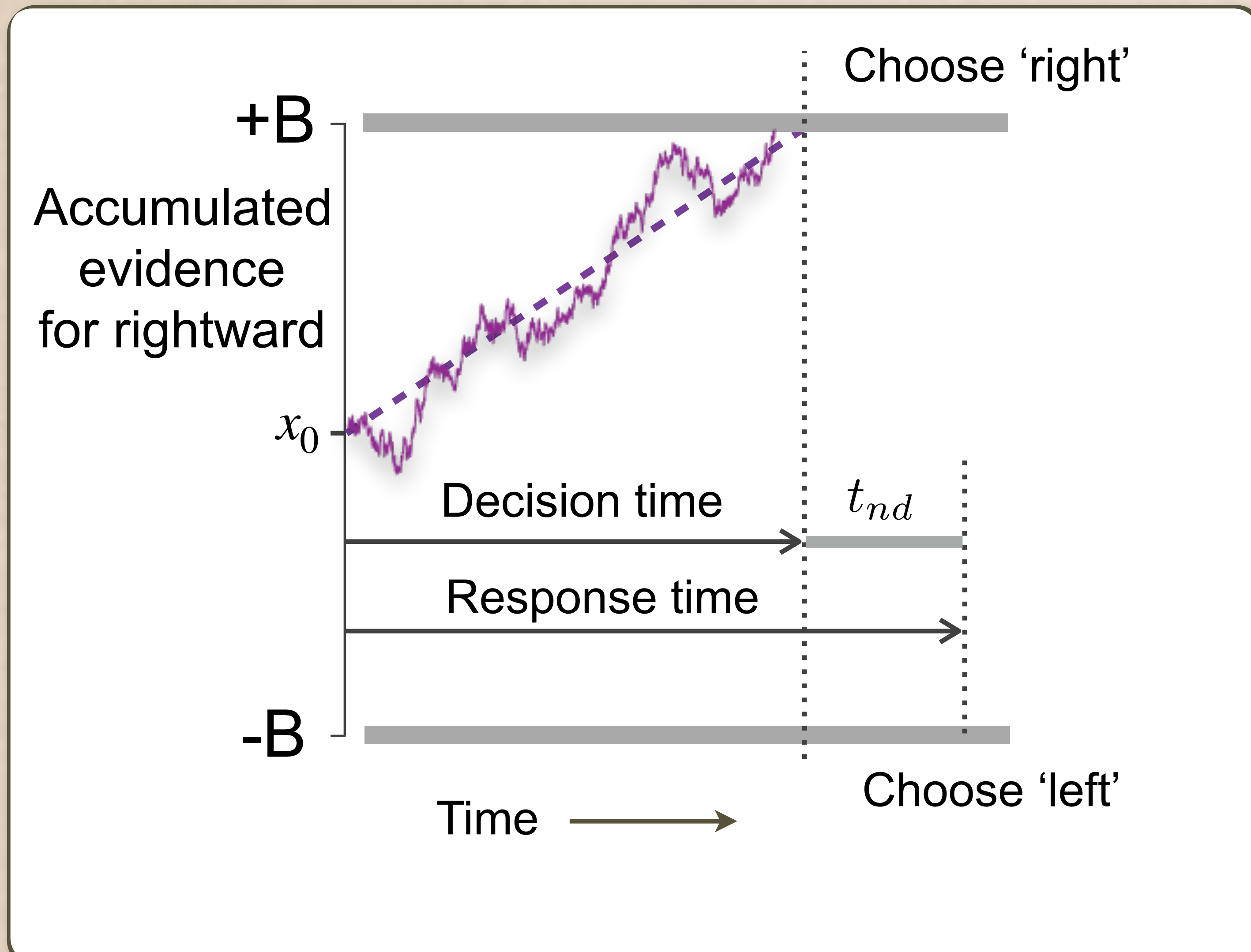


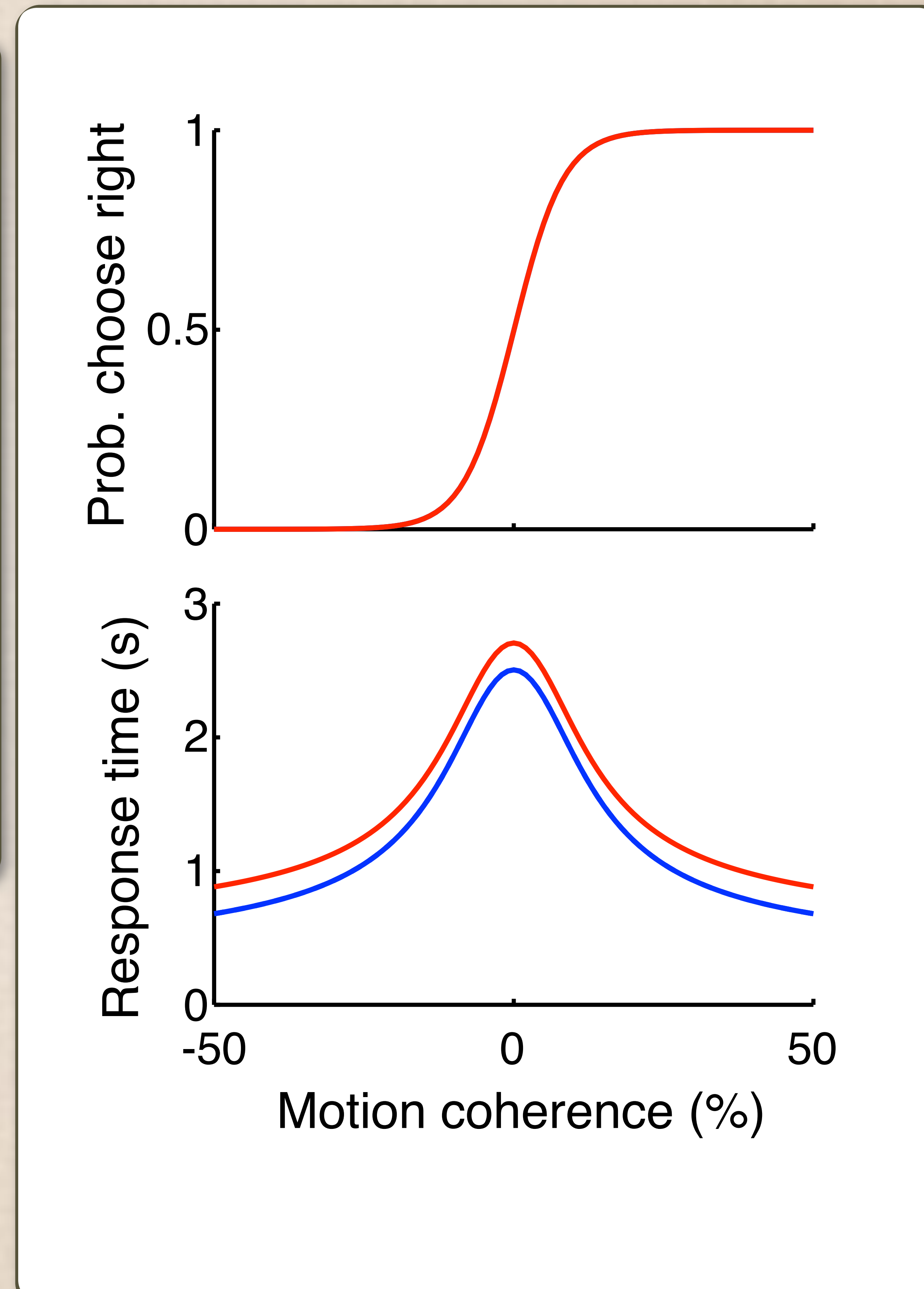
Change in non-decision time





Change in non-decision time

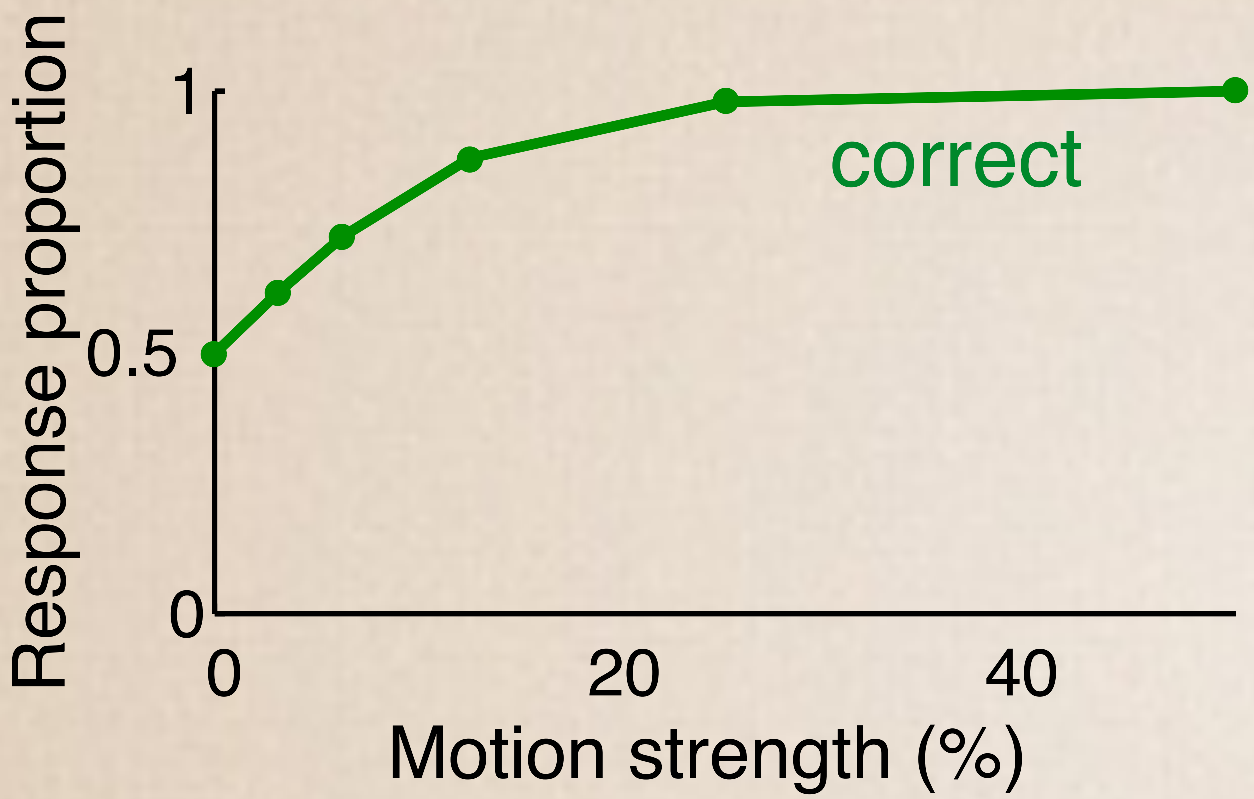




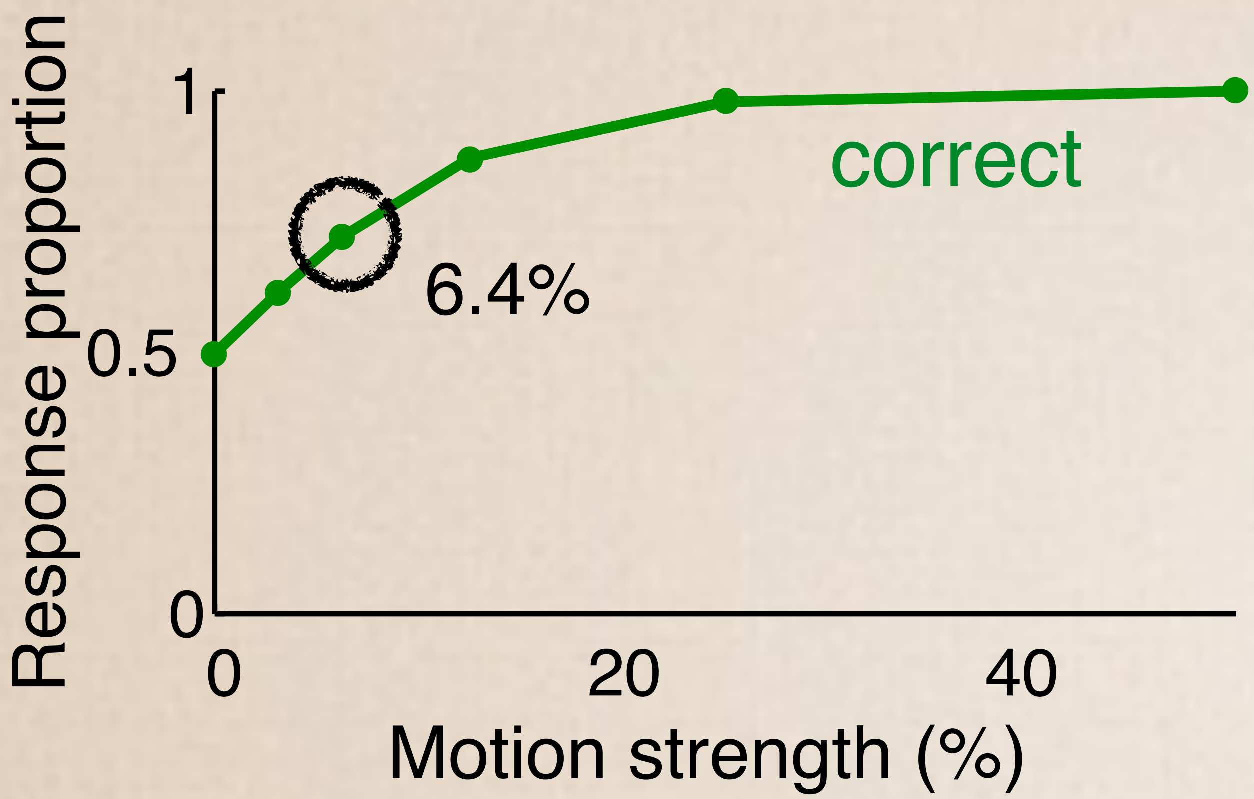
Quantile probability plot

Ratcliff et al., *Psychology and aging* (2001)

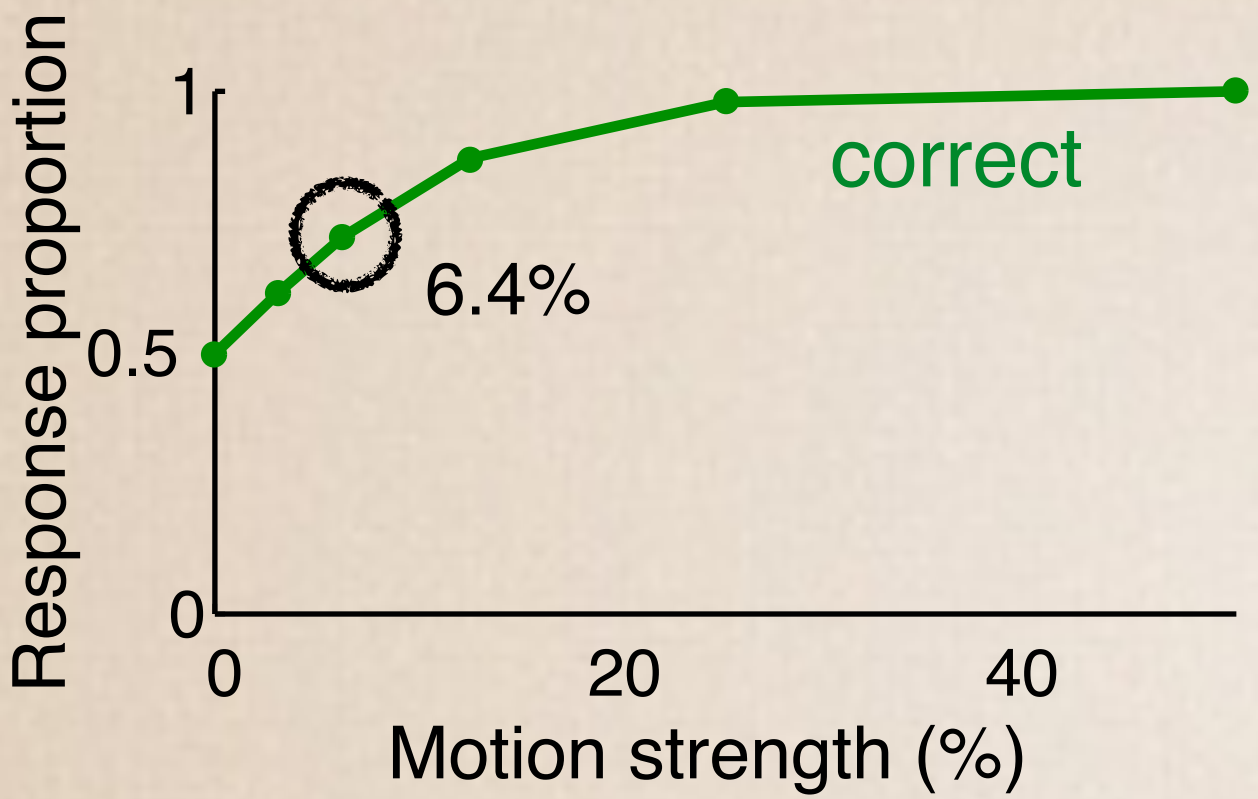
Quantile probability plot

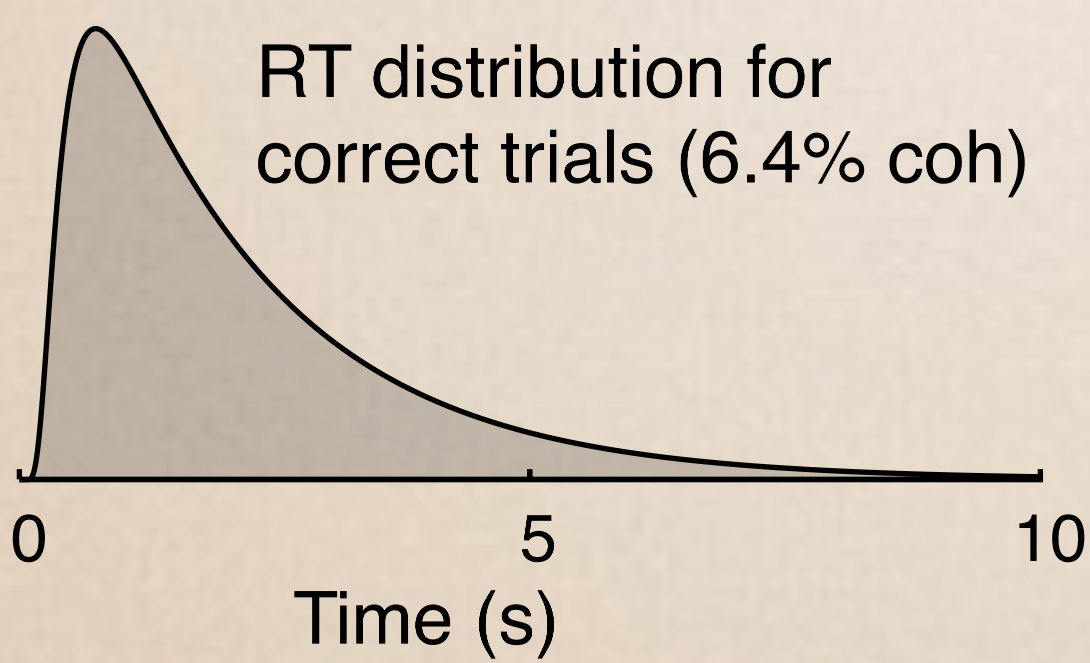


Quantile probability plot

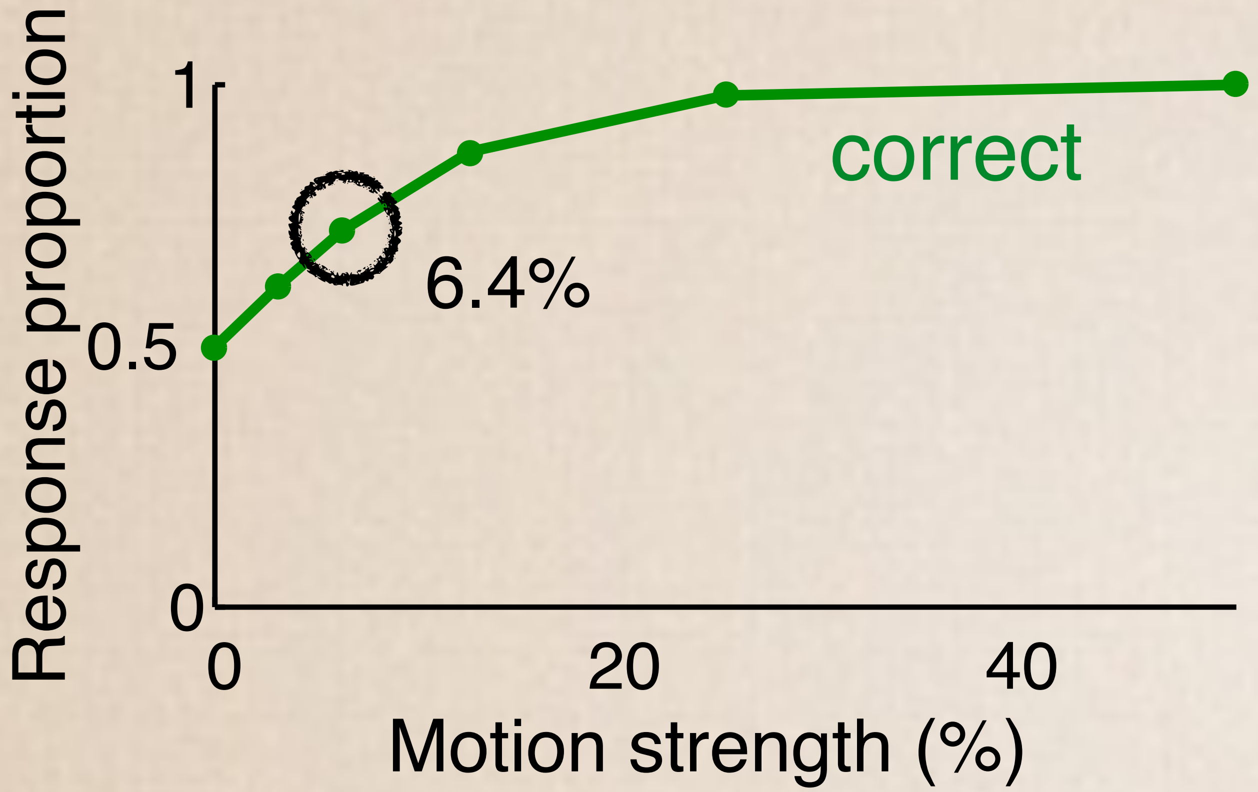


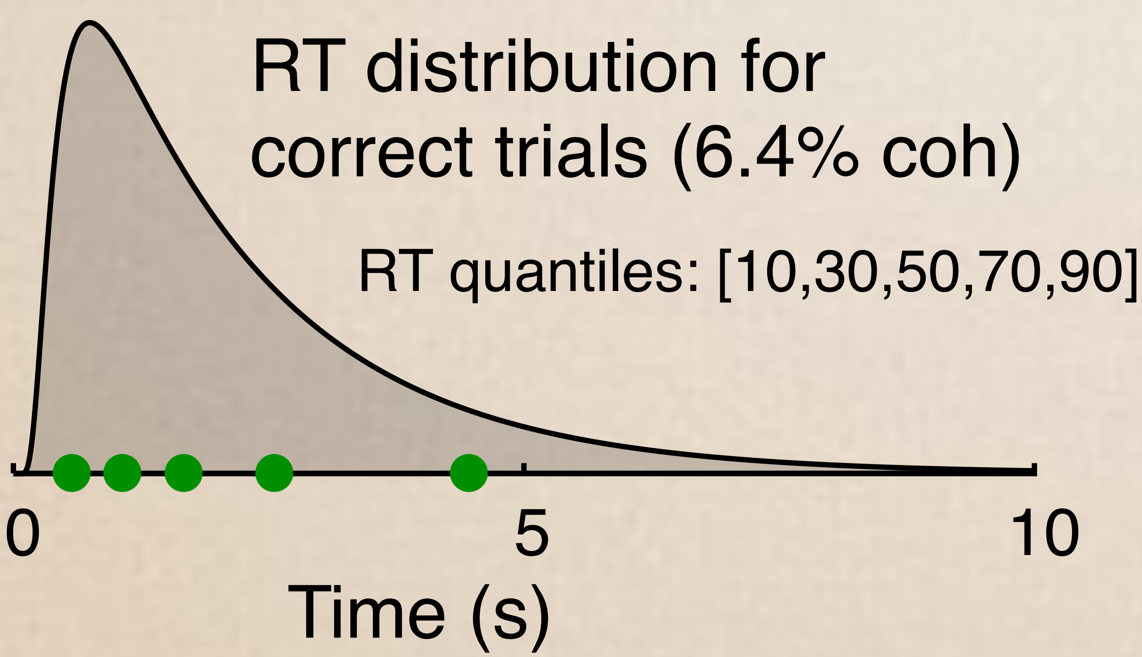
Quantile probability plot



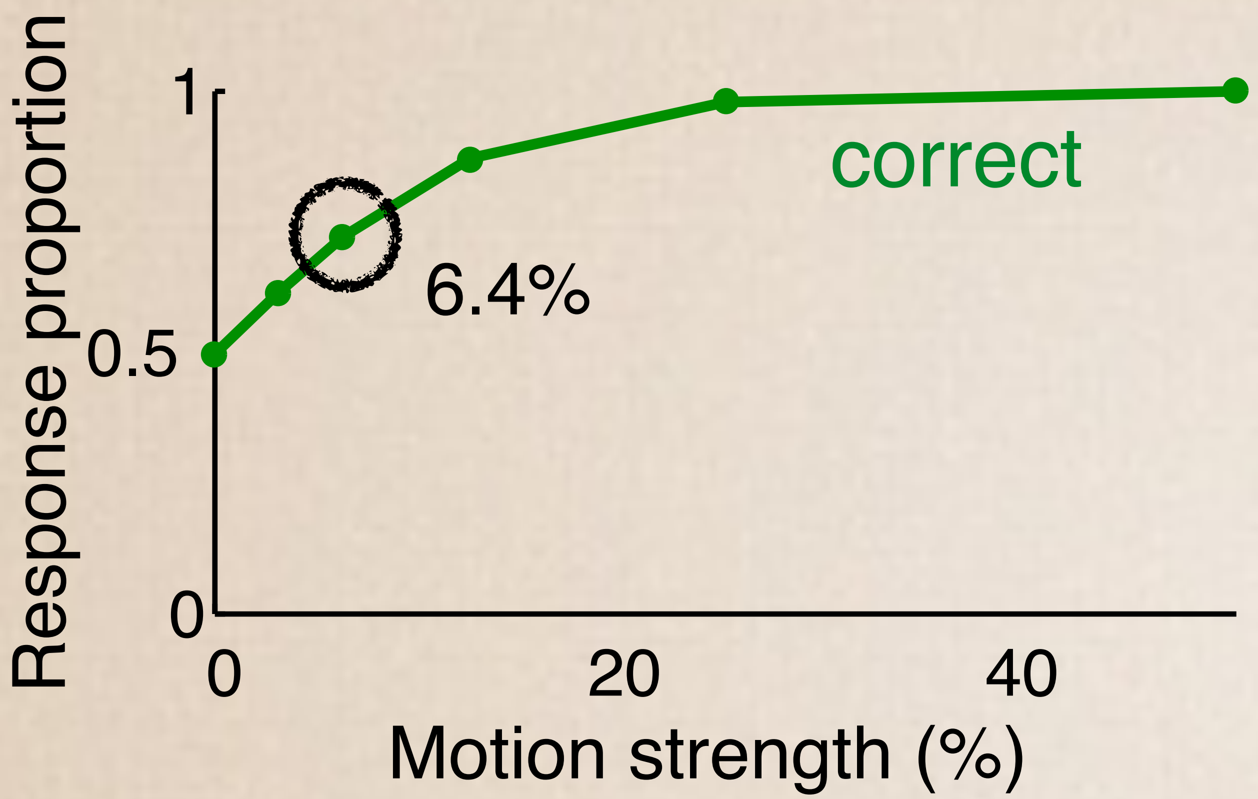


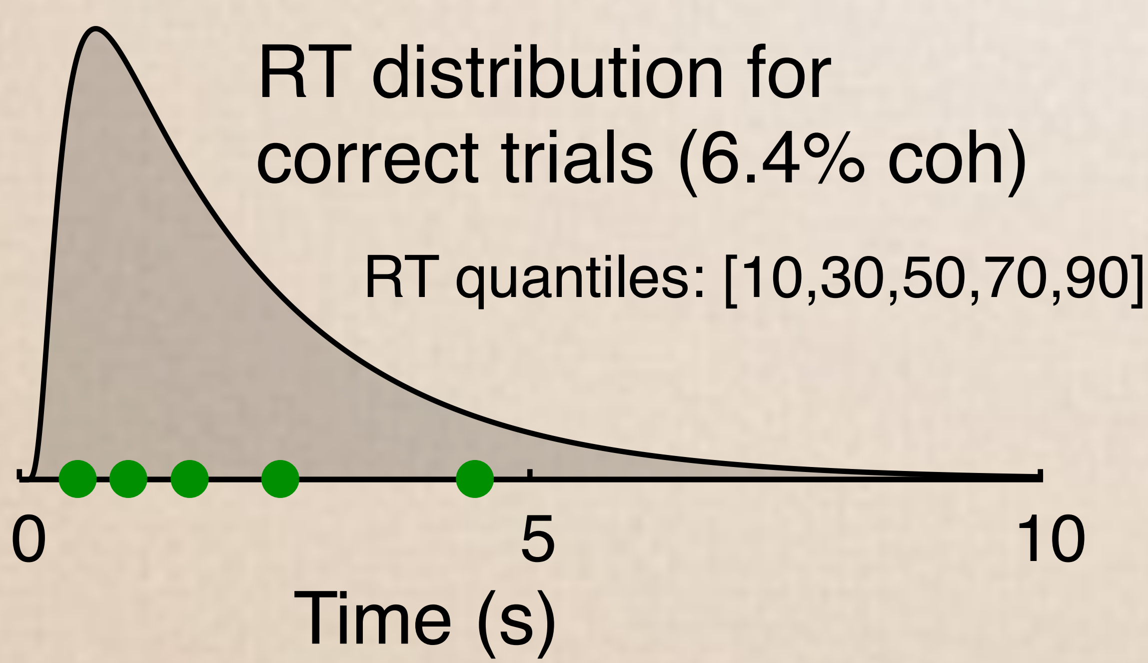
Quantile probability plot

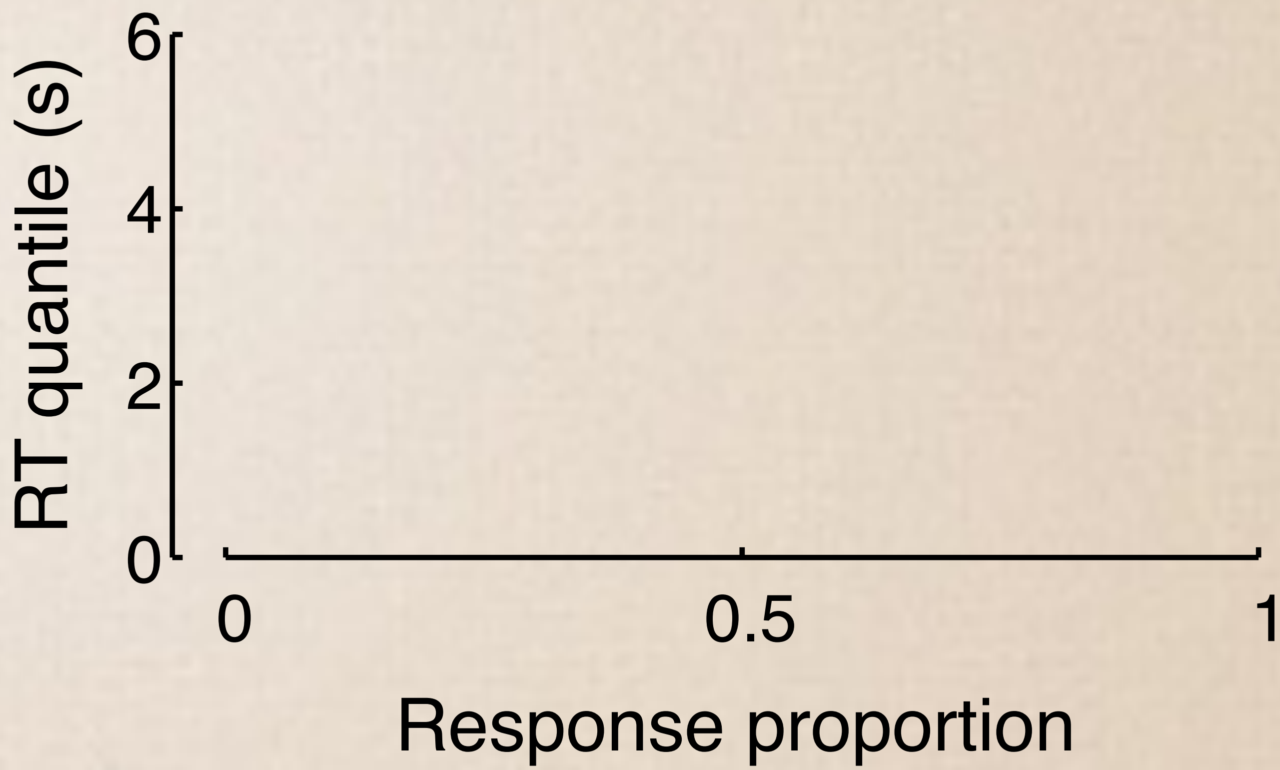




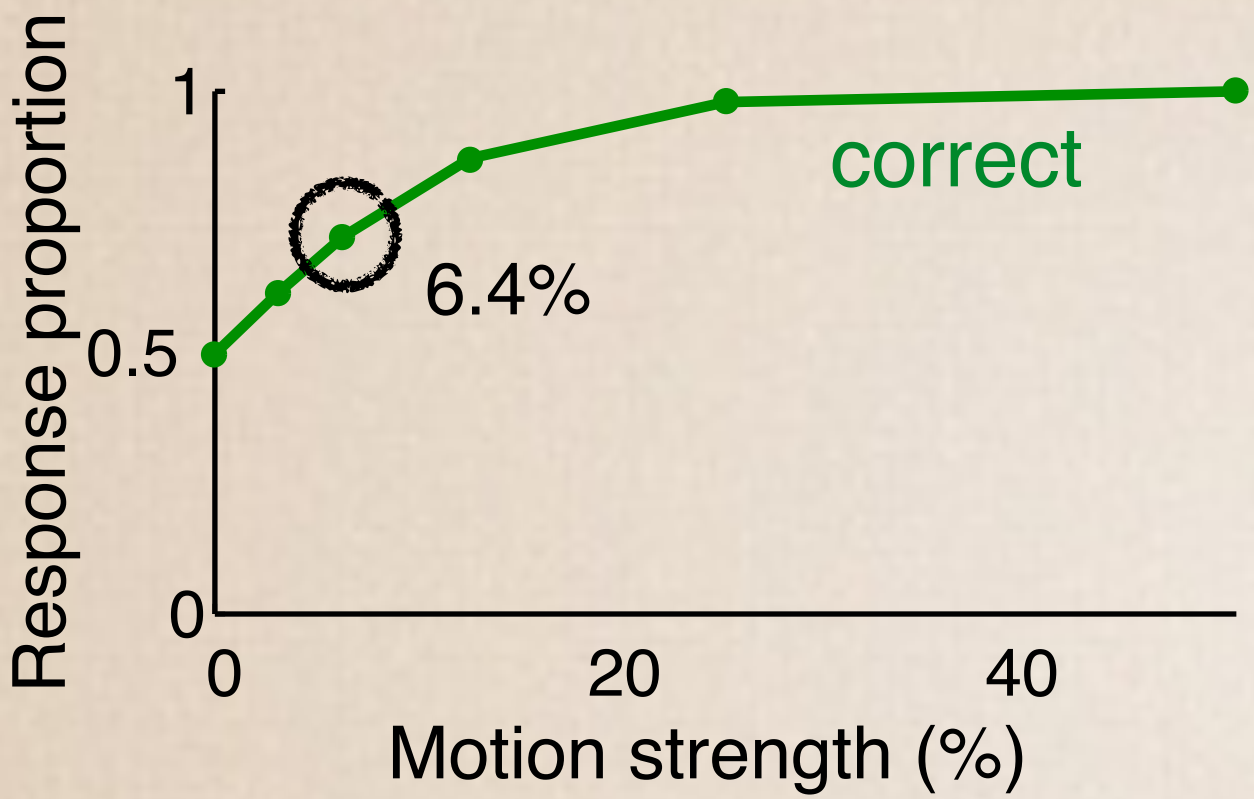
Quantile probability plot

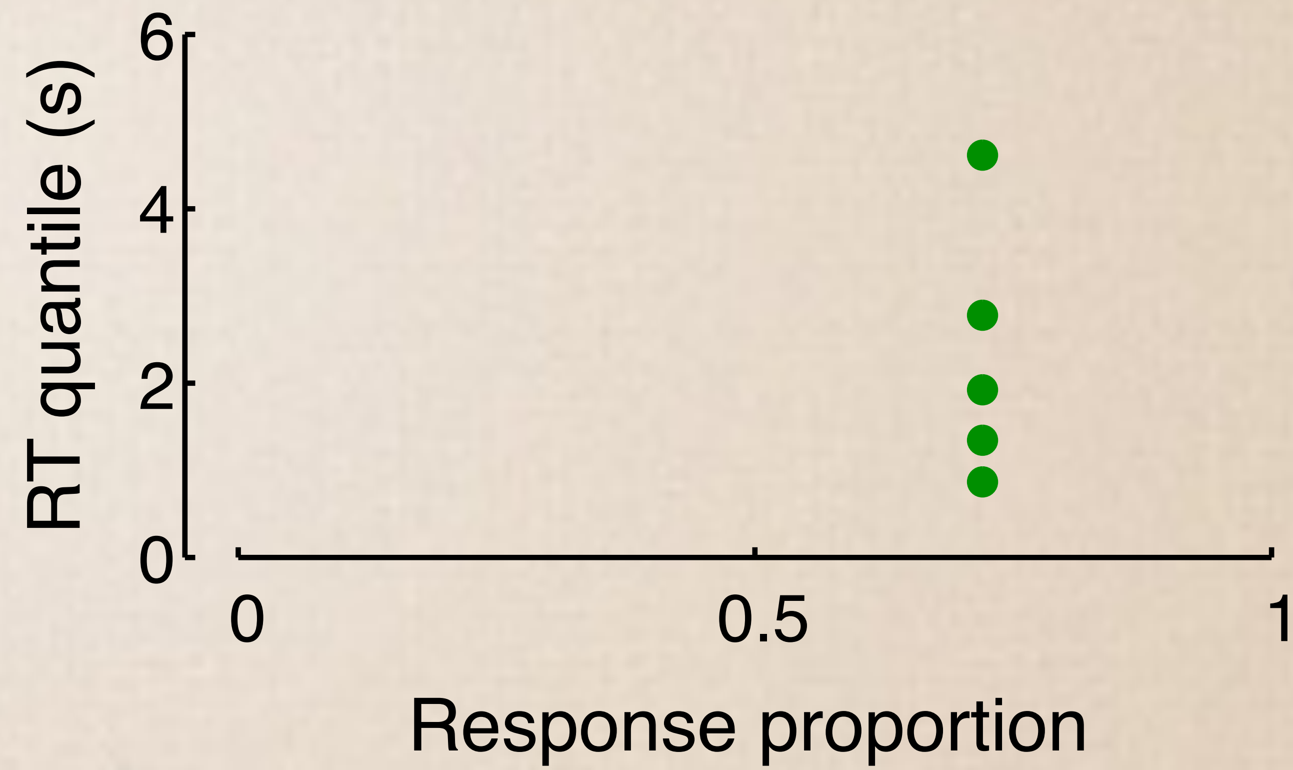


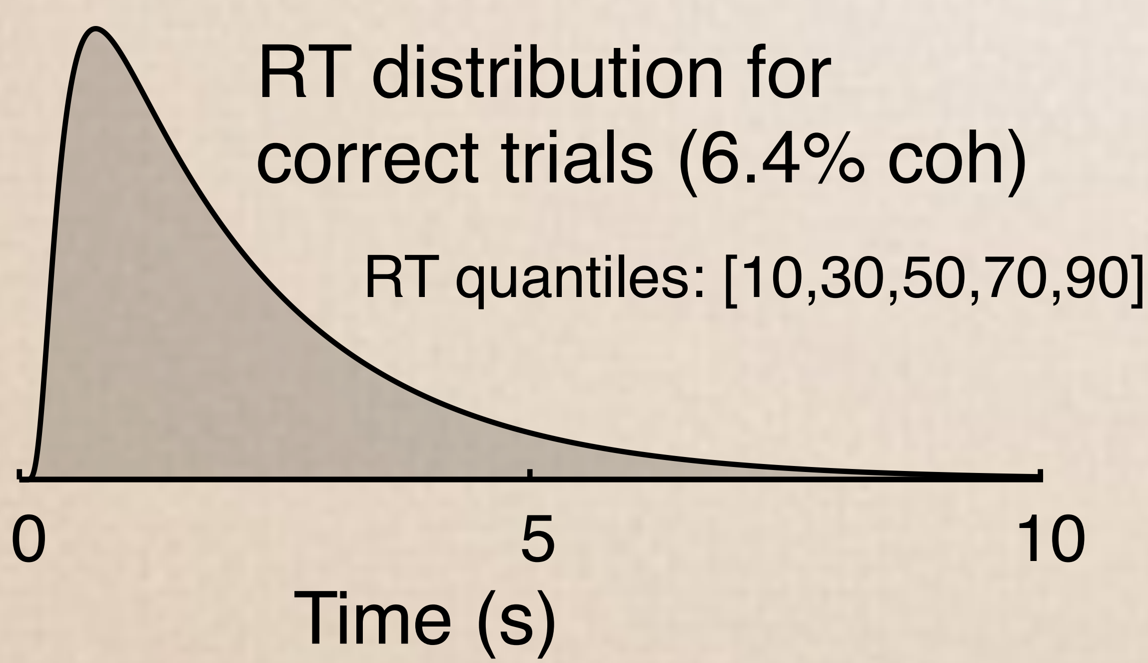




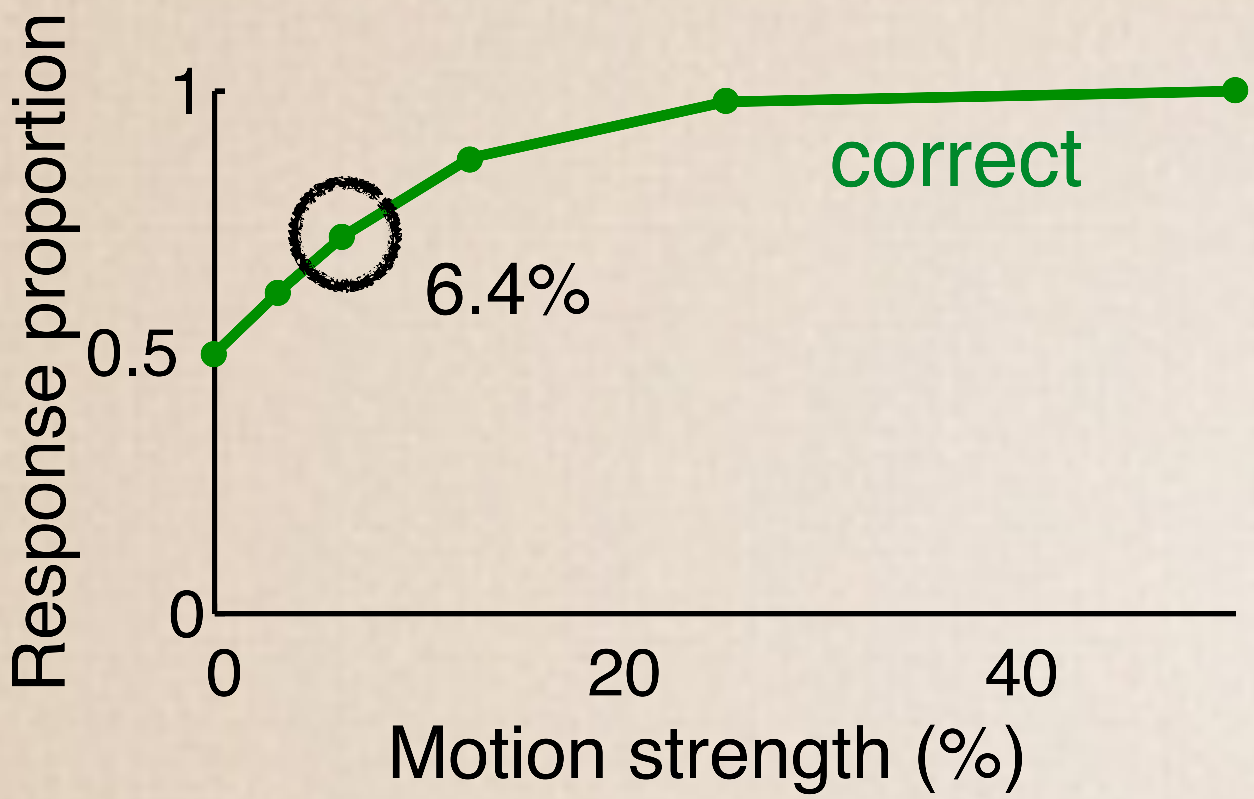
Quantile probability plot

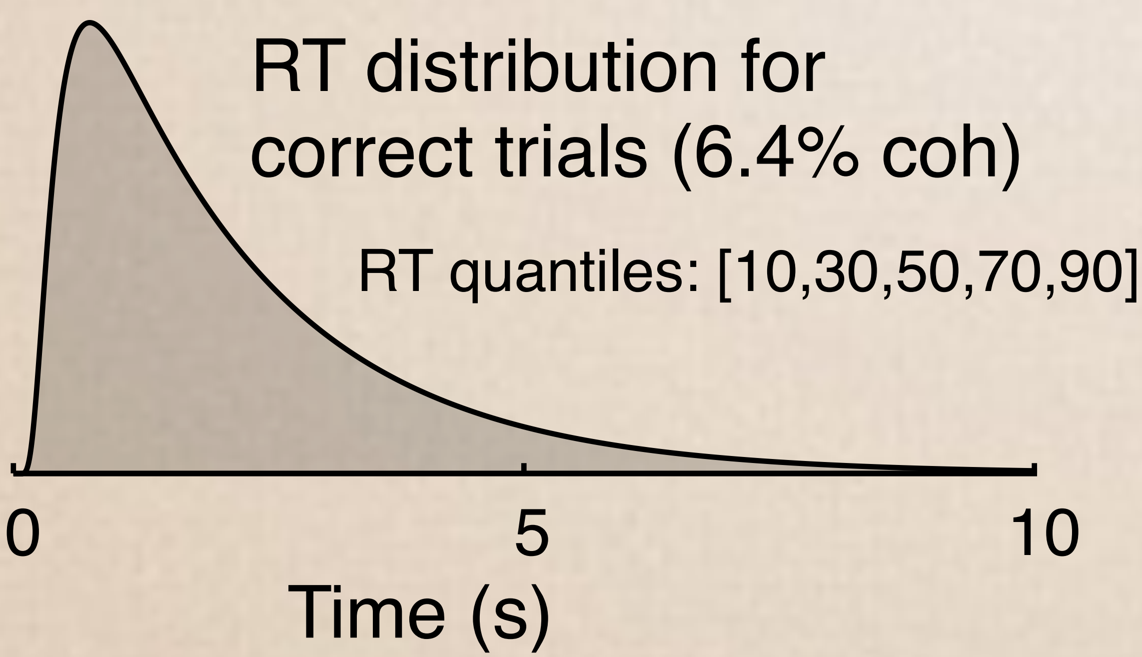


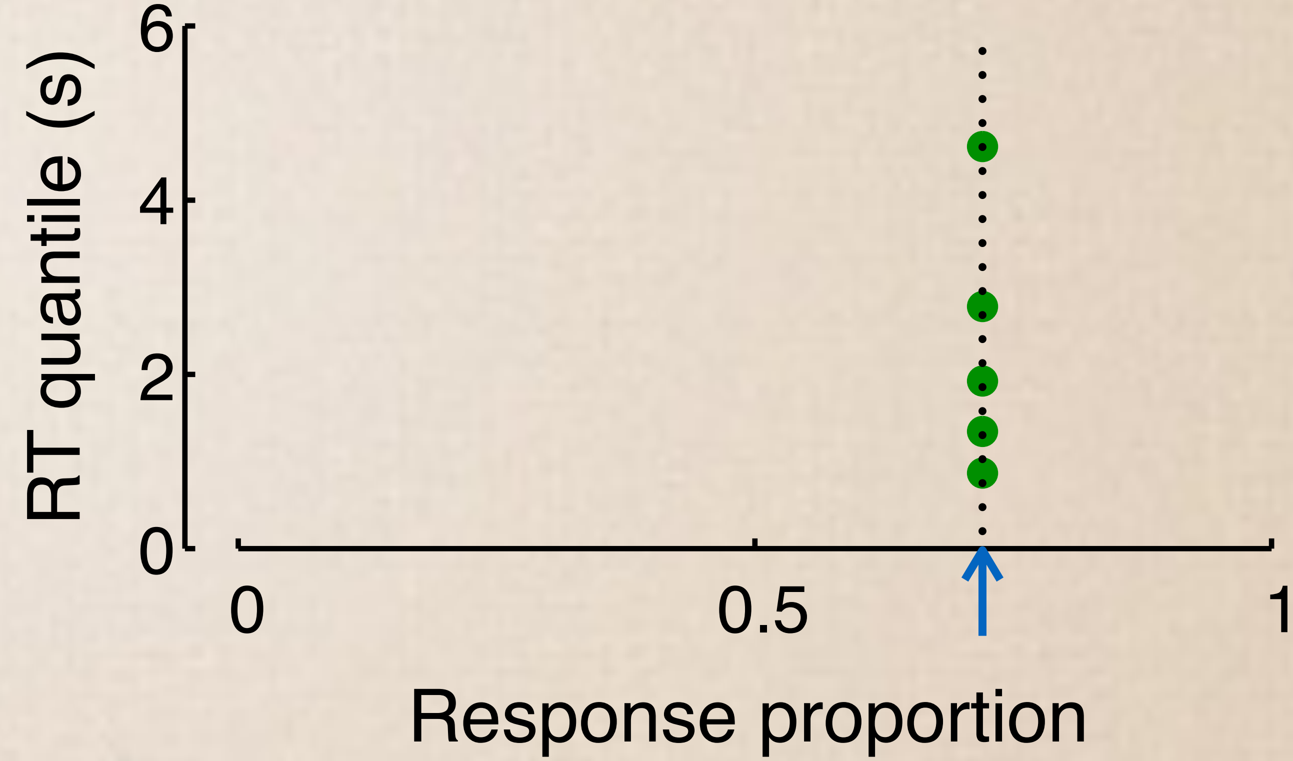




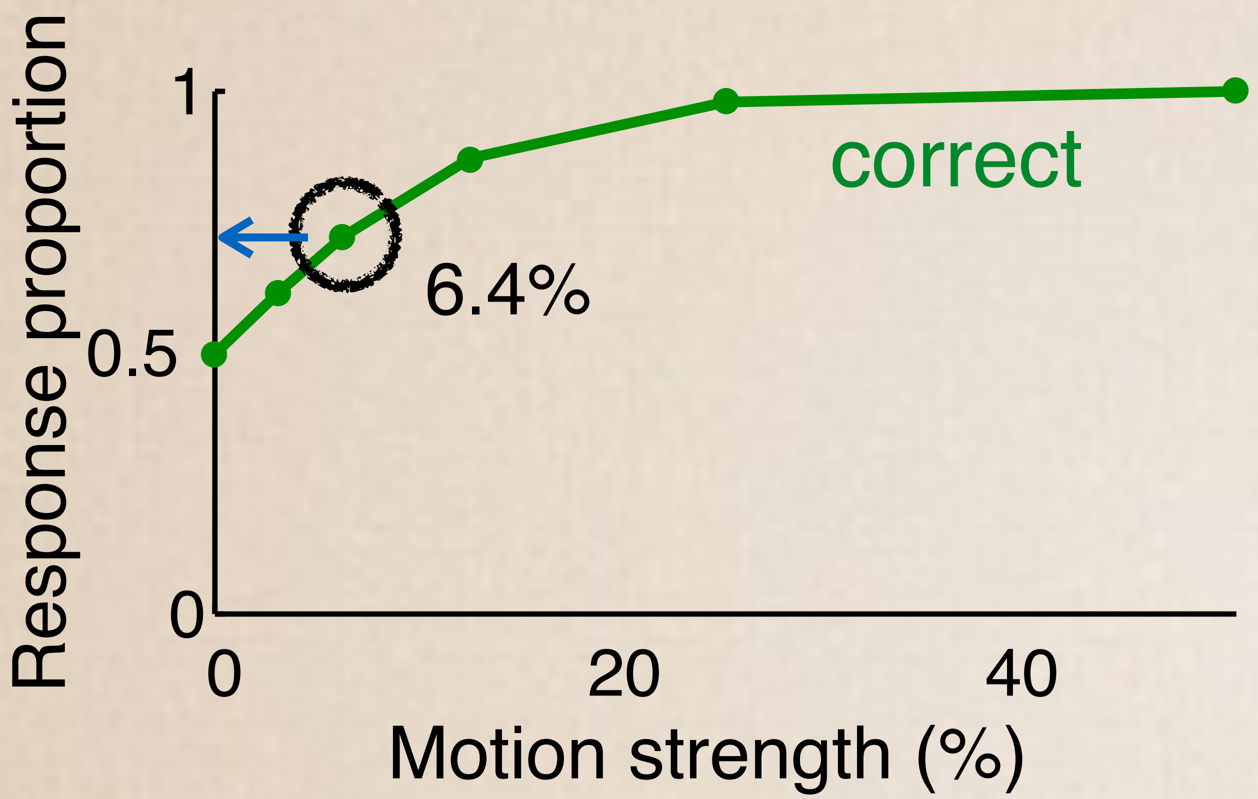
Quantile probability plot

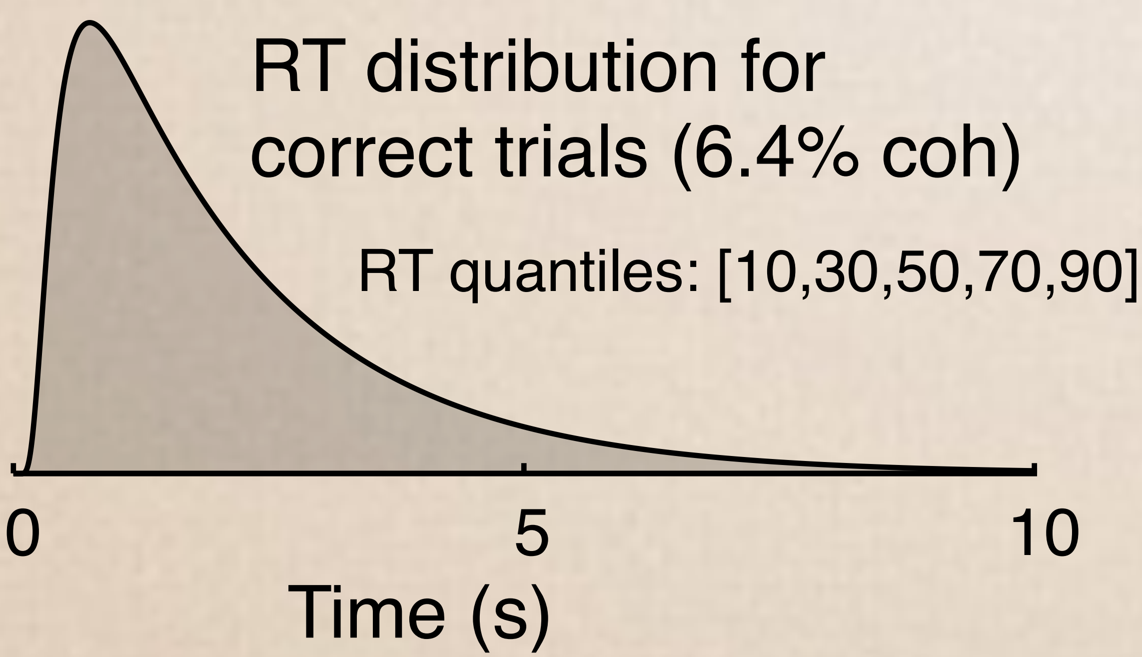


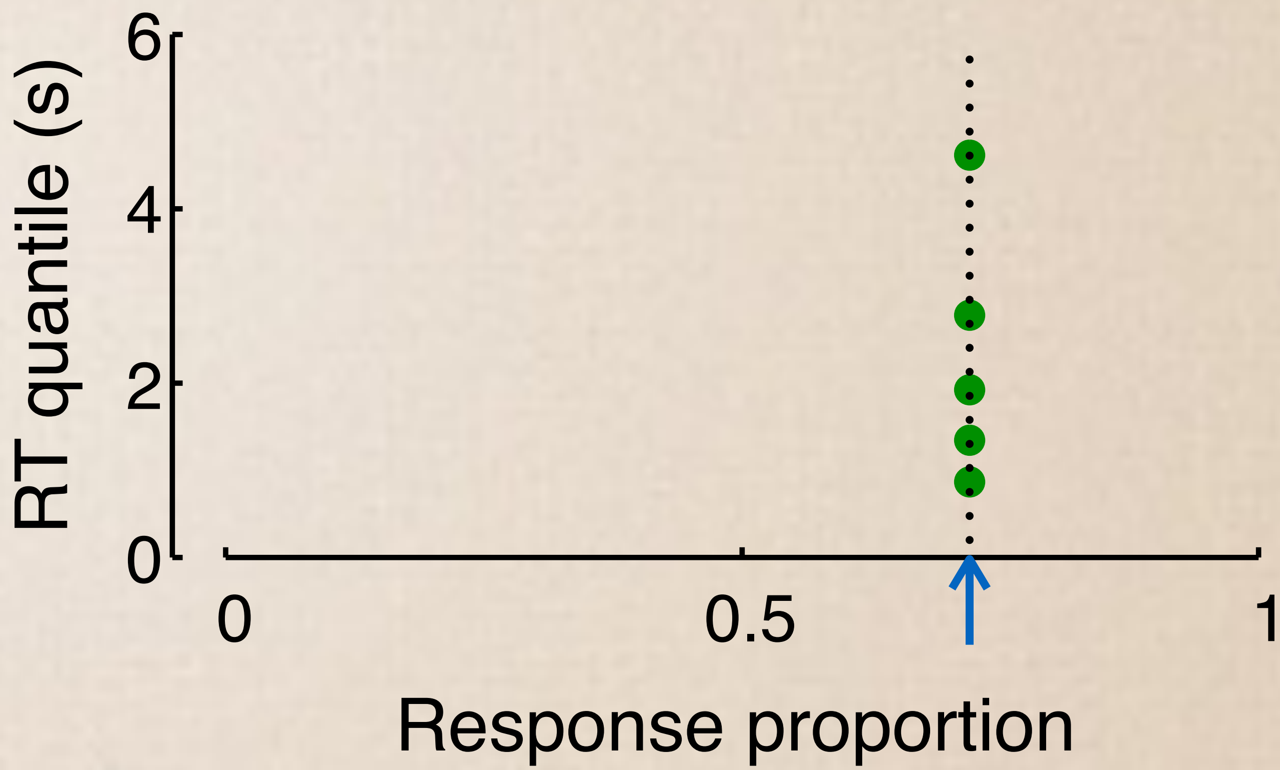




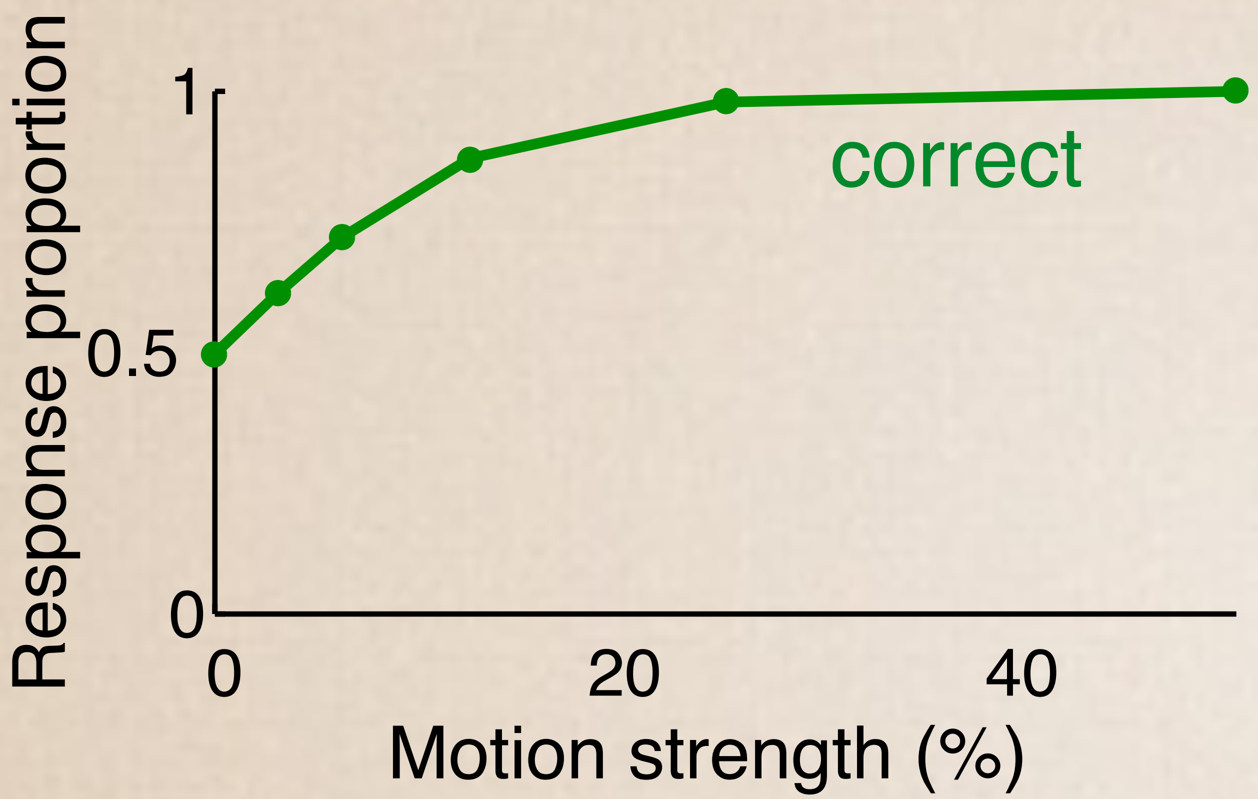
Quantile probability plot

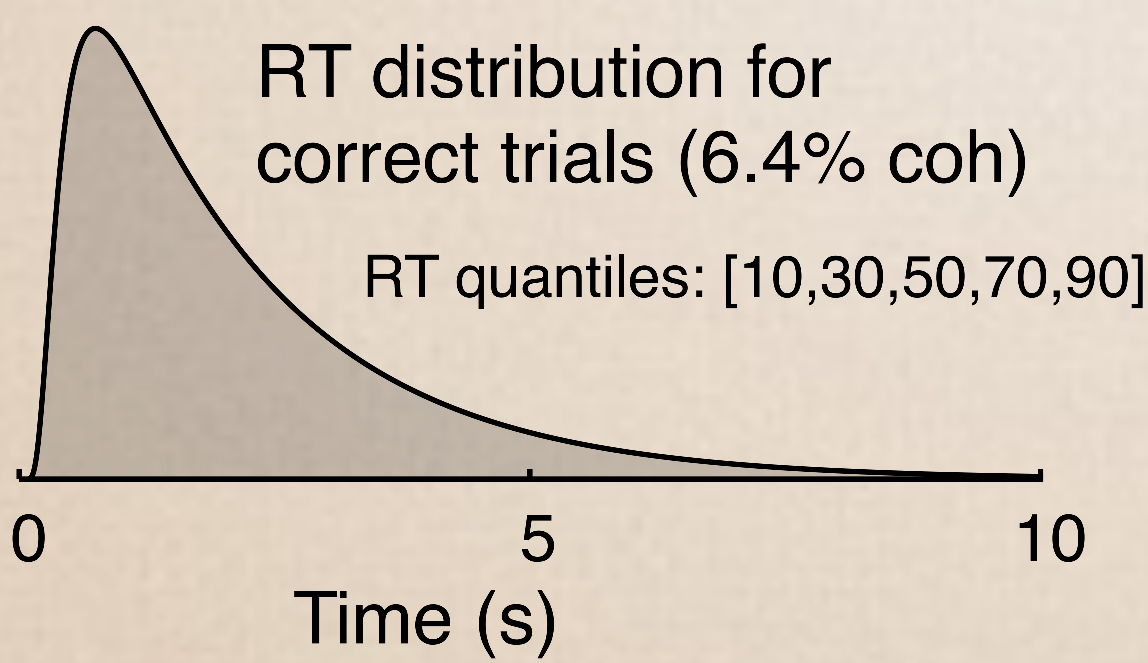






Quantile probability plot





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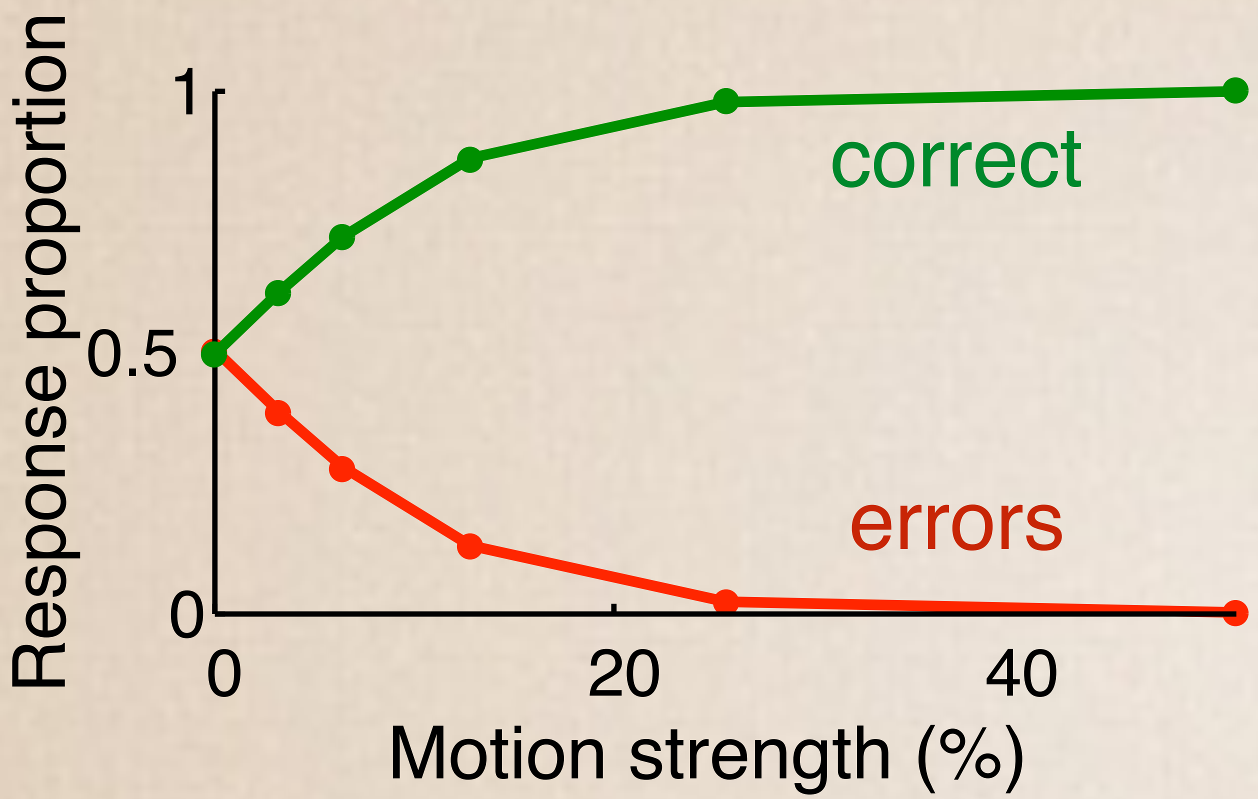
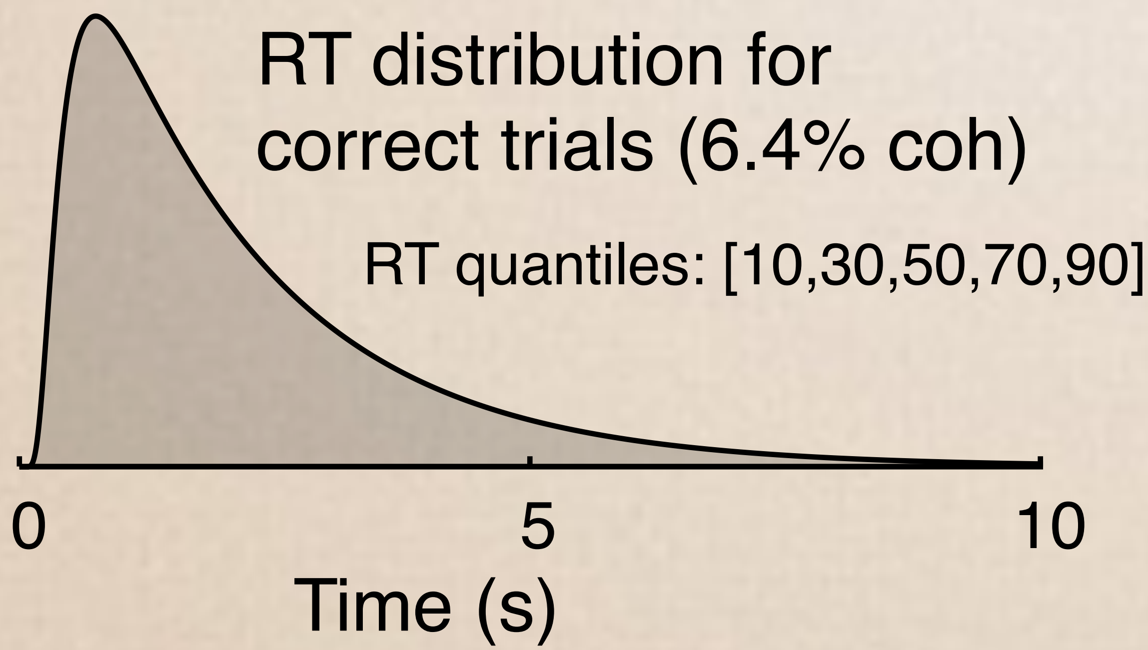
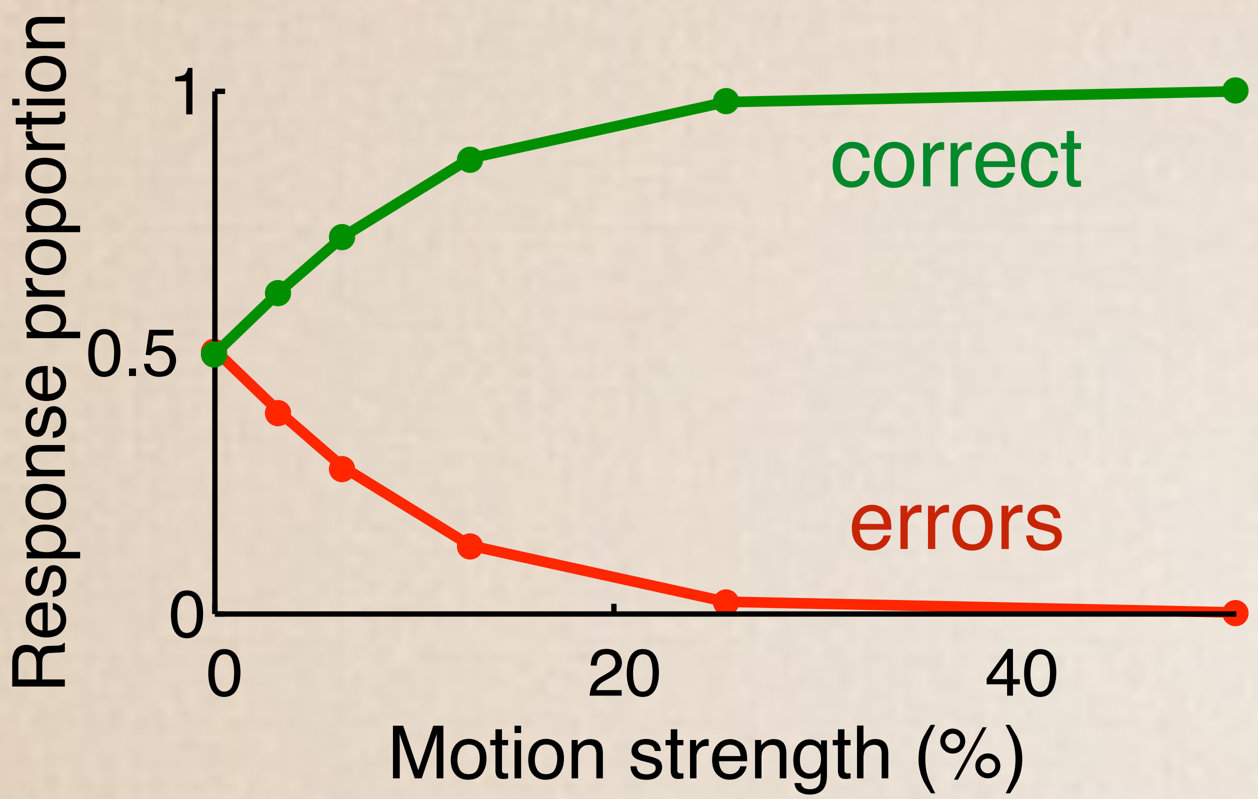
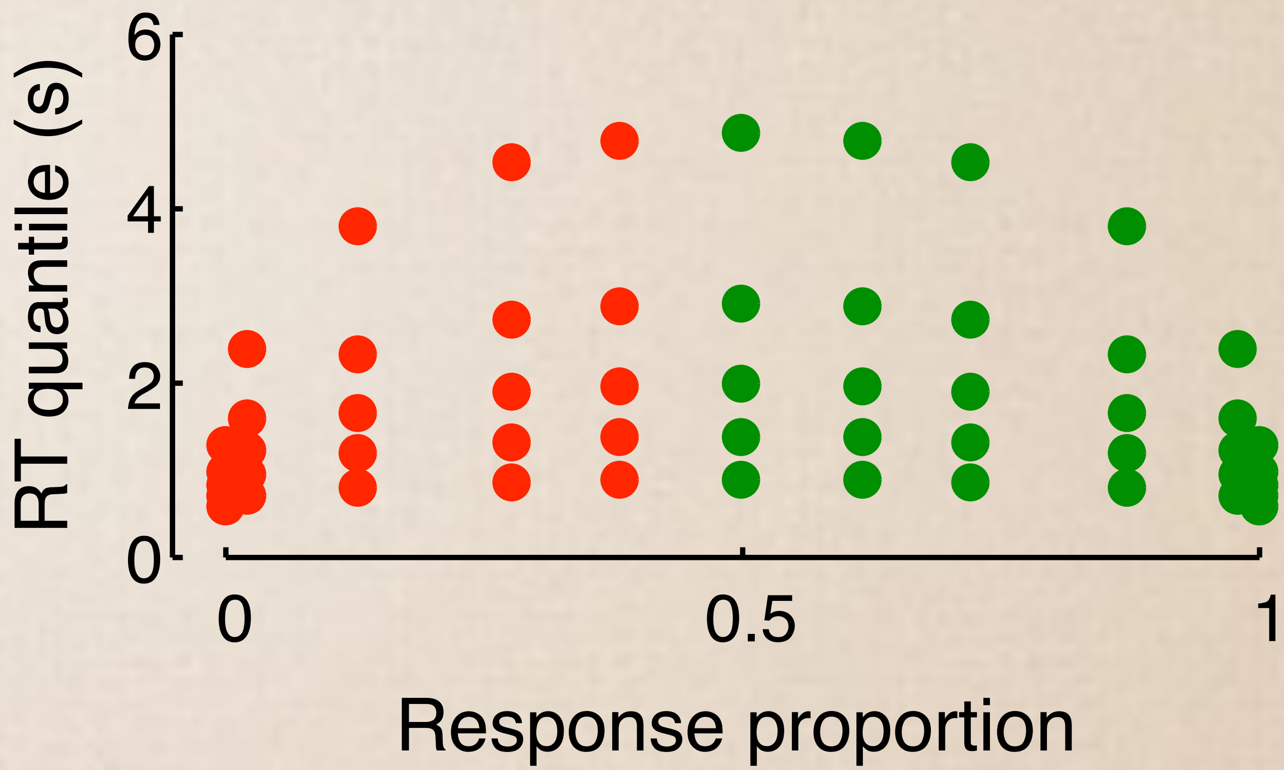
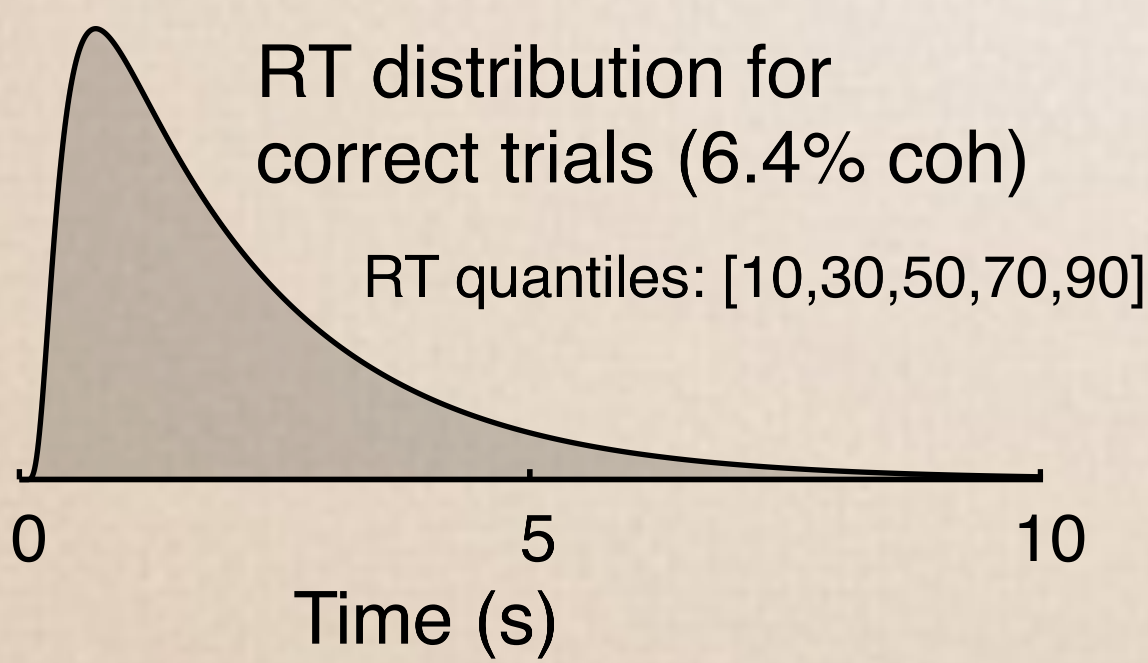




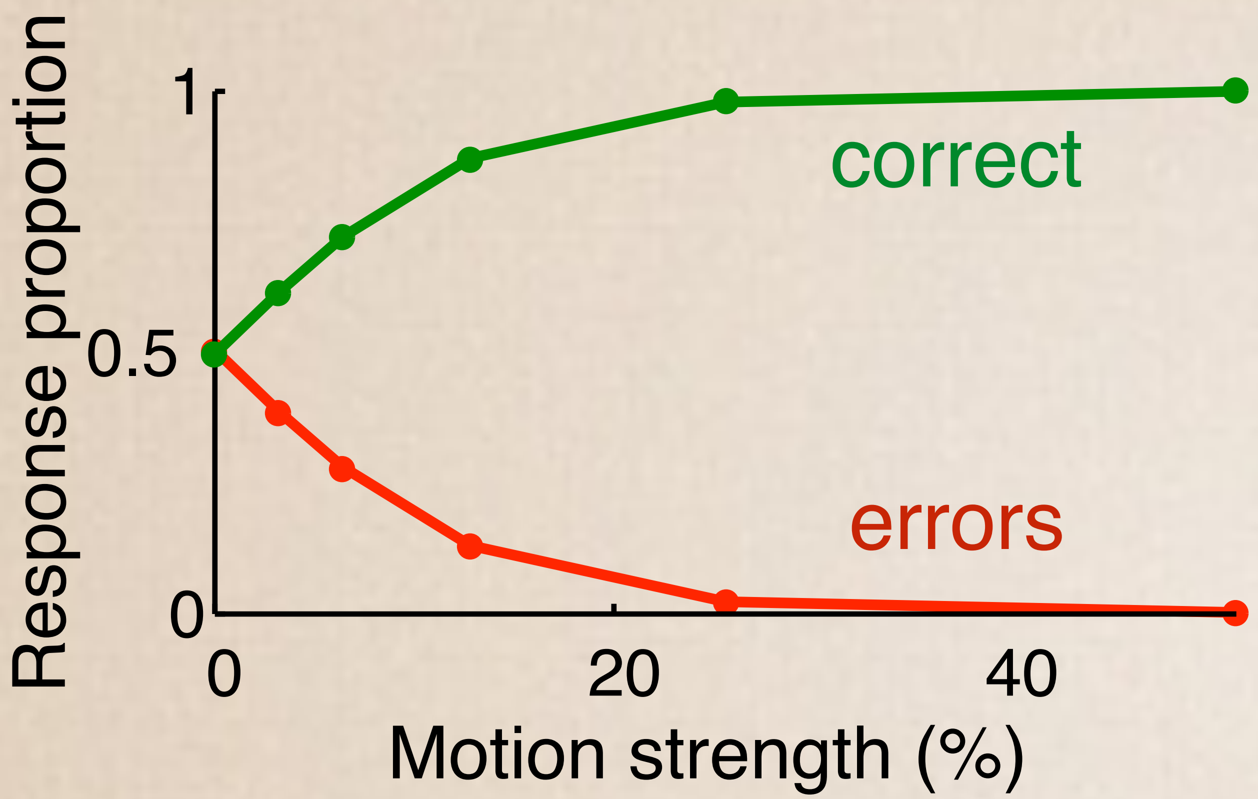
Quantile probability plot

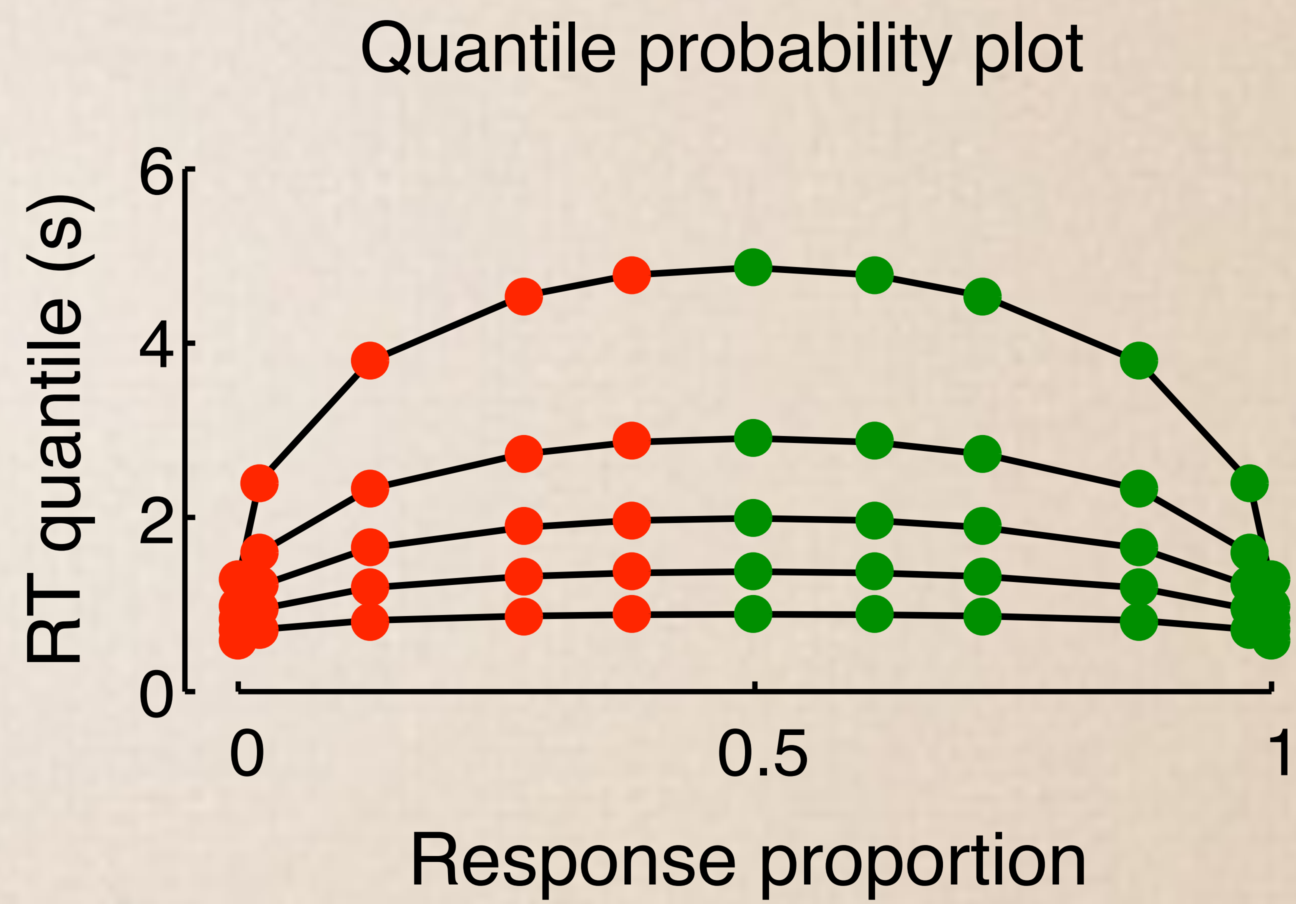


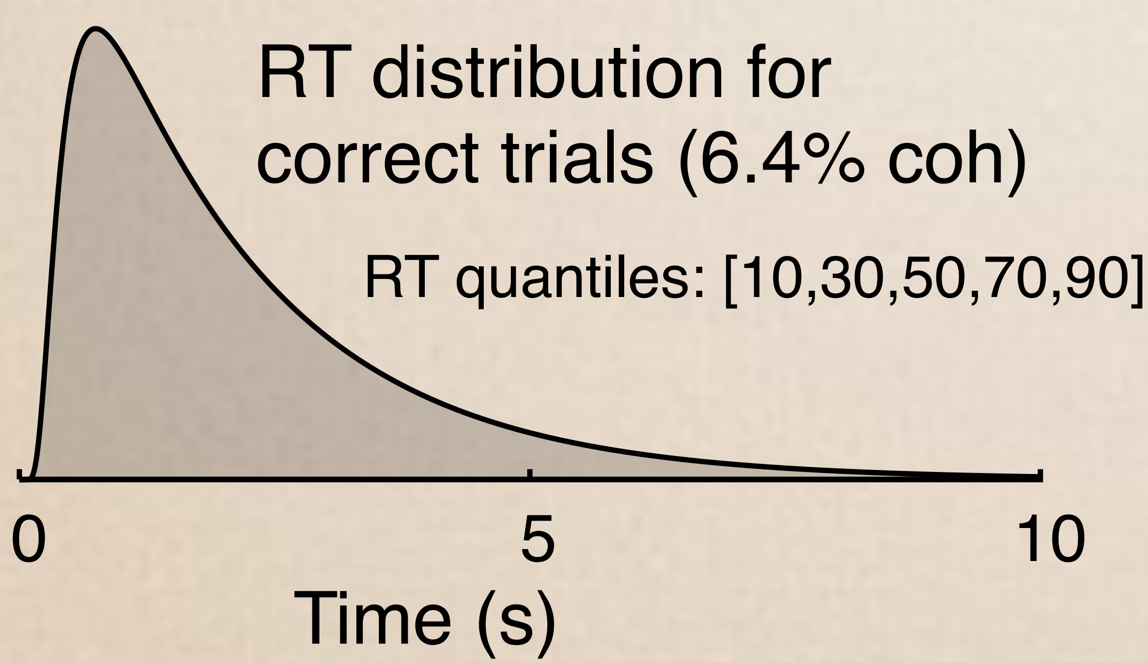




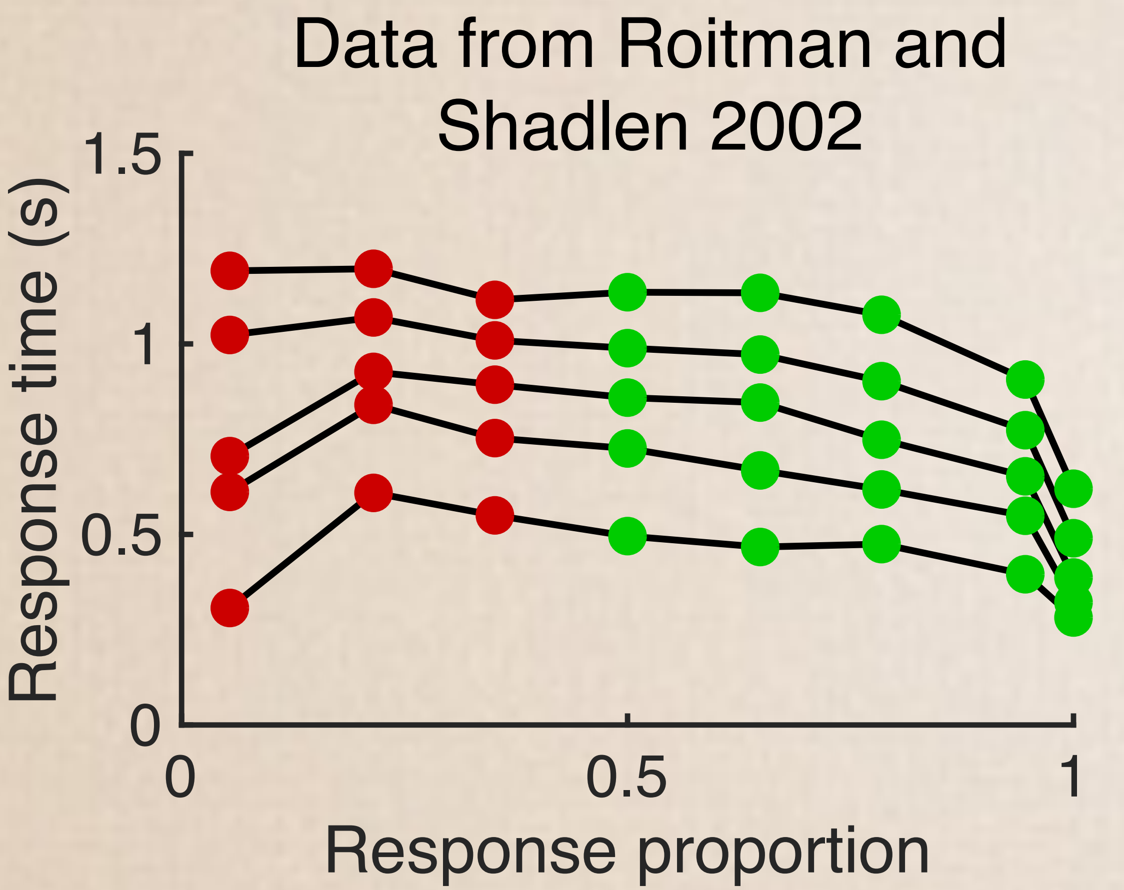
Quantile probability plot

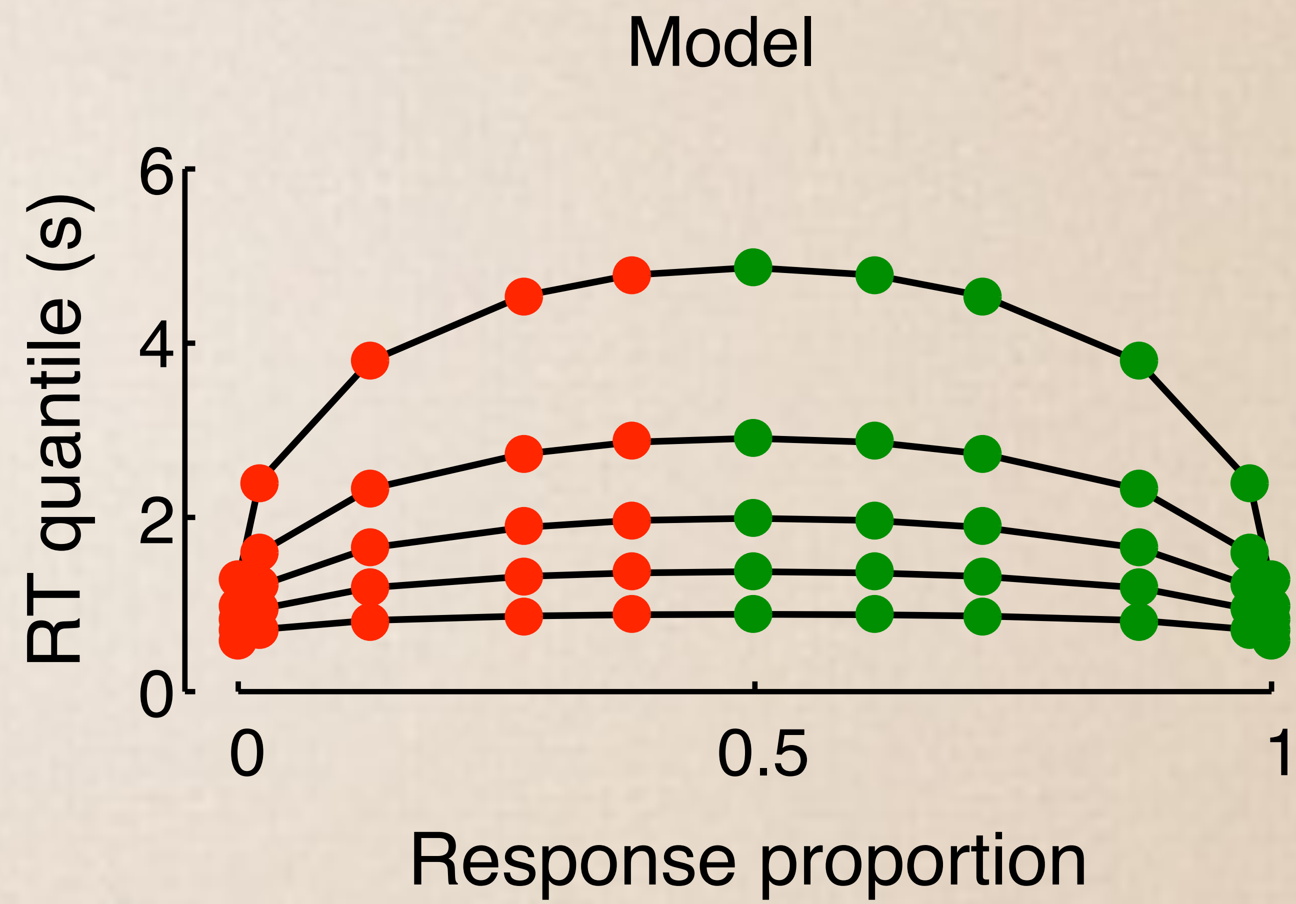




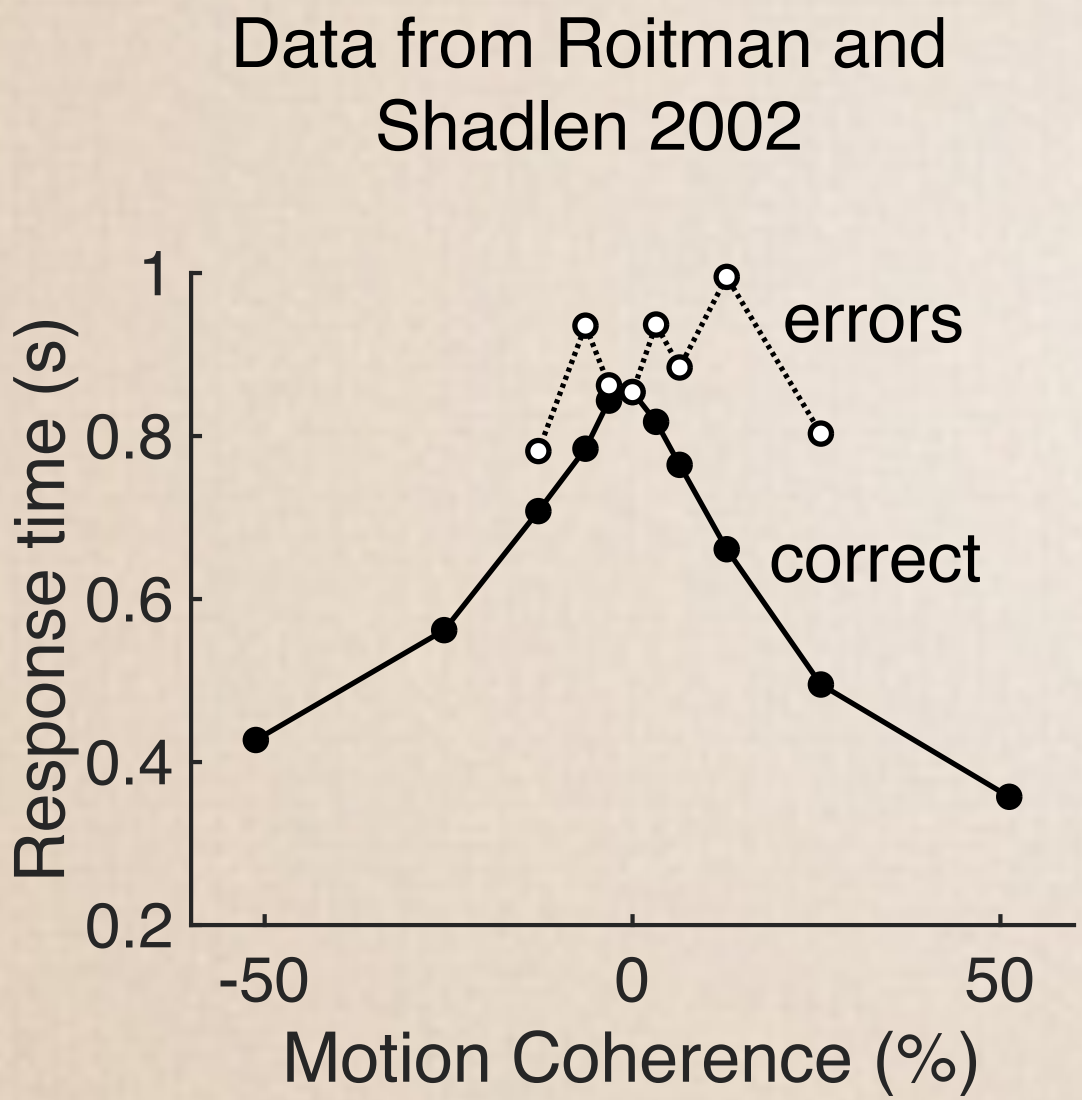


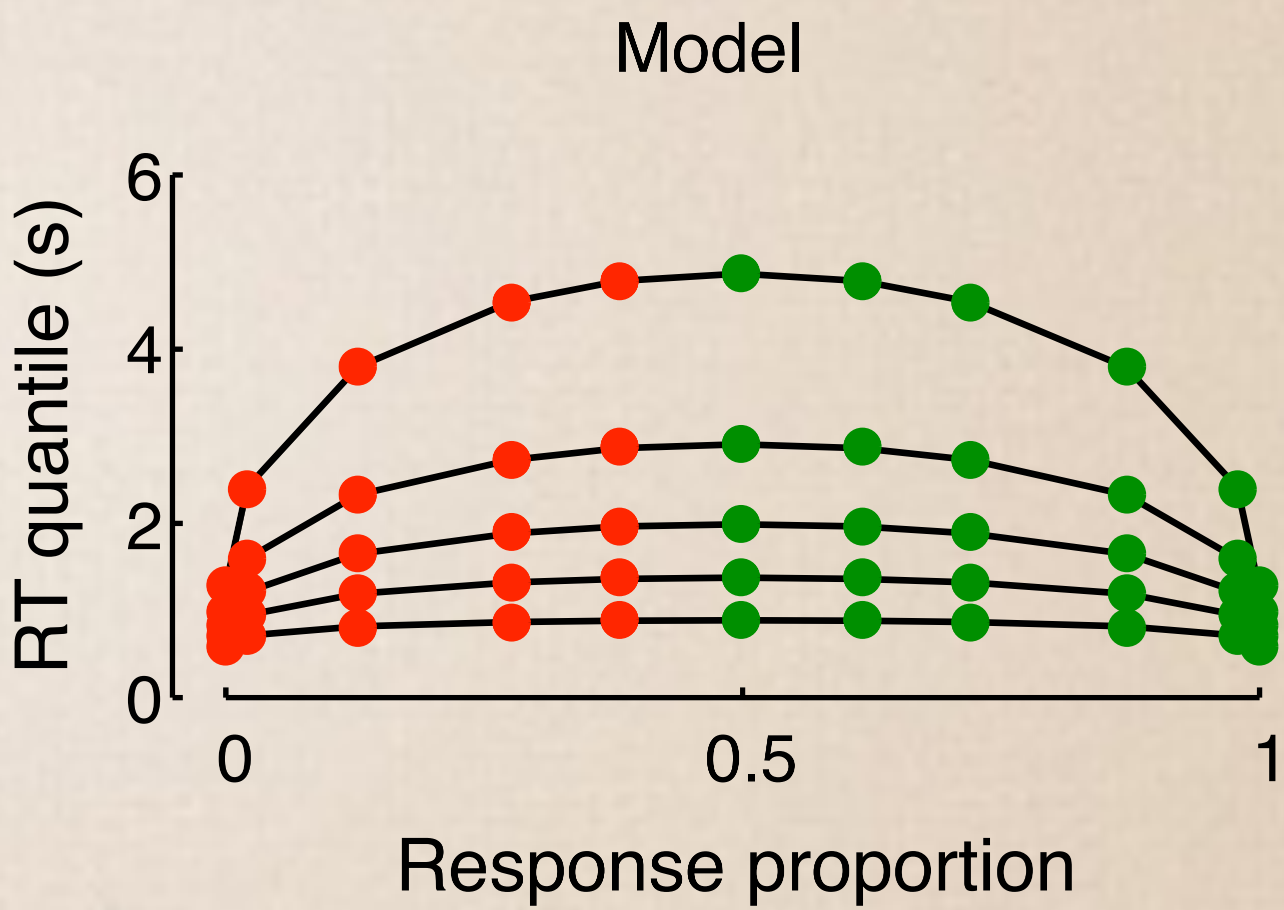
Problem with simple DDM: symmetric RT distributions



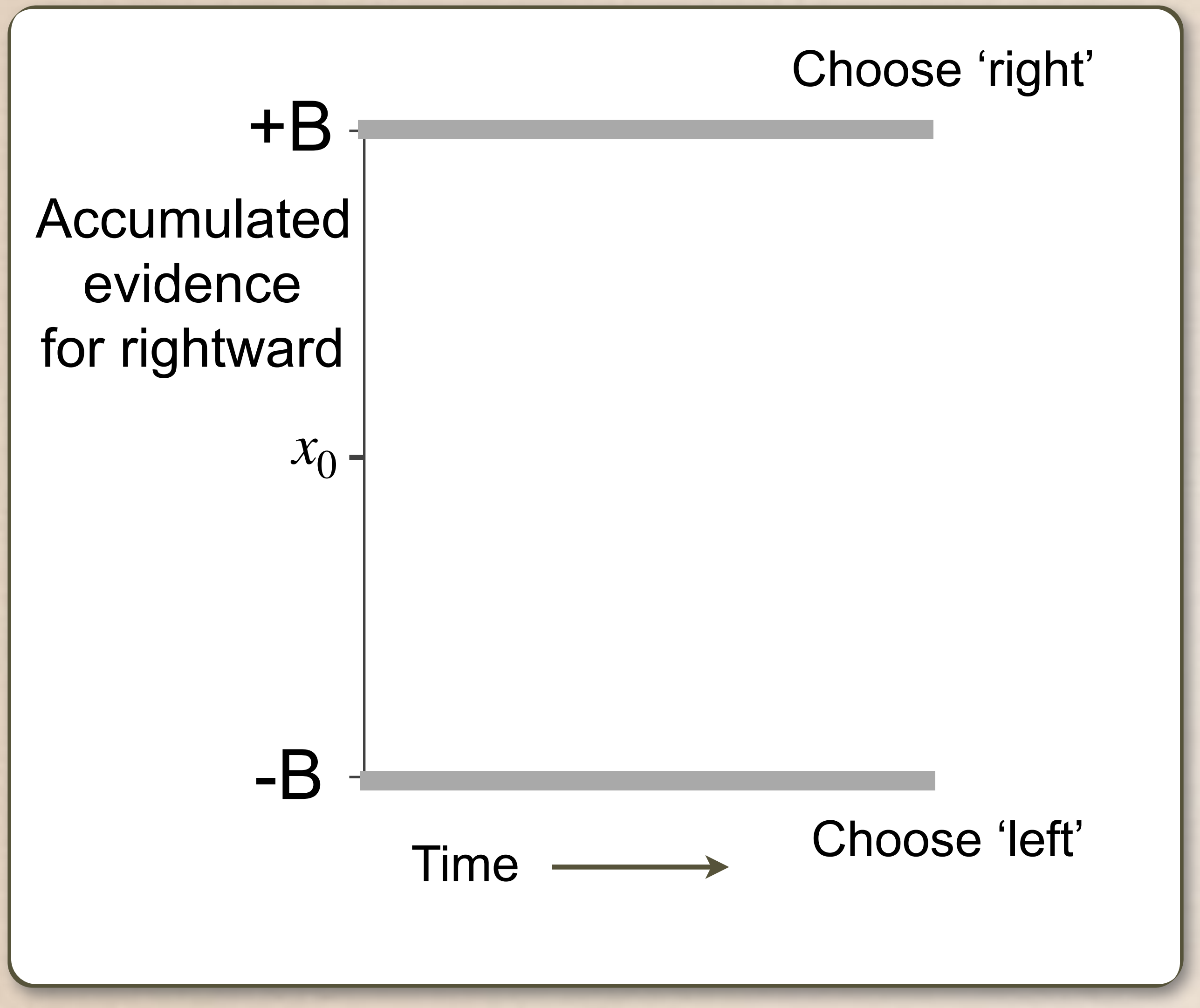


Problem with simple DDM: symmetric RT distributions





Why flat bounds lead to identical RT distributions for correct and incorrect decisions



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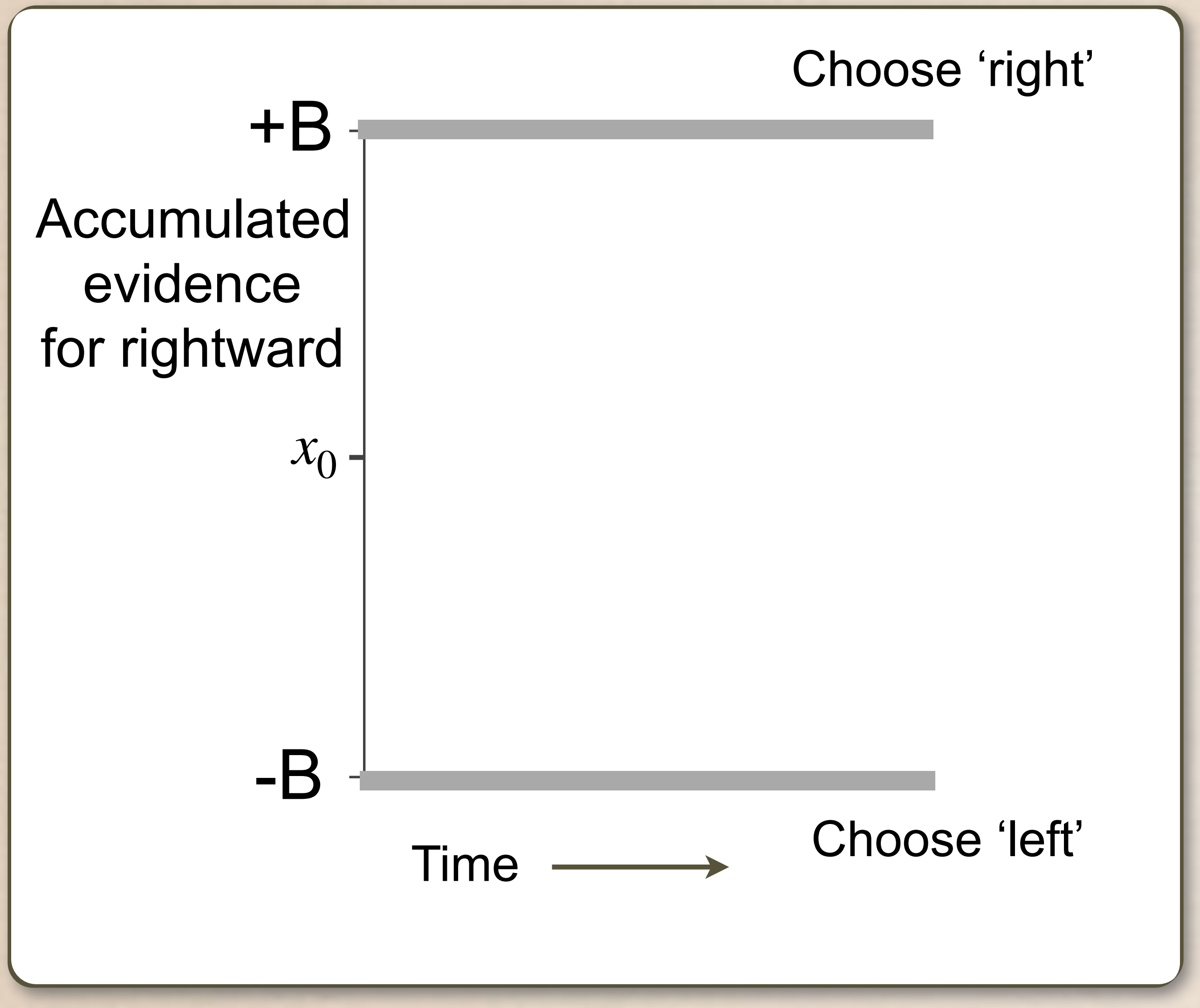
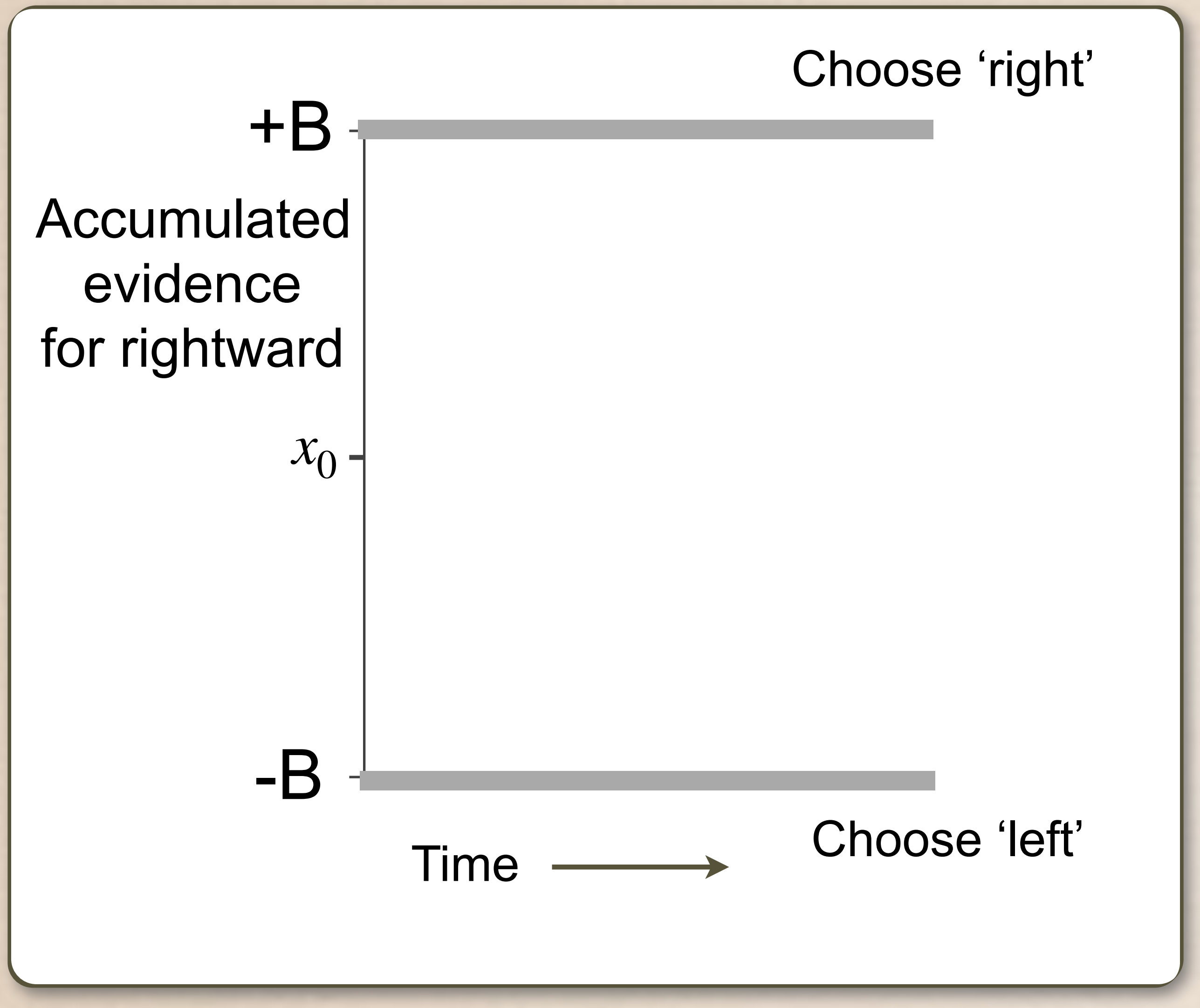


$\uparrow$ with probability p

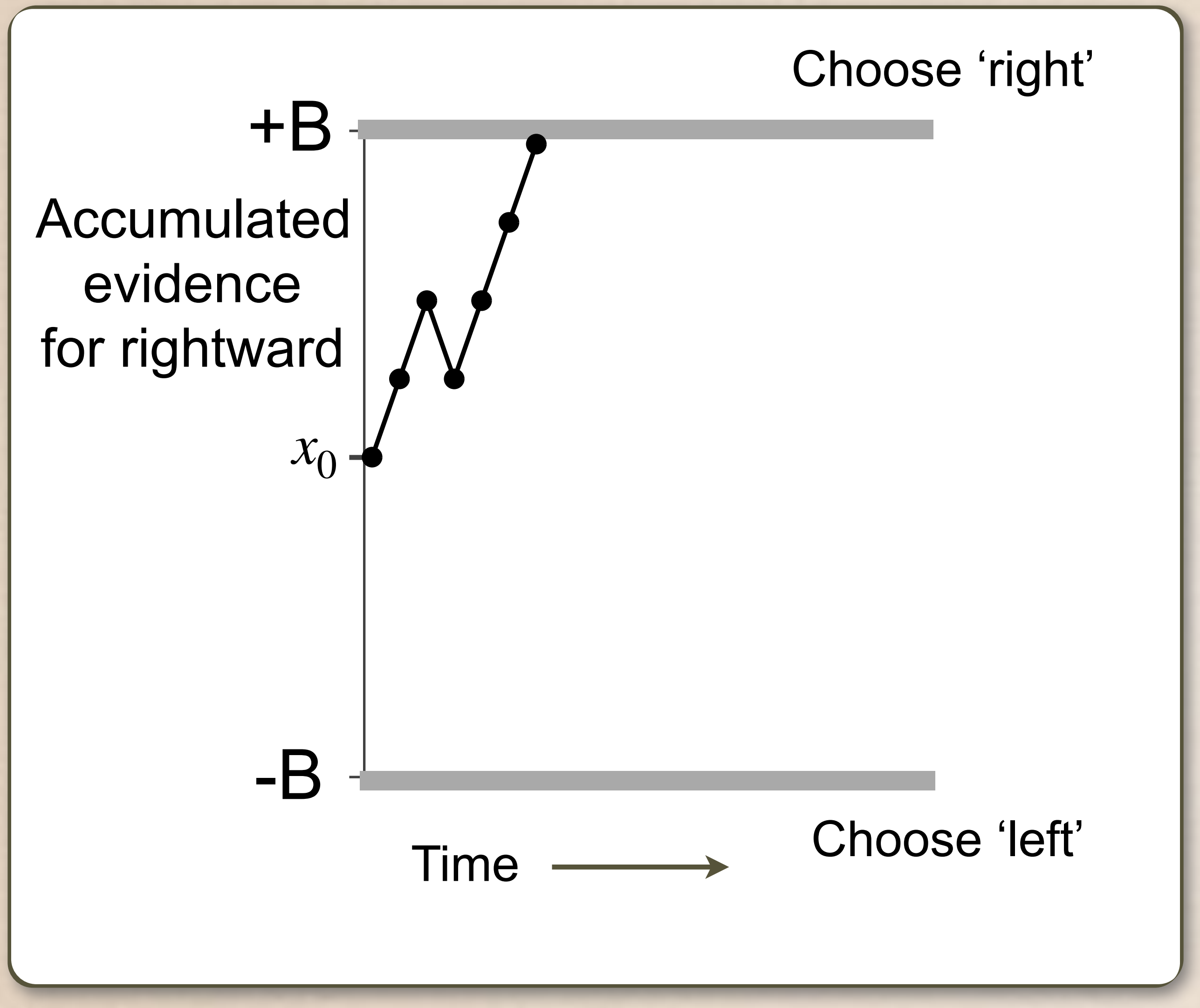
Why flat bounds lead to identical RT distributions for correct and incorrect decisions



$\nearrow$ with probability p

$\searrow$ with probability $q = 1 - p$

Why flat bounds lead to identical RT distributions for correct and incorrect decisions



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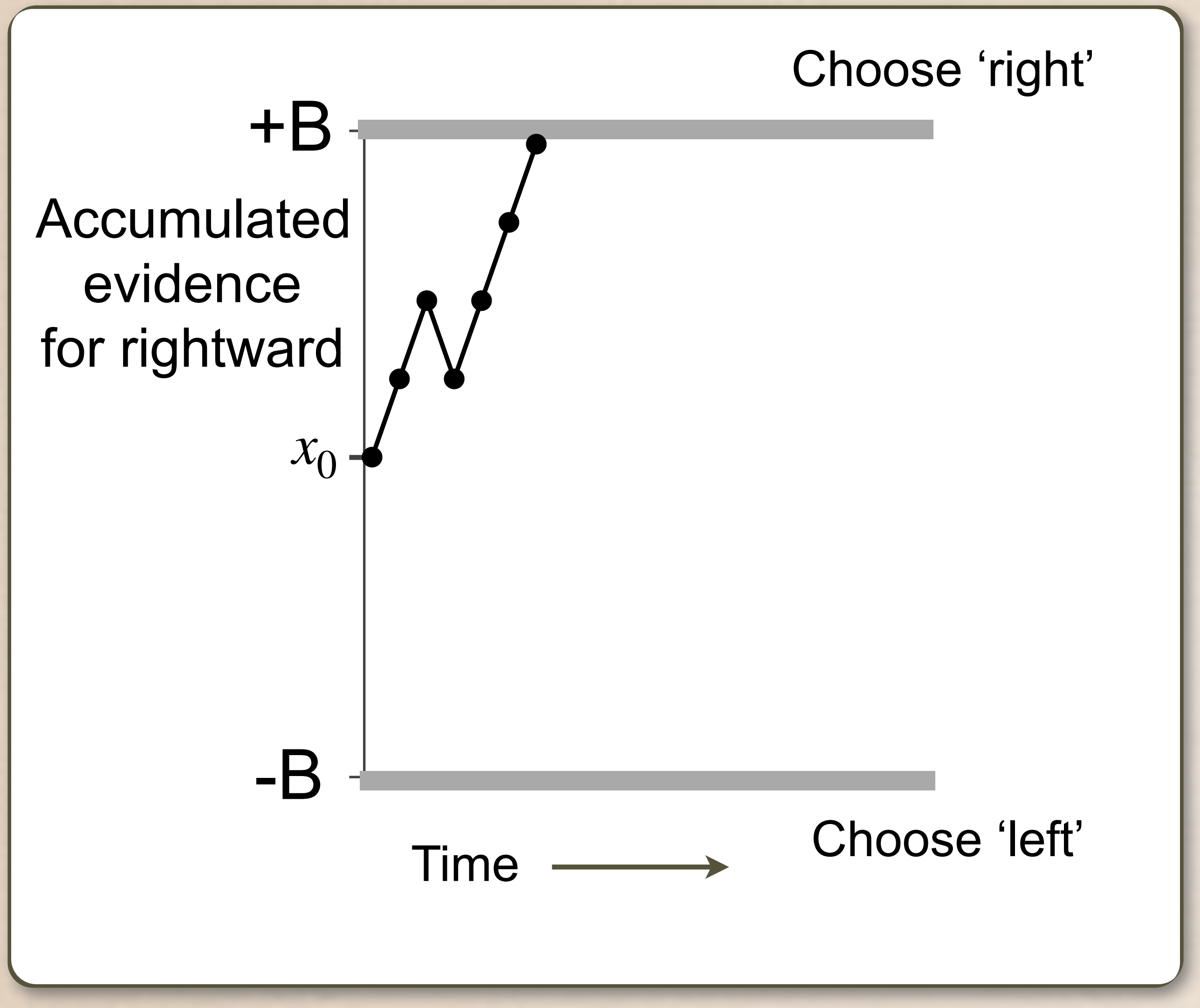
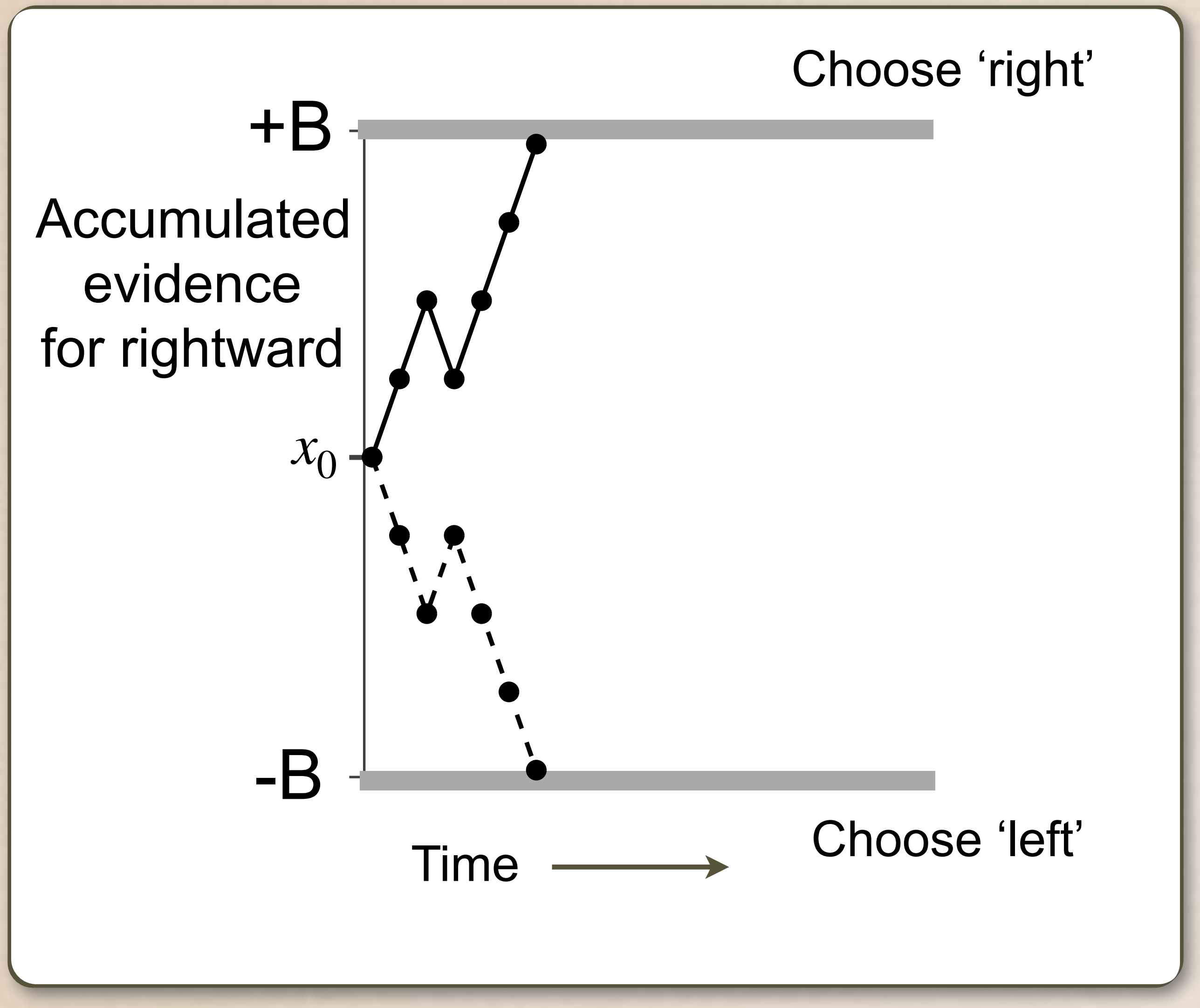


$\nearrow$ with probability p

$\searrow$ with probability $q = 1 - p$

Pr = $ppqppp$

Why flat bounds lead to identical RT distributions for correct and incorrect decisions



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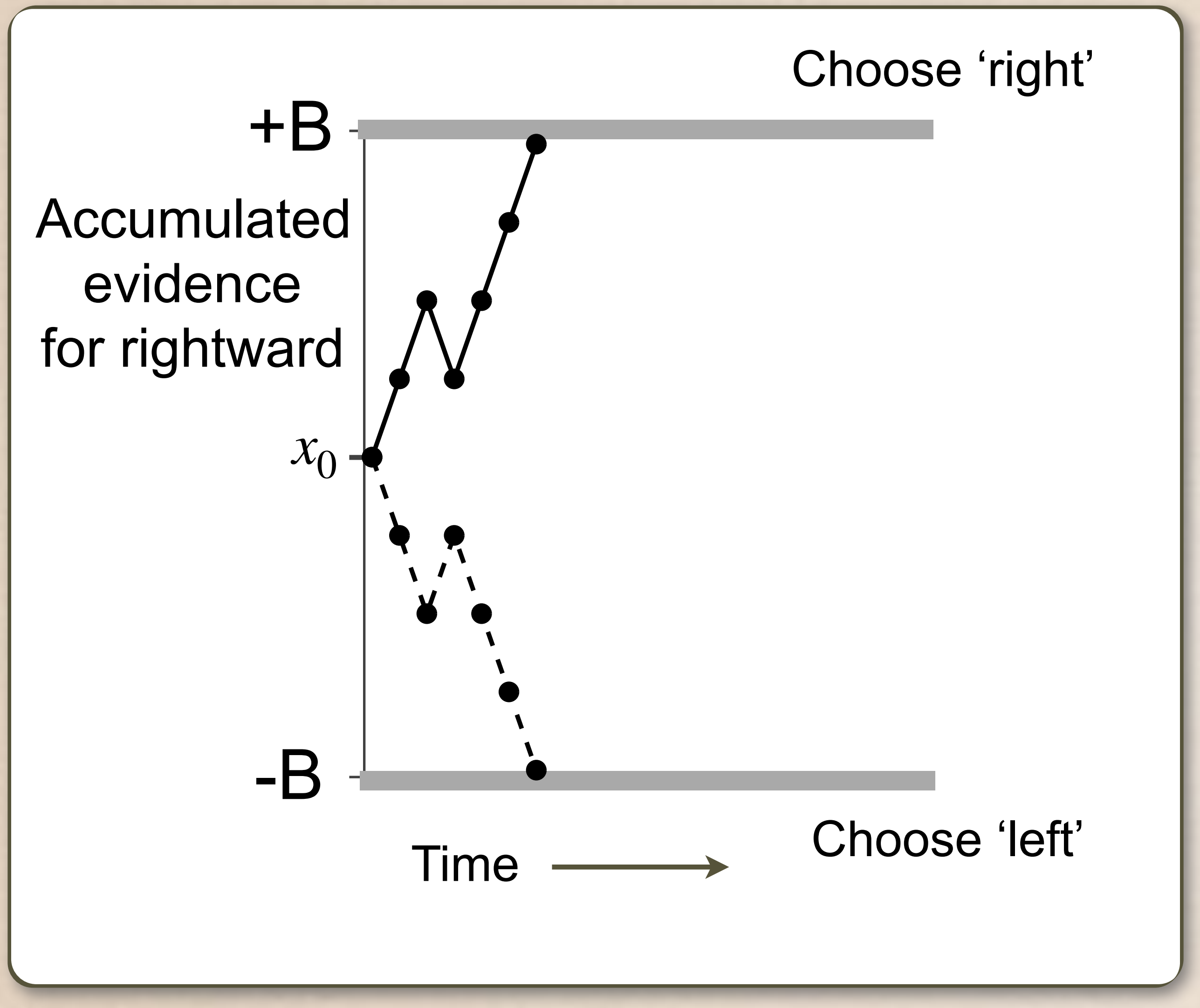


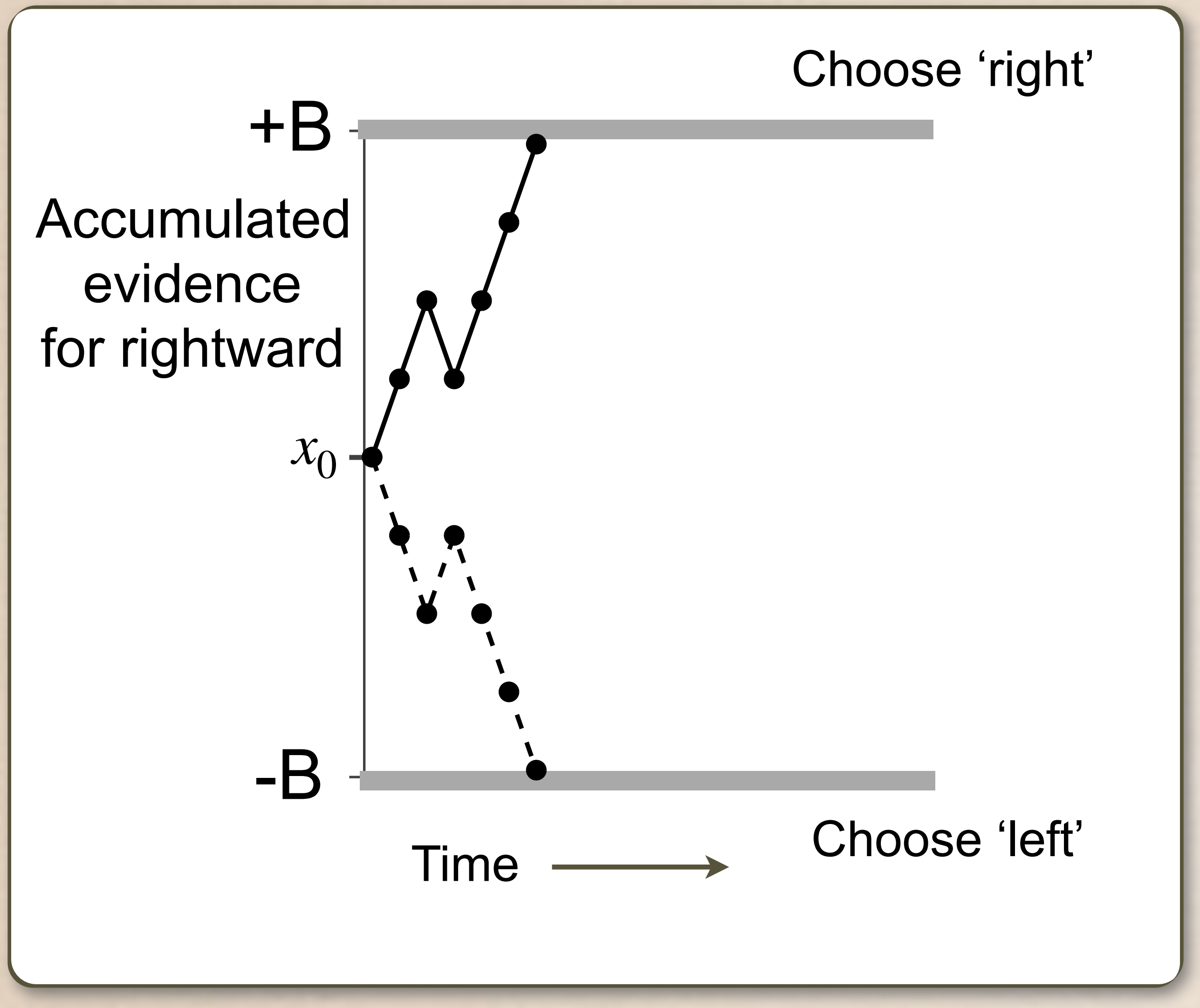
$\nearrow$ with probability p

$\searrow$ with probability $q = 1 - p$

Pr = $ppqppp$

Pr(mirror) = $qqpqqq$

Why flat bounds lead to identical RT distributions for correct and incorrect decisions



$\uparrow$ with probability p

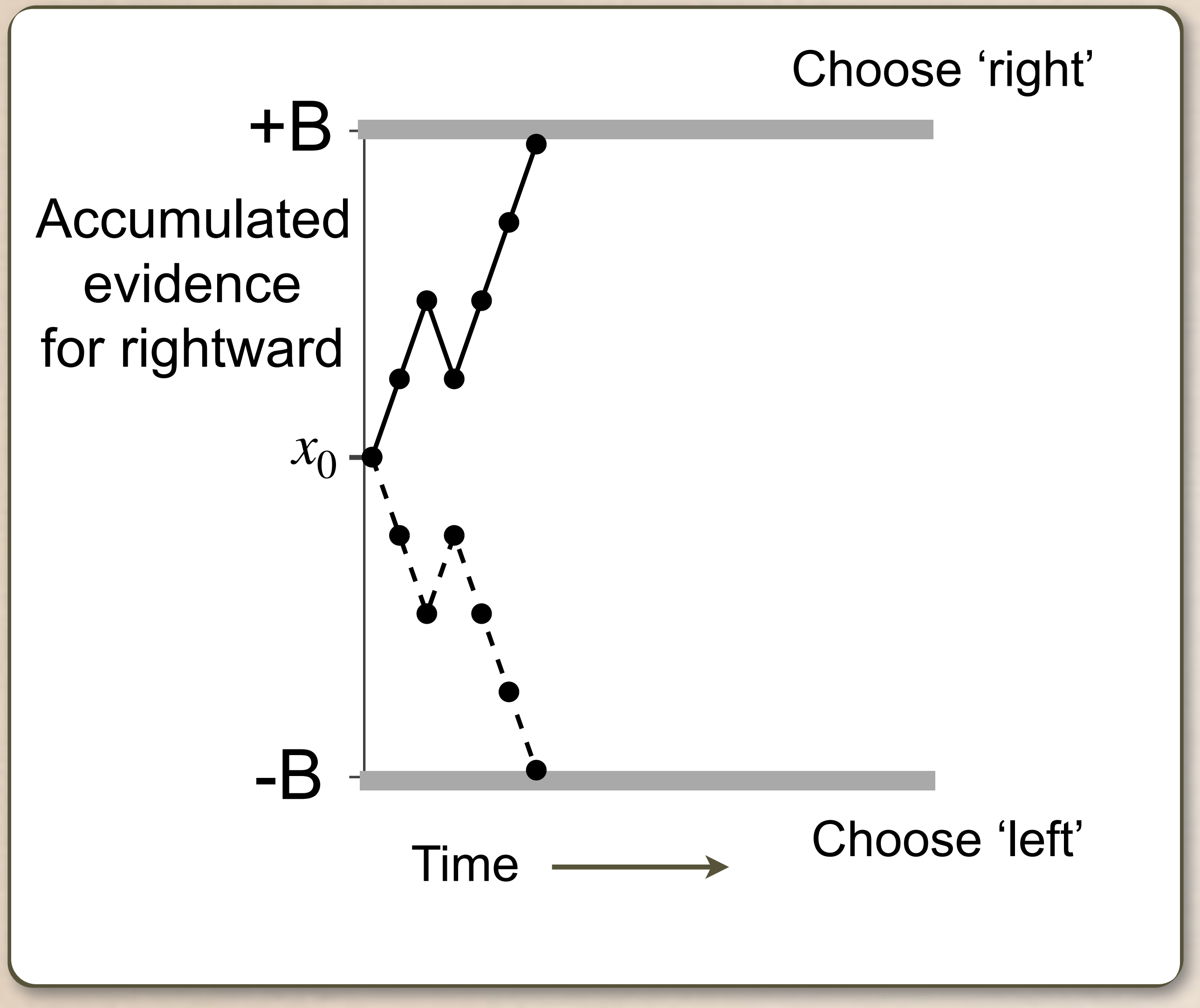
$\downarrow$ with probability $q = 1 - p$

Pr = $ppqppp$

Pr(mirror) = $qqpqqq$

$$\frac{\text{Pr}}{\text{Pr(mirror)}} = \frac{pppp}{qqqq} = \left(\frac{p}{1-p}\right)^{\gamma}$$

Why flat bounds lead to identical RT distributions for correct and incorrect decisions



$\nearrow$ with probability p

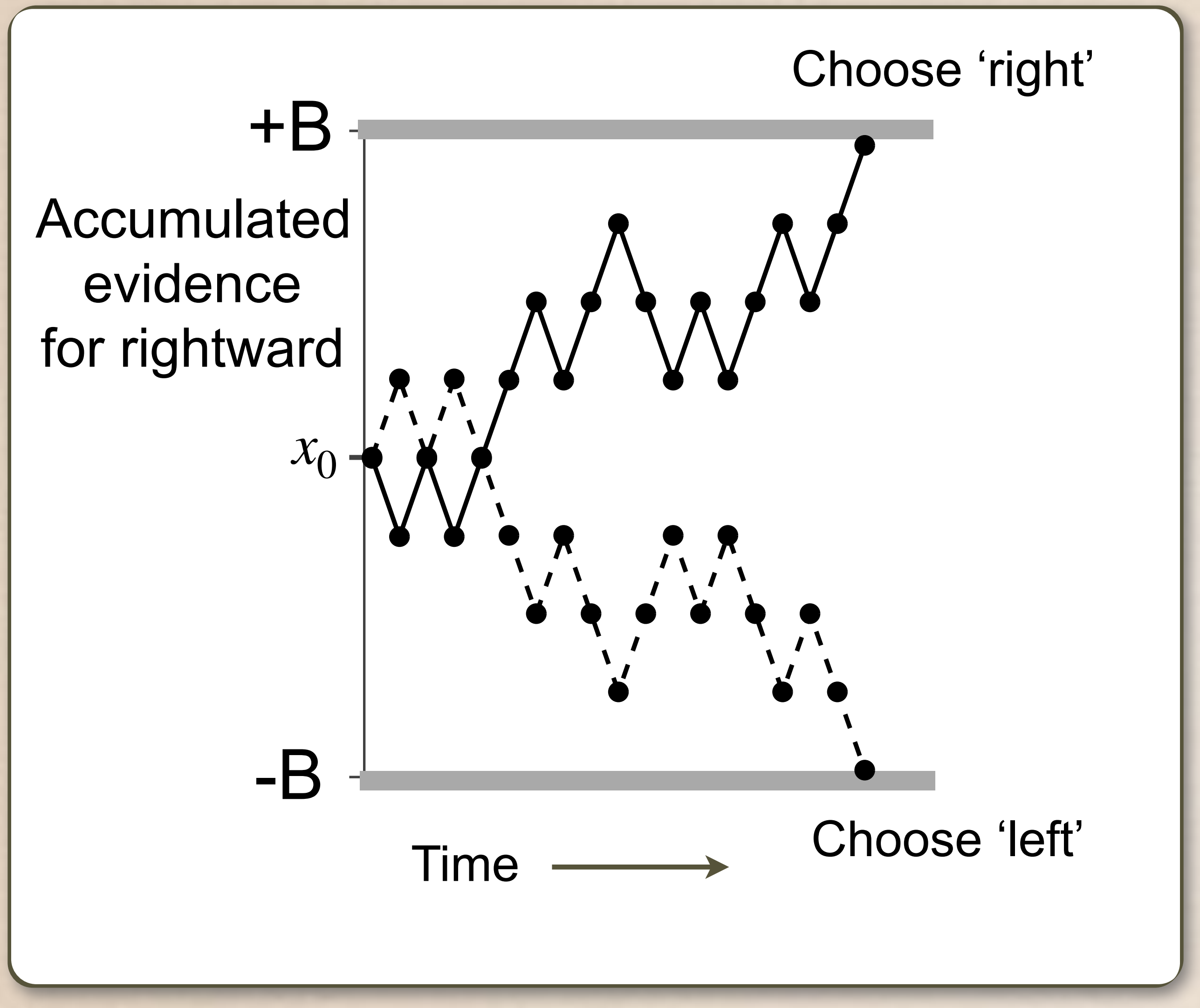
$\searrow$ with probability $q = 1 - p$

Pr = $ppqppp$

Pr(mirror) = $qqpqqq$

$$\frac{\text{Pr}}{\text{Pr(mirror)}} = \frac{pppp}{qqqq} = \left(\frac{p}{1-p}\right)^{\gamma = 4}$$

Why flat bounds lead to identical RT distributions for correct and incorrect decisions



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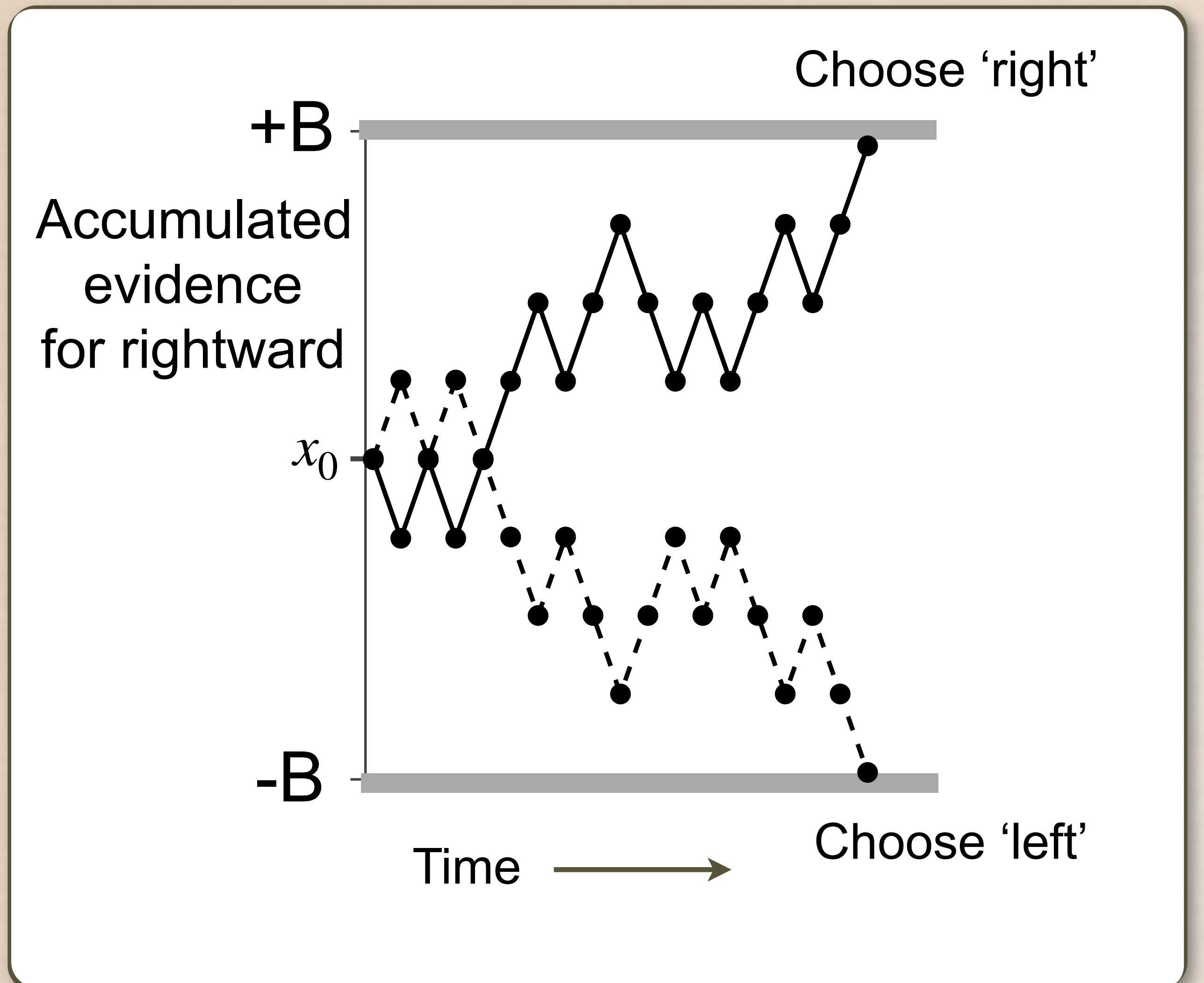
$$\frac{\text{Pr}}{\text{Pr(mirror)}} = \frac{pppp}{qqqq} = \left(\frac{p}{1-p}\right)^{\gamma \, =4}$$

Why flat bounds lead to identical RT distributions for correct and incorrect decisions



$\uparrow$ with probability p

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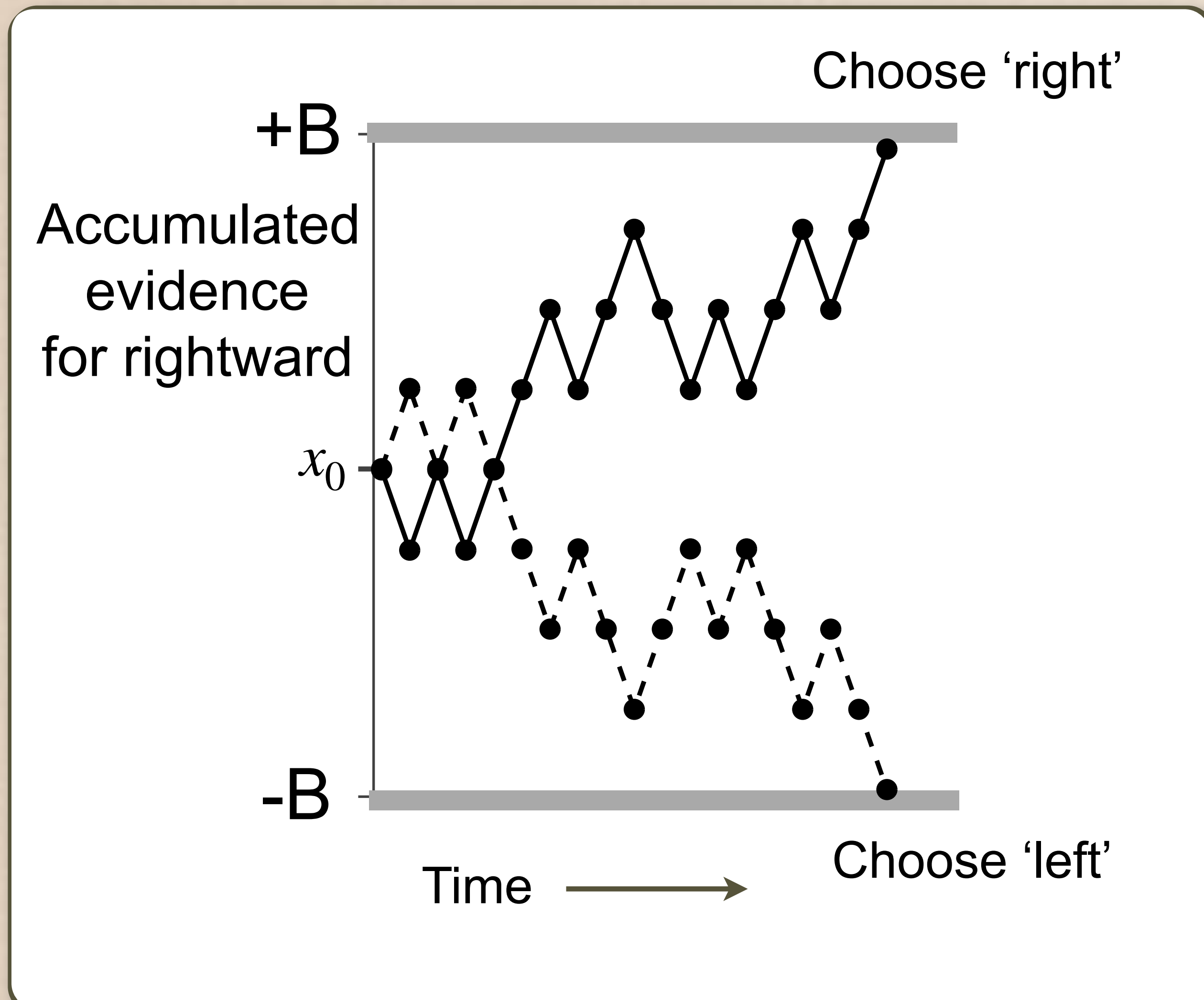
Pr = $qpqpppqppqqpqppqpp$

Pr(mirror) = $pqpqqqpqqppqpqqpqq$

$$\frac{\text{Pr}}{\text{Pr(mirror)}} = \frac{pppp}{qqqq} = \left(\frac{p}{1-p}\right)^{\gamma \,=4}$$

Why flat bounds lead to identical RT distributions for correct and incorrect decisions



$\nearrow$ with probability p

$\searrow$ with probability $q = 1 - p$

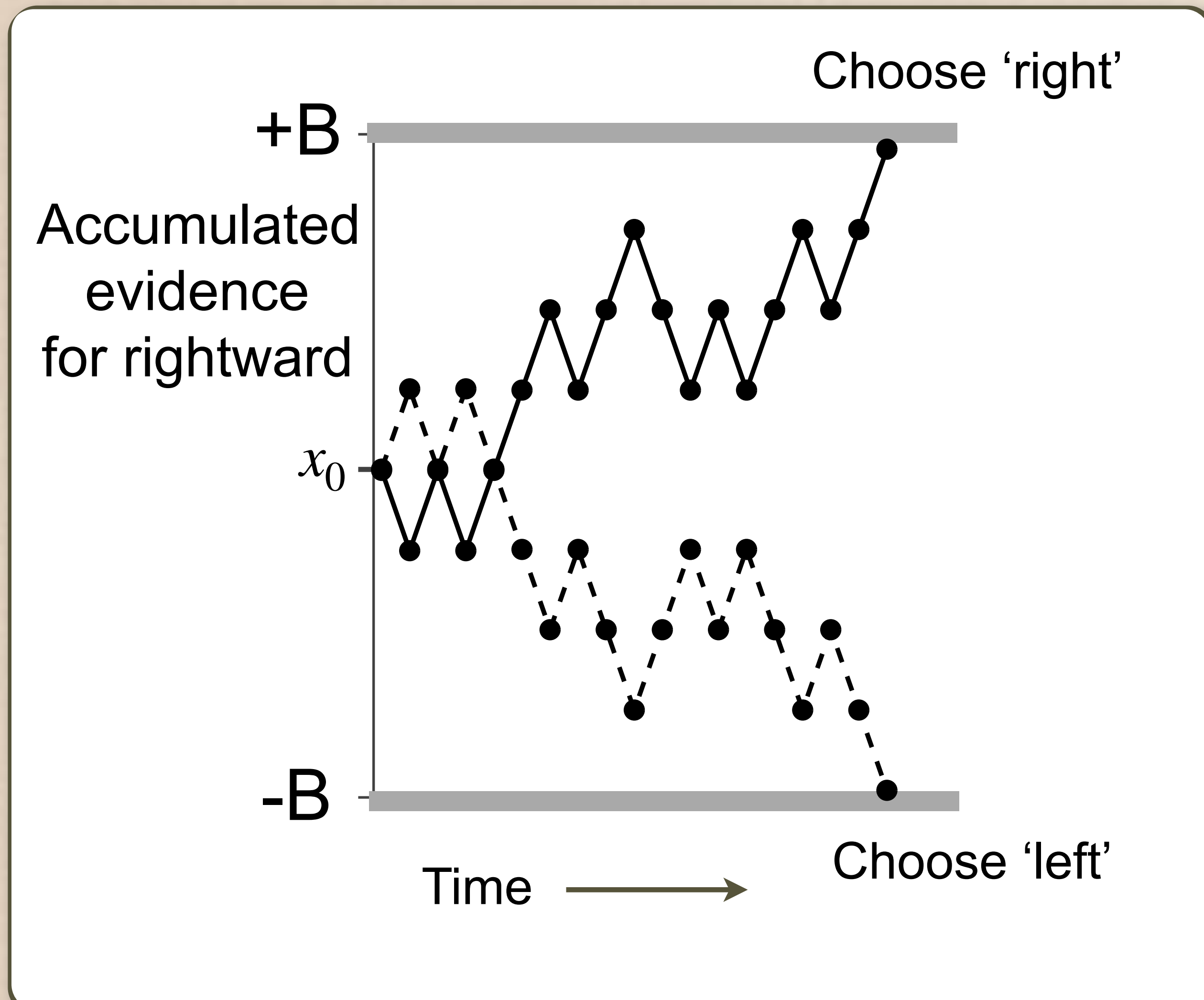
$\text{Pr} = qpqpppqppqqpqppqpp$

$\text{Pr(mirror)} = pqpqqqpqqppqpqqpqq$

$$\frac{\text{Pr}}{\text{Pr(mirror)}} = \frac{pppp}{qqqq} = \left(\frac{p}{1-p}\right)^{\gamma \, =4}$$

$$\boxed{P(t, \text{right}) = aP(t, \text{left})}$$

Why flat bounds lead to identical RT distributions for correct and incorrect decisions



$\nearrow$ with probability p

$\searrow$ with probability $q = 1 - p$

$$\text{Pr} = qpqpppqppqqpqppqpp$$

$$\text{Pr(mirror)} = pqpqqqpqqppqpqqpqq$$

$$\frac{\text{Pr}}{\text{Pr(mirror)}} = \frac{pppp}{qqqq} = \left(\frac{p}{1-p}\right)^{\gamma}$$

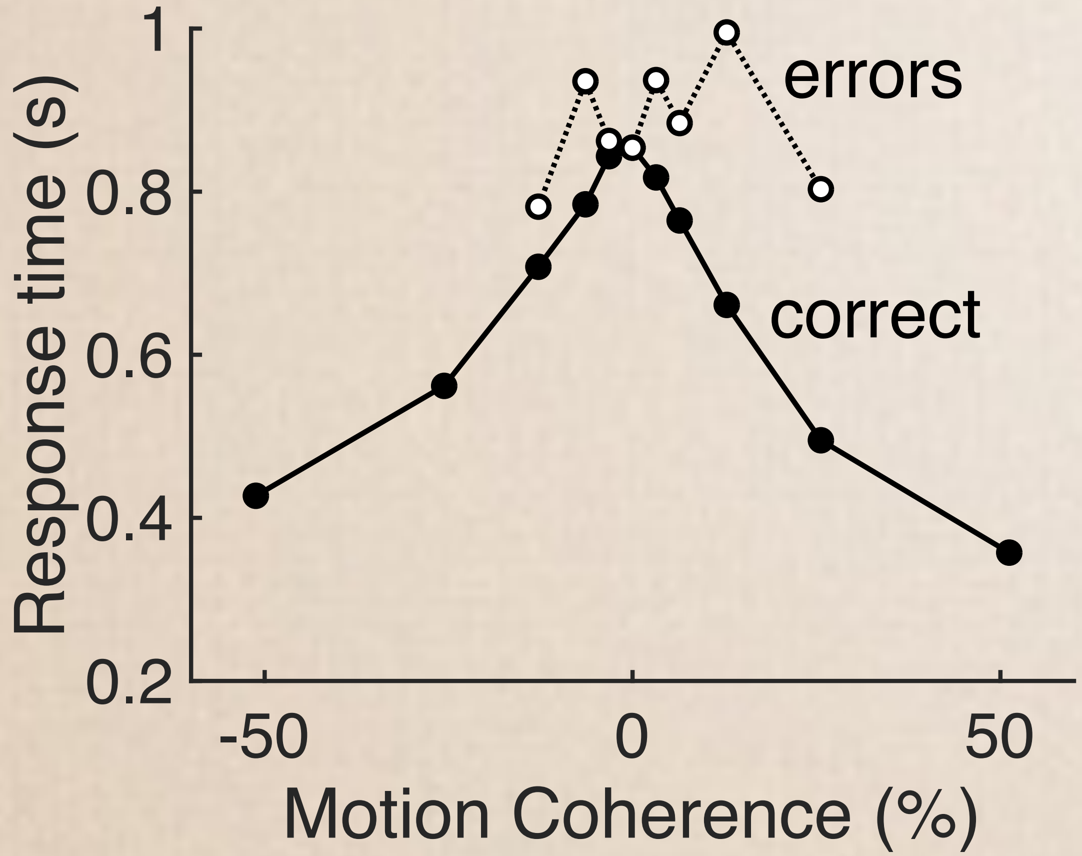
=4

After normalization:

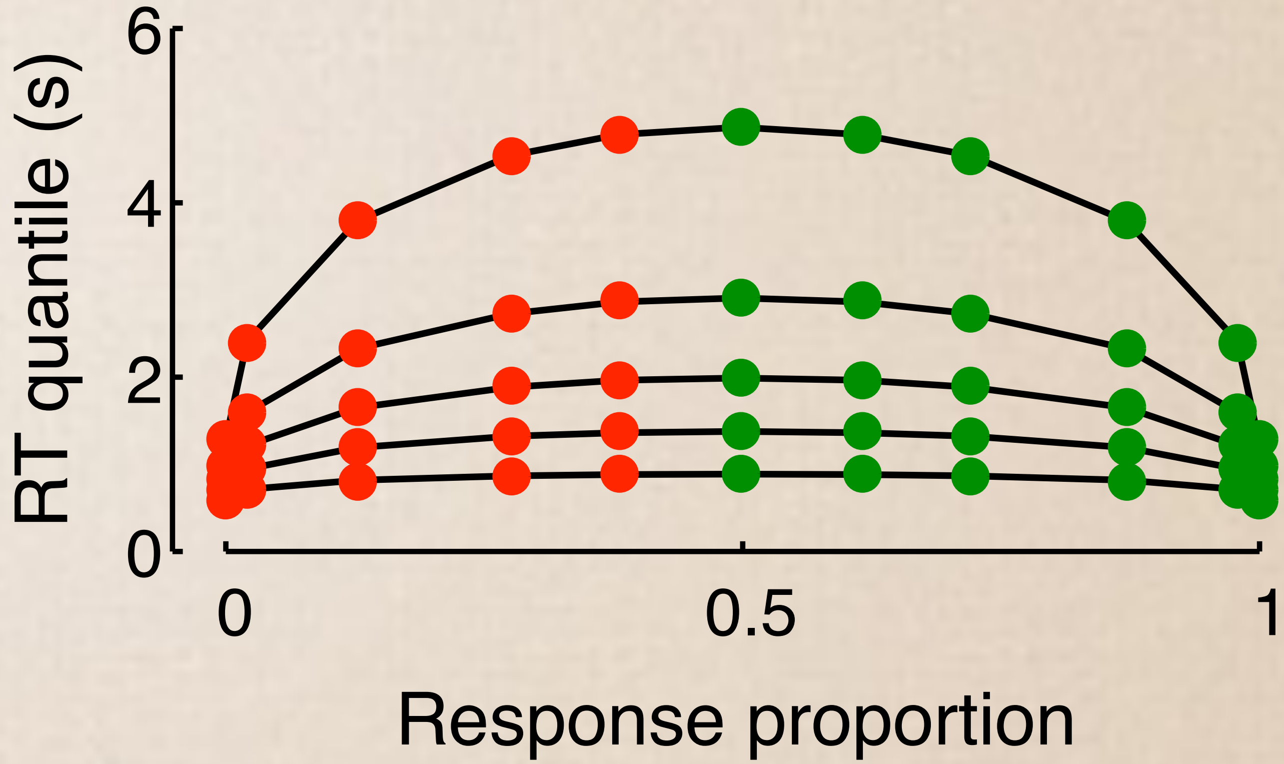
$$\boxed{P(t\,|\,\text{right}) = P(t\,|\,\text{left})}$$

Problem with simple DDM: symmetric RT distributions

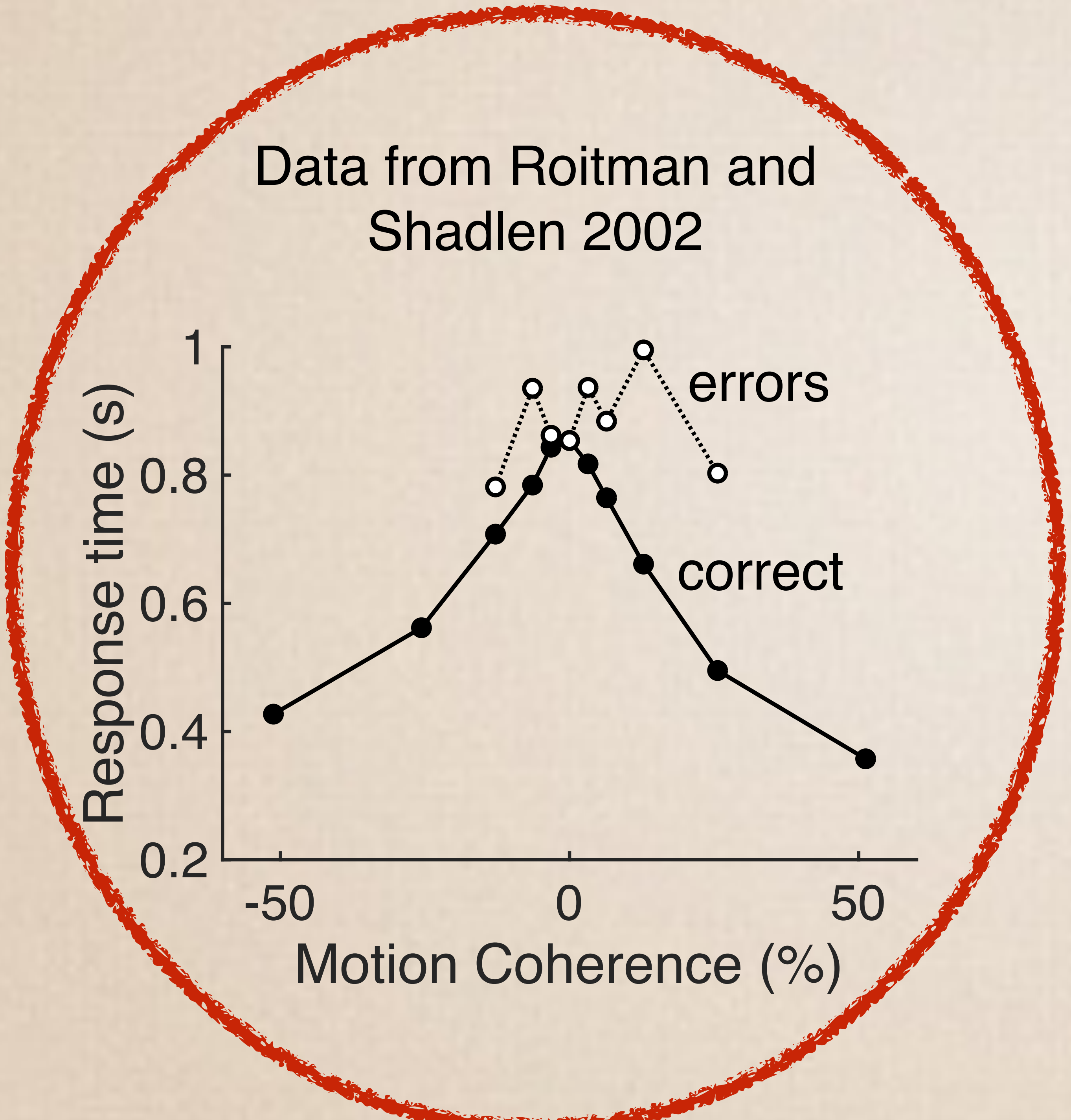
Data from Roitman and Shadlen 2002

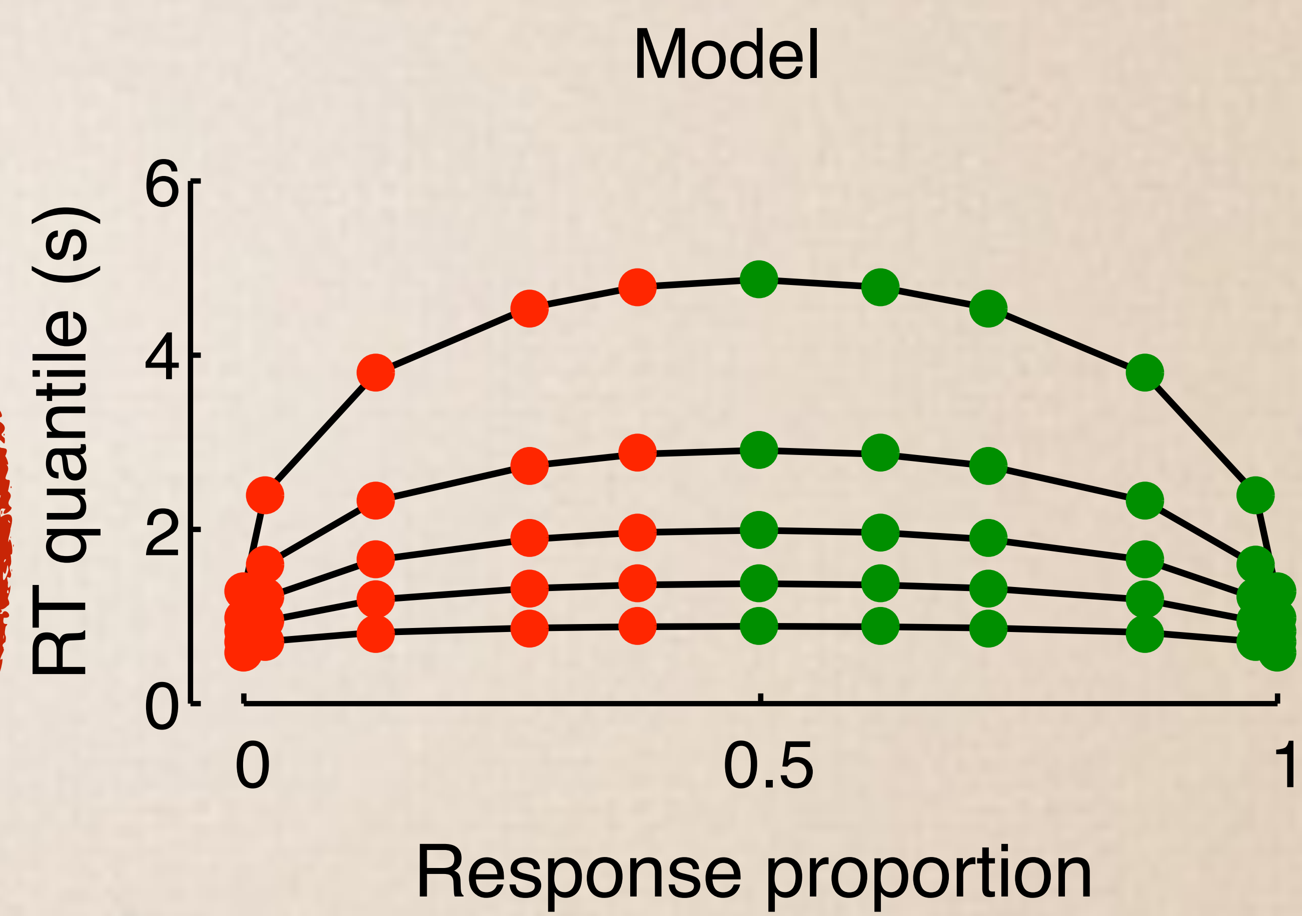


Model

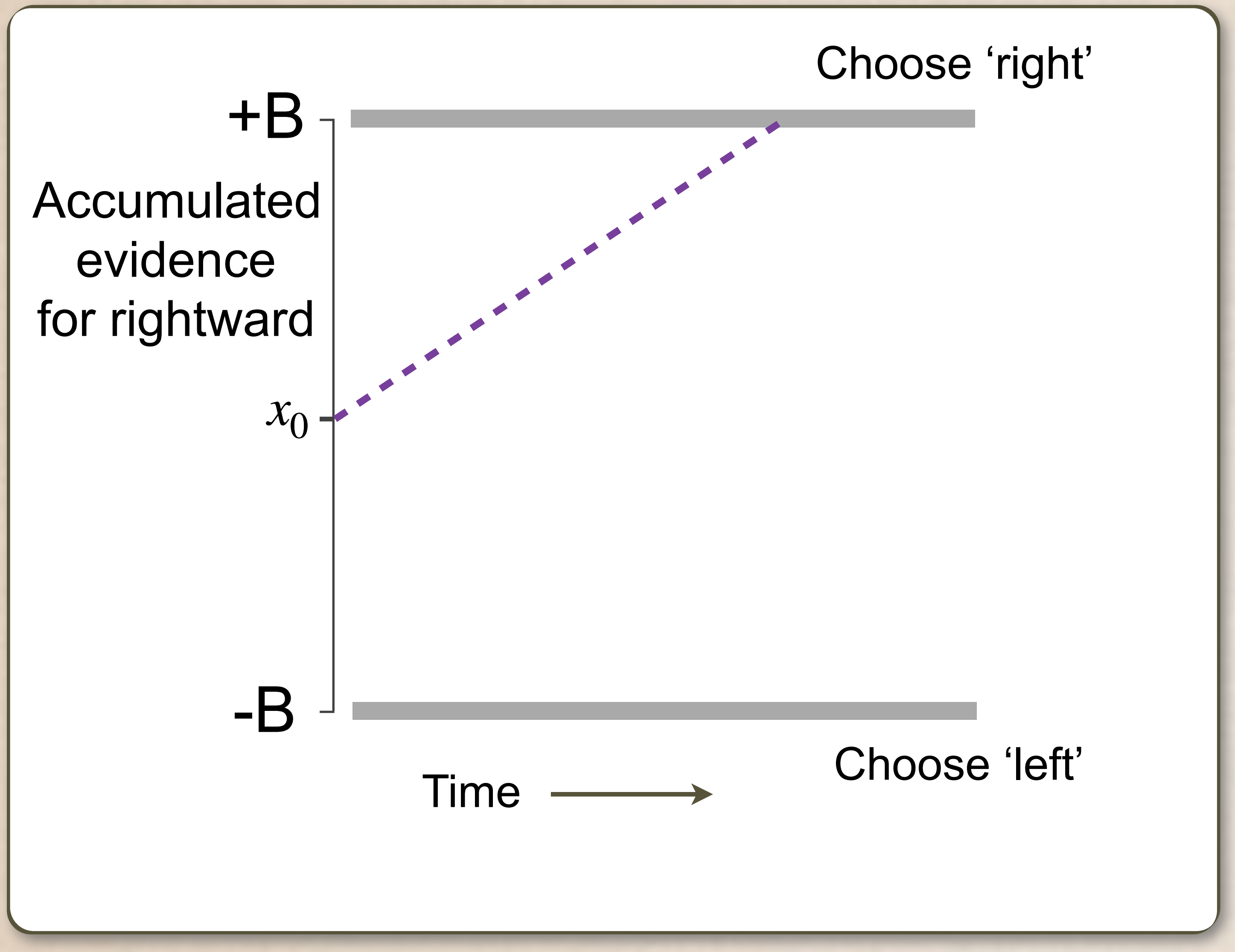


Problem with simple DDM: symmetric RT distributions



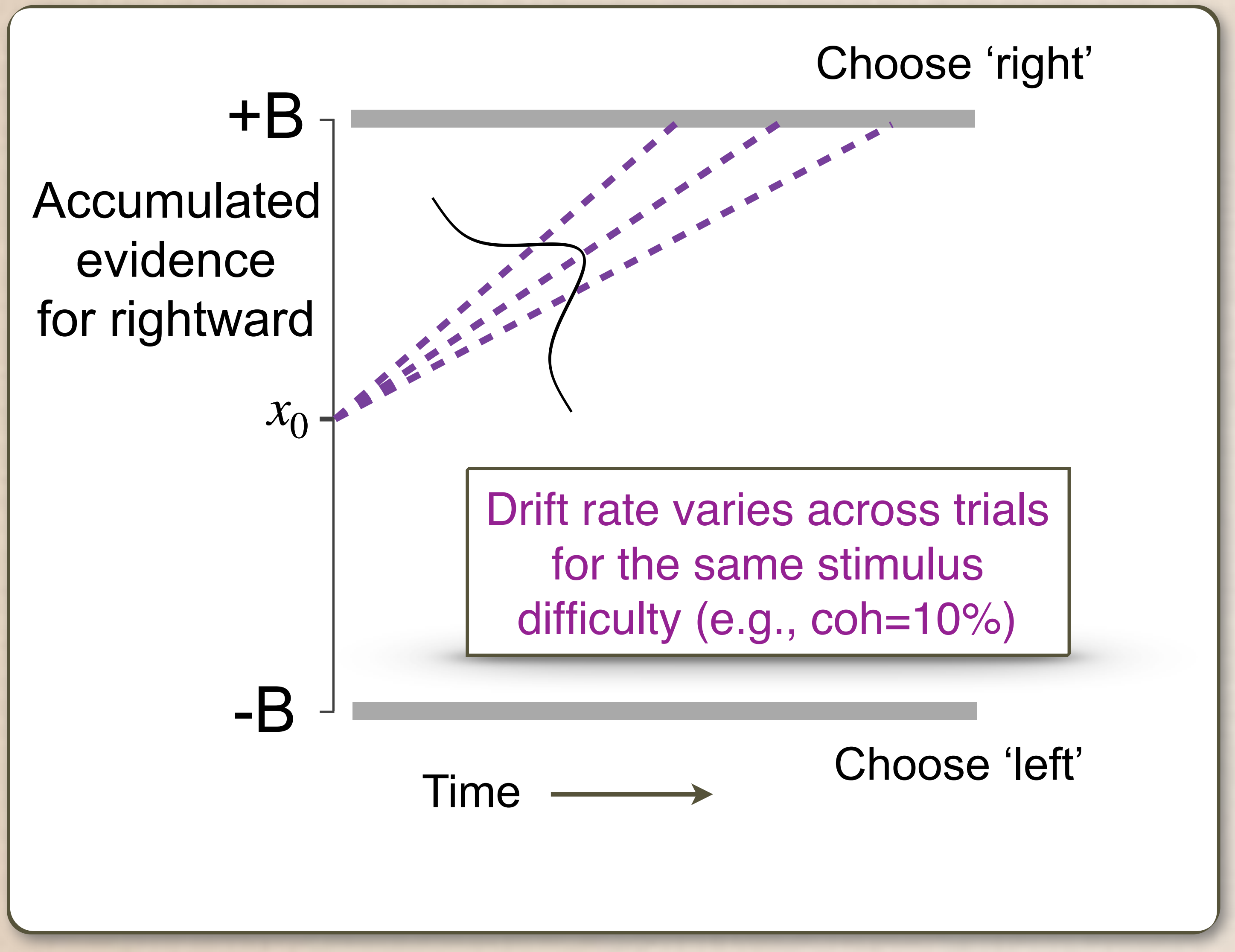


Heuristic solution: variability in drift-rate across trials



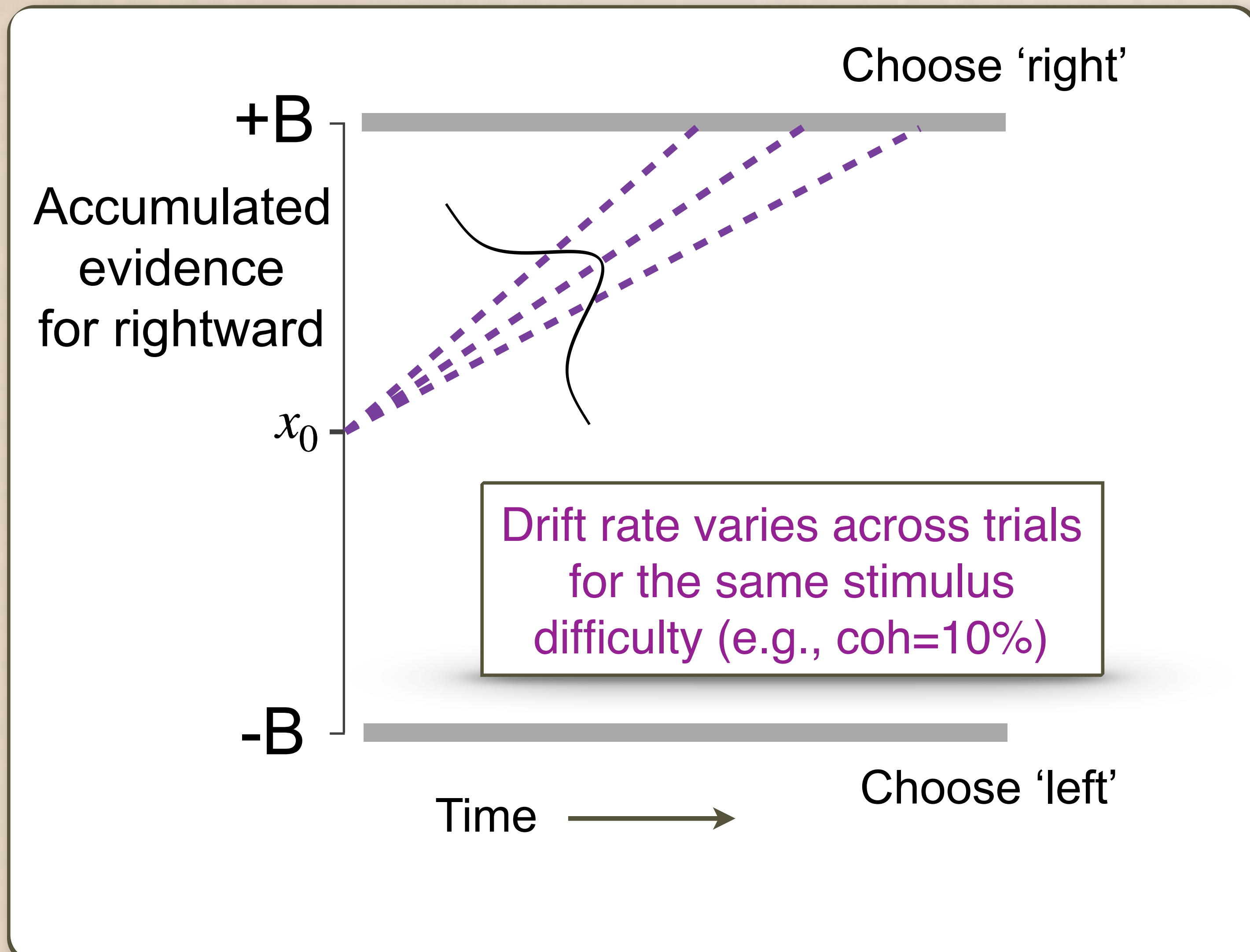
Ratcliff & McKoon (2008)

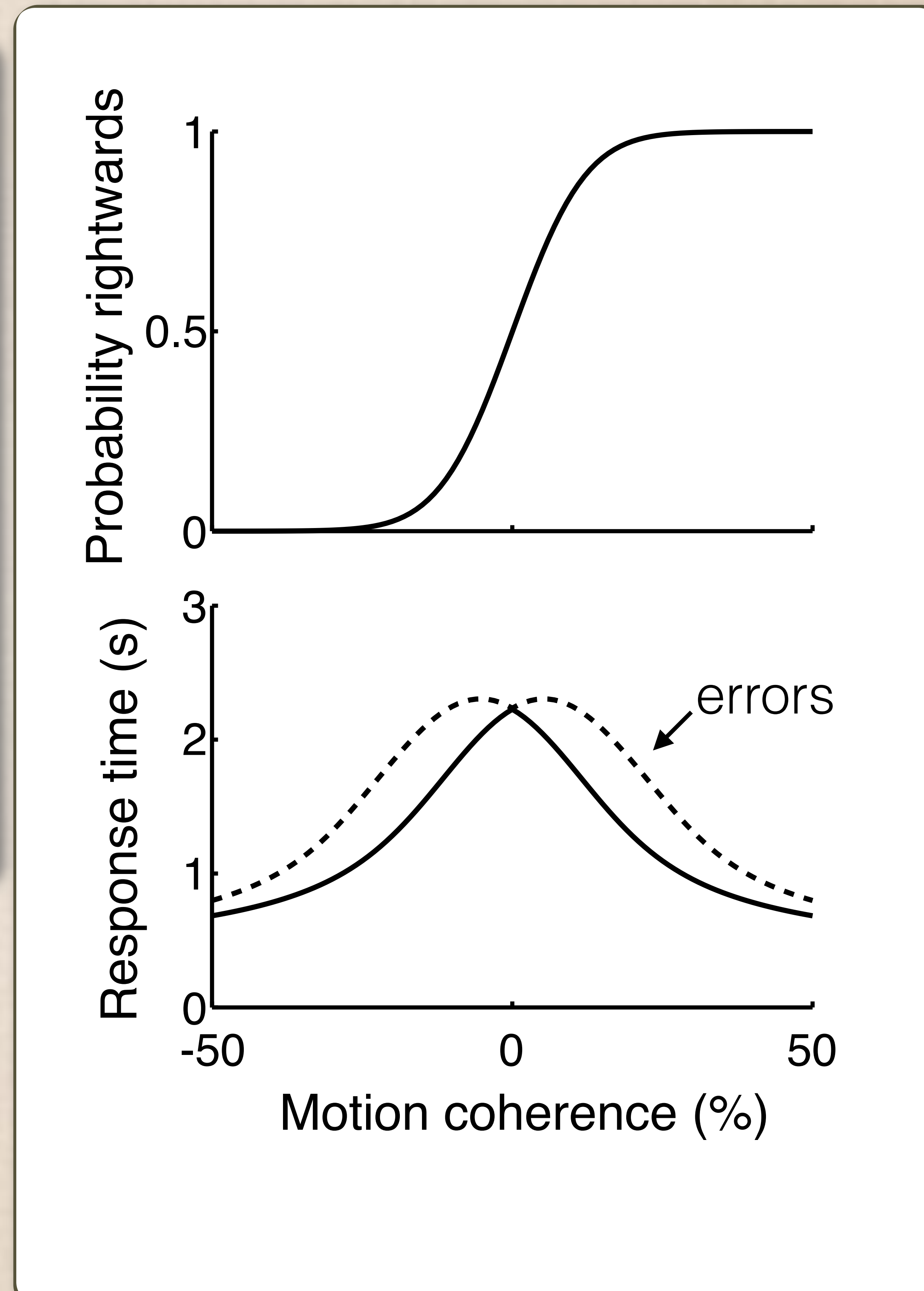
Heuristic solution: variability in drift-rate across trials



Ratcliff & McKoon (2008)

Heuristic solution: variability in drift-rate across trials





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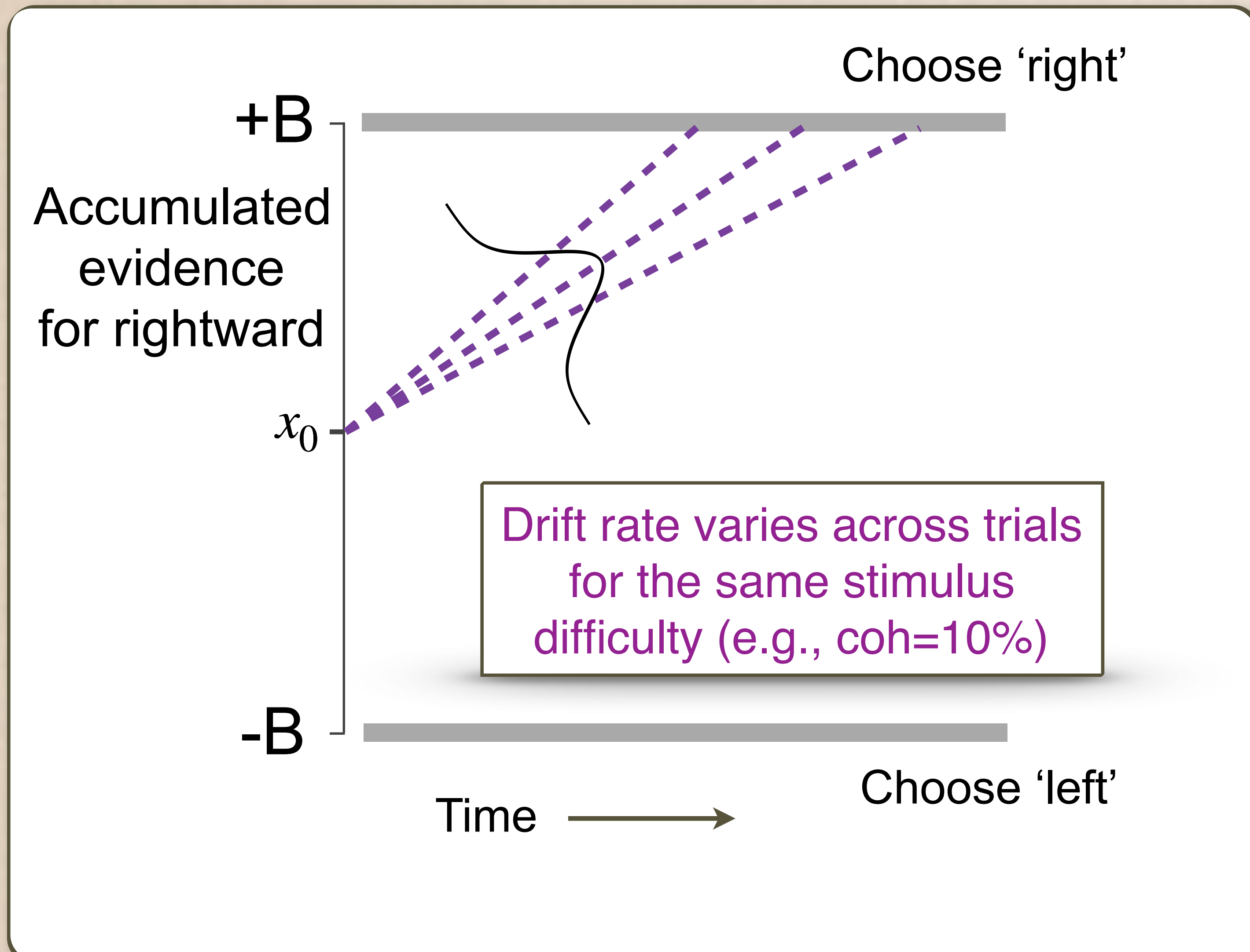
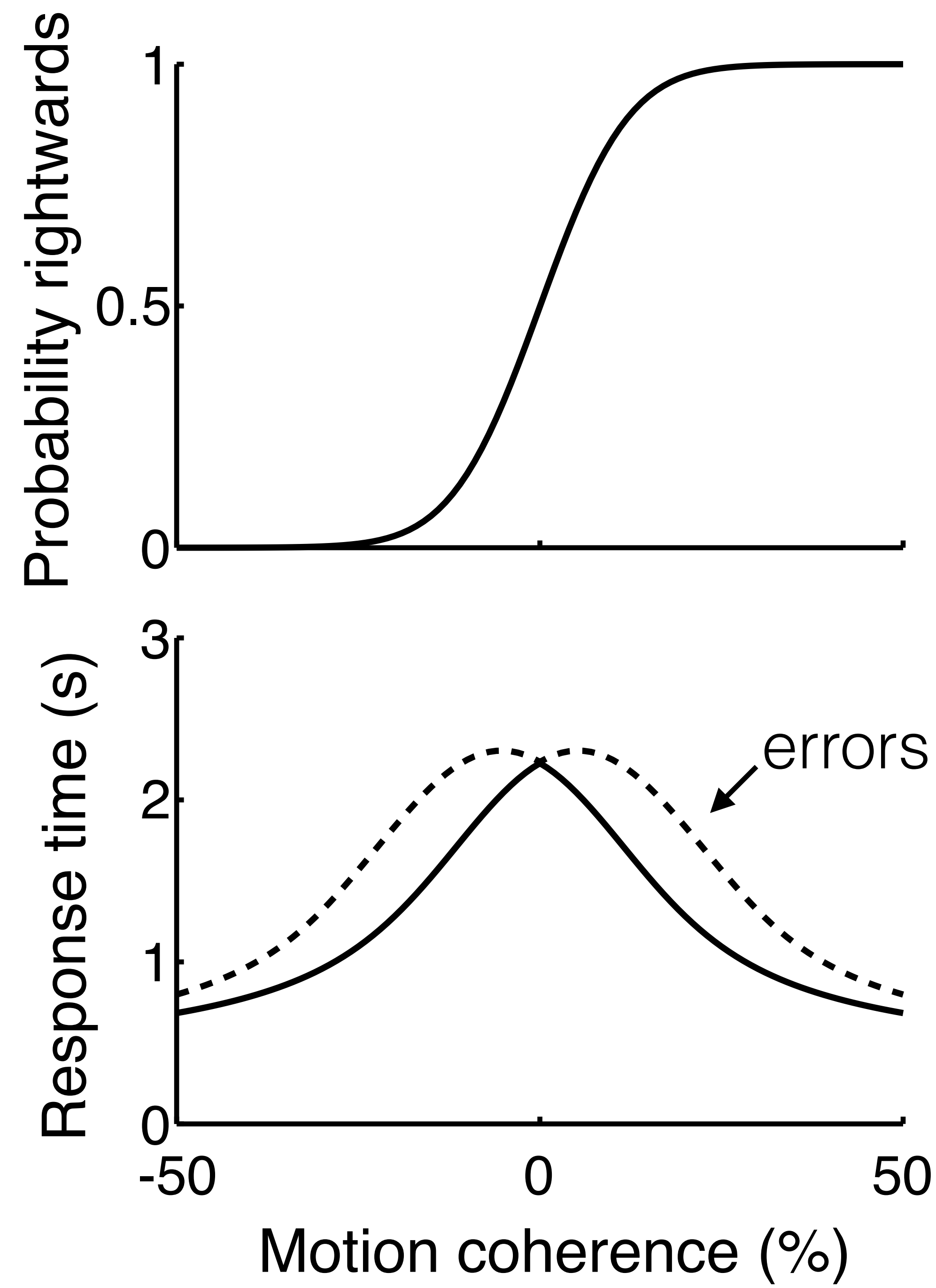
Heuristic solution: variability in drift-rate across trials



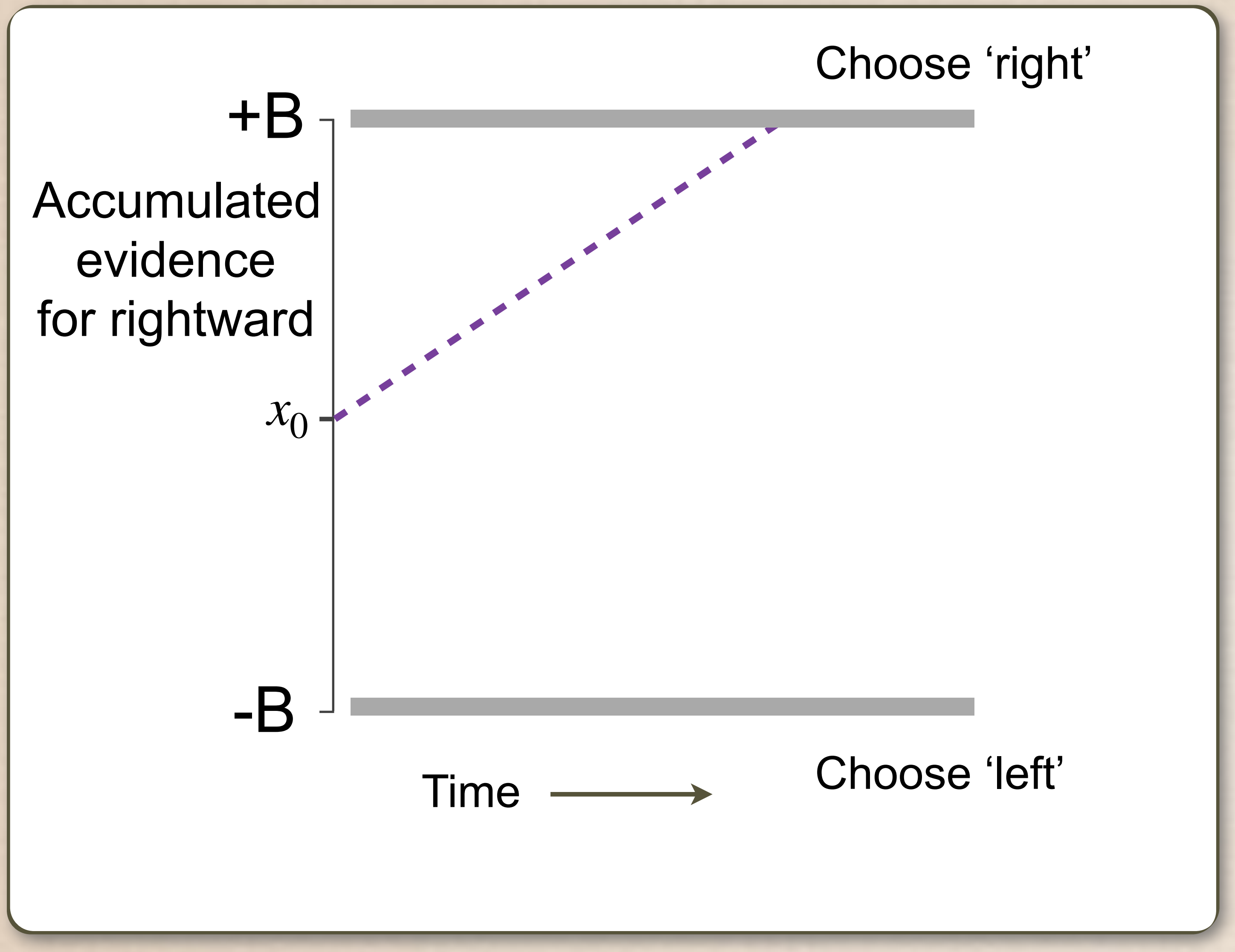


Drift-rates closer to zero → Slower responses

Drift-rates closer to zero → Higher error rates

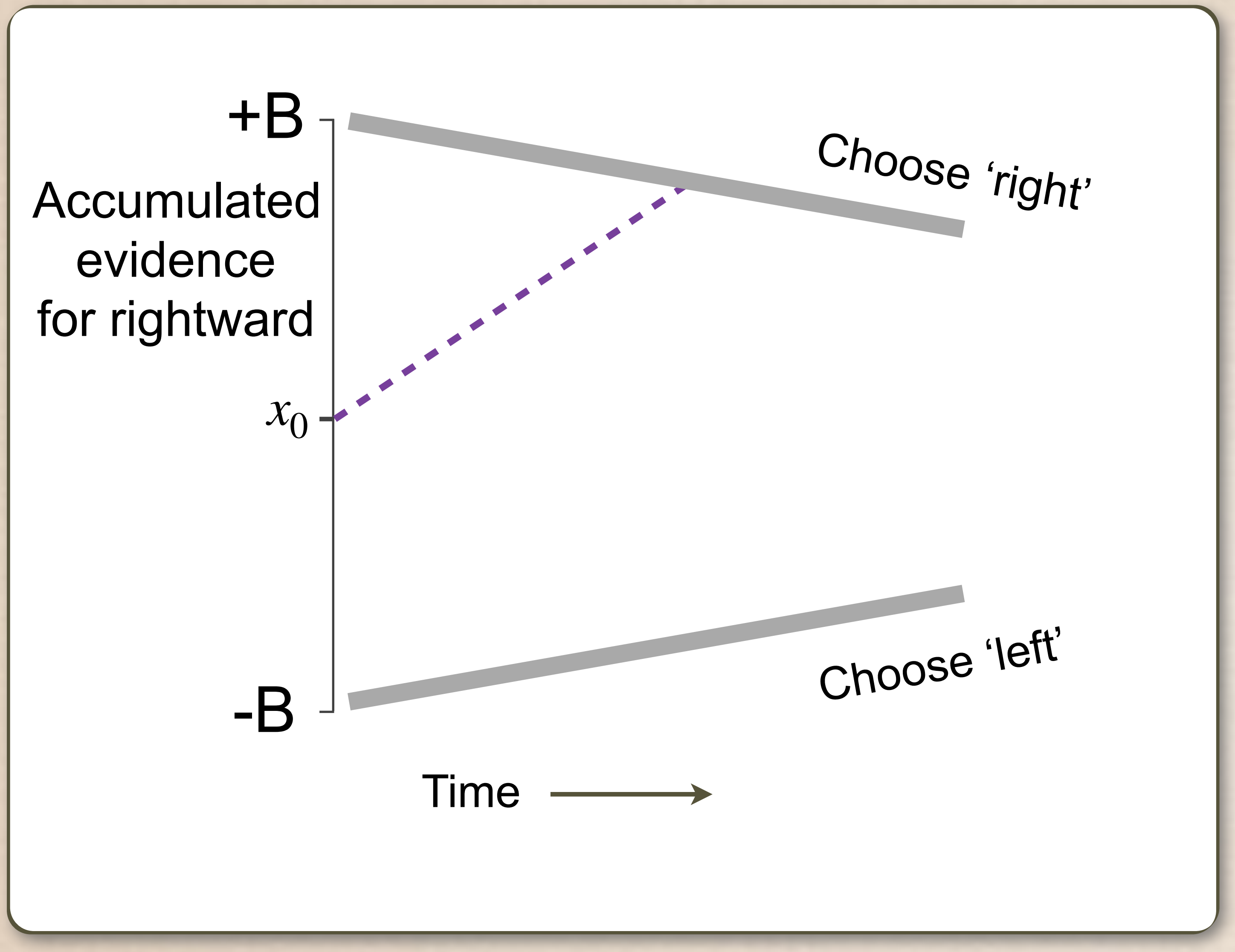
Ratcliff & McKoon (2008)

Collapsing bounds



MN Shadlen et al. (2006)

Collapsing bounds



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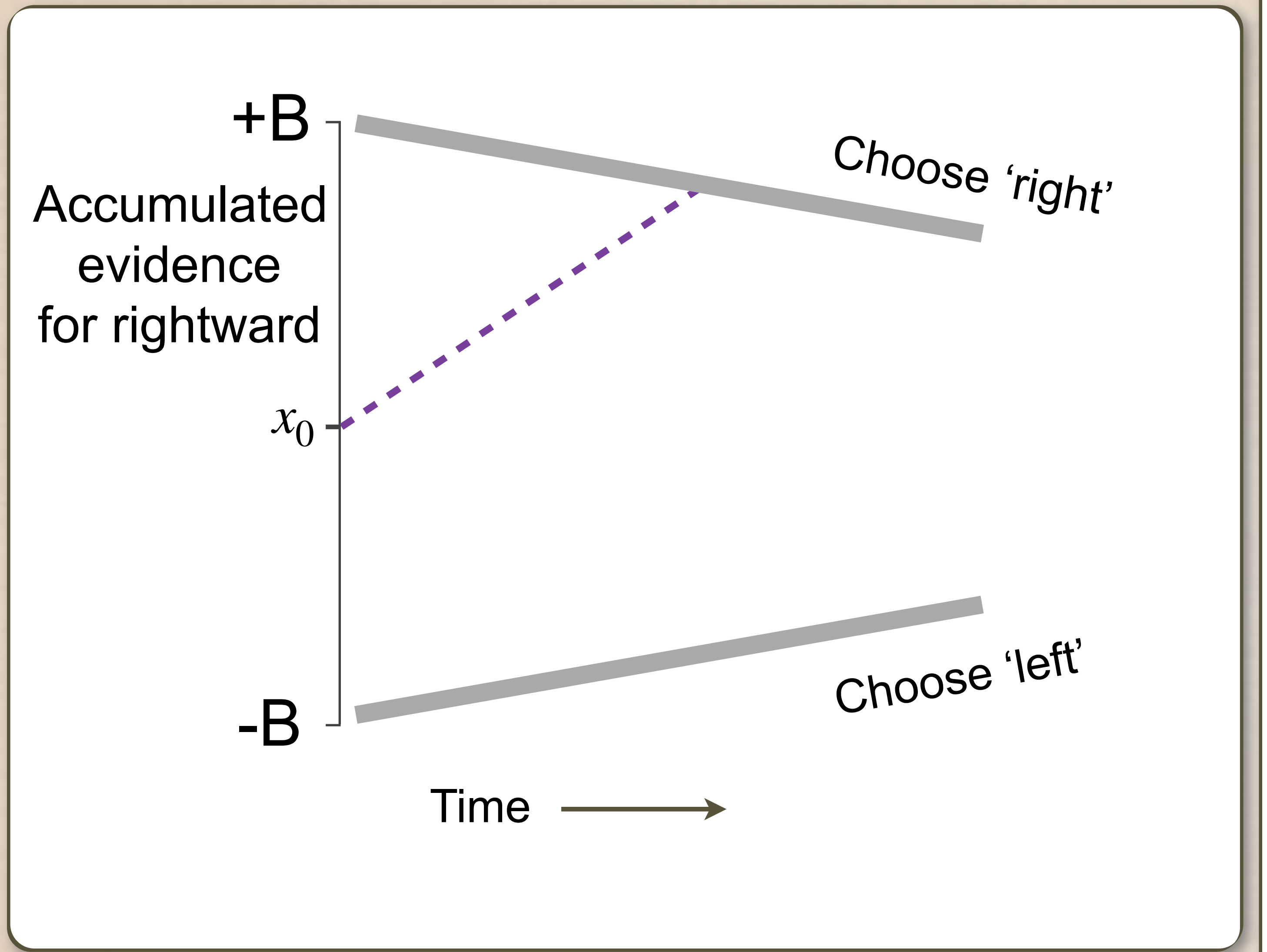

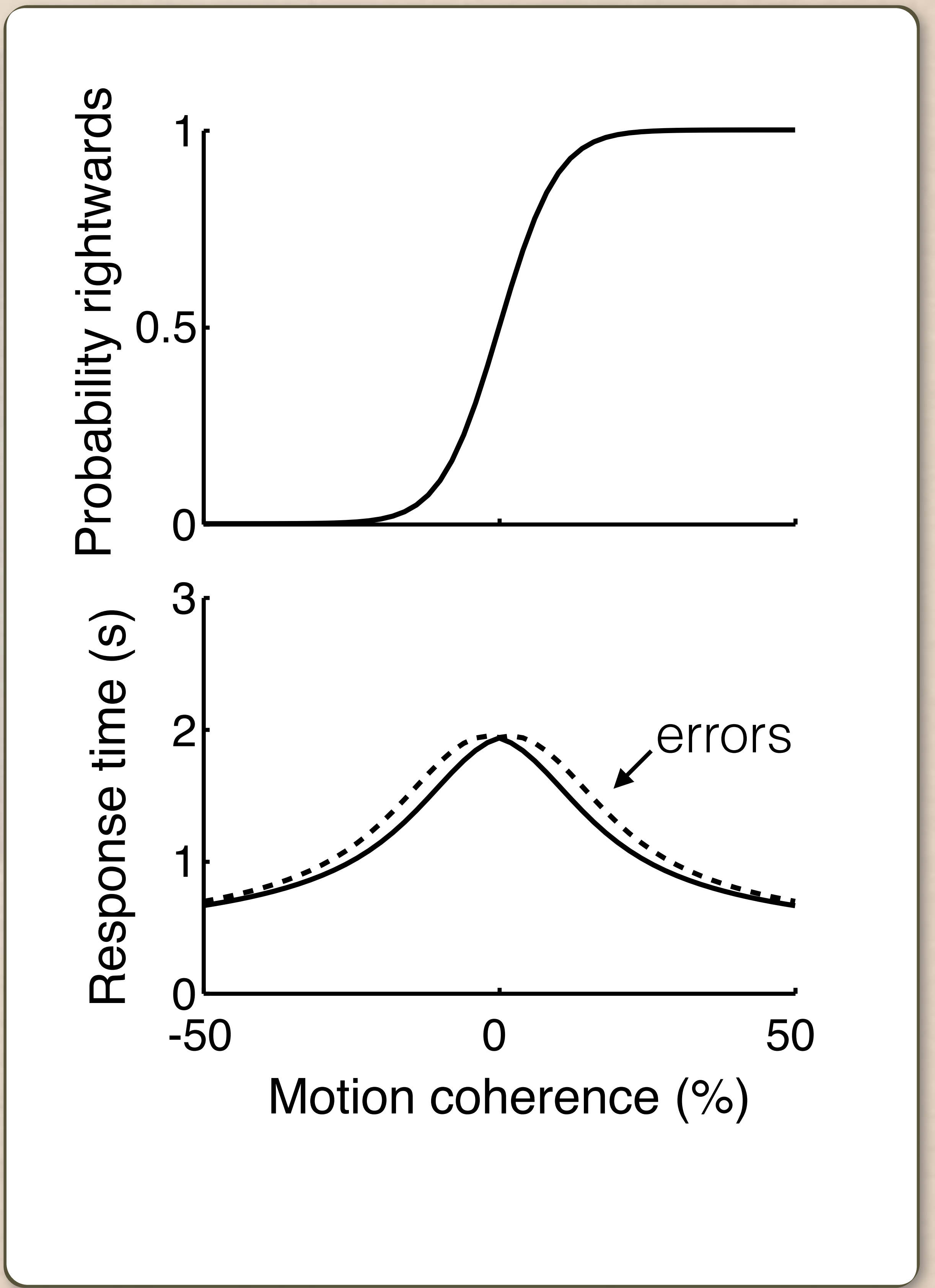

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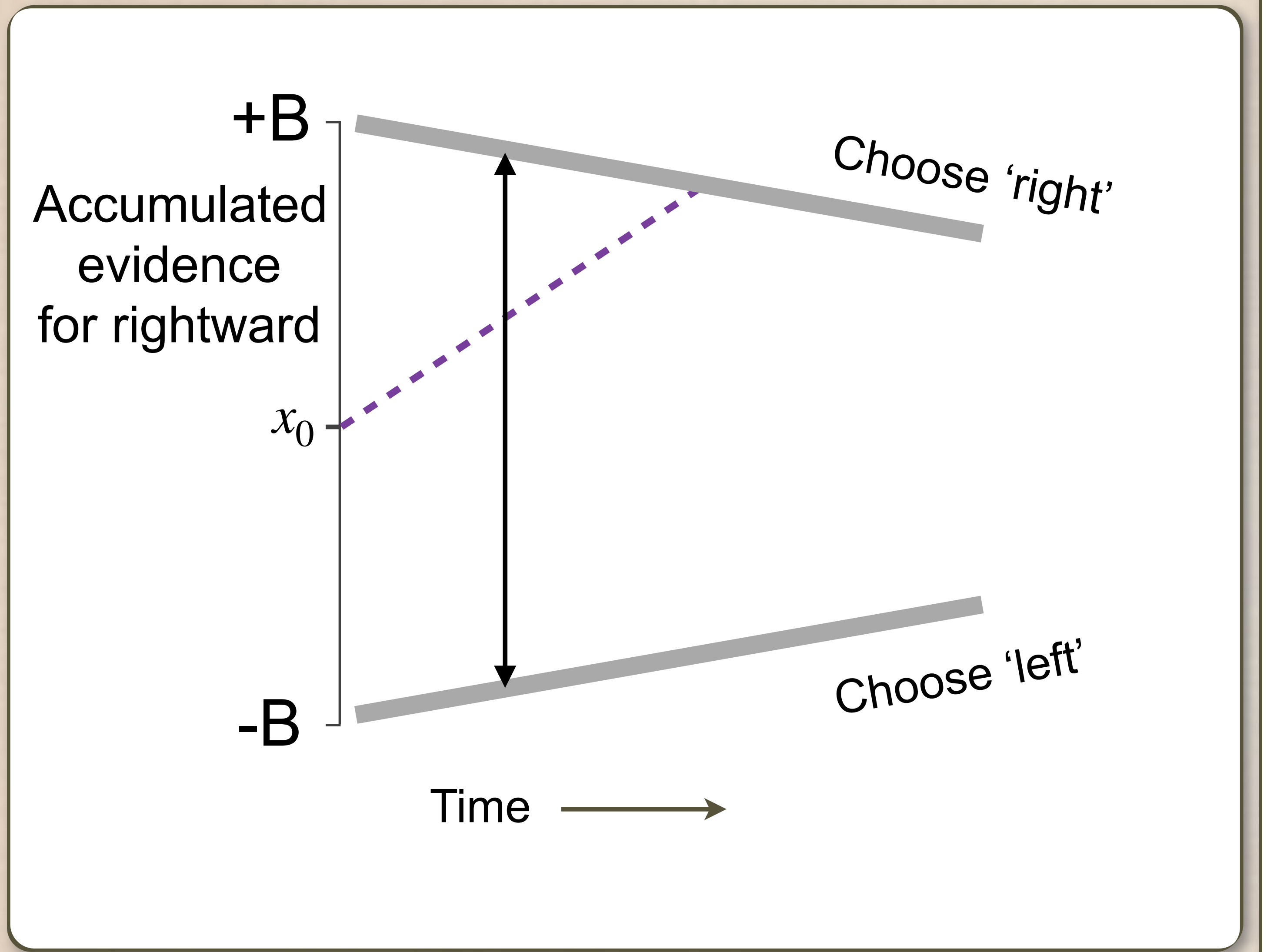

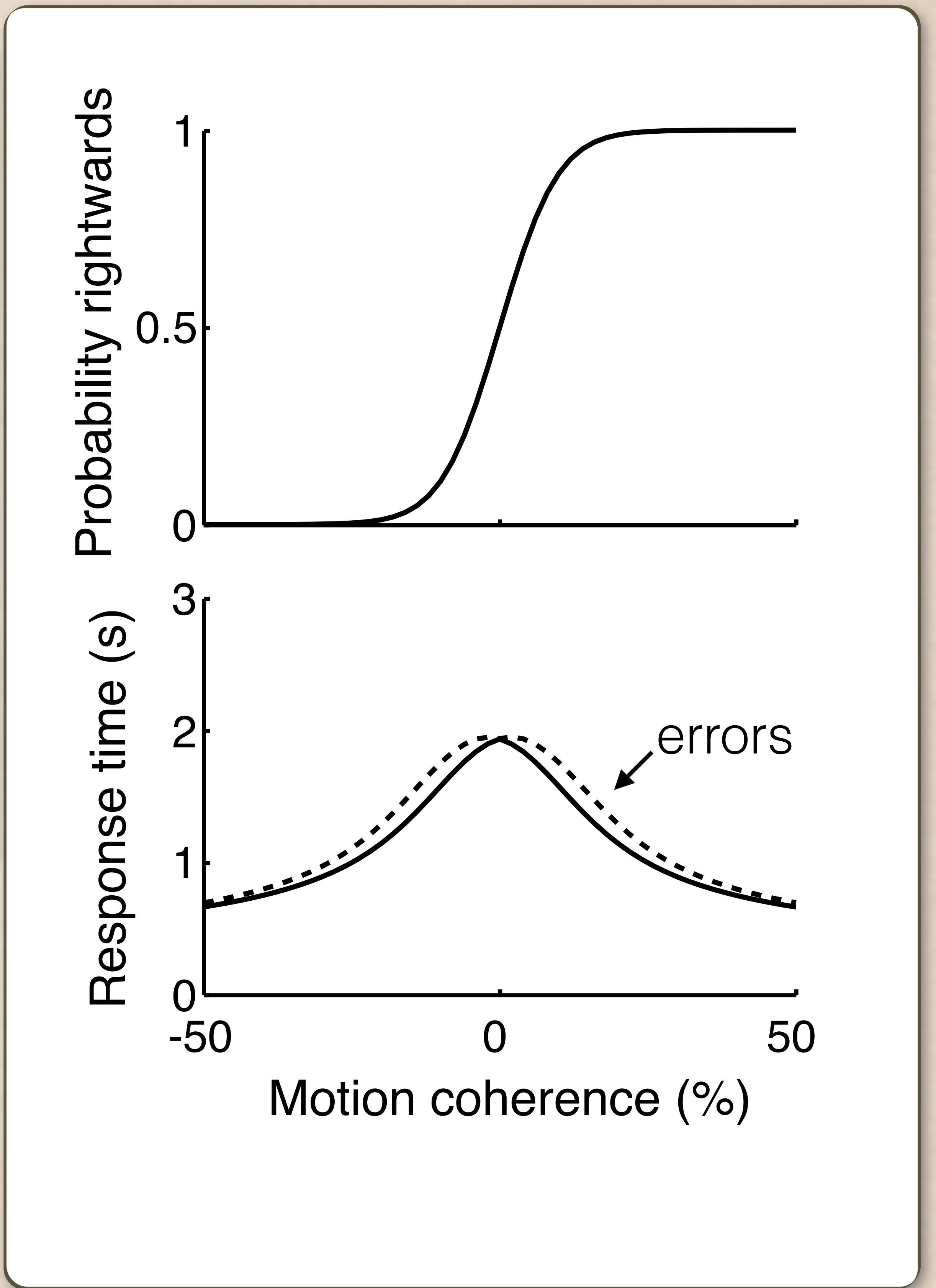

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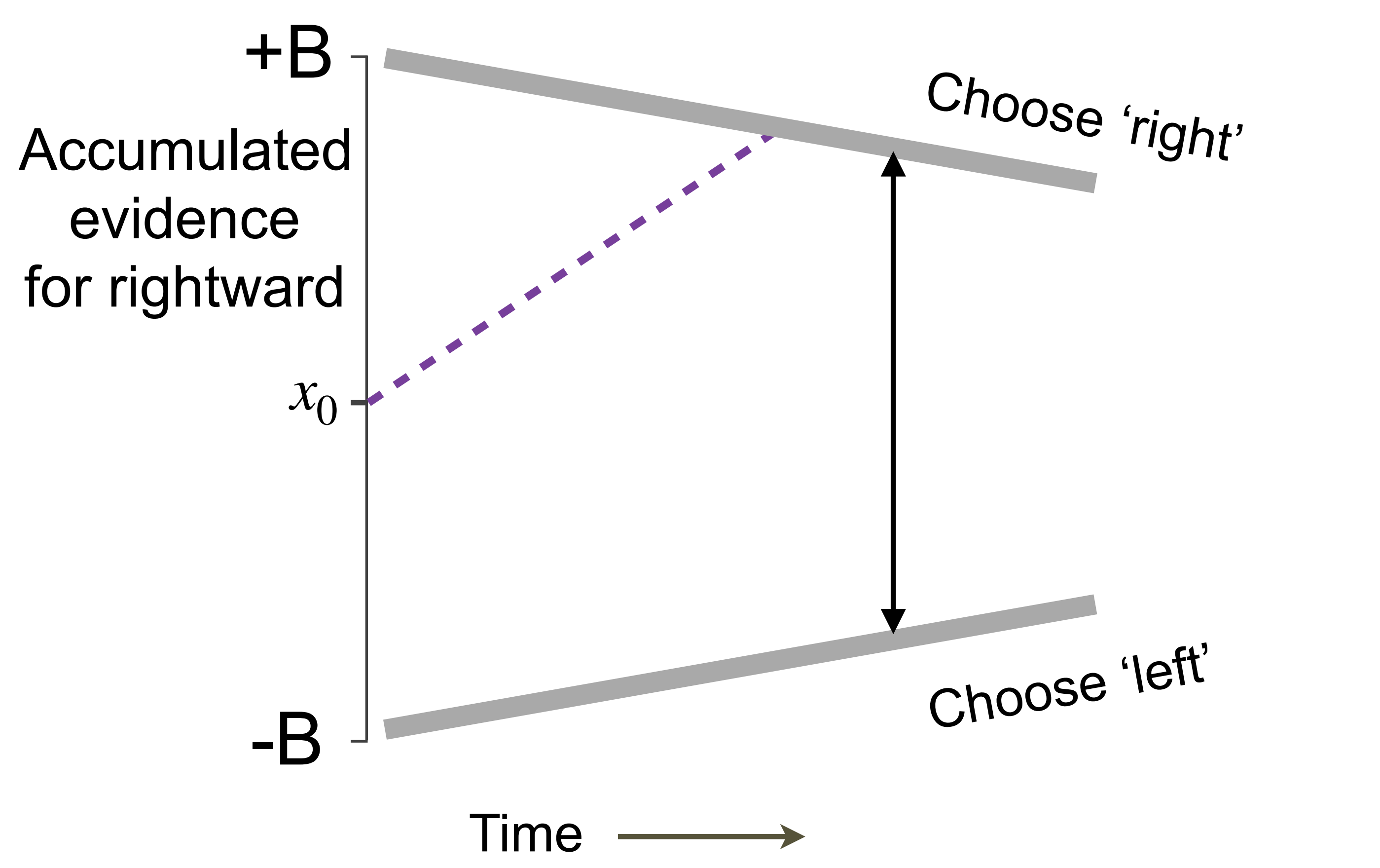

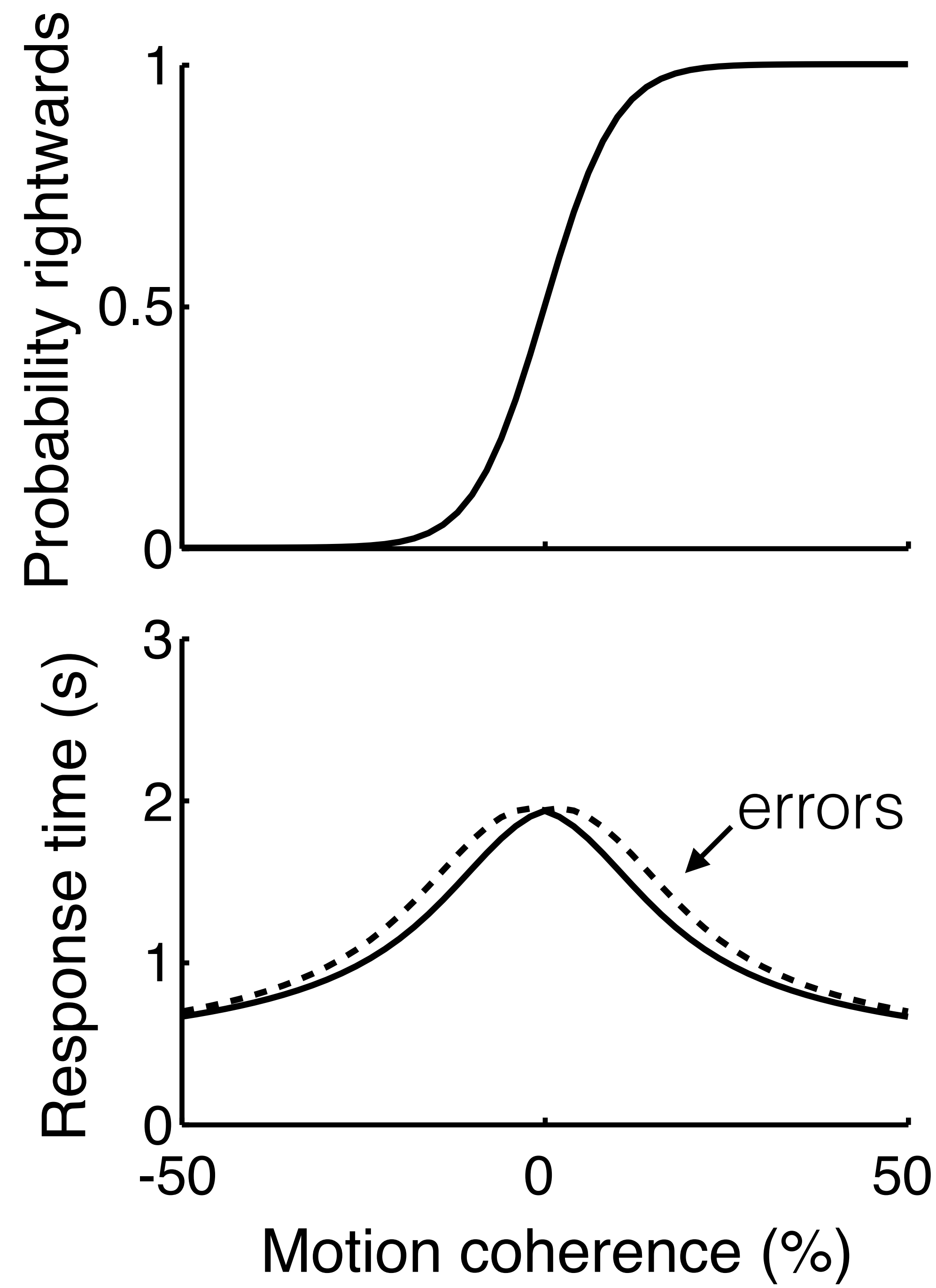

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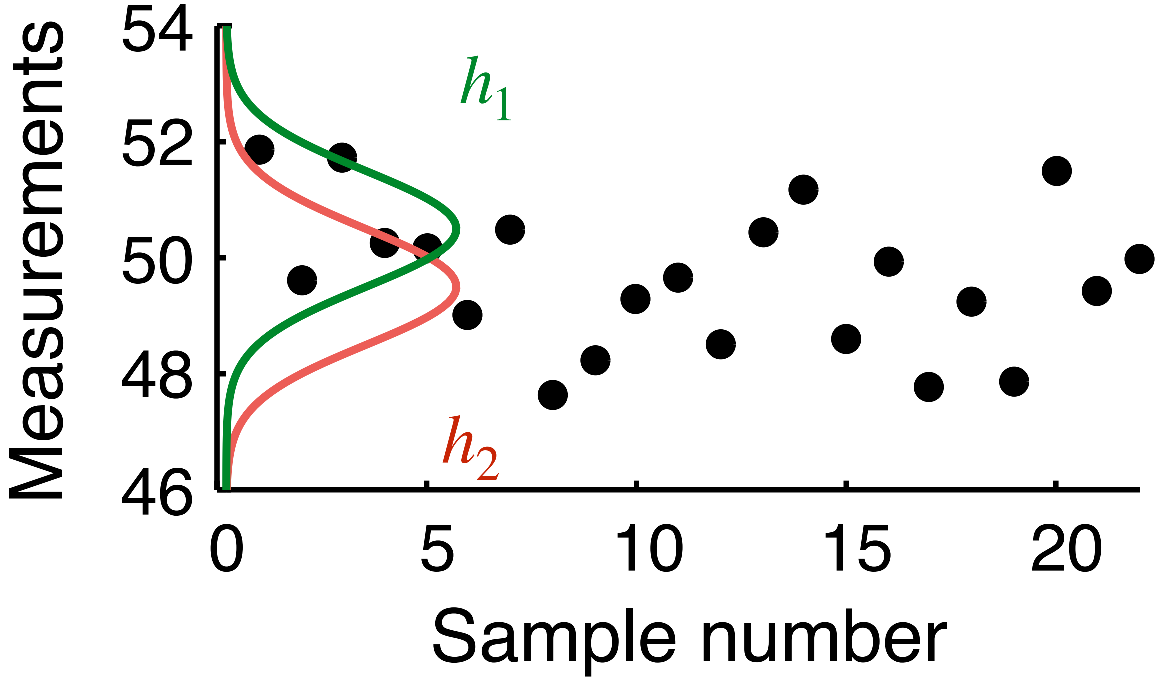
Collapsing bounds maximize reward-rate when difficulty varies across trials



Frazier & Yu, NiPS 2008
Drugowitsch et al., JNeuro 2012

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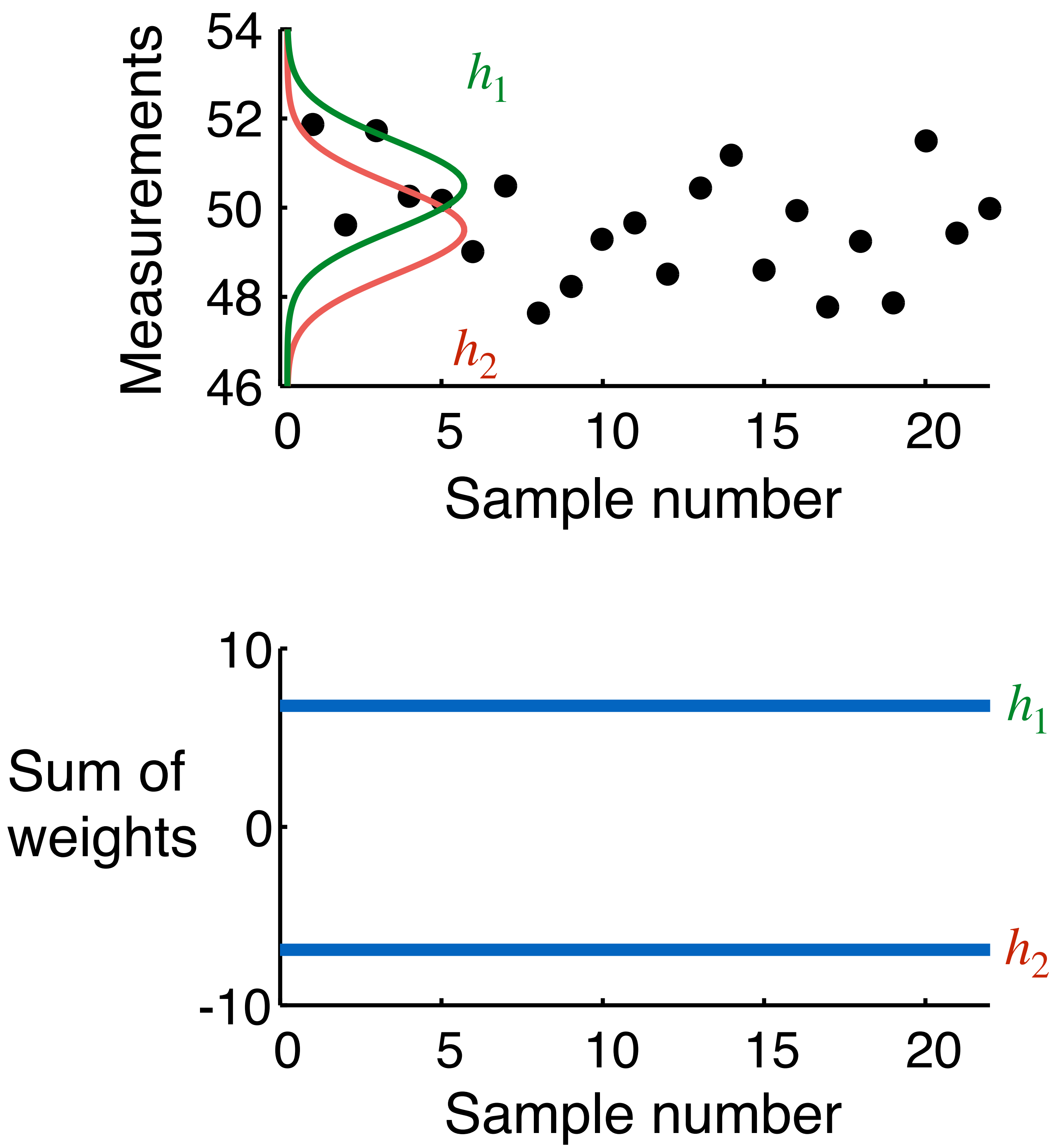

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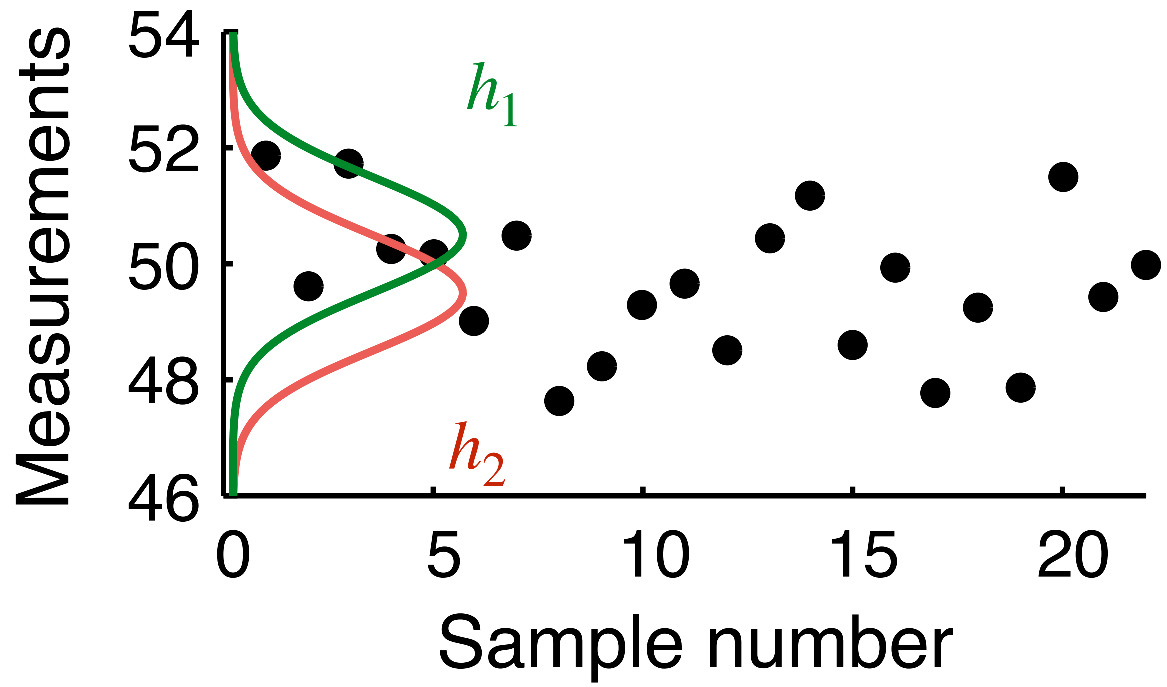

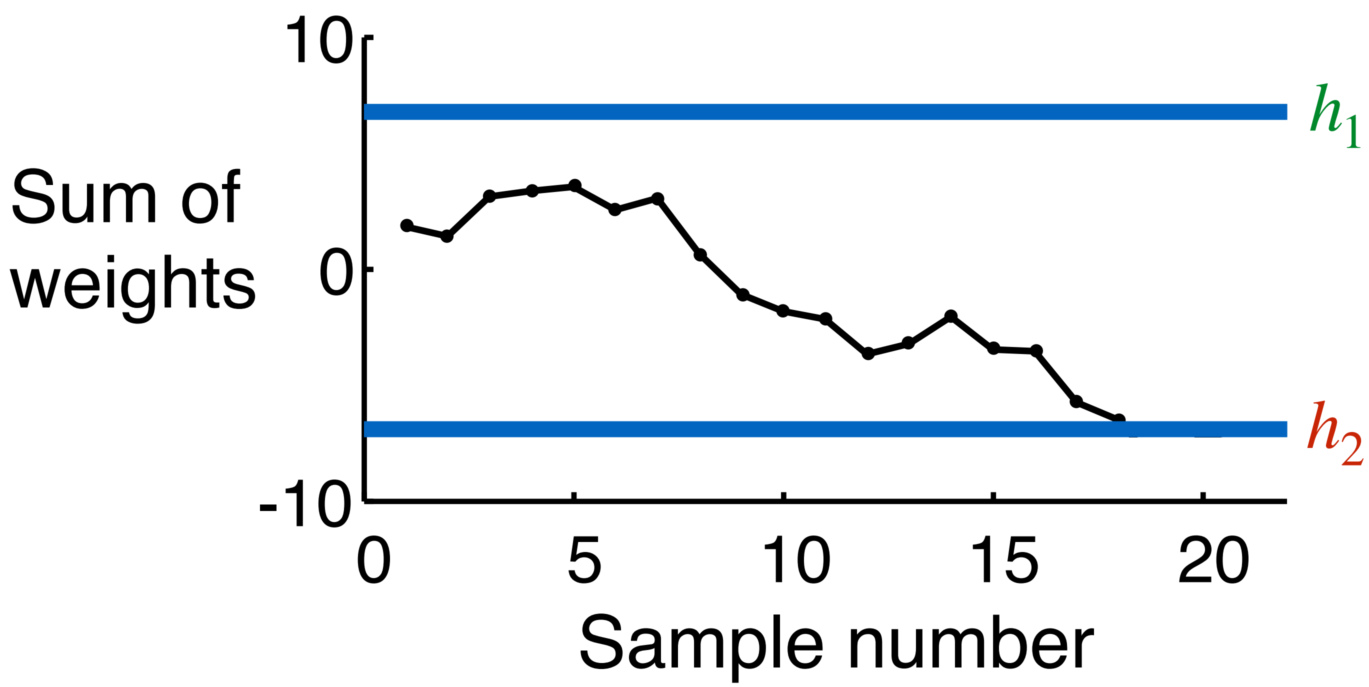

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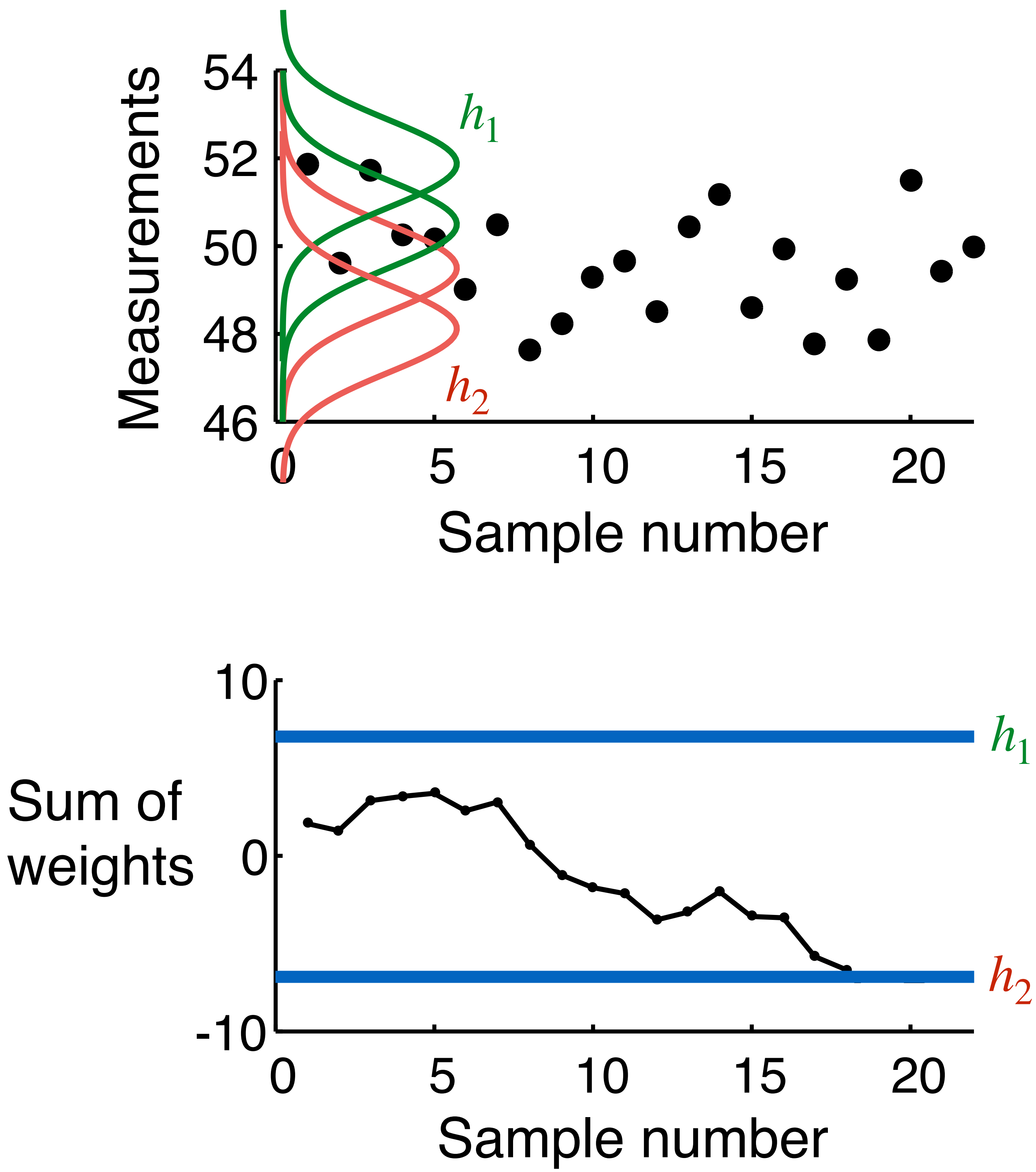

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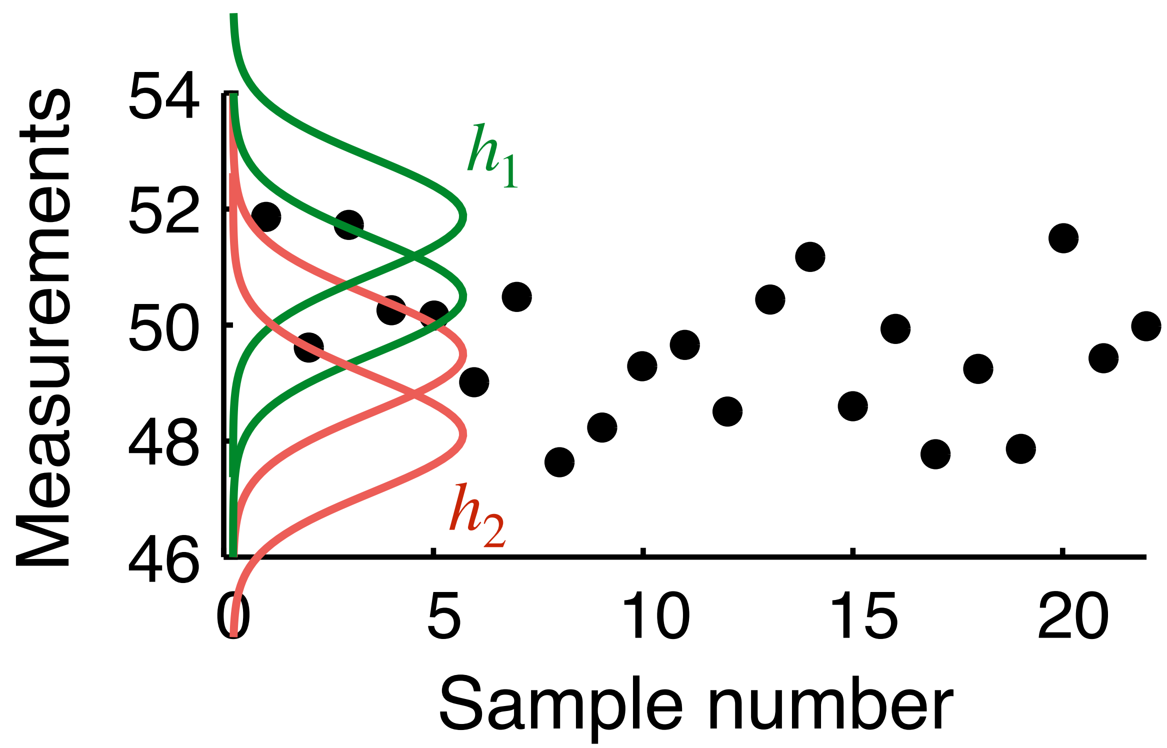

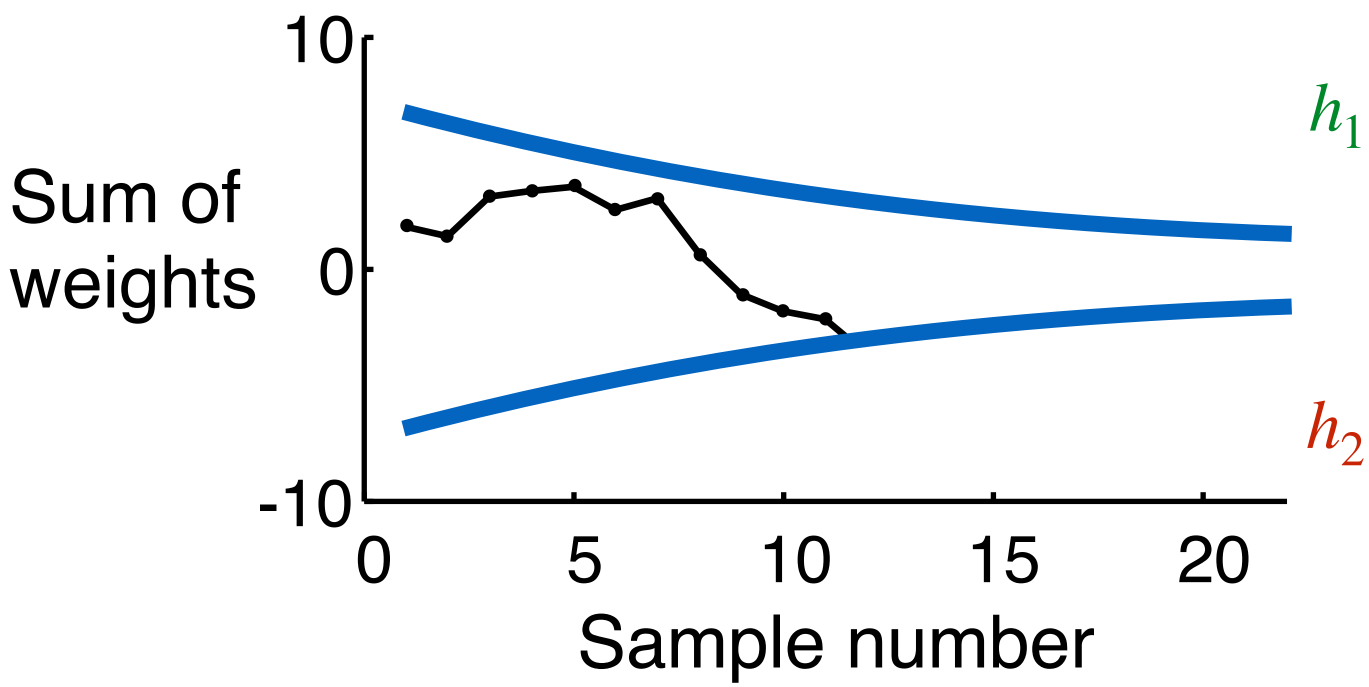

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Model fitting

Model fitting

https://github.com/arielzylberberg/CompPsychCourse_Zurich2019

test_data.mat

Motion Coherence (%)	Choice	RT (s)	Correct
-3.2	0	2.32	1
-12.8	0	0.76	1
0.0	0	1.31	0
51.2	1	0.39	1
-51.2	0	0.38	1
-25.6	0	0.55	1
-12.8	0	0.52	1
0.0	0	1.40	0
51.2	1	0.31	1
-3.2	0	2.05	1

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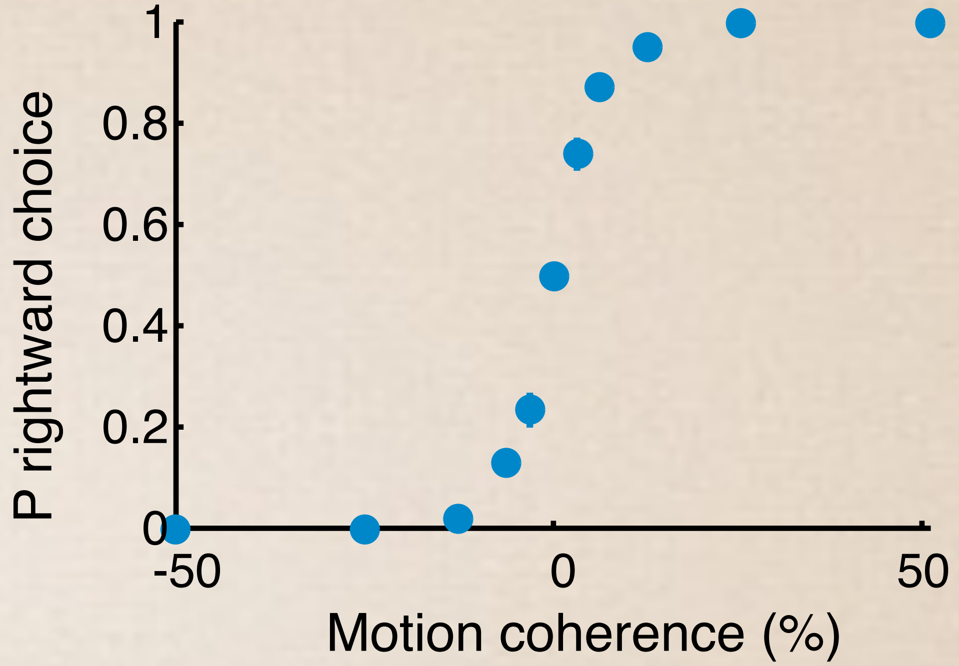

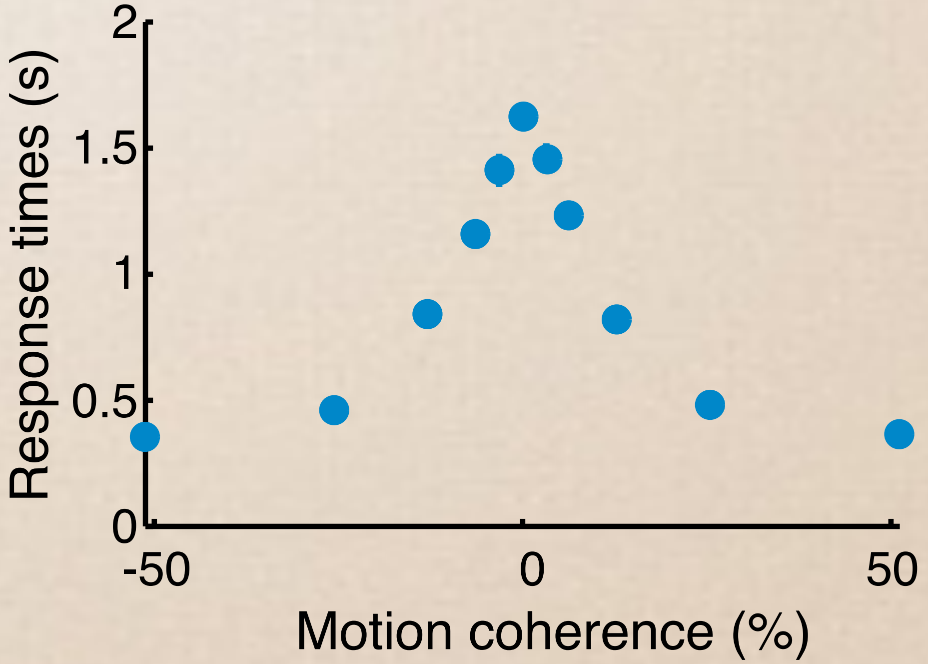

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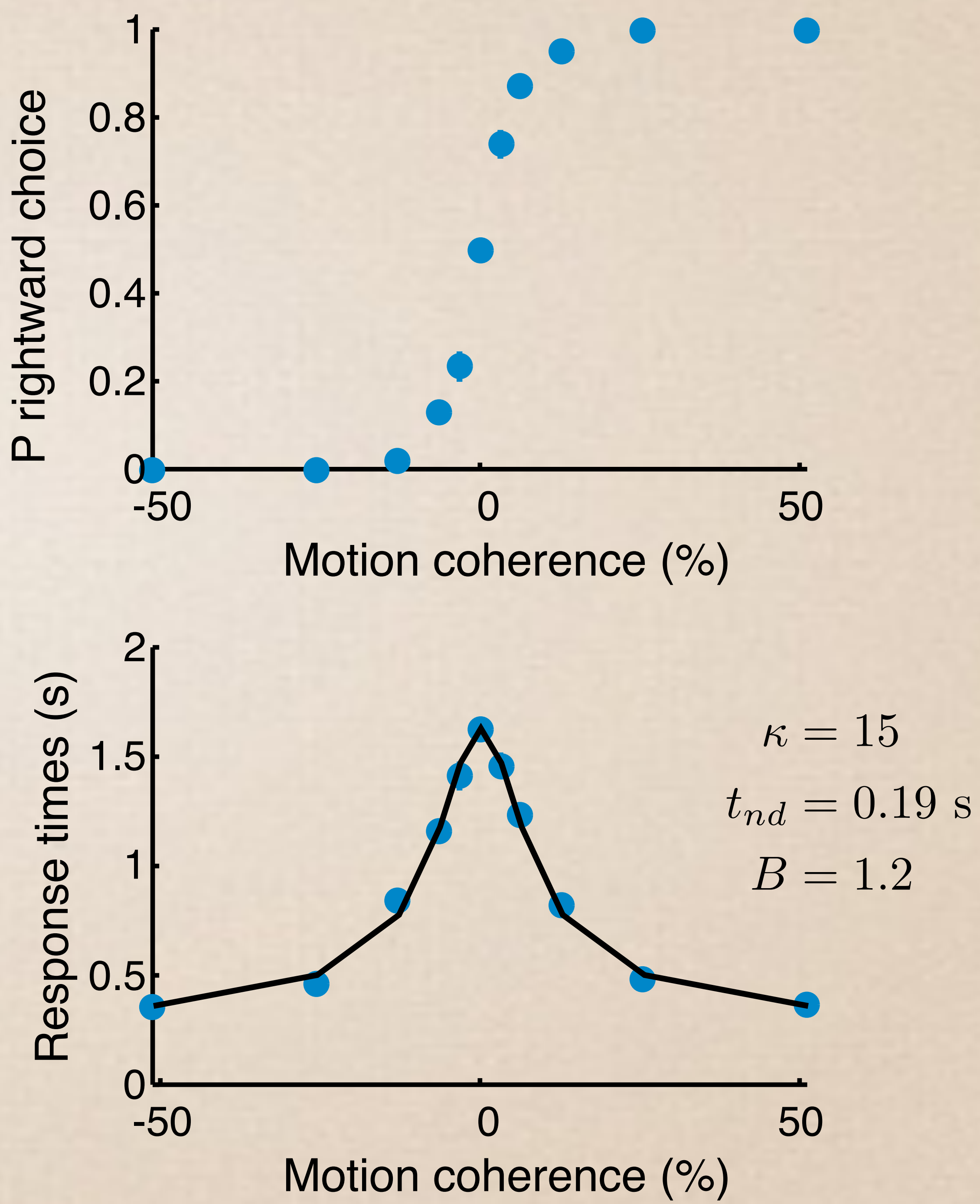

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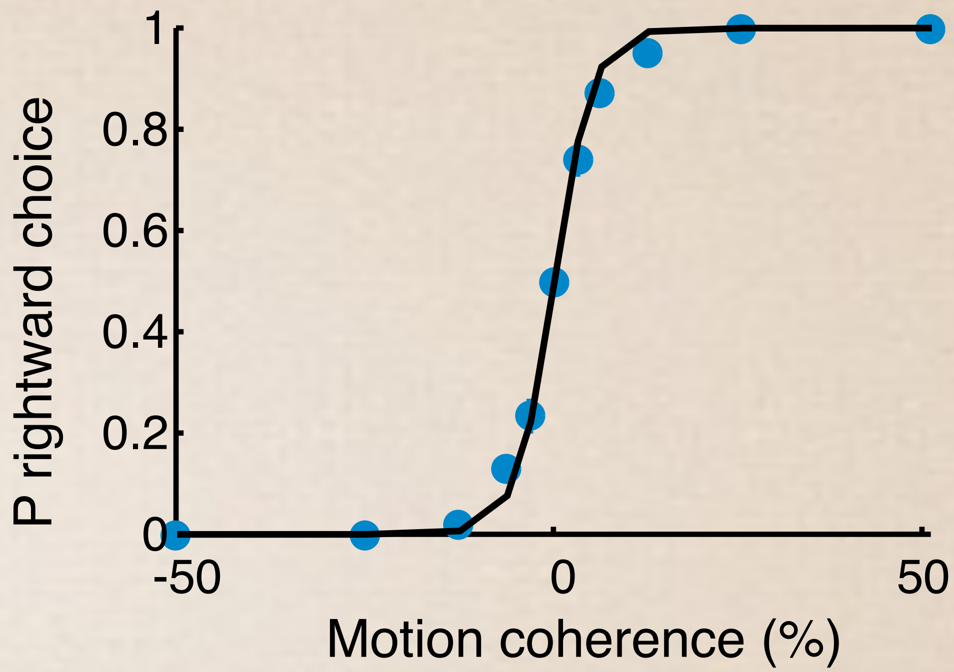

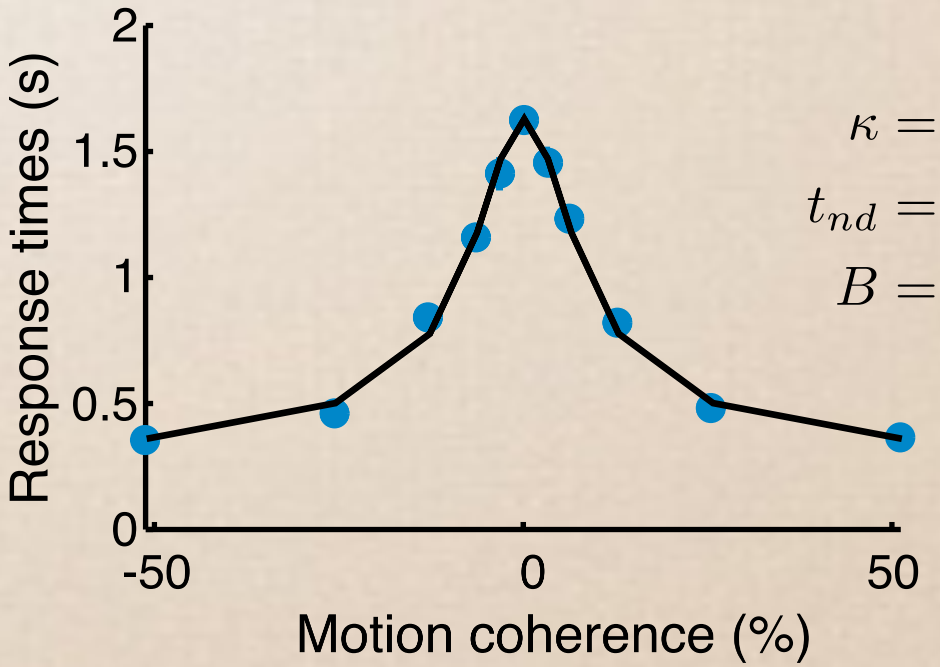

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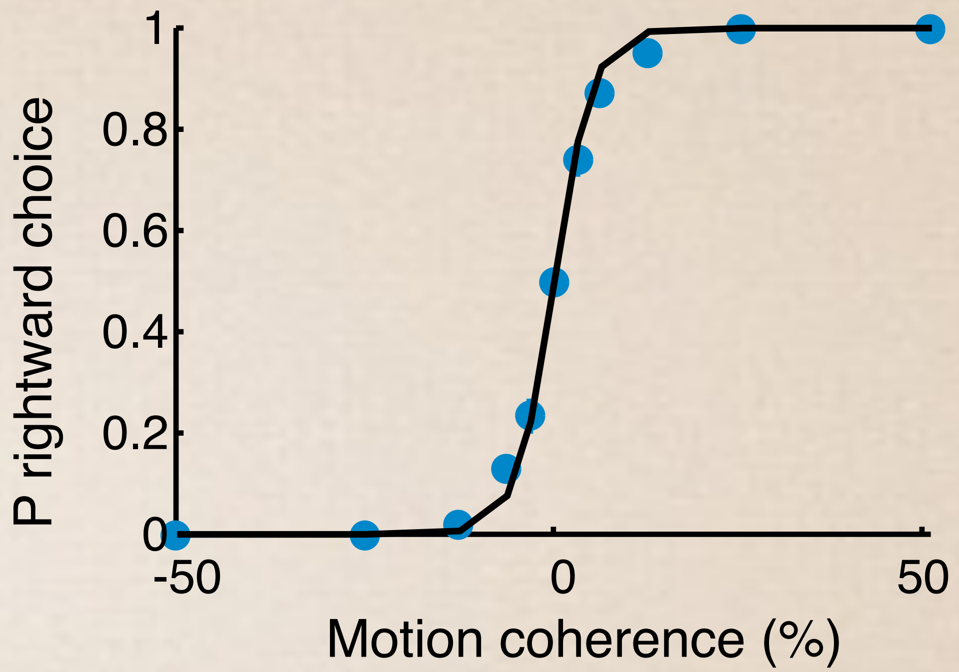

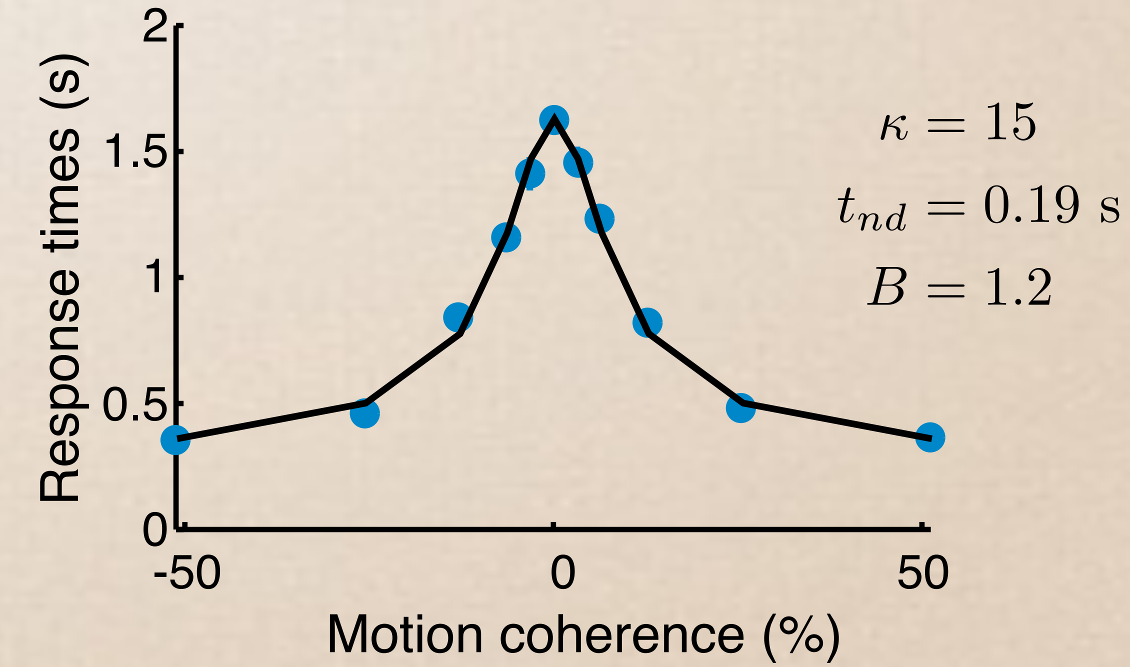

fit_means/main.m

Model fitting

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motion
coherence

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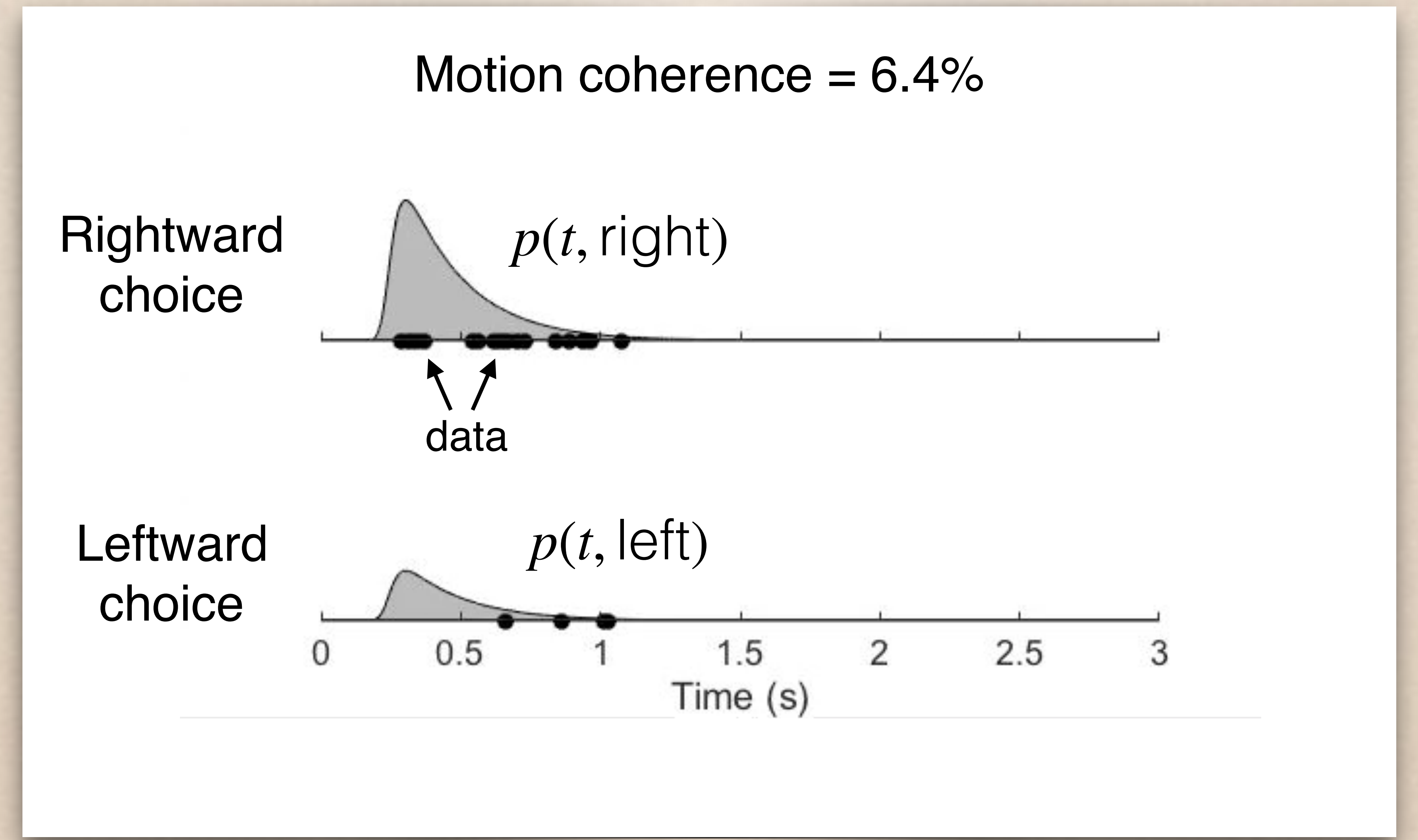
motion coherence



Analytic solution to the choice and response time distribution

Probability of terminating at the lower bound at time t:

$$p(t, \text{left}|\mu, B, \sigma, x_0) = p_-(\mu, B, \sigma, x_0) - \frac{\pi\sigma^2}{4B^2} e^{-\frac{(B+x_0)\mu}{\sigma^2}} \times \sum_{k=1}^{\infty} \frac{2k \ \sin(\frac{k\pi(B+x_0)}{2B}) e^{-\frac{1}{2}(\frac{\mu^2}{\sigma^2} + \frac{\pi^2 k^2 \sigma^2}{4B^2})t}}{\left(\frac{\mu^2}{\sigma^2} + \frac{\pi^2 k^2 \sigma^2}{4B^2}\right)}$$

, where

$$p_-(\mu, B, \sigma, x_0) = \frac{e^{-(4\mu B/\sigma^2)} - e^{-(2\mu(B+x_0)/\sigma^2)}}{e^{-(4\mu B/\sigma^2)} - 1}$$

Ratcliff 1978, Feller 1968

Fitting the DDM with flat bounds

1. HDDM

   http://ski.clps.brown.edu/hddm_docs/

2. DMAT

   https://ppw.kuleuven.be/okp/software/dmat/

3. fast-dm

   https://www.psychologie.uni-heidelberg.de/ae/meth/fast-dm/

4. EZ

   https://www.ejwagenmakers.com/2007/EZ.pdf

When analytical solutions are not available (e.g., collapsing bounds)

Fokker Planck equation:

$$\frac{\partial p(x,t)}{\partial t} = -\mu \frac{\partial p(x,t)}{\partial x} + \frac{\sigma^2}{2} \frac{\partial^2 p(x,t)}{\partial^2 x}$$

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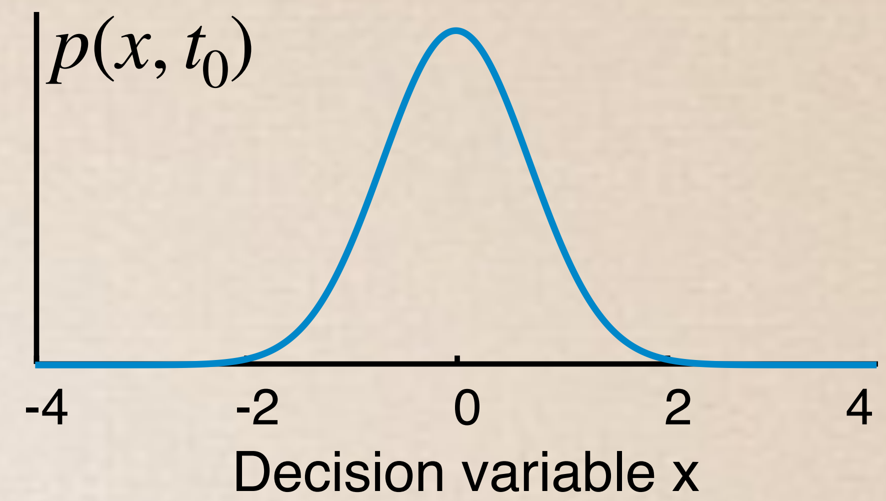

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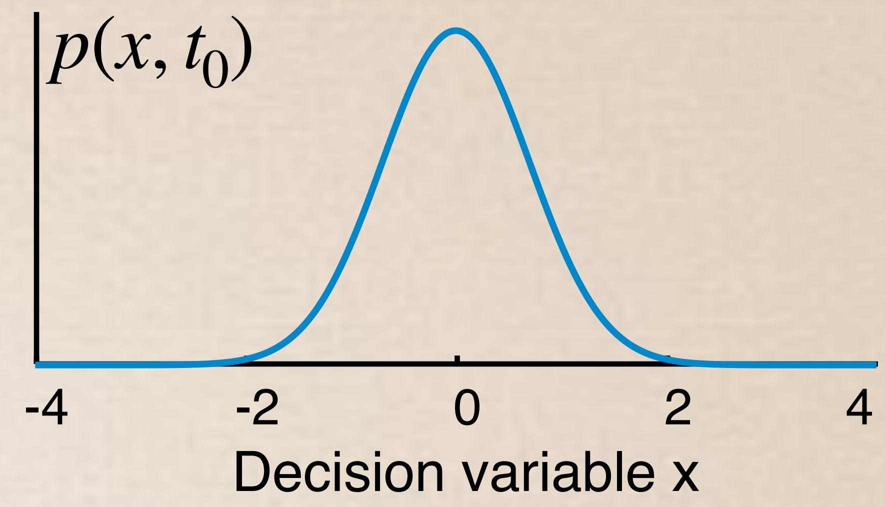

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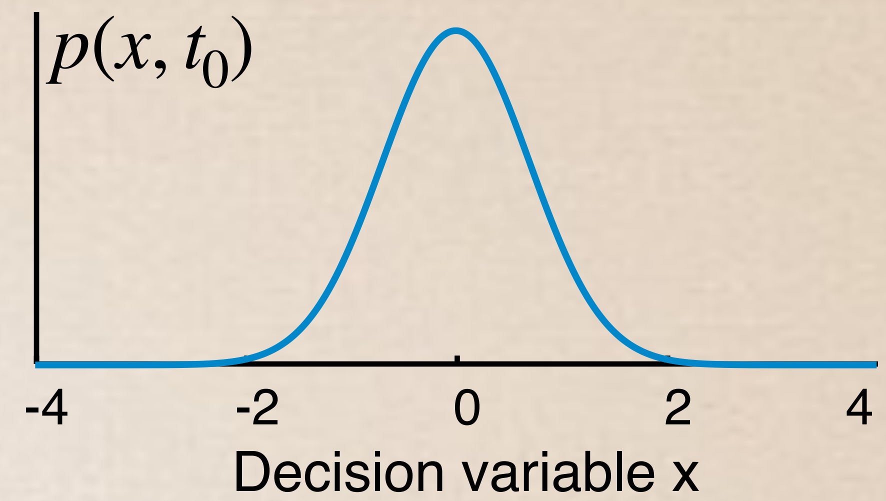

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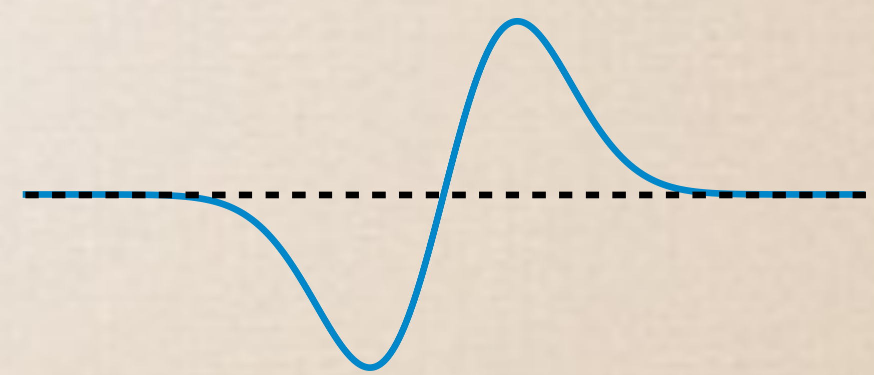

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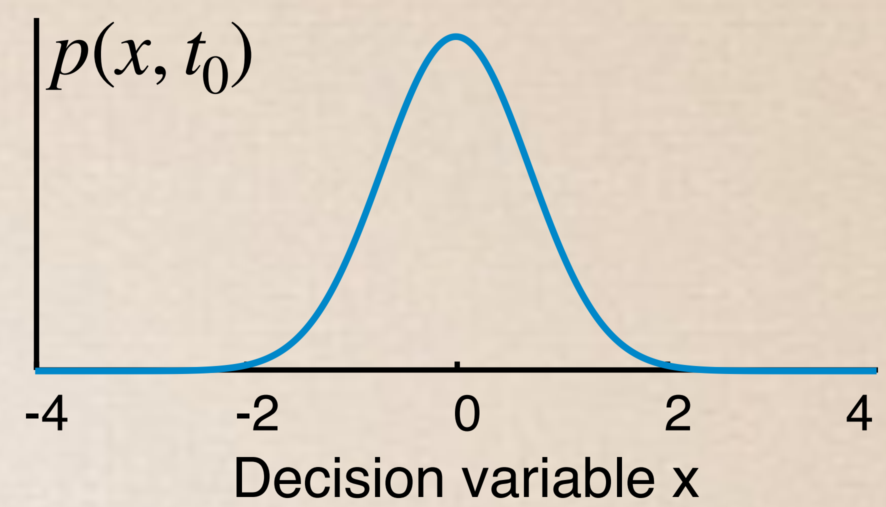

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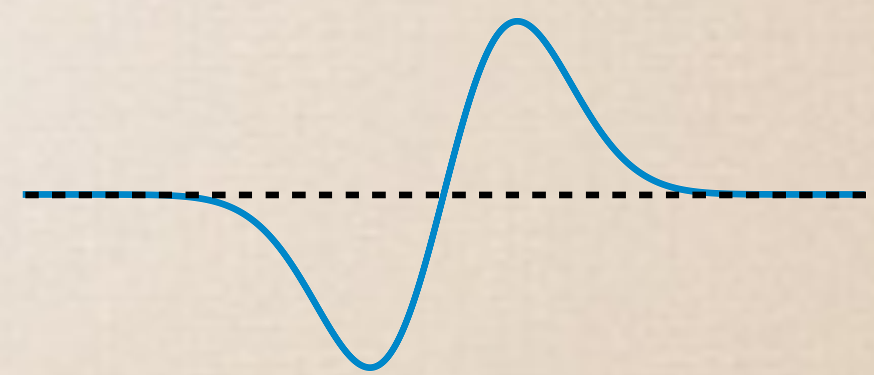

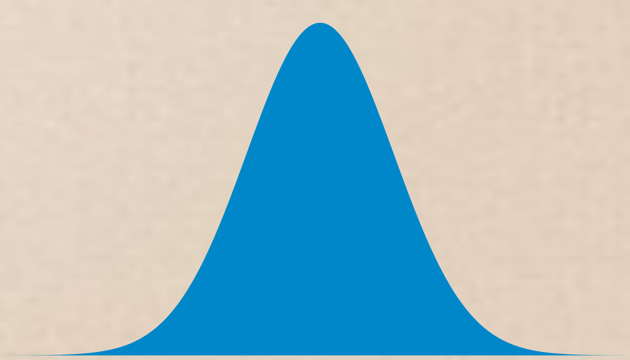

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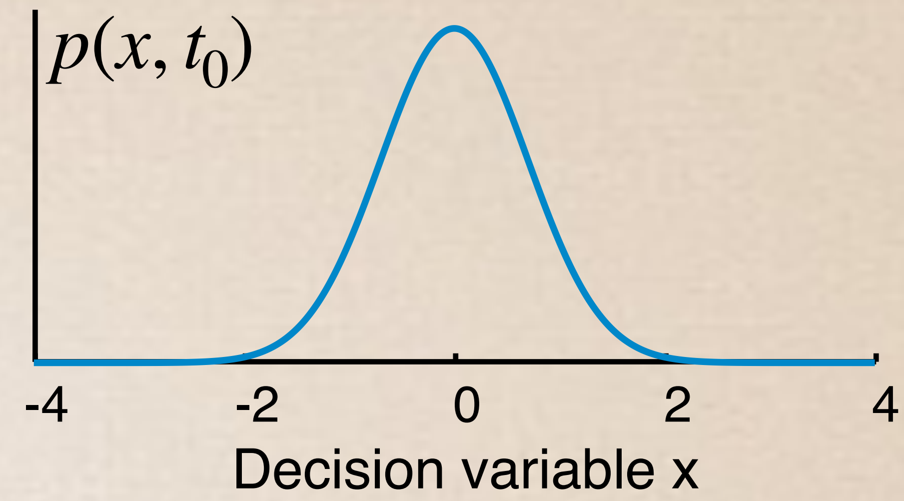

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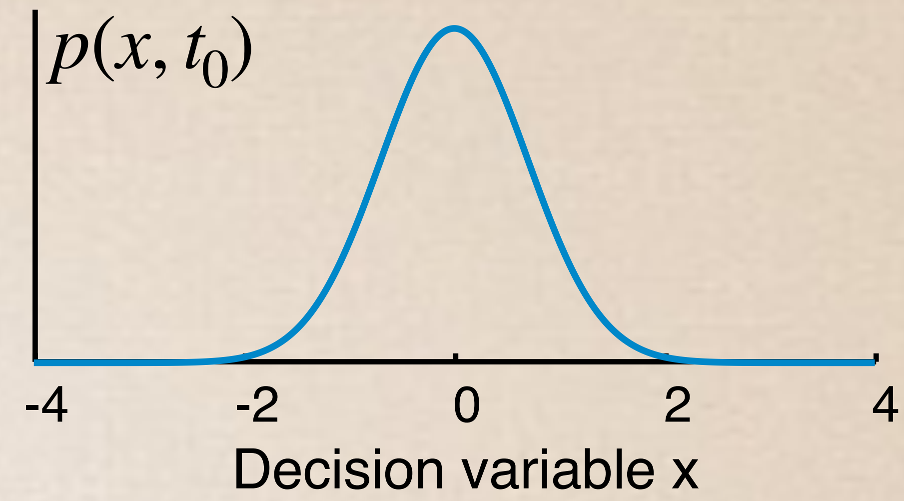

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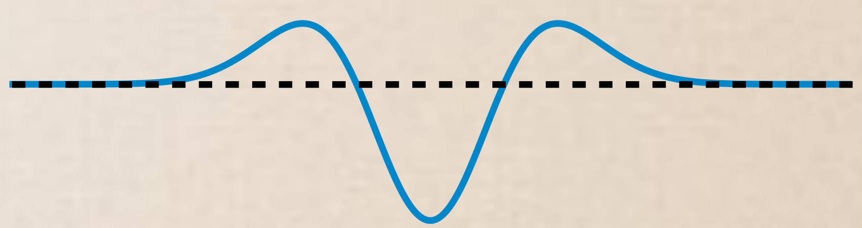

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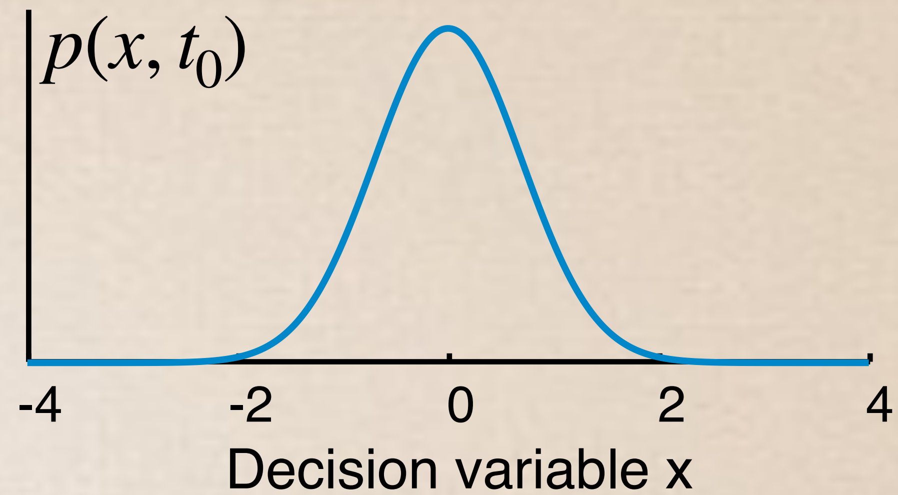

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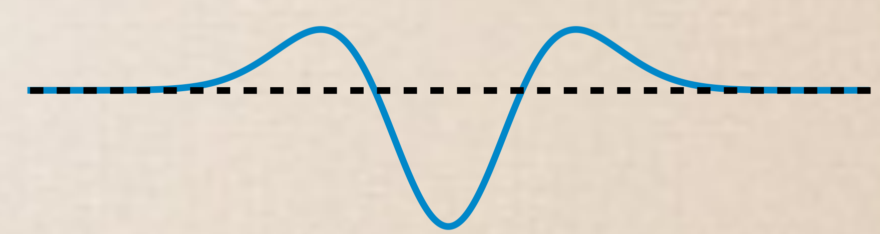

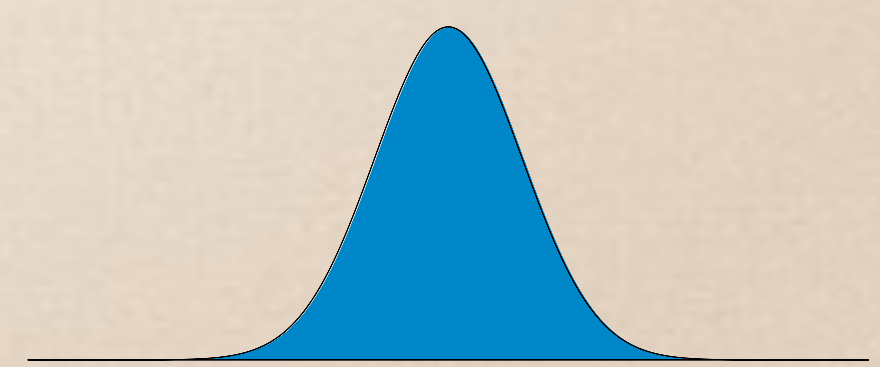

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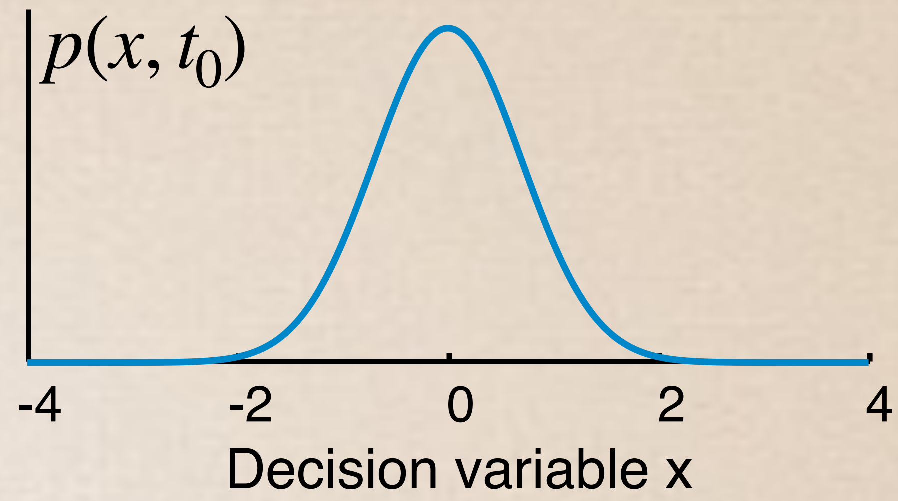

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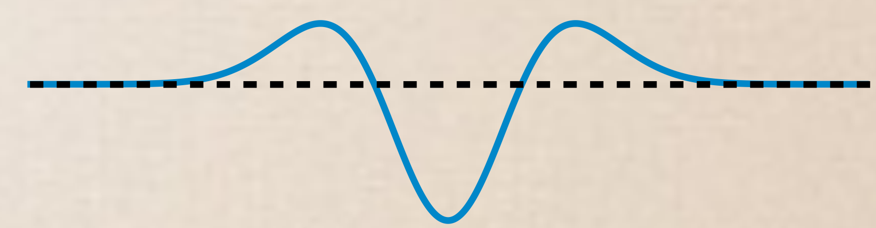

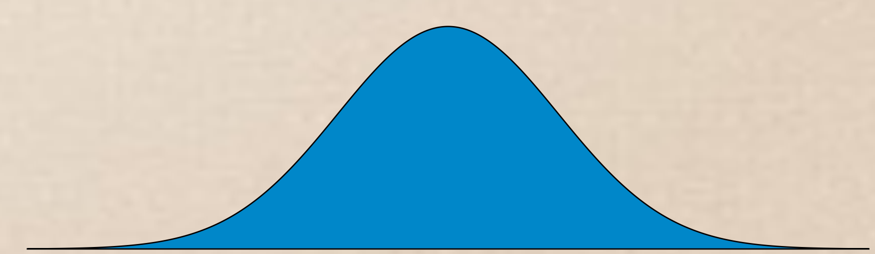

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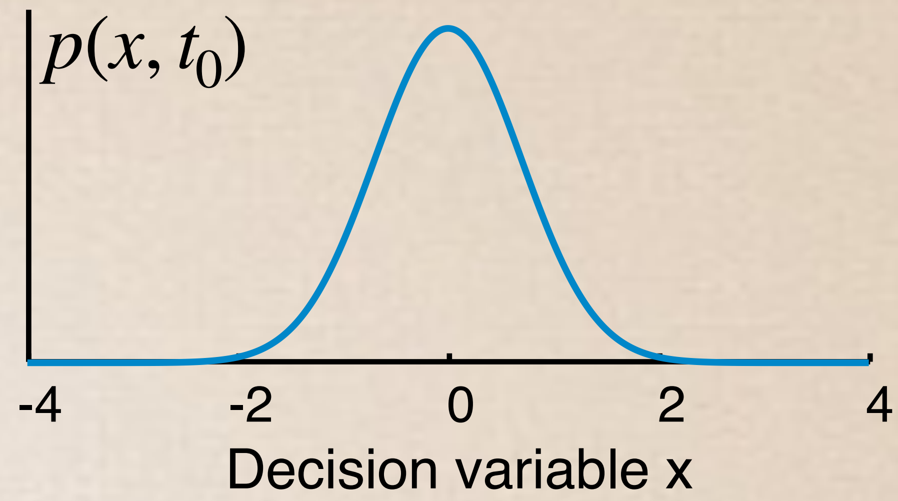

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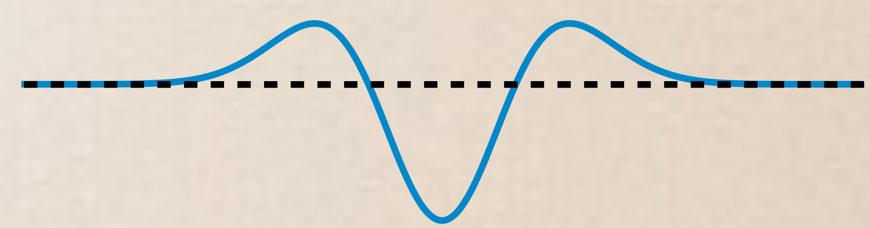

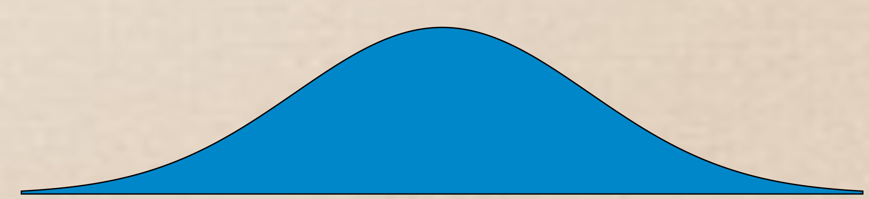

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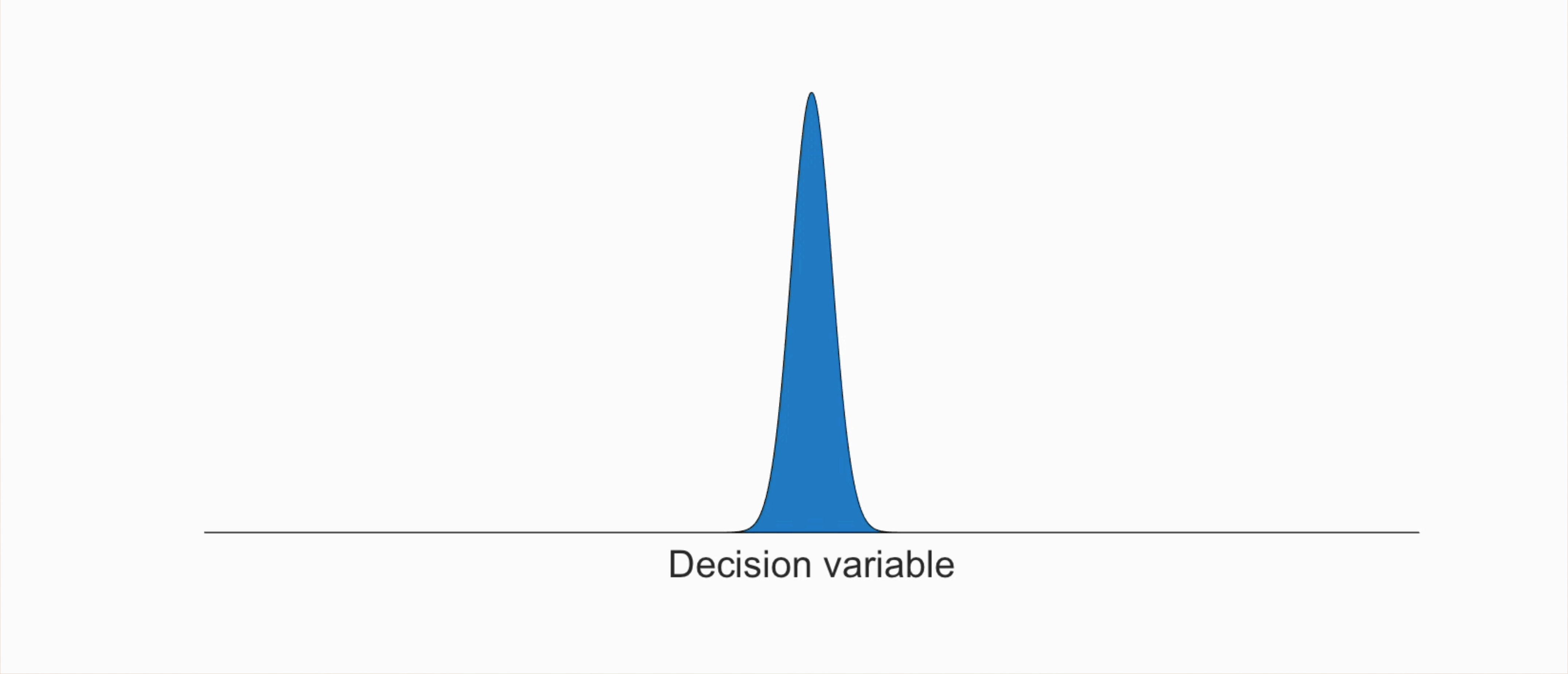

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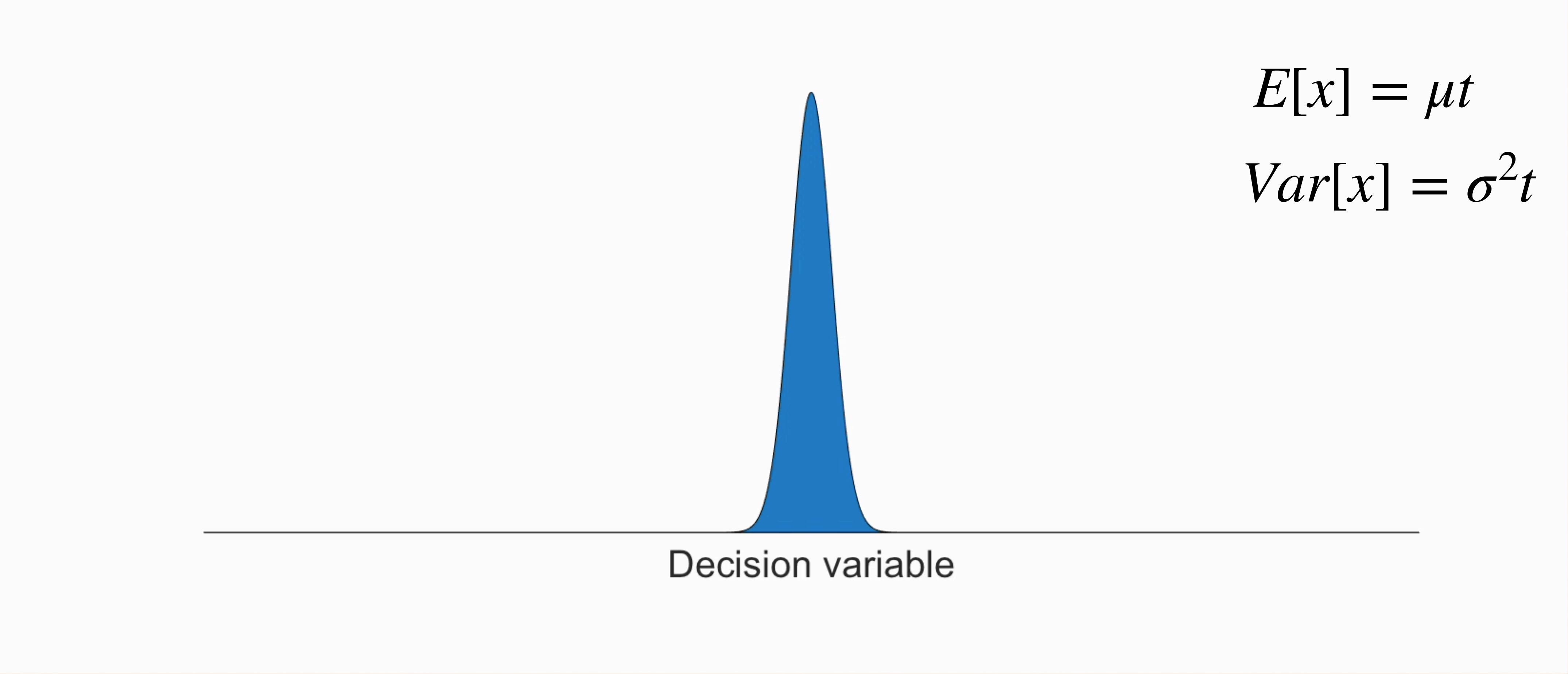

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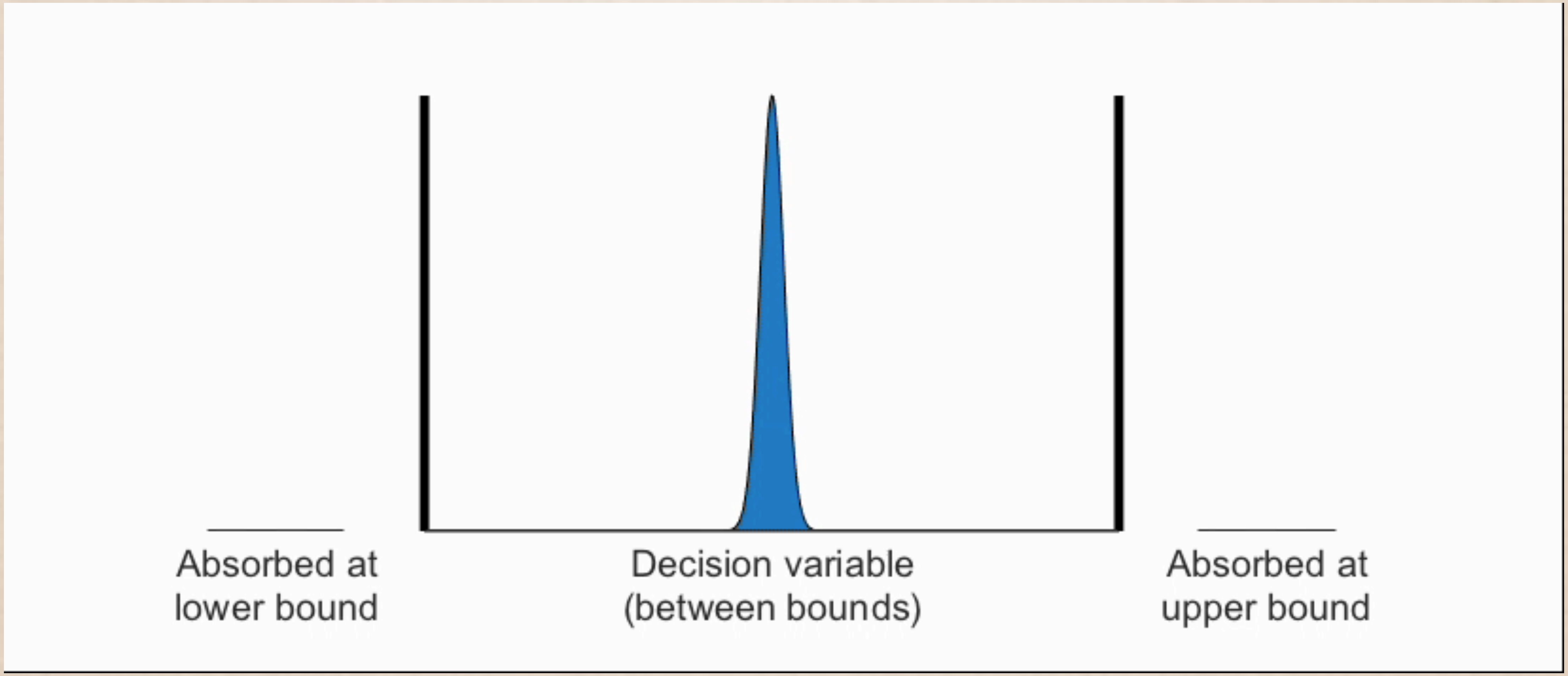

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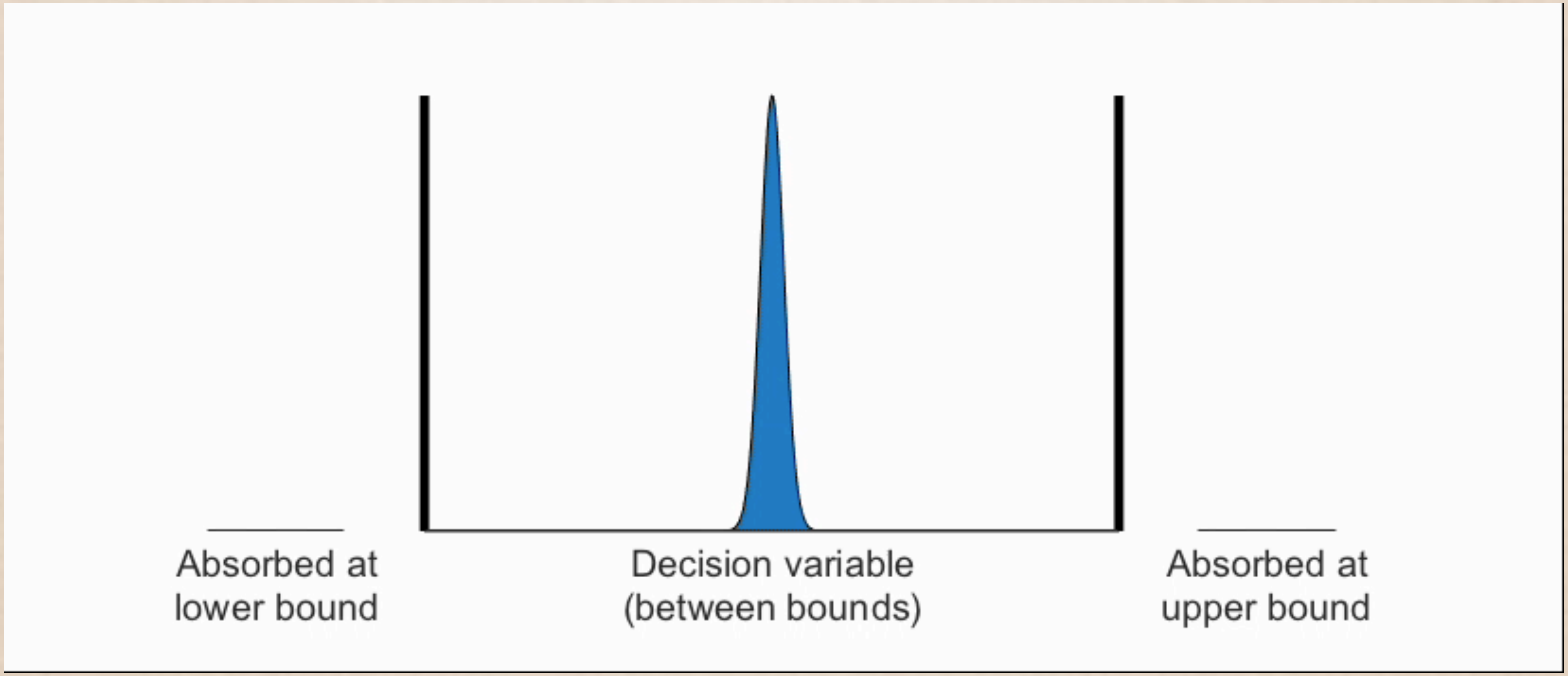

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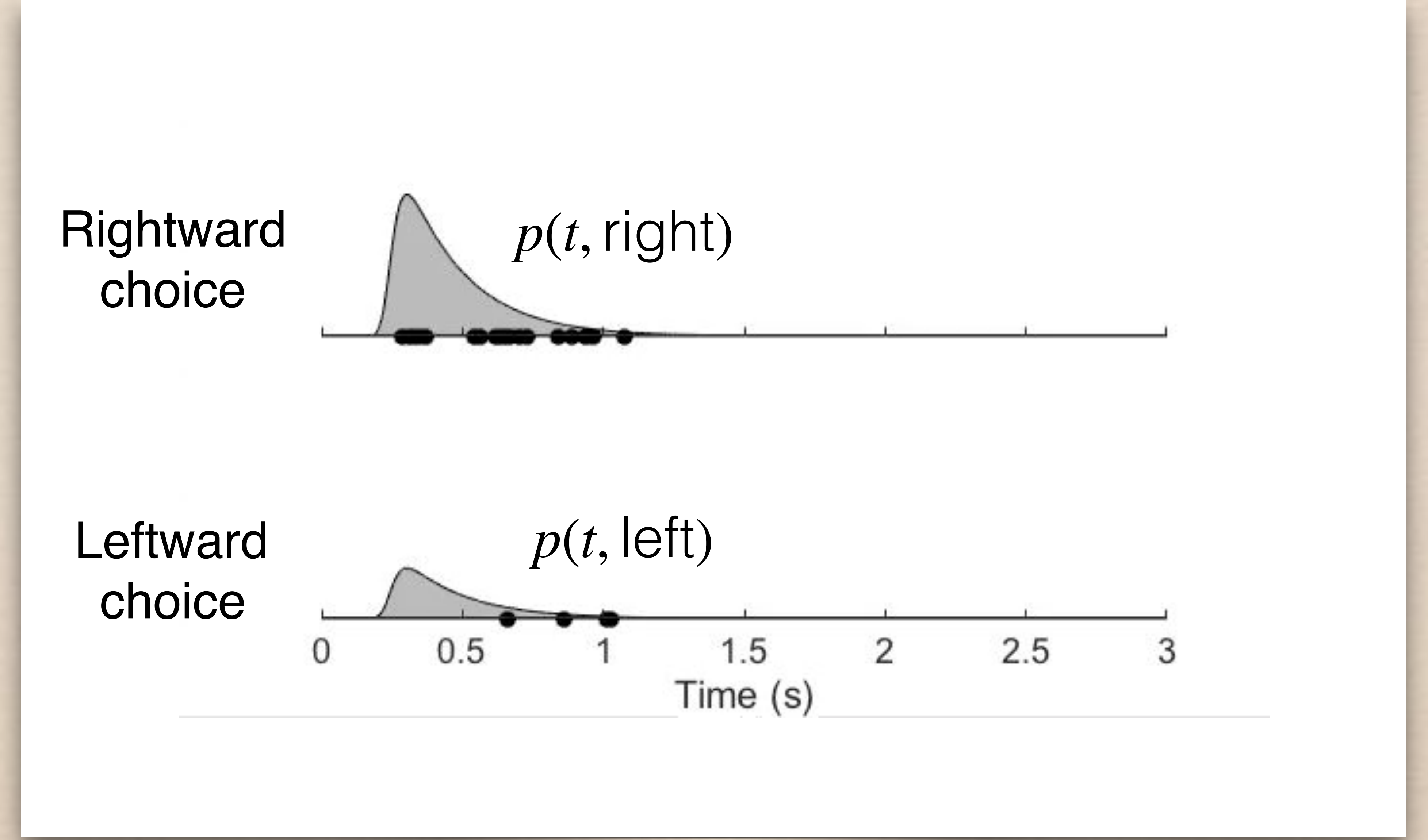
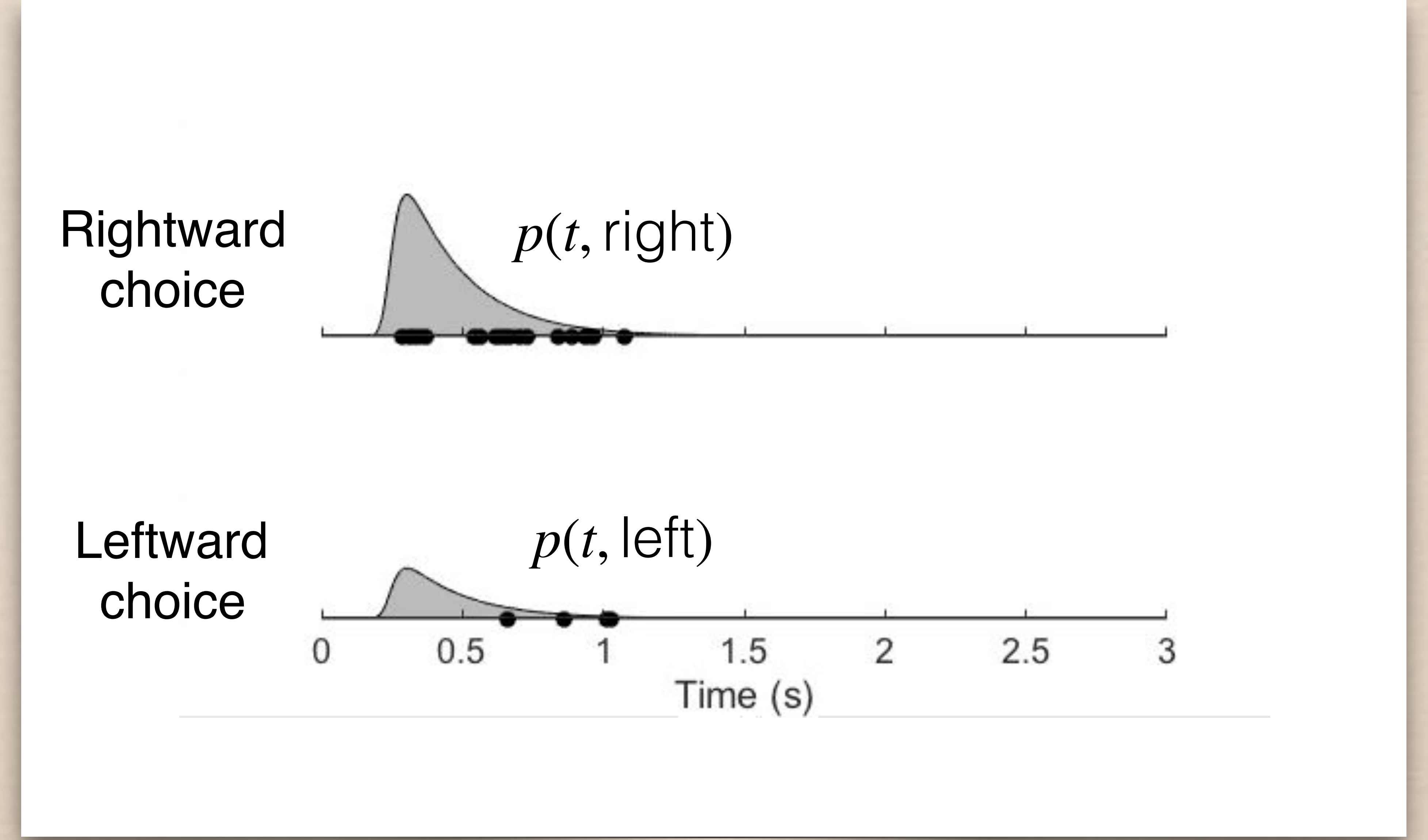


Code in Github uses Chang & Cooper's (1970) implicit integration method.

Model fitting

$$\hat{\theta}_{\mathrm{mle}} = \arg\max_{\theta} \left(\sum_{i=1}^{n_{tr}} p(\mathrm{RT}^{(i)}, \mathrm{choice}^{(i)} | c^{(i)}, \theta) \right)$$



Code in Github uses Chang & Cooper’s (1970) implicit integration method.

Parameter optimization

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- fminsearch (Nelder–Mead simplex method)
- fmincon
- Bayesian Adaptive Direct Search (BADS)
  - Luigi Acerbi (https://github.com/lacerbi/bads)

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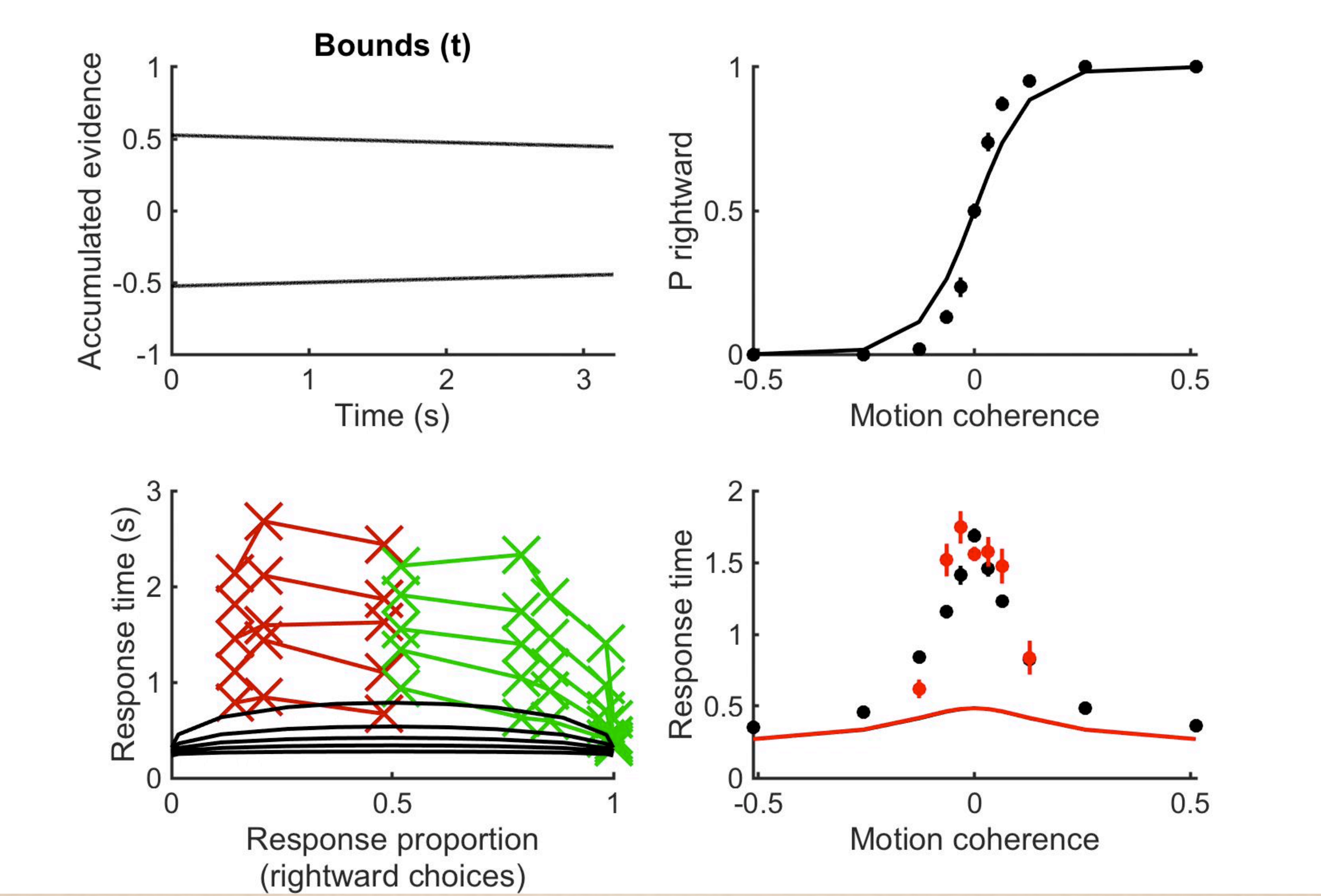
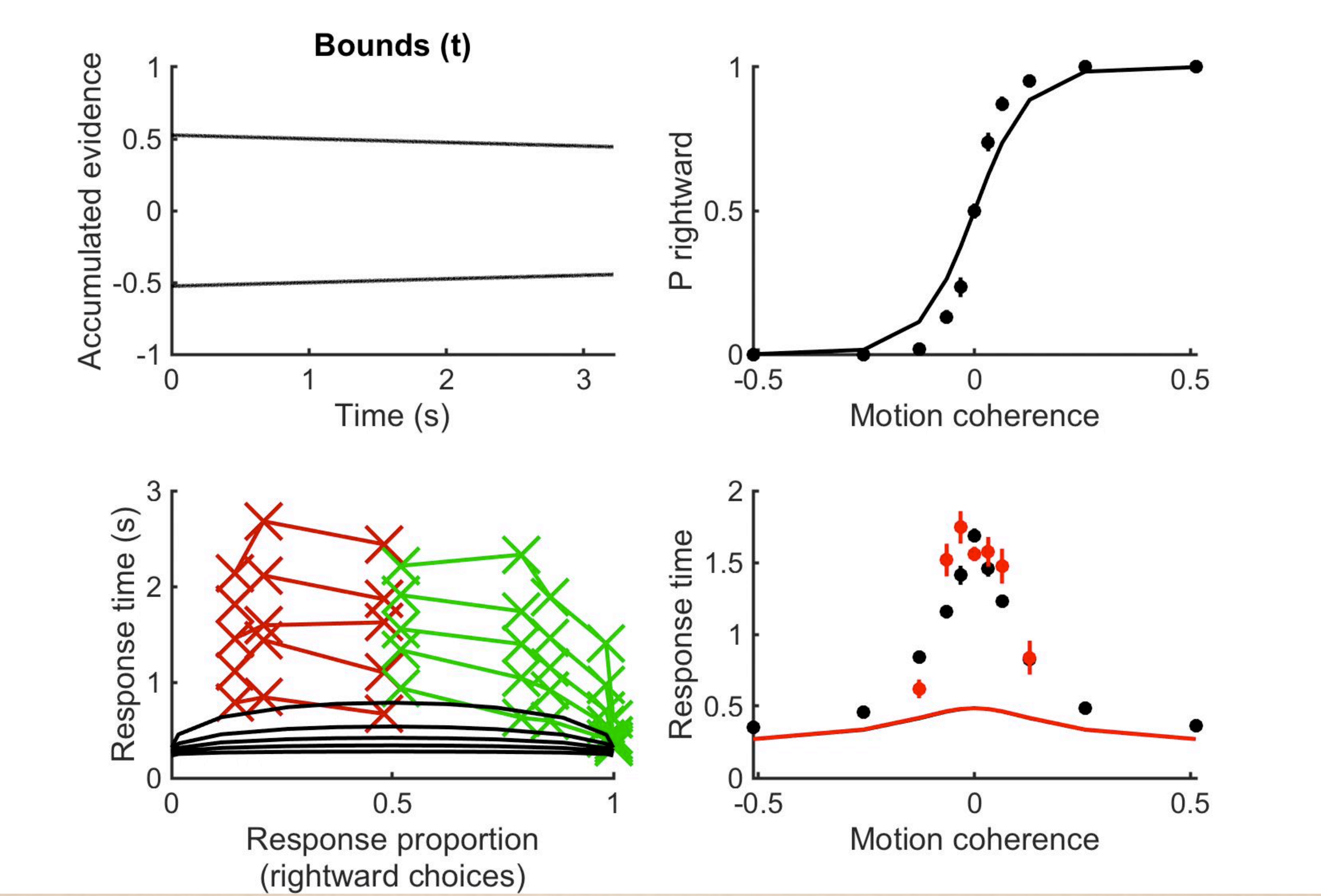
https://github.com/arielzylberberg/CompPsychCourse_Zurich2019/fit_single_trials/run_example.m



Six parameters:

signal-to-noise (κ), bound shape (3 params), non-decision time distribution (2 params) https://github.com/arielzylberberg/CompPsychCourse_Zurich2019/fit_single_trials/run_example.m



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Parameter recovery

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» Fit model to experimental data

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- Fit model to experimental data
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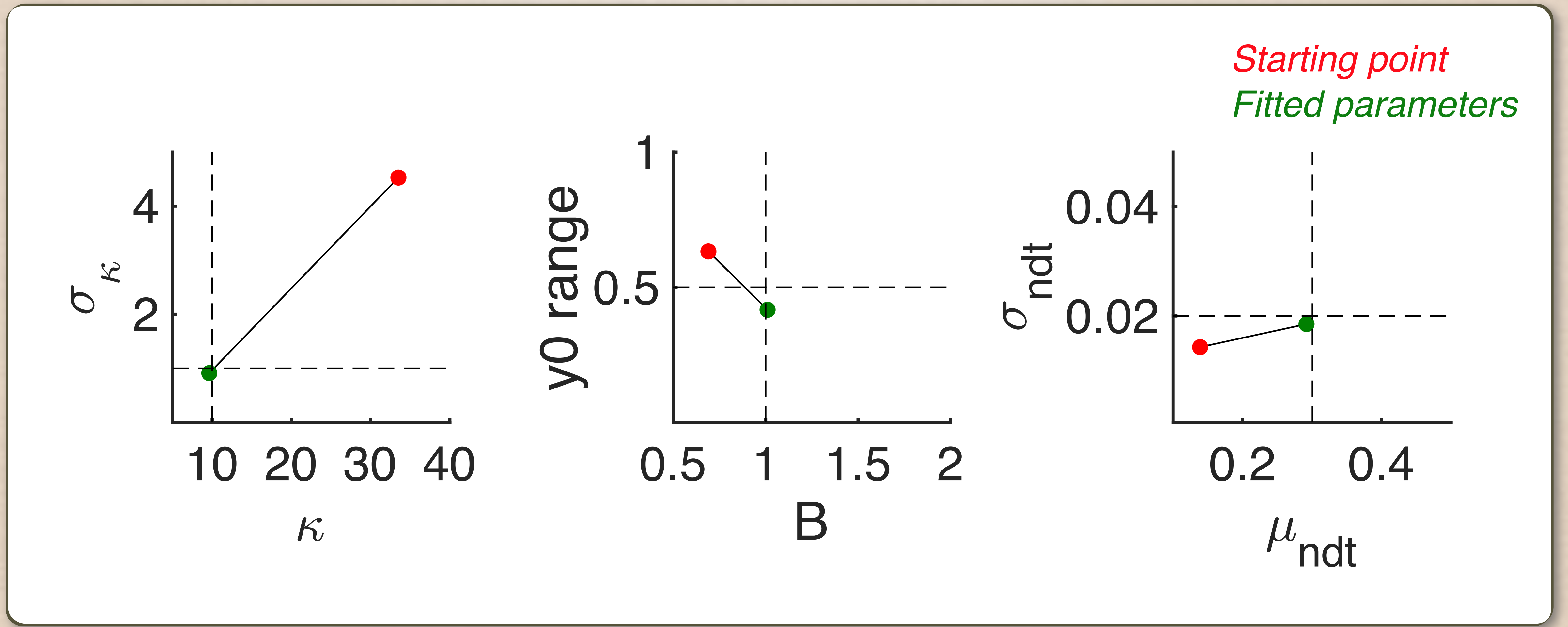

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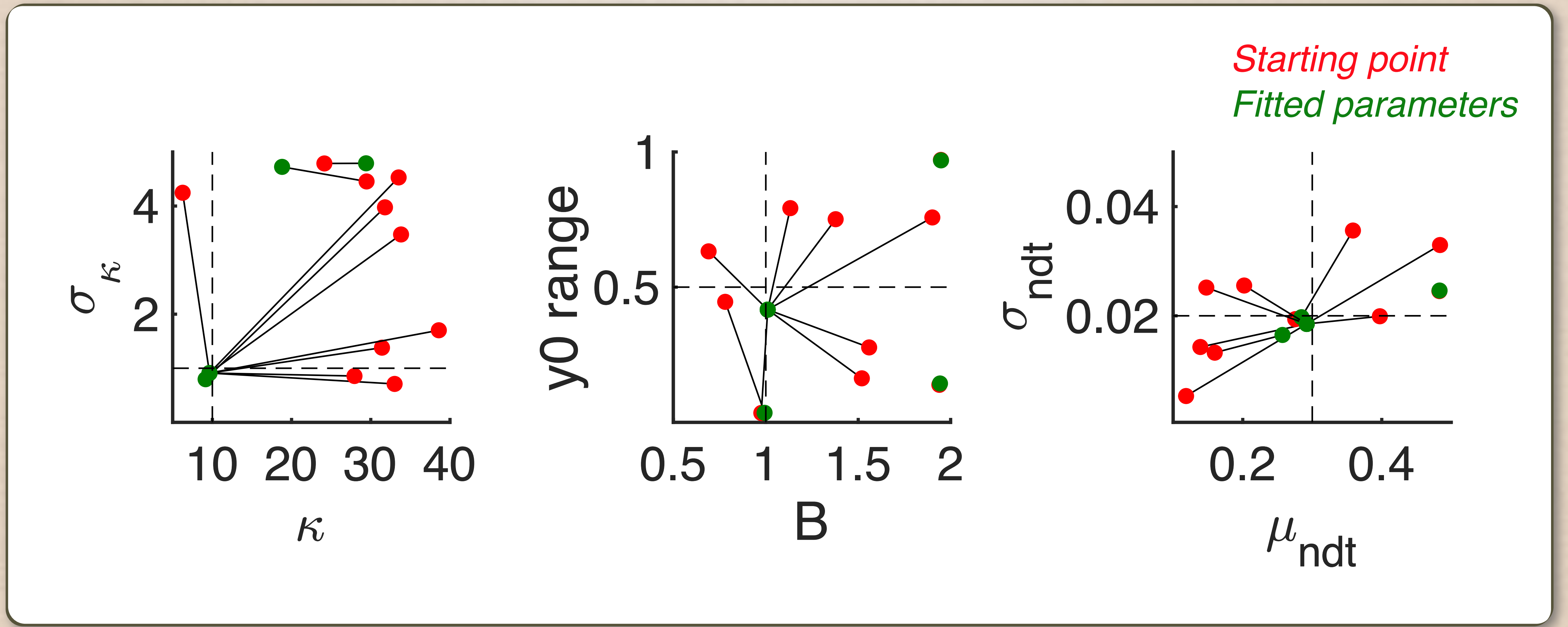
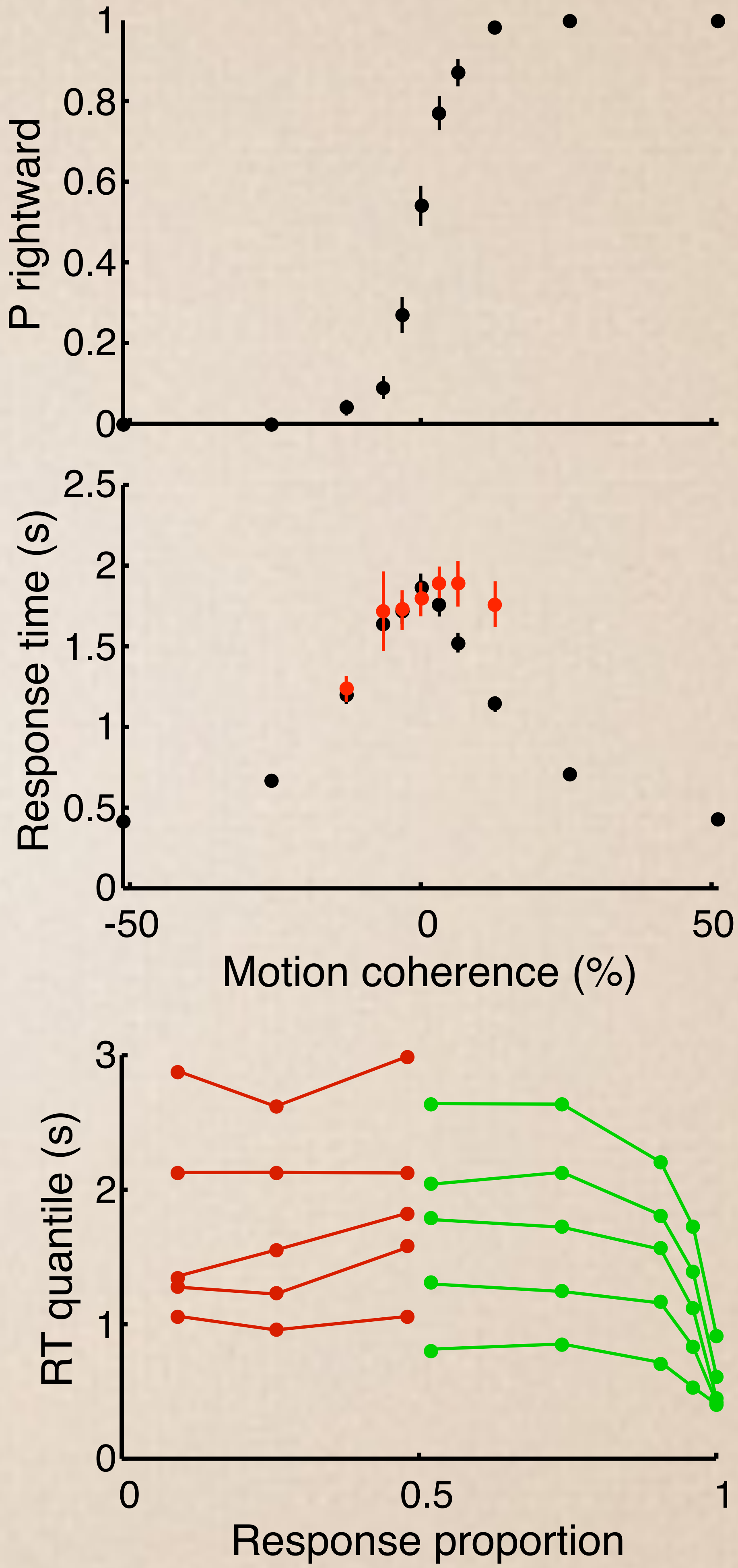


Model mimicry



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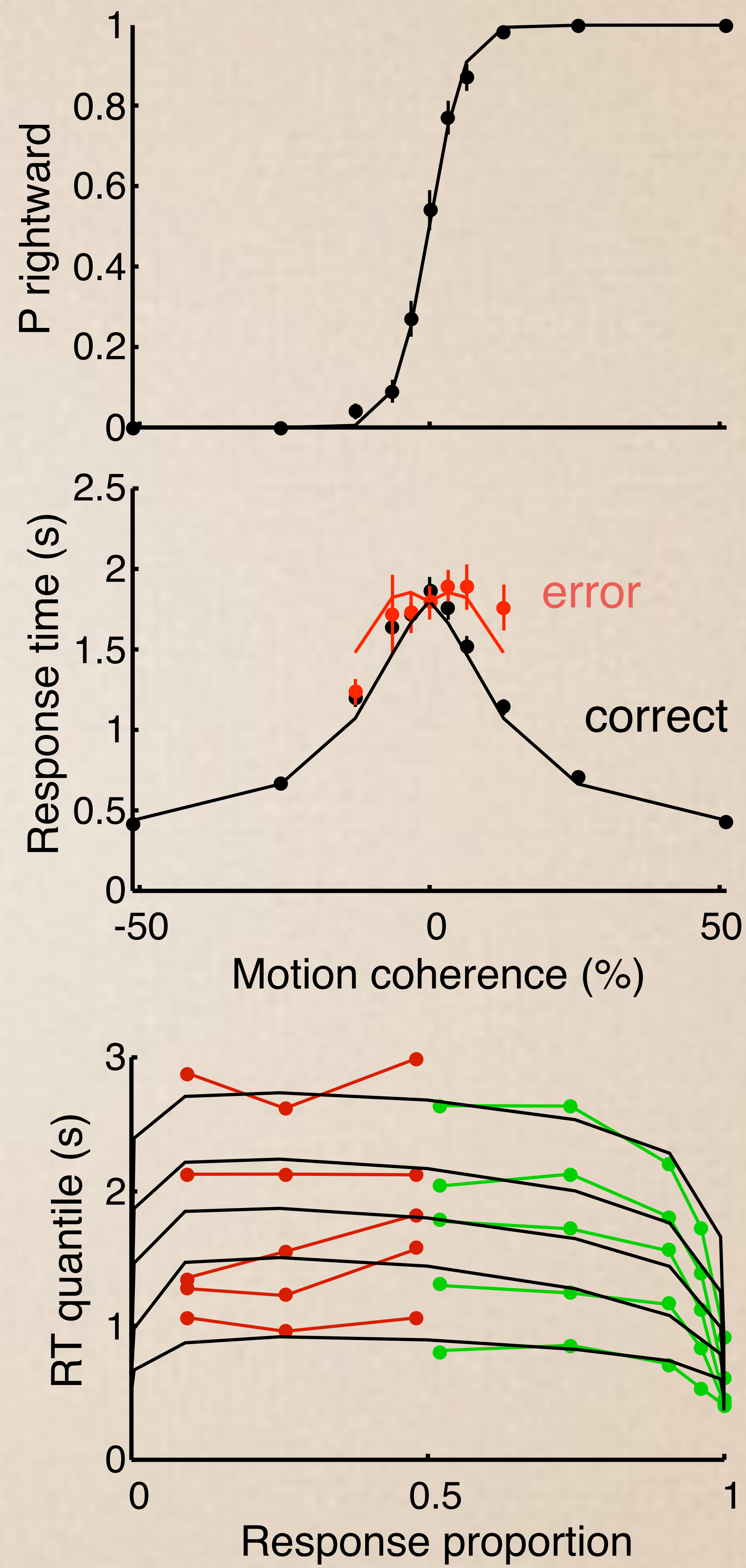
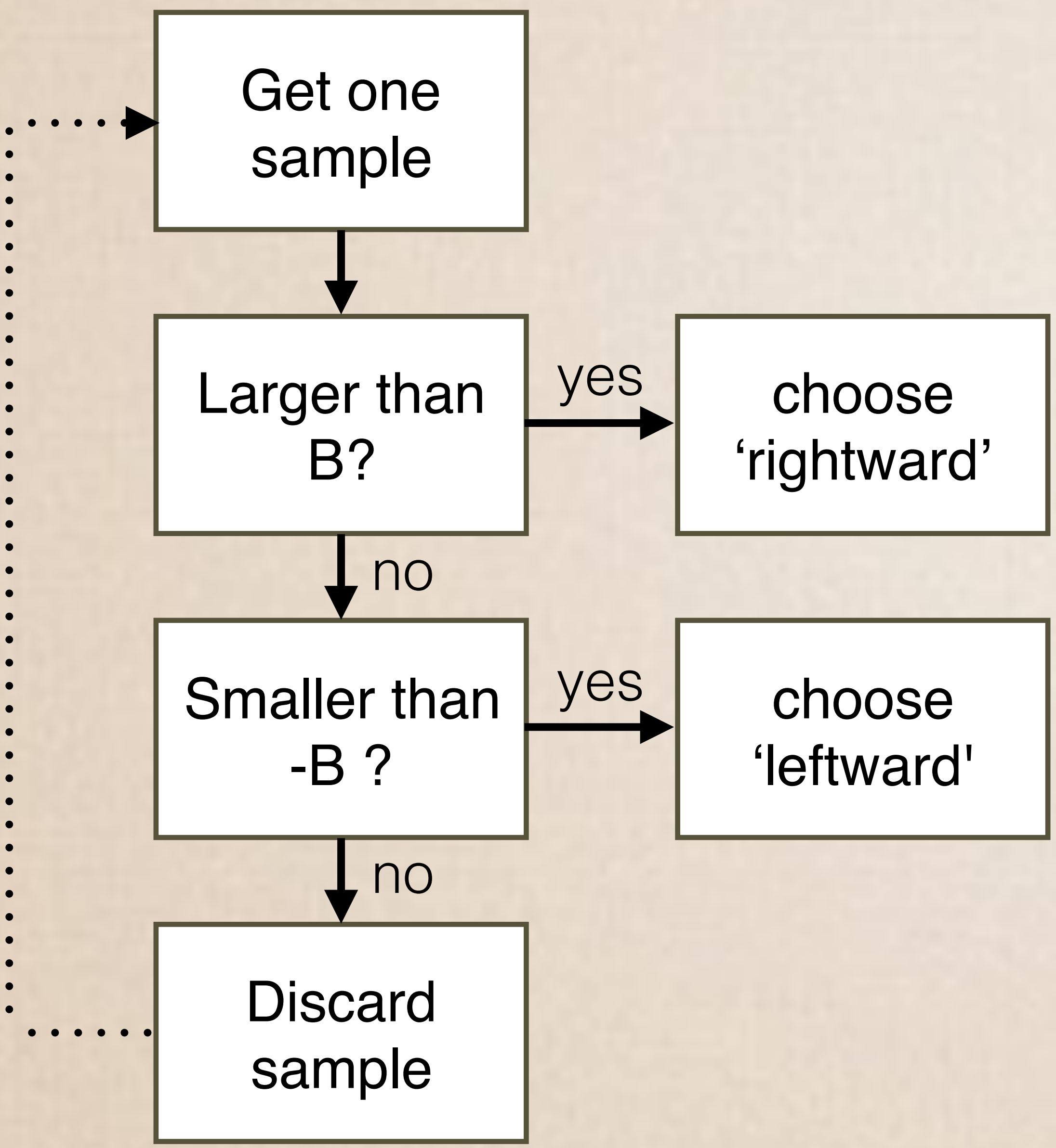
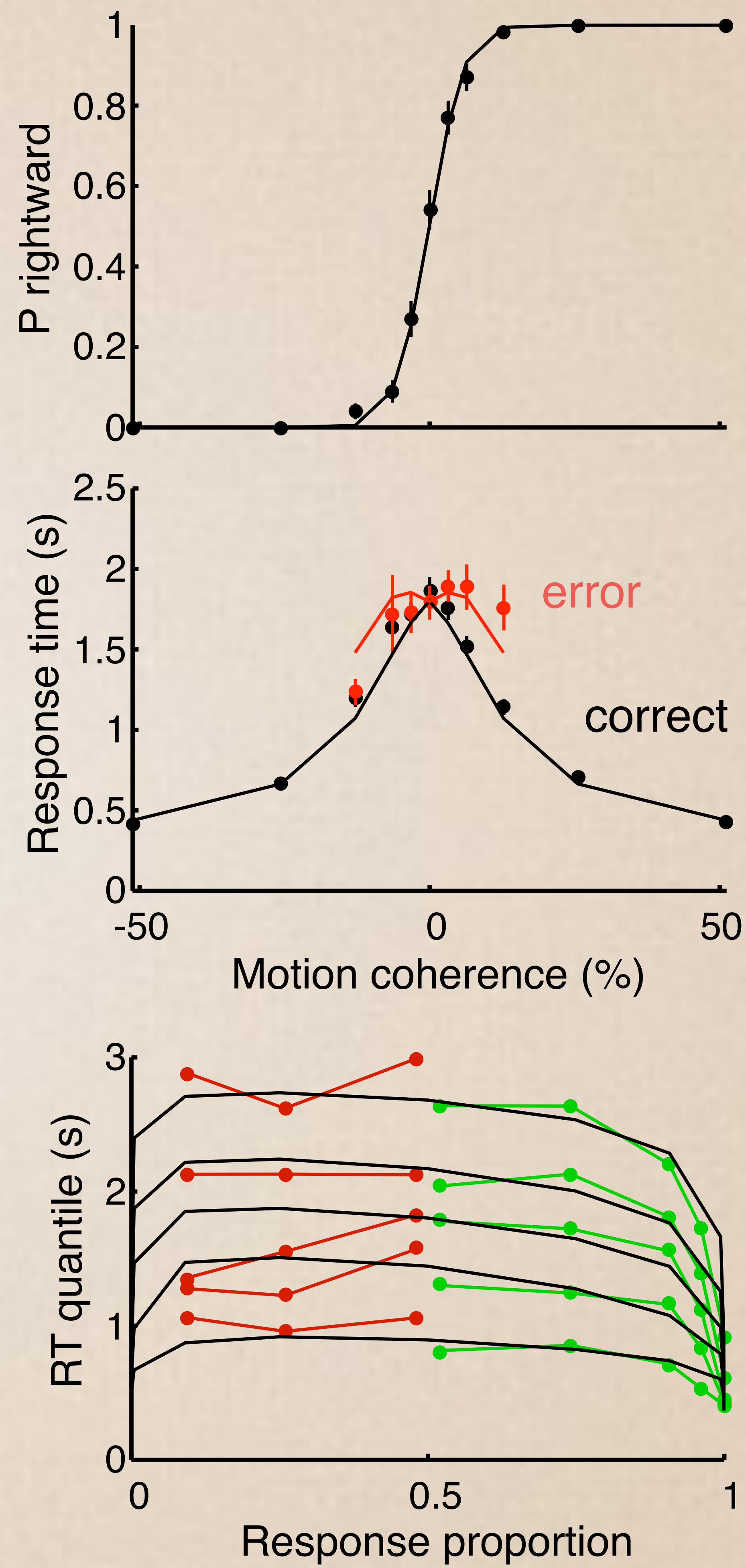


Model mimicry

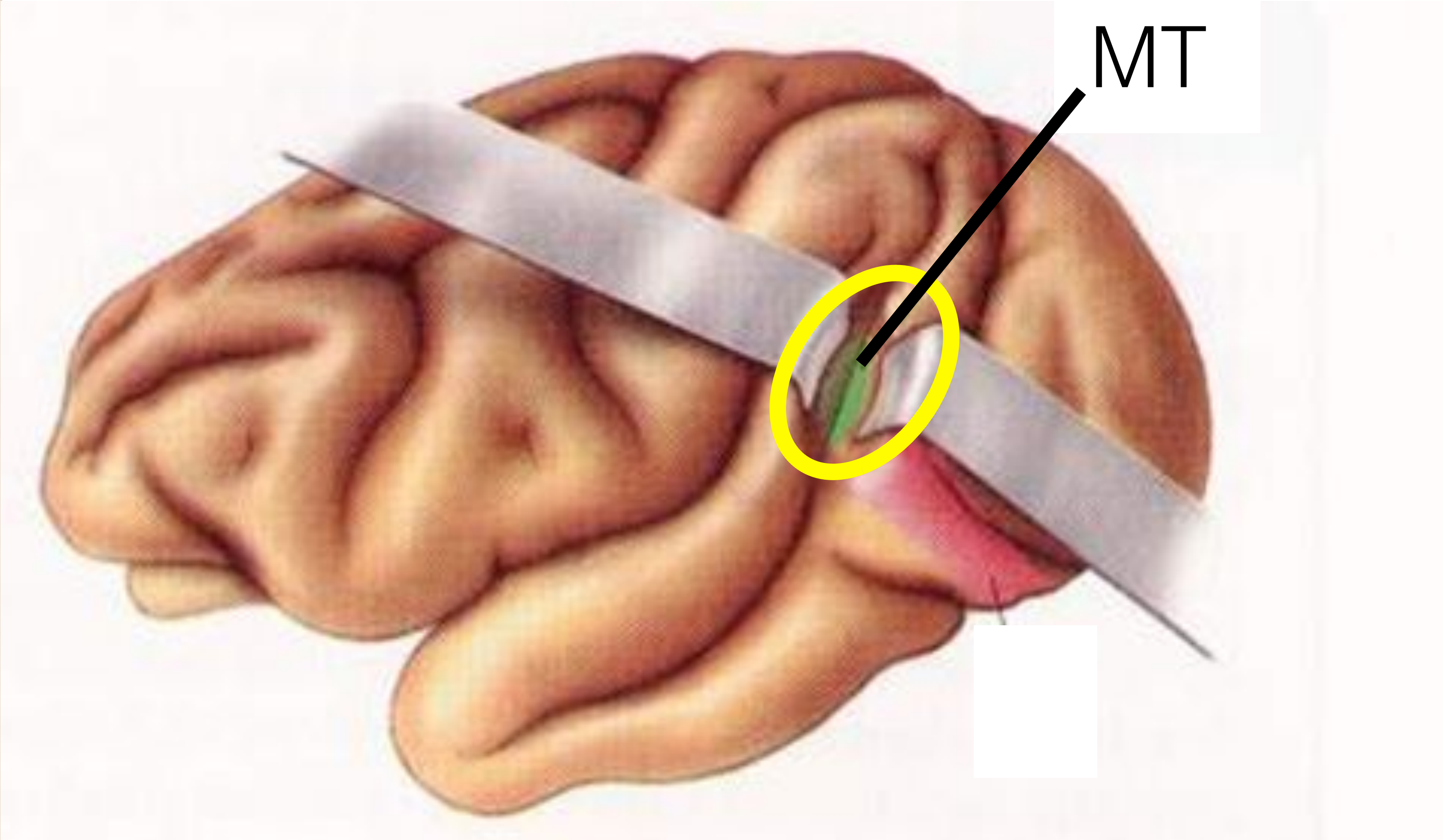
Cartwright and Festinger 1943





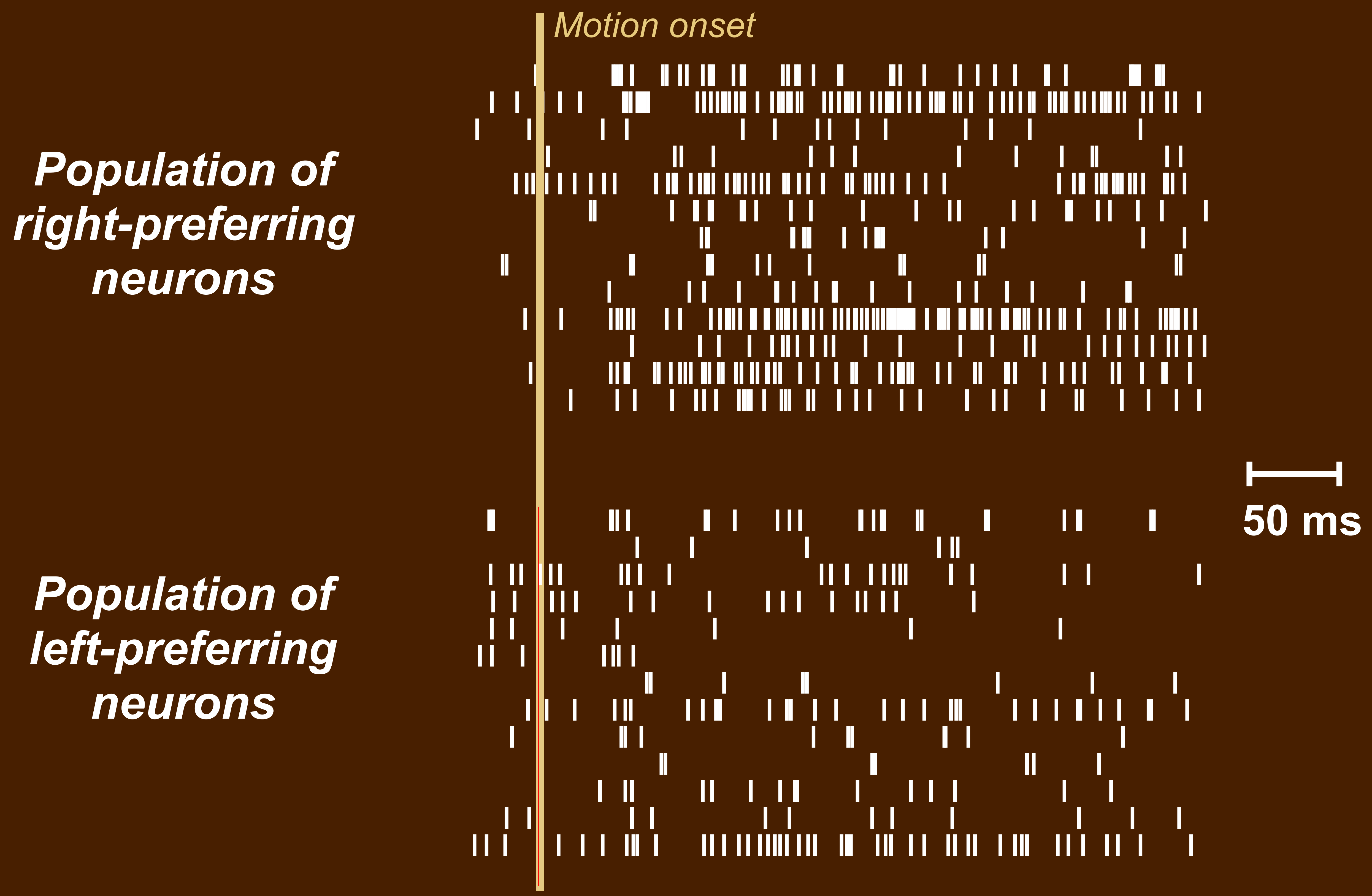
Neurophysiology

Direction-selective neurons in the visual cortex



Albright, Movshon, Newsome, Britten, …

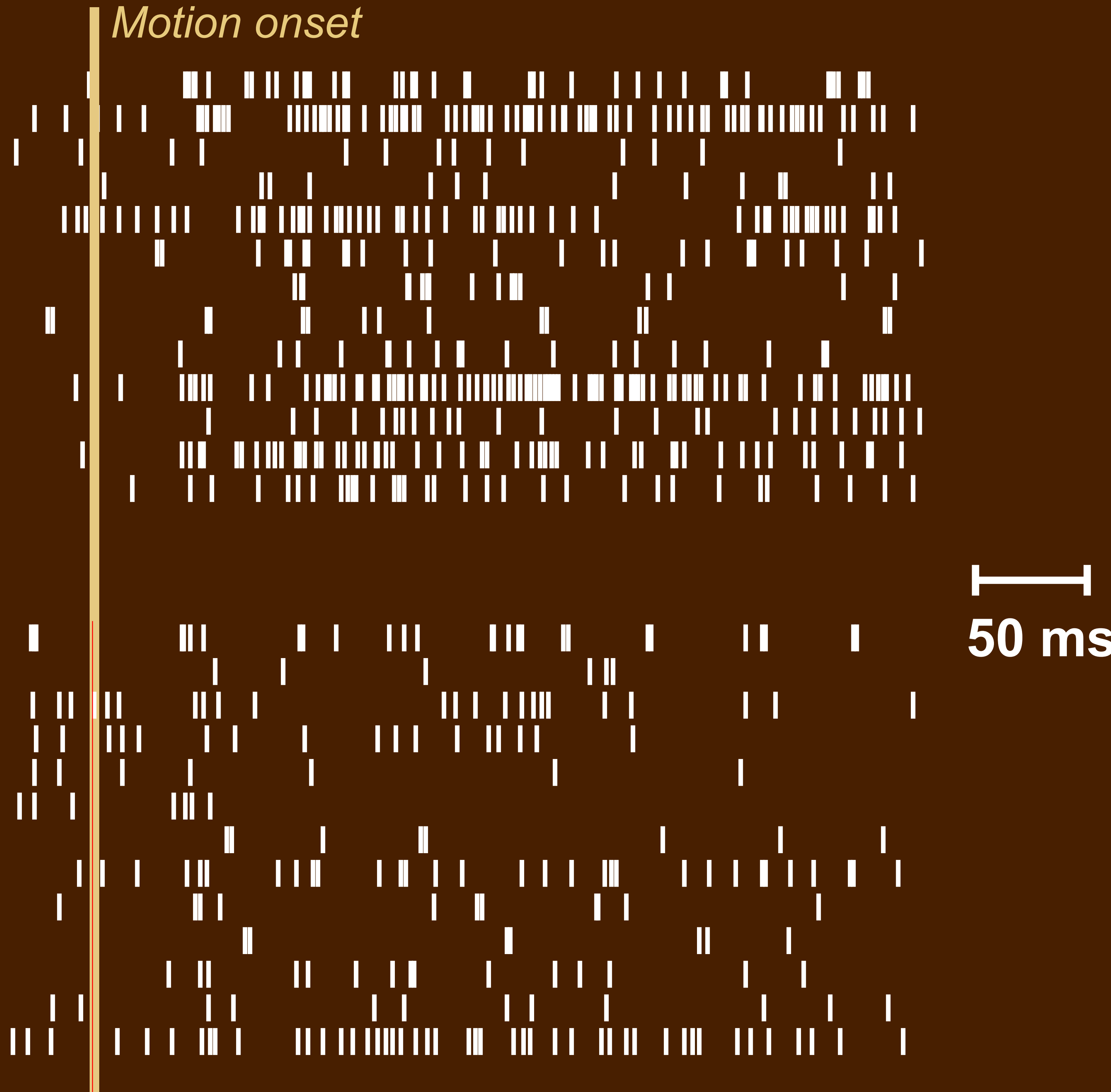
Is the motion rightward or leftward?



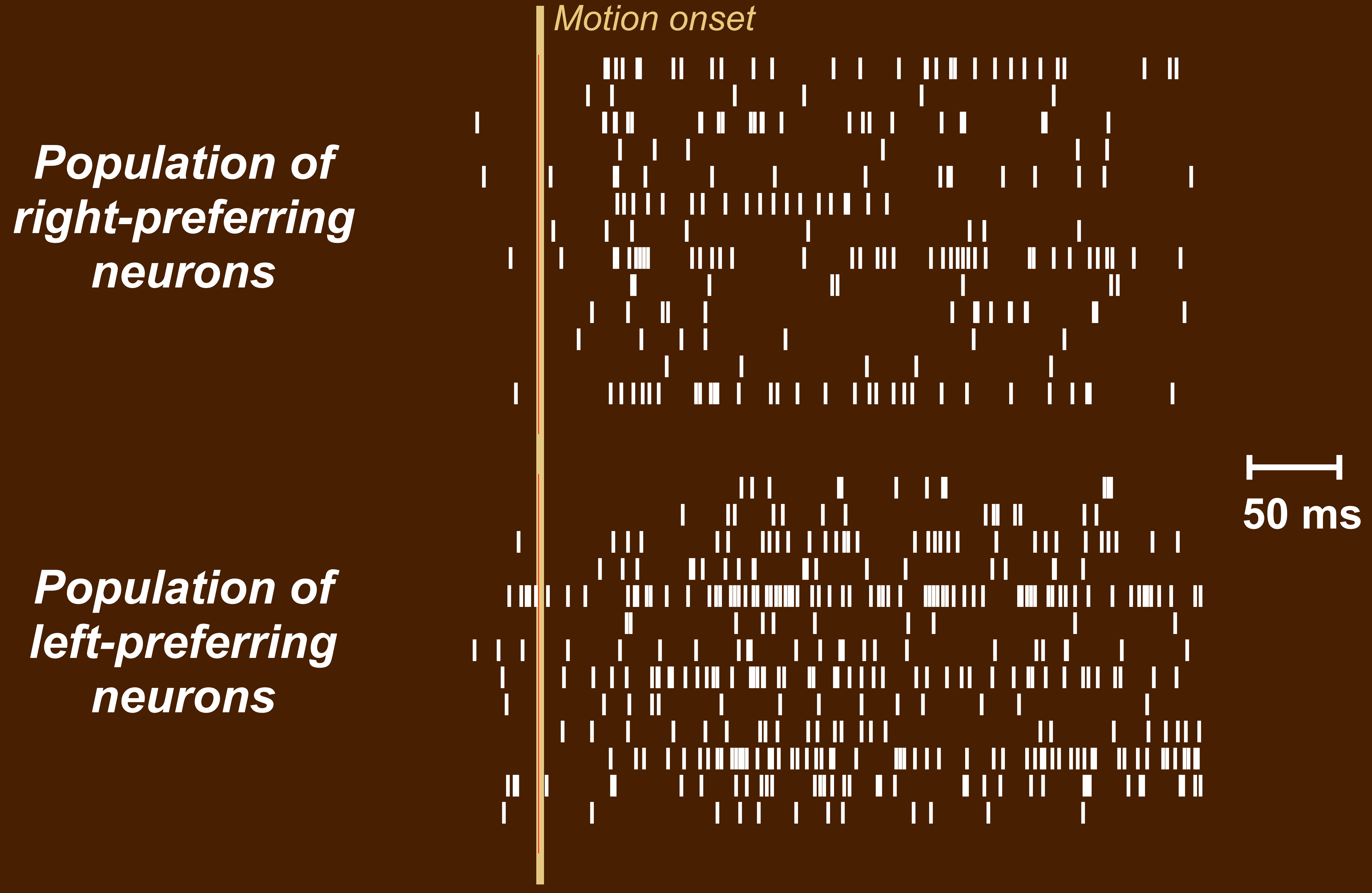
Is the motion rightward or leftward?

Population of right-preferring neurons

Population of left-preferring neurons



Is the motion rightward or leftward?



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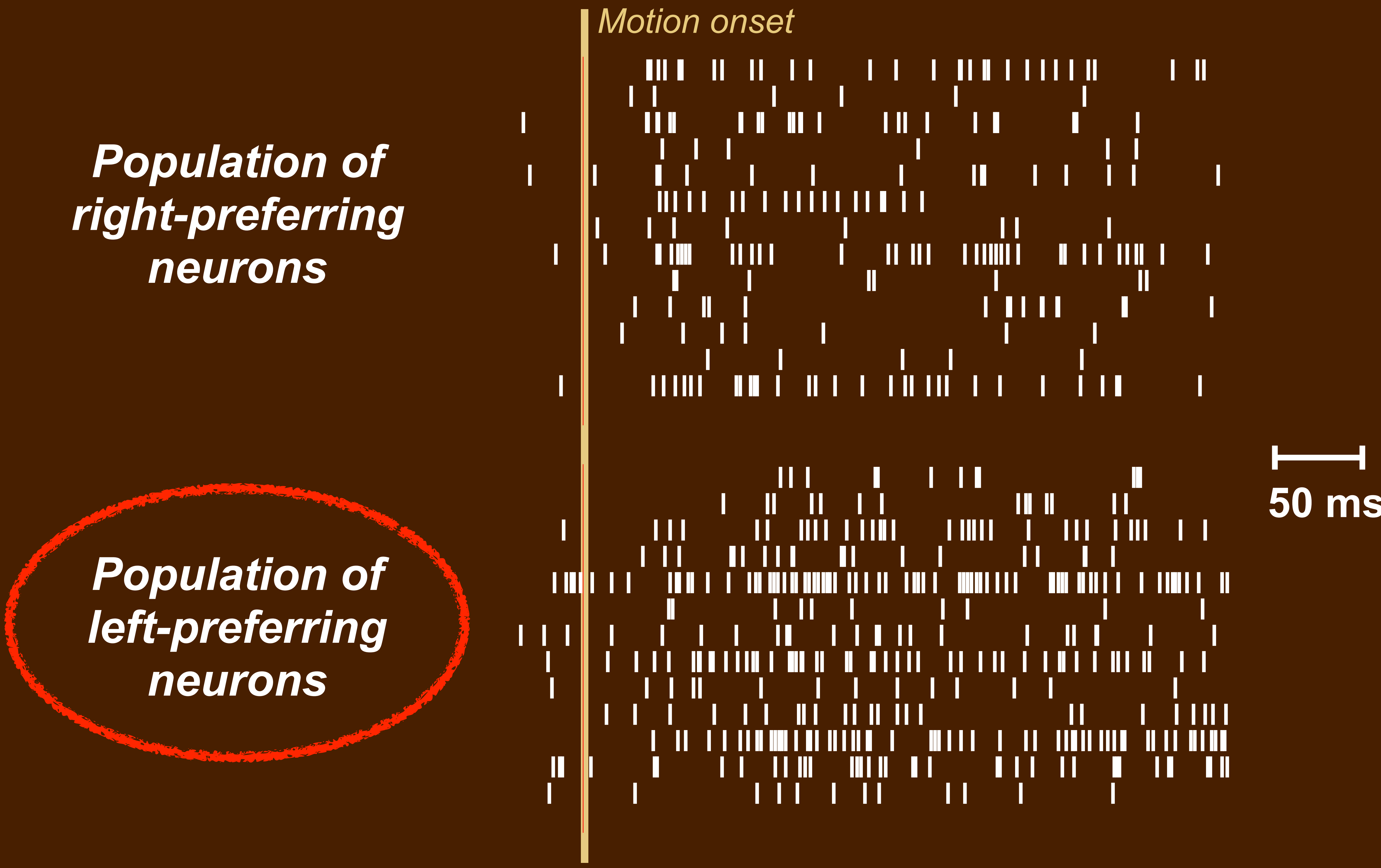


Decisions based on evidence streams

Population of right-preferring neurons

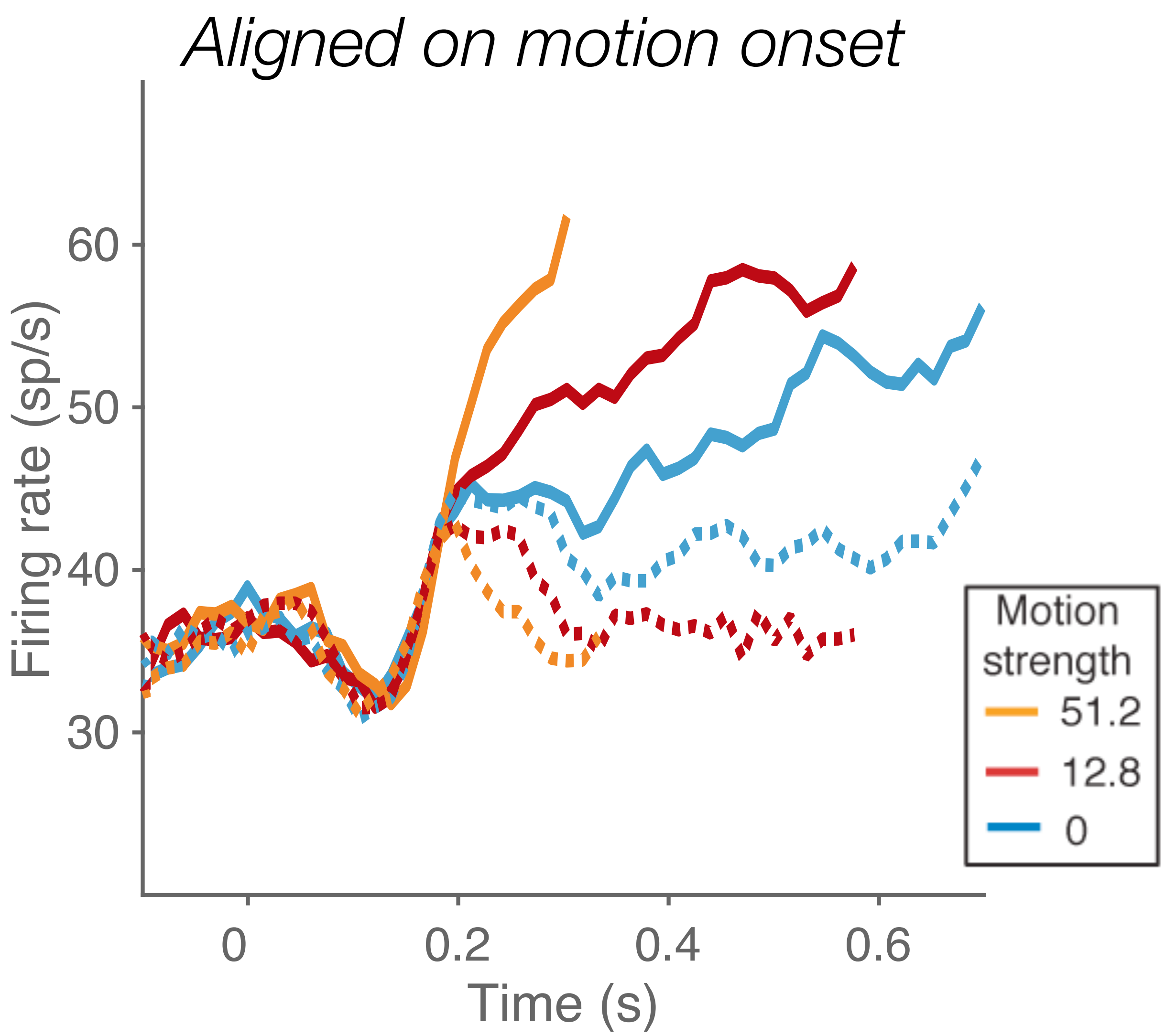
Population of left-preferring neurons

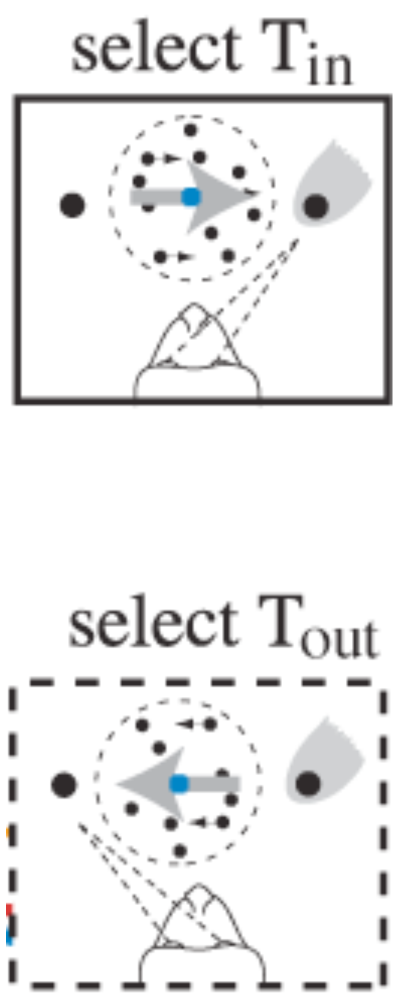
Decisions based on evidence streams

Population of right-preferring neurons

Population of left-preferring neurons

Average LIP activity in the random-dot motion task





Roitman & Shadlen, 2002 J. Neurosci.

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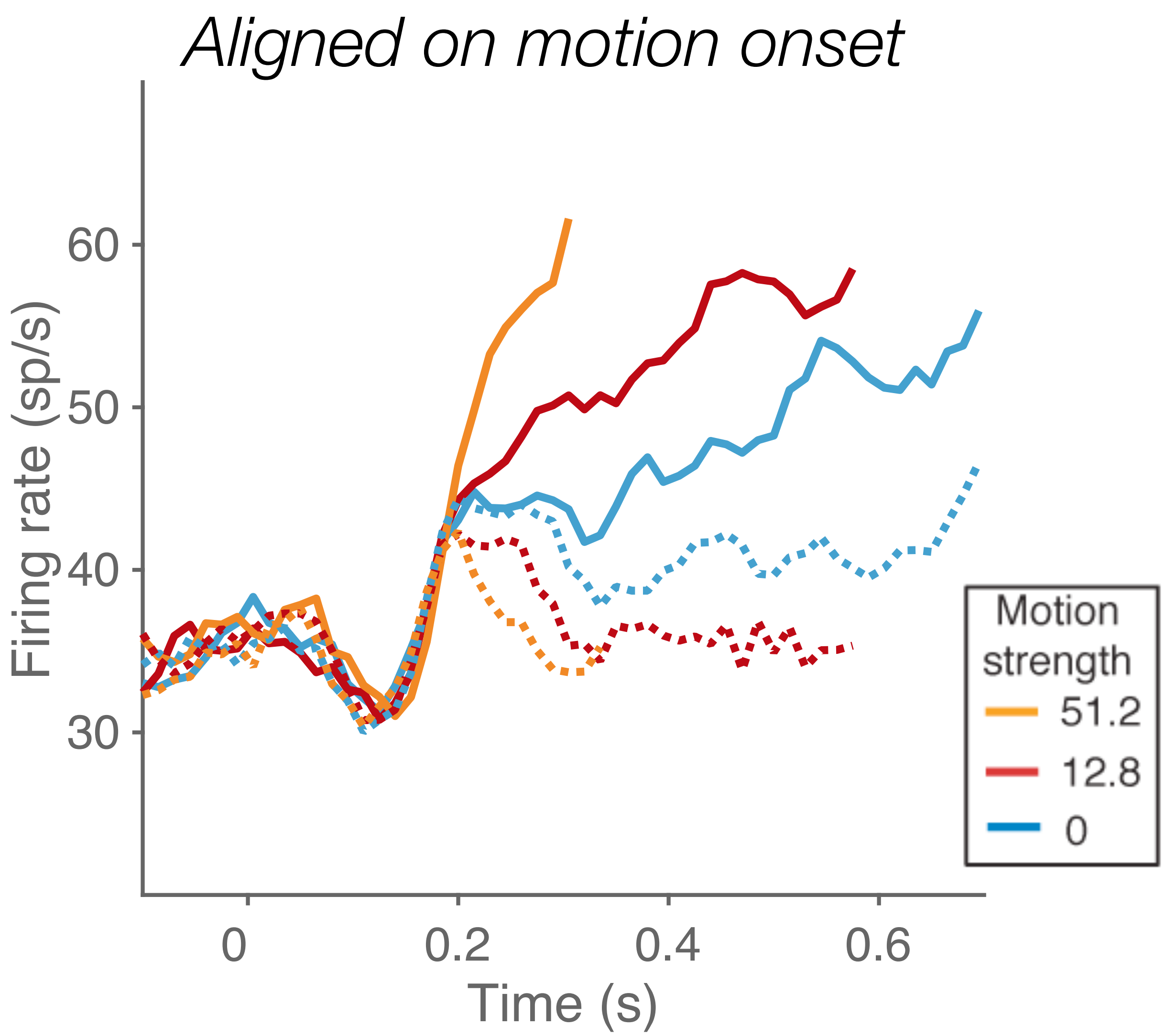

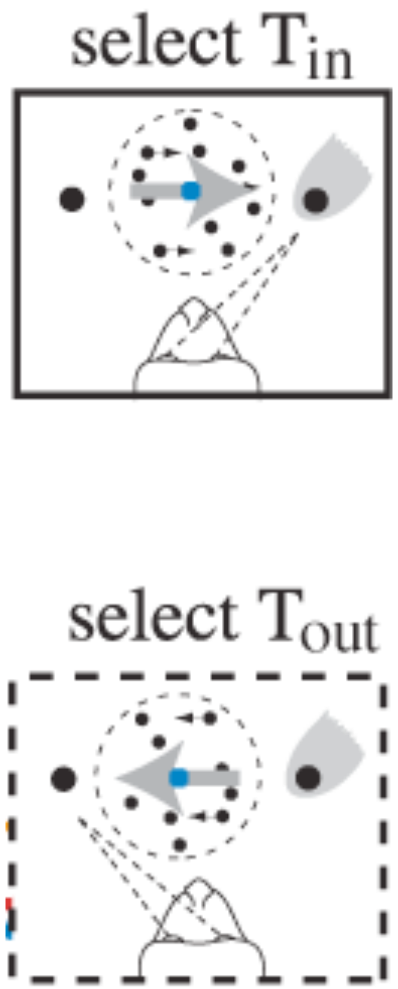

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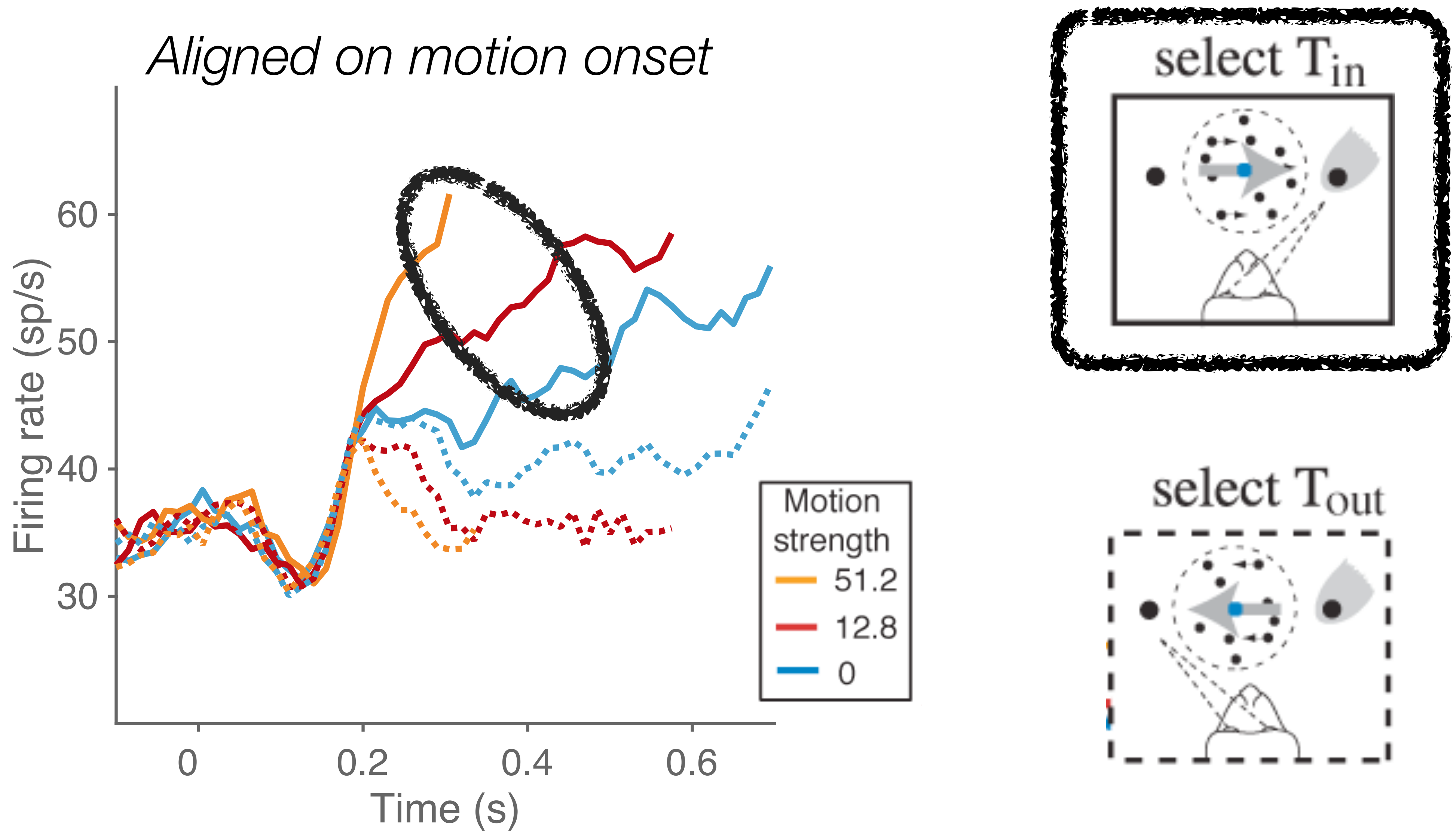

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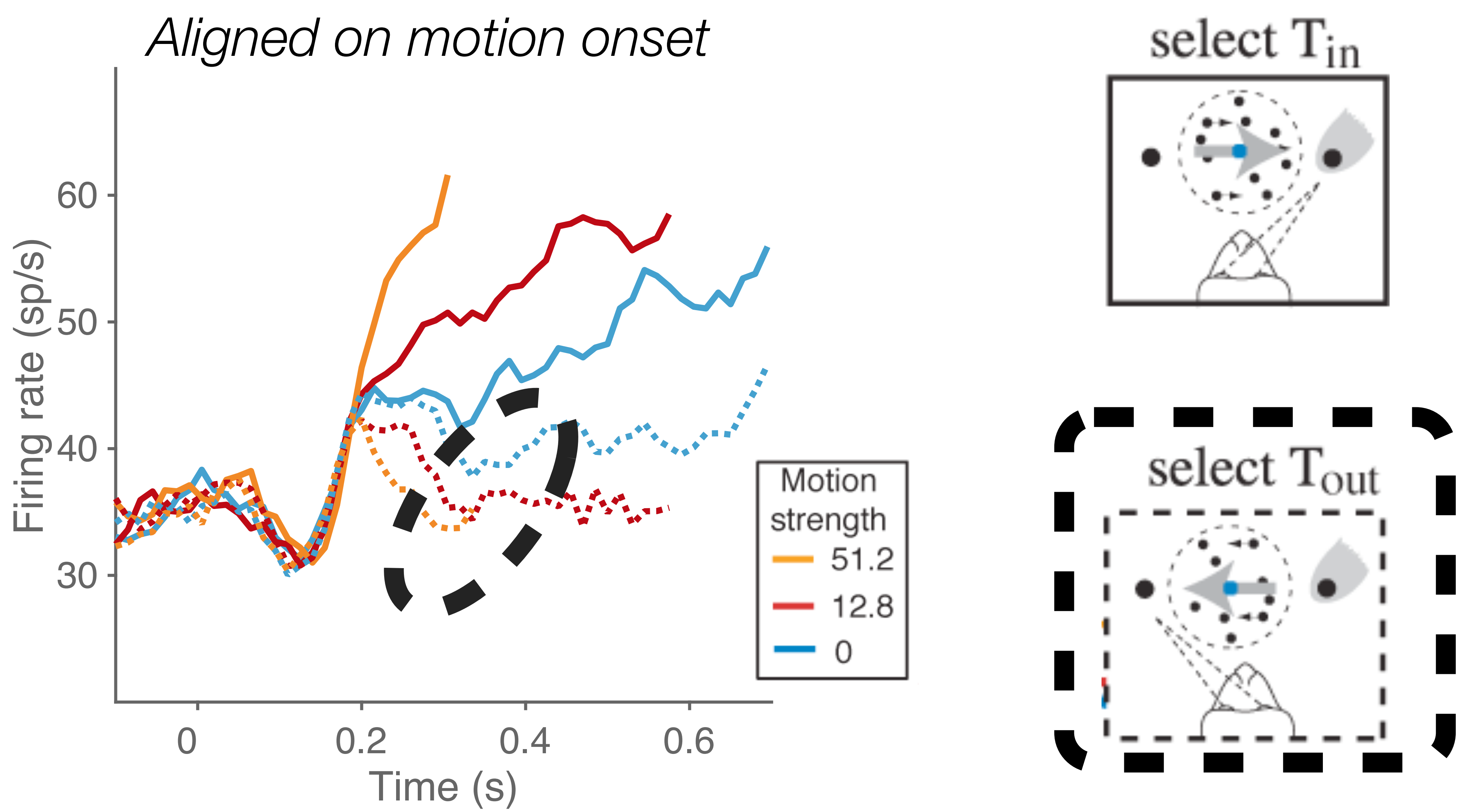

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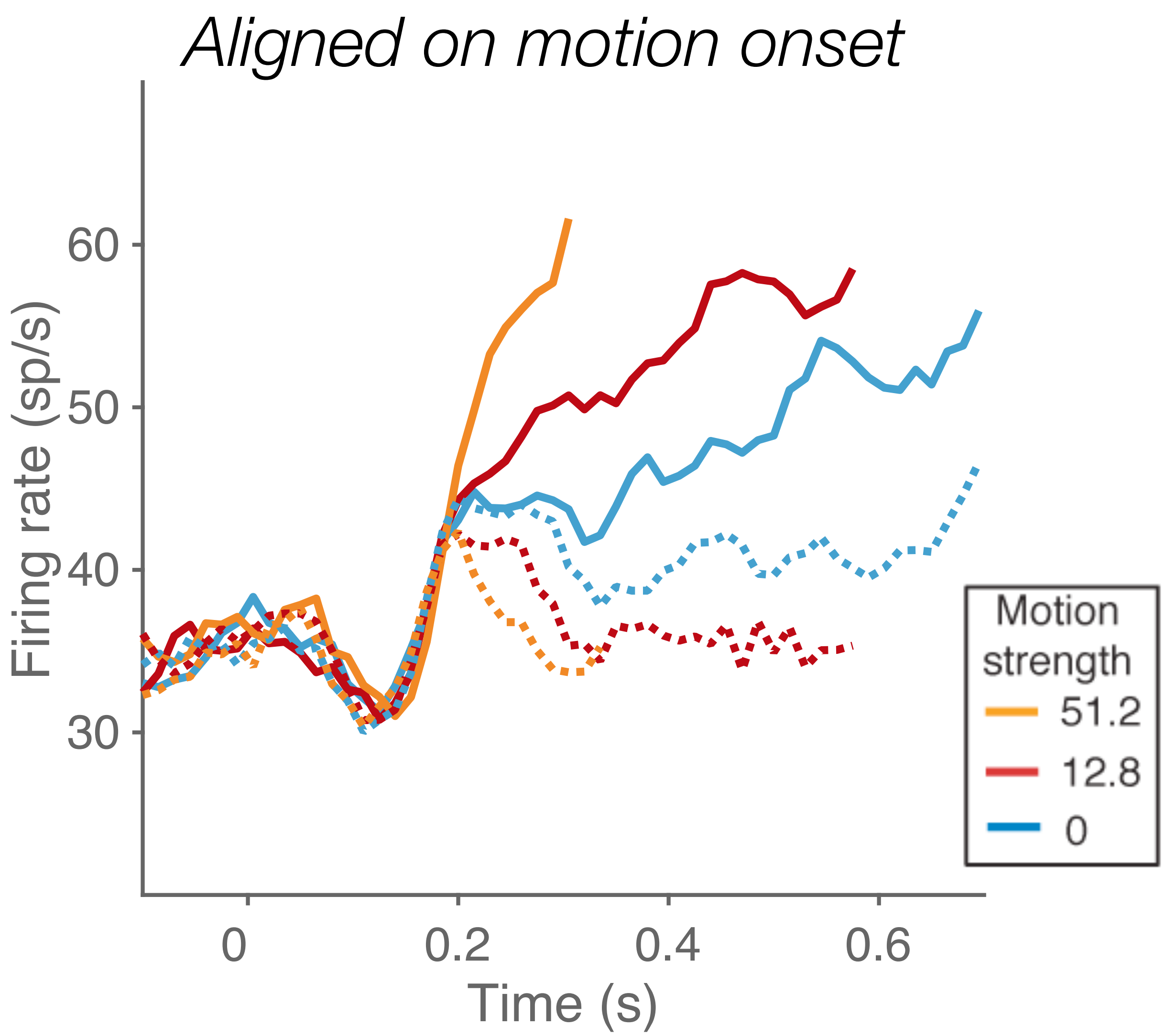

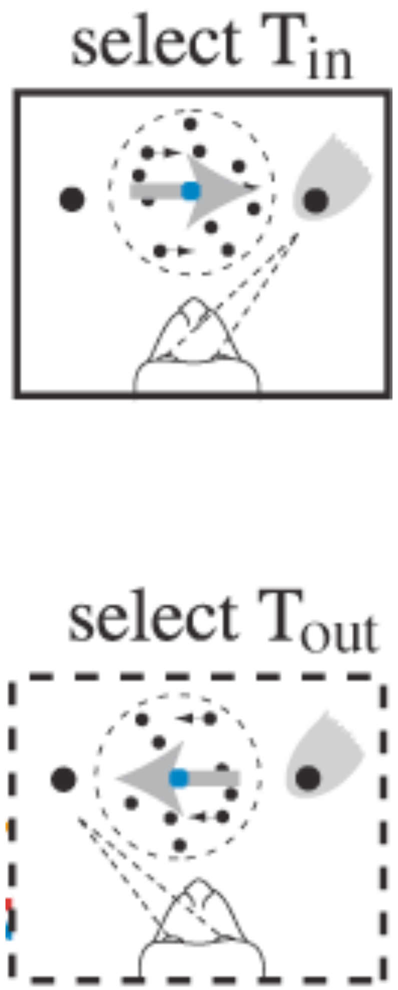

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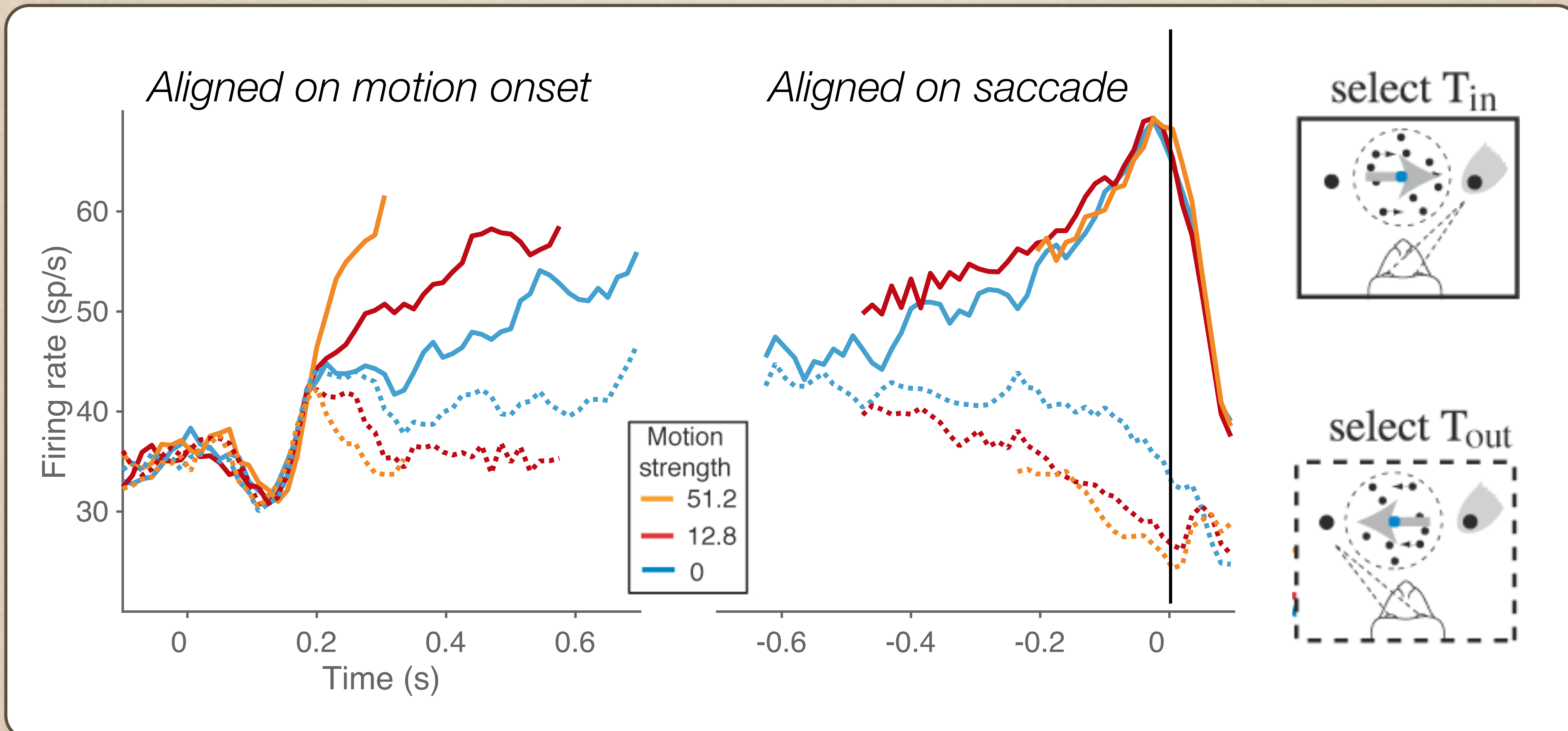

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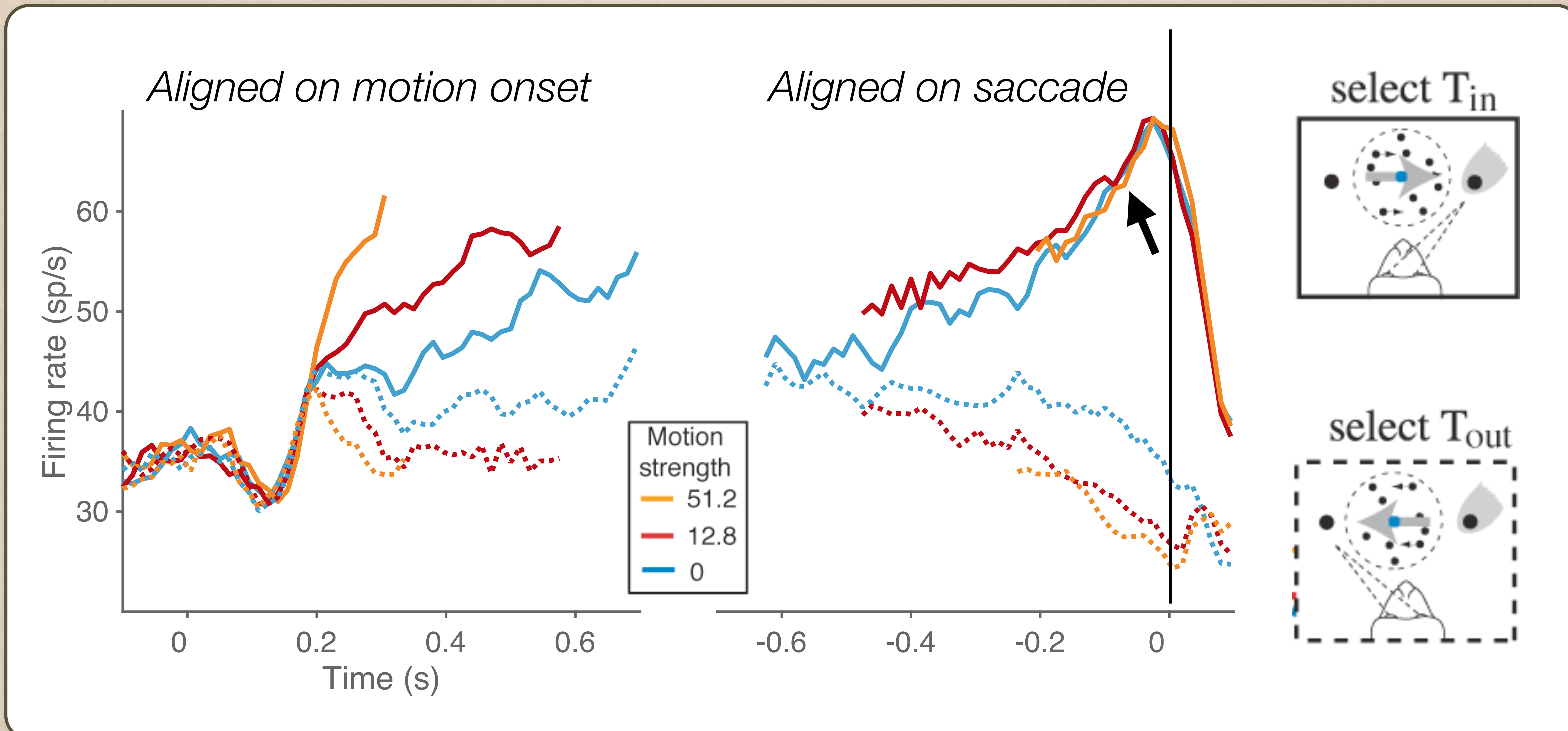

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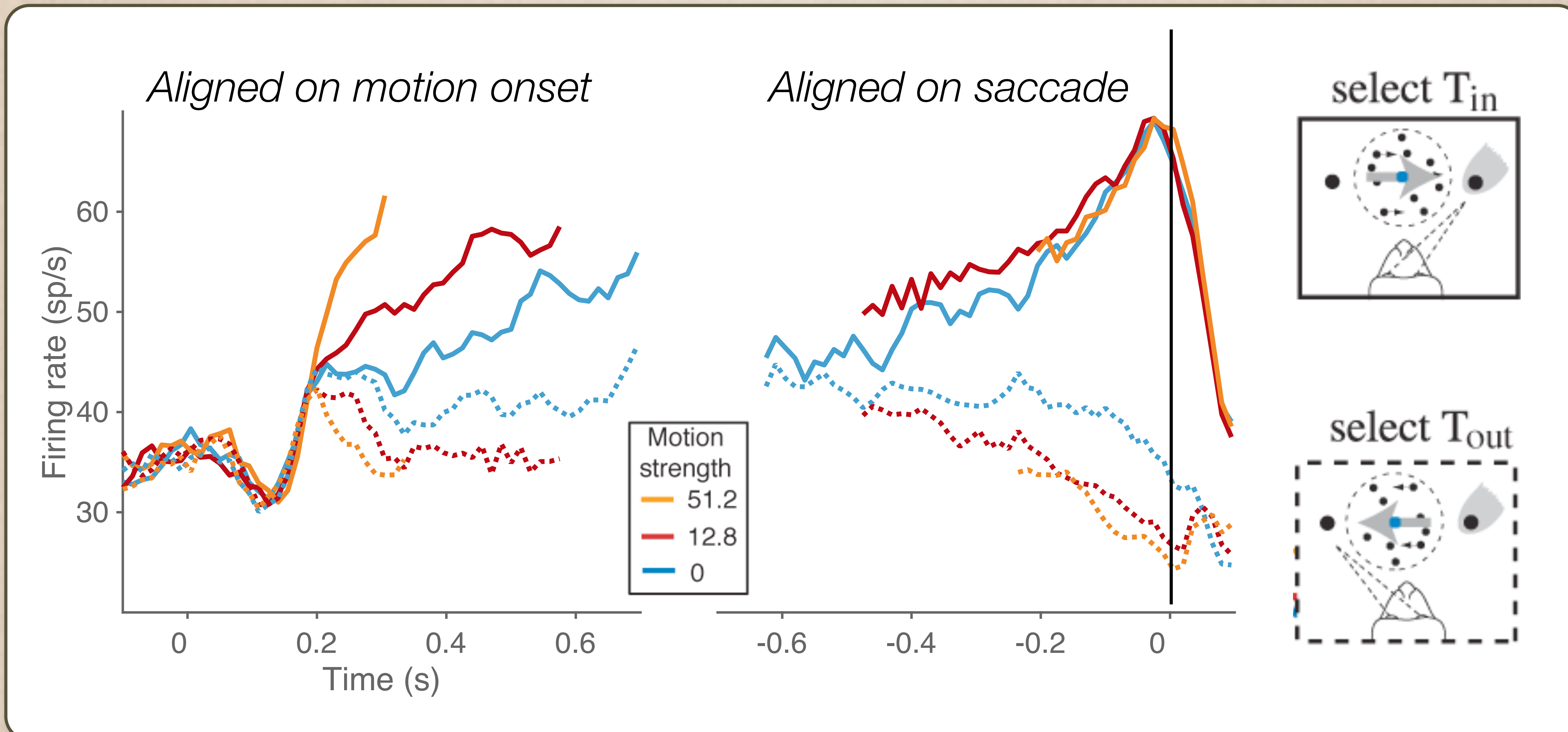

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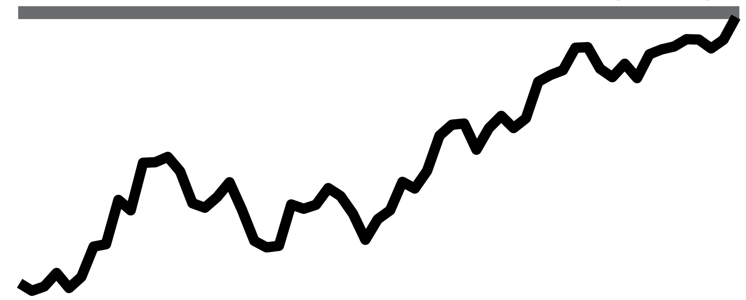
Competing races

$$e_{right} - e_{left} \xrightarrow{\int dt}$$

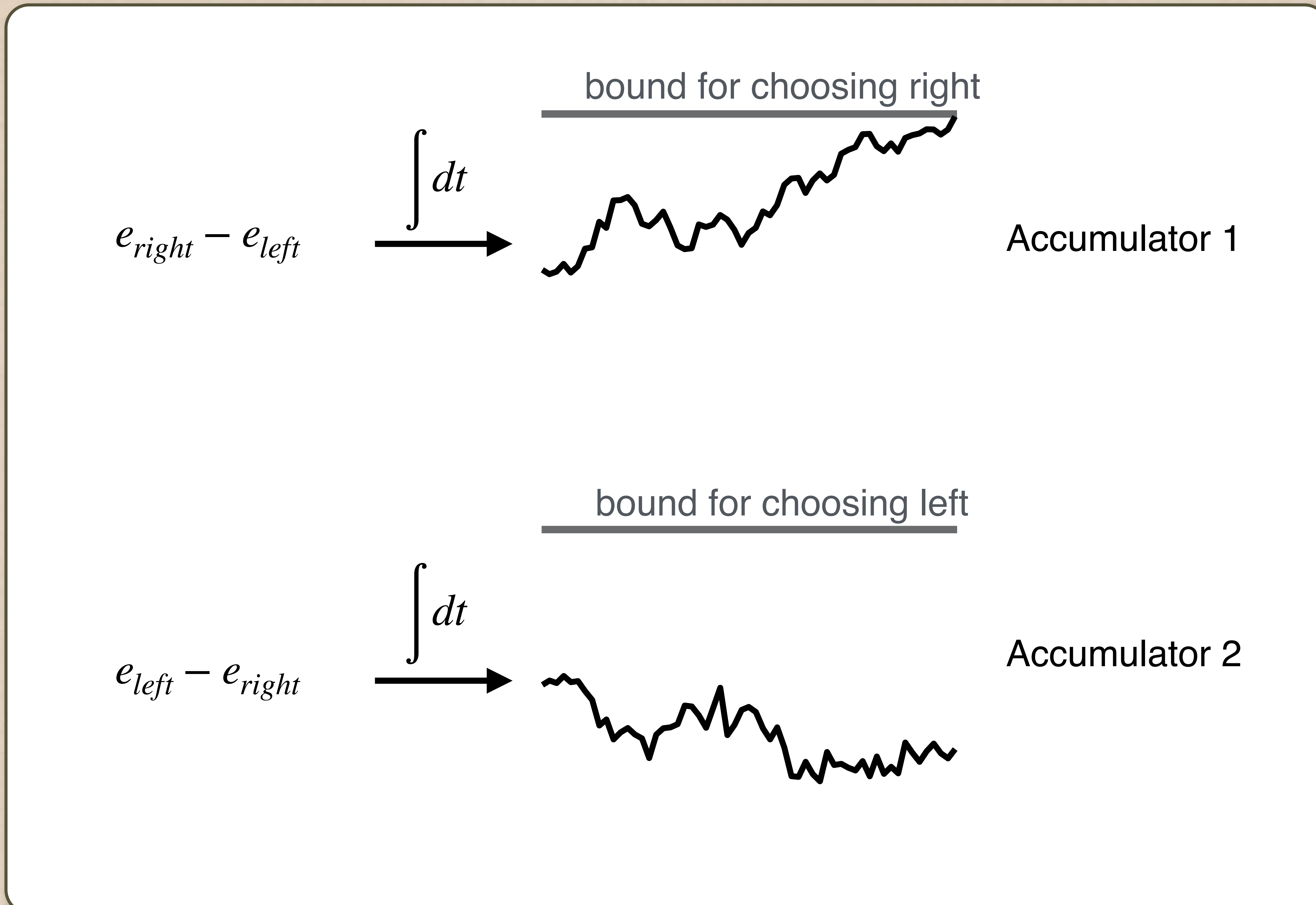
bound for choosing right



Accumulator 1

e.g., Usher and McClelland 2001, Kiani & Shadlen 2014

Competing races



e.g., Usher and McClelland 2001, Kiani & Shadlen 2014

Final summary

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Neurophysiology

- Neural responses from areas MT and LIP

Thank you!

Suggested reviews:

Gold and Shadlen, Neuron 2002,
"Banburismus and the Brain: Decoding Review the Relationship between Sensory Stimuli, Decisions, and Reward “

Ratcliff, Smith, Brown and McKoon, TiCS 2016,
“Diffusion Decision Model: Current Issues and History”

extra slides

Attempt to disambiguate between these alternatives using model comparison

Hawkings et al. (JNeurosci. 2015) “Revisiting the Evidence for Collapsing Boundaries and Urgency Signals in Perceptual Decision-Making”.