



Active inference

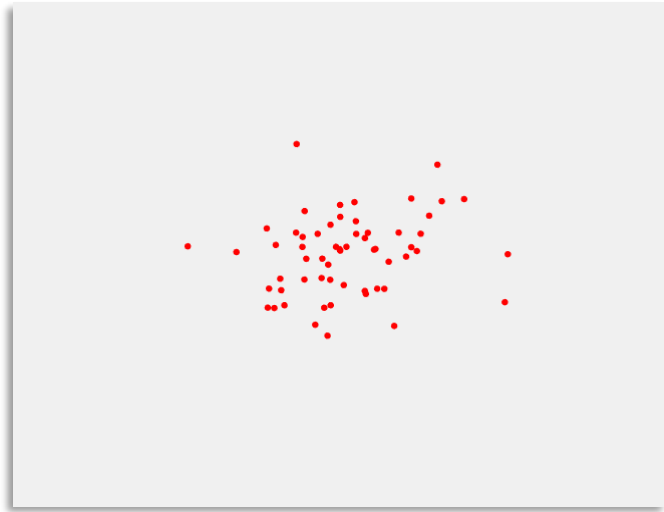
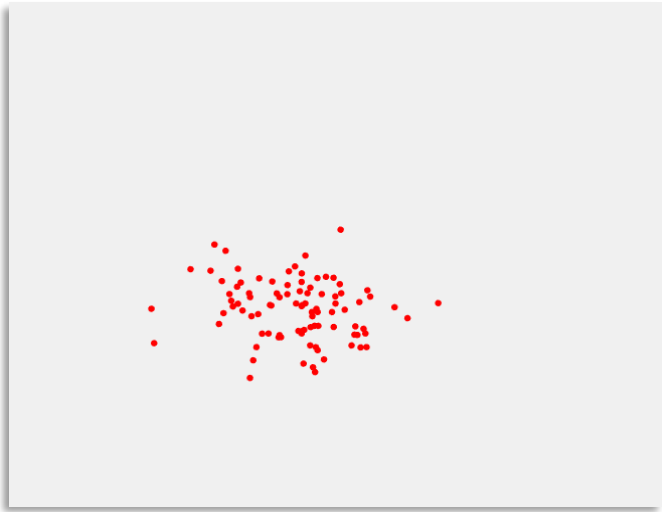
Computational psychiatry course

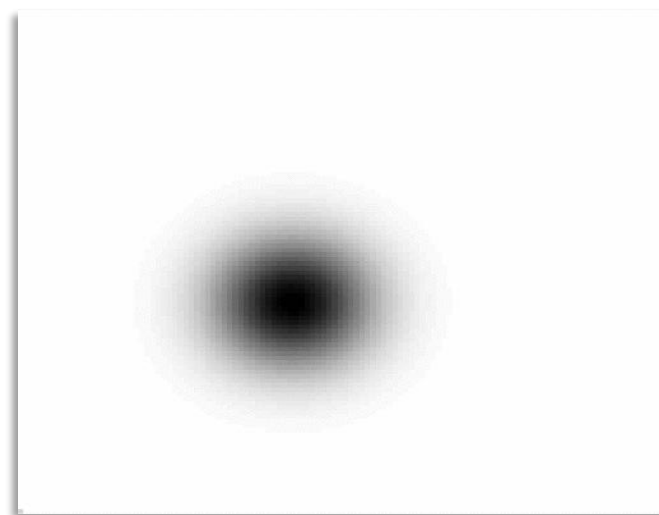
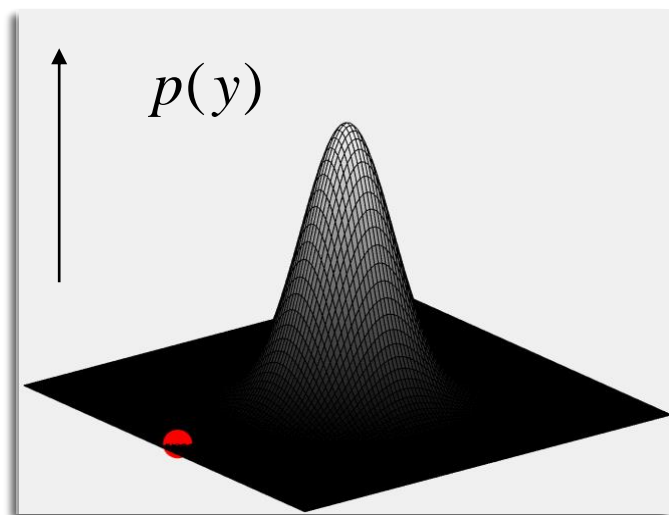


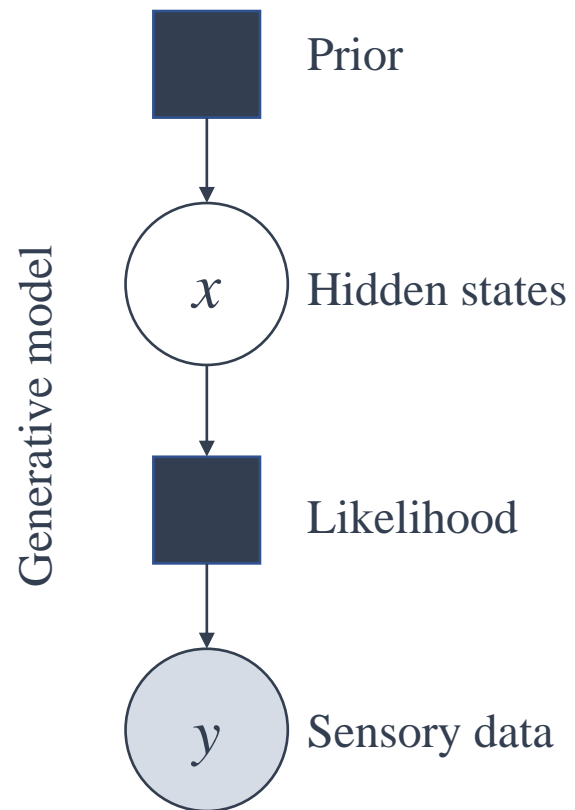
Active inference
Generative models
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Movement
Hierarchy



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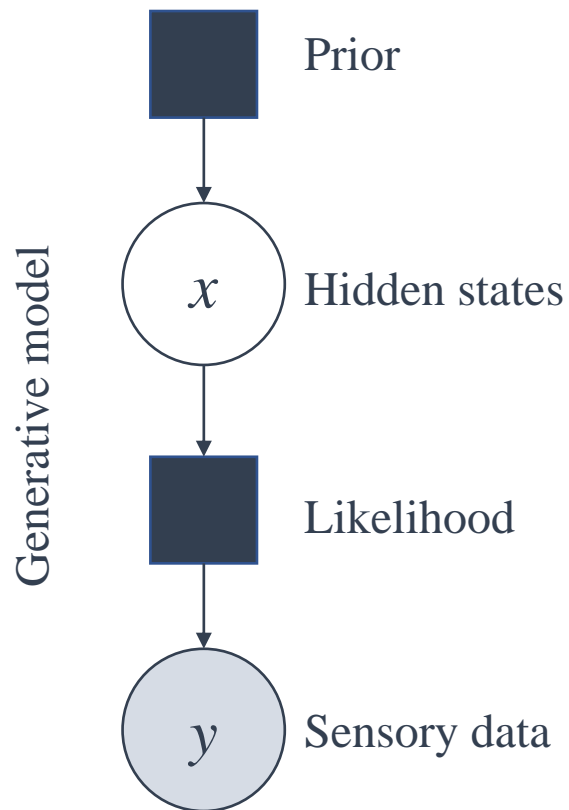






Model evidence

$$\ln p(y)$$



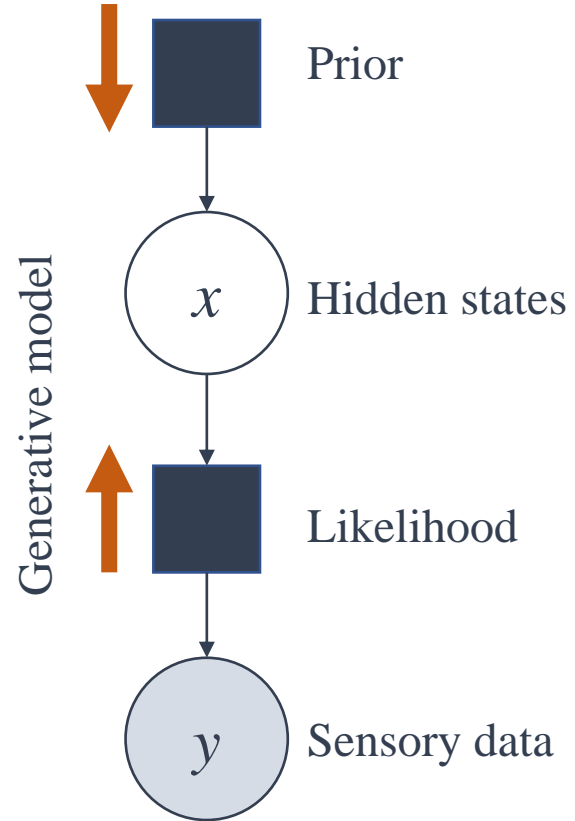
Free energy

$$\begin{aligned} -F(y) &= E_q[\ln p(x, y)] - E_q[\ln q(x)] \\ &= \ln p(y) + \underbrace{E_q[\ln p(x | y) - \ln q(x)]}_{\leq 0} \end{aligned}$$

Active inference

$$q(x) = \arg \min_q F[q, y]$$

$$a = \arg \min_a F[q, y(a)]$$



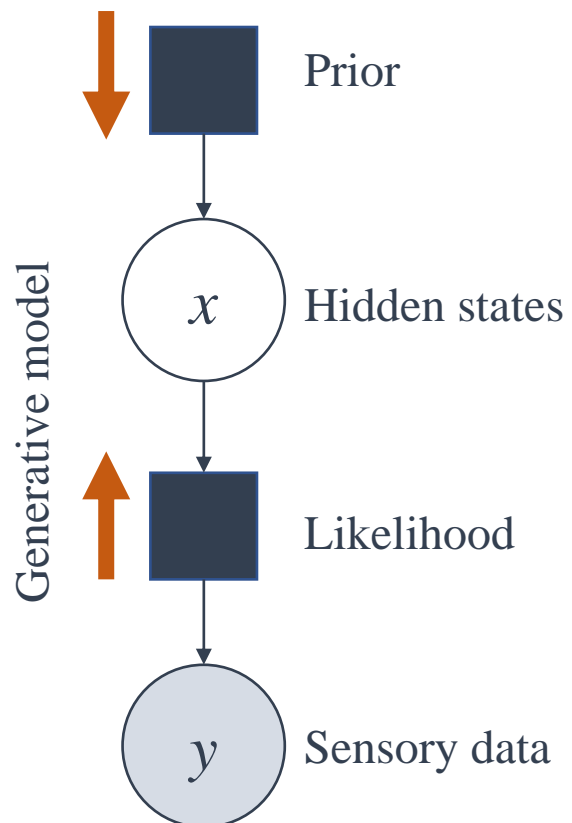
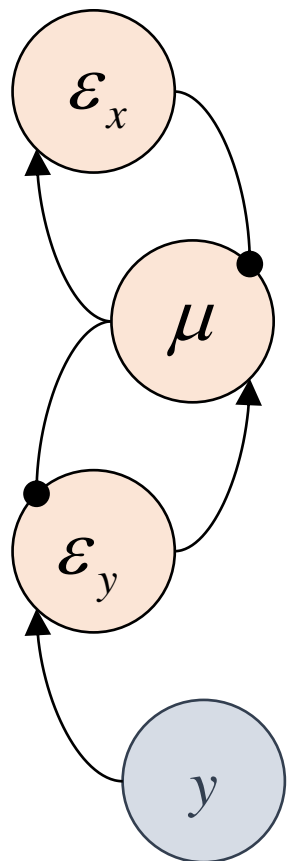
Free energy

$$\begin{aligned} -F(y) &= E_q[\ln p(x, y)] - E_q[\ln q(x)] \\ &= E_q[\ln p(y | x)] + E_q[\ln p(x)] \\ &\quad - E_q[\ln q(x)] \end{aligned}$$

Message passing

$$\delta_q F = 0 \Leftrightarrow$$

$$\ln q(x) = \ln p(y | x) + \ln p(x) + \text{const.}$$



Free energy

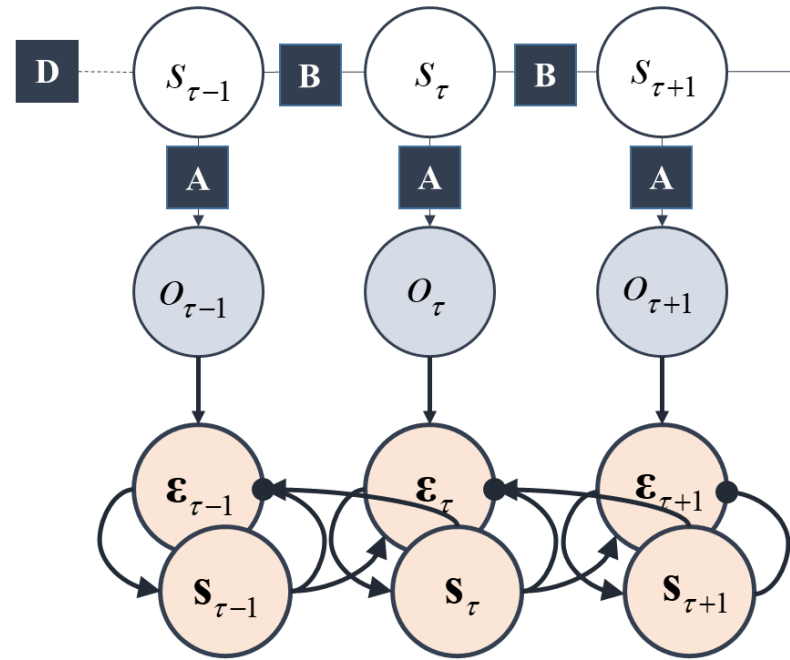
$$\begin{aligned}
 -F(y) &= E_q[\ln p(x, y)] - E_q[\ln q(x)] \\
 &= E_q[\ln p(y | x)] + E_q[\ln p(x)] \\
 &\quad - E_q[\ln q(x)]
 \end{aligned}$$

Message passing

$$\begin{aligned}
 \dot{\mu} &= -\nabla_{\mu} F \\
 &= \Pi_y \varepsilon_y - \Pi_x \varepsilon_x
 \end{aligned}$$



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Generative model

$$P(\tilde{o}, \tilde{s}) = P(s_1) \prod_{\tau} P(s_{\tau+1} | s_{\tau}) P(o_{\tau} | s_{\tau})$$

$$P(o_{\tau} | s_{\tau}) = \text{Cat}(\mathbf{A})$$

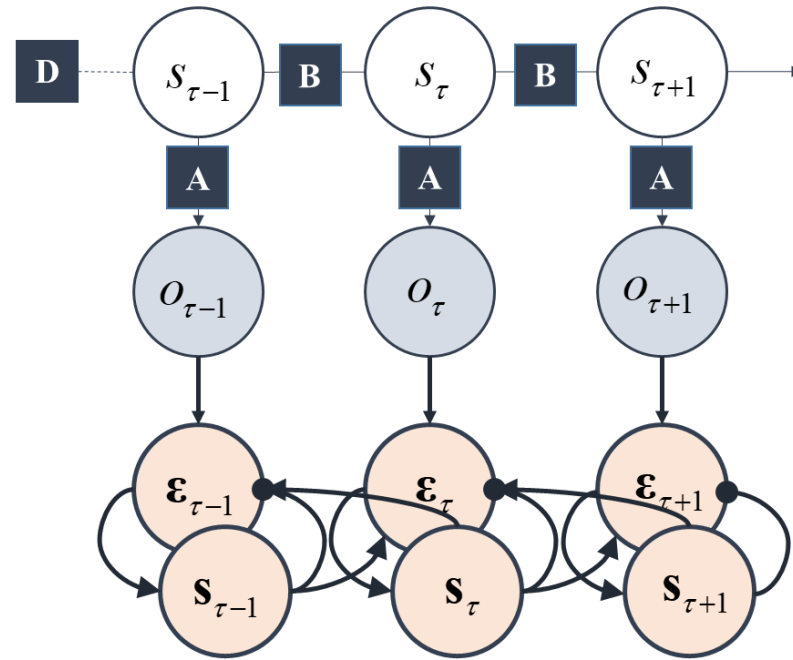
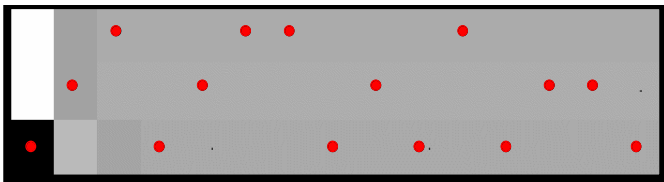
$$P(s_{\tau} | s_{\tau-1}) = \text{Cat}(\mathbf{B})$$

$$P(s_1) = \text{Cat}(\mathbf{D})$$

Bayesian message passing

$$\mathbf{s}_{\tau} = \sigma(\mathbf{v}_{\tau}); \quad \dot{\mathbf{v}}_{\tau} = \boldsymbol{\epsilon}_{\tau}$$

$$\boldsymbol{\epsilon}_{\tau} = \ln \mathbf{A} \cdot o_{\tau} + \frac{1}{2} \ln(\mathbf{B}_{\tau}^{\dagger} \mathbf{s}_{\tau}) + \frac{1}{2} \ln(\mathbf{B}_{\tau+1}^{\dagger} \mathbf{s}_{\tau+1}) - \ln \mathbf{s}_{\tau}$$



Generative model

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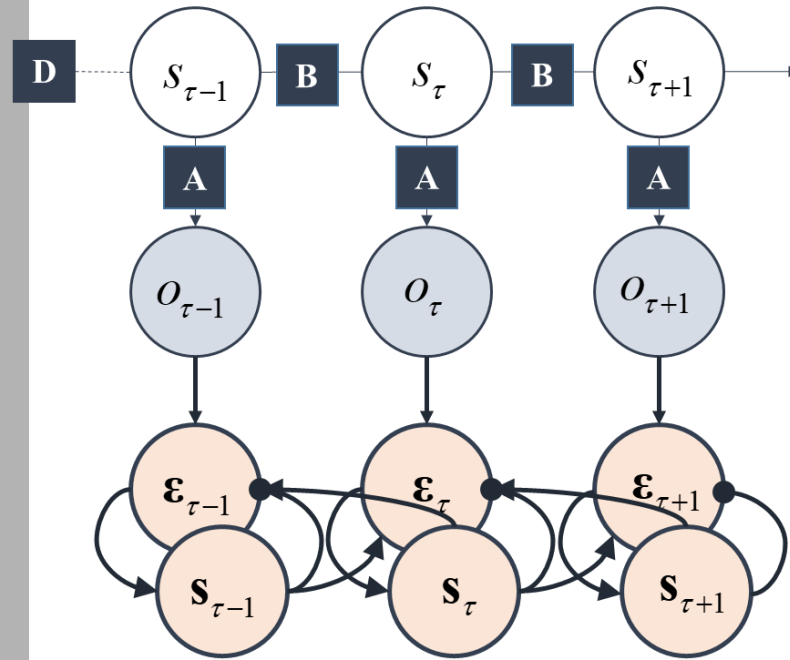
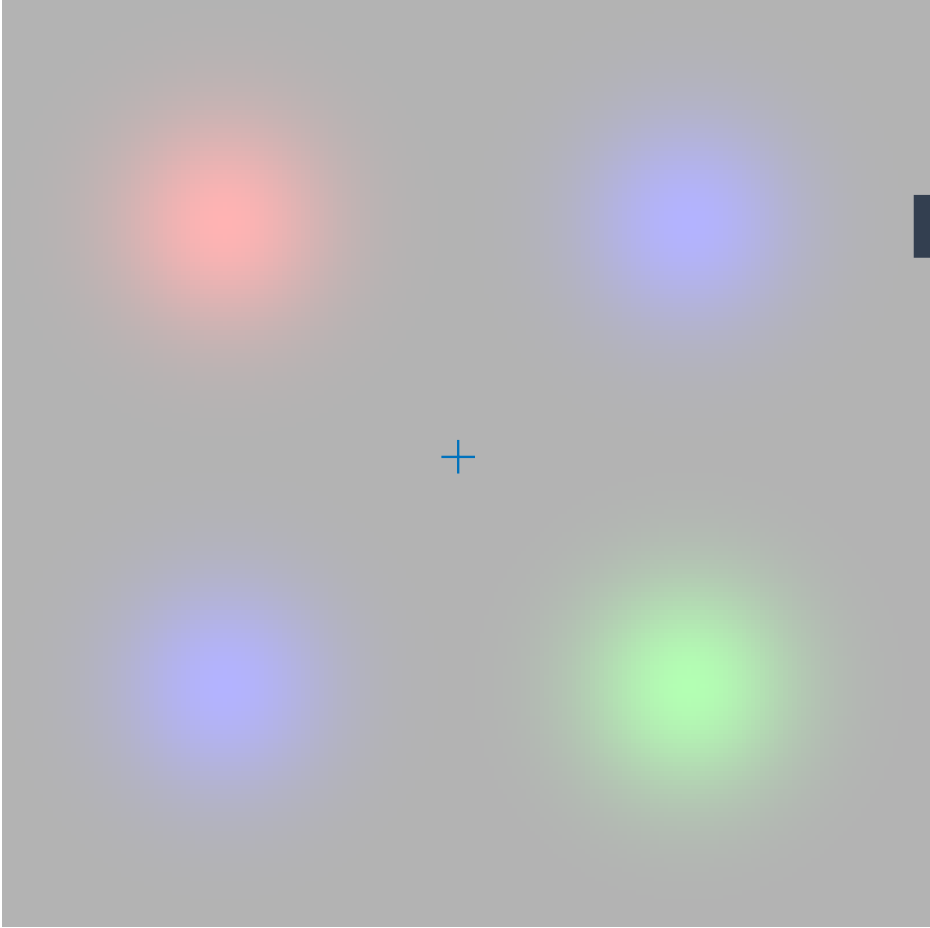
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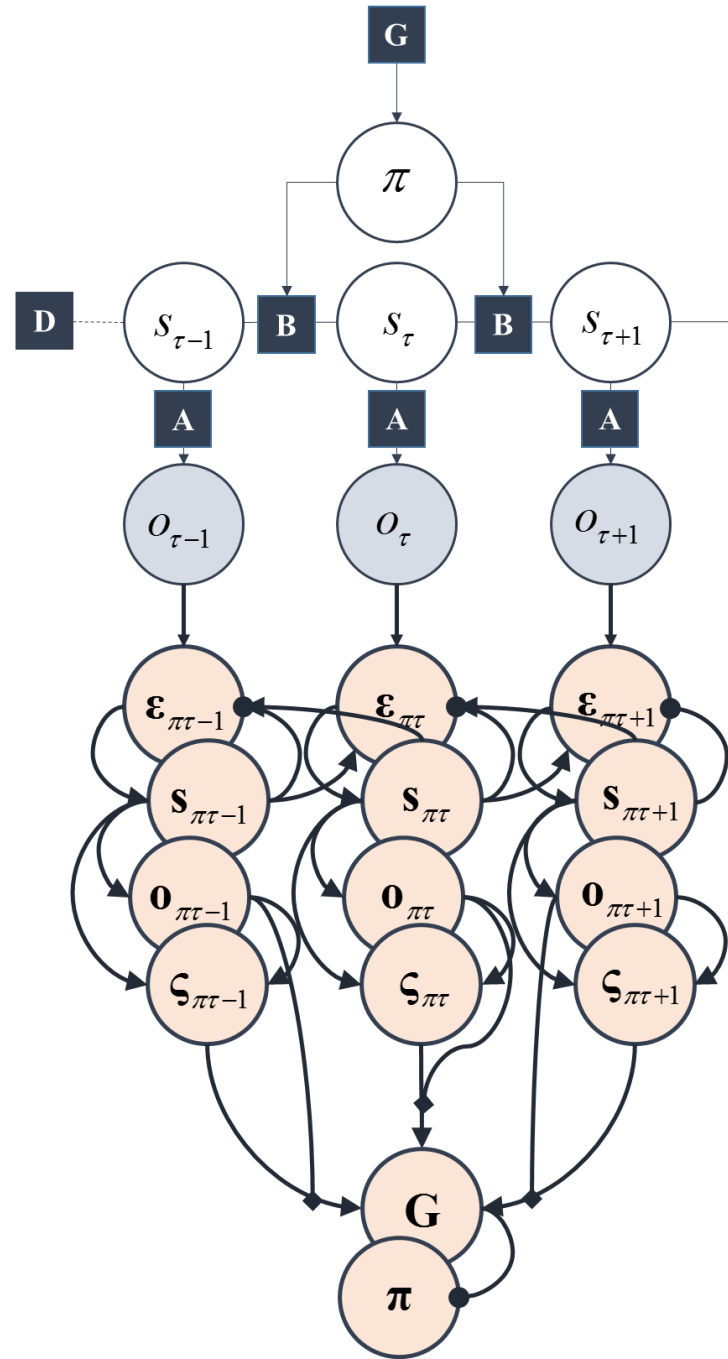
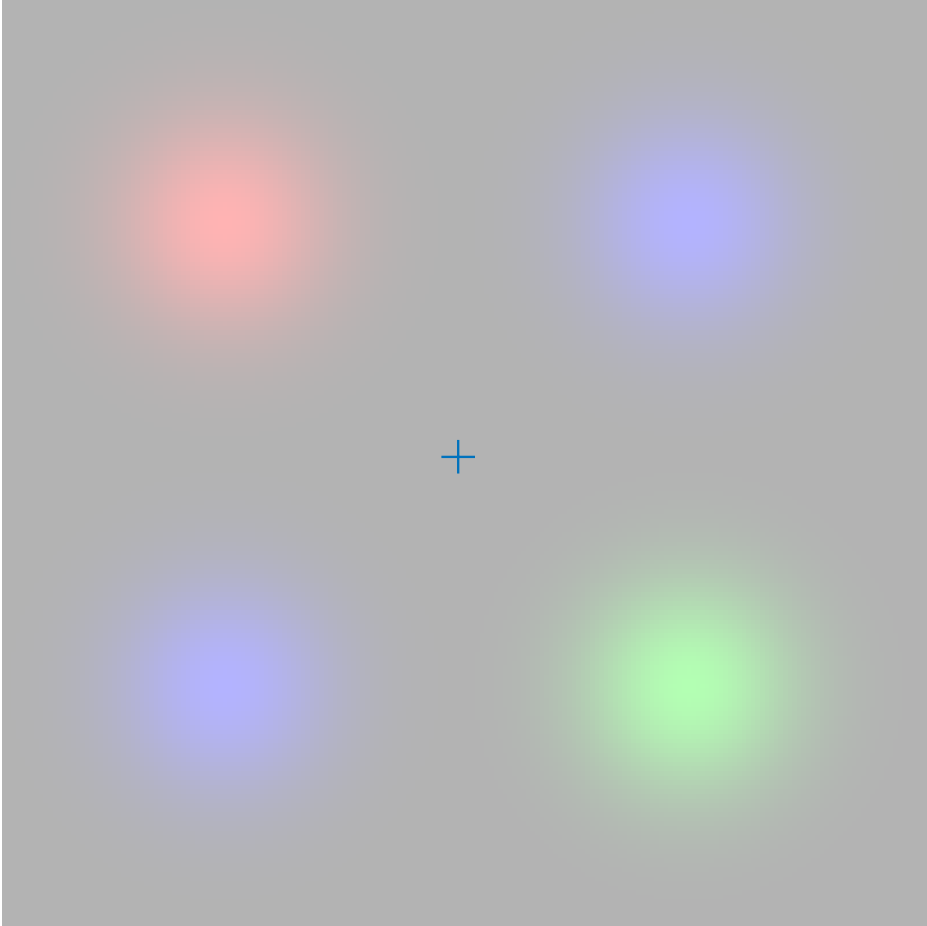
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$$P(\tilde{o}, \tilde{s}, \pi) = P(s_1)P(\pi) \prod_{\tau} P(s_{\tau+1} | s_{\tau}, \pi) P(o_{\tau} | s_{\tau})$$

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$$P(\pi) = \sigma(-\mathbf{G})$$

Bayesian message passing

$$\mathbf{s}_{\tau} = \pi \cdot \mathbf{s}_{\pi\tau}$$

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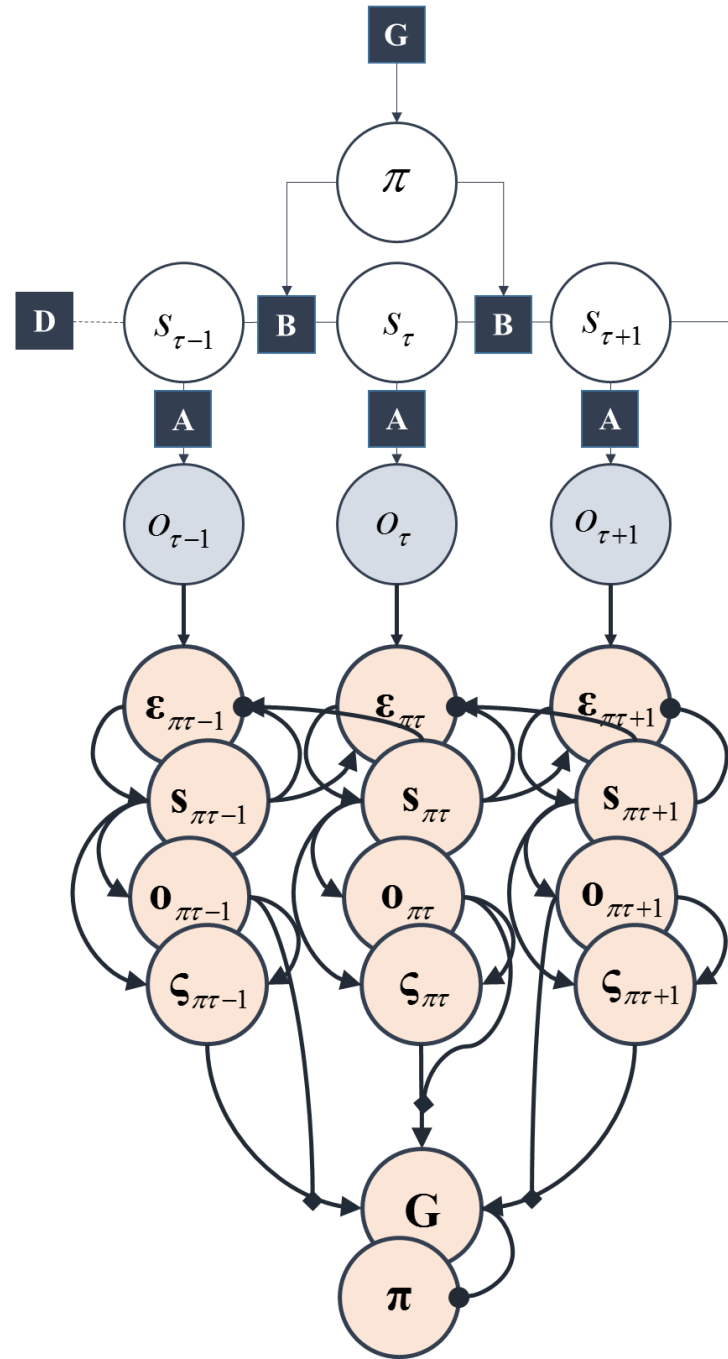
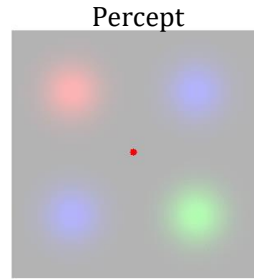
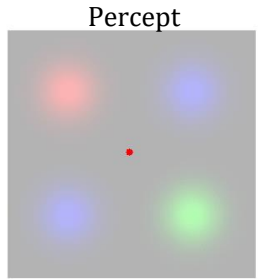
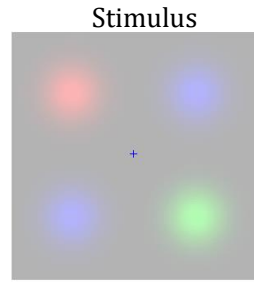
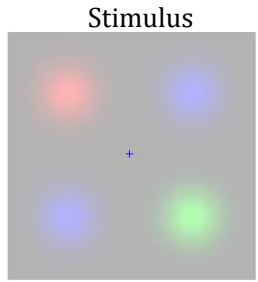
$$\mathbf{o}_{\pi\tau} = \mathbf{A} \mathbf{s}_{\pi\tau}$$

$$\boldsymbol{\zeta}_{\pi\tau} = \ln \mathbf{o}_{\pi\tau} - \ln \mathbf{C} + \mathbf{H} \cdot \mathbf{s}_{\pi\tau}$$

$$\mathbf{H} = -\text{diag}(\mathbf{A} \cdot \ln \mathbf{A})$$

$$\mathbf{G}_{\pi} = \mathbf{o}_{\pi\tau} \cdot \boldsymbol{\zeta}_{\pi\tau}$$

$$\pi = \sigma(-\mathbf{G})$$



Generative model

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Bayesian message passing

$$\mathbf{s}_{\tau} = \pi \cdot \mathbf{s}_{\pi\tau}$$

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$$\mathbf{\epsilon}_{\pi\tau} = \ln \mathbf{A} \cdot \mathbf{o}_{\tau} + \frac{1}{2} \ln(\mathbf{B}_{\pi\tau} \mathbf{s}_{\pi\tau}) + \frac{1}{2} \ln(\mathbf{B}_{\pi\tau+1}^{\dagger} \mathbf{s}_{\pi\tau+1}) - \ln \mathbf{s}_{\pi\tau}$$

$$\mathbf{o}_{\pi\tau} = \mathbf{A} \mathbf{s}_{\pi\tau}$$

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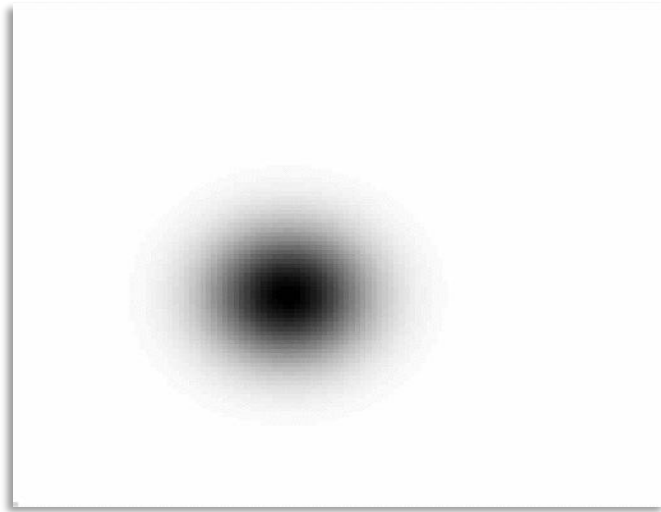
$$\mathbf{G}_{\pi} = \mathbf{o}_{\pi\tau} \cdot \mathbf{\varsigma}_{\pi\tau}$$

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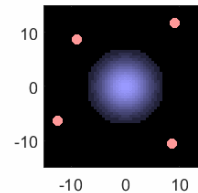
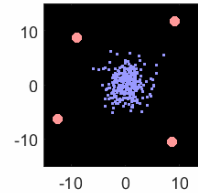
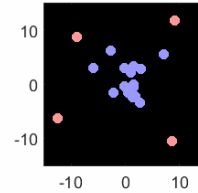
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Goals and steady state



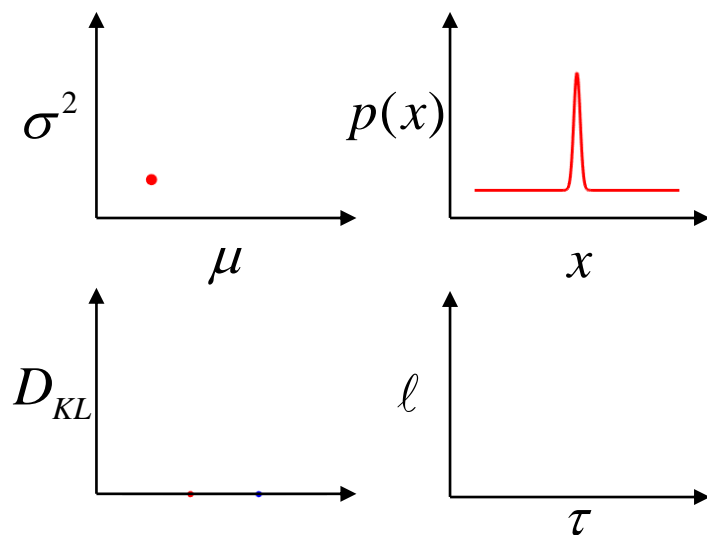
$$D_{KL} [P(o | \pi) || P(o | C)]$$

Goals and steady state



$$D_{KL} \left[P(o | \pi) \parallel P(o | C) \right]$$

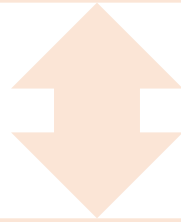
Goals and steady state



$$D_{KL} \left[P(o | \pi) \parallel P(o | C) \right]$$

Goals and steady state

$$-H[P(o|\pi)] - \mathbb{E}_{P(o|\pi)}[\ln P(o|C)]$$



$$D_{KL}[P(o|\pi) \parallel P(o|C)]$$



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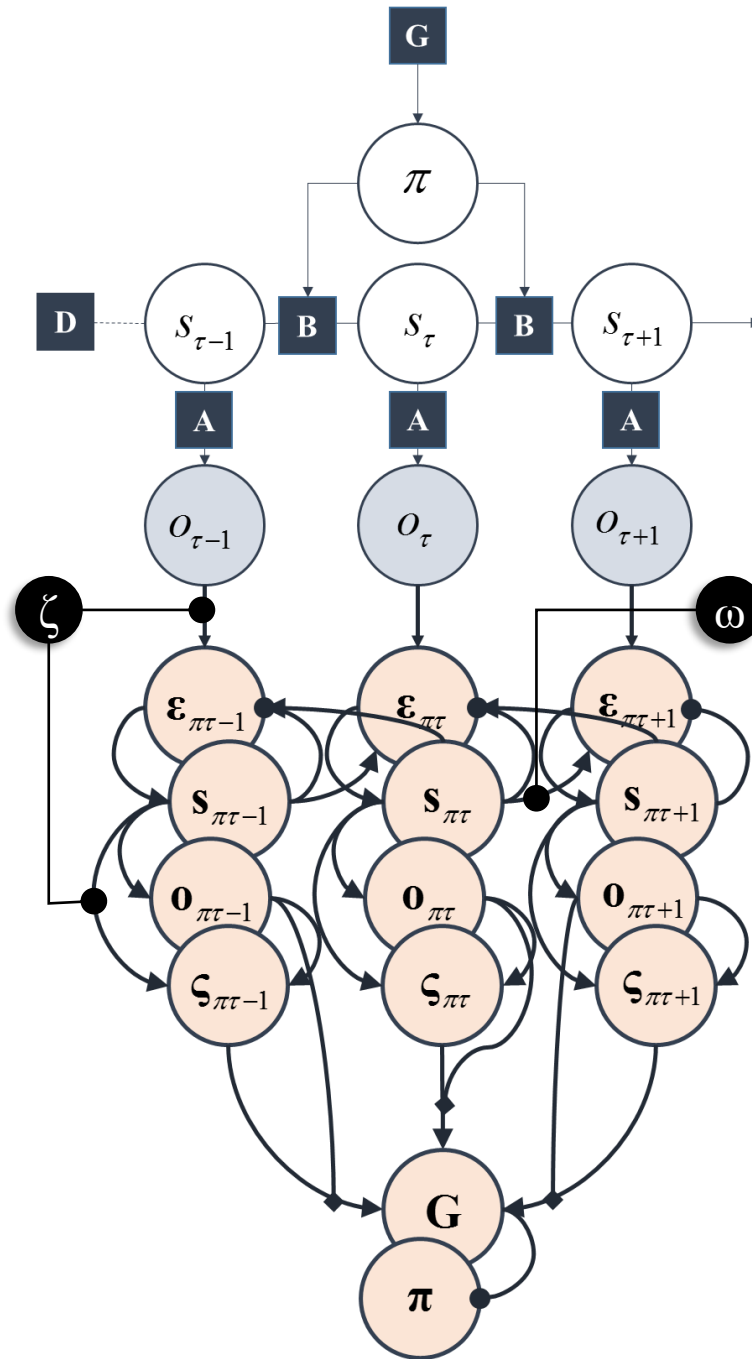
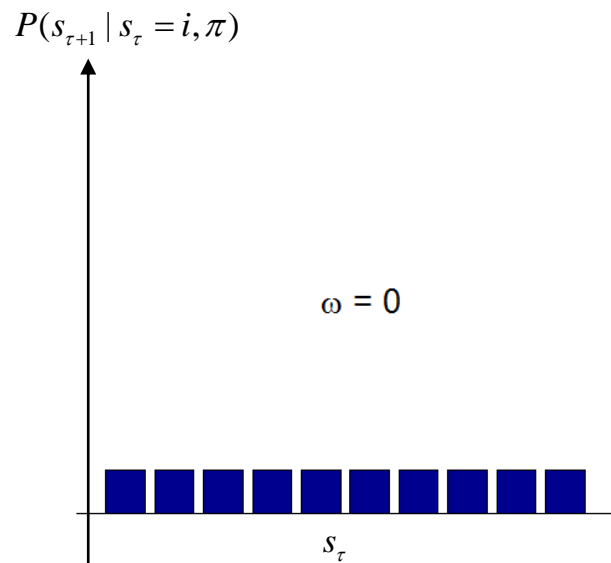
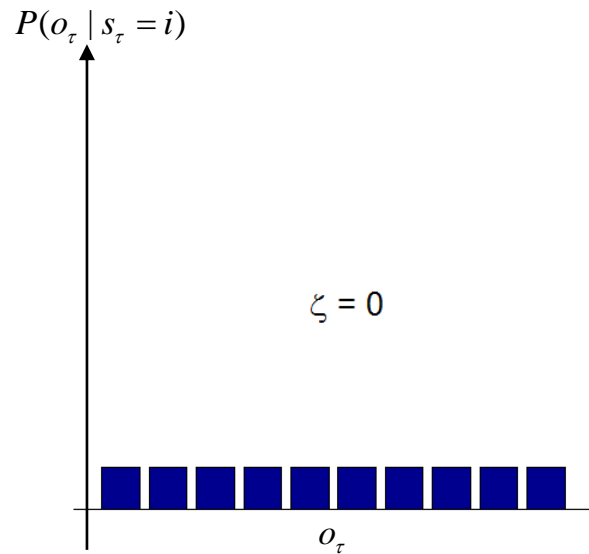
Information gain

$$\mathcal{I}(\pi) = D_{KL} \left[P(o, s | \pi) \parallel P(o | \pi) P(s | \pi) \right]$$

$$= \mathbb{E}_{P(o|\pi)} \left[D_{KL} \left[P(s | o, \pi) \parallel P(s | \pi) \right] \right]$$

$$= H \left[P(o | \pi) \right] - \mathbb{E}_{P(s|\pi)} \left[H \left[P(o | s) \right] \right]$$

$$P(o, s | \pi) = P(s | o, \pi) P(o | \pi) = P(o | s) P(s | \pi)$$



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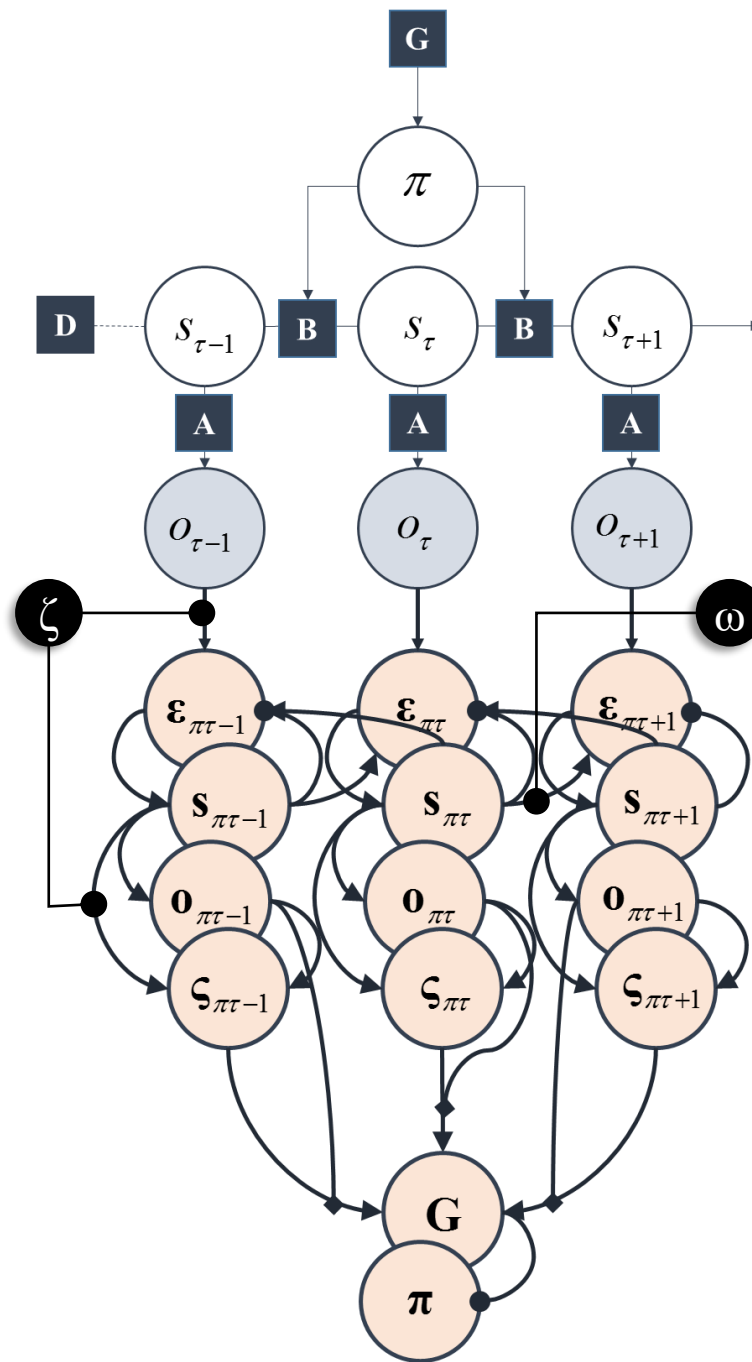
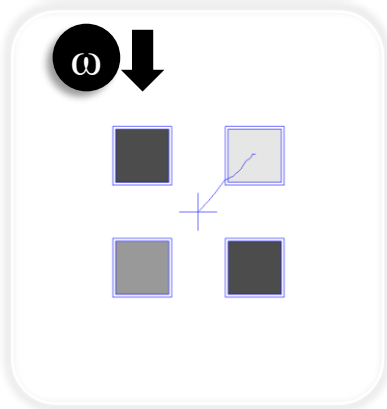
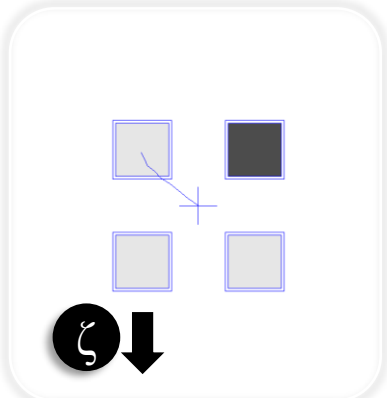
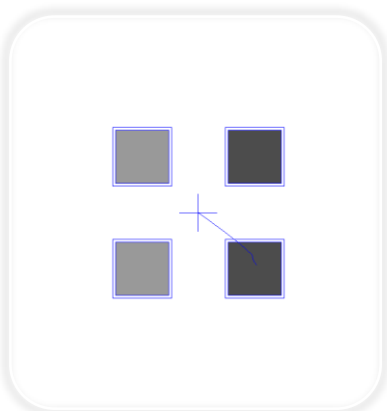
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Expected free energy

Explore

$$H[P(o|\pi)] - \mathbb{E}_{P(s|\pi)}[H[P(o|s)]]$$

$$\mathbb{E}_{P(o|\pi)}[\ln P(o|C)] + H[P(o|\pi)] - \mathbb{E}_{P(s|\pi)}[H[P(o|s)]]$$

$$\mathbb{E}_{P(o|\pi)}[\ln P(o|C)] + H[P(o|\pi)]$$

Exploit

Expected free energy

$$P(\pi) = \sigma[-G(\pi)]$$

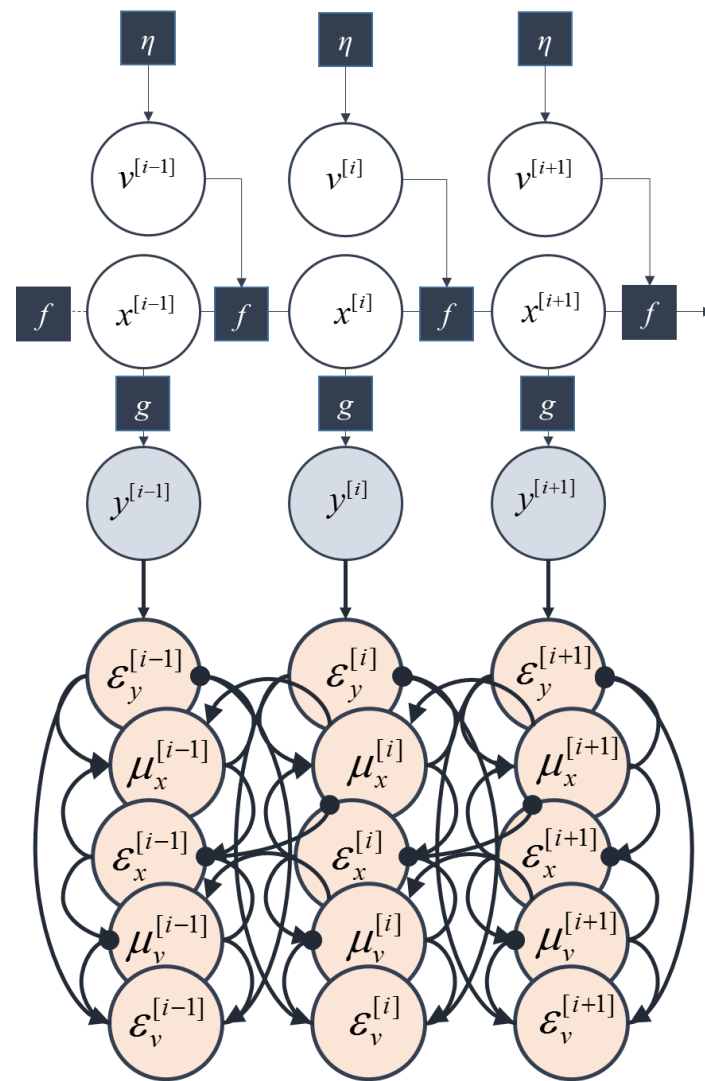
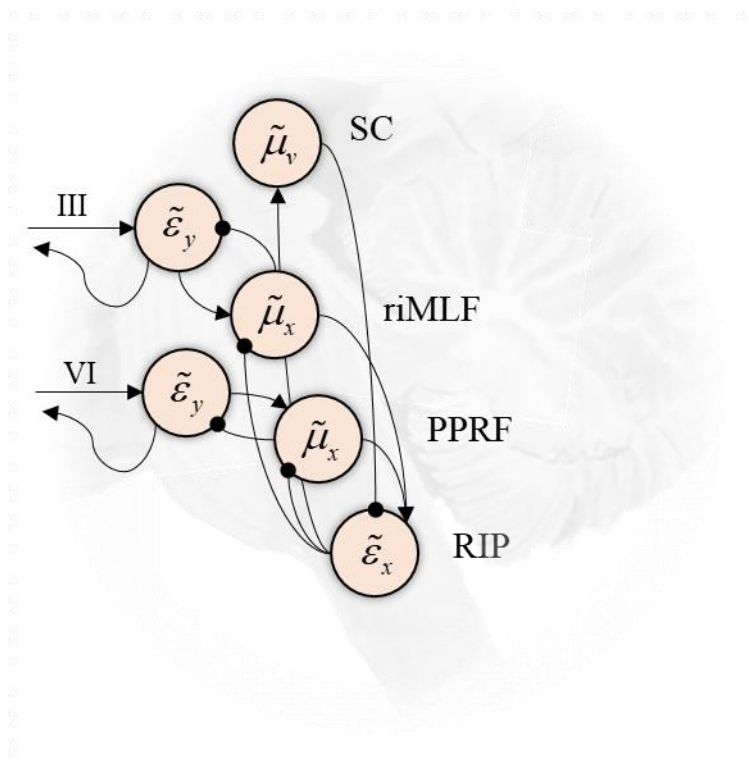
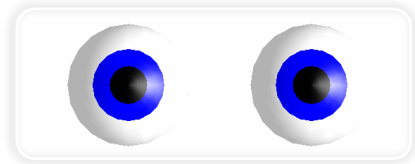
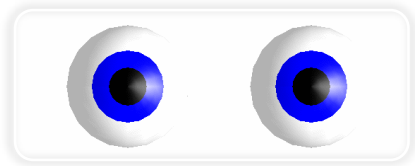
$$-G(\pi) = \mathbb{E}_{P(o|\pi)} [\ln P(o|C)] + H[P(o|\pi)] - \mathbb{E}_{P(s|\pi)} [H[P(o|s)]]$$

$$= \mathbb{E}_{P(o|\pi)} [\ln P(o|C)] + \mathbb{E}_{P(o,s|\pi)} [\ln P(s|\pi, o) - \ln P(s|\pi)]$$

$$-F(\pi) = \ln P(o) + \mathbb{E}_{Q(s|\pi)} [\ln P(s|\pi, o) - \ln Q(s|\pi)]$$



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Generative model

$$p(\tilde{y}, \tilde{x}, \tilde{v}) = \prod_i p(v^{[i]}) p(x^{[i+1]} | x^{[i]}, v^{[i]}) p(y^{[i]} | x^{[i]}, v^{[i]})$$

$$p(y^{[i]} | x^{[i]}, v^{[i]}) = \mathcal{N}(g^{[i]}(x^{[i]}, v^{[i]}), \Pi_y^{[i]})$$

$$p(x^{[i+1]} | x^{[i]}, v^{[i]}) = \mathcal{N}(f^{[i]}(x^{[i]}, v^{[i]}), \Pi_x^{[i]})$$

$$p(v^{[i]}) = \mathcal{N}(\eta^{[i]}, \Pi_v^{[i]})$$

Bayesian message passing

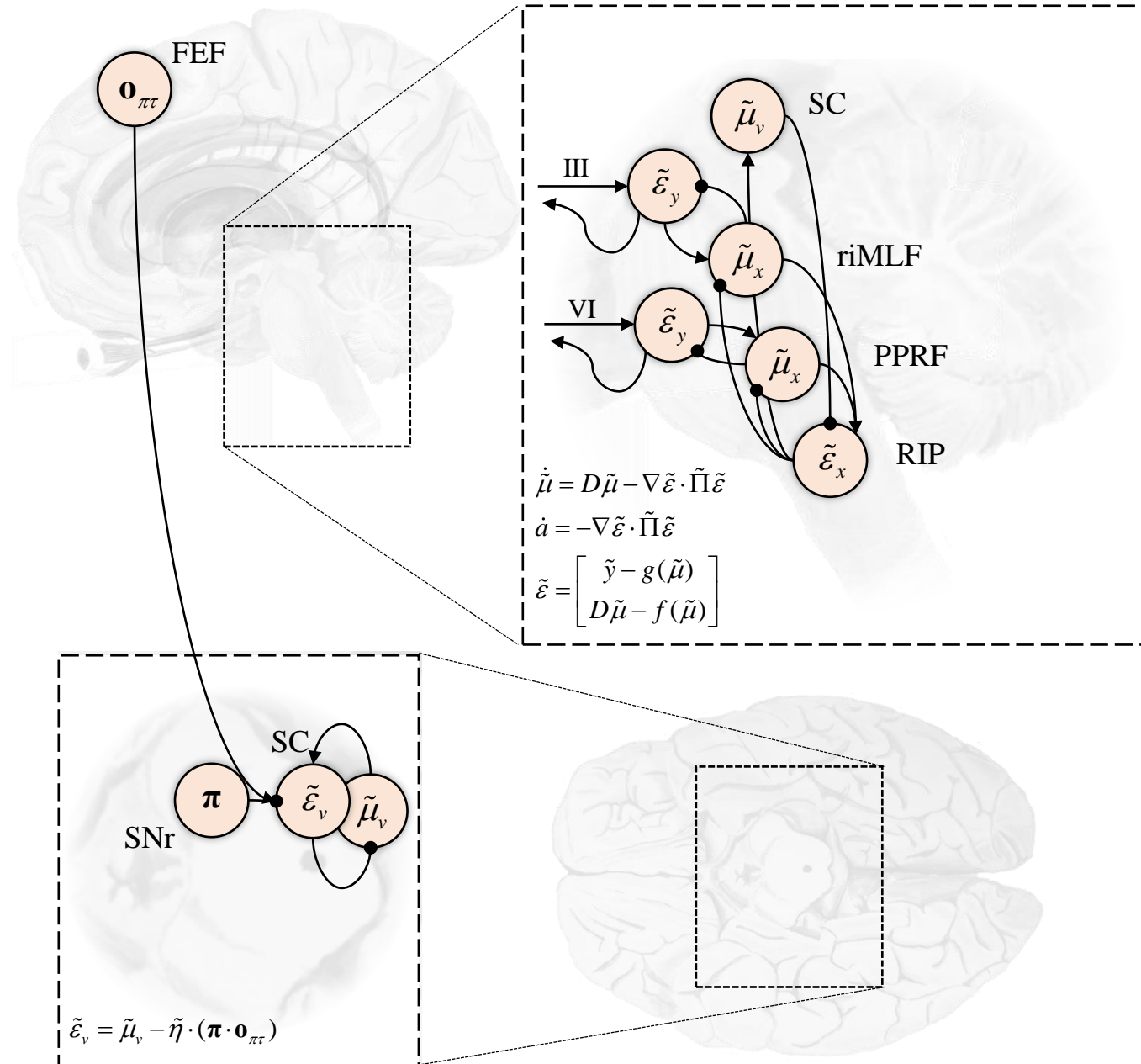
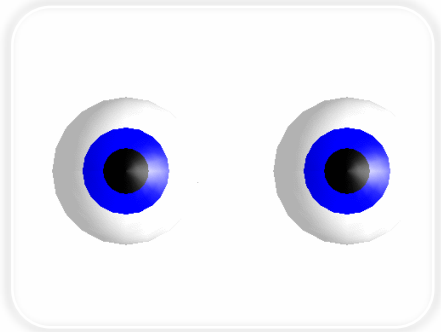
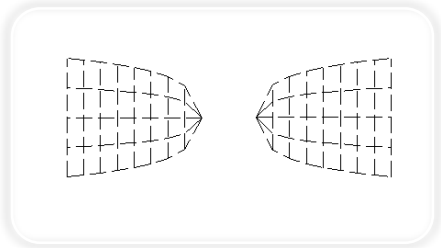
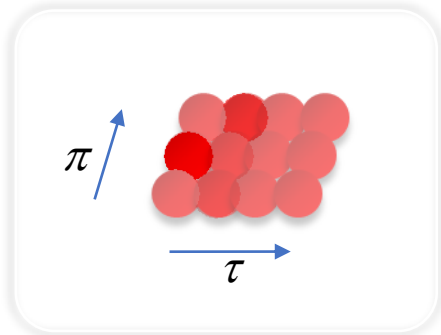
$$\epsilon_y^{[i]} = y^{[i]} - g^{[i]}(\mu_x^{[i]}, \mu_v^{[i]})$$

$$\epsilon_x^{[i]} = \mu_x^{[i+1]} - f^{[i]}(\mu_x^{[i]}, \mu_v^{[i]})$$

$$\epsilon_v^{[i]} = \mu_v^{[i]} - \eta^{[i]}$$

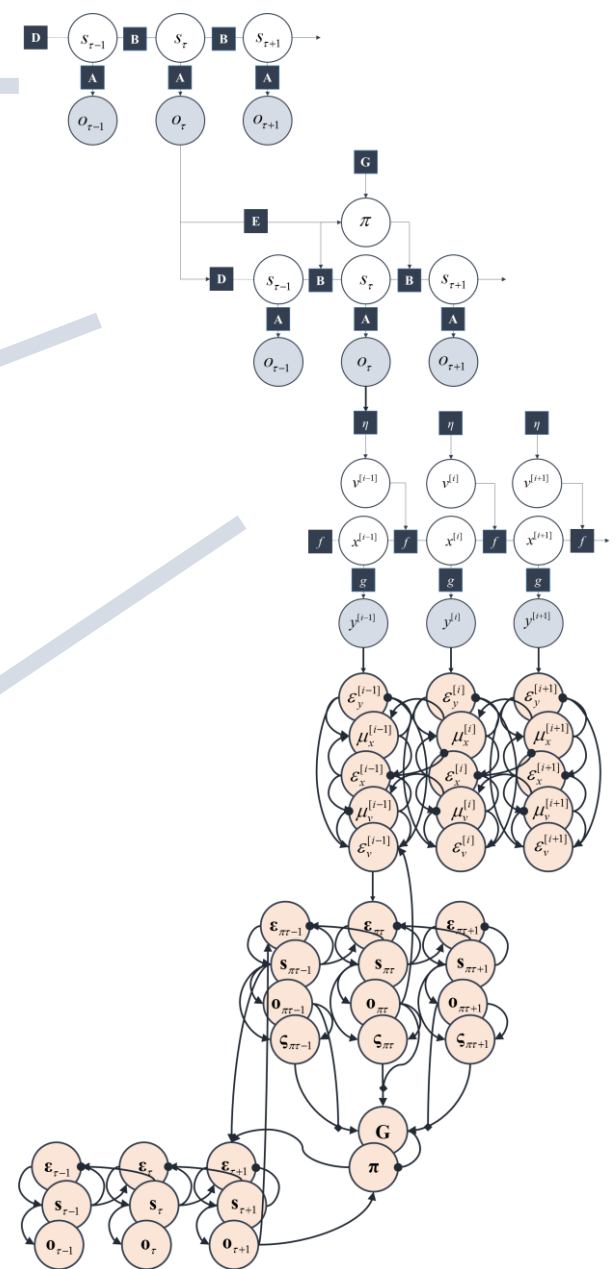
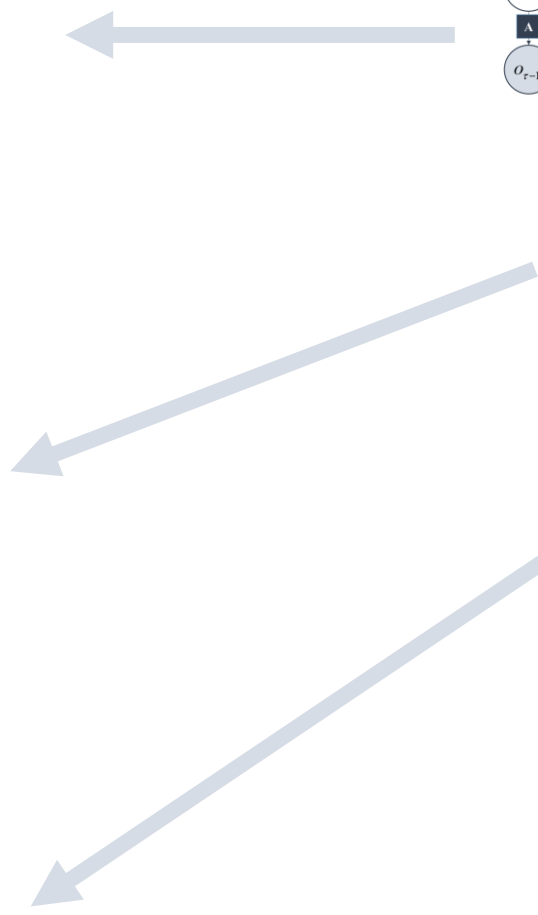
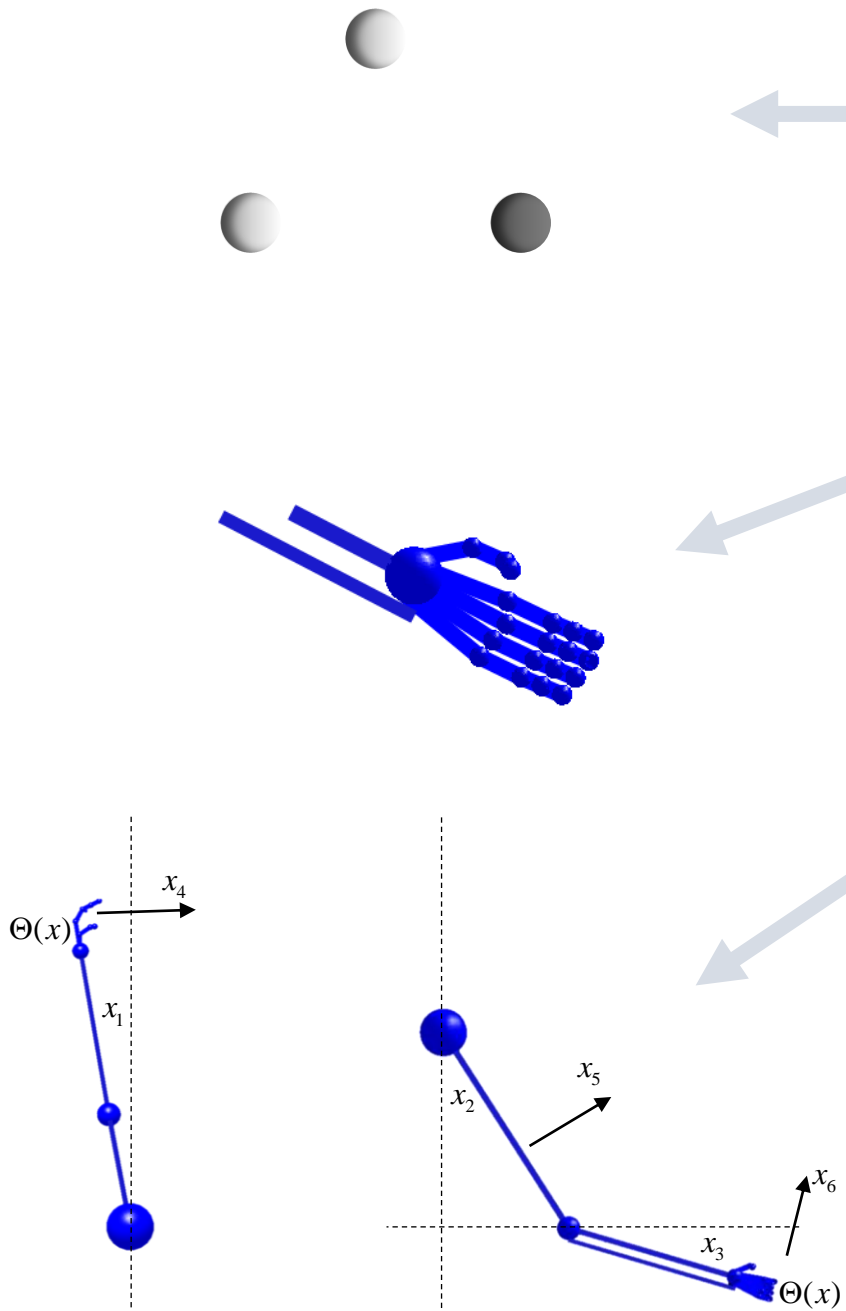
$$\begin{aligned} \dot{\mu}_x^{[i]} &= \mu_x^{[i+1]} \\ &\quad + \partial_{\mu_x^{[i]}} g^{[i]} \cdot \Pi_y^{[i]} \epsilon_y^{[i]} - \Pi_x^{[i-1]} \epsilon_x^{[i-1]} + \partial_{\mu_x^{[i]}} f^{[i]} \cdot \Pi_x^{[i]} \epsilon_x^{[i]} \end{aligned}$$

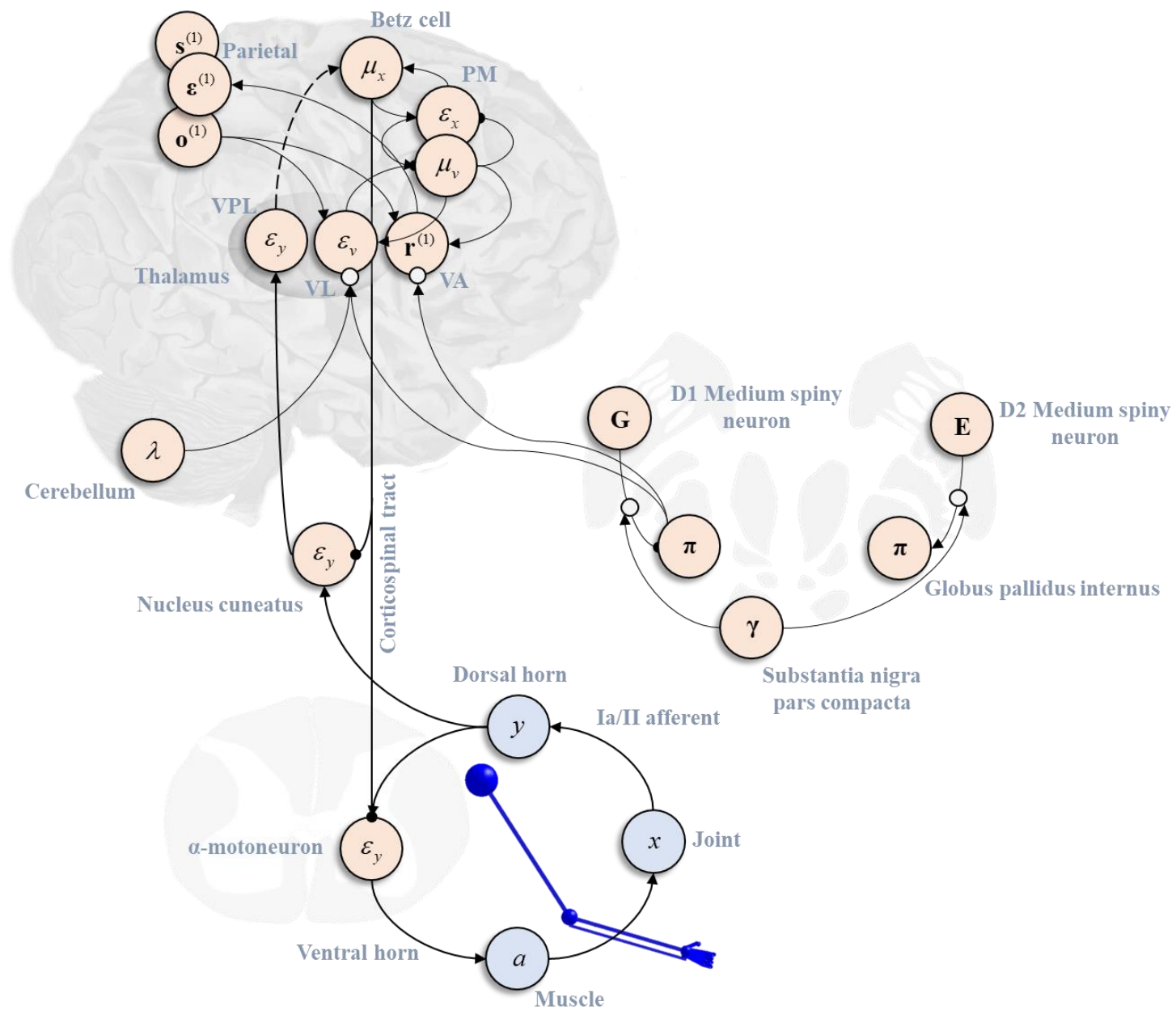
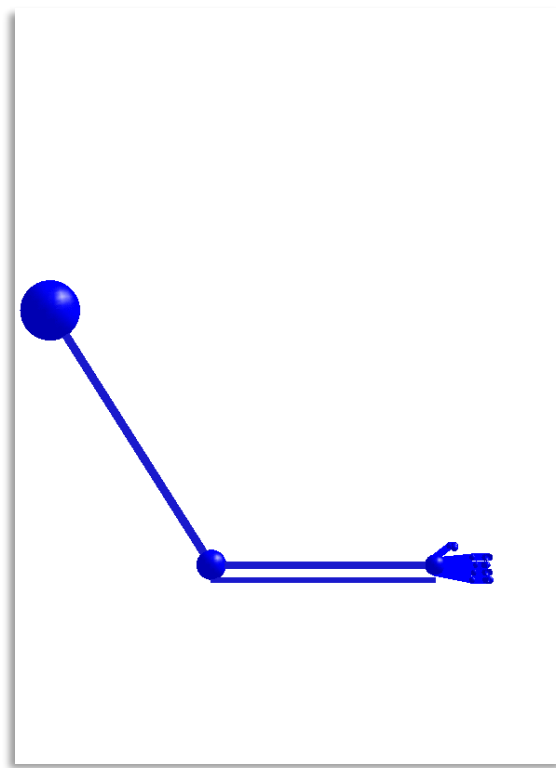
$$\begin{aligned} \dot{\mu}_v^{[i]} &= \mu_v^{[i+1]} \\ &\quad + \partial_{\mu_v^{[i]}} g^{[i]} \cdot \Pi_y^{[i]} \epsilon_y^{[i]} + \partial_{\mu_v^{[i]}} f^{[i]} \cdot \Pi_x^{[i]} \epsilon_x^{[i]} - \Pi_v^{[i]} \epsilon_v^{[i]} \end{aligned}$$

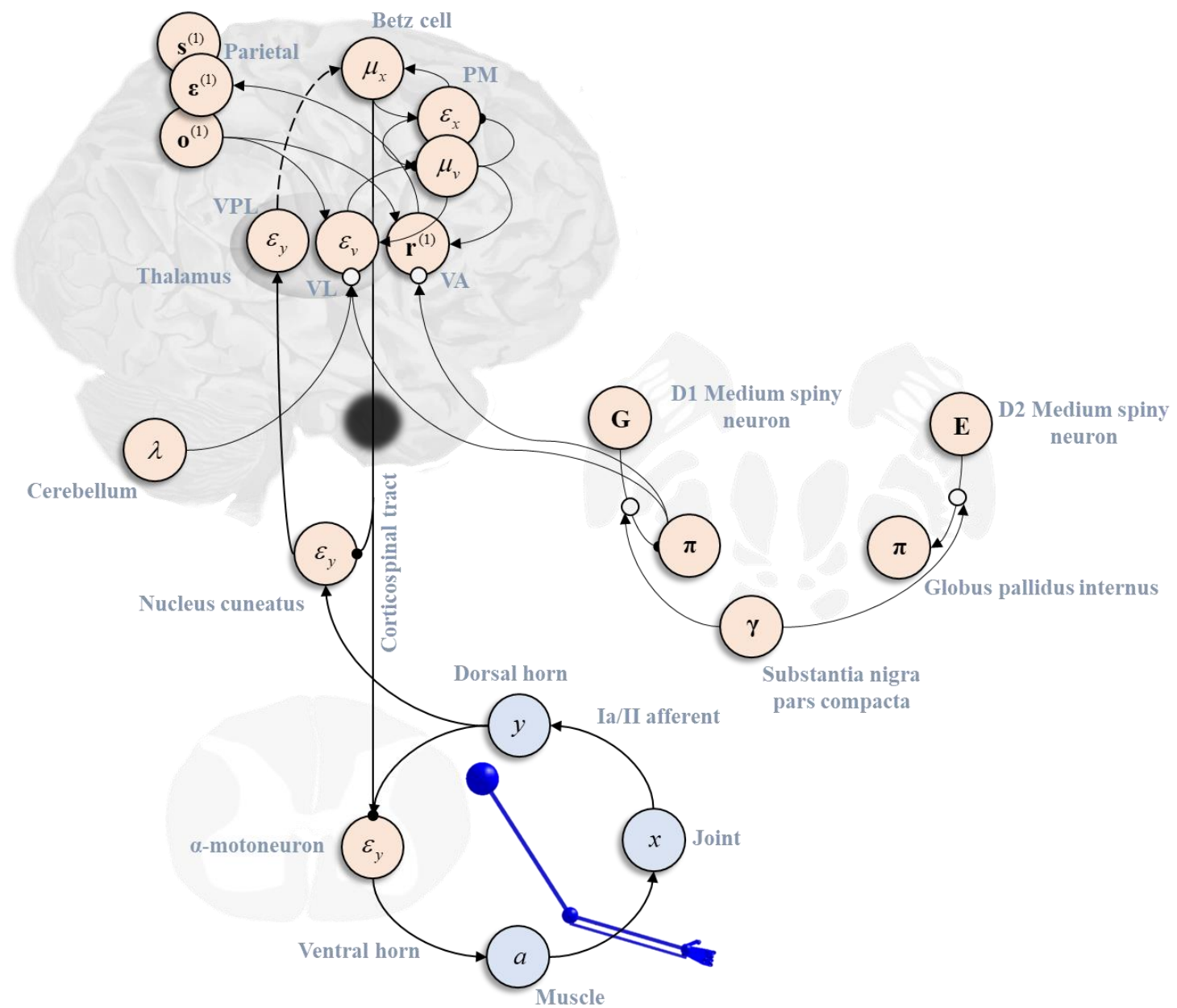
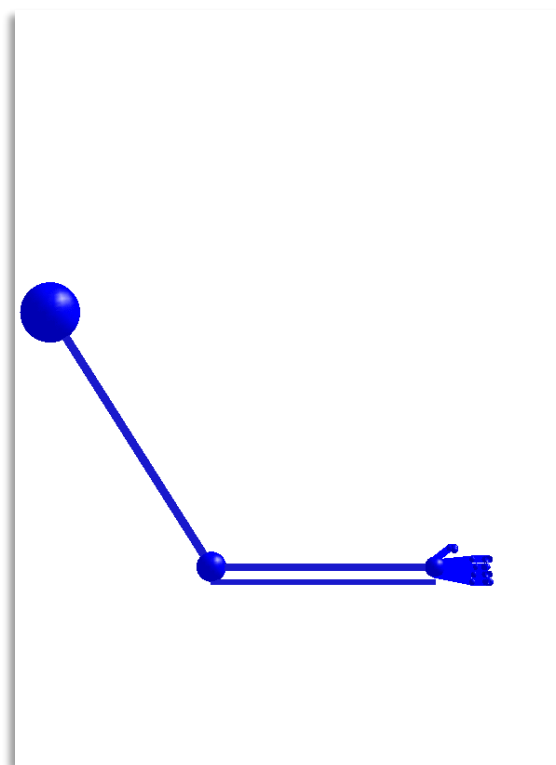


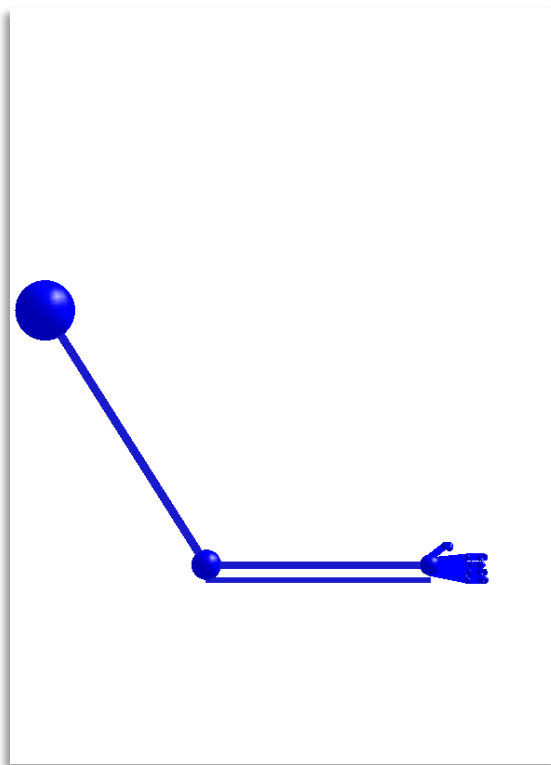
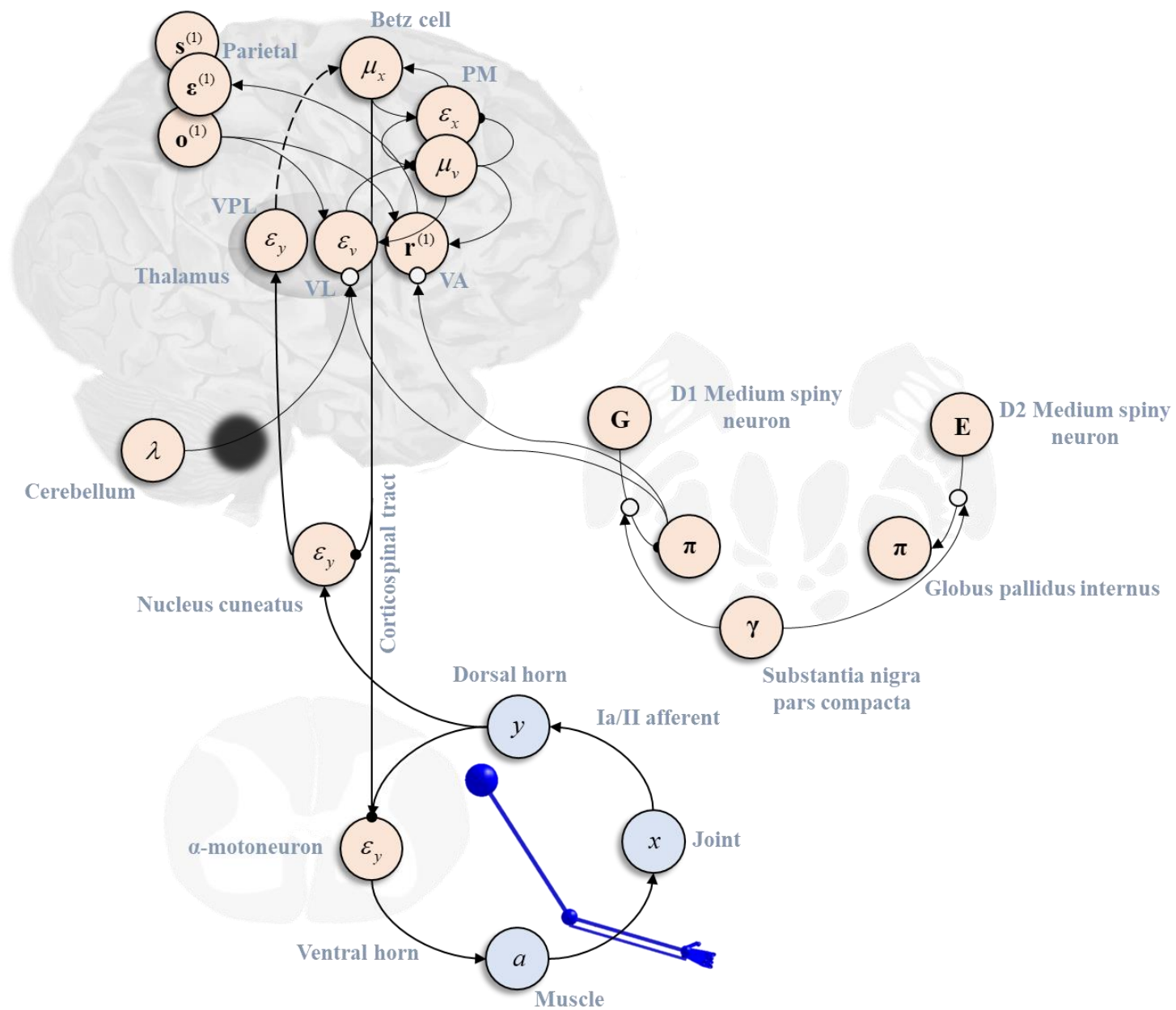


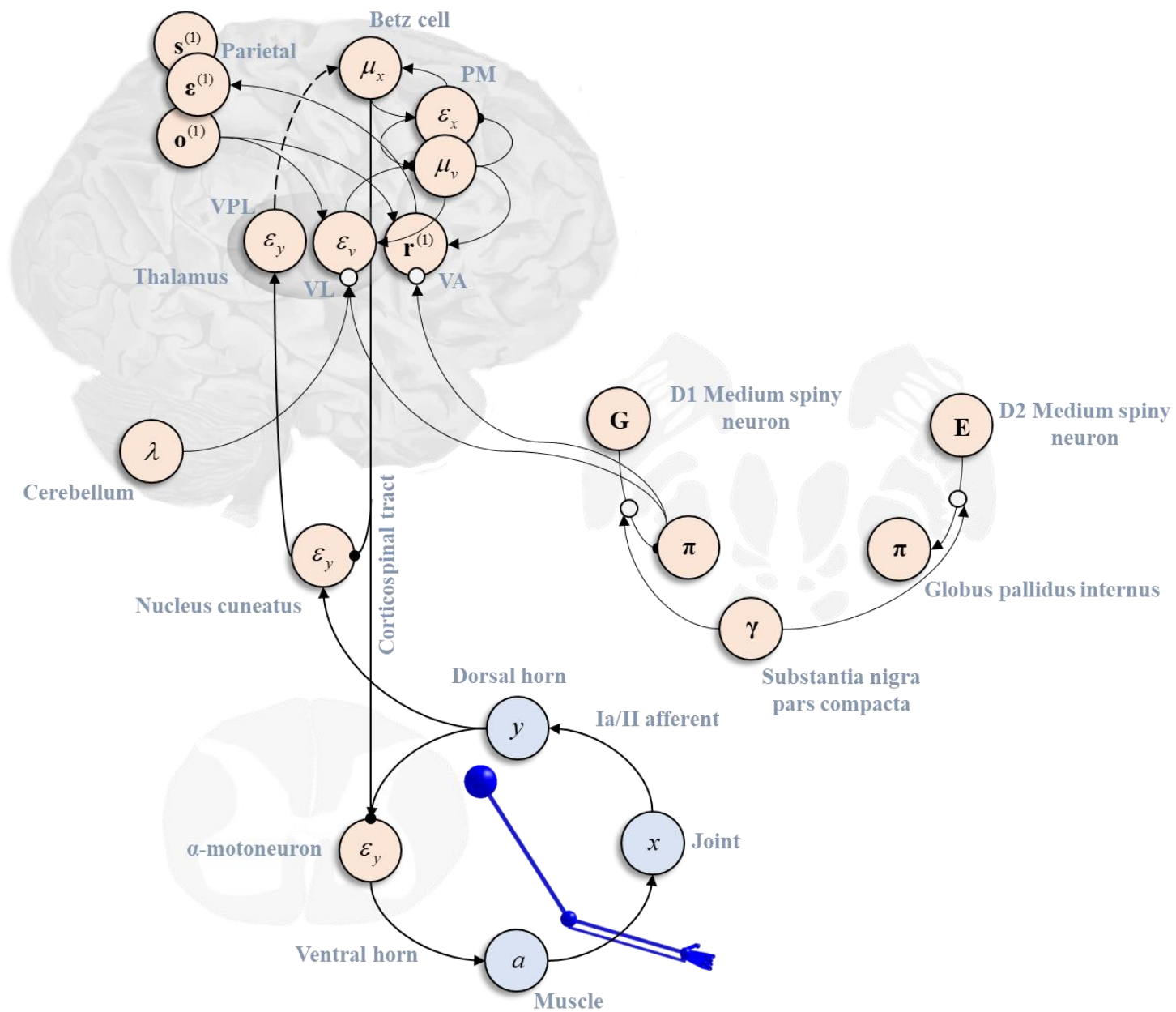
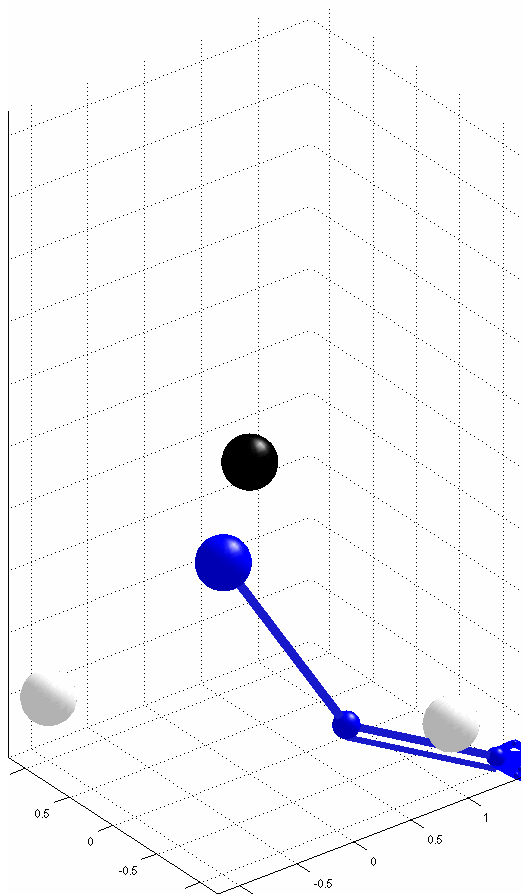
Active inference
Generative models
Exploitation
Exploration
Movement
Hierarchy

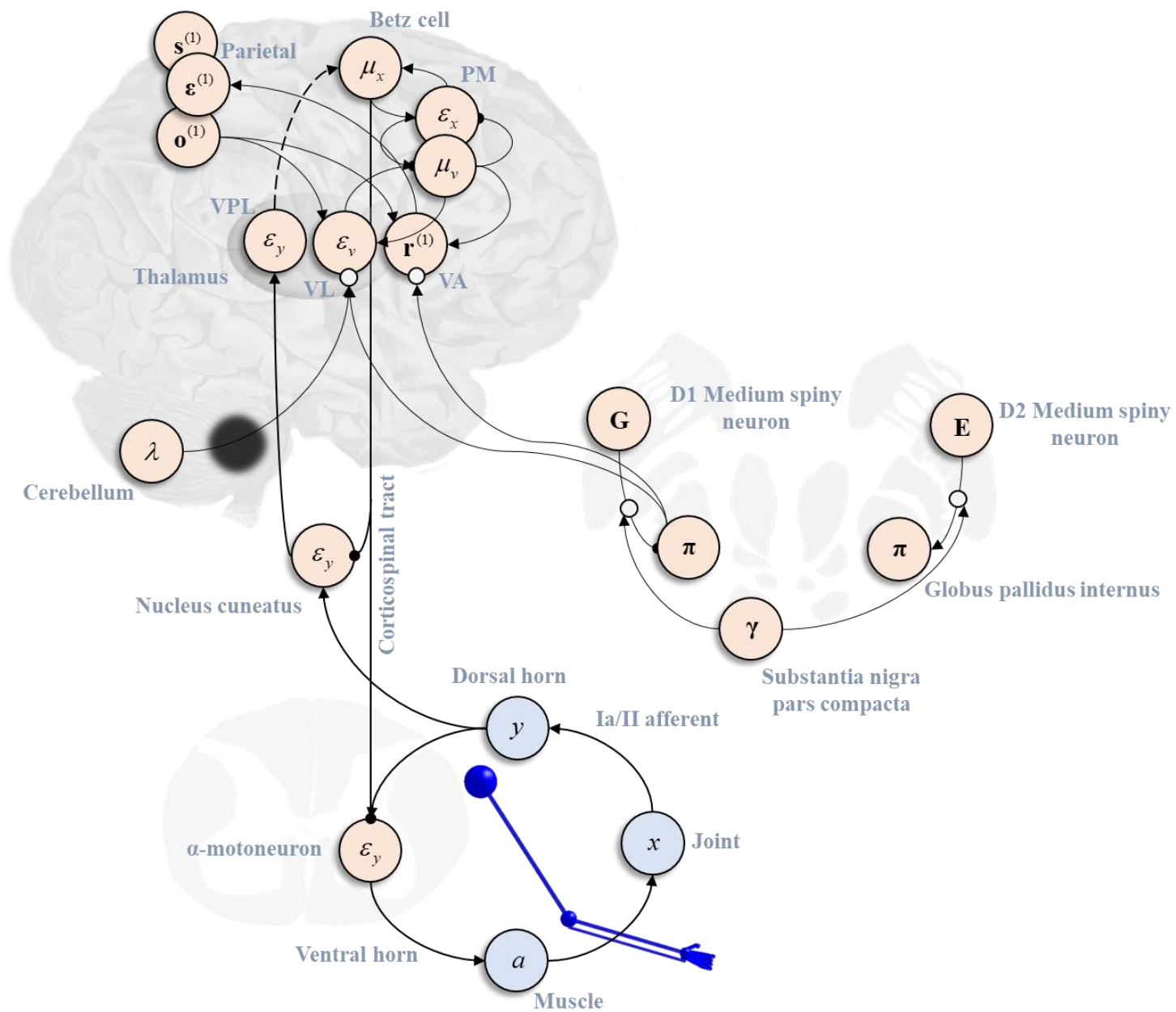
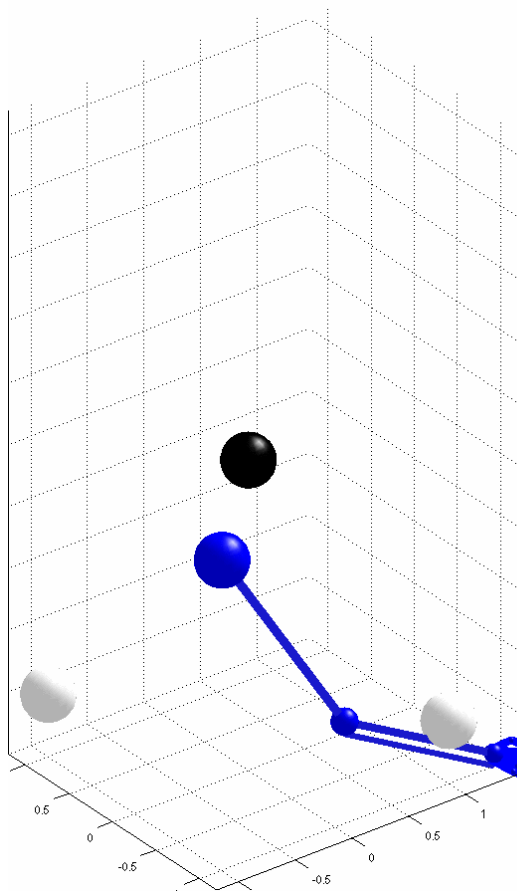


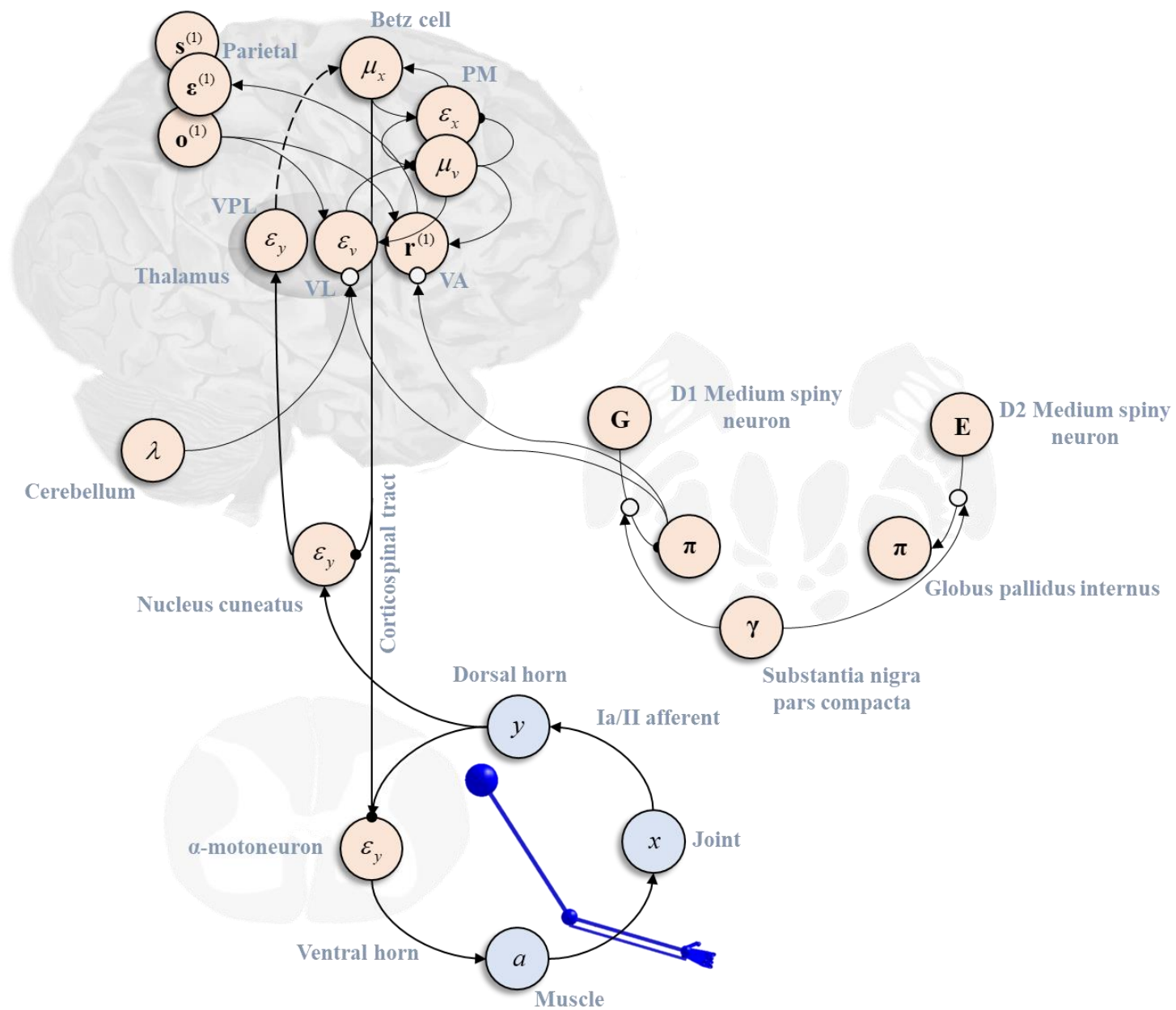


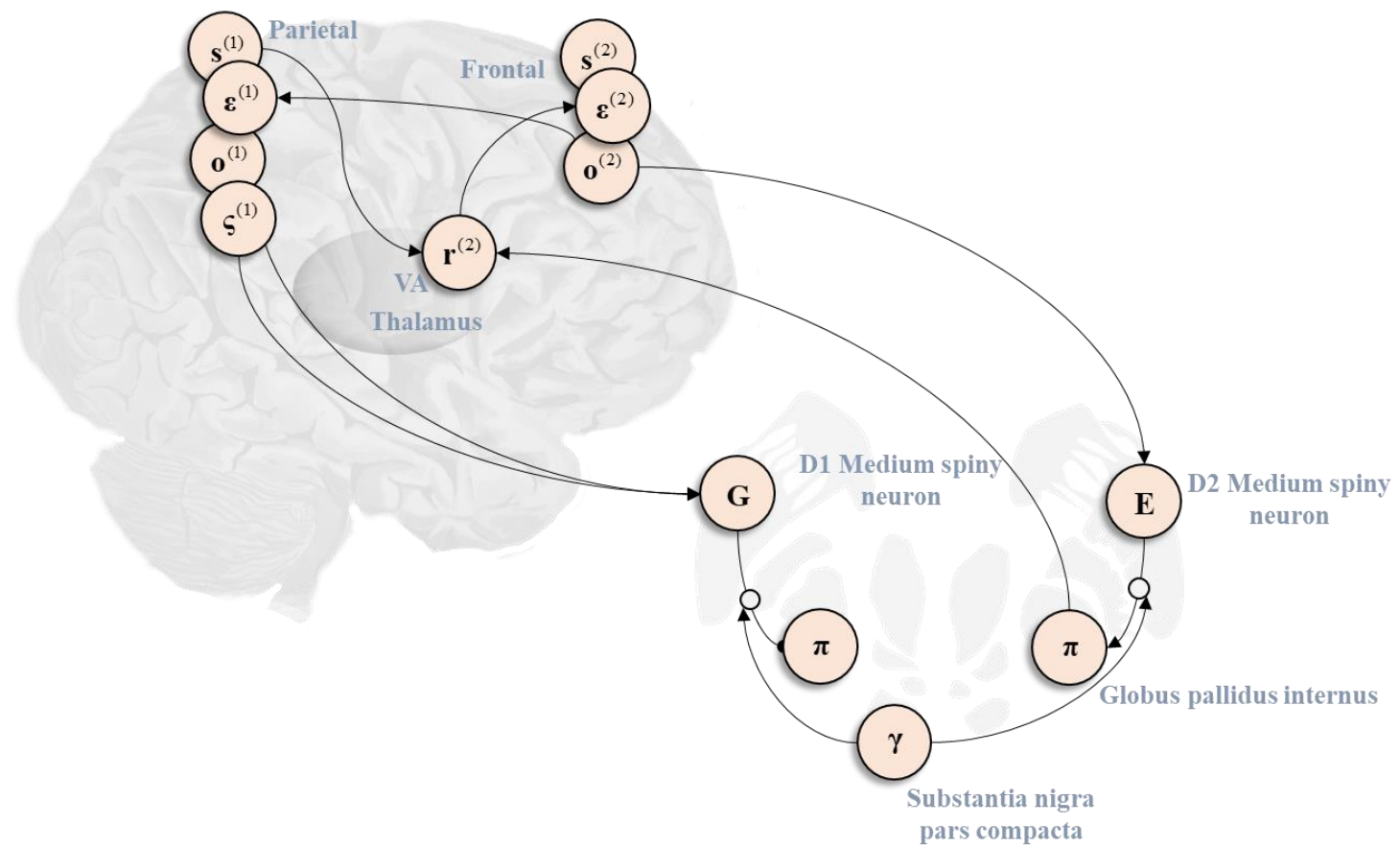


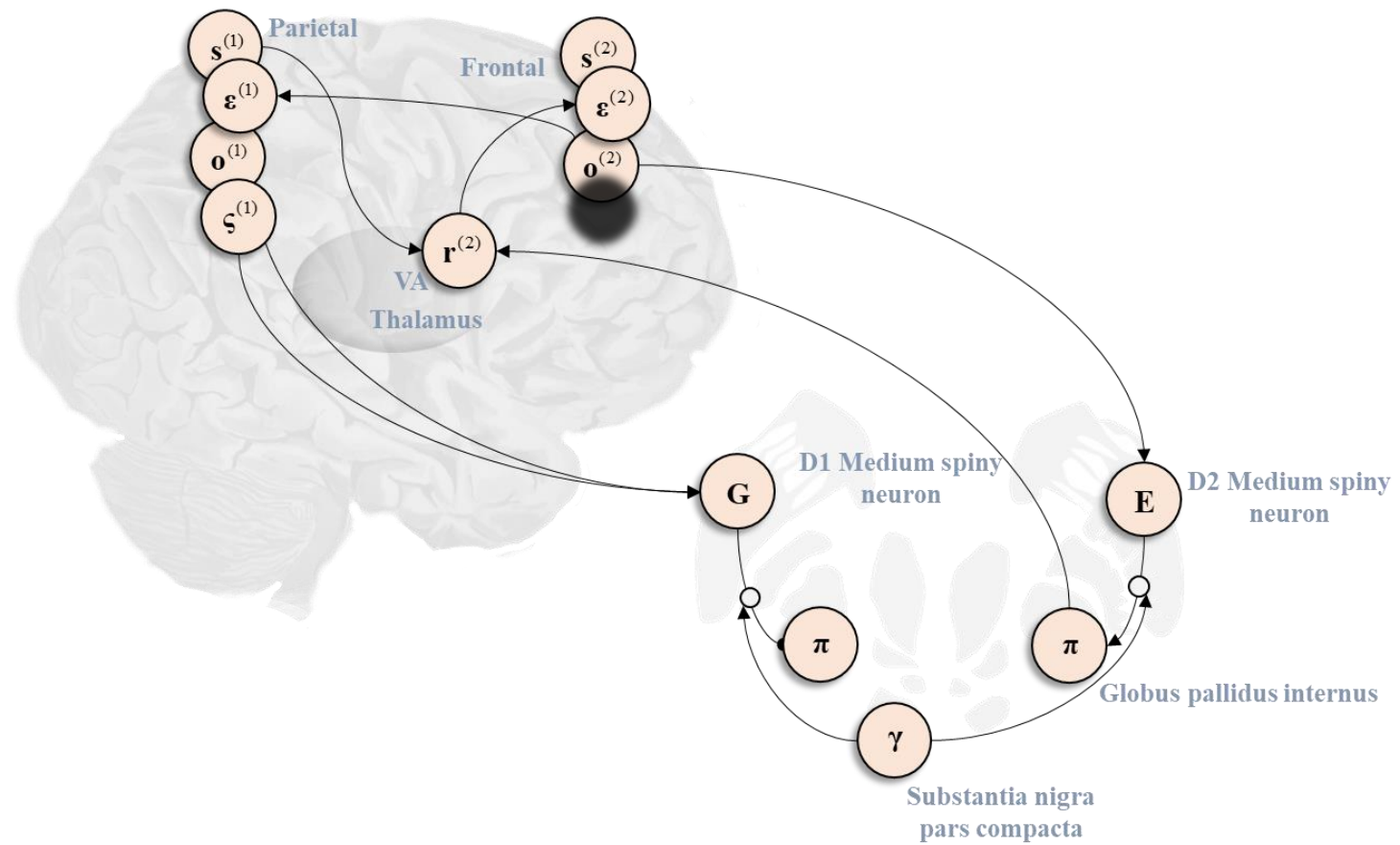
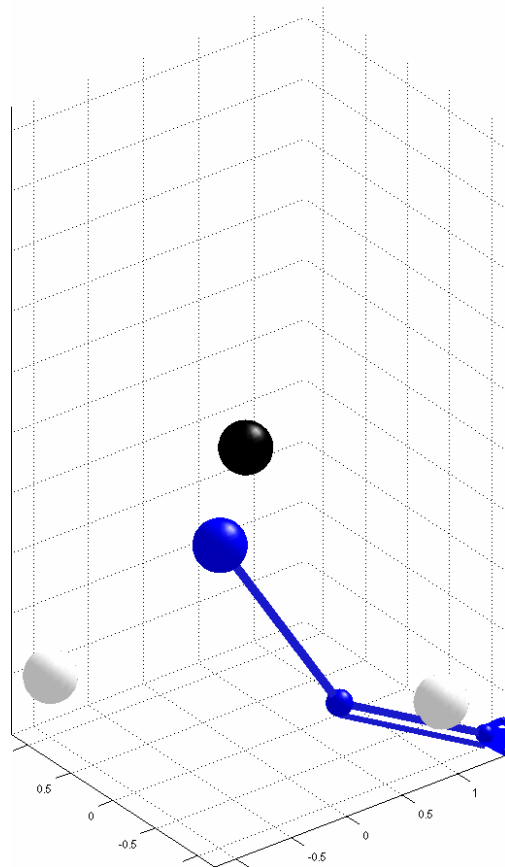


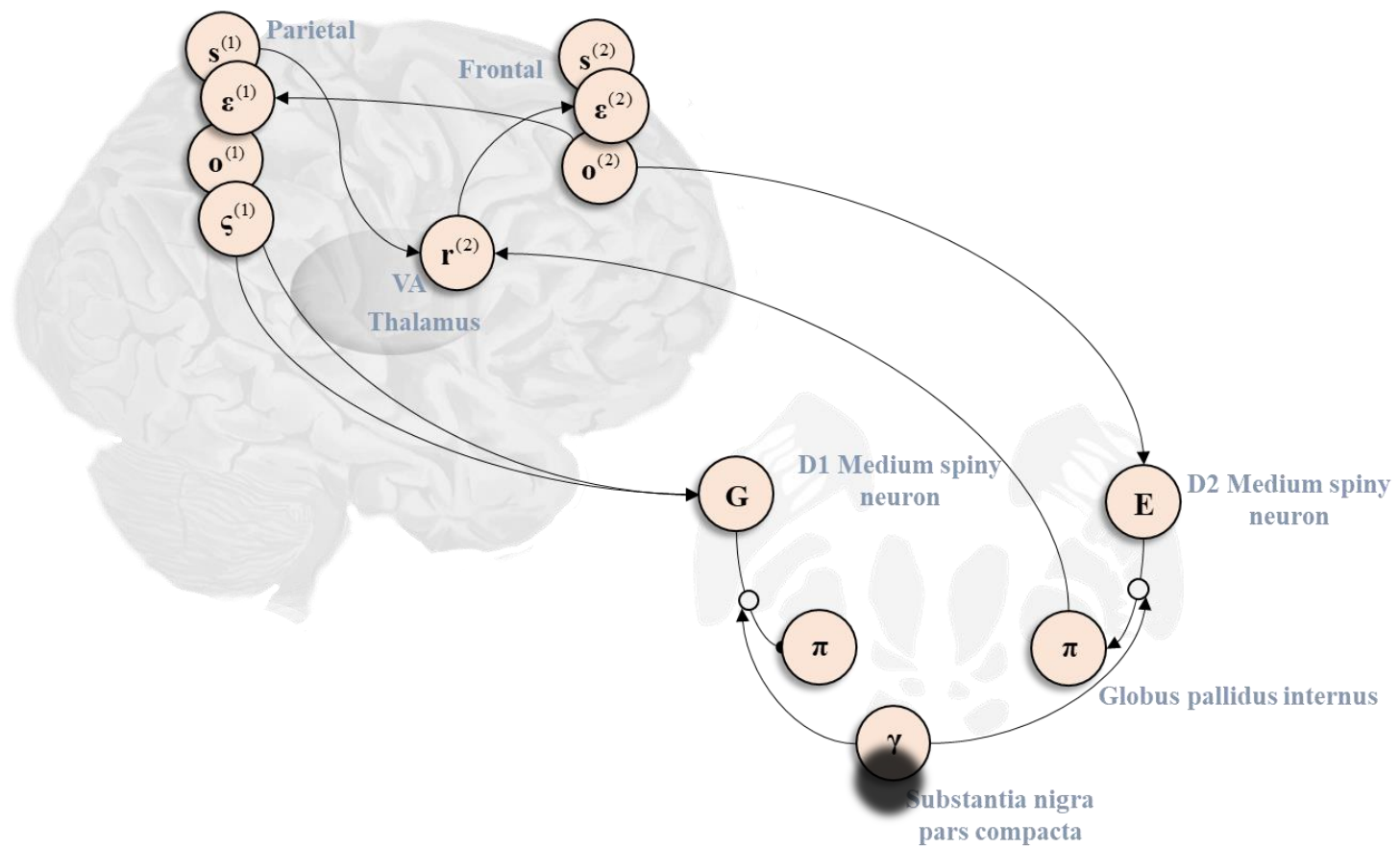
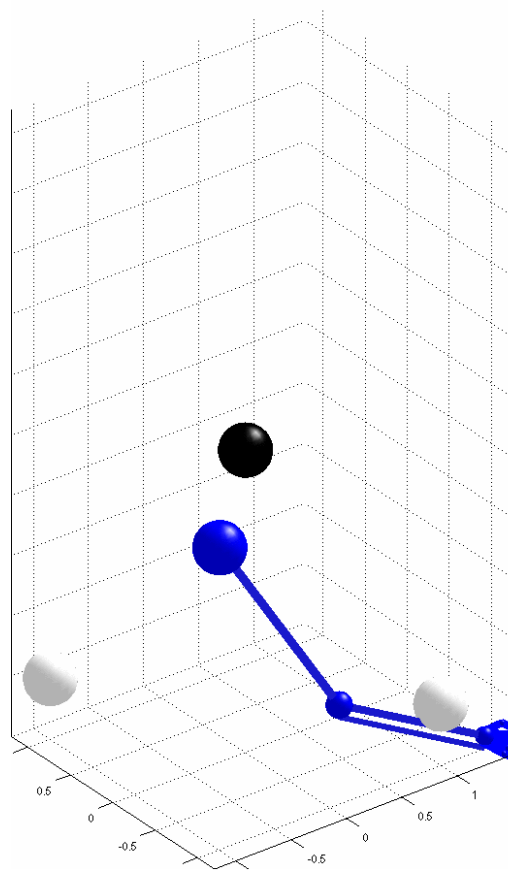


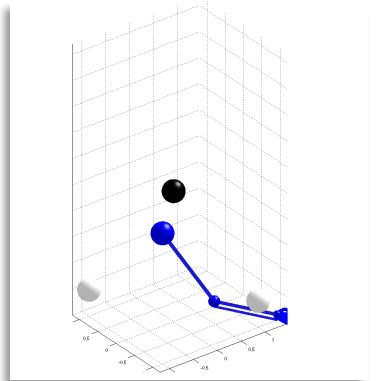
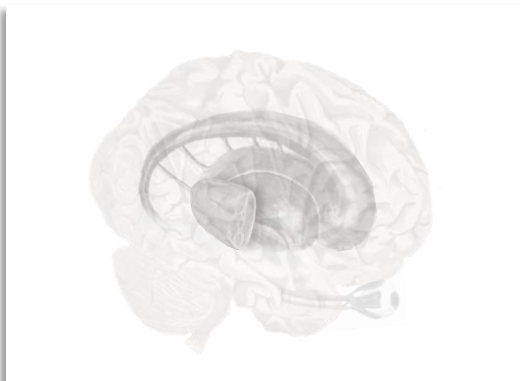
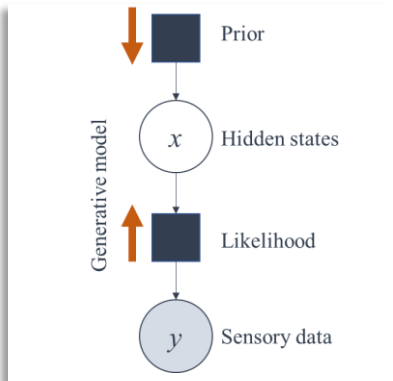
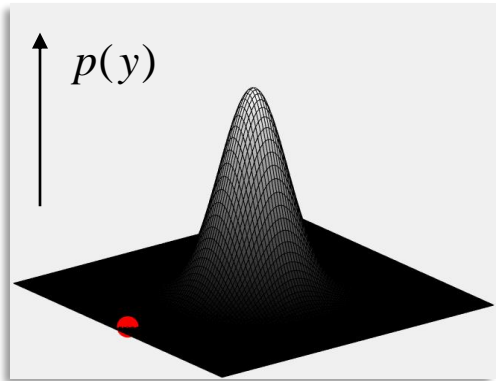
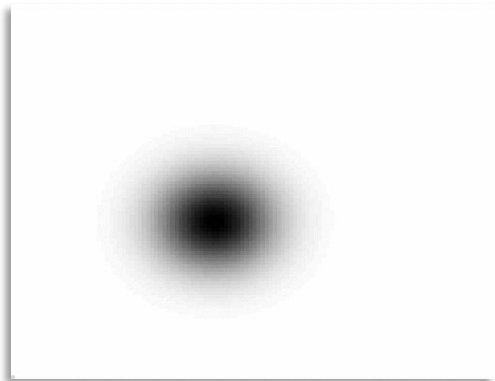












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