Dynamic Causal Modelling for M/EEG: introduction

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Overview

1 DCM: introduction

2 Dynamical systems theory

3 Neural states dynamics

4 Bayesian inference

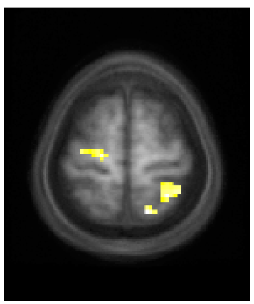
5 Conclusion

Overview

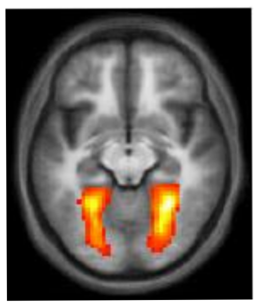
Introduction

from functional segregation to functional integration

localizing brain activity:
functional segregation



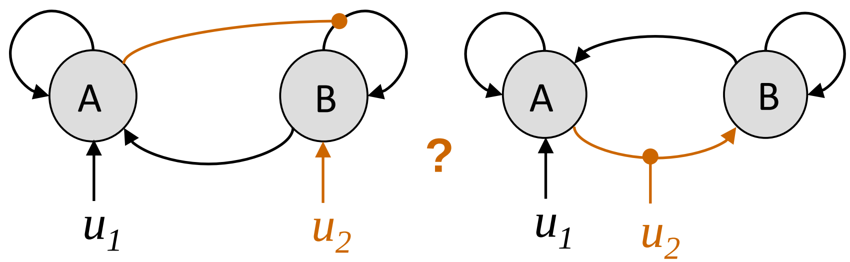
u_1



$u_1 X u_2$

« *Where, in the brain, did my experimental manipulation have an effect?* »

effective connectivity analysis:
functional integration



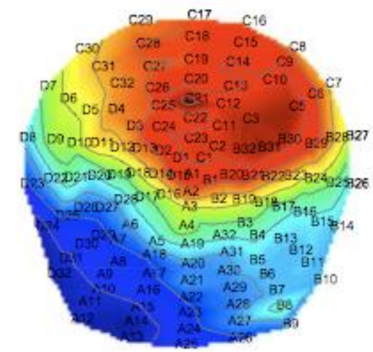
« *How did my experimental manipulation propagate through the network?* »

Introduction

DCM: evolution and observation mappings

Hemodynamic
observation model:
temporal convolution

Electromagnetic
observation model:
spatial convolution



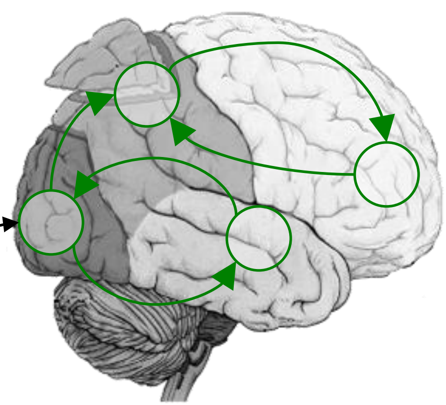
neural states dynamics

$$\dot{x} = f(x, u, \theta)$$

fMRI

EEG/MEG

- simple neuronal model
- realistic observation model



inputs

- realistic neuronal model
- simple observation model

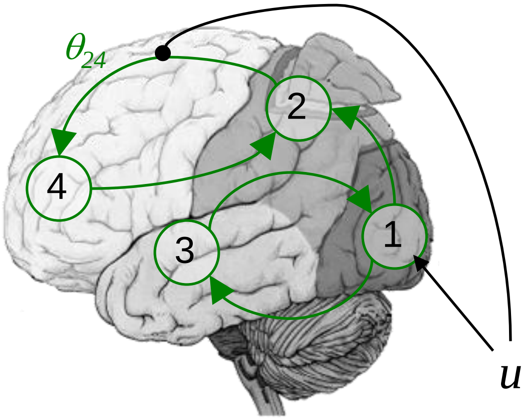
Introduction

DCM: a parametric statistical approach

- DCM: model structure

$$\begin{cases} y = g(x,\varphi) + \varepsilon \\ \dot{x} = f(x,u,\theta) \end{cases} \quad \Rightarrow p(y|\theta,\varphi,m)$$

likelihood



- DCM: Bayesian inference

parameter estimate: $\hat{\theta} = E[\theta|y,m]$

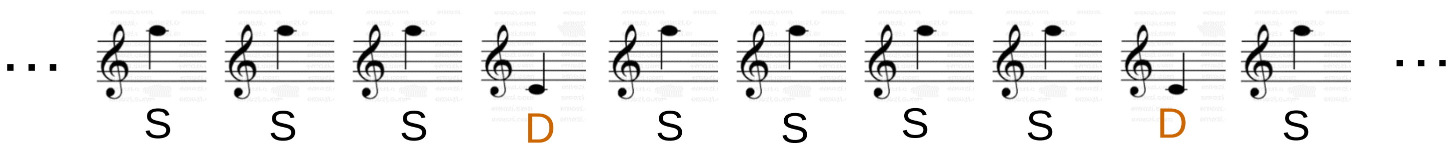
priors on parameters

model evidence: $p(y|m) = \int p(y|\theta,\varphi,m)\, p(\theta|m)\, p(\varphi|m)\, d\varphi d\theta$

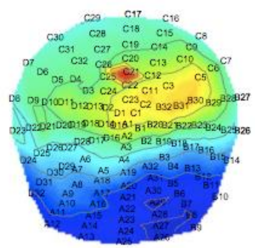
Introduction

DCM for EEG-MEG: auditory mismatch negativity

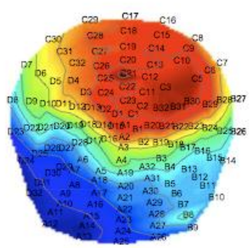
sequence of auditory stimuli



standard condition (S)

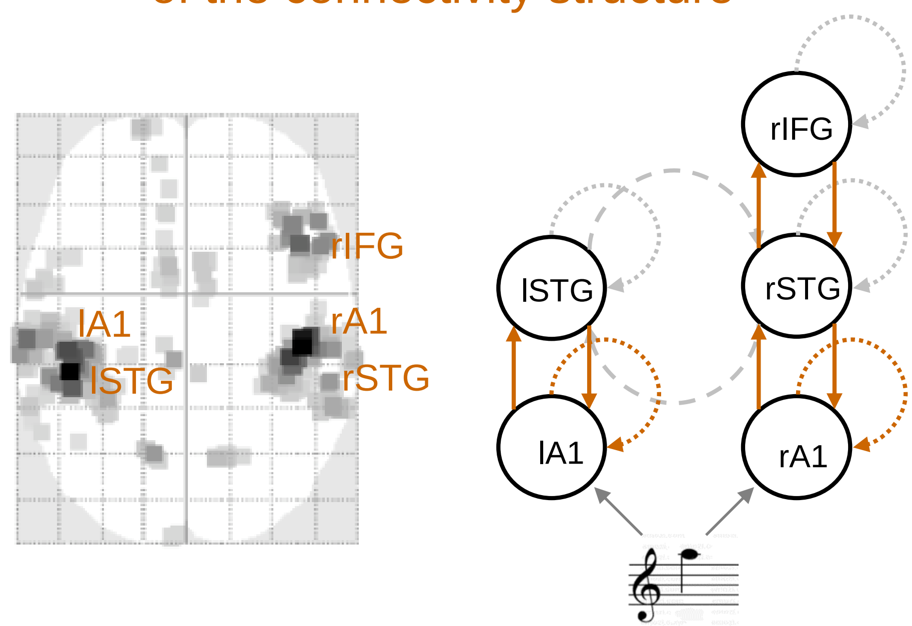


deviant condition (D)



t ~ 200 ms

S-D: **reorganisation**
of the connectivity structure



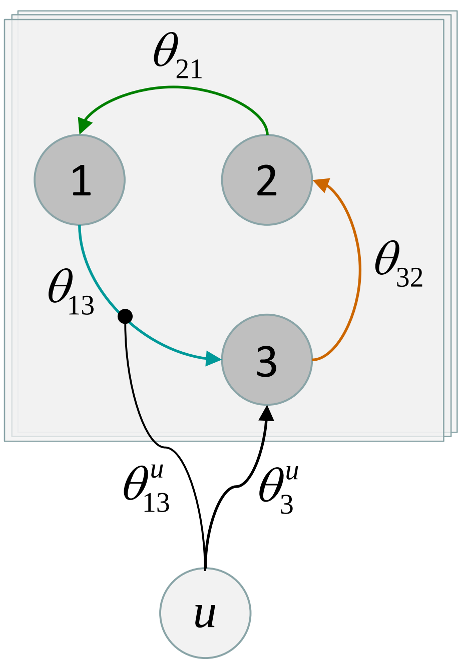
Daunizeau, Kiebel et al., Neuroimage, 2009

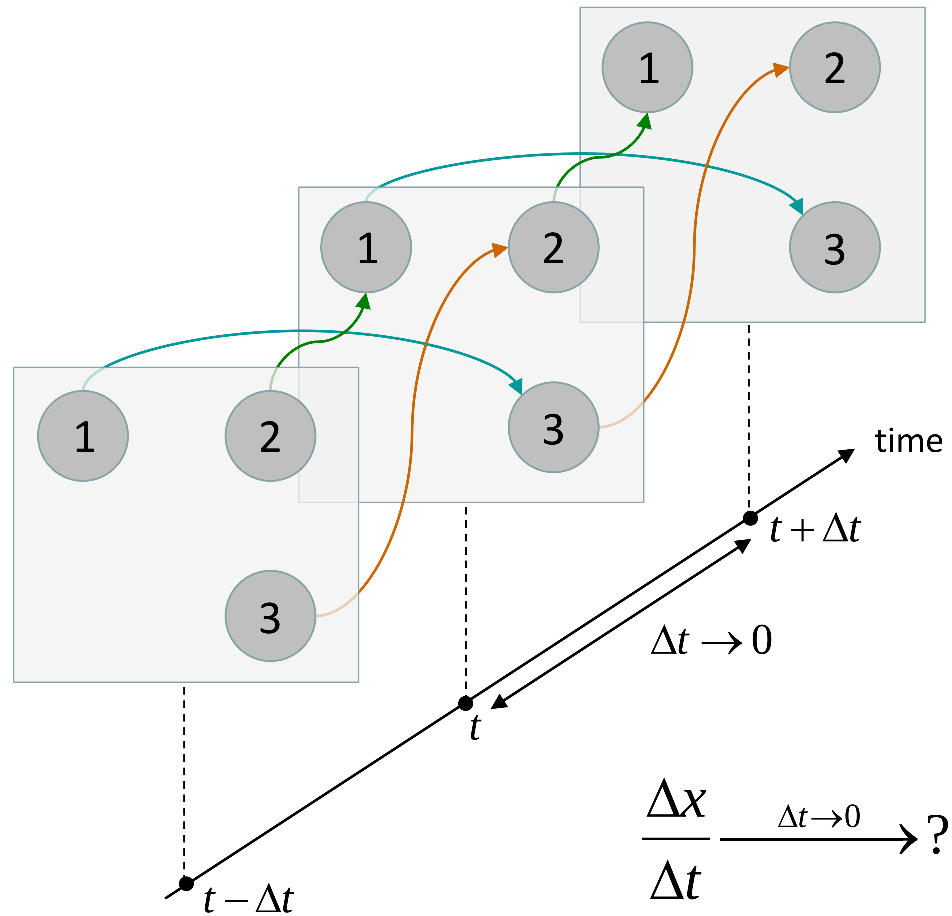
Overview

Dynamical systems theory

dealing with feedback loops

$$u \xrightarrow{\theta} x \xrightarrow{\varphi} y$$





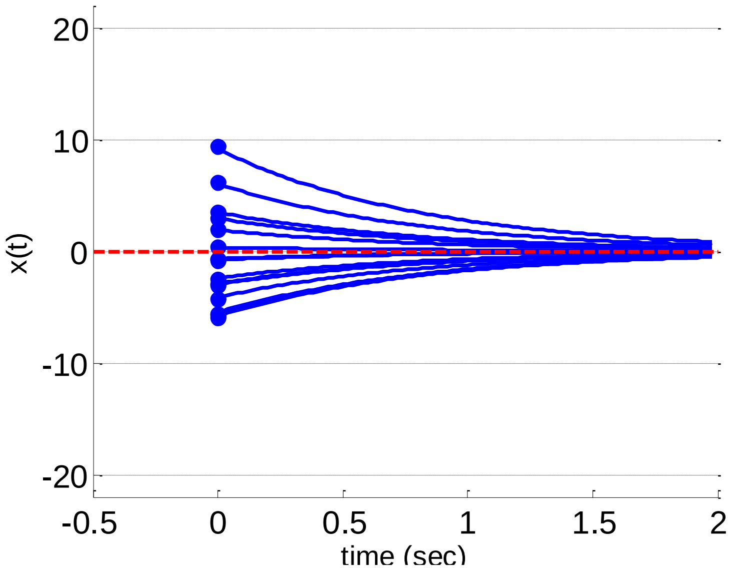
$$\frac{\Delta x}{\Delta t} \xrightarrow{\Delta t \to 0} ?$$

Dynamical systems theory

1D linear dynamical system

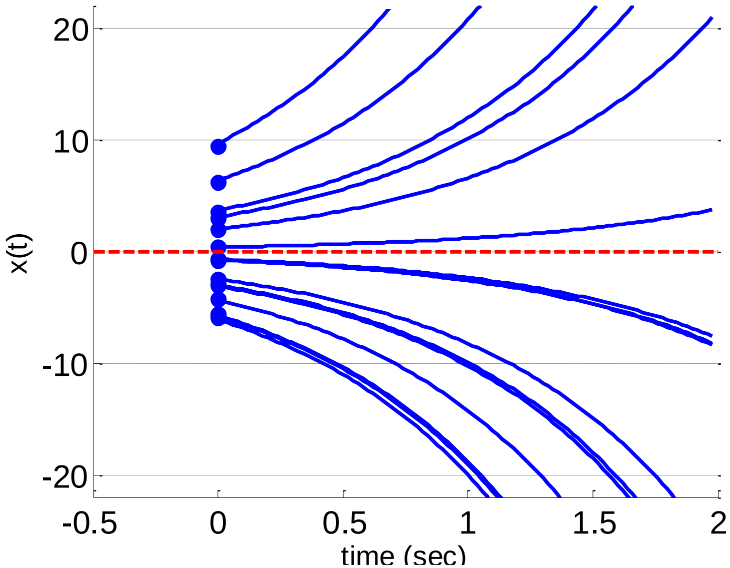
Ordinary Differential Equation (ODE): $\dot{x} = a \times x$

a<0



fixed point = stable

a>0

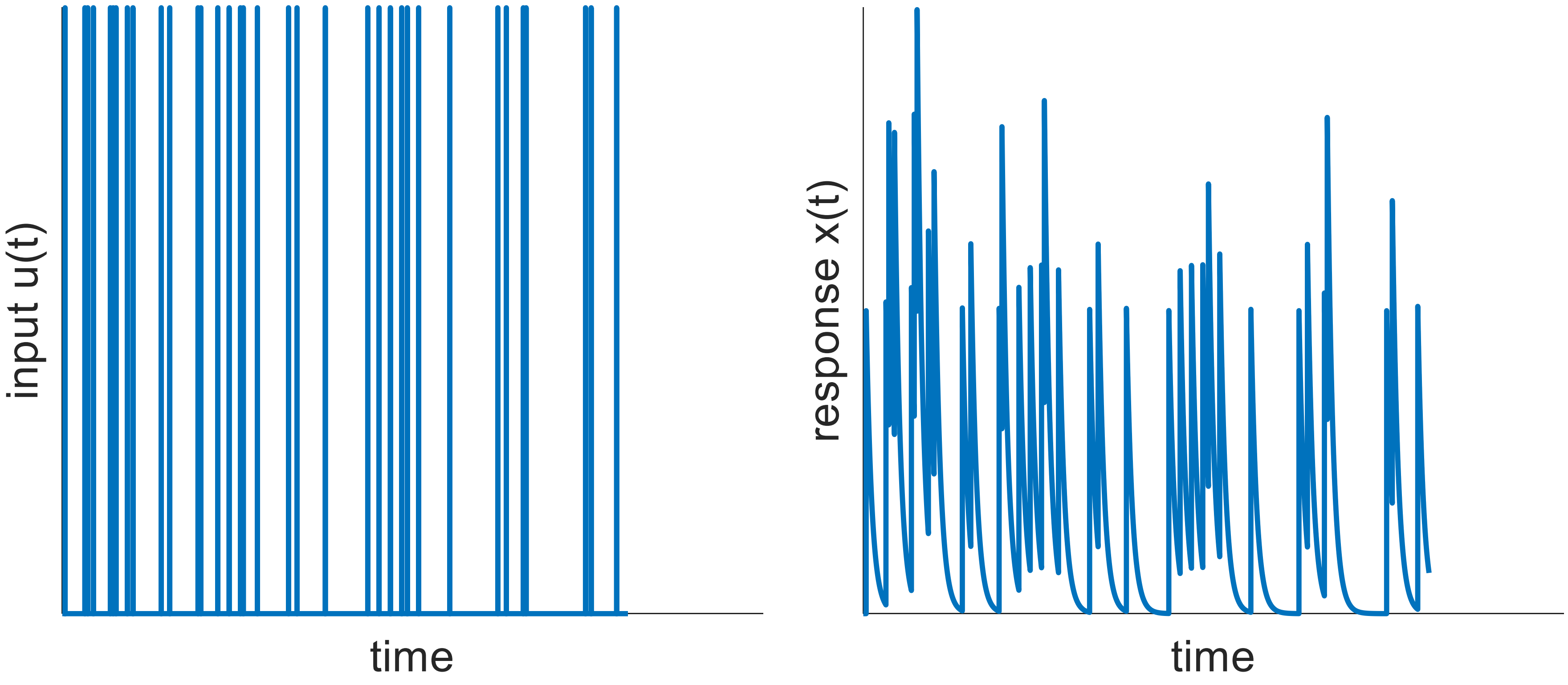


fixed point = unstable

Dynamical systems theory

1D linear dynamical system: input history effects

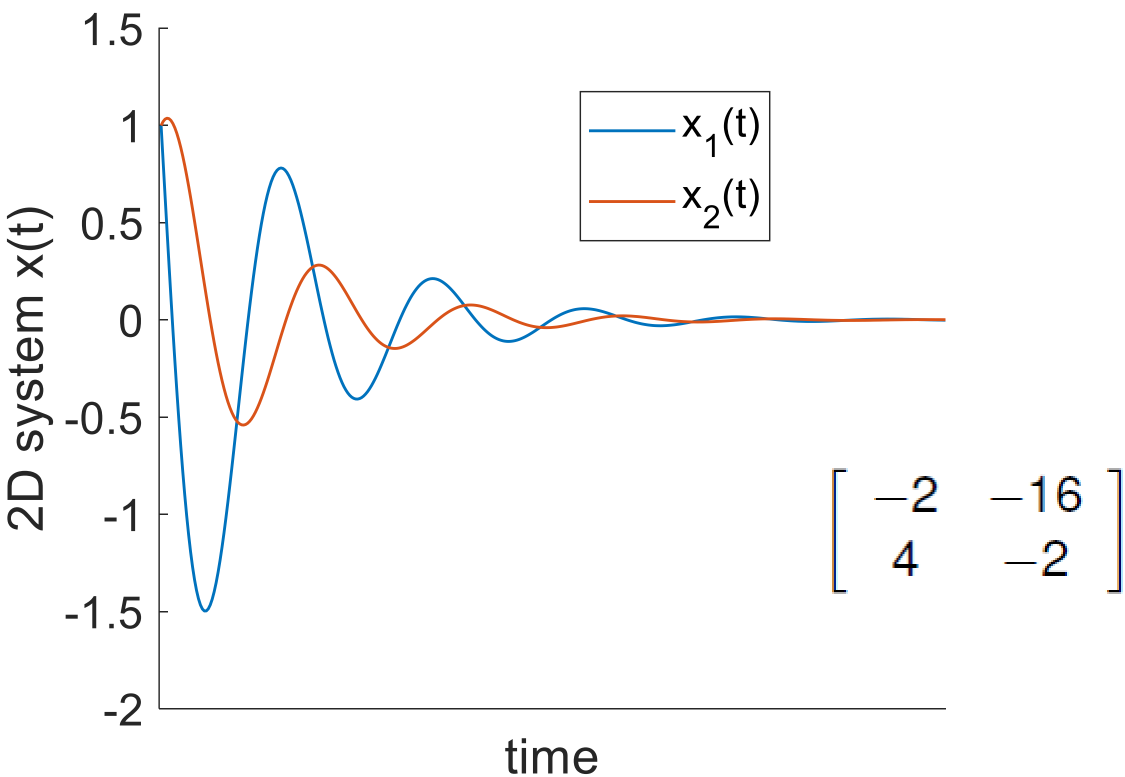
Impact of inputs on the system: $\dot{x} = u - a \times x$



Dynamical systems theory

2D linear dynamical system

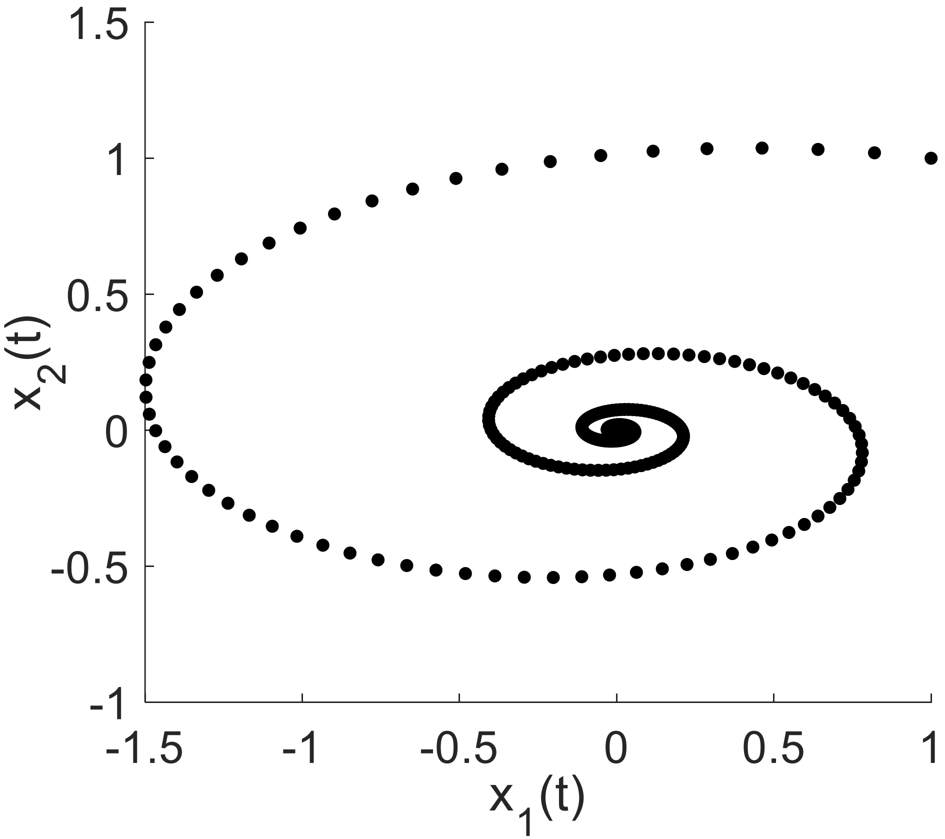
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

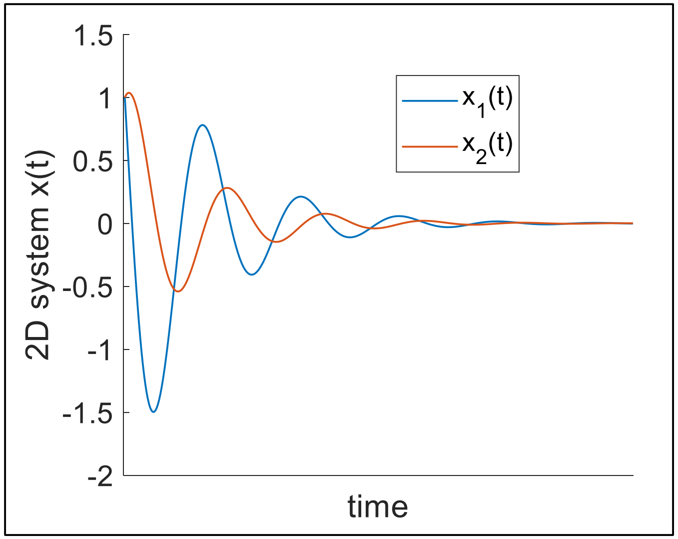


Dynamical systems theory

2D linear dynamical system: states' correlation structure

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



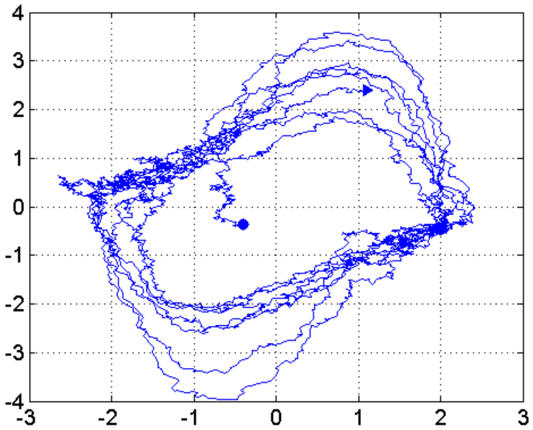


Dynamical systems theory
summary

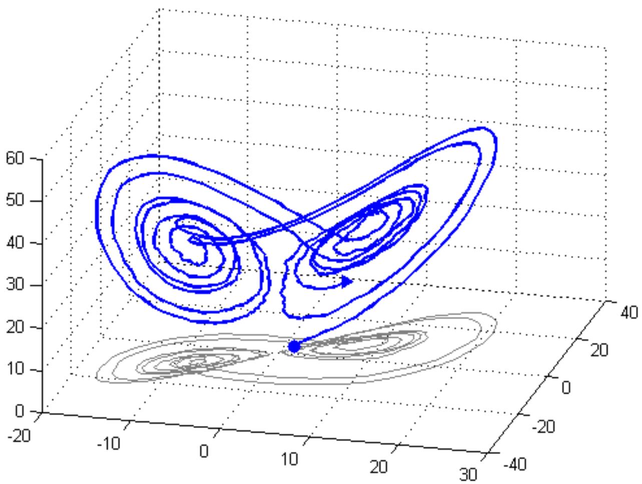
- Motivation: modelling reciprocal influences (feedback loops)

- Linear dynamical systems can be described in terms of their impulse response

- Dynamical *repertoire* depend on the system's dimension (and nonlinearities):
    - D>0: fixed points
    - D>1: spirals
    - D>1: limit cycles (e.g., action potentials)
    - D>2: metastability (e.g., winnerless competition)



limit cycle (Vand Der Pol)



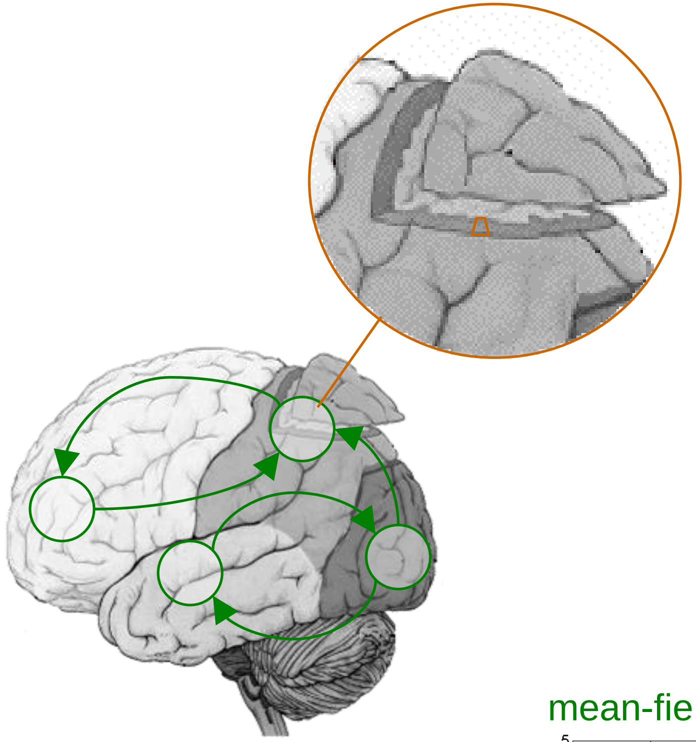
strange attractor (Lorenz)

Overview

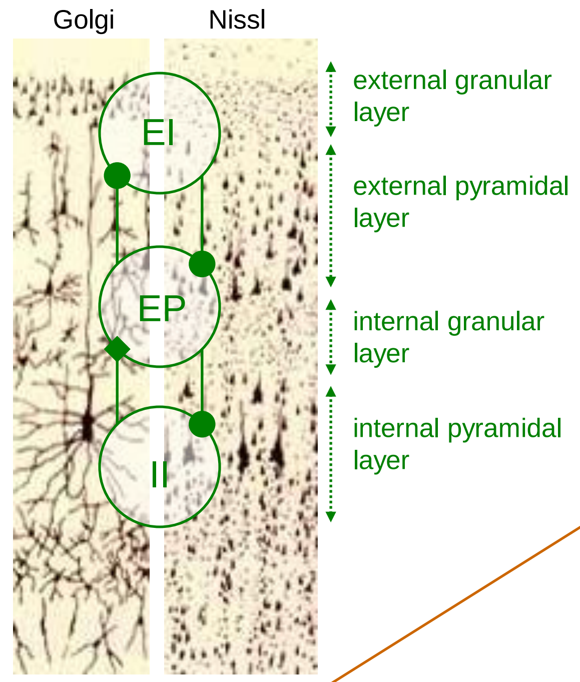
Neural ensembles dynamics

DCM for M/EEG: *systems of neural populations*

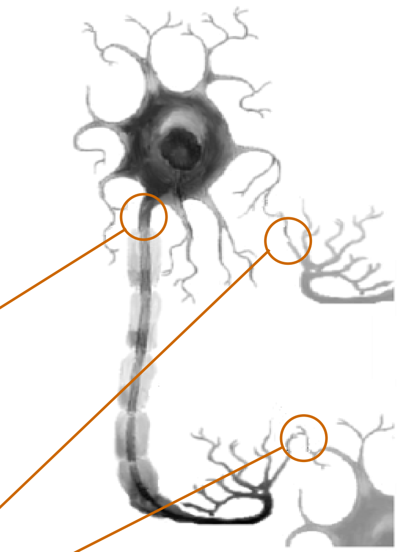
macro-scale



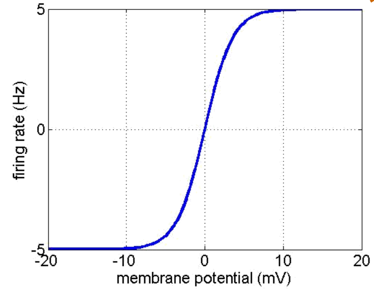
meso-scale



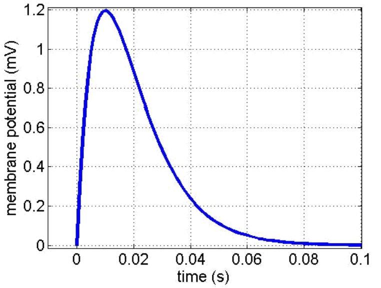
micro-scale



mean-field firing rate

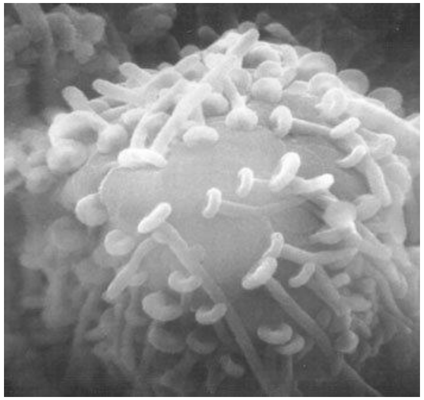


synaptic dynamics



Neural ensembles dynamics

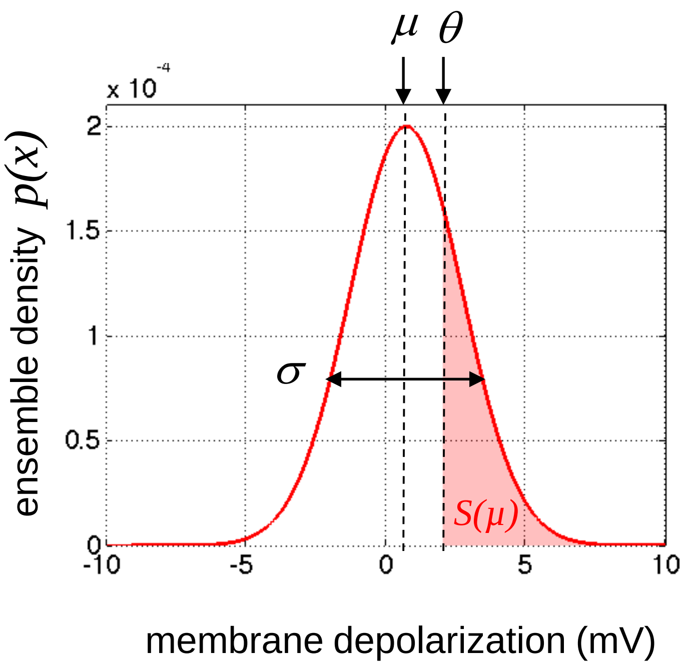
DCM for M/EEG: *from micro- to meso-scale*

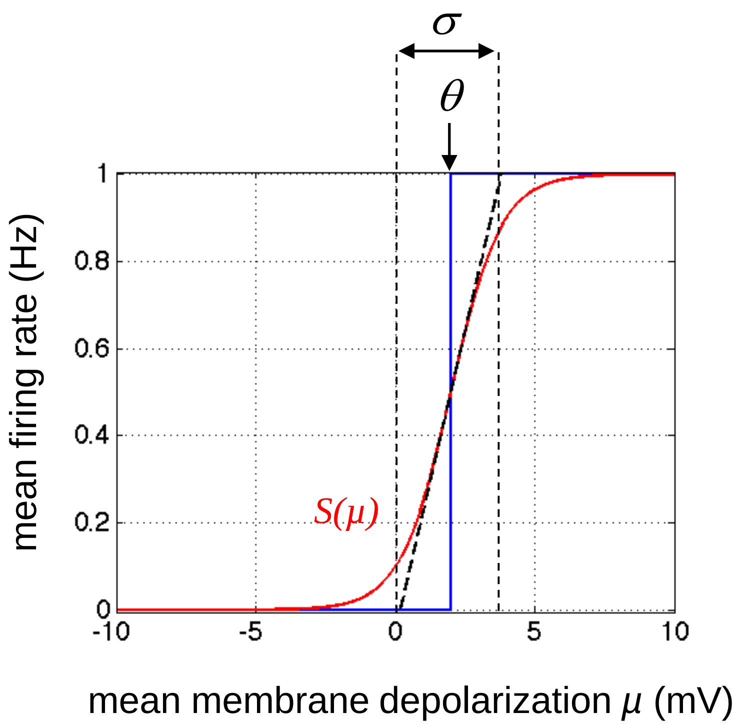


$x_j(t)$: post-synaptic potential of j^{th} neuron within its ensemble

$$\frac{1}{N-1}\sum_{j'\neq j} H\left(x_{j'}(t)-\theta\right) \xrightarrow{N\to\infty} \int H\left(x(t)-\theta\right) p\left(x(t)\right) dx$$

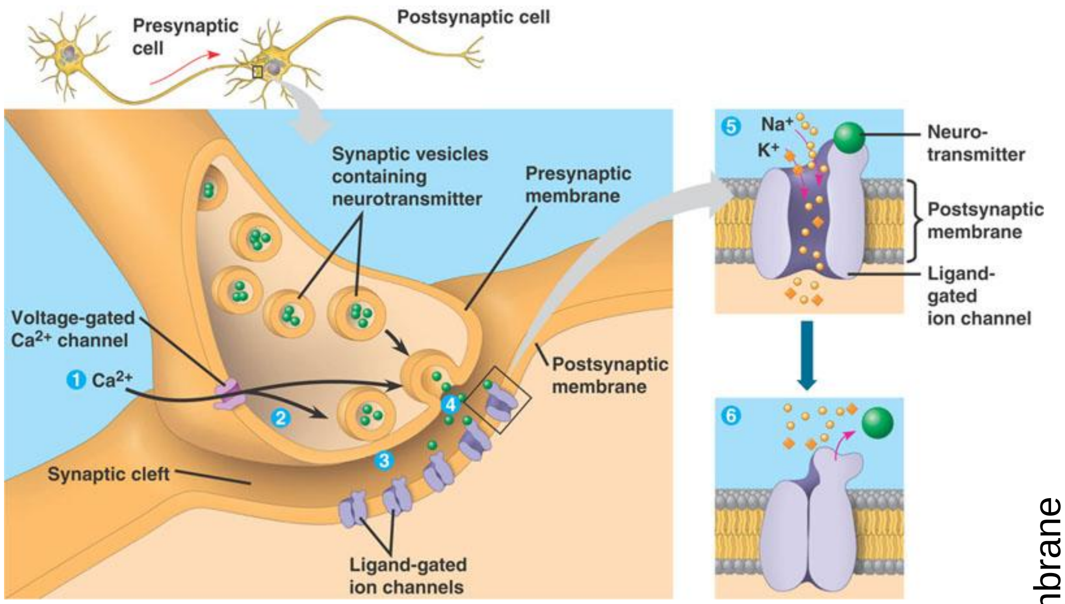
$\approx S(\mu)$ mean-field firing rate



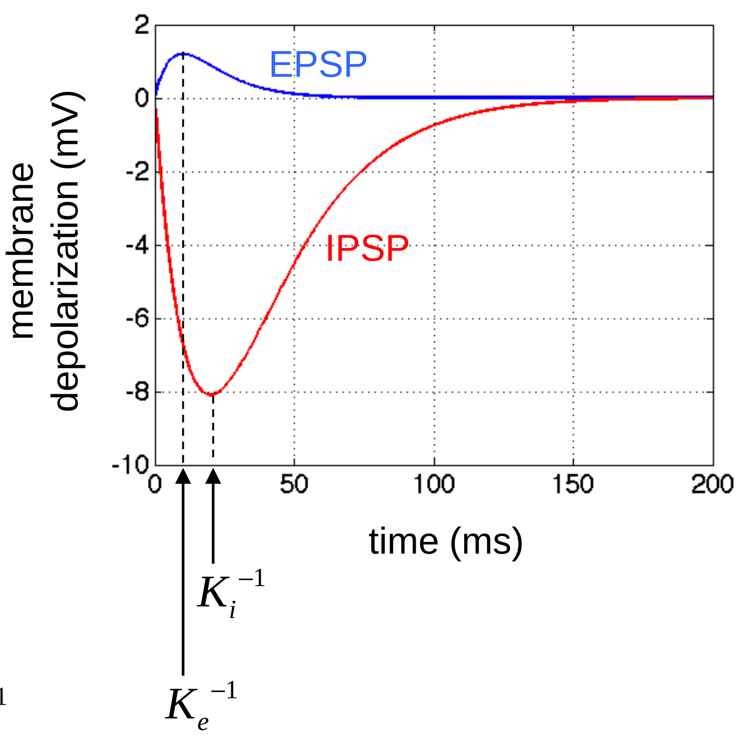


Neural ensembles dynamics

DCM for M/EEG: *synaptic dynamics*



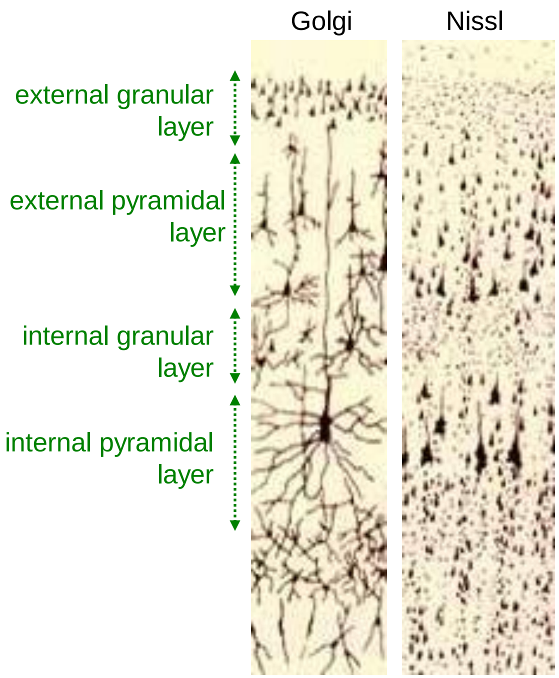
post-synaptic potential

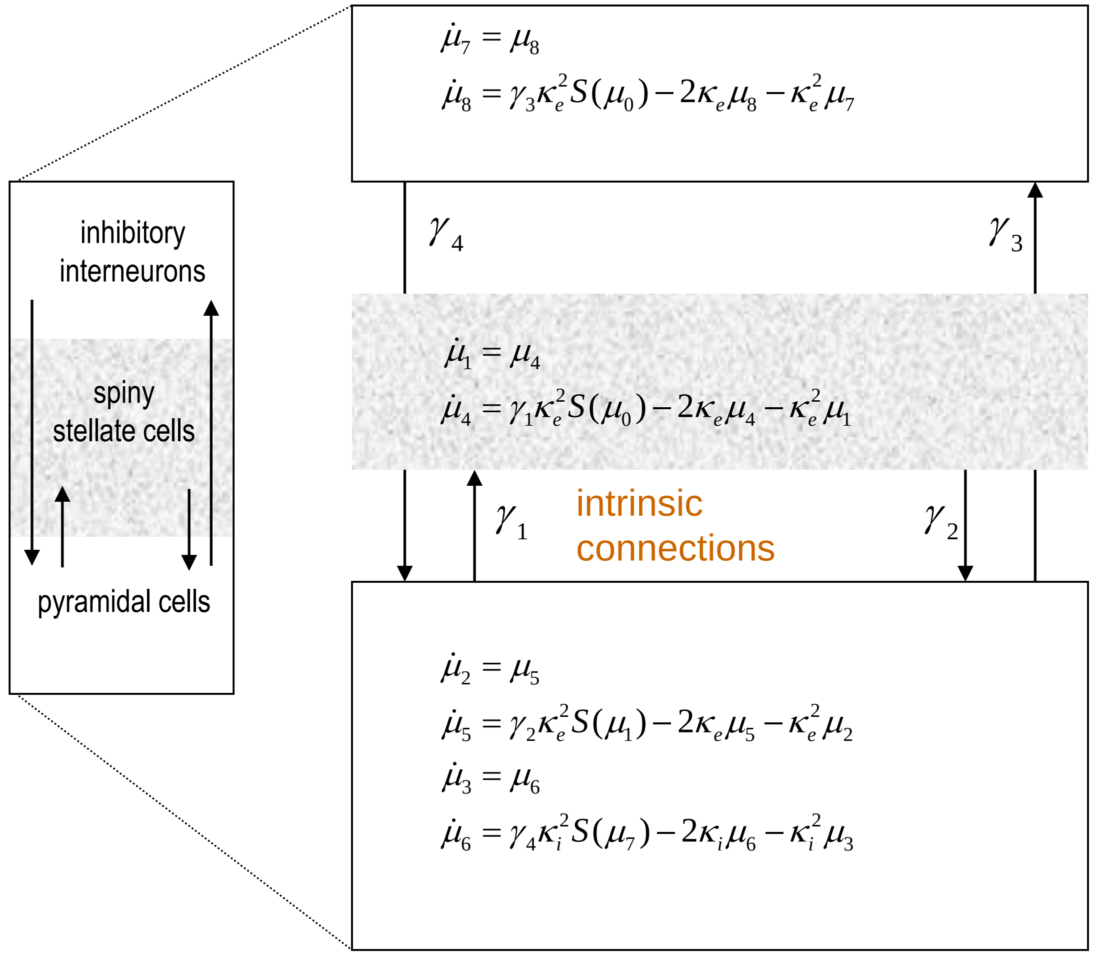


$$
\begin{cases}
\dot{\mu}_1 = \mu_2 \\
\dot{\mu}_2 = \kappa_{i/e}^2 \, S(\bullet) - 2\kappa_{i/e}\,\mu_2 - \kappa_{i/e}^2\,\mu_1
\end{cases}
$$

Neural ensembles dynamics

DCM for M/EEG: *intrinsic connections within the cortical column*



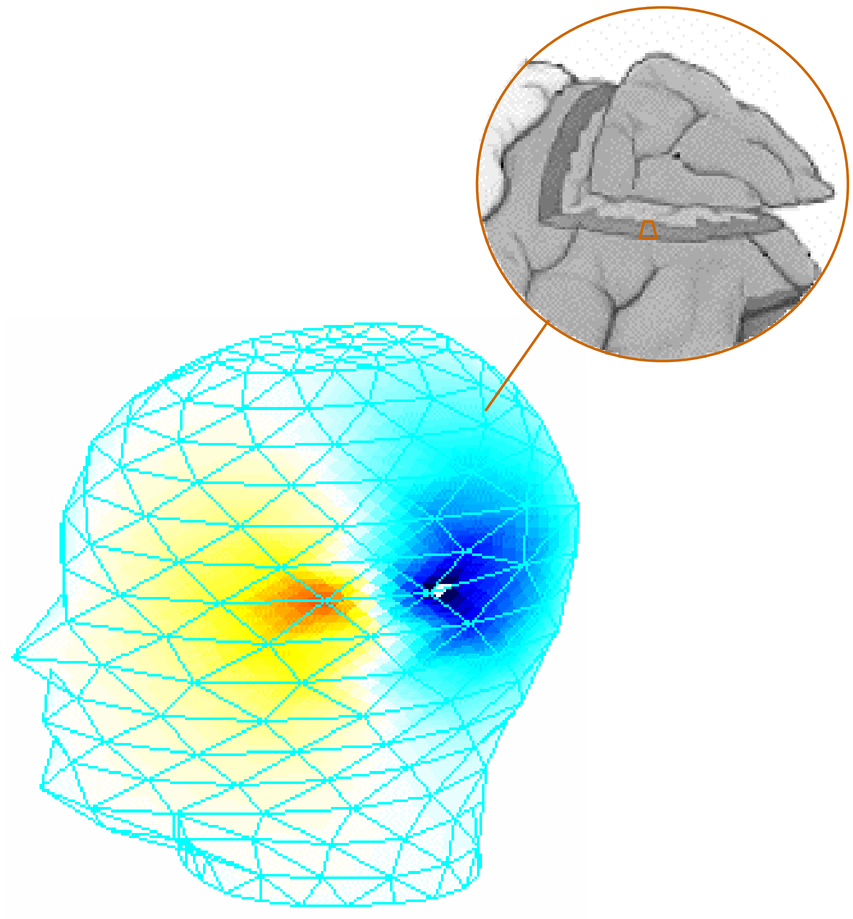


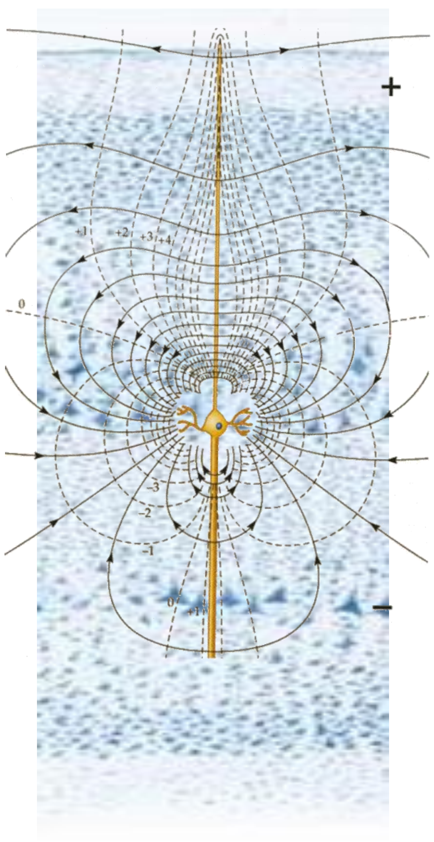
Overview

Observation mappings

DCM for M/EEG: the electromagnetic forward problem

$$\mathbf{y}(t) = \sum_i \mathbf{L}^{(i)} \sum_j \beta_j \mu^{(ij)}(t)$$





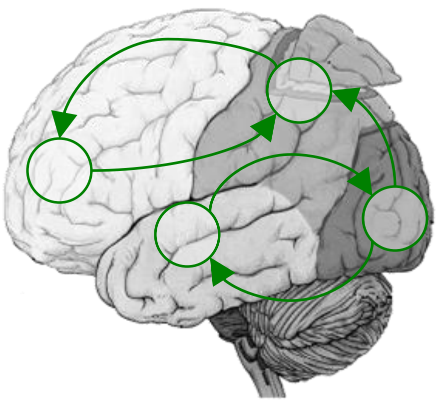
Bayesian inference

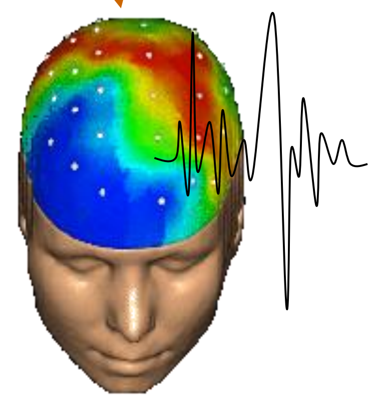
forward and inverse problems

forward problem

$$p\left(y \middle| \vartheta, m\right)$$

likelihood





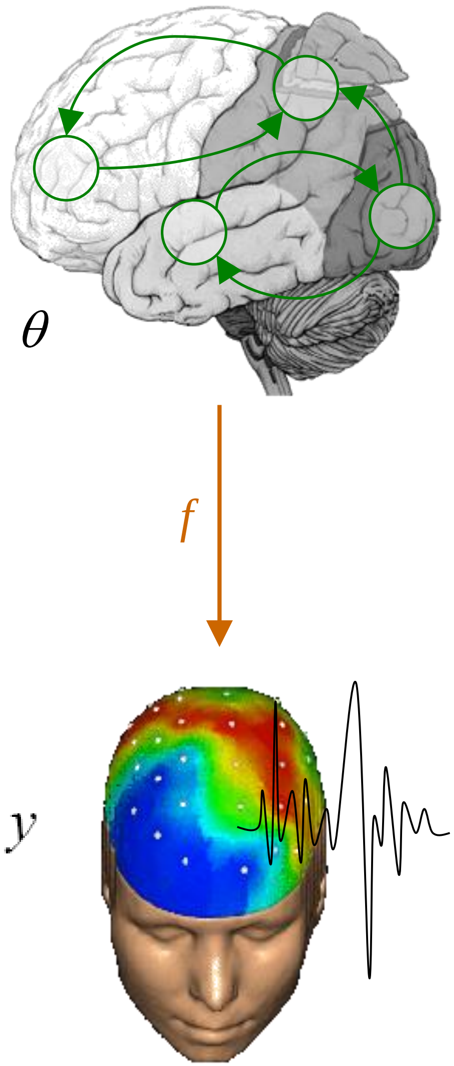
posterior distribution

$$p\left(\vartheta \middle| y, m\right)$$

inverse problem

Bayesian paradigm

deriving the likelihood function



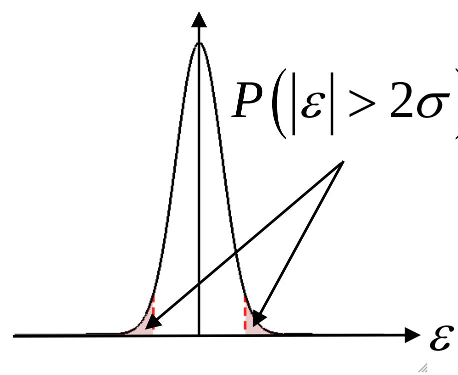
- Model of data with unknown parameters:

$$y = f(\theta) \qquad \text{e.g., GLM:} \quad f(\theta) = X\theta$$

- But data is noisy: $y = f(\theta) + \varepsilon$

- Assume noise/residuals is 'small':

$$p(\varepsilon) \propto \exp\left(-\frac{1}{2\sigma^2}\varepsilon^2\right)$$



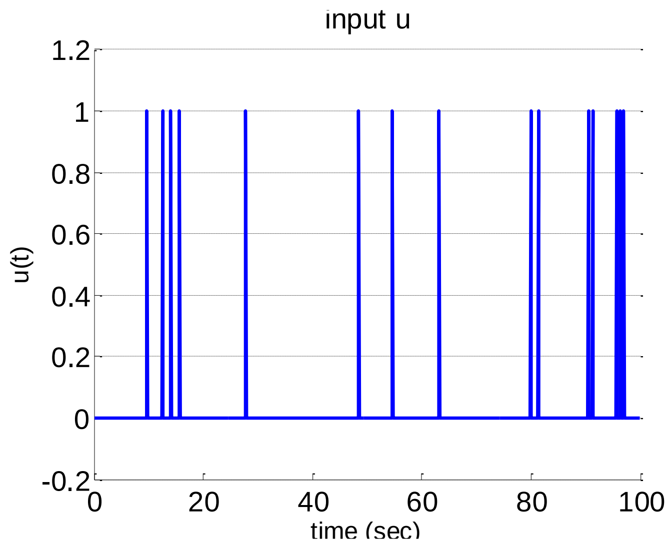
$$P\left(|\varepsilon| > 2\sigma\right) \approx 0.05$$

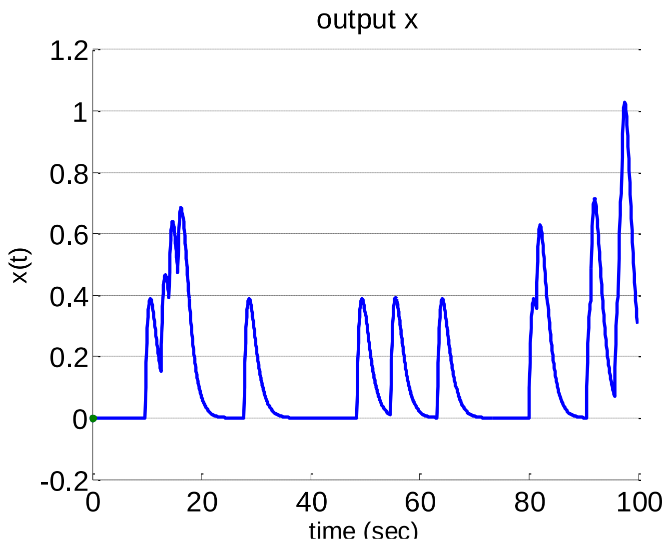
→ Distribution of data, *given fixed parameters*:

$$p(y|\theta) \propto \exp\left(-\frac{1}{2\sigma^2}\left(y - f(\theta)\right)^2\right)$$

Bayesian paradigm

the likelihood function of an alpha kernel



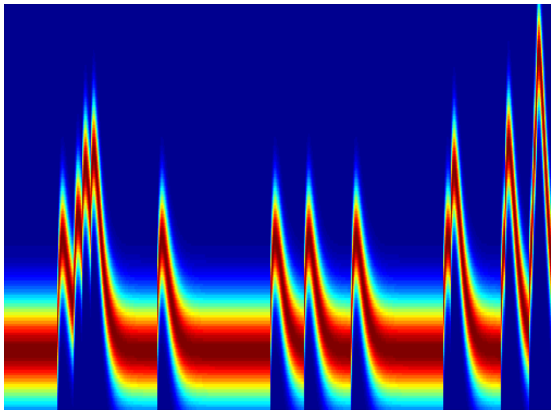


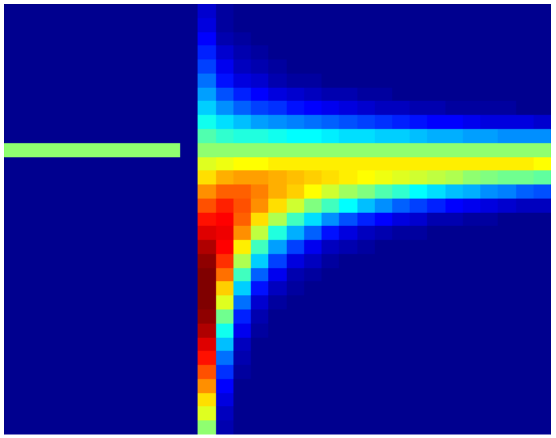
$$p\left(y\middle|\theta,m\right)$$

holding the parameters fixed

$$p\left(y\middle|\theta,m\right)$$

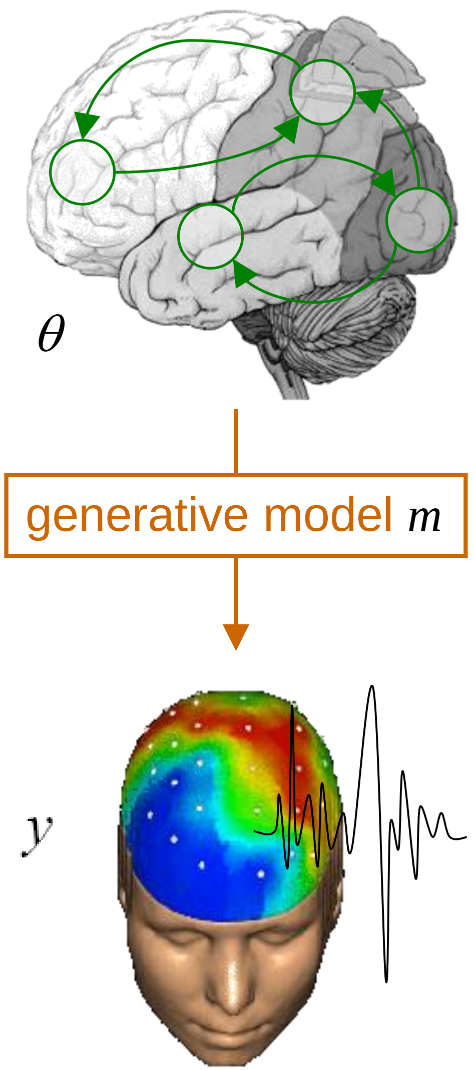
holding the data fixed





Bayesian paradigm

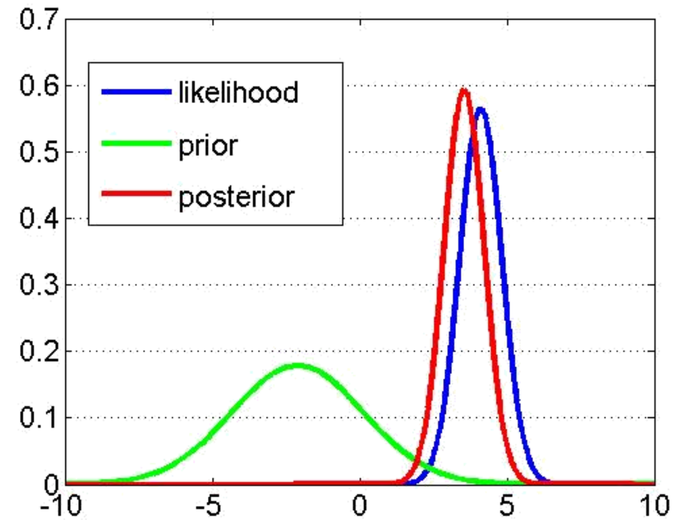
likelihood, priors and the model evidence



Likelihood: $p(y|\theta, m)$

Prior: $p(\theta|m)$

Bayes rule: $p(\theta|y,m) = \dfrac{p(y|\theta,m)\,p(\theta|m)}{p(y|m)}$



Bayesian inference

type, role and impact of priors

- Types of priors:
  - ✓ Explicit priors on *model parameters* (e.g., connection strengths)
  - ✓ Implicit priors on *model functional form* (e.g., system dynamics)
  - ✓ Choice of “interesting” *data features* (e.g., ERP vs phase data)

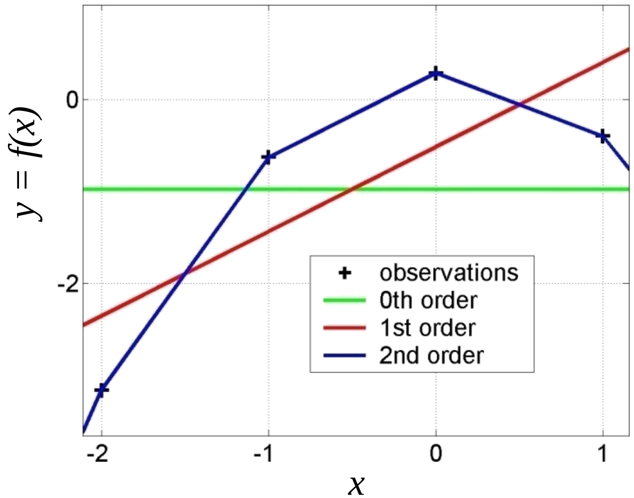
- Role of priors (on model parameters):
  - ✓ Resolving the *ill-posedness* of the inverse problem
  - ✓ Avoiding *overfitting* (cf. generalization error)

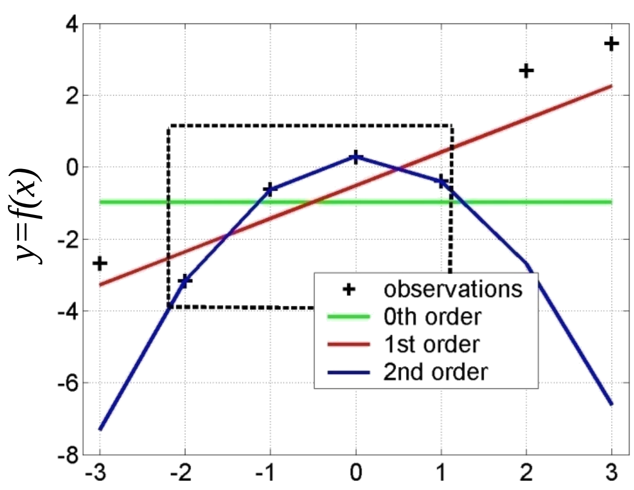
- Impact of priors:
  - ✓ On parameter posterior distributions (cf. “shrinkage to the mean” effect)
  - ✓ On model evidence (cf. “Occam’s razor”)
  - ✓ On free-energy landscape (cf. Laplace approximation)

Bayesian inference

model comparison

Principle of parsimony :
« plurality should not be assumed without necessity »

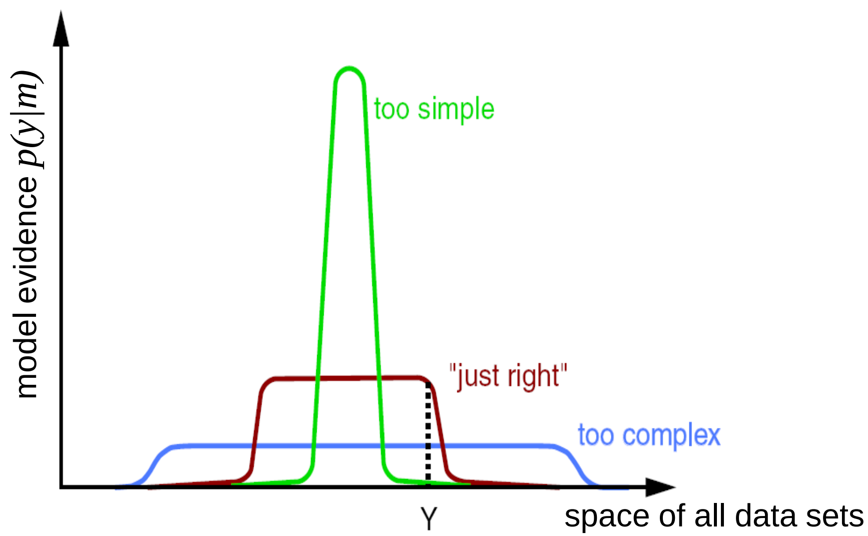




Model evidence:

$$p(y|m) = \int p(y|\vartheta, m)\, p(\vartheta|m)\, d\vartheta$$

"Occam's razor" :

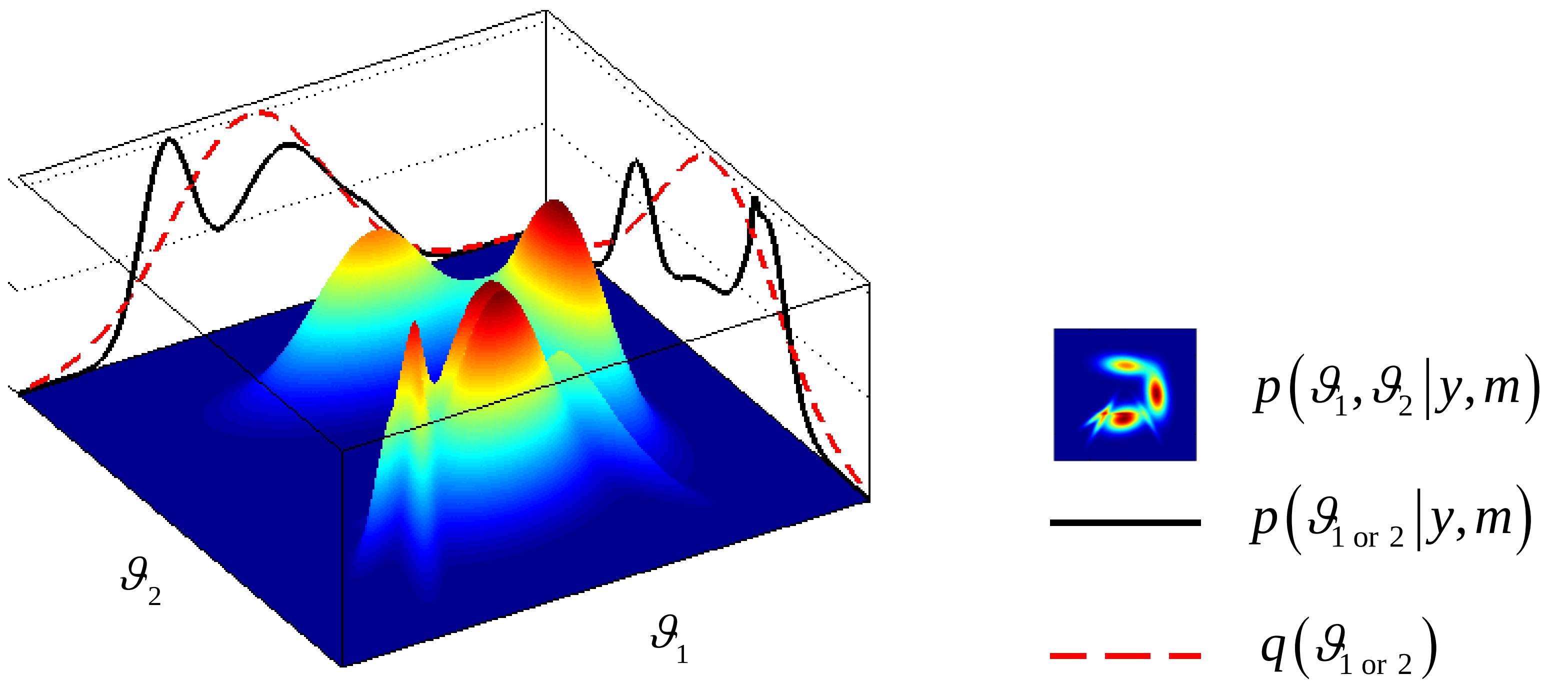


Bayesian inference

the variational Bayesian approach

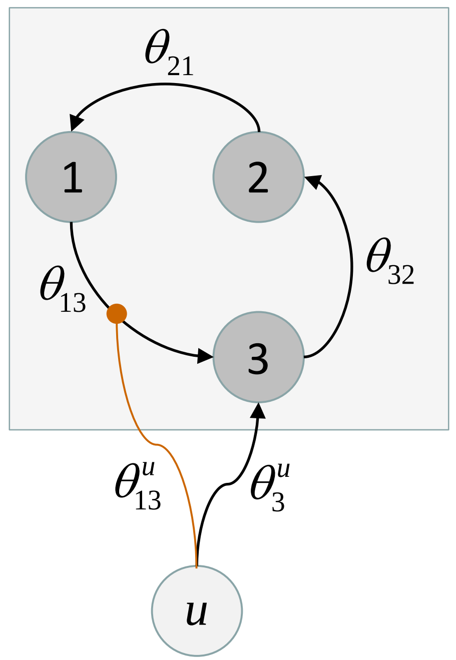
$$\ln p\left(y|m\right) = \underbrace{\left\langle \ln p\left(\vartheta, y|m\right)\right\rangle_q + S\left(q\right)}_{\text{free energy : functional of } q} + D_{KL}\left(q\left(\vartheta\right); p\left(\vartheta|y,m\right)\right)$$

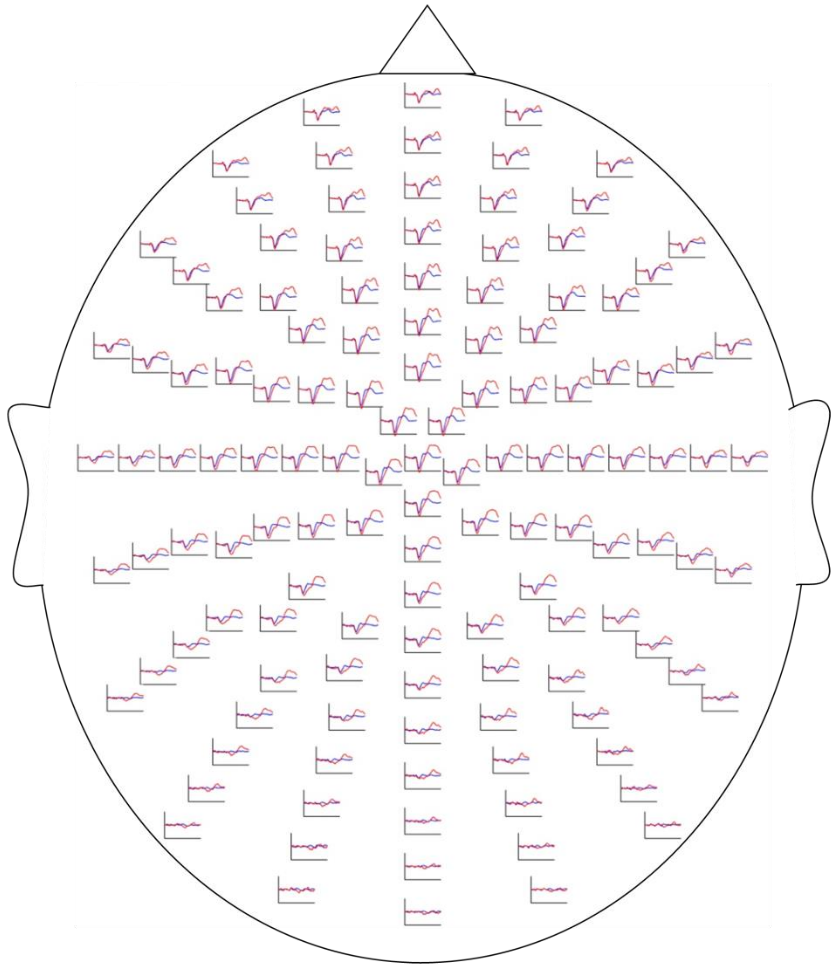
mean-field: approximate marginal posterior distributions: $\left\{q\left(\vartheta_1\right), q\left(\vartheta_2\right)\right\}$



Bayesian inference

DCM: key model parameters





$(\theta_{21}, \theta_{32}, \theta_{13})$ state-state coupling

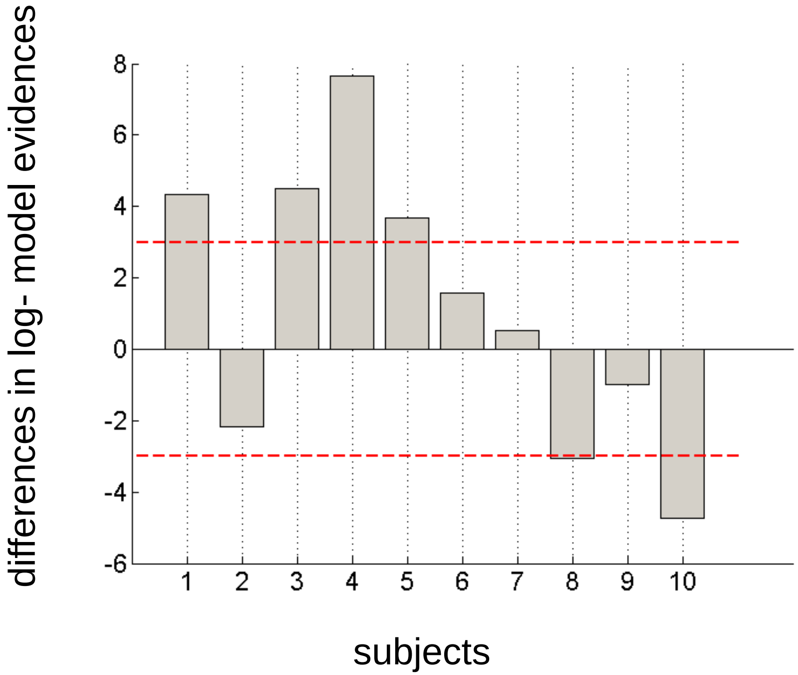
θ_3^u input-state coupling

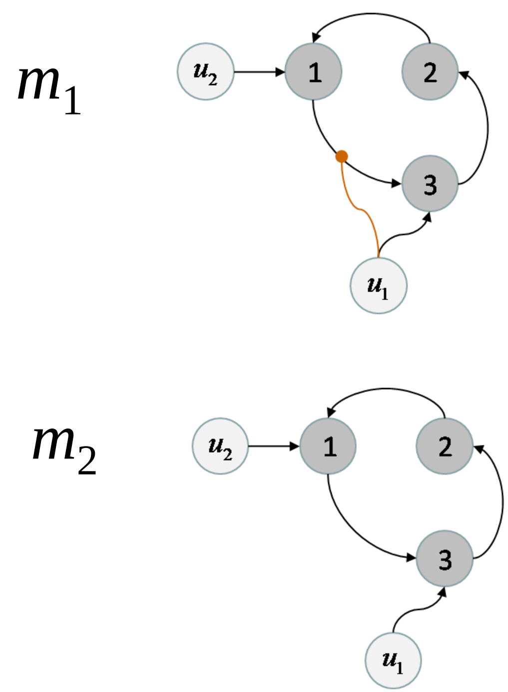
θ_{13}^u input-dependent modulatory effect

Bayesian inference

model comparison for group studies

$$\ln p\left(y \middle| m_1\right) - \ln p\left(y \middle| m_2\right)$$





fixed effect assume all subjects correspond to the same model

random effect assume different subjects might correspond to different models

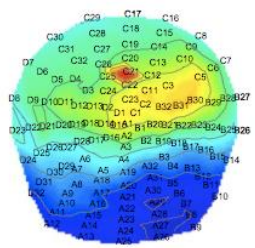
Overview

Conclusion

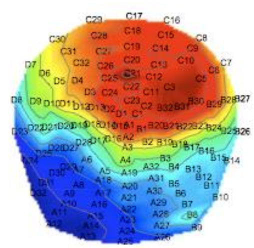
back to the auditory mismatch negativity

sequence of auditory stimuli

standard condition (S)

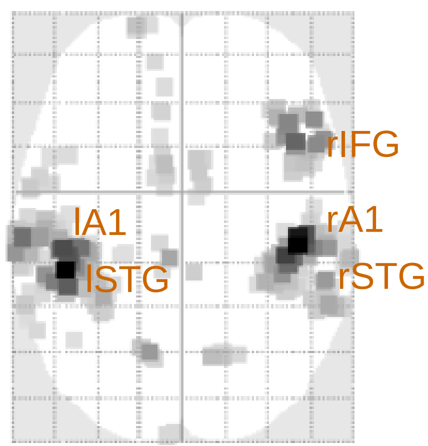


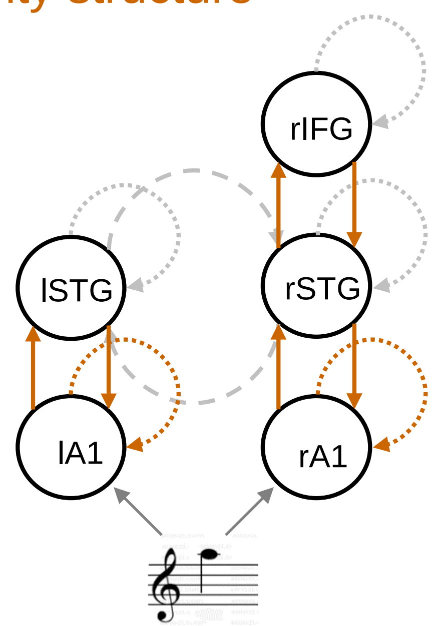
deviant condition (D)



t ~ 200 ms

S-D: **reorganisation** of the connectivity structure

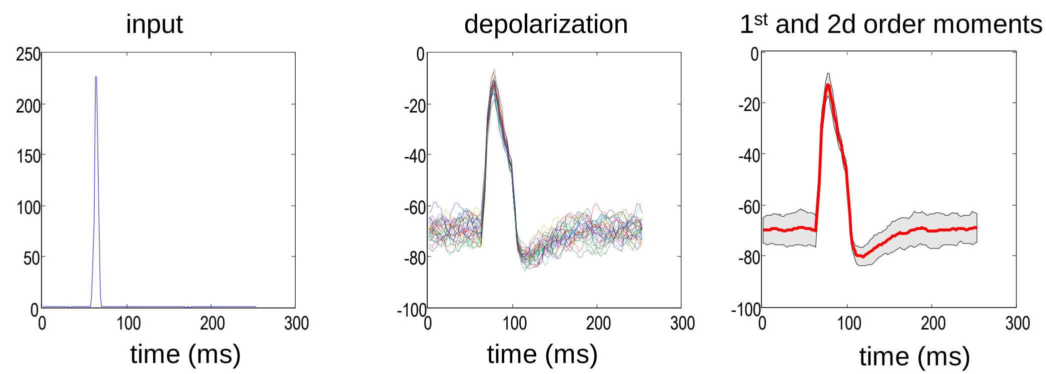




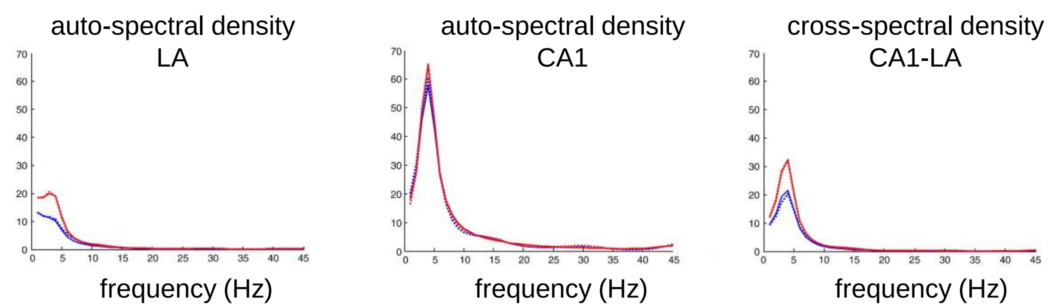
Conclusion

DCM for M/EEG: variants

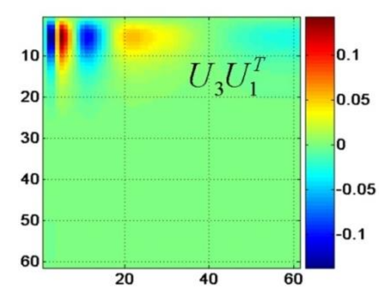
- second-order mean-field DCM



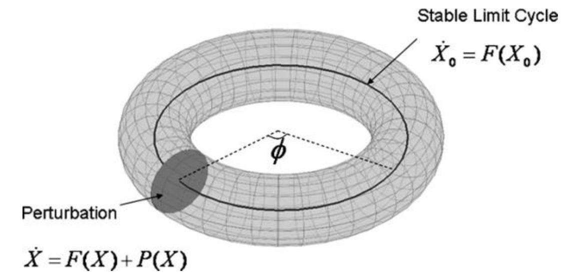
- DCM for steady-state responses



- DCM for induced responses



- DCM for phase coupling



Conclusions

- Objectives of a DCM study
  - Compare candidate interpretations of an effect of interest
  - Assess micro- and/or meso-scopic mechanisms ("mathematical microscope")
  - …
- Assumptions
  - Biophysical/neurophysiological (e.g., neural ensemble dynamics…)
  - Statistical (e.g., Gaussian residuals…)
  - Algorithmic (e.g., ODE integration scheme, VB,…)
- Limitations
  - Robustness to violations of assumptions
    - → Family inference, validity analysis,…
  - Reliability of statistical inference
    - → Parameter recovery and/or confusion analysis
  - Interpretation

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