# Models of Connectivity: Advanced Hierarchical Unsupervised Generative Embedding (HUGE)

Yu Yao & Stefan Frässle



Computational Psychiatry Course 2020 Zurich | 10<sup>th</sup> September 2020





Eidgenössische Technische Hochschule Zürich Swiss Federal Institute of Technology Zurich

### Introduction

**Hierarchical** 

**U**nsupervised

Generative

**E**mbedding

### Introduction

Hierarchical

Unsupervised

**G**enerative

**E**mbedding

- model-based dimensionality reduction
- mechanically interpretable features
- stratification of heterogeneous cohorts

# Generative Embedding for Model-Based Classification of fMRI Data

Kay H. Brodersen<sup>1,2</sup>\*, Thomas M. Schofield<sup>3</sup>, Alexander P. Leff<sup>3</sup>, Cheng Soon Ong<sup>1</sup>, Ekaterina I. Lomakina<sup>1,2</sup>, Joachim M. Buhmann<sup>1</sup>, Klaas E. Stephan<sup>2,3</sup>

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#### Abstract

Decoding models, such as those underlying multivariate classification algorithms, have been increasingly used to infer cognitive or clinical brain states from measures of brain activity obtained by functional magnetic resonance imaging (fMRI). The practicality of current classifiers, however, is restricted by two major challenges. First, due to the high data dimensionality and low sample size, algorithms struggle to separate informative from uninformative features, resulting in poor generalization performance. Second, popular discriminative methods such as support vector machines (SVMs) rarely afford mechanistic interpretability. In this paper, we address these issues by proposing a novel generative-embedding approach that incorporates neurobiologically interpretable generative models into discriminative classifiers. Our approach extends previous work on trial-by-trial classification for electrophysiological recordings to subject-by-subject classification for fMRI and offers two key advantages over conventional methods: it may provide more accurate predictions by exploiting discriminative information encoded in 'hidden' physiological quantities such as synaptic connection strengths; and it affords mechanistic interpretability of clinical classifications. Here, we introduce generative embedding for fMRI using a

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Model: DCM; effective connectivity

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- Model: DCM; effective connectivity
- classification: supervised learning

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- Model: DCM; effective connectivity
- classification: supervised learning
- data: fMRI

#### **Dataset:**

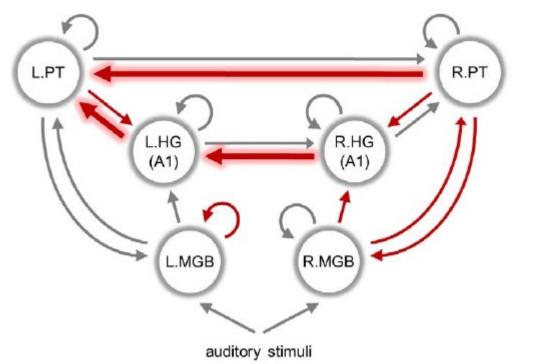
- patients (N=11) vs. controls (N=26)
  - moderate aphasia due to stroke
- passive speech listening
  - (i) normal speech
  - (ii) time-reversed speech
- 1.5 T, TR 3.15 s

Leff AP, Schofield TM, Stephan KE, Crinion JT, Friston KJ, et al. (2008) The cortical dynamics of intelligible speech. J Neurosci 28: 13209–13215.



### model specification

- specify a DCM network structure
- language network
- non-lesioned part of brain



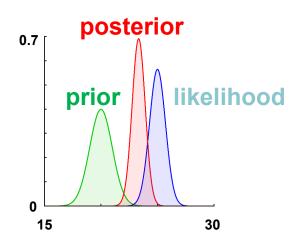


#### model inversion

- VBL (Variational Bayes under Laplace approx.)
- posterior over connectivity

$$posterior \\ p(\theta|y,m) = \frac{p(y|\theta,m)p(\theta|m)}{p(y|m)}$$

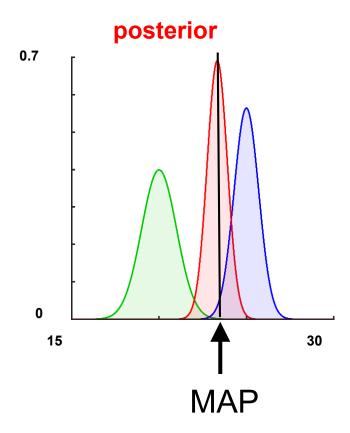
model evidence





### extracting point estimates

MAP (Maximum a posteriori)

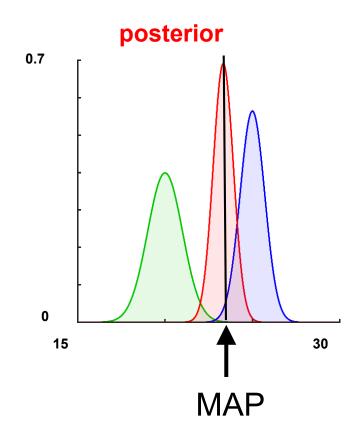




### extracting point estimates

- MAP (Maximum a posteriori)
- represent data from a subject as a single point in the space of model parameters

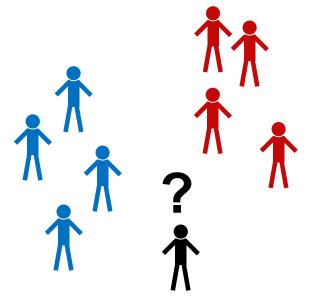






### classification

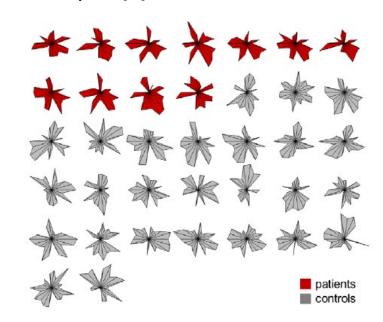
- supervised learning
- SVM (Support Vector Machine)

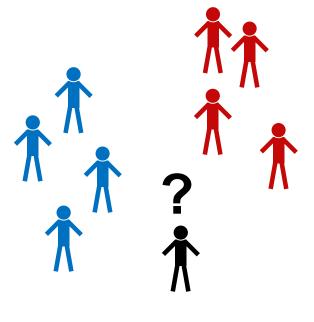




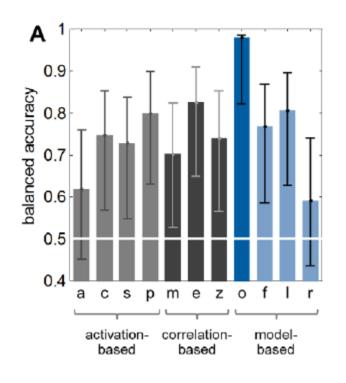
### classification

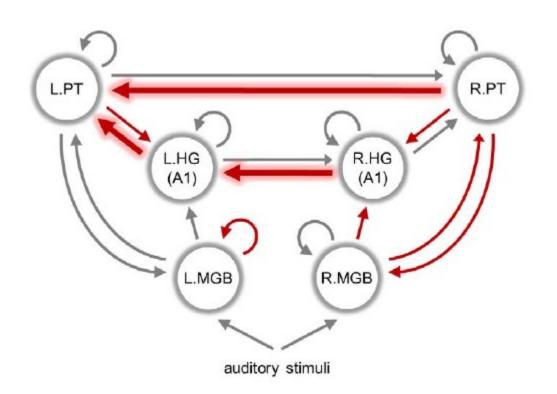
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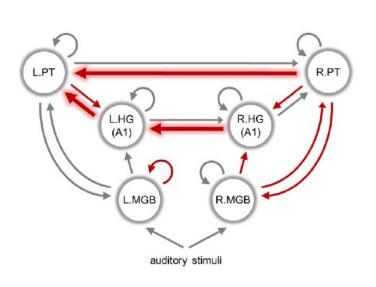


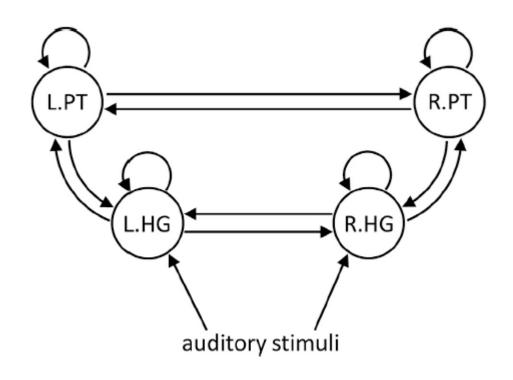






### model specification



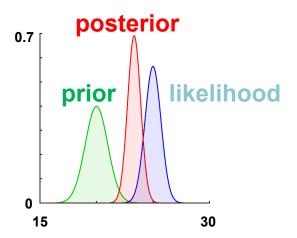




#### model inversion

- Variational Bayes
- conjugate priors

Yao et al. (2018). Variational Bayesian Inversion for Hierarchical Unsupervised Generative Embedding (HUGE). NeuroImage

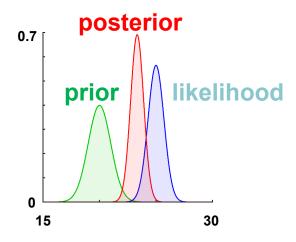




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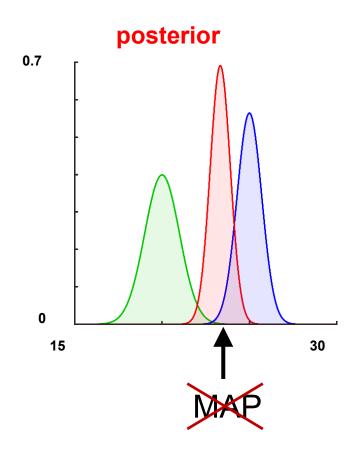
Yao et al. (2018). Variational Bayesian Inversion for Hierarchical Unsupervised Generative Embedding (HUGE). NeuroImage





### extracting point estimates

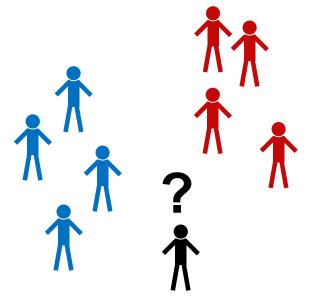
- HUGE is a hierarchical model
- uses information from the entire posterior instead of a point estimate

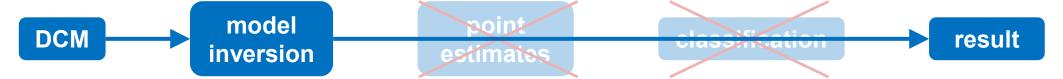




#### classification

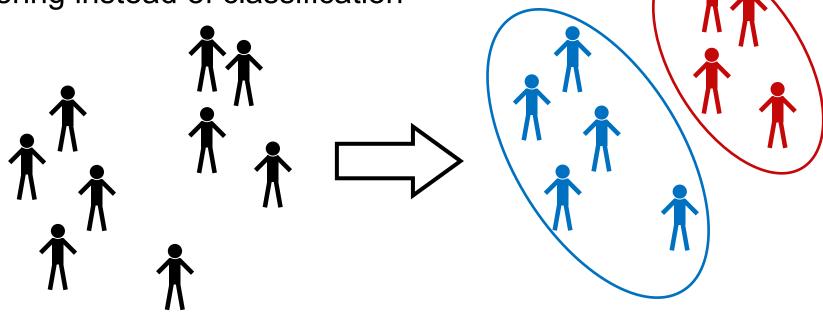
- HUGE is unsupervised
- clustering instead of classification





### classification

- HUGE is unsupervised
- clustering instead of classification





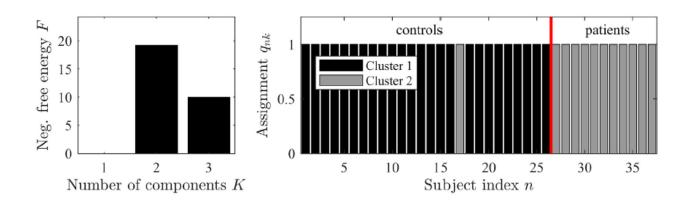
#### **MATLAB** interface

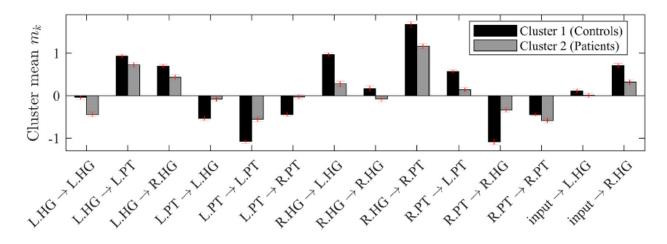
```
model = tapas_Huge('dcm',list_of_dcm); % build HUGE model
model.estimate('K',2); % fit model to data
```



#### result

- HUGE detects existence of two subgroups
- subgroups match onto patients and controls with a balanced purity of 96%

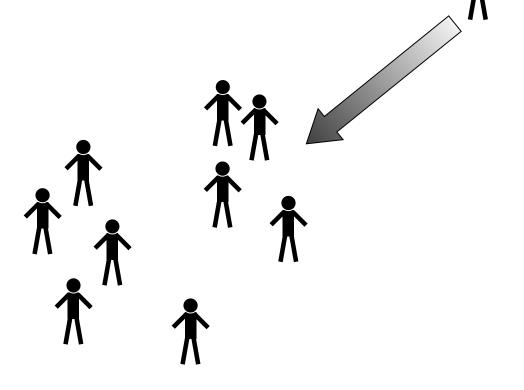






### beyond GE

- HUGE can also be used for Empirical Bayes
- "estimate" prior from data
- regularize outliers



# **HUGE Model Equations**

### assignment indicator

$$p(d_n = k|\pi) = \operatorname{Cat}(k|\pi) = \pi_k$$

### subject-level

### connectivity parameters

$$p\left(\theta_n^{(c)}\middle|d_n=k,\mu_k,\Sigma_k\right)=N\left(\theta_n^{(c)}\middle|\mu_k,\Sigma_k\right)$$

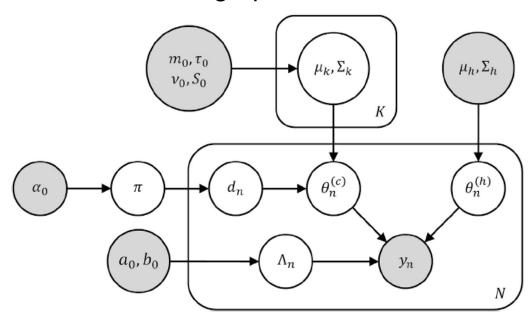
### hemodynamic parameters

$$p\left(\theta_n^{(h)}\big|\mu_h, \Sigma_h\right) = N\left(\theta_n^{(h)}\big|\mu_h, \Sigma_h\right)$$

#### observation

$$y_n = g(u, \theta_n) + \eta_n$$

#### graphical model



#### cluster-level

### cluster weights

$$\pi \sim D(\pi | \alpha_0)$$

#### cluster mean and variance

$$\mu_k, \Sigma_k \sim \text{NW}^{-1}(\mu_k, \Sigma_k | m_0, \tau_0, \nu_0, S_0)$$

# **HUGE Model Equations**

Yao et al. (2018). Variational Bayesian Inversion for Hierarchical Unsupervised Generative Embedding (HUGE). Neurolmage

### assignment indicator

$$p(d_n = k|\pi) = \operatorname{Cat}(k|\pi) = \pi$$

Variational Bayesian inversion for hierarchical unsupervised generative embedding (HUGE)

### subject-level

Yu Yao a,\*, Sudhir S. Raman , Michael Schiek , Alex Leff , Stefan Frässle , Klaas E. Stephan , Klaas E. Stephan

connectivity parameters

$$p(\theta_n^{(c)}|d_n = k, \mu_k, \Sigma_k) = N(\theta_n^{(c)})$$

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hemodynamic parameters

$$p\left(\theta_n^{(h)} \middle| \mu_h, \Sigma_h\right) = N\big(\theta_n^{(h)} \middle| \mu_h, \Sigma_h\big)$$

cluster-level

Raman et al. 2016. "A hierarchical model for integrating unsupervised generative embedding and empirical Bayes." J. Neurosci. Meth. 269, 6–20.

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b Central Institute ZEA-2 Electronic Systems, Research Center Jülich, 52425 Jülich, Germany

<sup>&</sup>lt;sup>c</sup> Wellcome Trust Centre for Neuroimaging, University College Landon, London, WC1N 3BG, United Kingdom

# **HUGE Model Equations**

### assignment indicator

$$p(d_n = k|\pi) = \operatorname{Cat}(k|\pi) = \pi_k$$

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#### connectivity parameters

$$p\left(\theta_n^{(c)}\middle|d_n=k,\mu_k,\Sigma_k\right)=N\left(\theta_n^{(c)}\middle|\mu_k,\Sigma_k\right)$$

#### hemodynamic parameters

$$p\left(\theta_n^{(h)}\big|\mu_h, \Sigma_h\right) = N\big(\theta_n^{(h)}\big|\mu_h, \Sigma_h\big)$$

#### observation

$$y_n = g(u, \theta_n) + \eta_n$$

#### SATURDAY - 12th September 2020

11:45 Practical Tutorials: Morning

Tutorial A: Bayesian Learning

Tore Erdmann, Sandra Iglesias, Lilian Weber

Tutorial B: Active Inference

Thomas Parr, Philipp Schwartenbeck

Tutorial C: Reinforcement Learning

Woo-Young Ahn, Nathaniel Haines, Jaeyeong Yang

Tutorial D: Model Inversion

Eduardo Aponte, Lionel Rigoux

Tutorial E: Machine Learning

Thomas Wolfers, Saige Rutherford

Tutorial F: Dynamic Causal Modeling

Jakob Heinzle, Herman Galioulline

Tutorial G: rDCM

Stefan Frässle, Cao Tri Do

Tutorial H: HUGE

Yu Yao, Matthias Müller-Schrader

#### cluster weights

$$\pi \sim D(\pi | \alpha_0)$$

cluster-

#### cluster mean and variance

$$\mu_k, \Sigma_k \sim \text{NW}^{-1}(\mu_k, \Sigma_k | m_0, \tau_0, \nu_0, S_0)$$

standard model

*likelihood*: encodes generative process *prior*: encodes model assumptions

joint likelihood prior
$$p(y,\theta|\lambda,m) = p(y|\theta,\lambda,m)p(\theta|\lambda,m)$$

y: data

 $\theta$ : model parameters

 $\lambda$ : prior parameters

standard model

*likelihood*: encodes generative process *prior*: encodes model assumptions

$$\frac{p(y,\theta|\lambda,m)}{p(y|\theta,\lambda,m)} = p(y|\theta,\lambda,m)p(\theta|\lambda,m)$$

Remember:

p(x, y, z) = p(x|y, z)p(y|z)p(z)

y: data

 $\theta$ : model parameters

 $\lambda$ : prior parameters

hierarchical model

*likelihood*: encodes generative process *prior*: encodes model assumptions

joint likelihood prior hyperprior 
$$p(y,\theta,\lambda|\phi,m) = p(y|\theta,\lambda,\phi,m)p(\theta|\lambda,\phi,m)p(\lambda|\phi,m)$$

Remember:

p(x, y, z) = p(x|y, z)p(y|z)p(z)

y: data

 $\theta$ : model parameters

 $\lambda$ : prior parameters

 $\phi$ : parameters of hyperprior

hierarchical model

*likelihood*: encodes generative process *prior*: encodes a clustering model

joint likelihood prior hyperprior 
$$p(y,\theta,\lambda|\phi,m) = p(y|\theta,\lambda,\phi,m)p(\theta|\lambda,\phi,m)p(\lambda|\phi,m)$$

mixture of Gaussians

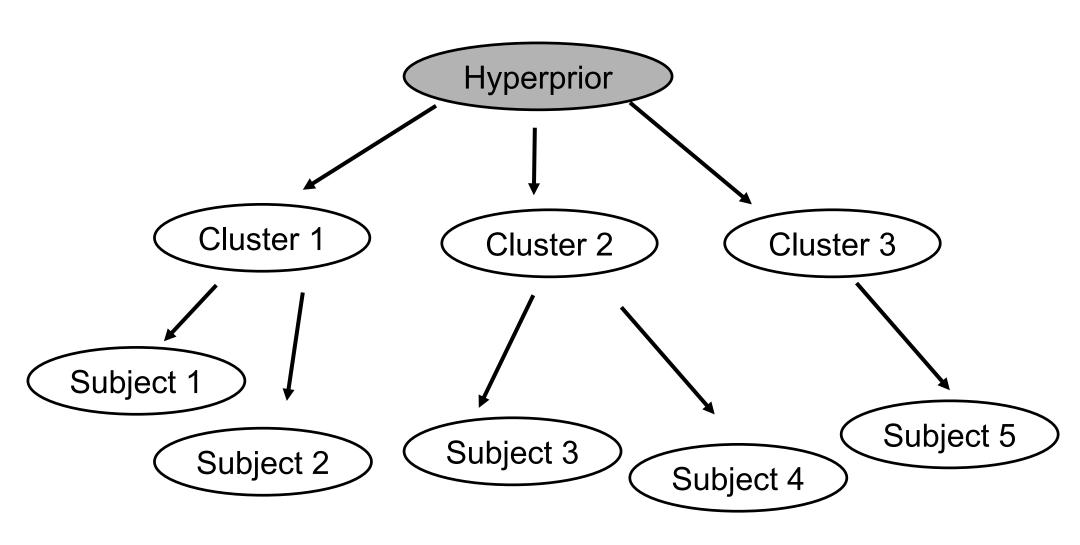
$$p(\theta|\lambda,\phi,m) = \sum_{k} \pi_{k} N(\theta|\mu_{k}, \Sigma_{k})$$
$$\lambda = \{\pi,\mu,\Sigma\}$$

y: data

 $\theta$ : model parameters

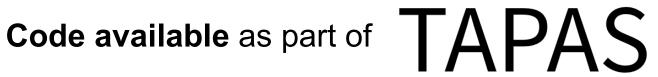
 $\lambda$ : prior parameters

 $\phi$ : parameters of hyperprior



### Software





www.translationalneuromodeling.org/tapas

**CPC Practical Session** (Saturday)

**Tutorial H: HUGE** 

### References

Brodersen, K.H., Schofield, T.M., Leff, A.P., Ong, C.S., Lomakina, E.I., Buhmann, J.M., Stephan, K.E., 2011. Generative embedding for model-based classification of fMRI data. PLoS Comput. Biol. 7.

Brodersen, K.H., Deserno, L., Schlagenhauf, F., Lin, Z., Penny, W.D., Buhmann, J.M., Stephan, K.E., 2014. Dissecting psychiatric spectrum disorders by generative embedding. Neuroimage: Clinica 4, 98–111.

Raman, S., Deserno, L., Schlagenhauf, F., Stephan, K.E., 2016. A hierarchical model for integrating unsupervised generative embedding and empirical Bayes. J. Neurosci. Meth. 269, 6–20.

Yao Y, Raman SS, Schiek M, Leff A, Frässle S, Stephan KE. Variational Bayesian Inversion for Hierarchical Unsupervised Generative Embedding (HUGE);179:604–619.

# Thank you