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Translational Neuromodeling Unit
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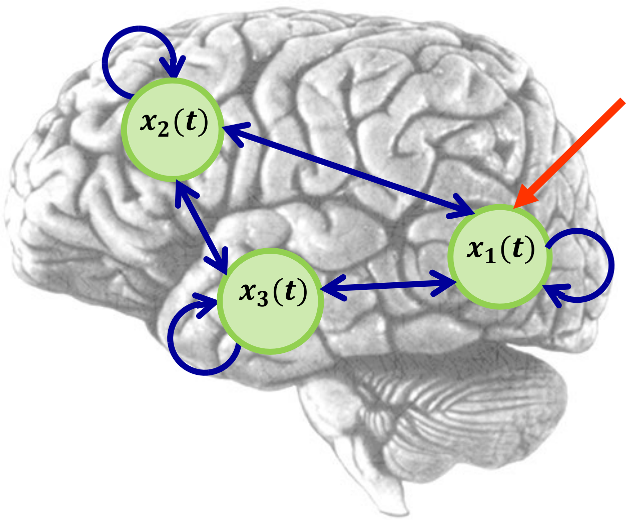

Modeling connectivity:
Dynamic Causal Modeling for fMRI

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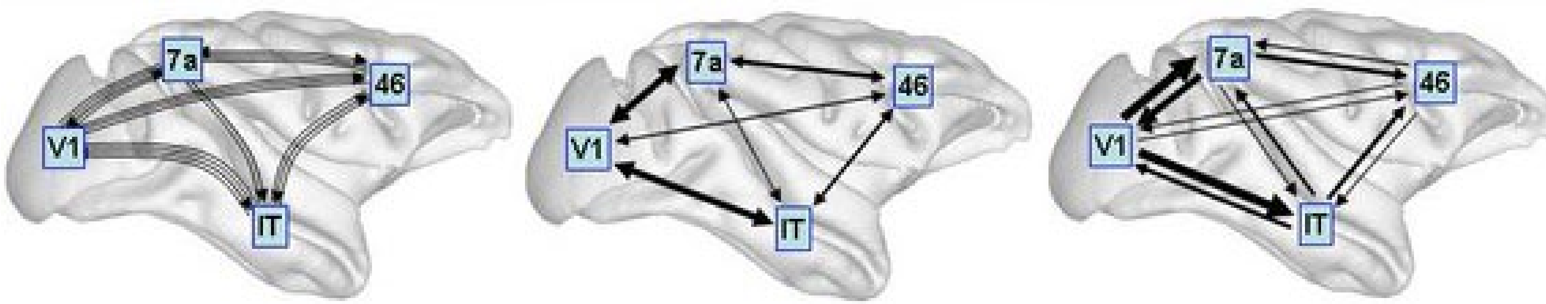
Institute for Biomedical Engineering
University and ETH Zürich

CP Course 2022, Zürich, Switzerland

Structural, functional & effective connectivity



Sporns 2007, *Scholarpedia*

anatomical/structural

- presence of physical connections

→ *DWI, tractography, tracer studies (animals)*

functional

- statistical dependency between regional time series

→ *correlations, ICA*

effective

- direct influences between neuronal populations

→ *DCM*

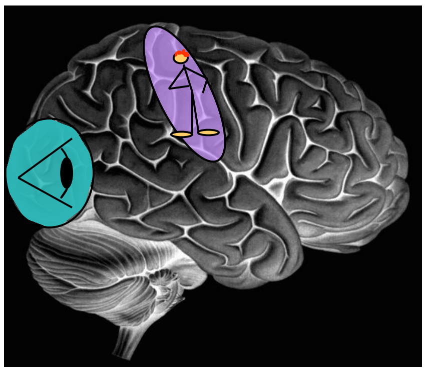
Context-independent

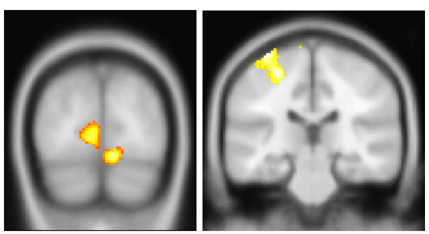
Mechanism - free

Mechanistic

Specialisation vs. Integration

Functional Specialisation

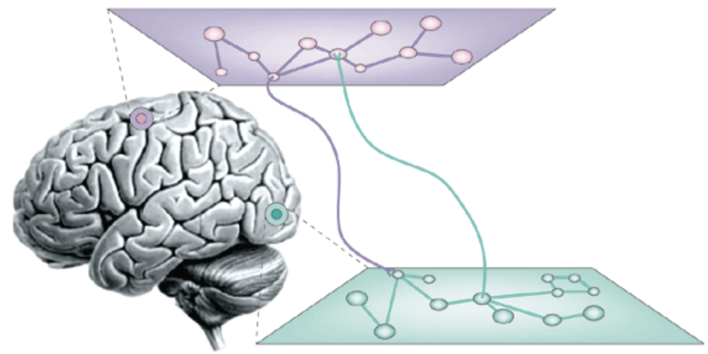


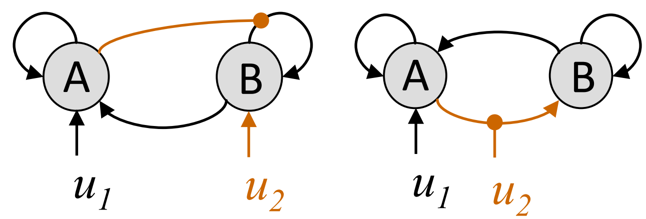


u_1 $u_1 \times u_2$

*«**Where**, in the brain, did my experimental manipulation have **an effect**?»*

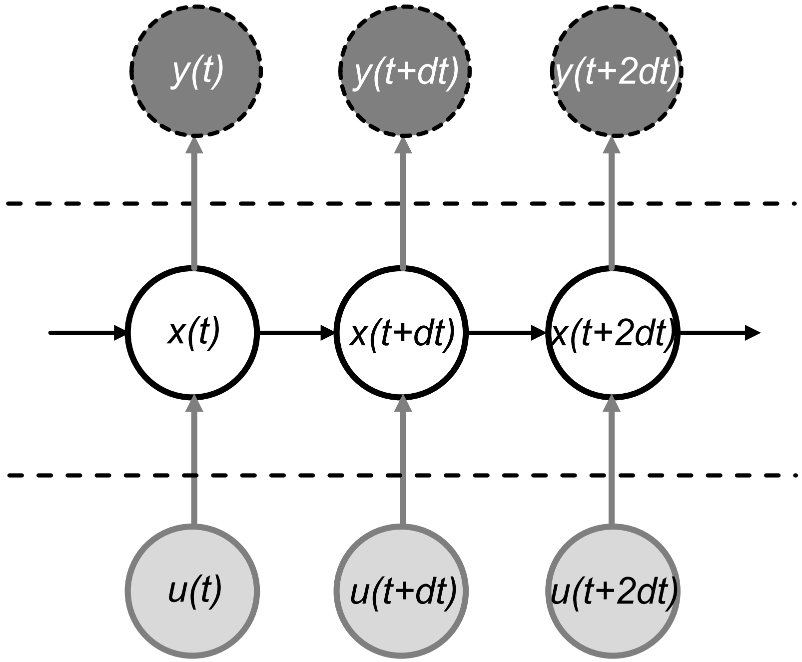
Functional Integration





*«**How** did my experimental manipulation **propagate through the network**?»*

A reminder – generative models



Observed data (fMRI)

$$y = g(x, \theta) + \varepsilon$$

Hidden states (Brain activity)

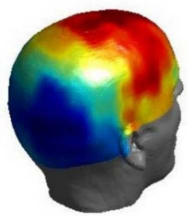
$$\frac{dx}{dt} = f(x, u, \theta) + \omega$$

Inputs (Exp. manipulations)

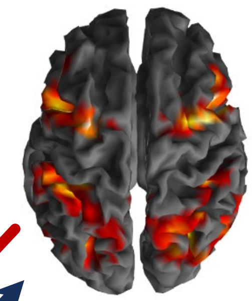
$$u(t)$$

Dynamic causal modelling

EEG, MEG



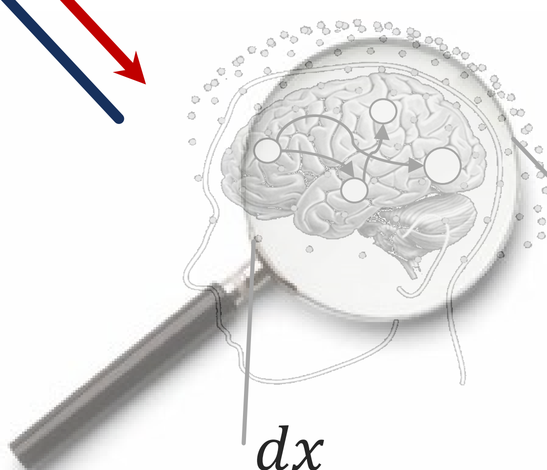
fMRI



Model inversion:
Estimating neuronal mechanisms

Forward model:
Predicting measured activity

$$y = g(x, \theta) + \varepsilon$$



$$\frac{dx}{dt} = f(x, u, \theta) + \omega$$

State equation:
Describing neuronal dynamics (and hemodynamics)

DCM for EEG
→ later today
→ Rosalyn Moran

Friston et al., 2003, NeuroImage; David et al., 2006, NeuroImage

Dynamic causal modelling

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Dynamic causal modelling

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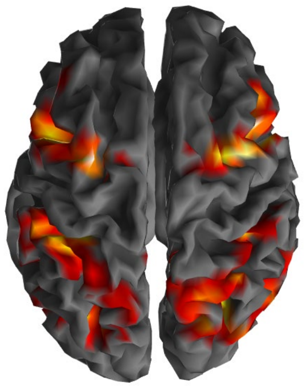
Received 18 October 2002; revised 7 March 2003; accepted 2 April 2003

DCM for fMRI - overview



fMRI

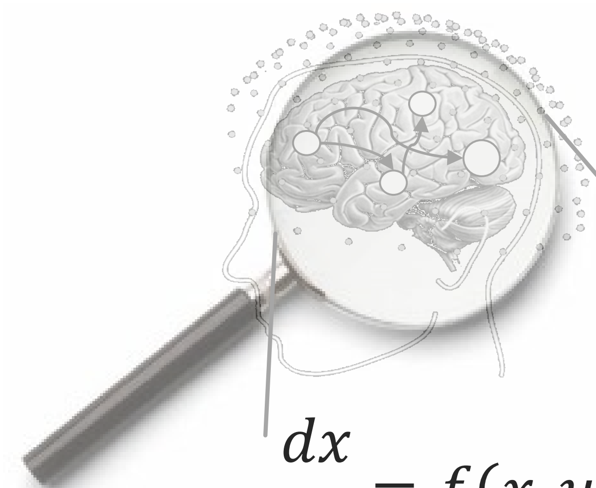
Model inversion:

Estimating
neuronal
mechanisms

Forward model:

Predicting
measured activity

$$y = g(x, \theta) + \varepsilon$$

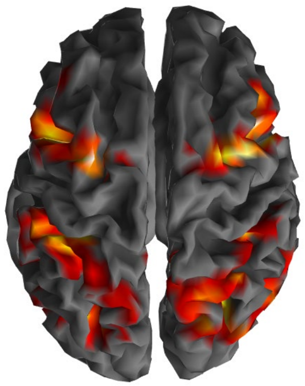


$$\frac{dx}{dt} = f(x, u, \theta)$$

Neural state equation:

Describing neuronal
dynamics

DCM for fMRI - overview



fMRI

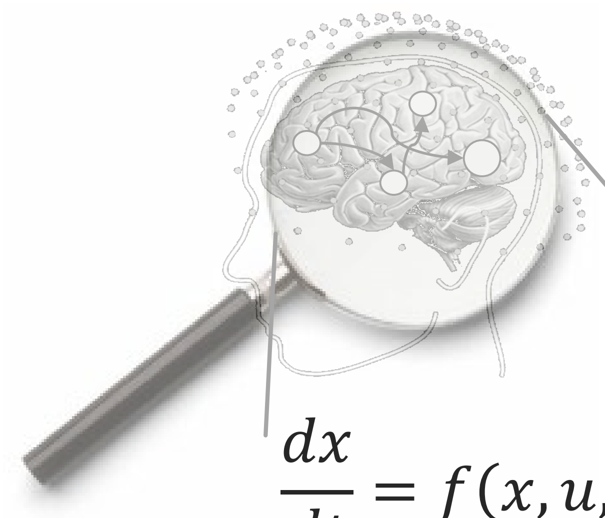
Model inversion:

Estimating neuronal mechanisms

Forward model:

Predicting measured activity

$$y = g(x, \theta) + \varepsilon$$



$$\frac{dx}{dt} = f(x, u, \theta)$$

Neural state equation:

Describing neuronal dynamics

Neuronal state equations

$$\frac{dx}{dt} = f(x,u)$$

Neuronal state equations

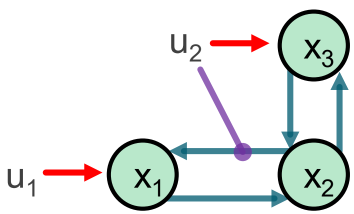
$$\frac{dx}{dt} = f(x,u) \approx \boxed{f(x_0,0) + \frac{\partial f}{\partial x}x + \frac{\partial f}{\partial u}u + \frac{\partial^2 f}{\partial x \partial u}ux} + \frac{\partial^2 f}{\partial x^2}\frac{x^2}{2} + \cdots$$

bilinear model

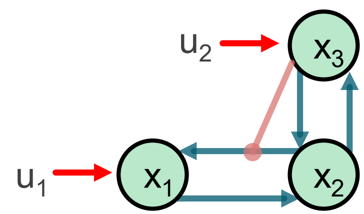
Neuronal state equations

$$\frac{dx}{dt} = f(x,u) \approx f(x_0,0) + \overbrace{\frac{\partial f}{\partial x}}^{A} x + \overbrace{\frac{\partial f}{\partial u}}^{C} u + \overbrace{\frac{\partial^2 f}{\partial x \partial u}}^{B} ux + \overbrace{\frac{\partial^2 f}{\partial x^2}}^{D} \frac{x^2}{2} + \cdots$$

bilinear model

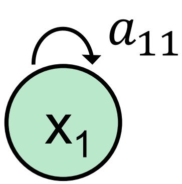


nonlinear model

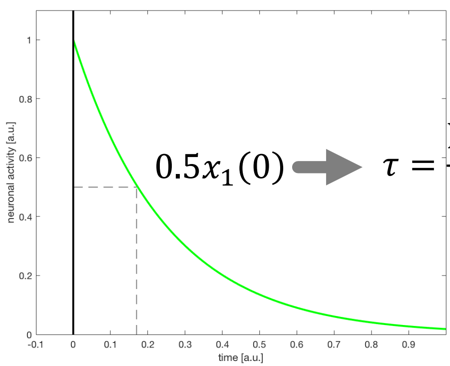


Neuronal state equations

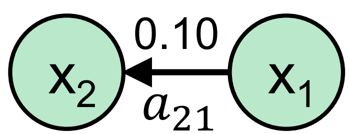
DCM effective connectivity parameters are rate constants



$$\frac{dx_1}{dt} = a_{11} x_1 \quad \longrightarrow \quad x_1(t) = x_1(0) \cdot exp(a_{11} t)$$



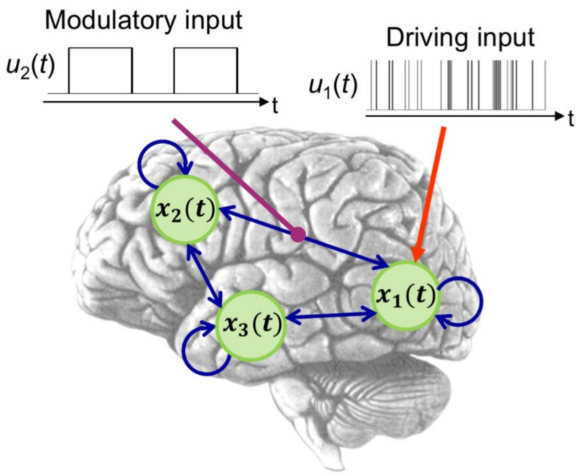
$$0.5 x_1(0) \longrightarrow \tau = \frac{\ln 2}{a_{11}}$$



If a_{21} is $0.10 s^{-1}$, this means that, per unit time, the increase in activity in x_2 corresponds to 10% of the current activity in x_1

Neuronal state equations

Interim summary: bilinear neuronal state equation



State change

External inputs

Current state

$$\frac{dx}{dt} = \underbrace{\left(A + \sum_{j=1}^{m} u_j B^{(j)}\right)}_{\text{connectivity}} x + Cu$$

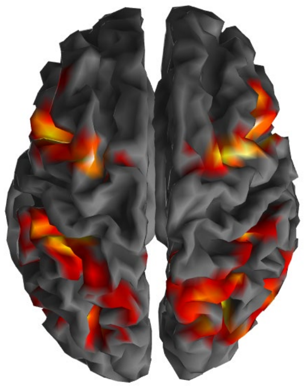
$$\theta = \{A, B^{(1)}, \dots, B^{(m)}, C\}$$

Endogenous connectivity

Modulatory connectivity

Driving input weights

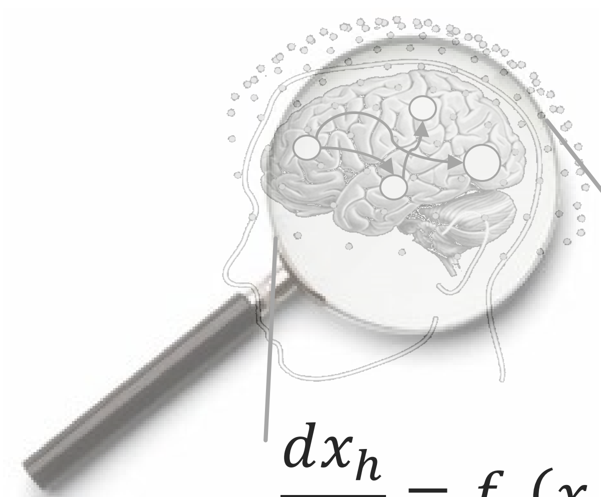
DCM for fMRI - overview



Model inversion:
Estimating
neuronal
mechanisms

Forward model:
Predicting
measured activity

$$y = g(x, \theta) + \varepsilon$$



$$\frac{dx_h}{dt} = f_h(x_h, x, \theta)$$

Hemodynamic state equation:
Describing hemodynamics

The hemodynamic response

Neuronal dynamics only indirectly observable via hemodynamic response

↑ neuronal activity

↑ blood flow

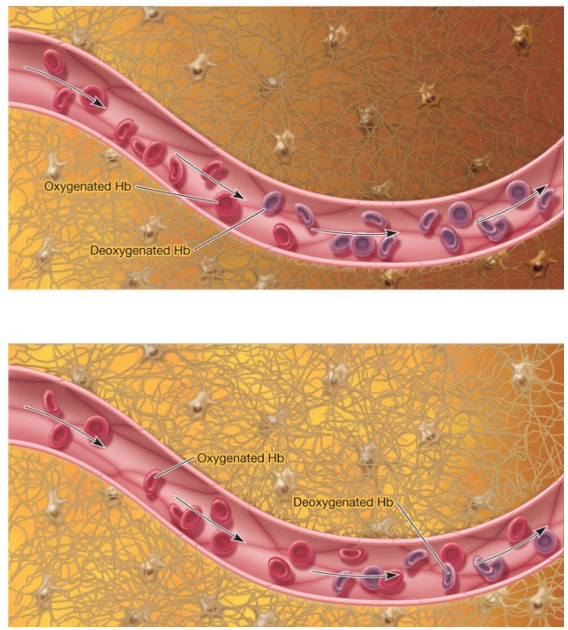
↑ oxygenated Hb

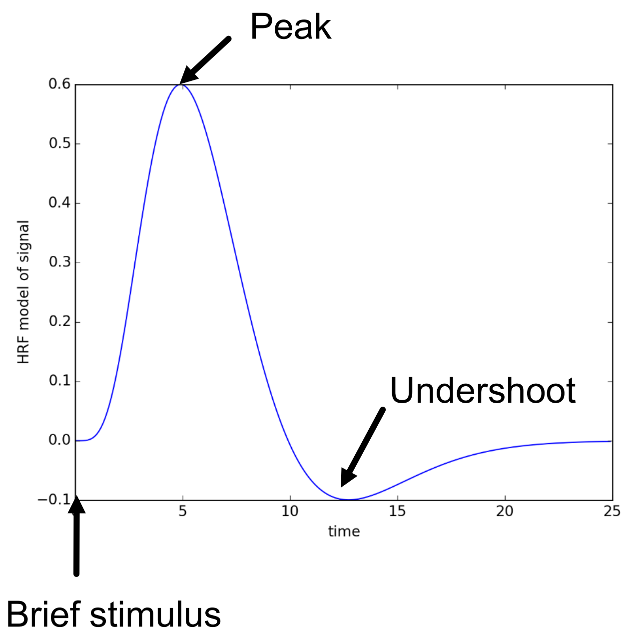
↑ T2*

↑ fMRI signal

Rest

Activity





Huettel et al, 2004, fMRI (Book)

The hemodynamic model

6 parameters:

$$\theta^h = \{\kappa, \gamma, \tau, \alpha, \rho, \varepsilon\}$$

Important for model fitting, but typically of no interest for statistical inference.

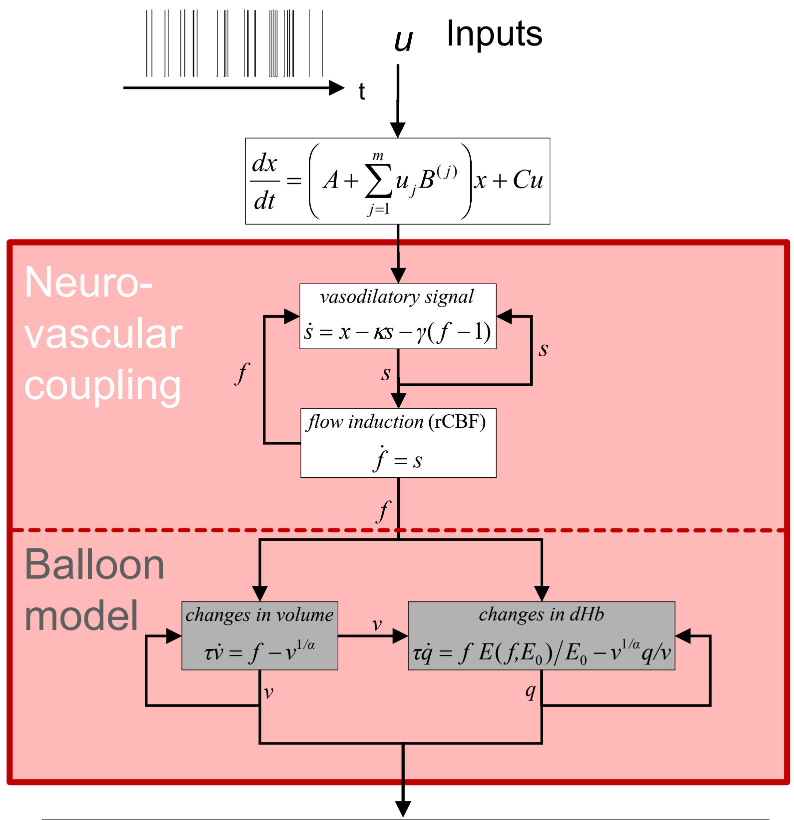
Region specific HRF

→ Parameters computed separately for each region

State equation

neural

hemodynamic



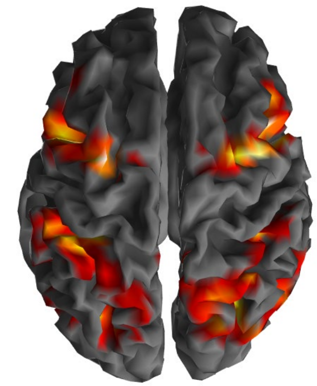
$$\lambda(q,v) = \frac{\Delta S}{S_0} \approx V_0\left[k_1(1-q) + k_2\left(1-\frac{q}{v}\right) + k_3(1-v)\right]$$

$$k_1 = 4.3\vartheta_0 E_0 TE$$
$$k_2 = \varepsilon r_0 E_0 TE$$
$$k_3 = 1 - \varepsilon$$

BOLD signal change equation

Stephan et al., 2007, NeuroImage

DCM for fMRI - overview



fMRI

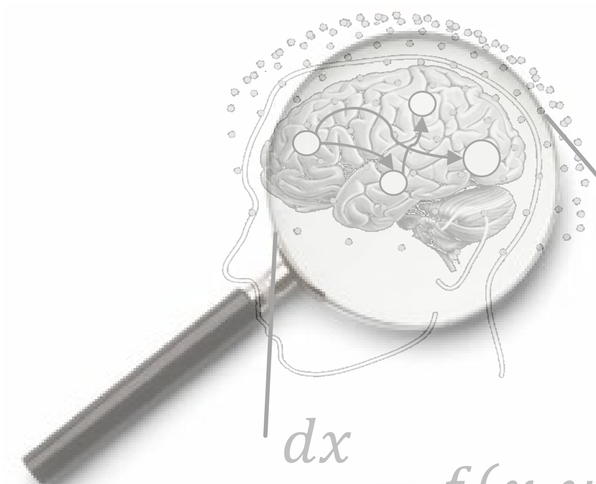
Model inversion:

Estimating neuronal mechanisms

Forward model:

Predicting measured activity

$$y = g(x, \theta) + \varepsilon$$



$$\frac{dx}{dt} = f(x, u, \theta) + \omega$$

Neural state equation:

Describing neuronal dynamics

Friston et al., 2003, NeuroImage; David et al., 2006, NeuroImage

The BOLD signal equation

Resting blood volume

Deoxyhemoglobin content

Blood volume

$$\lambda(q,v) = \frac{\Delta S}{S_0} \approx V_0 \left[k_1(1-q) + k_2\left(1 - \frac{q}{v}\right) + k_3(1-v) \right]$$

BOLD-Signal Parameters:

$k_1 = 4.3\vartheta_0 E_0 TE$

$k_2 = \varepsilon r_0 E_0 TE$

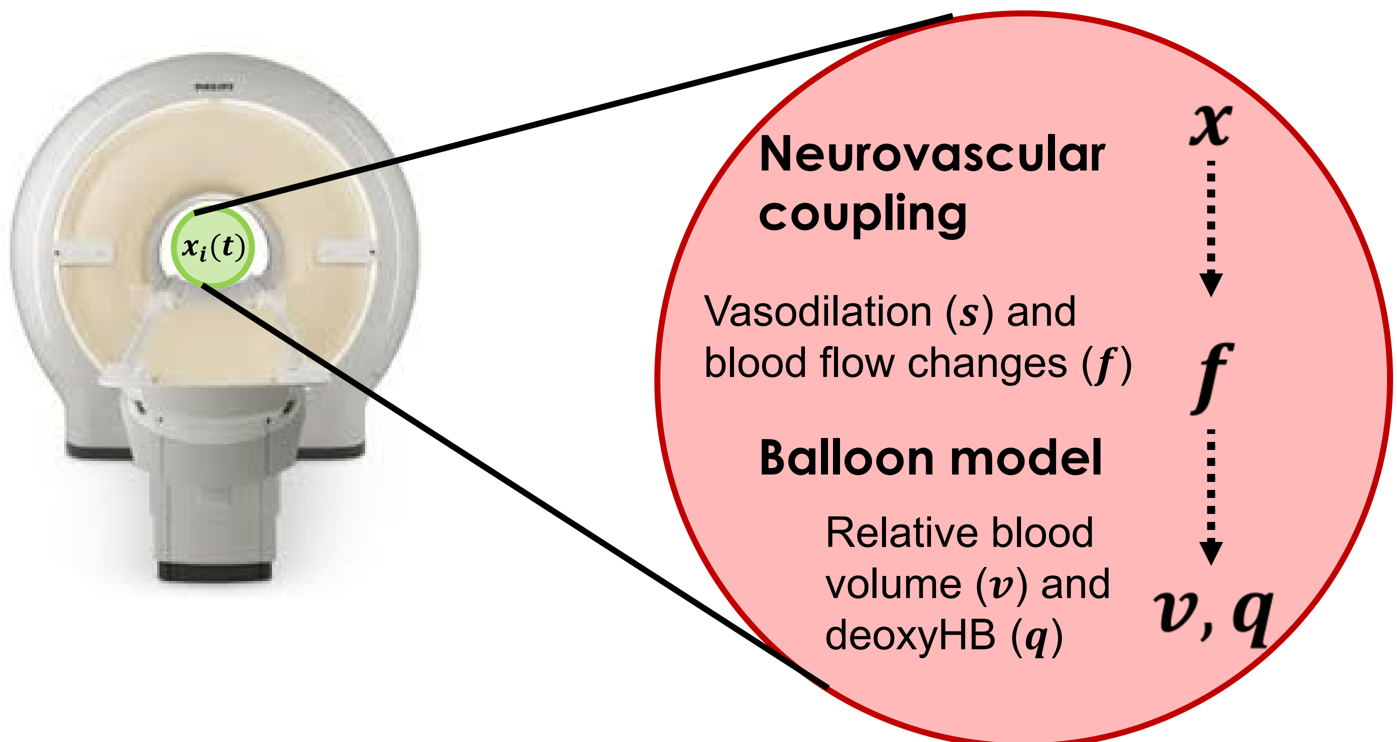
$k_3 = 1 - \varepsilon$

$V_0 = 0.04 \qquad E_0 = 0.32 - 0.4$

At 1.5 Tesla	At 3 Tesla	At 7 Tesla
$\vartheta_0 \approx 40.3\ \mathrm{s}^{-1}$	$\vartheta_0 \approx 80.6\ \mathrm{s}^{-1}$	$\vartheta_0 \approx 188\ \mathrm{s}^{-1}$
$r_0 \approx 25\ \mathrm{s}^{-1}$	$r_0 \approx 110\ \mathrm{s}^{-1}$	$r_0 \approx 340\ \mathrm{s}^{-1}$
$TE \approx 0.04\ s$	$TE \approx 0.035\ s$	$TE \approx 0.025\ s$
$\varepsilon \approx 1.28$	$\varepsilon \approx 0.47$	$\varepsilon \approx 0.026$

Heinzle et al., 2016, NeuroImage

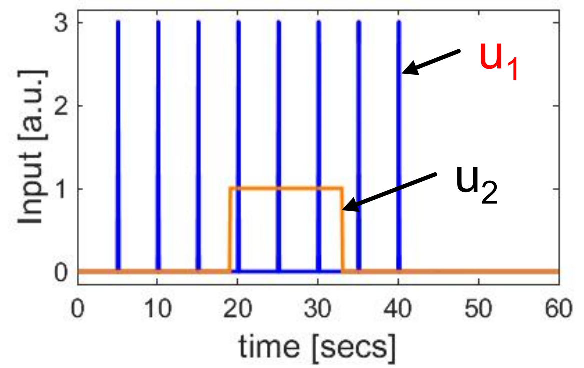
From neural activity to the BOLD signal: summary



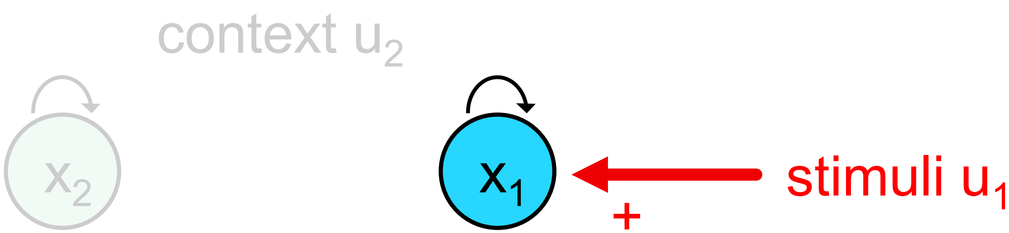
BOLD signal is a **direct function** of v and q

$$y = \frac{\Delta S}{S_0} = g(v, q) + \varepsilon$$

Simulation example: What can DCM explain?



Example: single node

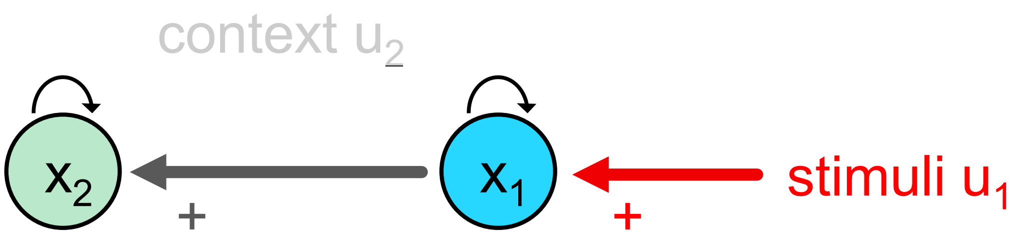


$$\frac{dx}{dt} = (A + u_2 B^{(2)})x + C u_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \left(\begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} + u_2 \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right) \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} c_{11} & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

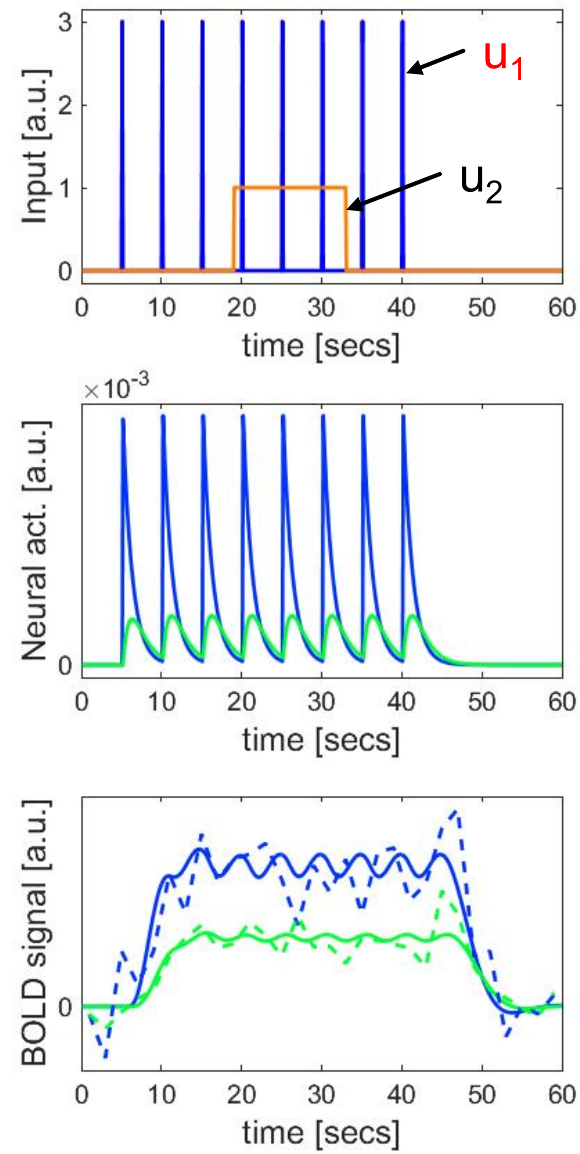
Simulation example: What can DCM explain?

Example: two connected node



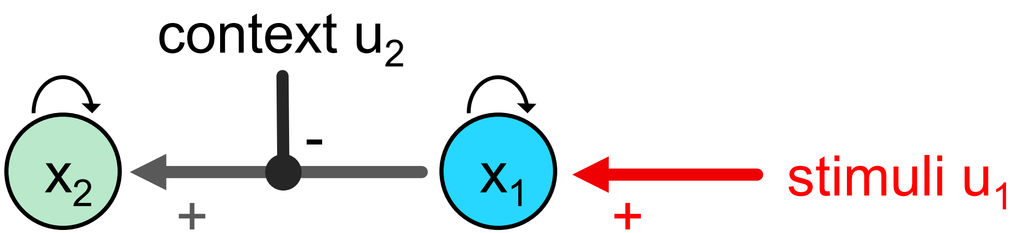
$$\frac{dx}{dt} = \left(A + u_2 B^{(2)}\right)x + C u_1$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \left(\begin{bmatrix}a_{11} & 0\\ \boldsymbol{a_{21}} & a_{22}\end{bmatrix} + u_2\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}\right)\cdot\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}c_{11} & 0\\ 0 & 0\end{bmatrix}\cdot\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$



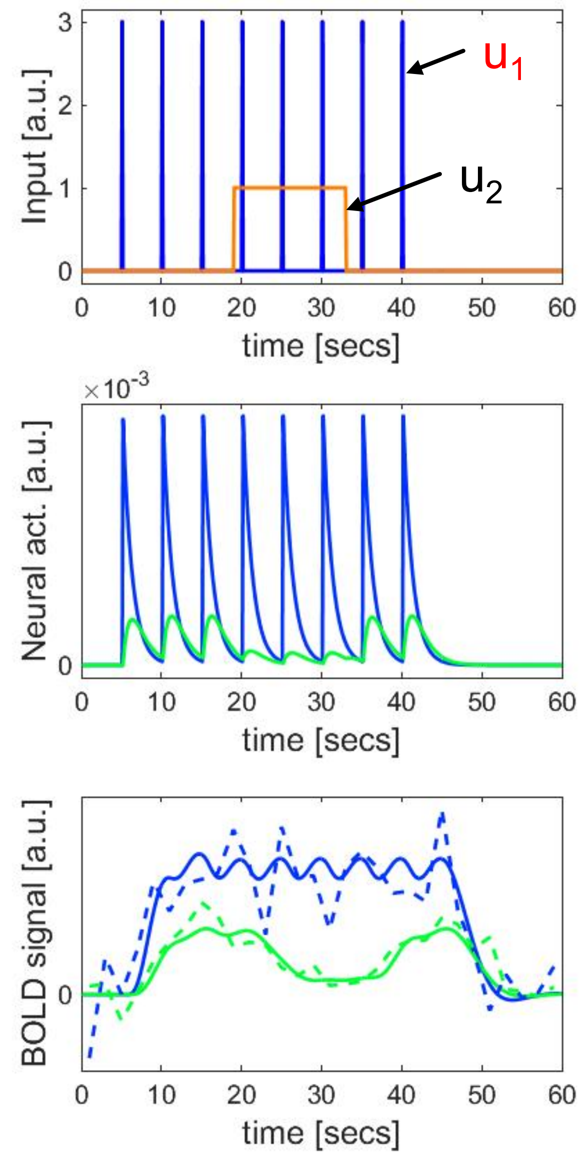
Simulation example: What can DCM explain?

Example: modulation of connection



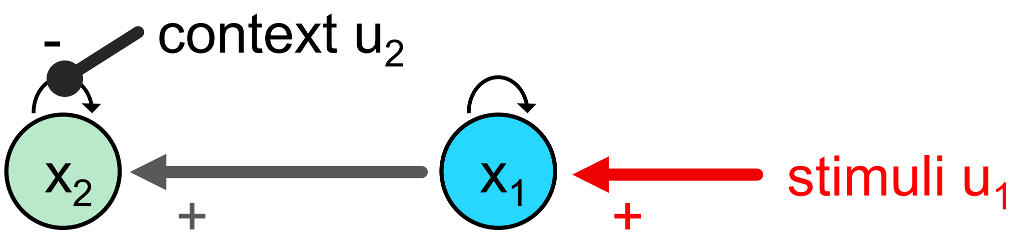
$$\frac{dx}{dt} = (A + u_2 B^{(2)})x + Cu_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \left(\begin{bmatrix} a_{11} & 0 \\ \boldsymbol{a_{21}} & a_{22} \end{bmatrix} + u_2 \begin{bmatrix} 0 & 0 \\ \boldsymbol{b_{21}^{(2)}} & \boldsymbol{0} \end{bmatrix} \right) \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} c_{11} & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$



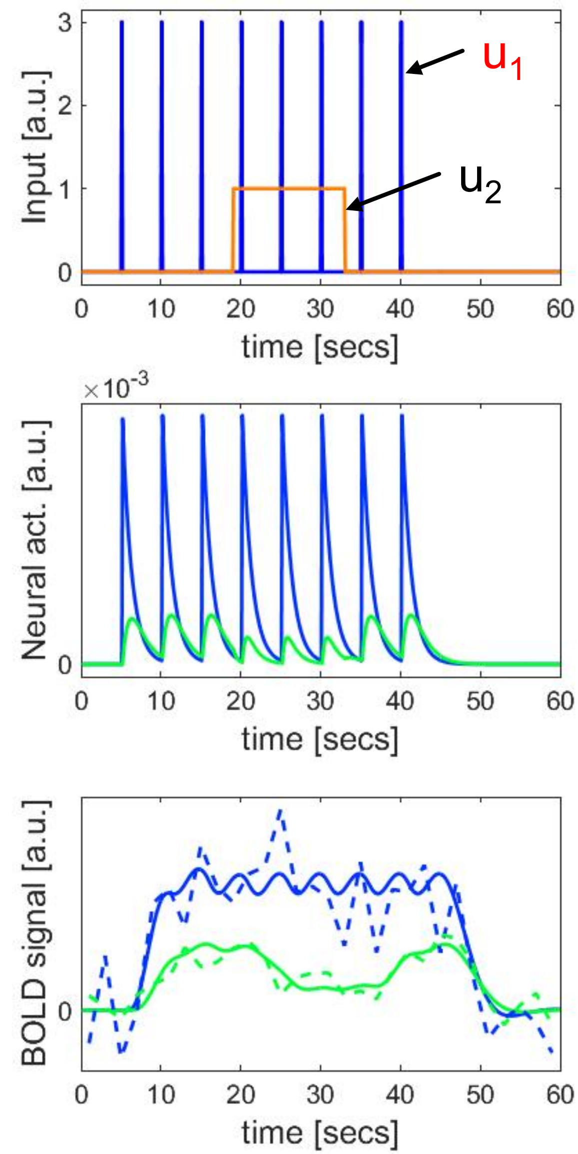
Simulation example: What can DCM explain?

Example: modulation of inhibitory self-connection



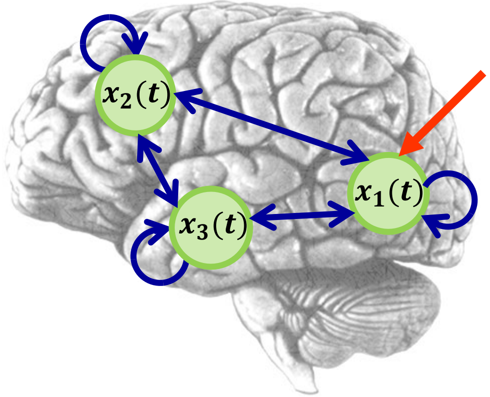
$$\frac{dx}{dt} = \left(A + u_2 B^{(2)}\right)x + Cu_1$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \left(\begin{bmatrix}a_{11} & 0\\ \boldsymbol{a_{21}} & a_{22}\end{bmatrix} + u_2\begin{bmatrix}0 & 0\\ \mathbf{0} & \boldsymbol{b_{22}^{(2)}}\end{bmatrix}\right)\cdot\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}c_{11} & 0\\ 0 & 0\end{bmatrix}\cdot\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$



DCM for fMRI

A simple model of a neural network …



$x_i(t)$ Neural node

Input

Connections

… described as a dynamical system …

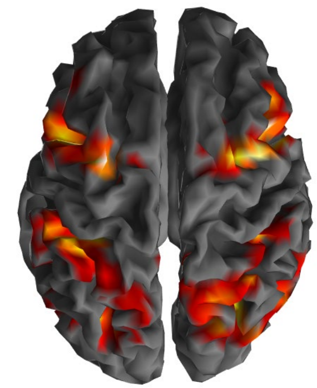
$$\dot{x} = f(x, u, \theta)$$

… causes the data (BOLD signal).

$$y = g(x, \theta) + \varepsilon$$

Simulate the system with input u and parameters θ
→ BOLD signal time course y that can be compared to measured data.

DCM for fMRI - overview



fMRI

Model inversion:

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Forward model:

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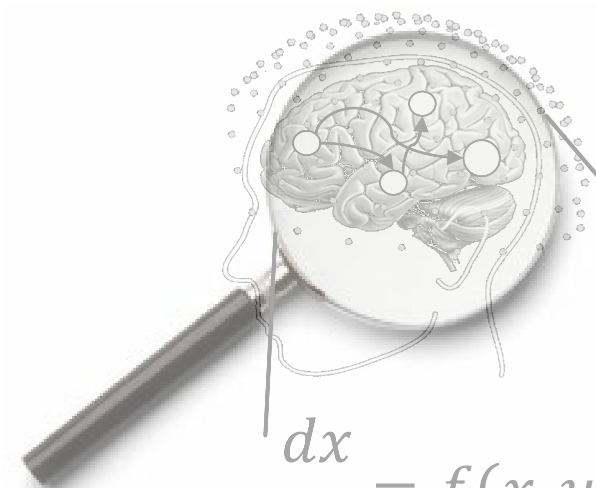
$$y = g(x, \theta) + \varepsilon$$



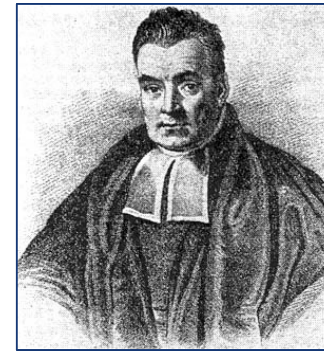
$$\frac{dx}{dt} = f(x, u, \theta) + \omega$$

Neural state equation:

Describing neuronal dynamics

Friston et al., 2003, NeuroImage; David et al., 2006, NeuroImage

Bayes' theorem



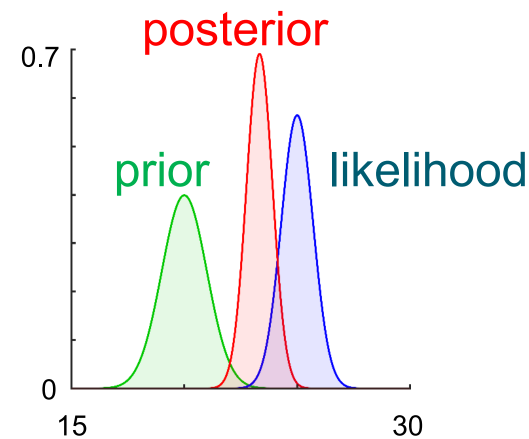
Reverend Thomas Bayes
(1702-1761)

likelihood prior

posterior

$$p(\theta|y,m) = \frac{p(y|\theta,m)\,p(\theta|m)}{p(y|m)}$$

model evidence



The likelihood function for DCM

$$\boxed{p(y(t)|\theta, m)} = \mathcal{N}\left(y(t); g\left(\theta^{n}, \theta^{h}, u(t)\right), \theta^{\sigma}\right)$$

likelihood

Assume data is normally distributed around the prediction from the dynamical model (Gaussian noise)

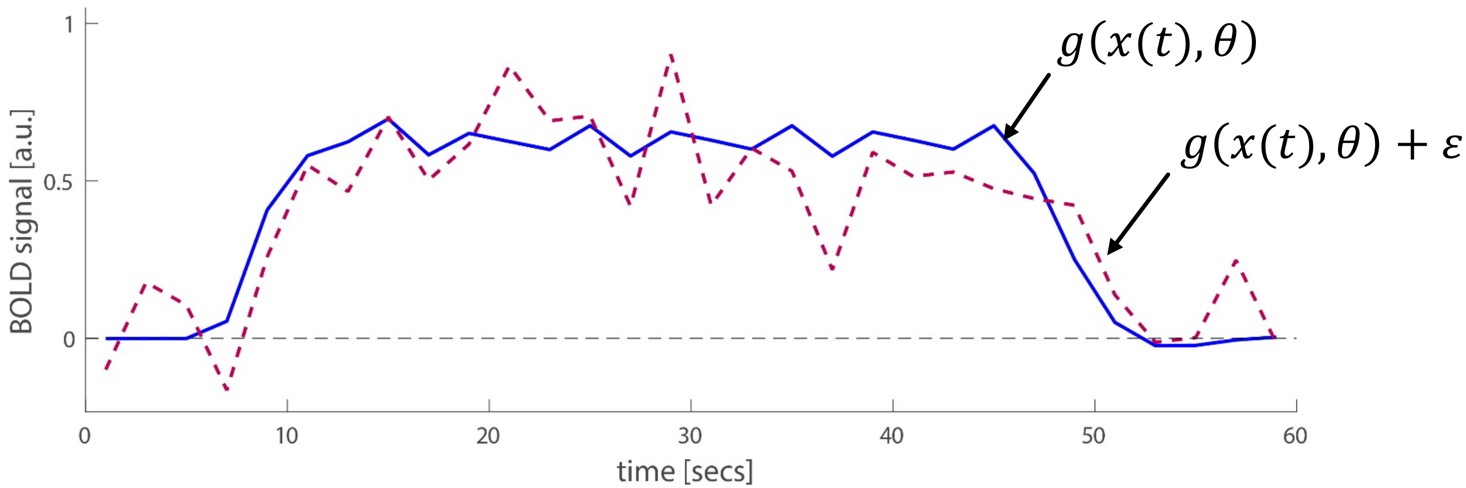
$$y(t) = g(x(t), \theta) + \varepsilon$$

$$\varepsilon \sim \mathcal{N}(0, \sigma^{2} I)$$

Data is prediction plus Gaussian noise

The likelihood function for DCM

$$\underbrace{p(y(t)|\theta,m)}_{\text{likelihood}} = \mathcal{N}\left(y(t); g\left(\theta^n, \theta^h, u\right), \theta^\sigma\right)$$



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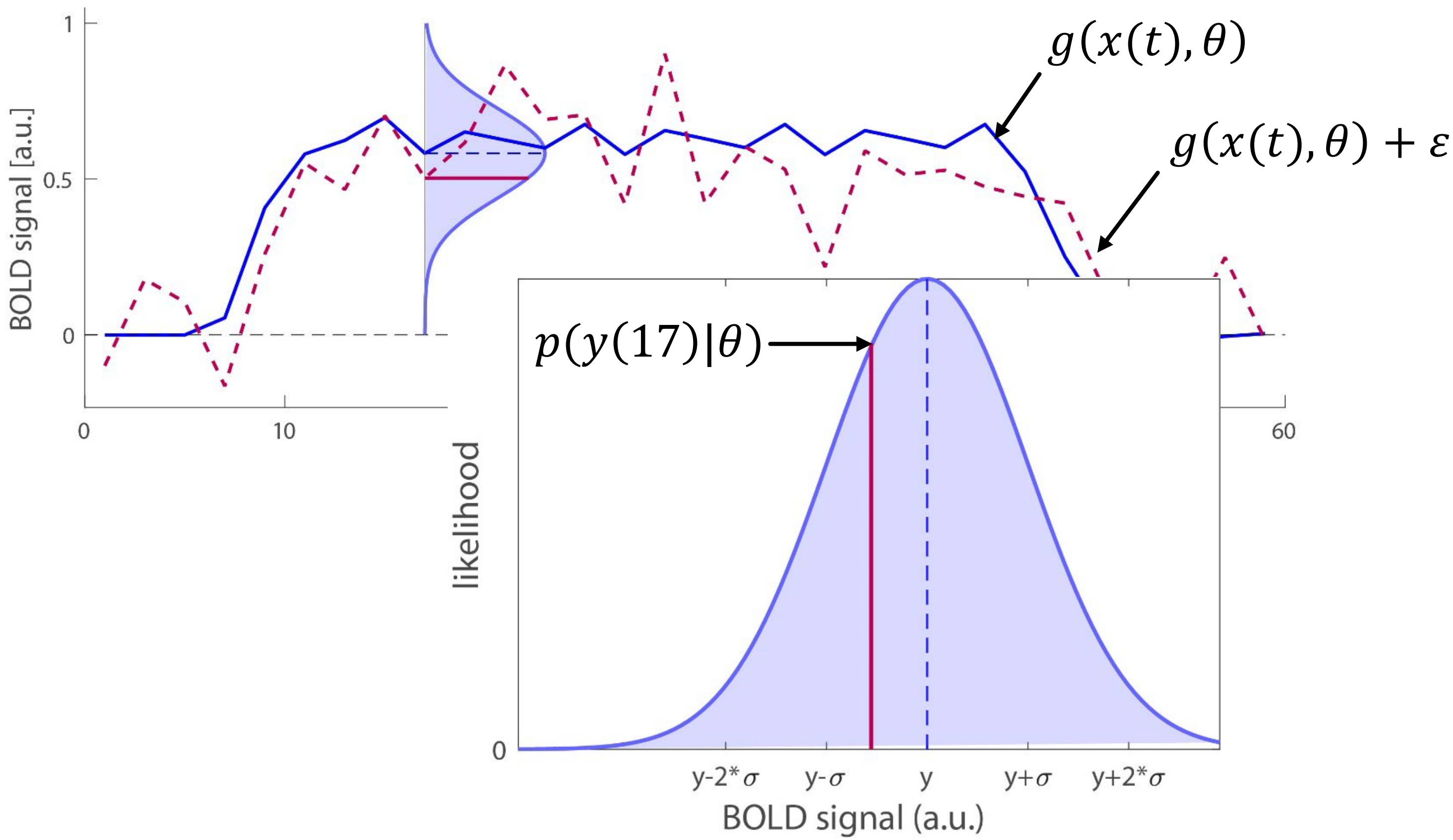
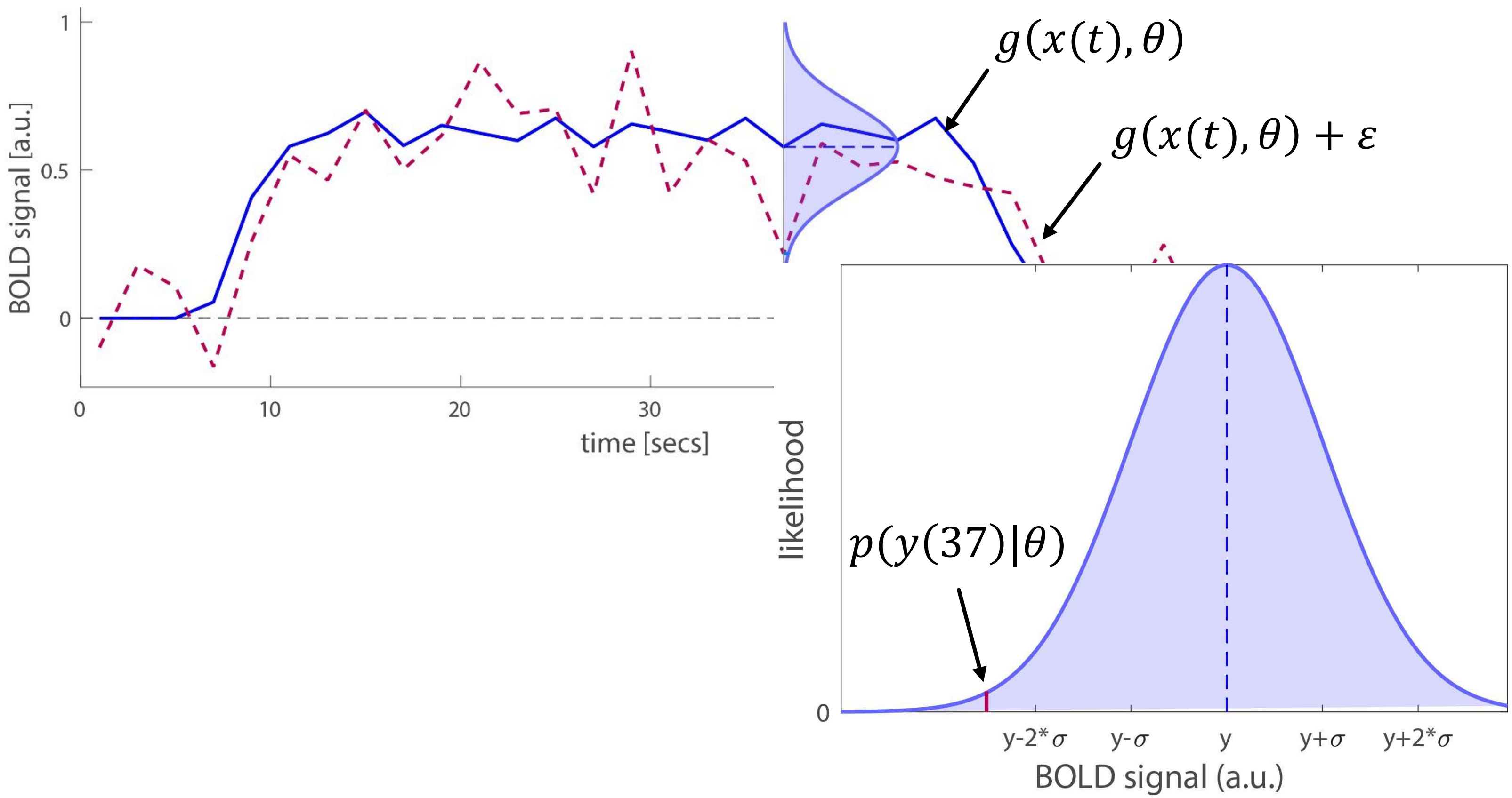


The likelihood function for DCM

$$\underbrace{p(y(t)|\theta, m)}_{\text{likelihood}} = \mathcal{N}\left(y(t); g\left(\theta^n, \theta^h, u\right), \theta^\sigma\right)$$



Priors

prior

$$p(\theta|y,m) = \frac{p(y|\theta,m)\,p(\theta|m)}{p(y|m)}$$

Neuronal parameters:

- self-connections: principled (to “ensure” that the system is stable)
- other parameters (between—region connections, modulation, inputs): shrinkage priors

Hemodynamic parameters:

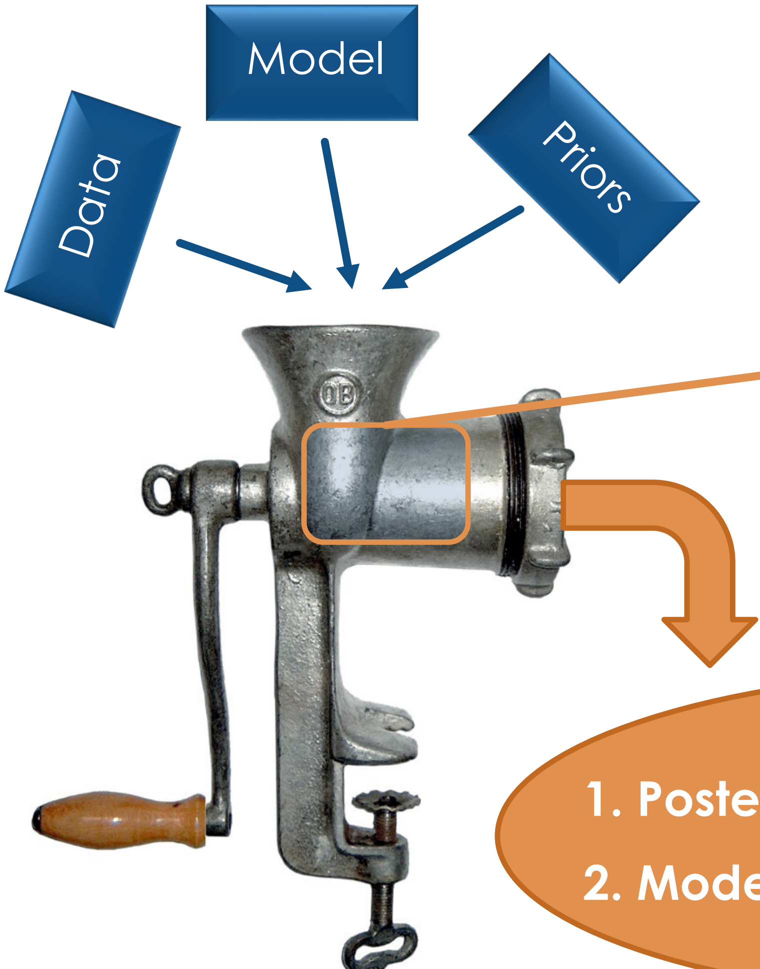
- empirical

Noise prior:

- assume relatively noisy data

(not default in SPM12 → set DCM.options.hE = 0; DCM.options.hC = 1)

Model estimation: running the machinery



Free energy approximation
Variational Bayes

Thermodynamic integration
MCMC

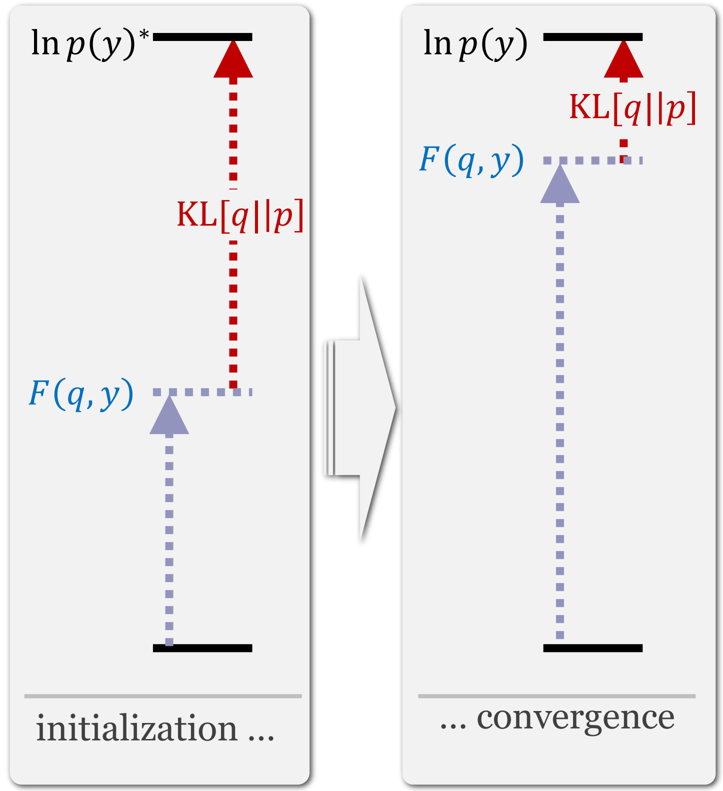
1. Posterior parameters - $p(\theta|y,m)$
2. Model evidence - $p(y|m)$

Inversion – variational Free Energy approximation to model evidence

model evidence

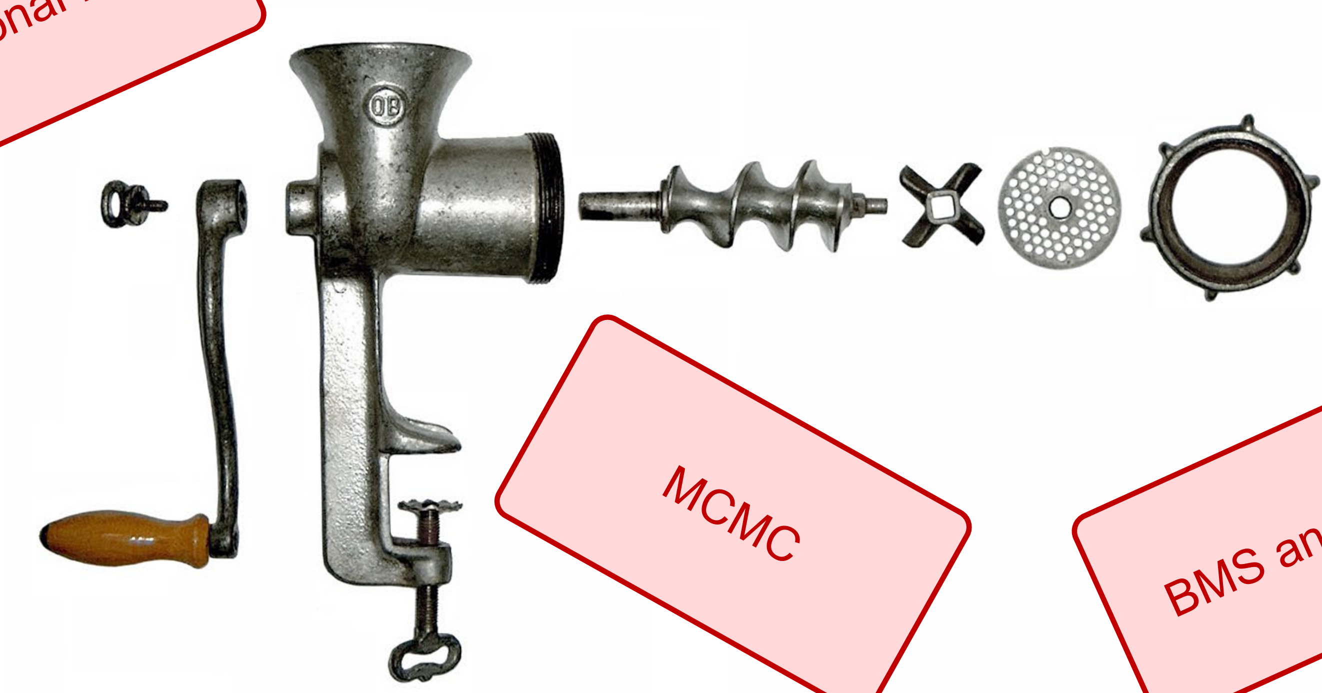
$$\ln p(y) = \underbrace{\mathrm{KL}[q||p]}_{\substack{\textbf{divergence} \\ \geq 0 \\ \text{(unknown)}}} + \underbrace{F(q,y)}_{\substack{\textbf{neg. free energy} \\ \text{(easy to evaluate} \\ \text{for a given } q)}}$$

When $F(q,y)$ is maximized, $q(\theta)$ is our best estimate of the true posterior.



Model estimation: running the machinery

Variational Bayes

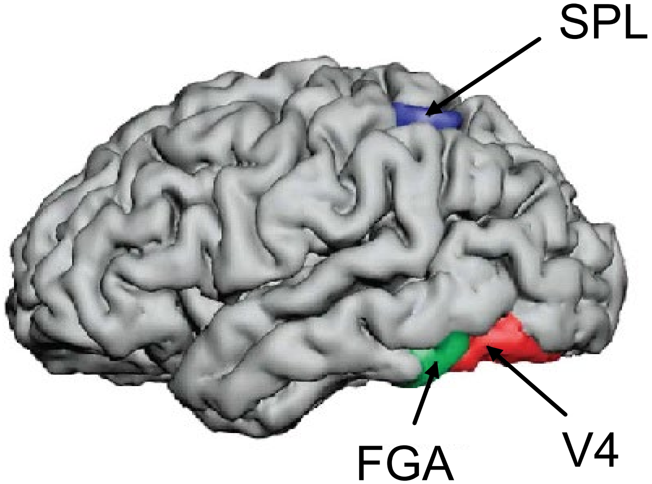


MCMC

BMS and BMA

Model selection example: Synesthesia

- Specific sensory stimuli lead to unusual, additional experiences
- Grapheme-color synesthesia: color
- Involuntary, automatic; stable over time, prevalence ~4%
- Potential cause: aberrant **cross-activation/coupling** between brain areas
  - grapheme encoding area (FGA)
  - color area (V4)
  - superior parietal lobule (SPL)

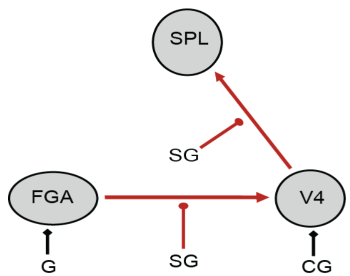


Hubbard, 2007

Bottom-up or Top-down "cross-activation"?

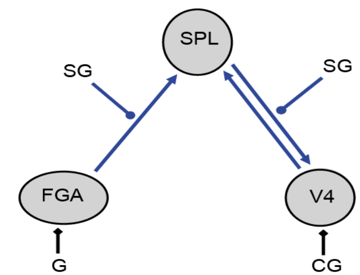
Bottom-up

(*Ramachandran & Hubbard, 2001*)



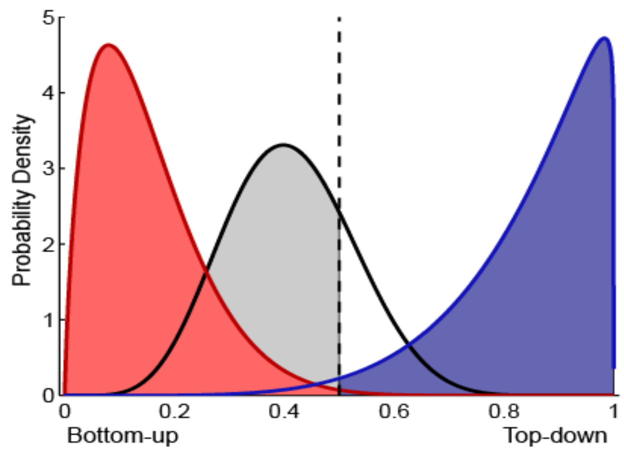
Top-down

(*Grossenbacher & Lovelace, 2001*)

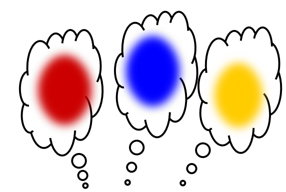


Projectors

ABC

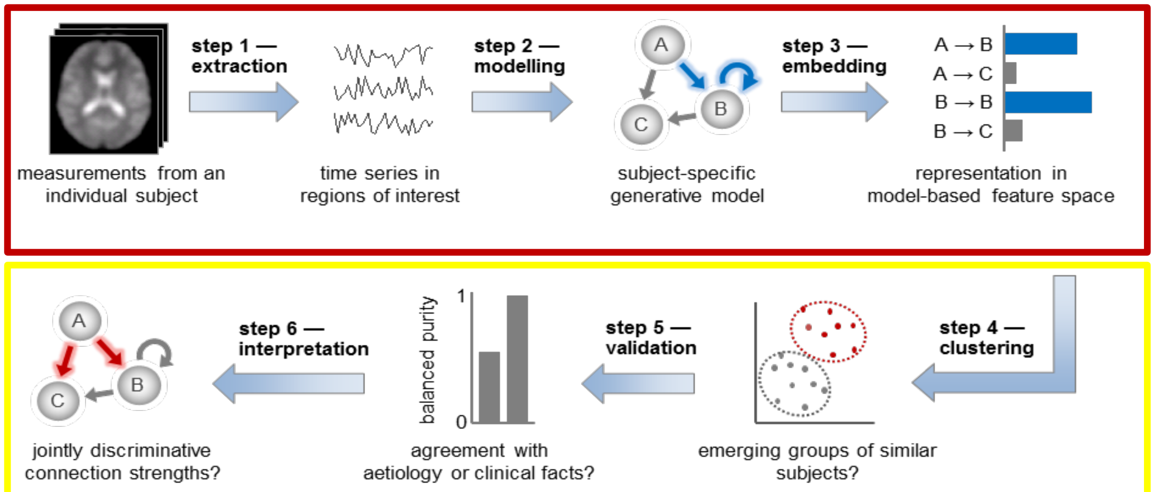


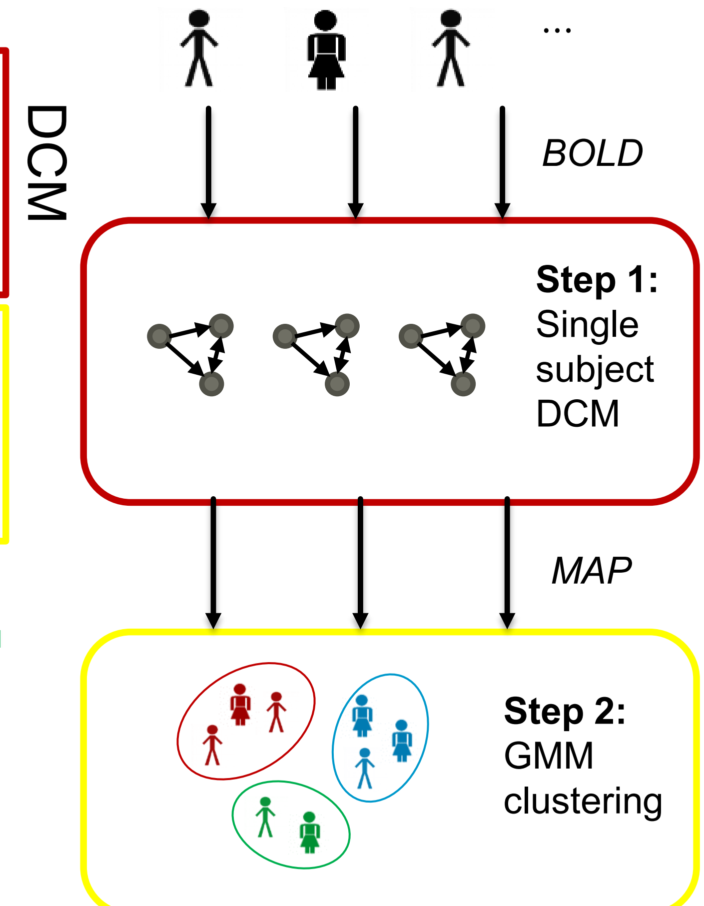
Associators

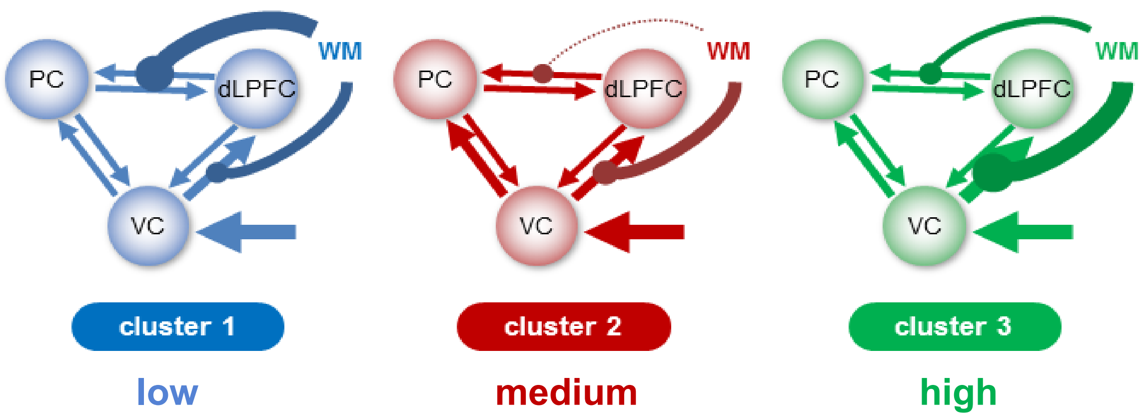


ABC

Example: DCM for physiologically plausible feature extraction (generative embedding)







negative symptoms (NS) subscale of the PANSS score

Brodersen et al., NeuroImage: Clinical, 2014

What questions can we answer using DCM?

Model comparison

What is the functional architecture of a network of brain regions?

→ Synesthesia

Are optimal models different between groups?

→ Synesthesia

Which connections are modulated by experimental manipulations?

Parameter inference

Are parameters different between individuals/groups?

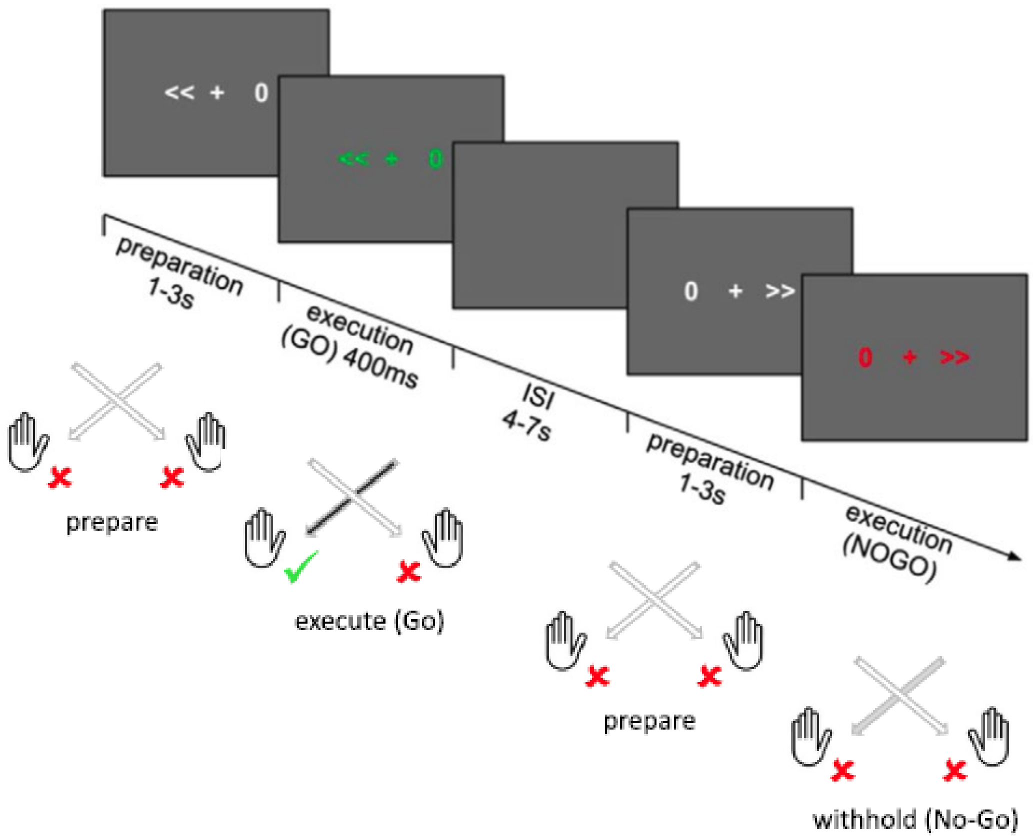
Use parameters as physiologically informed summary statistics
→ Generative embedding

... and of course many more!

Limitations

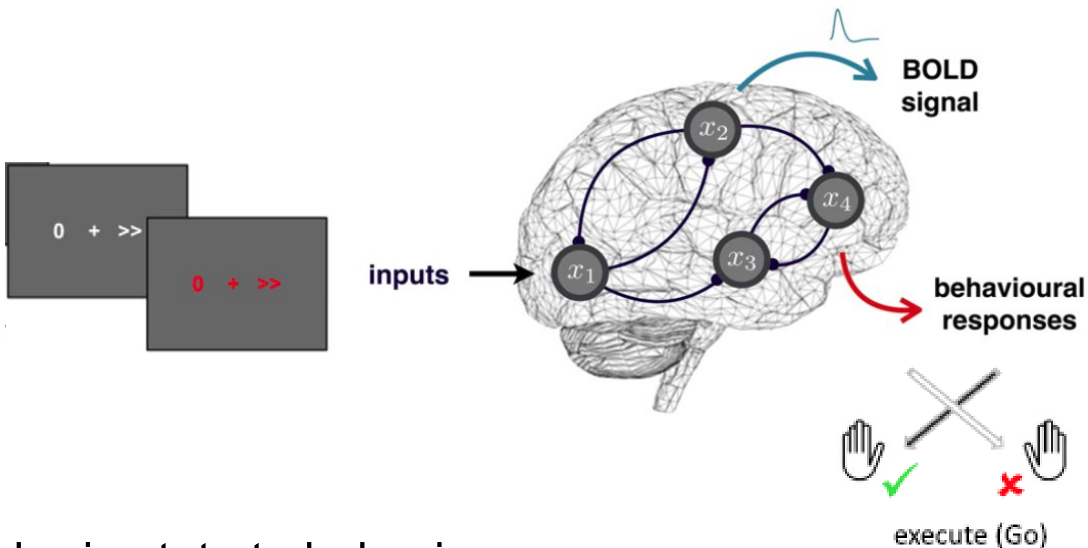
- DCMs only have inputs, no outputs
  - Limits the study of behavioral paradigms
- Local minima
  - Variational approximation can get stuck in local minima of free energy
- Size of networks
  - Standard inversion too slow for large networks (>10 nodes).
- Regularization through fixed priors:
  - Regularization based on other data → empirical Bayes.

Behavioral DCM – a step towards a neurocomputational model

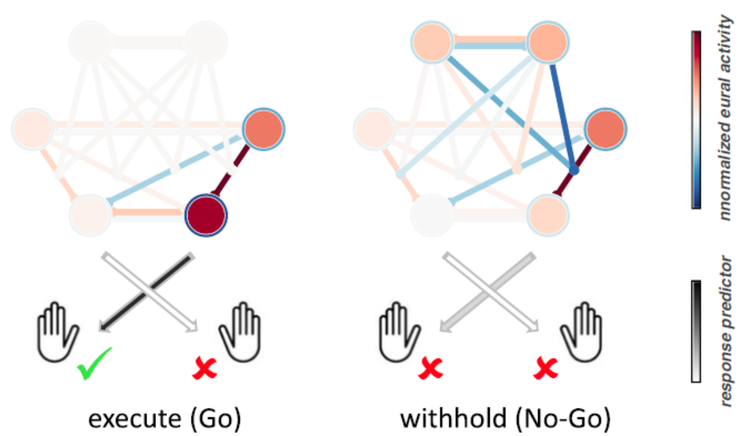


Rigoux and Daunizeau, Neuroimage, 2015

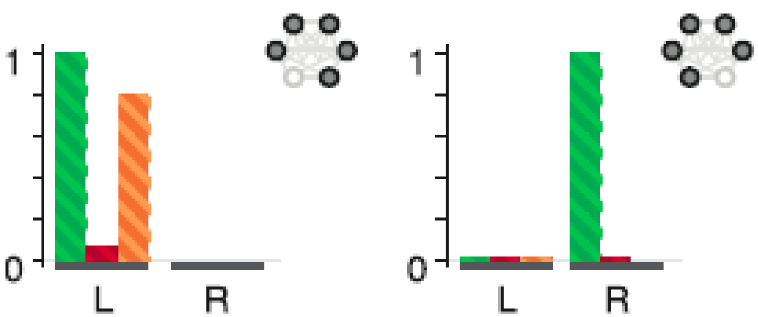
Behavioral DCM – a step towards a neurocomputational model



Mapping brain state to behavior

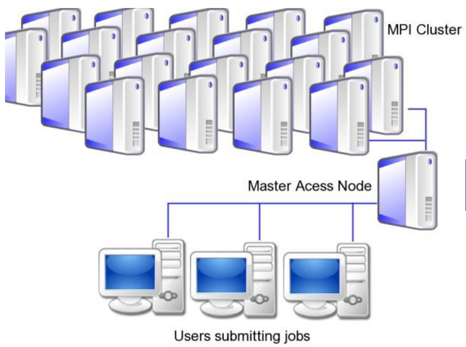


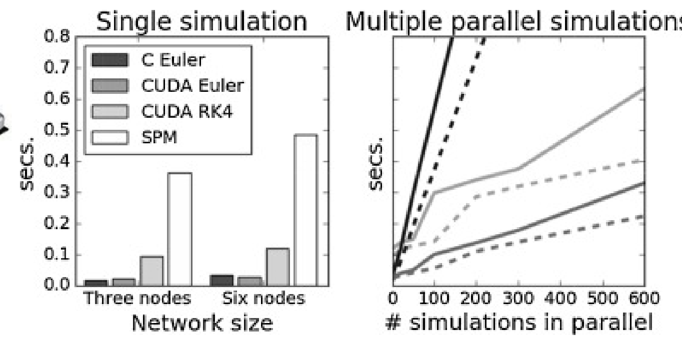
Lesion simulations



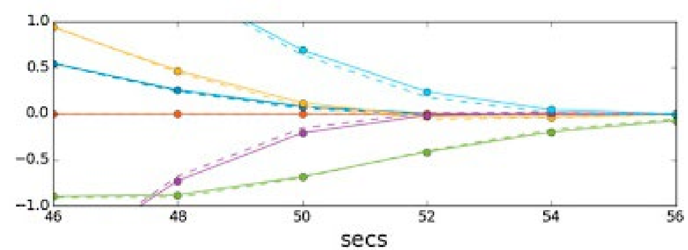
Rigoux and Daunizeau, Neuroimage, 2015

MCMC inversion of DCMs: Massively Parallel DCM - mpdcm





$$\left.\begin{array}{l}\dot{x} = f(x, u_1, \theta_1) \\ \dot{x} = f(x, u_2, \theta_2) \\ \vdots \\ \dot{x} = f(x, u_1, \theta_1)\end{array}\right\} \quad \text{mpdcm\_integrate(dcms)} \quad \left\{\begin{array}{l} y_1 \\ y_2 \\ \vdots \\ y_3 \end{array}\right.$$



- Fast inversion of DCMs
- MCMC based inversion possible

→ **Thermodynamic Integration** (alternative to negative Free Energy)

Aponte et al., Cogn. Neurodyn., 2021,Aponte et al., J. Neurosci. Meth., 2015

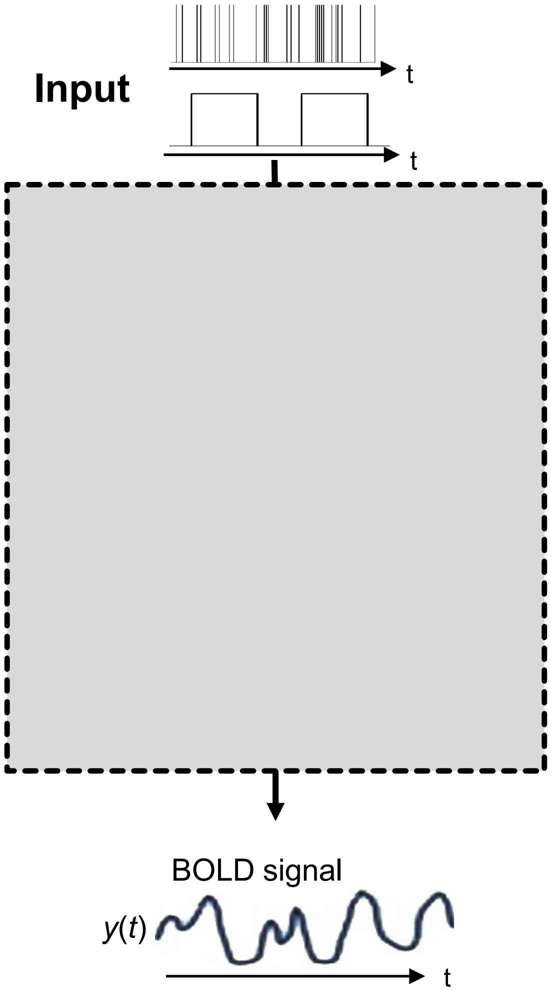
Recent additions to DCM for fMRI

- Massively parallel dynamic causal modelling
  - mpdcm Aponte et al., J Neuroscience Methods, 2016
- Regression dynamic causal modelling
  - rDCM Frässle et al., Neuroimage, 2017
- Hierarchical unsupervised generative embedding
  - HUGE Yu et al., Neuroimage, 2019

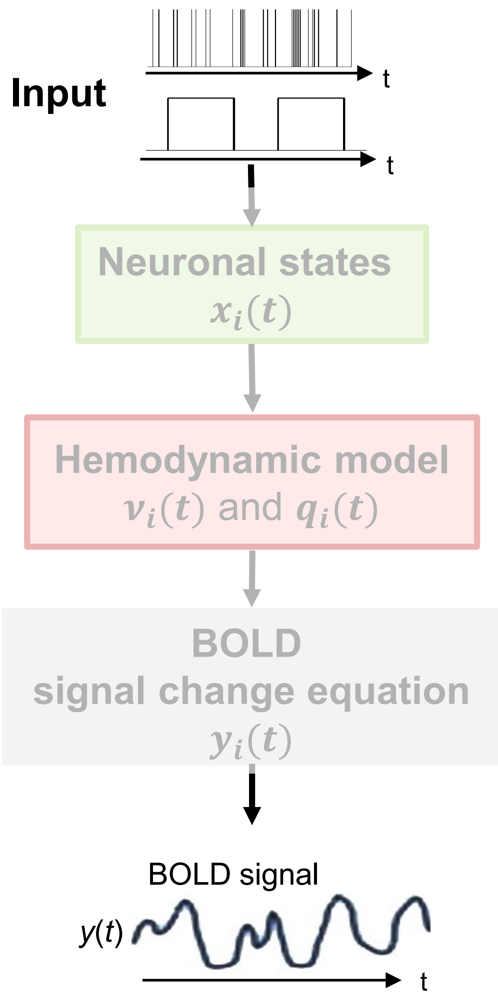
Modeling Connectivity: Advanced DCM for fMRI
→ Stefan Frässle

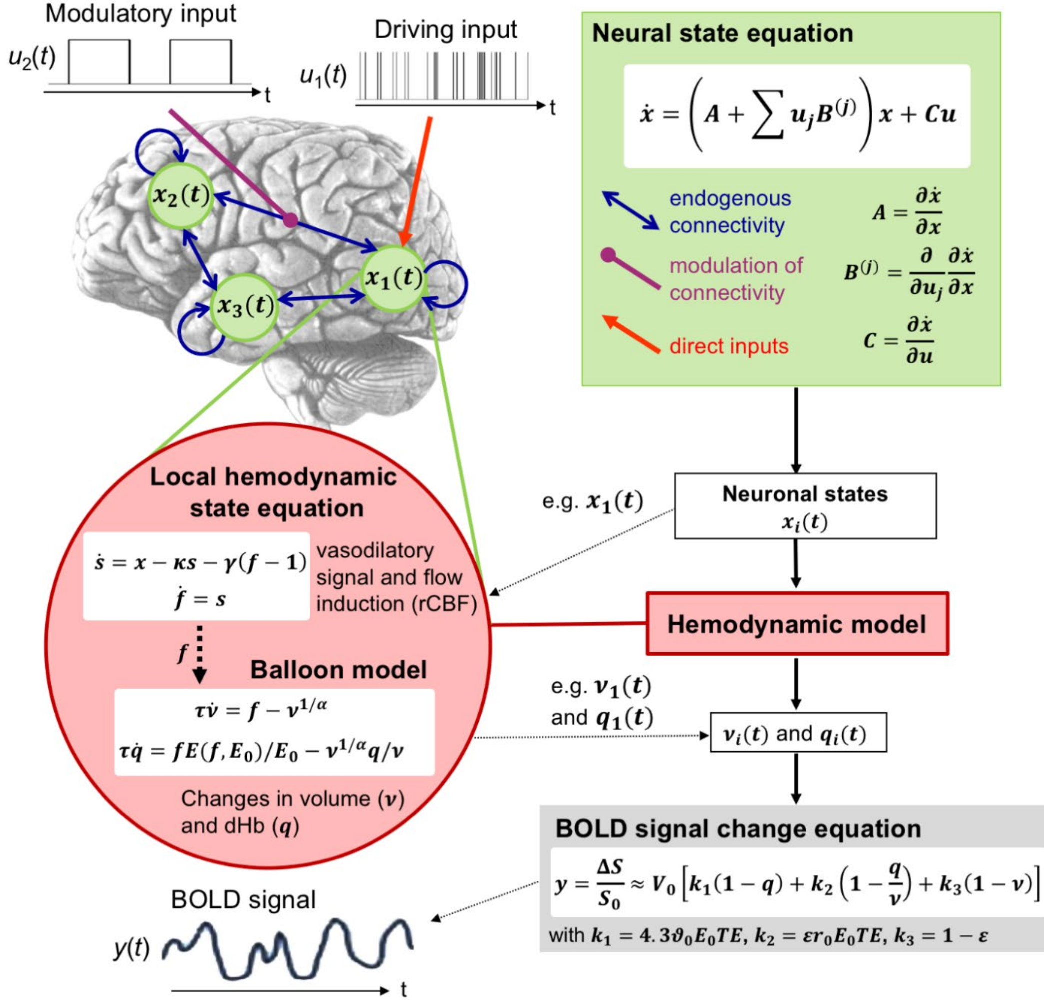
Available in TAPAS:
www.translationalneuromodeling.org/tapas

Summary – generative model



Summary – generative model





Stephan et al., 2015, Neuron

Summary - Bayesian system identification

$$u(t)$$

Neural (and hemo-) dynamics

$$dx/dt = f(x,u,\theta)$$

Observer function

$$y = g(x,\theta) + \varepsilon$$

$$p(y \mid \theta, m) = N(g(\theta), \Sigma(\theta))$$

$$p(\theta, m) = N(\mu_\theta, \Sigma_\theta)$$

Inference on model structure

$$p(y \mid m) = \int p(y \mid \theta, m) p(\theta) d\theta$$

Inference on parameters

$$p(\theta \mid y, m) = \frac{p(y \mid \theta, m) p(\theta, m)}{p(y \mid m)}$$

Design experimental inputs

Define likelihood model

Specify priors

Invert model

Make inferences

DCM software note

Basic functionality for DCM for fMRI is provided within

SPM
https://www.fil.ion.ucl.ac.uk/spm/

Thank you!

Many thanks to Stefan Frässle,
Klaas Enno Stephan, Hanneke den Ouden
and Jean Daunizeau for many of the slides!

List with suggested DCM literature in Appendix of this presentation!

DCM literature (1)

- **Aponte EA, Raman S, Sengupta B, Penny WD, Stephan KE, Heinzle J (2016). mpdcm: A Toolbox for Massively Parallel Dynamic Causal Modeling. Journal of Neuroscience Methods 257: 7-16.**
- **Aponte EA, Yao Y, Raman S, Frässle S, Heinzle J, Penny WD, Stephan KE (2021). An introduction to thermodynamic integration and application. Cognitive Neurodynamics 16: 1-15.**
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- Frässle S, Yao Y, Schöbi S, Aponte EA, Heinzle J, Stephan KE (in press) Generative models for clinical applications in computational psychiatry. Wiley Interdisciplinary Reviews: Cognitive Science.
- **Frässle S, Lomakina EI, Razi A, Friston KJ, Buhmann JM, Stephan KE (2017) Regression DCM for fMRI. NeuroImage 155:406-421.**

DCM literature (2)

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- Li B, Daunizeau J, Stephan KE, Penny WD, Friston KJ (2011). Stochastic DCM and generalised filtering. NeuroImage 58: 442-457
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- Penny WD, Stephan KE, Mechelli A, Friston KJ (2004a) Comparing dynamic causal models. NeuroImage 22:1157-1172.
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- Penny WD (2012) Comparing dynamic causal models using AIC, BIC and free energy. Neuroimage 59: 319-330
- **Raman S, Deserno L, Schlagenhauf F, Stephan KE (2016). A hierarchical model for integrating unsupervised generative embedding and empirical Bayes. Journal of Neuroscience Methods 269: 6-20.**
- Rigoux L, Stephan KE, Friston KJ, Daunizeau J (2014). Bayesian model selection for group studies – revisited. NeuroImage 84: 971-985.
- **Rigoux L and Daunizeau J (2015). Dynamic causal modelling of brain–behaviour relationships. NeuroImage 117:202-221**
- Stephan KE, Harrison LM, Penny WD, Friston KJ (2004) Biophysical models of fMRI responses. Curr Opin Neurobiol 14:629-635.
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- Stephan KE, Harrison LM, Kiebel SJ, David O, Penny WD, Friston KJ (2007) Dynamic causal models of neural system dynamics: current state and future extensions. J Biosci 32:129-144.

DCM literature (3)

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- **Stephan KE, Penny WD, Moran RJ, den Ouden HEM, Daunizeau J, Friston KJ (2010) Ten simple rules for Dynamic Causal Modelling. NeuroImage 49: 3099-3109.**
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- **Yao Y, Raman SS, Schiek M, Leff A, Frässle S, Stephan KE (2018) Variational Bayesian Inversion for Hierarchical Unsupervised Generative Embedding (HUGE). NeuroImage, 179: 604-619**
- **Zeidman P, Jafarian A, Corbin N, Seghier ML, Razi A, Price CJ, Friston KJ (2019) A guide to group effective connectivity analysis, part 1: First level analysis with DCM for fMRI. NeuroImage, DOI: 10.1016/j.neuroimage.2019.06.031**
- **Zeidman P, Jafarian A, Seghier ML , Litvak V, Cagnan H , Price CJ, Friston KJ (2019) A guide to group effective connectivity analysis, part 2: Second level analysis with PEB. NeuroImage, DOI: 10.1016/j.neuroimage.2019.06.032**