Drift Diffusion Model

Computational Psychiatry Course Zurich
September 14, 2022
Matt Nassar

Perceptual Decisions

Perceptual Decisions

Perceptual Decisions

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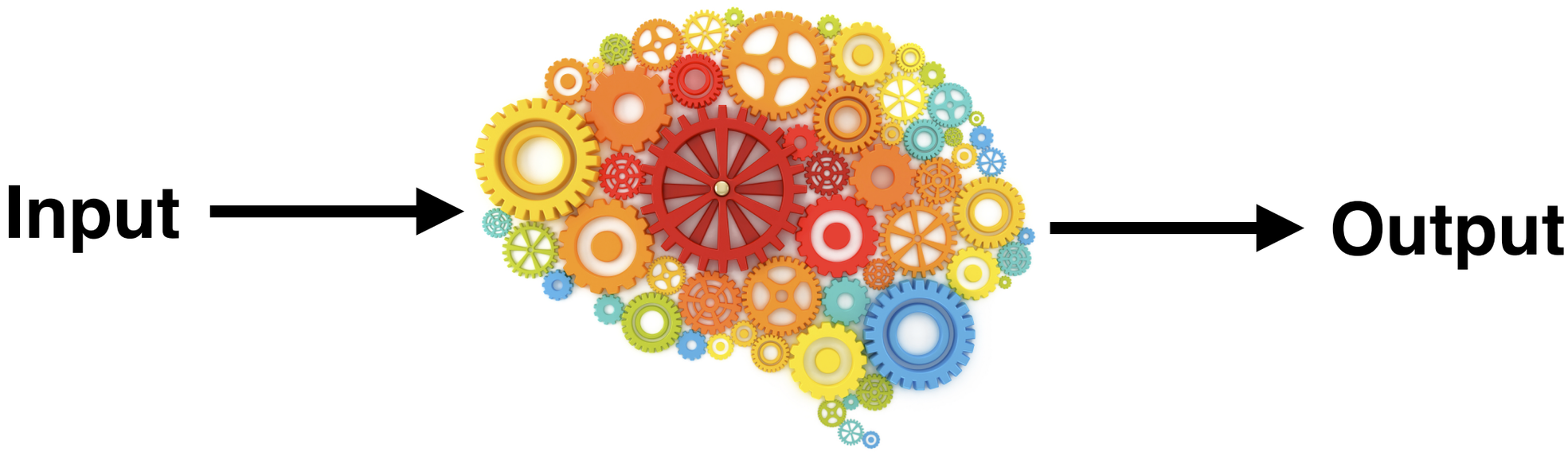
Perceptual Decisions

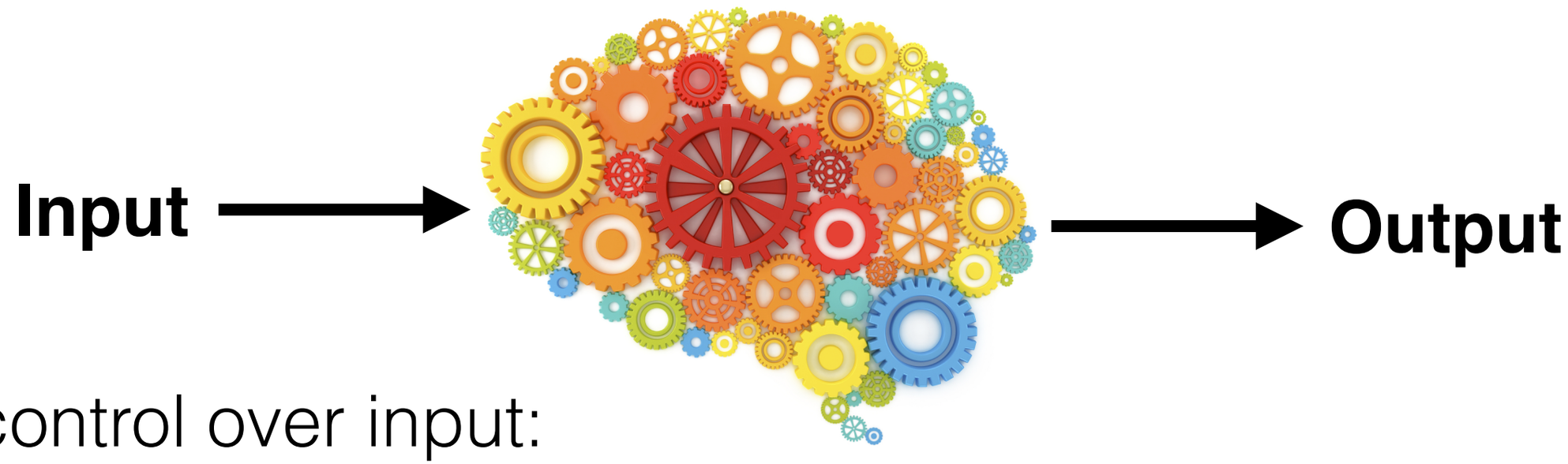
Perceptual Decisions

Important for survival!!!

Perceptual Decisions

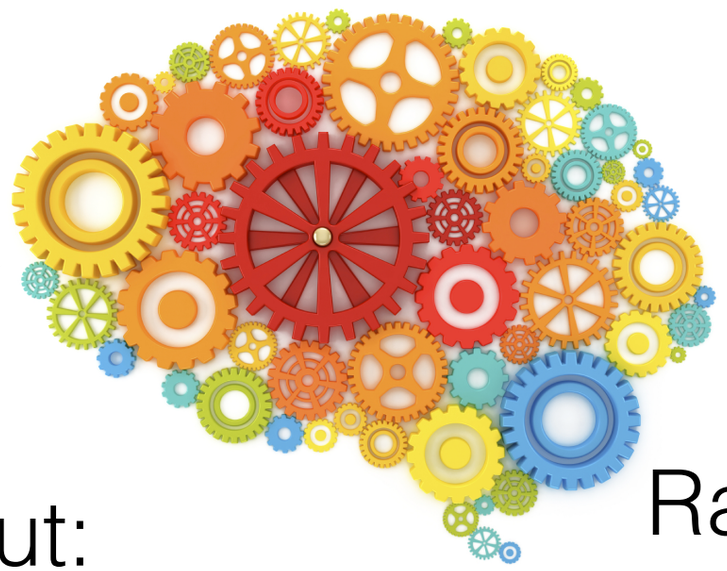


Perceptual Decisions



Total control over input:

Perceptual Decisions

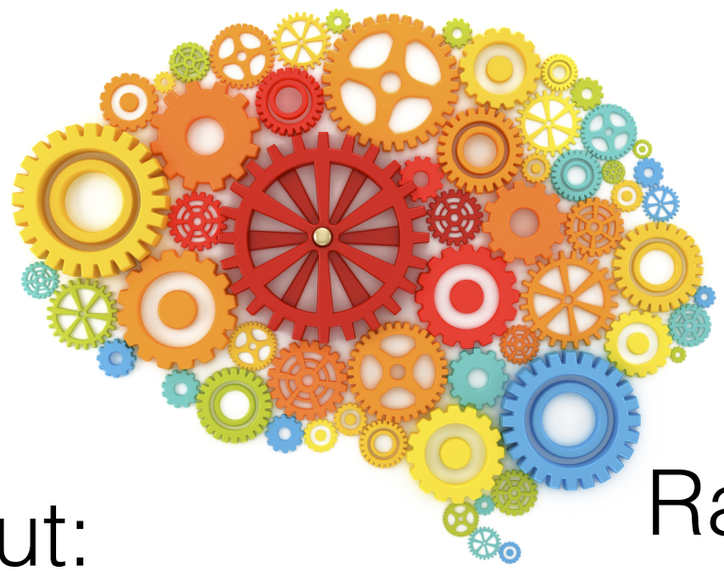
Input →  → **Output**

Total control over input:

Rapid measurement of decision & timing

a,s,d,f

Perceptual Decisions

Input →  → **Output**

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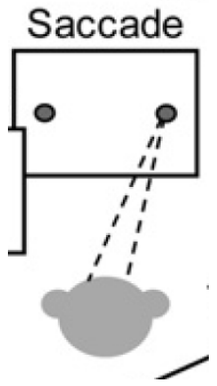
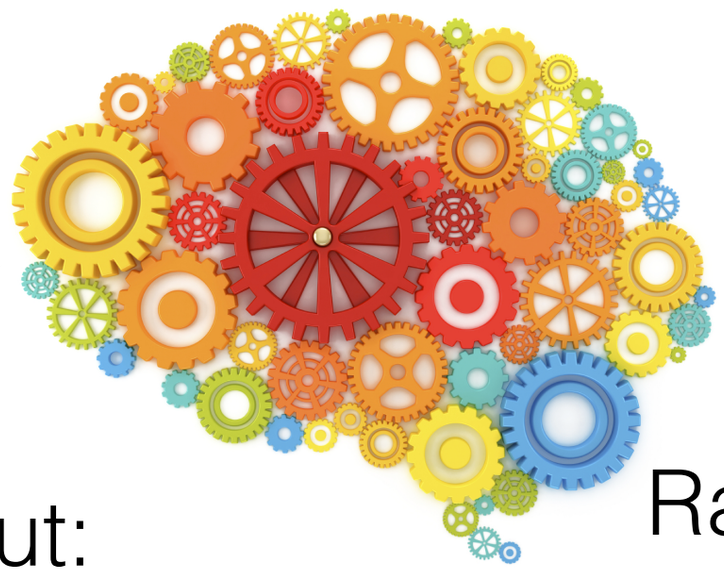


Perceptual Decisions

Paradigms conducive to animal research

Input →  → **Output**

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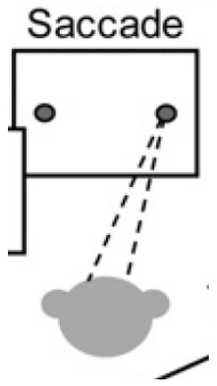


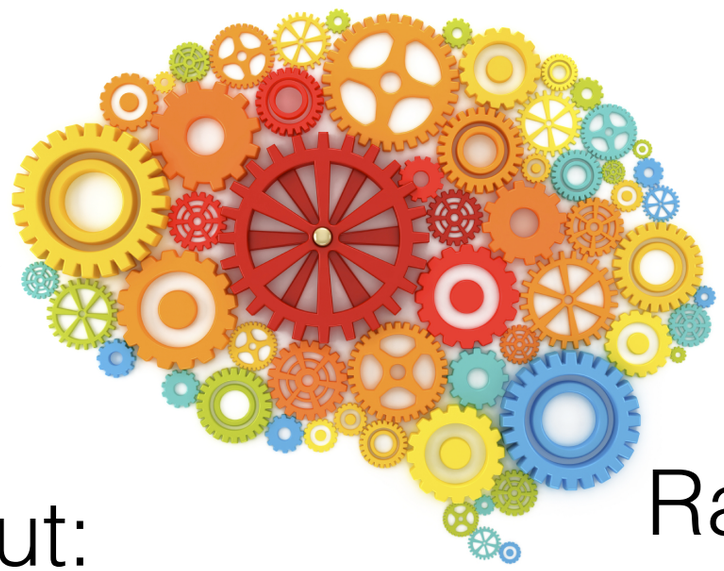
Perceptual Decisions

Paradigms conducive to animal research

Record neurons!

Silence neurons!

Activate neurons!

Input →  → **Output**

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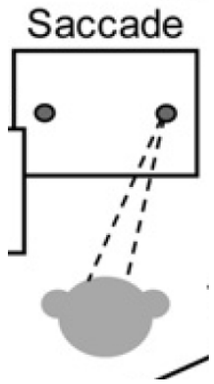

Perceptual Decisions

Paradigms conducive to animal research

We have a good
idea where to look for
sensory inputs and motor outputs

LIP MT

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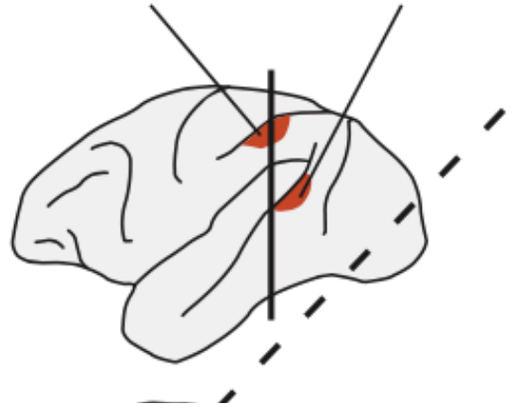
Input →  → **Output**

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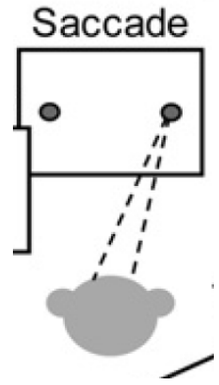
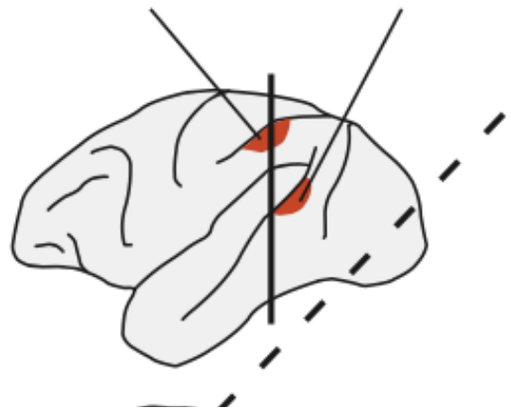


Perceptual Decisions

Amenable to good experiments!!!

We have a good idea where to look for sensory inputs and motor outputs

LIP MT



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Total control over input:

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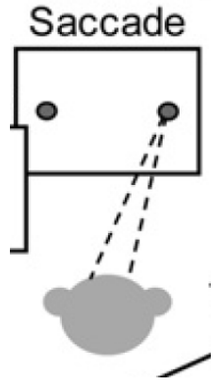


Which way are dots moving?

Which way are dots moving?

A simple model of perceptual decision making:

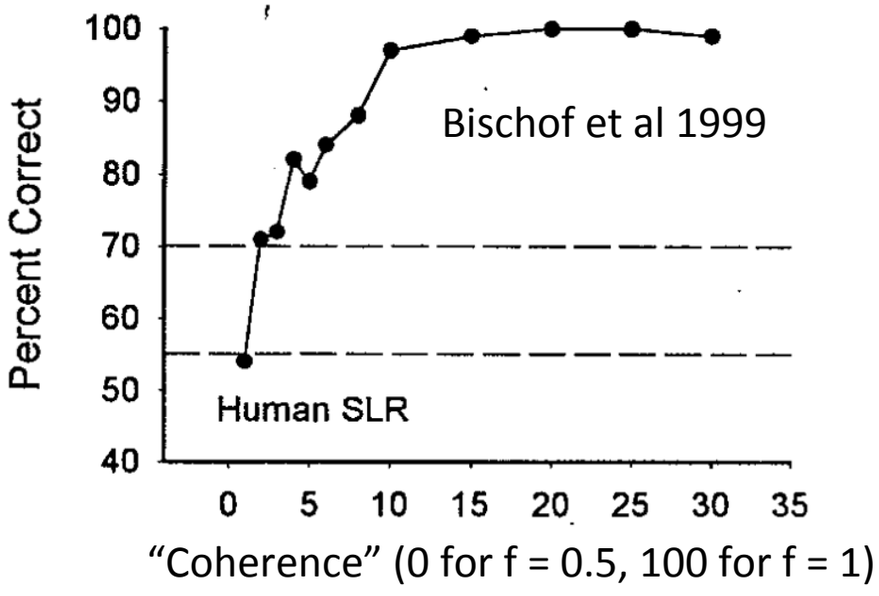
- Observe one frame (timestep) of dot motion

A simple model of perceptual decision making:

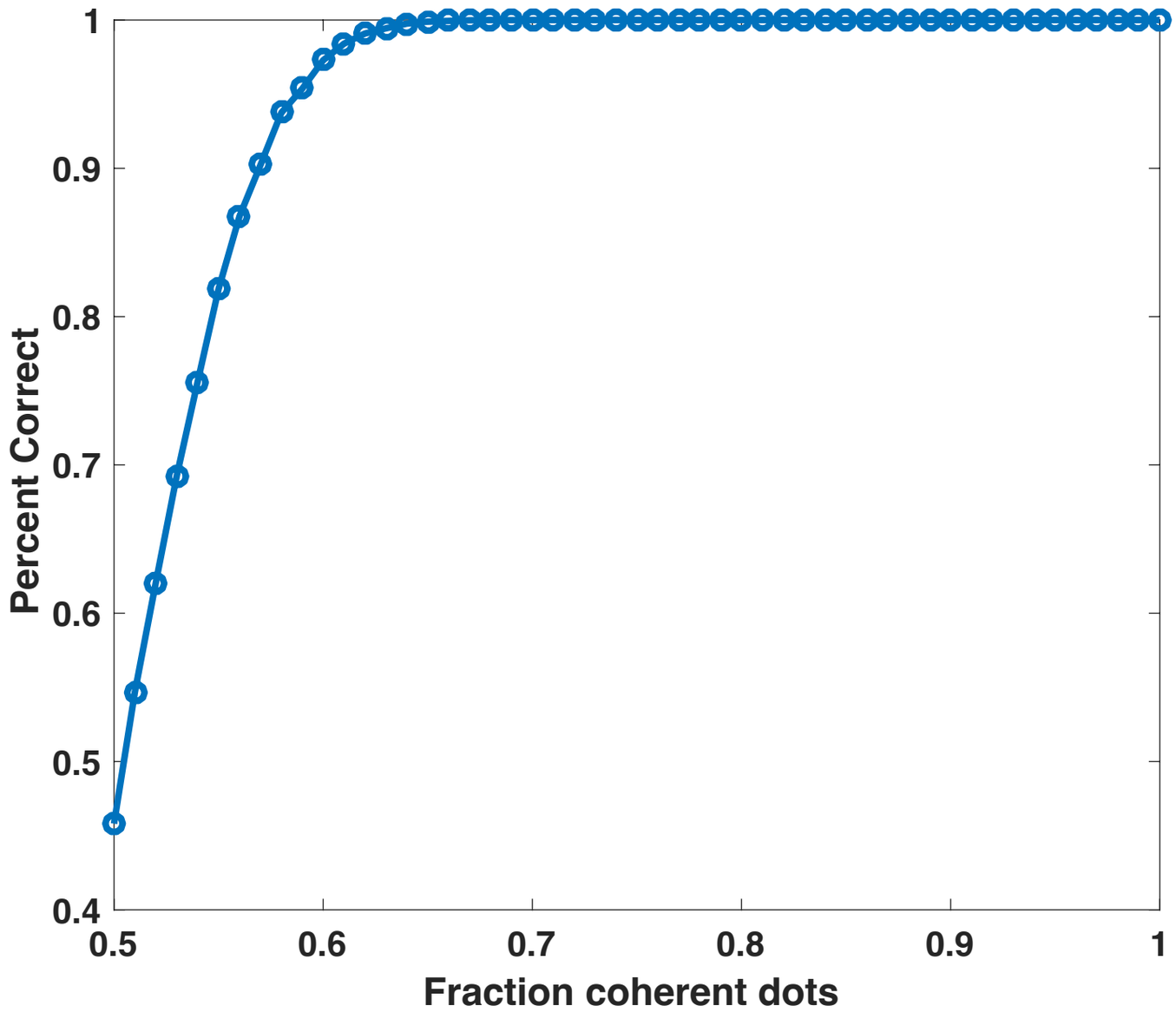
- Observe one frame (timestep) of dot motion
- Count ones that are moving rightward
- If that is greater than 1/2 the dots, say right
- Otherwise say left

Our simple model makes a prediction

One human's performance



"Coherence" (0 for f = 0.5, 100 for f = 1)



Which way are dots moving?

When *should* you stop to make a decision?

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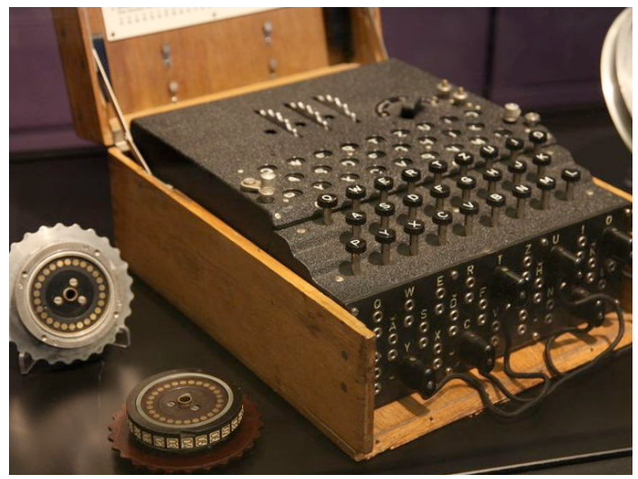
The “enigma”



Weight of evidence =

$$\log\left[\frac{\Pr(m|h_1)}{\Pr(m|h_0)}\right] + \log\left[\frac{\Pr(m|h_1)}{\Pr(m|h_0)}\right] + \ldots$$

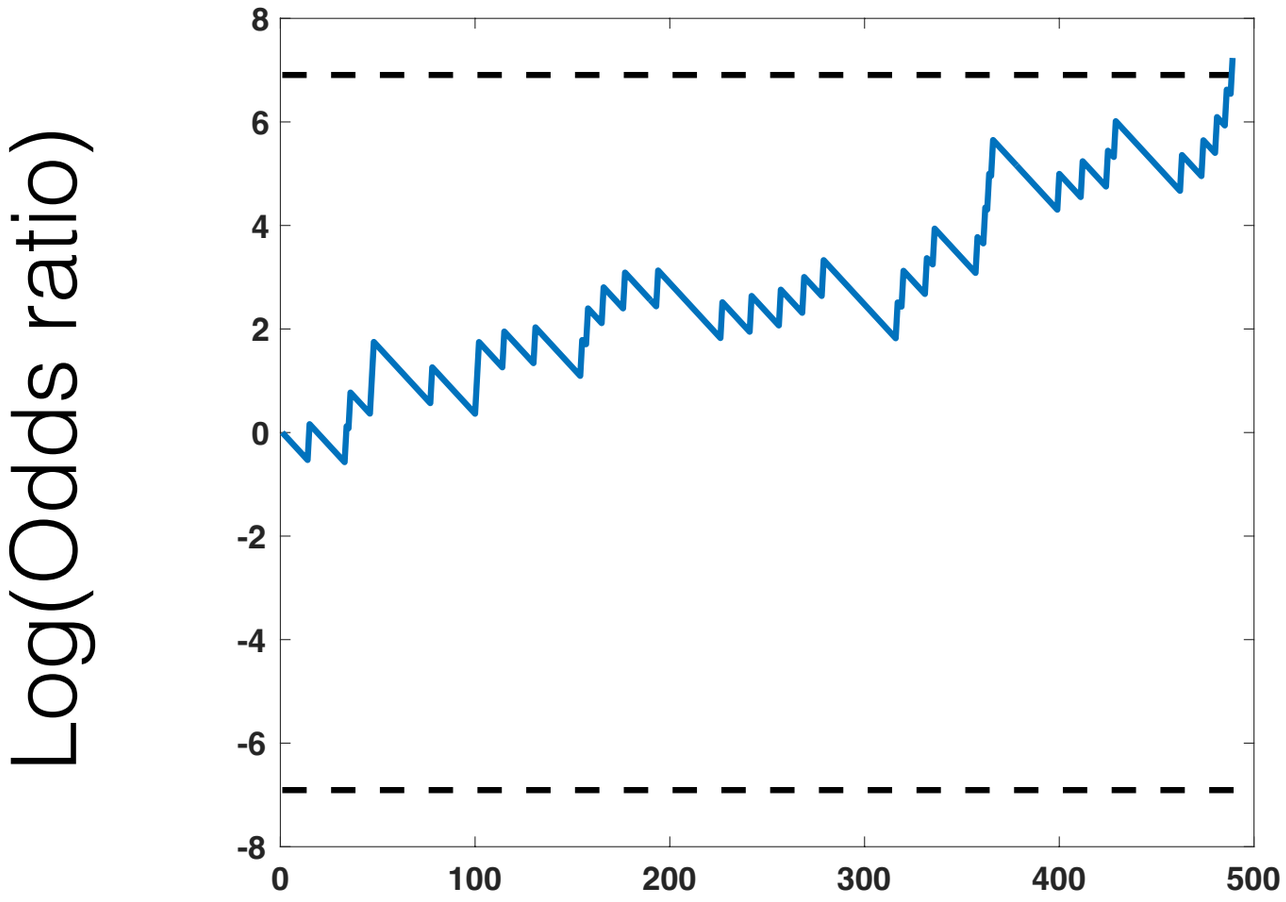
Alan Turing

Accumulate to some log odds ratio

Stop at preferred level of certainty

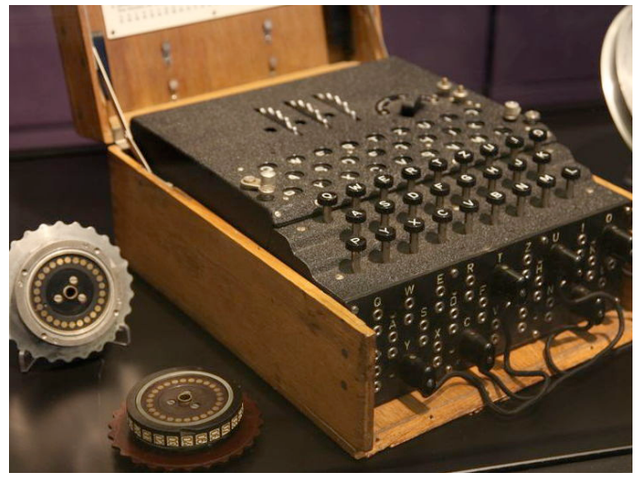
When *should* you stop to make a decision?

SAME STATE



DIFFERENT STATE

The “enigma”



Alan Turing

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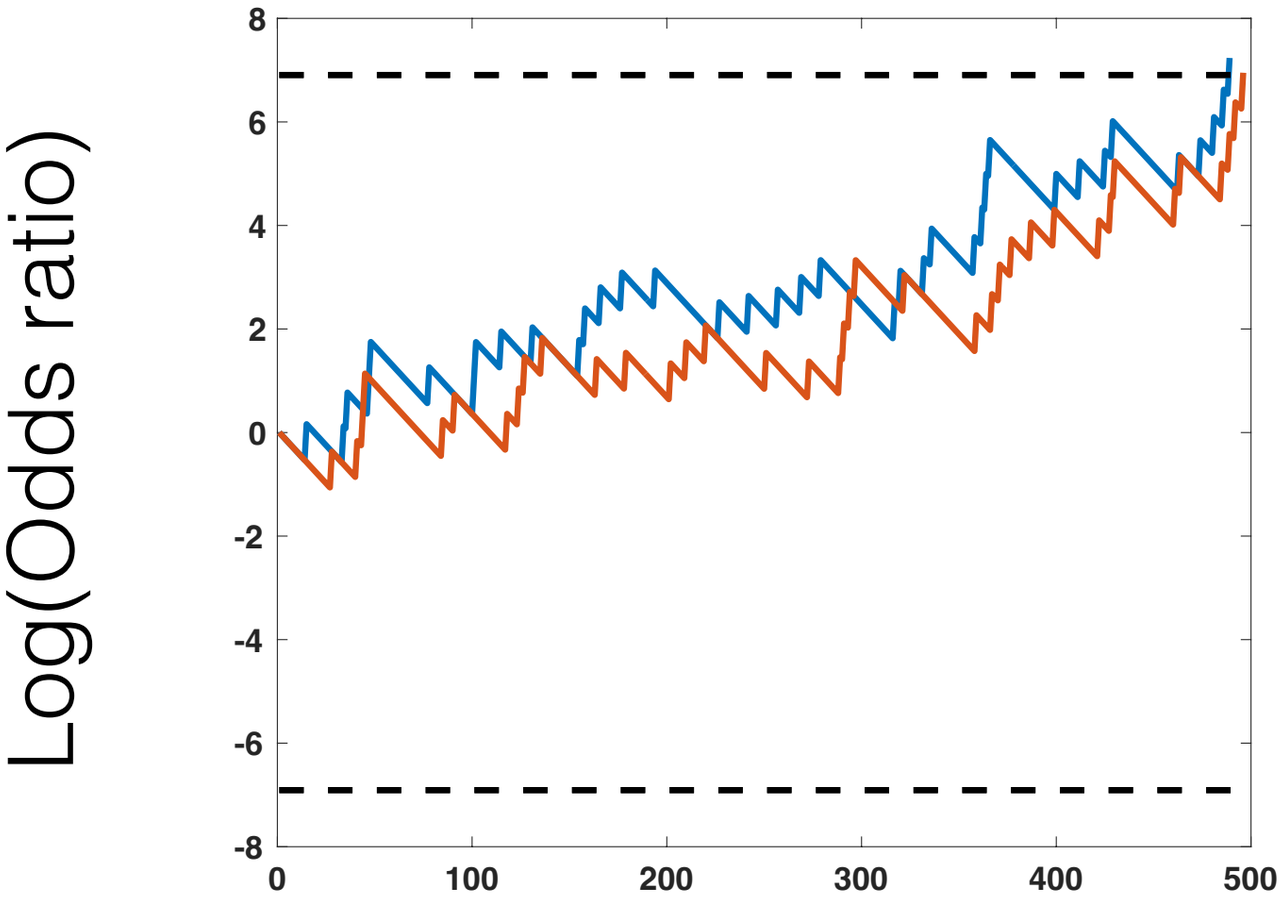

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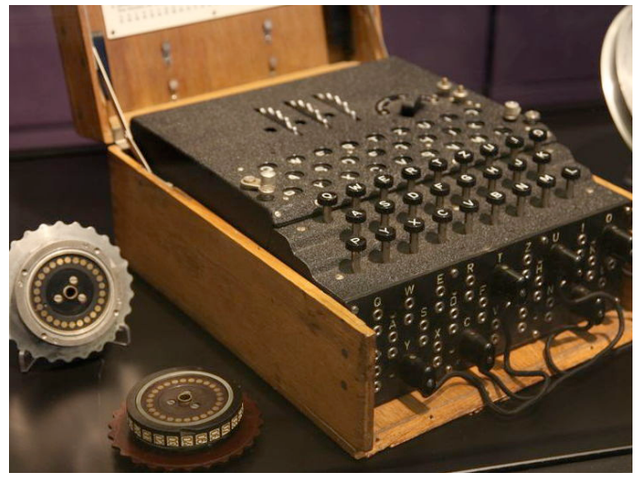

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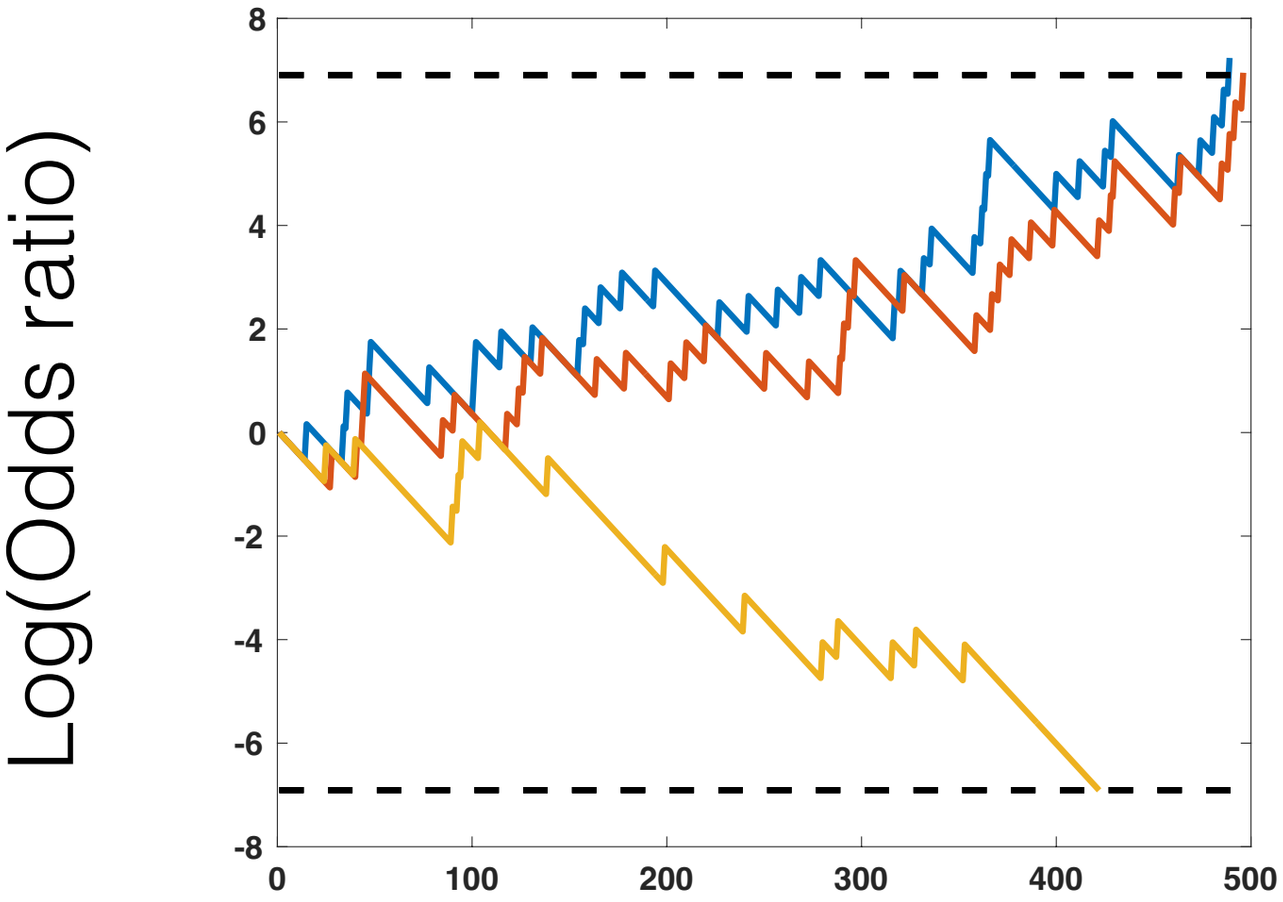

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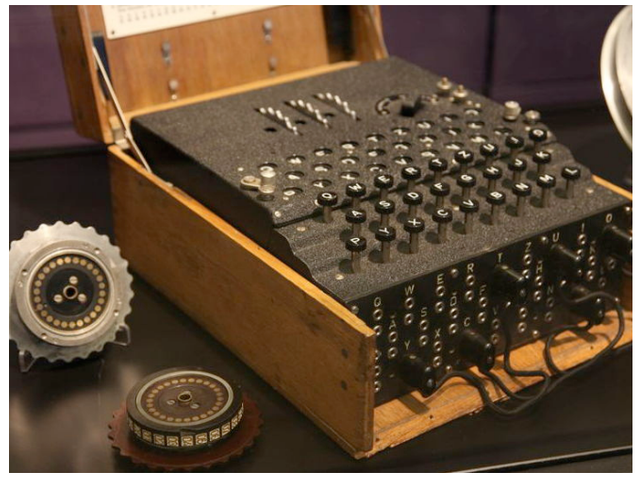

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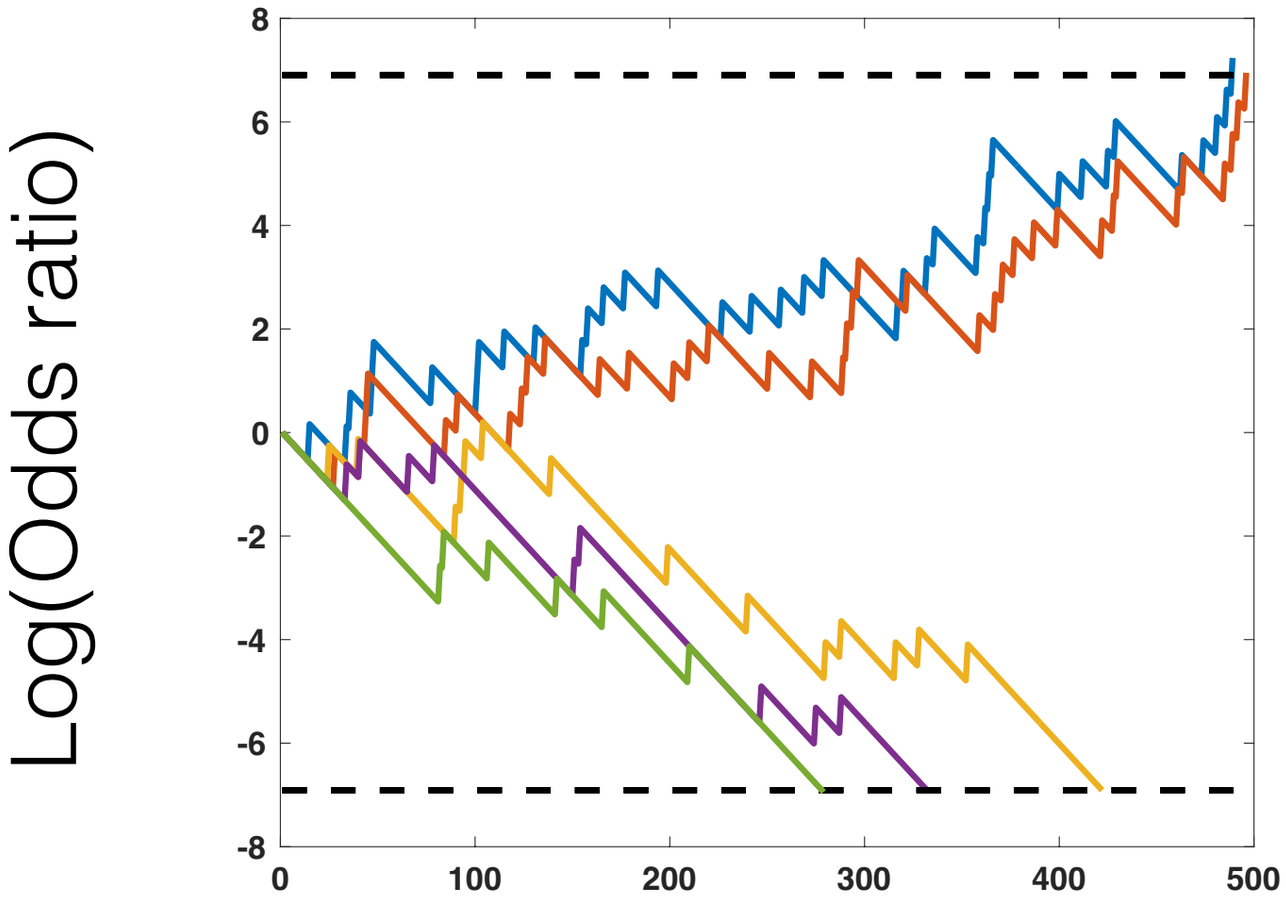
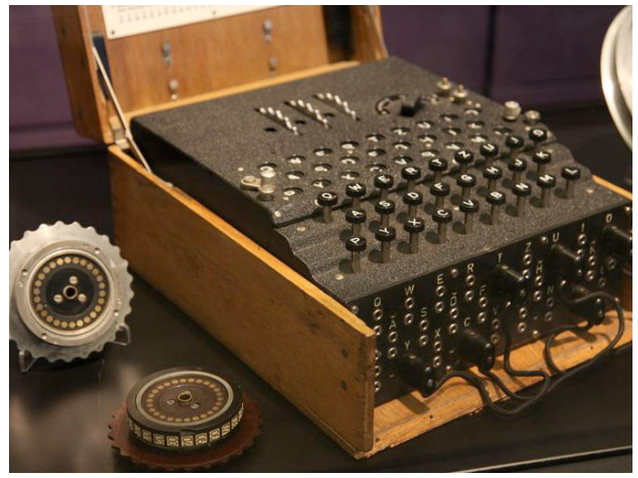


DIFFERENT STATE

Symbols Checked

The “enigma”

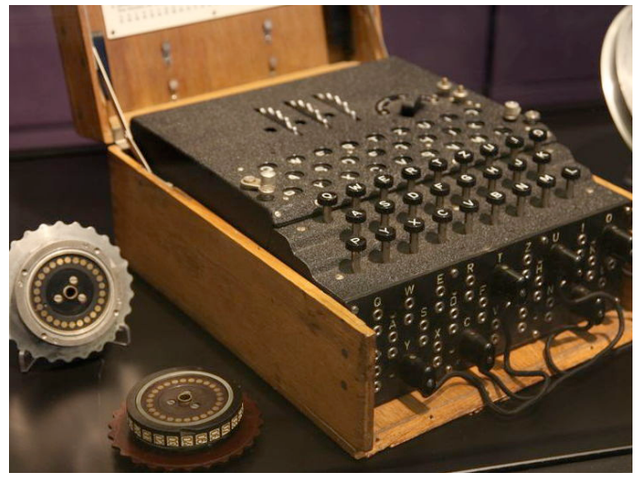


Alan Turing

When *should* you stop to make a decision?

Key insights:

The “enigma”



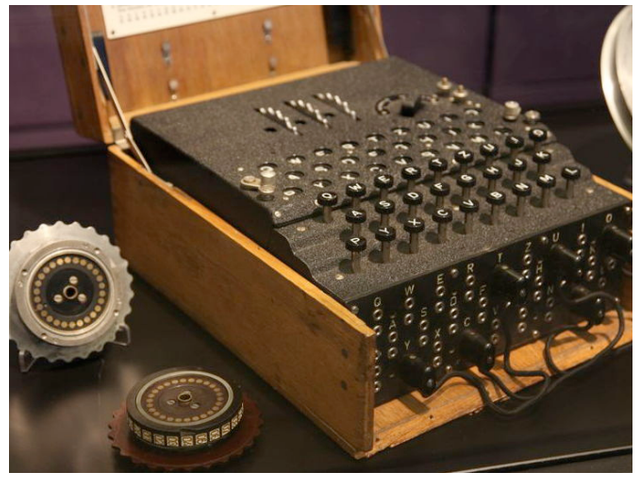
Alan Turing

When *should* you stop to make a decision?

Key insights:

Evidence can be quantified as log likelihood ratio

The “enigma”



Alan Turing

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Key insights:

Evidence can be ***accumulated*** over time

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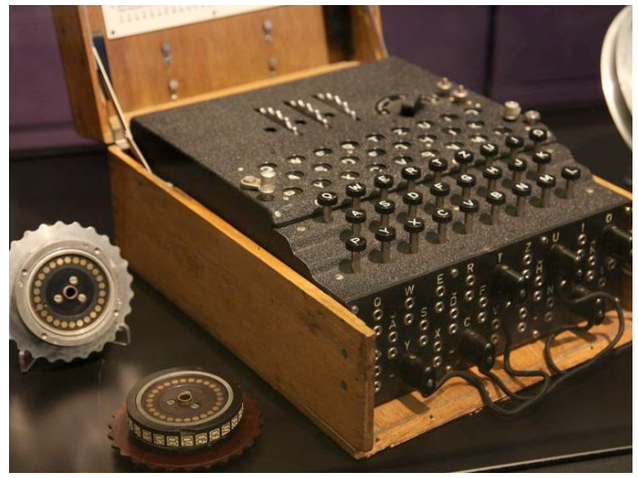


Alan Turing

When *should* you stop to make a decision?

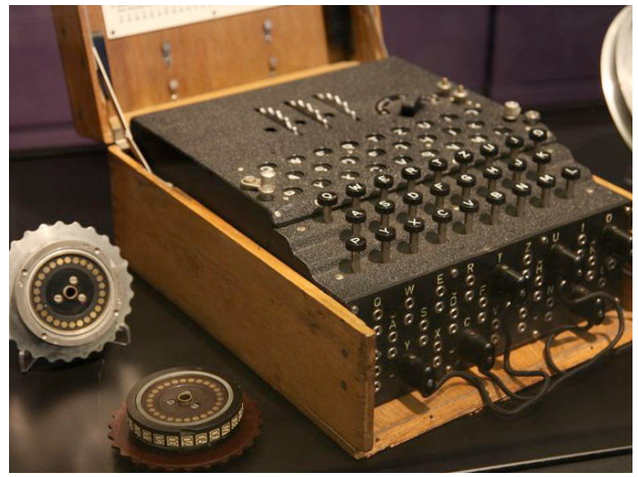
Key insights:

Evidence can be ***accumulated*** over time

A fixed ***stopping criterion*** on accumulated evidence ensures the shortest time to a given level of decision certainty

Wald 1947

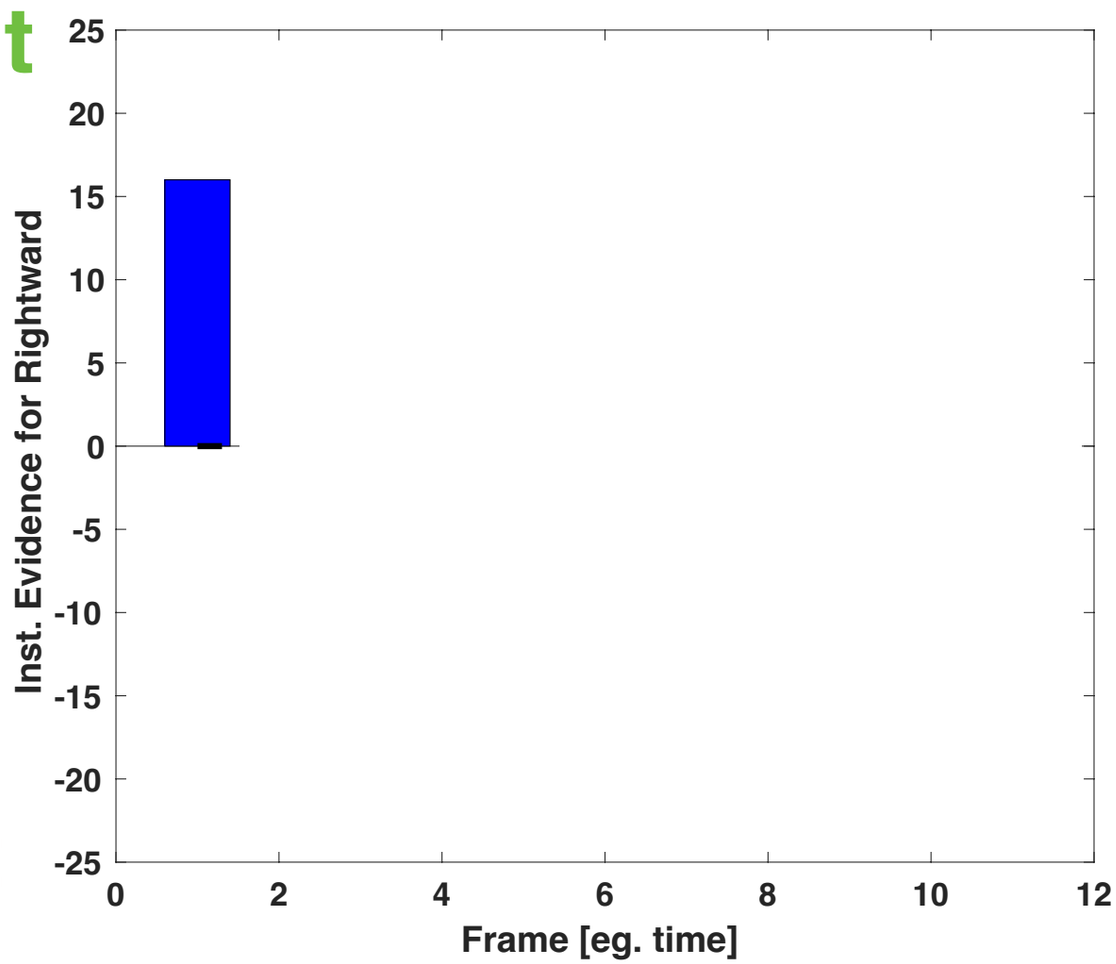
The “enigma”



Alan Turing

Back to the perceptual decision…

More right



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Back to the perceptual decision…

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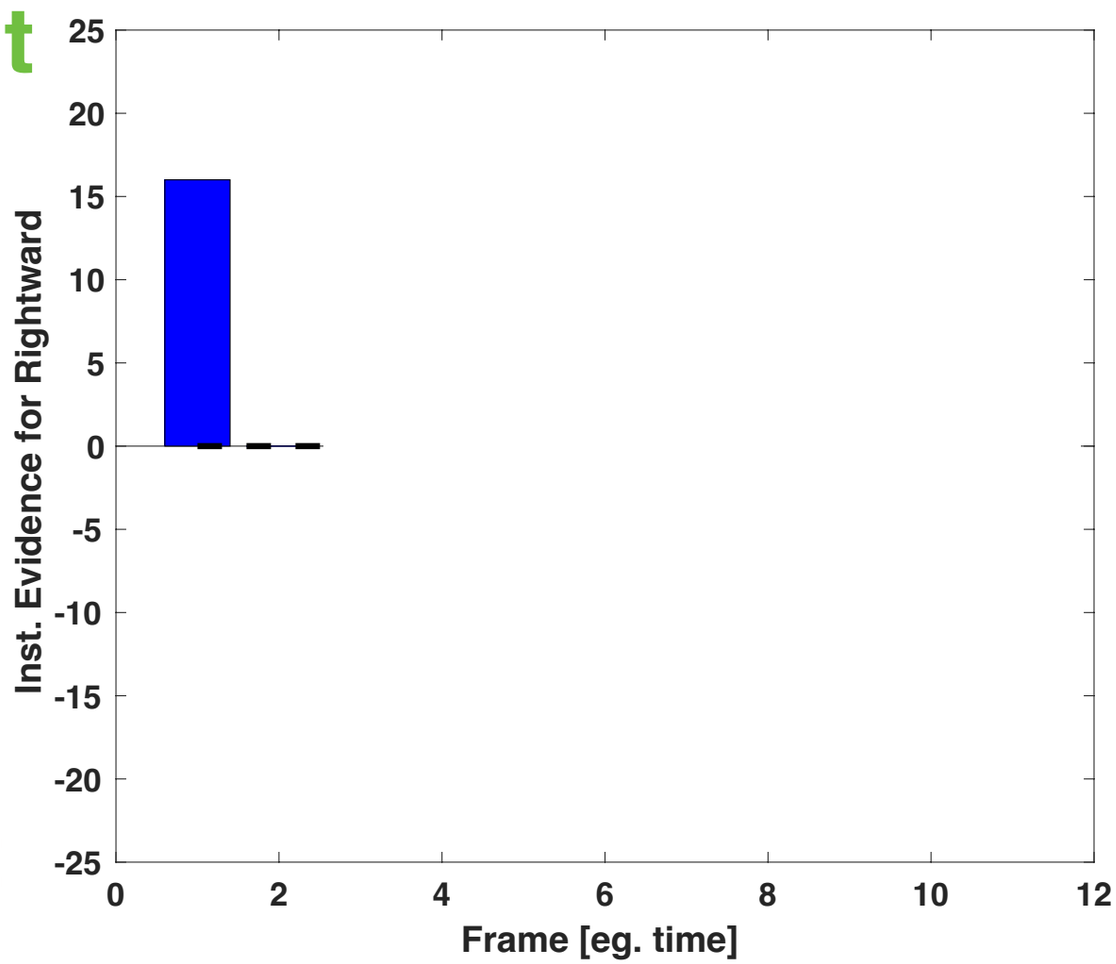

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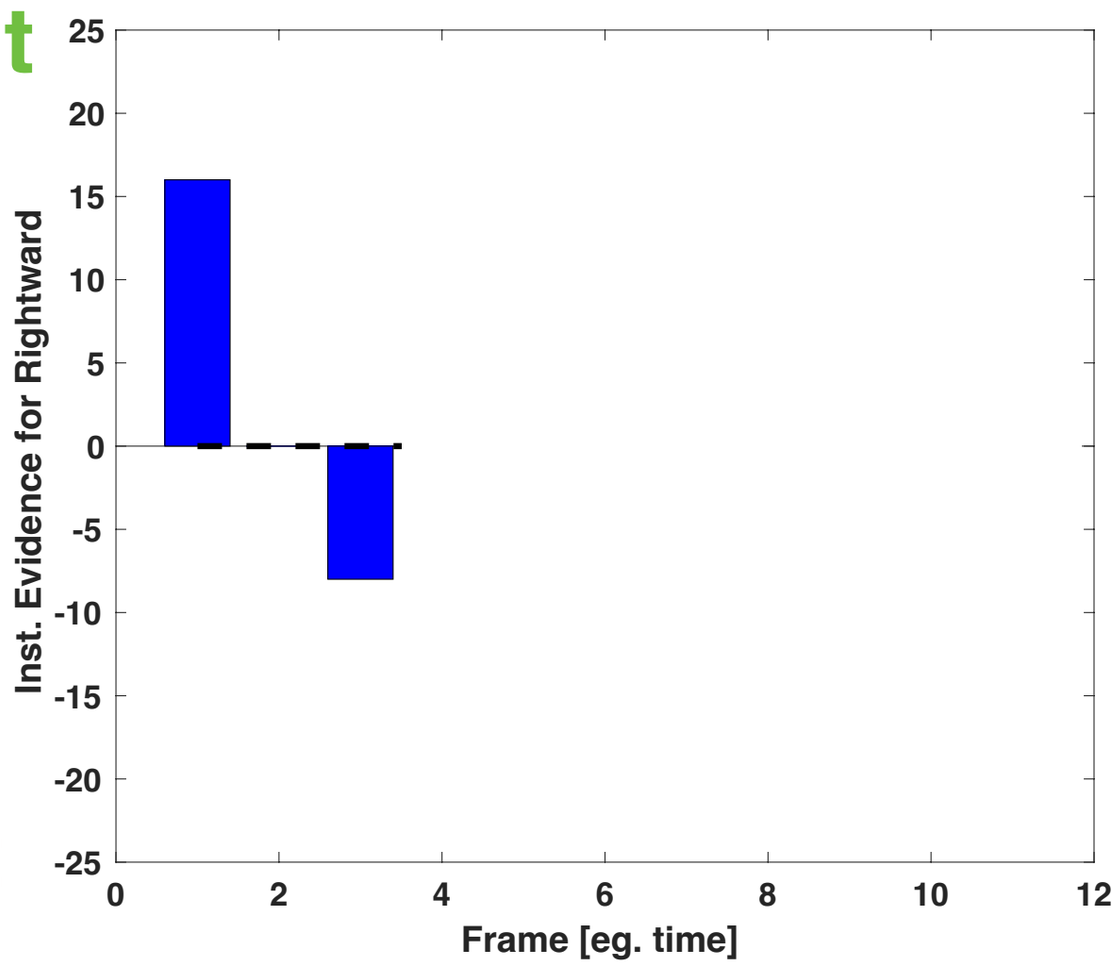

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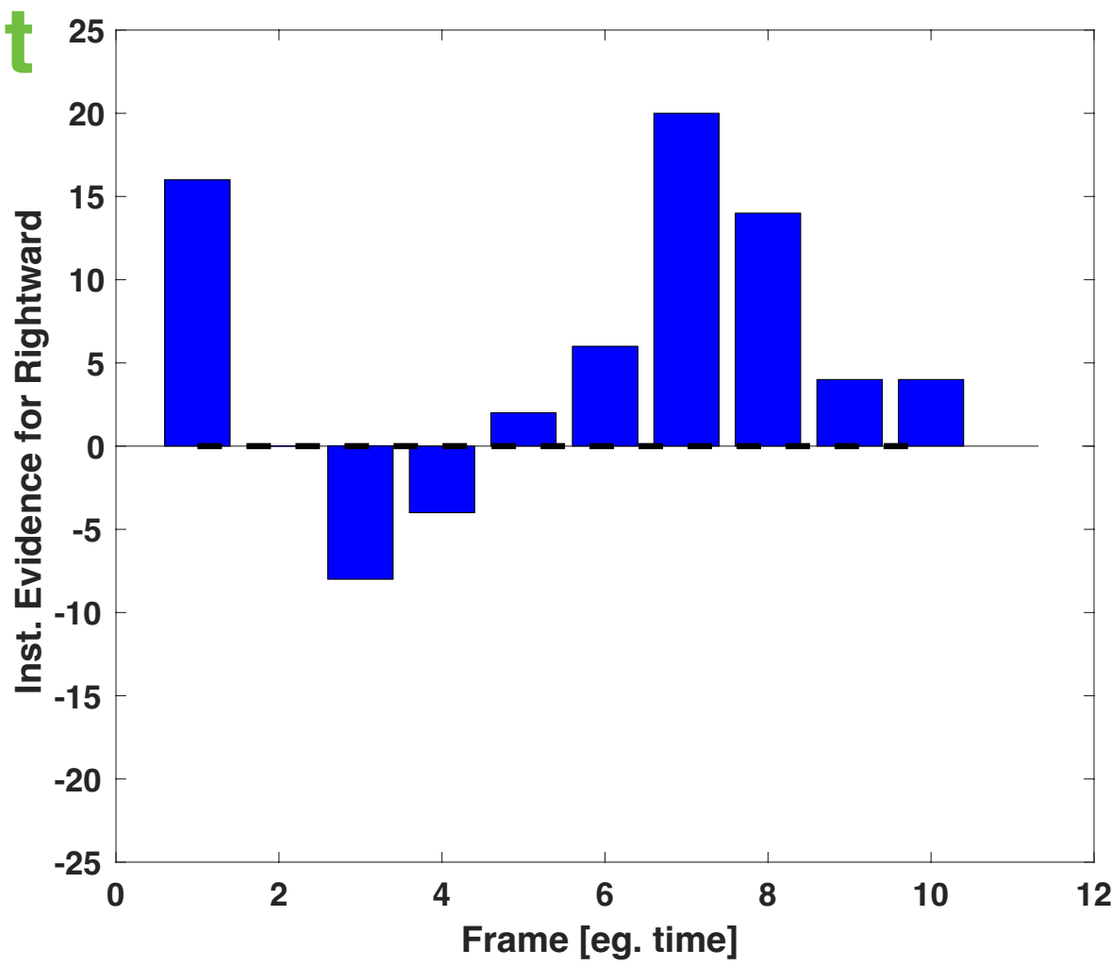
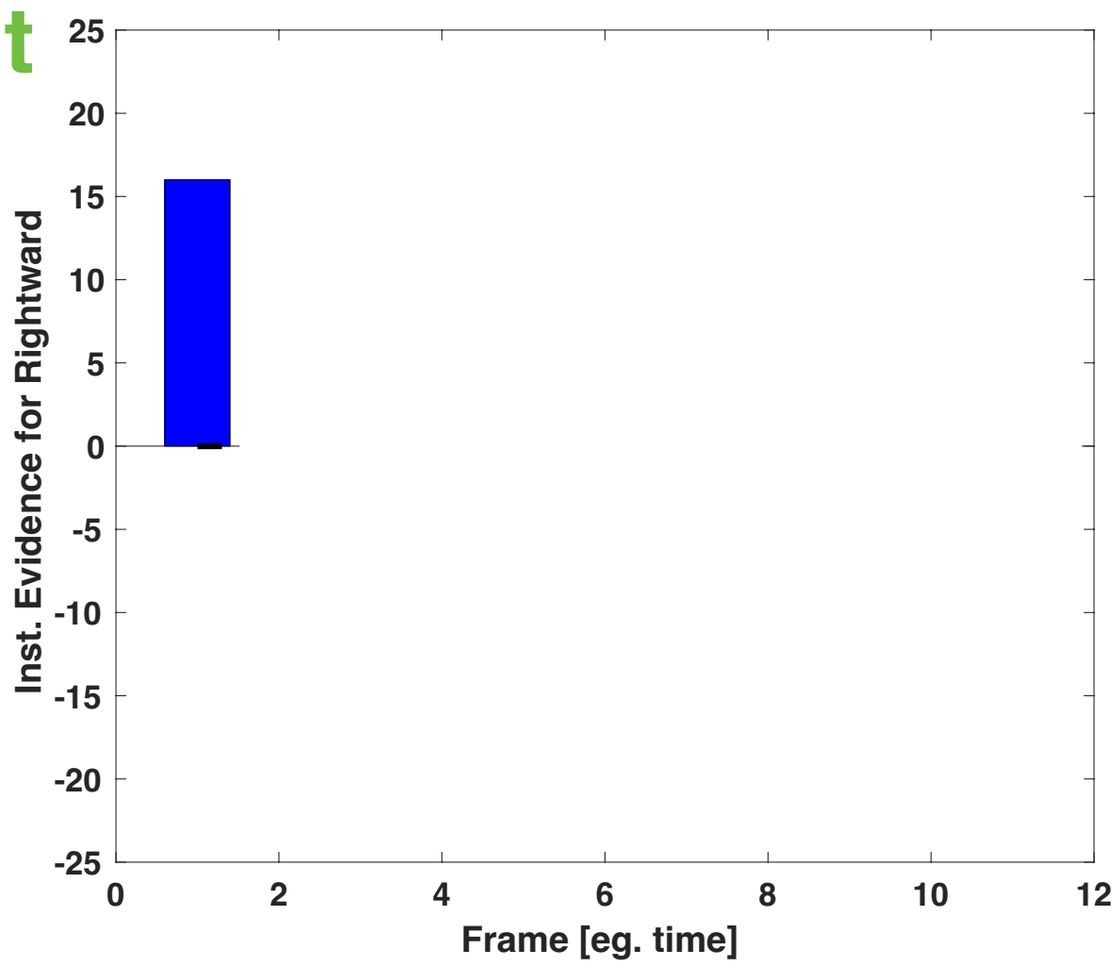


More left

Back to the perceptual decision…

Instantaneous evidence

More right



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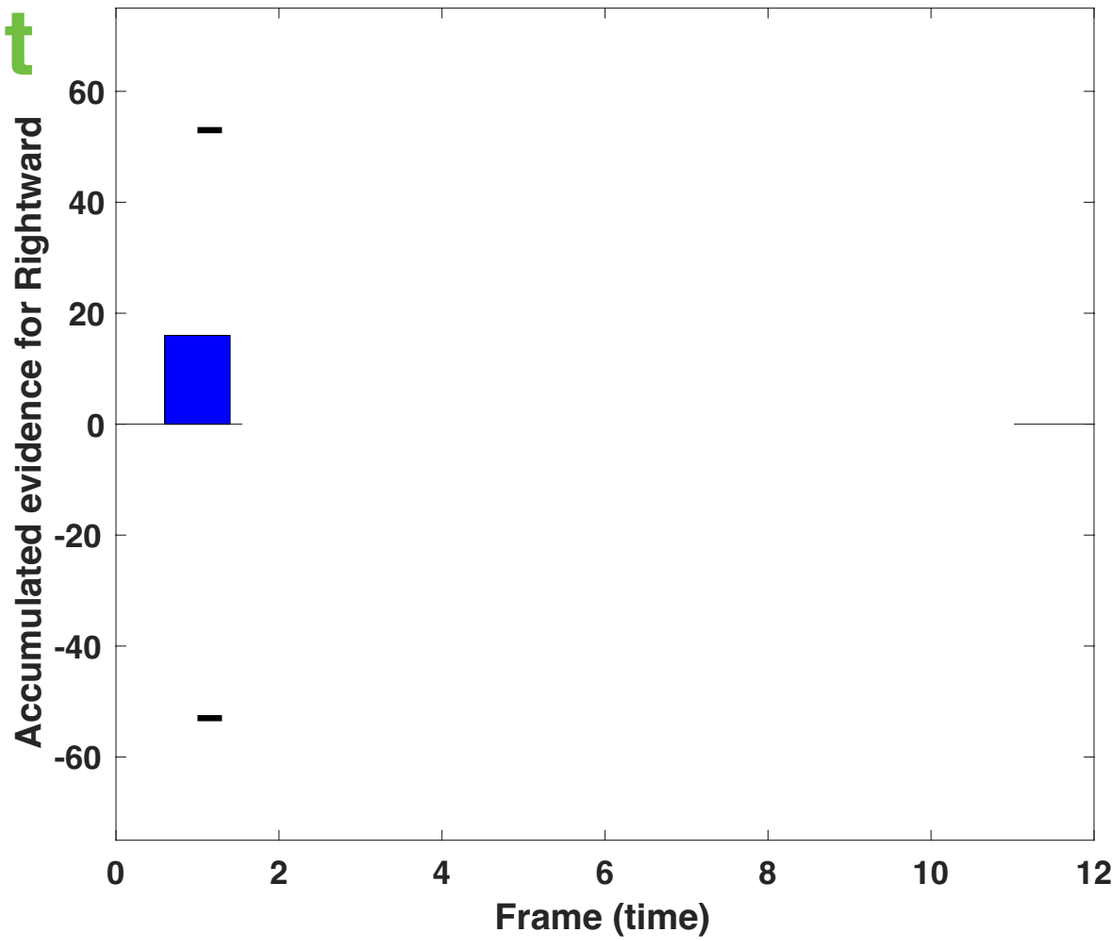
Accumulated evidence

More right



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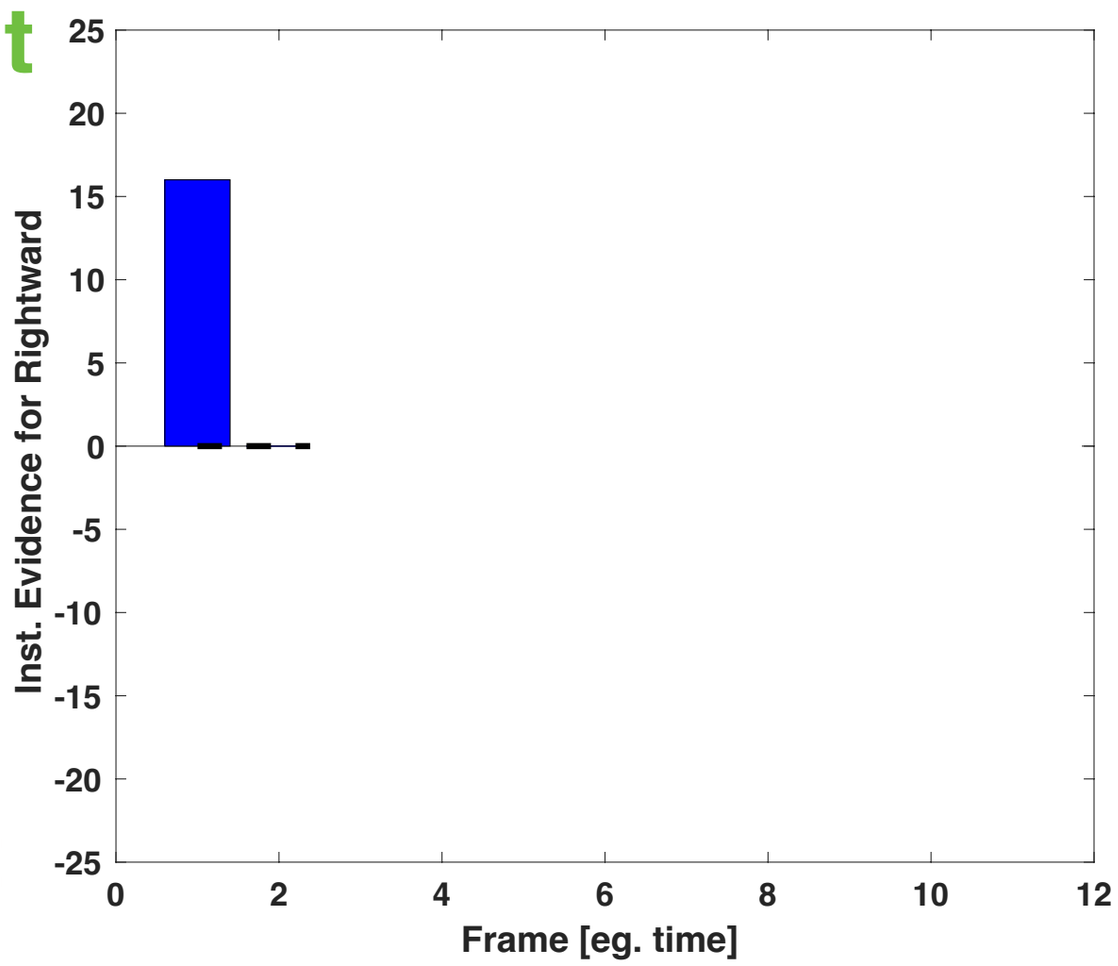

More left

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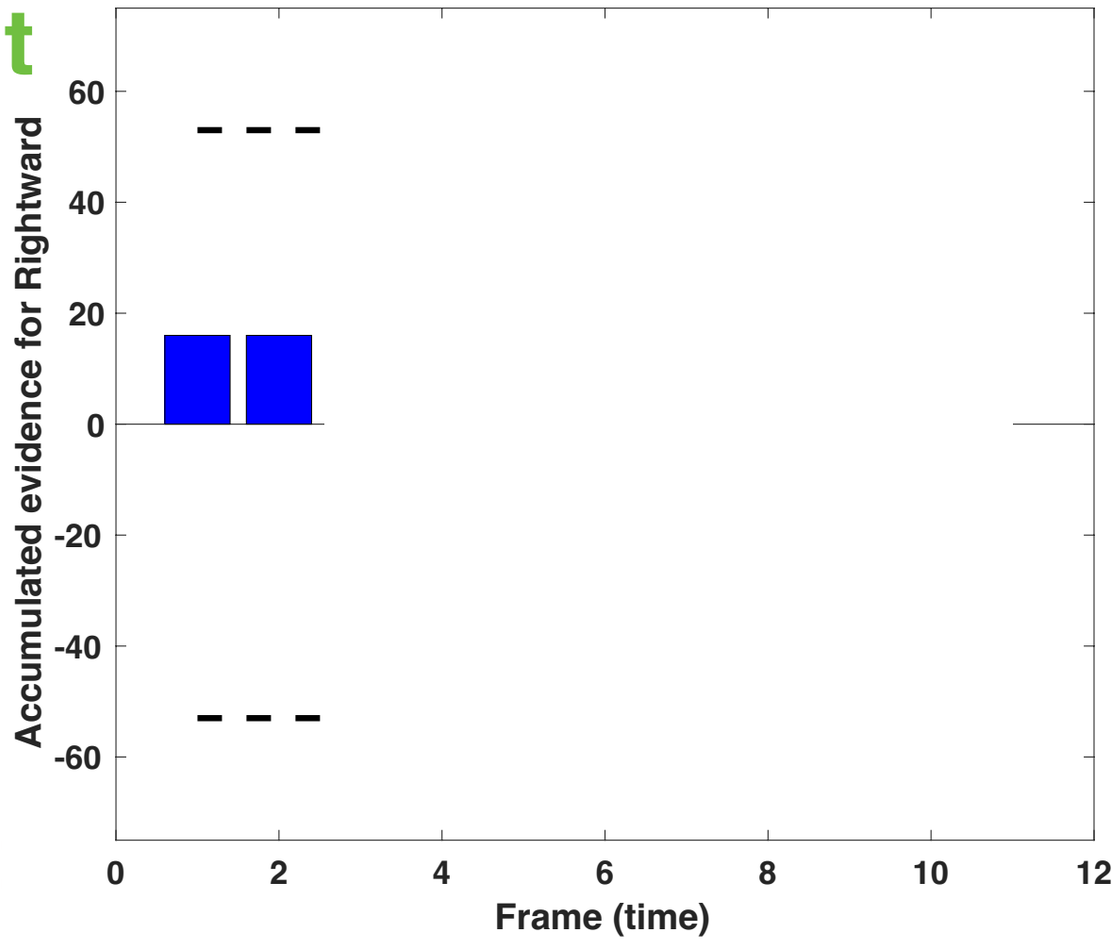

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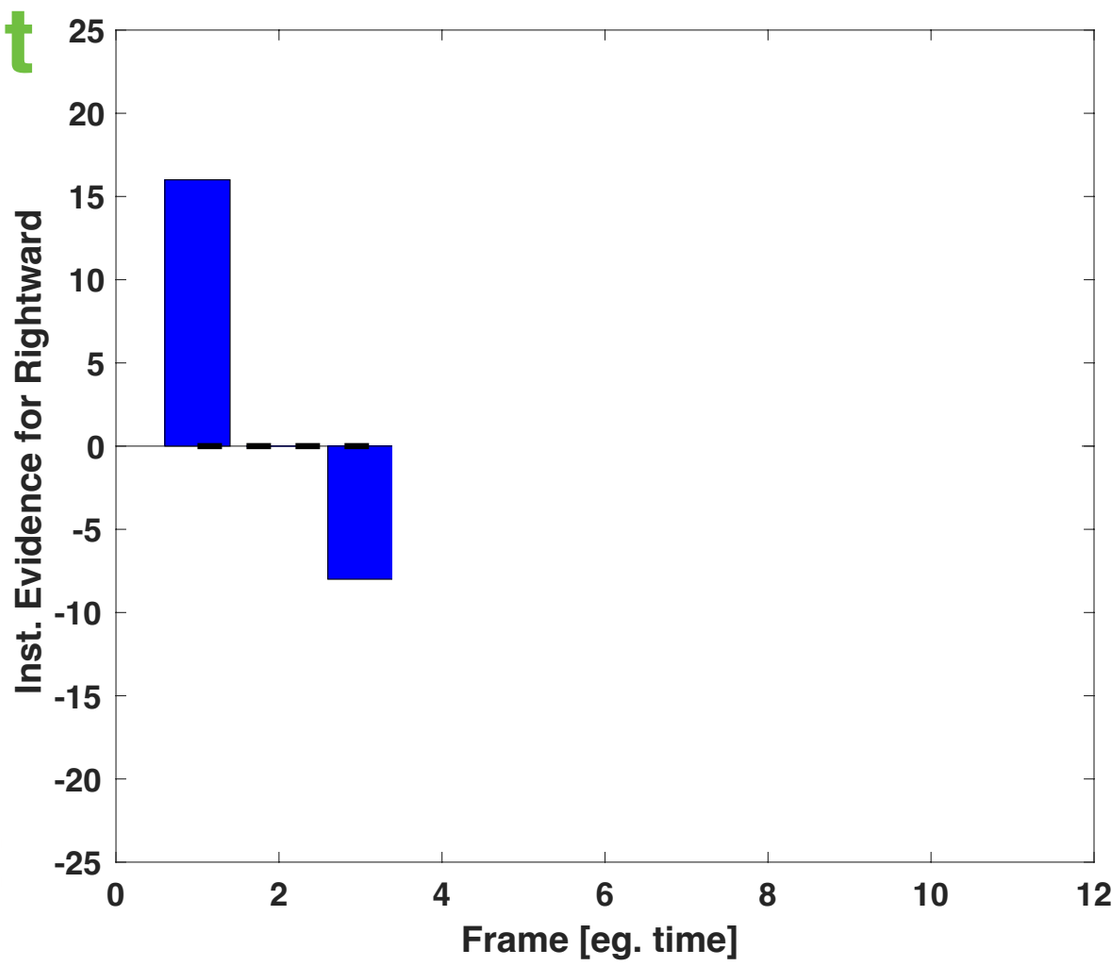

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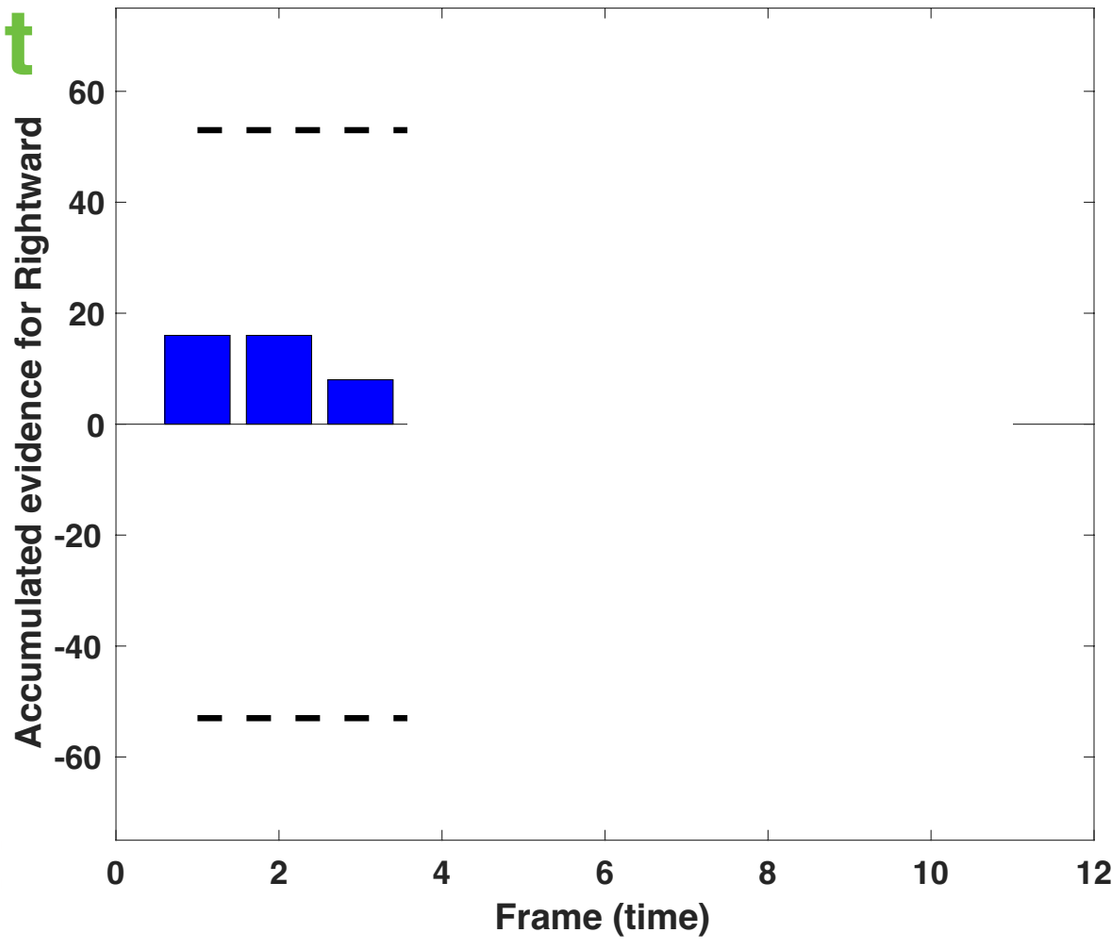

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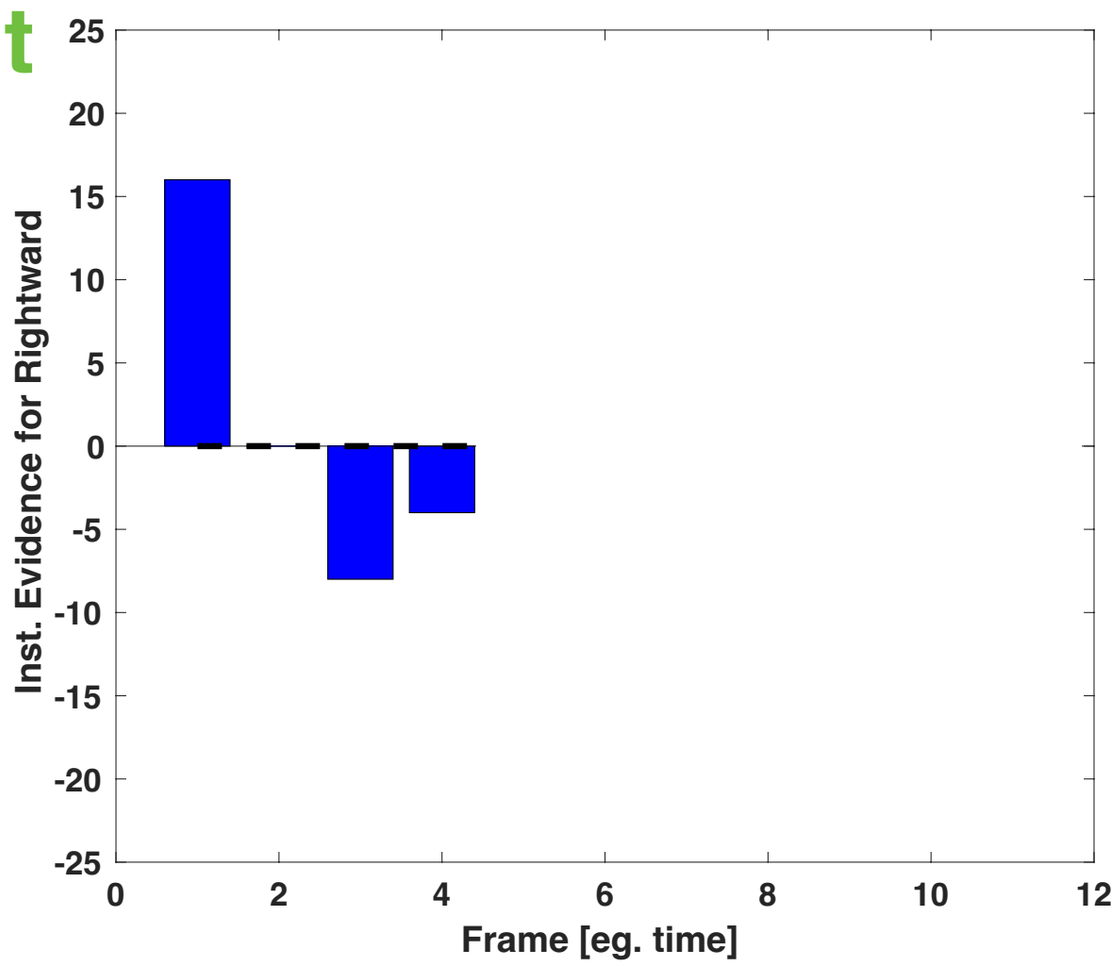

Accumulated evidence

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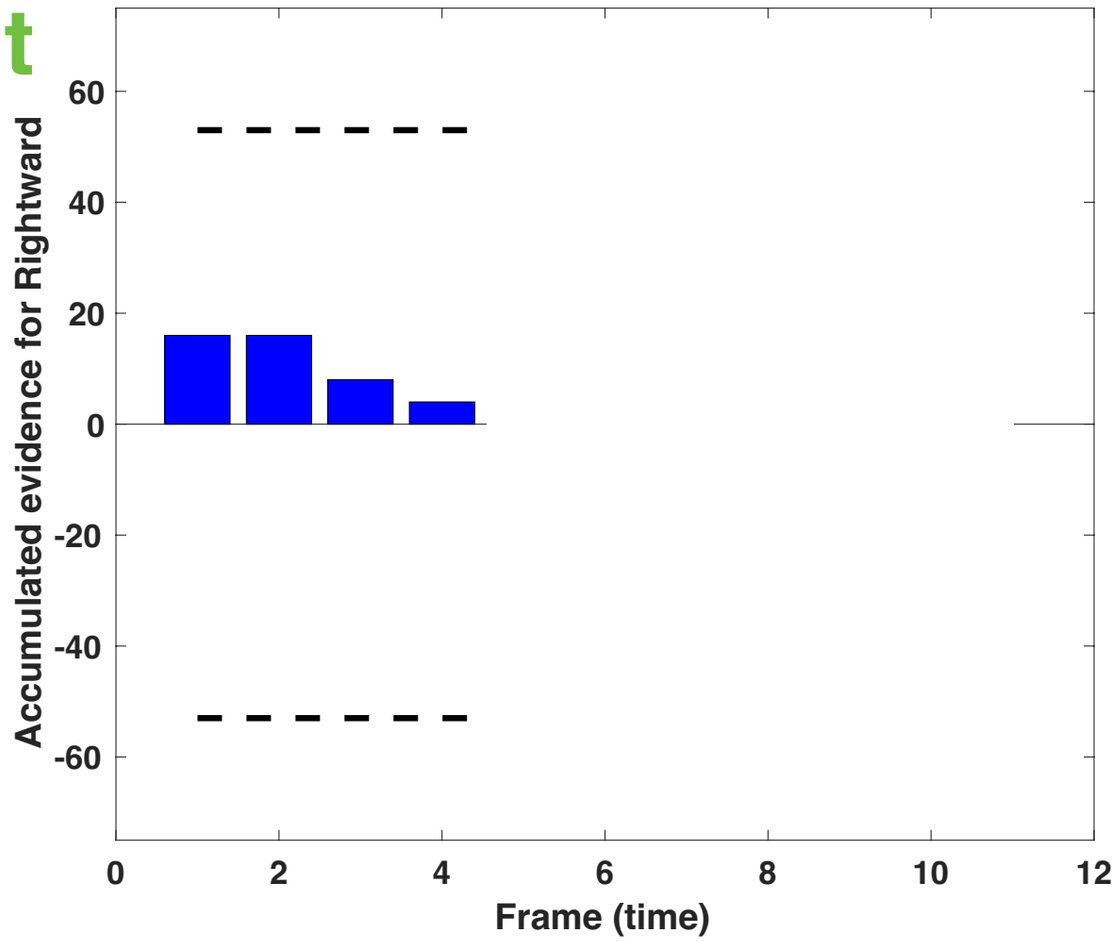

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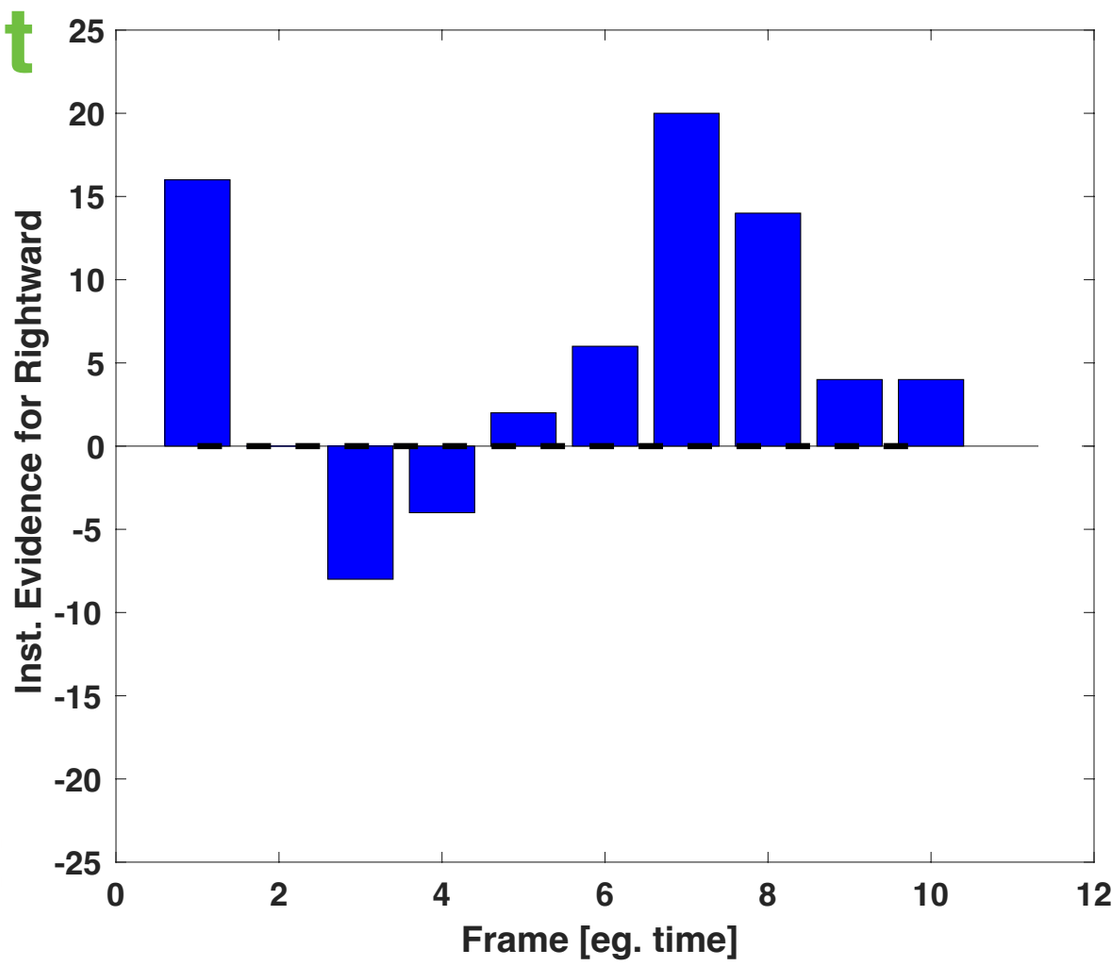
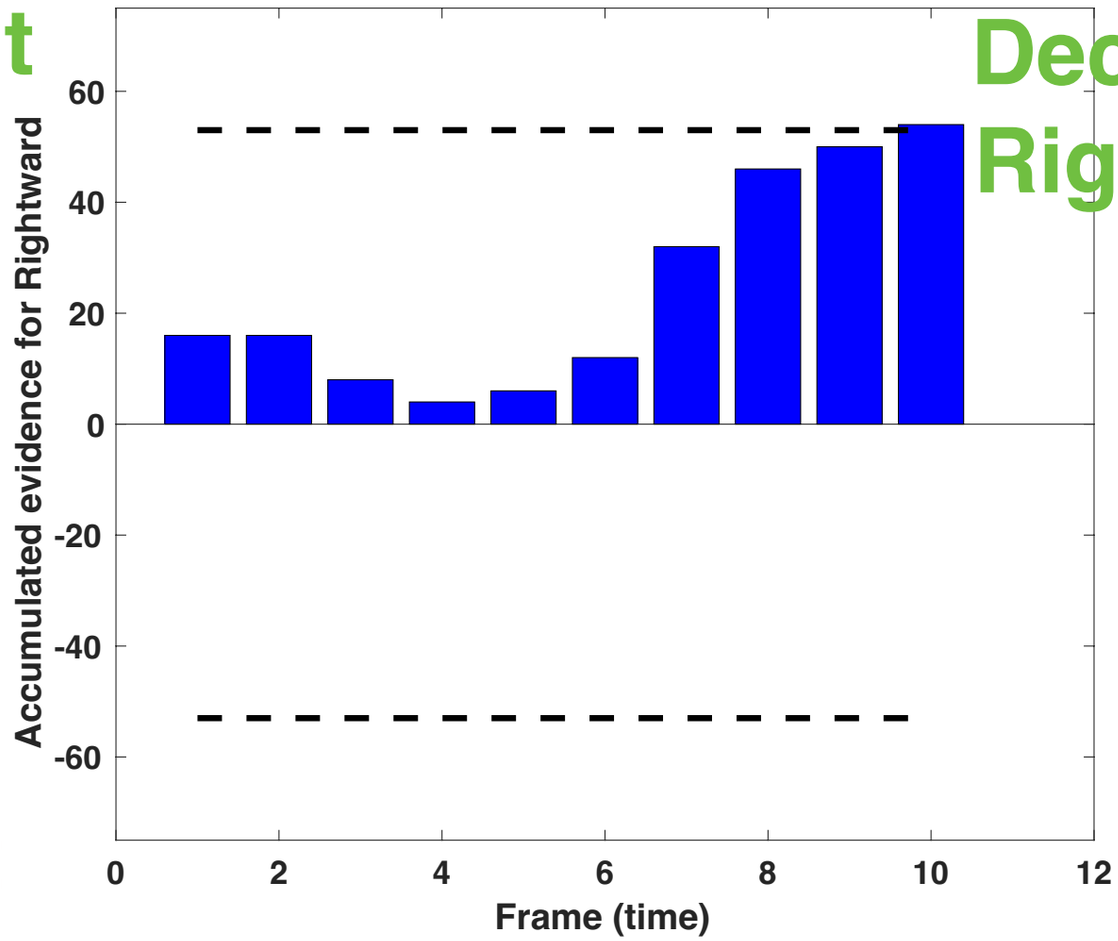


Accumulated evidence

More right

More left

Decide Right!!



Instantaneous evidence

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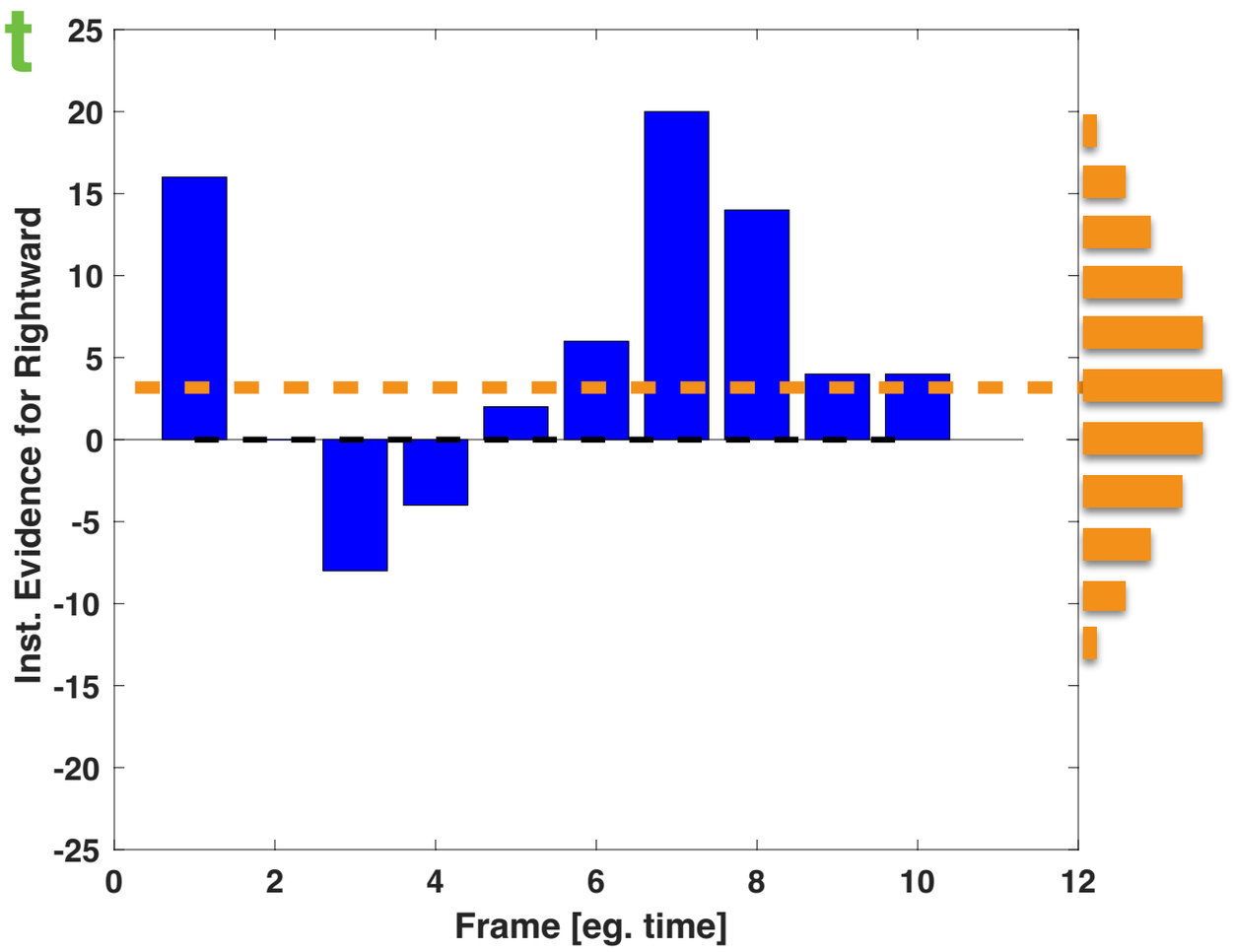

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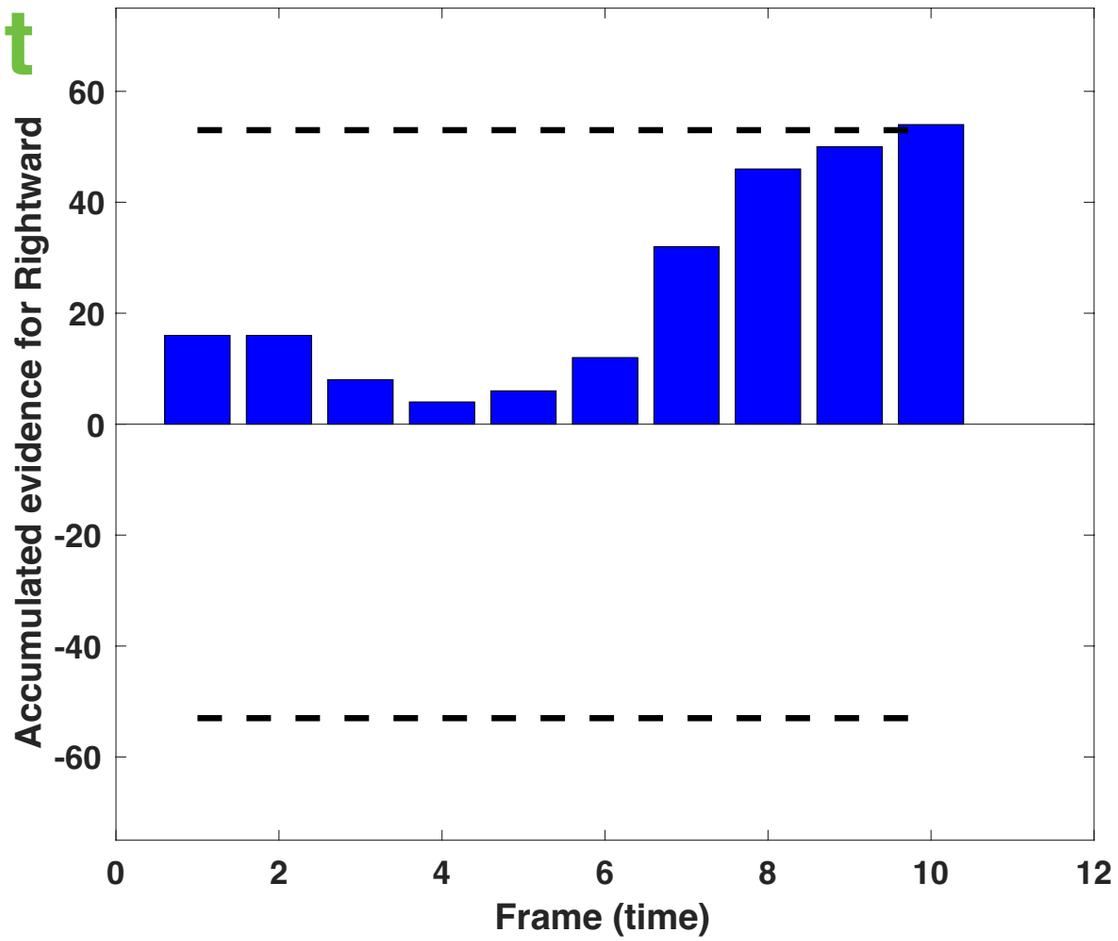

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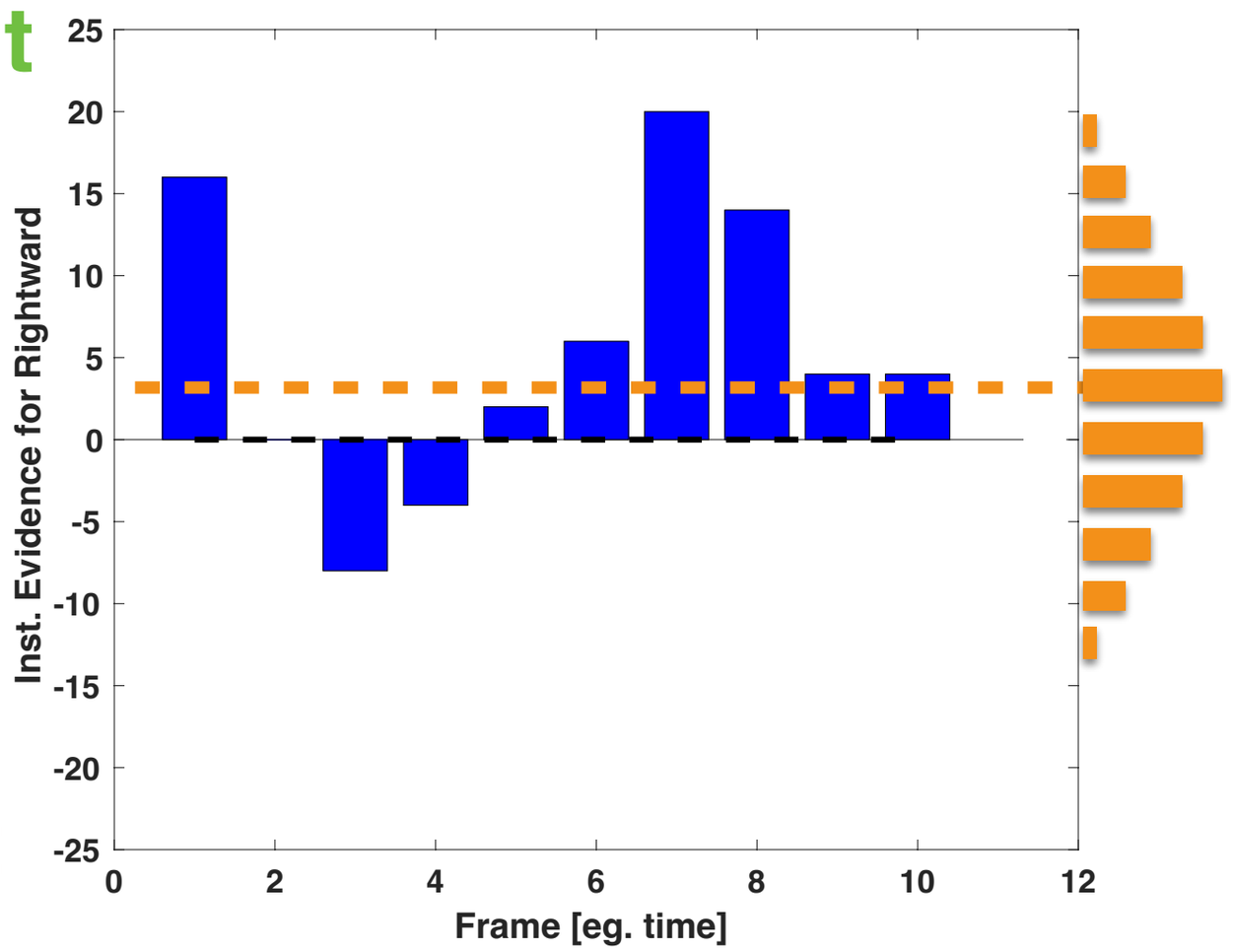

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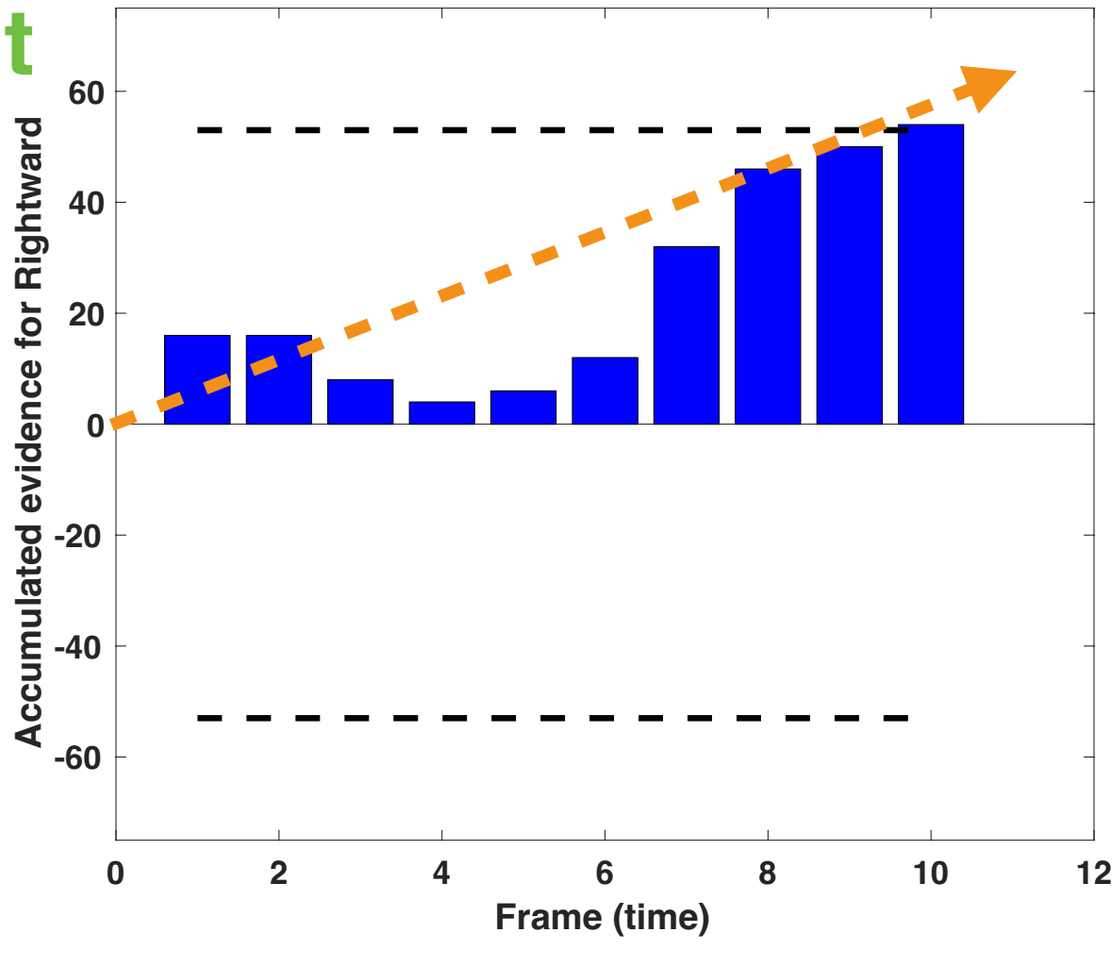

Drift diffusion model:

Drift diffusion model:

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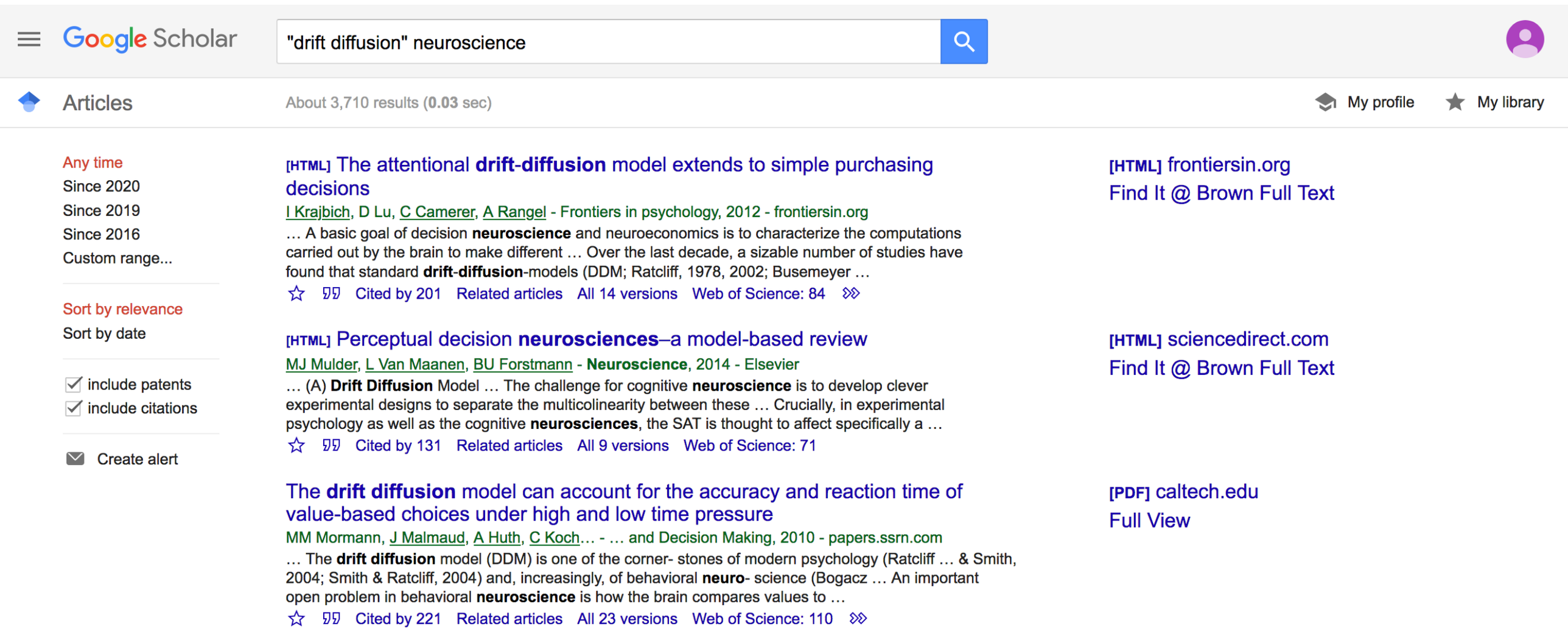
Drift diffusion model:



Drift diffusion model:

Key advance:

Model both choice AND reaction time!!!

Drift diffusion model:

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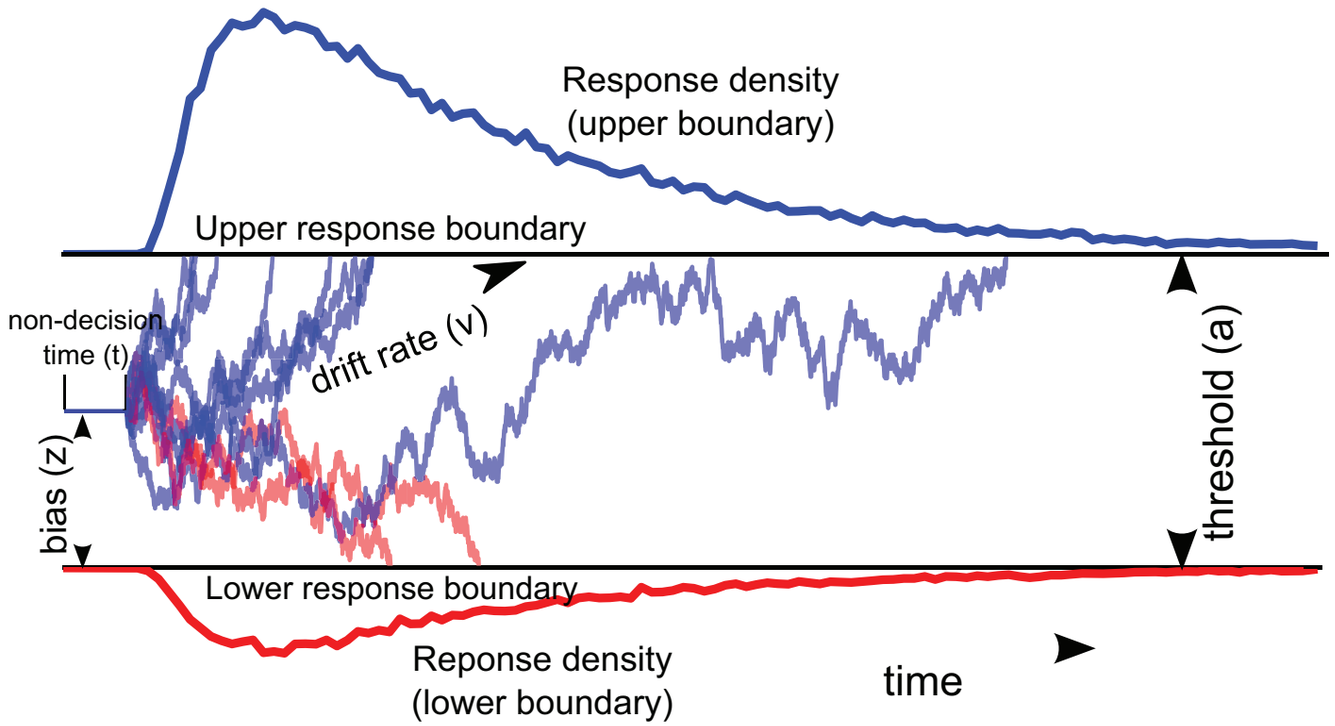

Wiecki & Frank

Drift diffusion model:

Our simple model with the following changes:

Drift diffusion model:

Our simple model with the following changes:

1) accumulate evidence over time 

Drift diffusion model:

Our simple model with the following changes:

1) accumulate evidence over time
2) make a decision after reaching a criterion

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1) accumulate evidence over time
2) make a decision after reaching a criterion
3) normal distribution of evidence

Drift diffusion model formalism:

Parameters:

A = Drift rate
y0 = Starting point
c = Drift noise (std)
z = Decision threshold

Variables:

y = Accumulated evidence
t = timestep

Drift diffusion model formalism:

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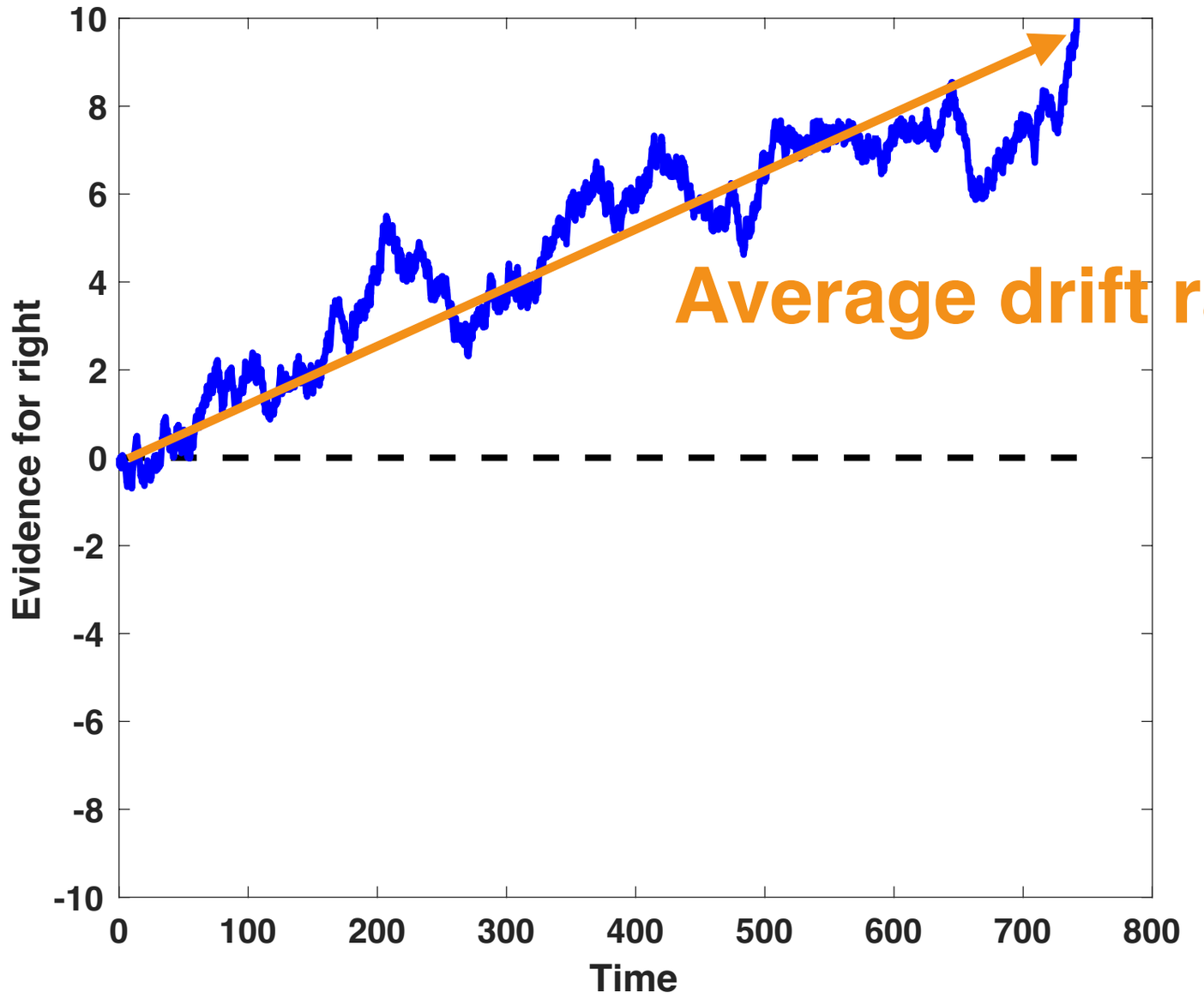
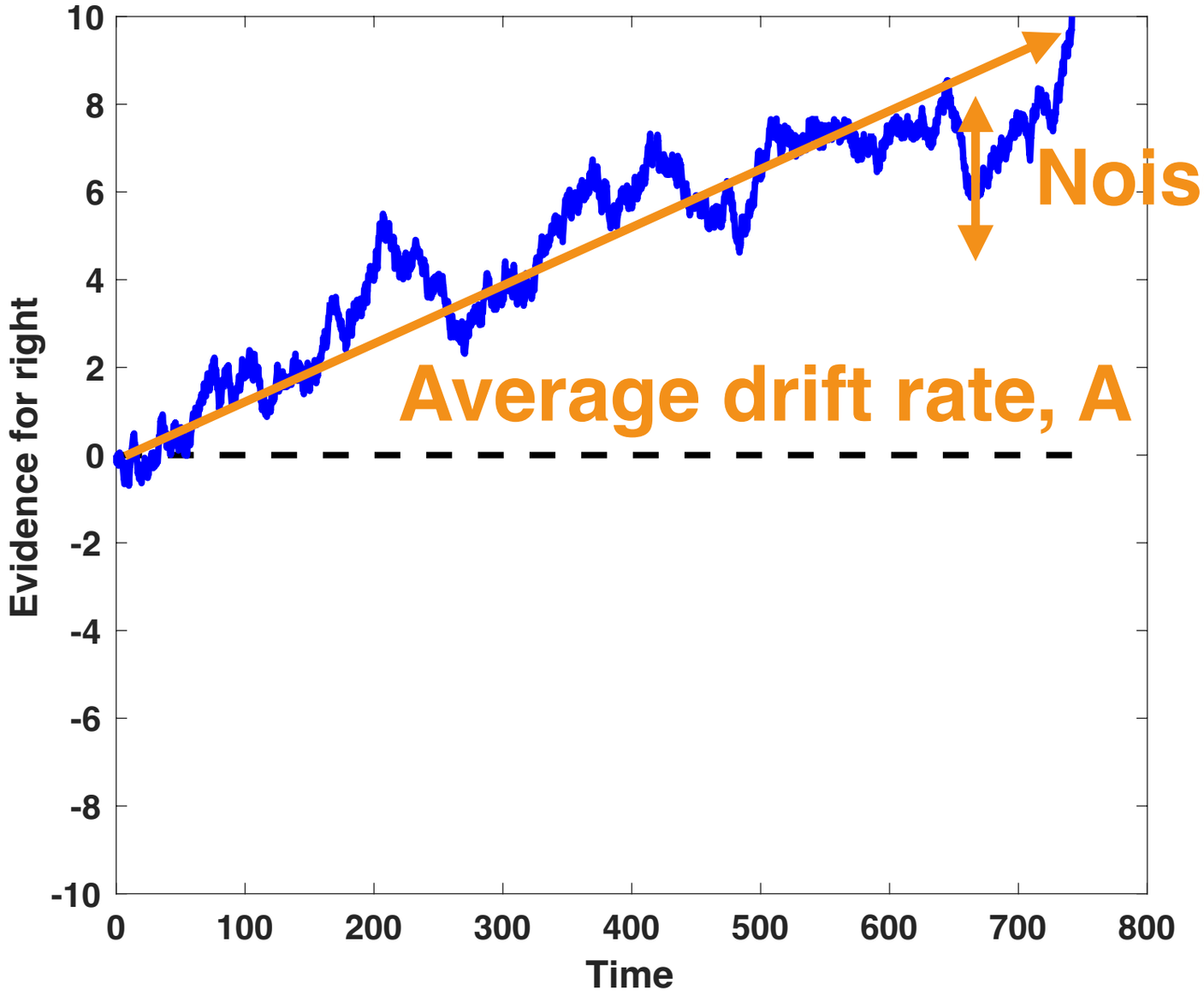
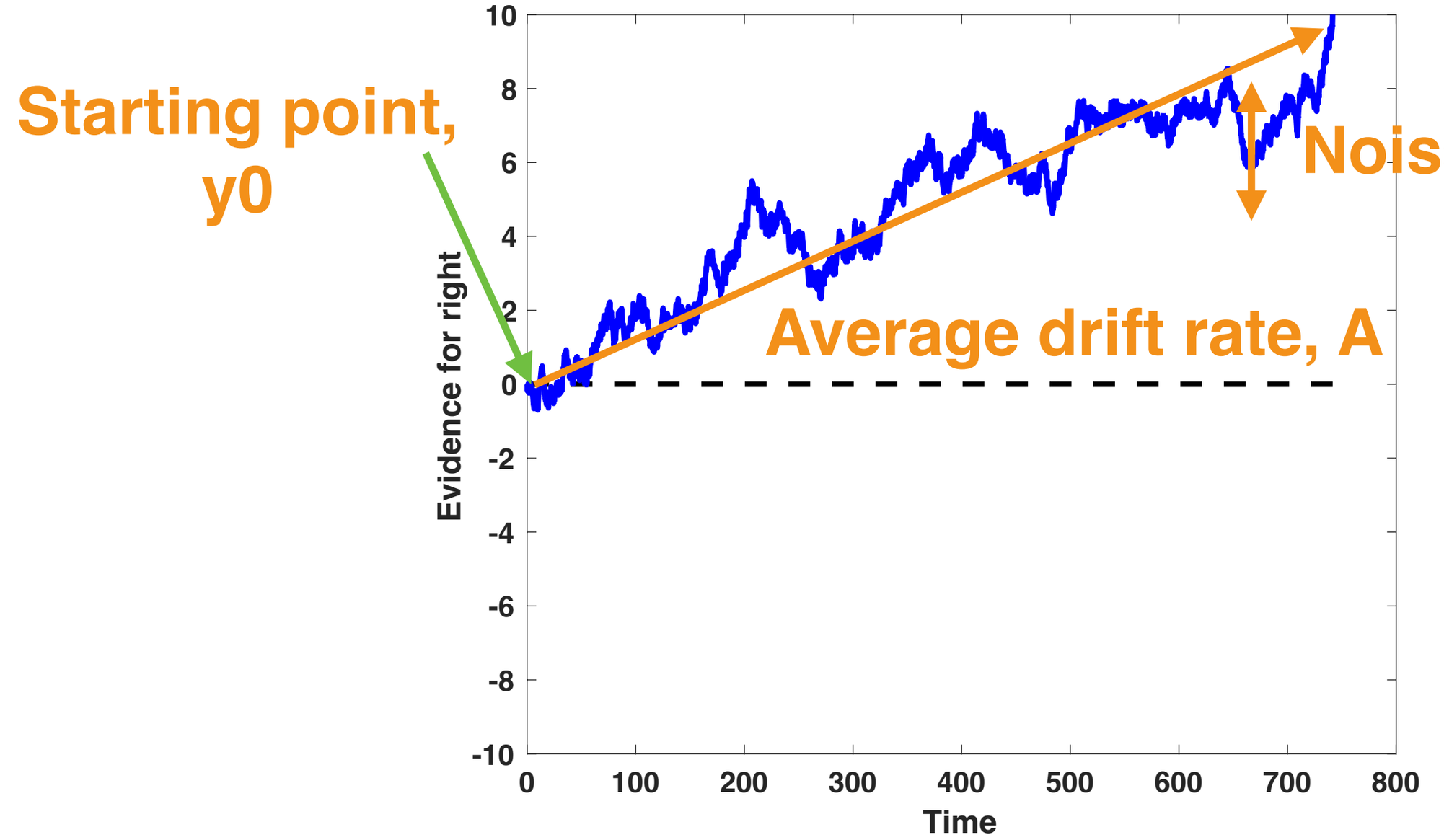
Drift diffusion model:



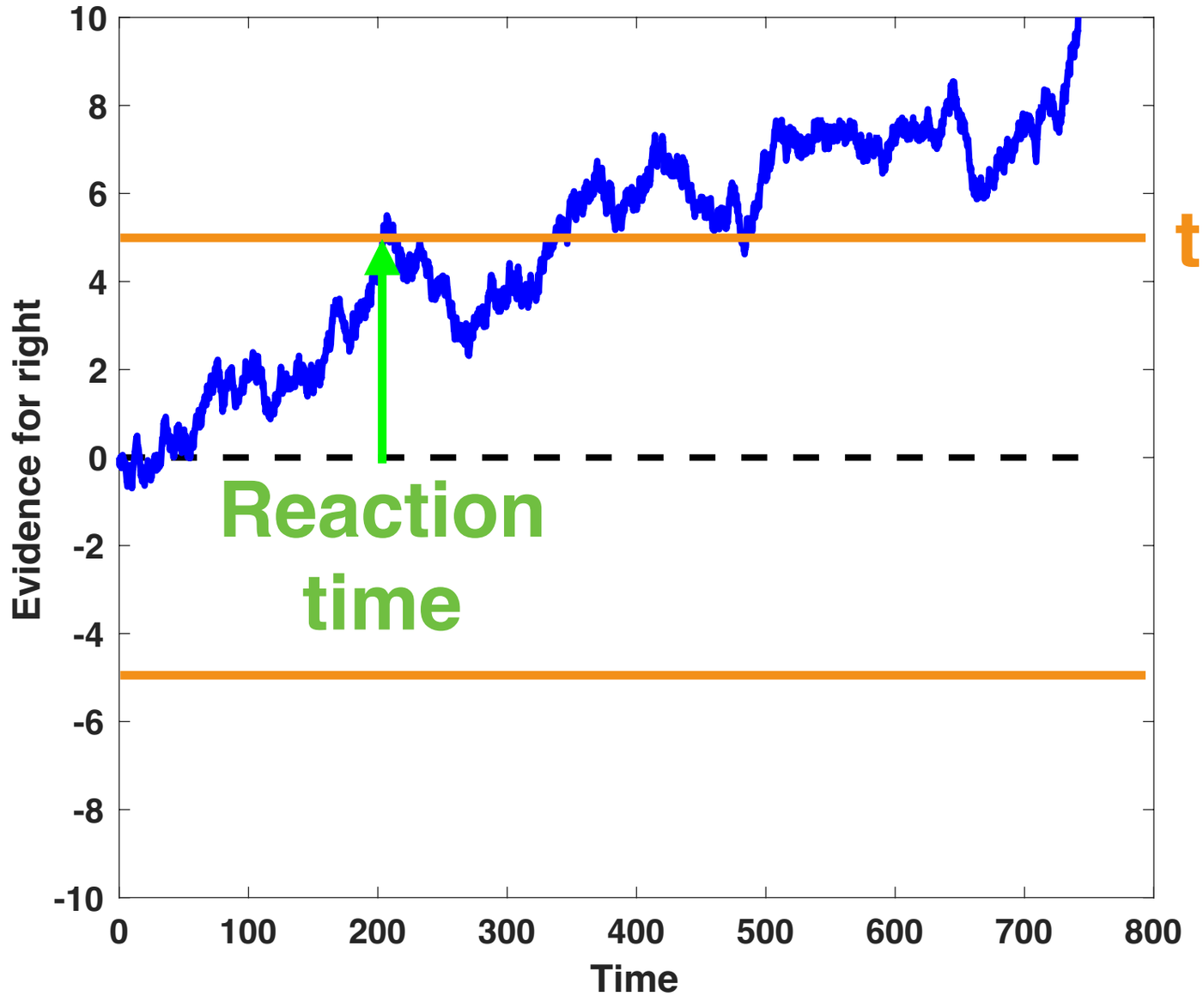
Drift diffusion model:



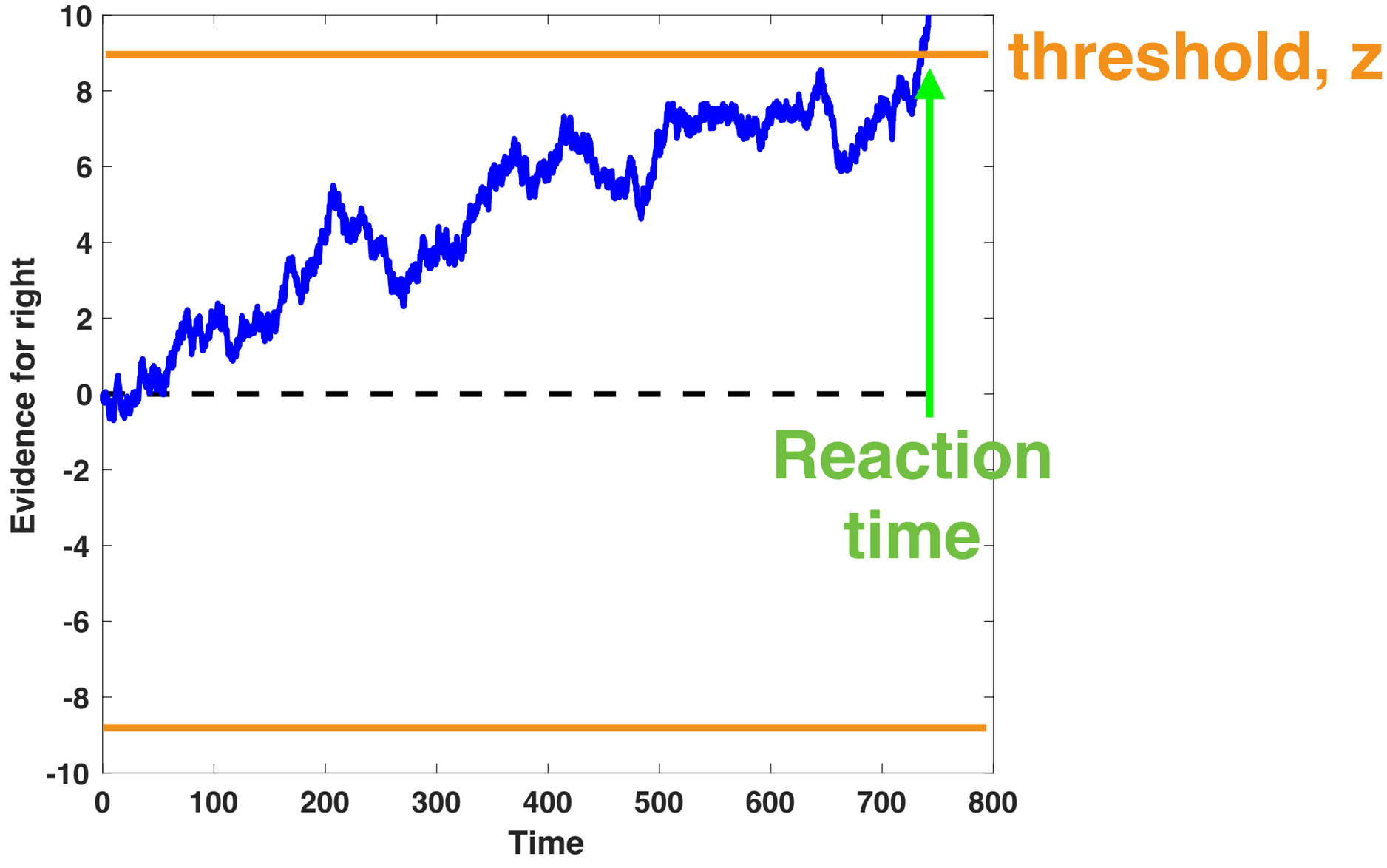
Drift diffusion model:



Drift diffusion model:



Drift diffusion model:



Drift diffusion model formalism:

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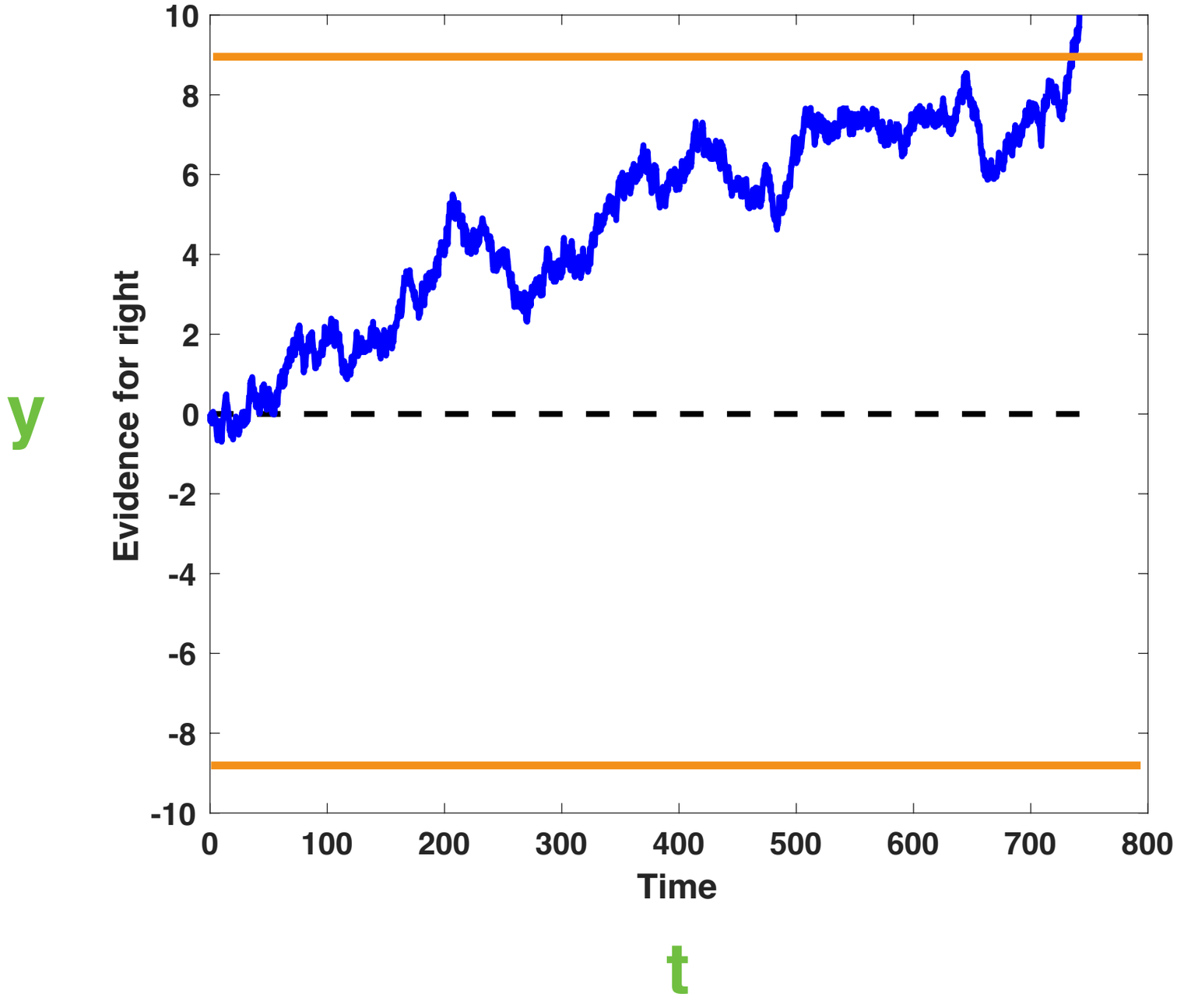
Drift diffusion model:



Simulation?

Drift diffusion model formalism:

Parameters:

A = Drift rate
y0 = Starting point
c = Drift noise (std)
z = Decision threshold

Variables:

y = Accumulated evidence
t = timestep

Drift diffusion model formalism:

Parameters:

A = Drift rate
y0 = Starting point
c = Drift noise (std)
z = Decision threshold

Additional parameters:

Non-decision time
Start point variability
Drift rate variability

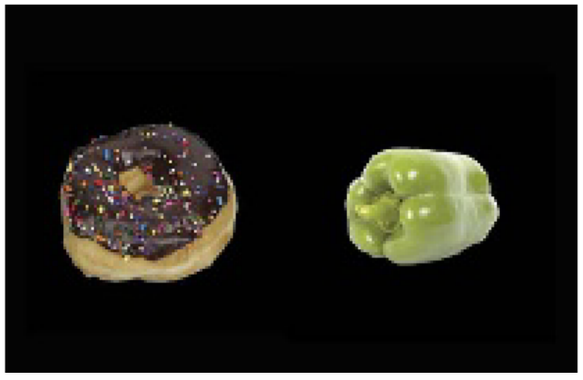
Variables:

y = Accumulated evidence
t = timestep

Drift diffusion for other sorts of decisions

Which do you want?

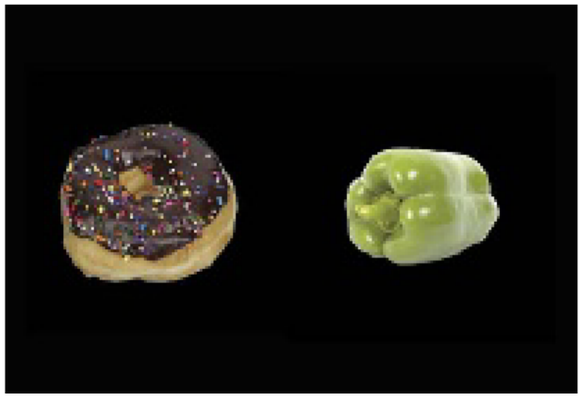
‘Which do you want?’



Tajima 2019

Which do you want?

‘Which do you want?’

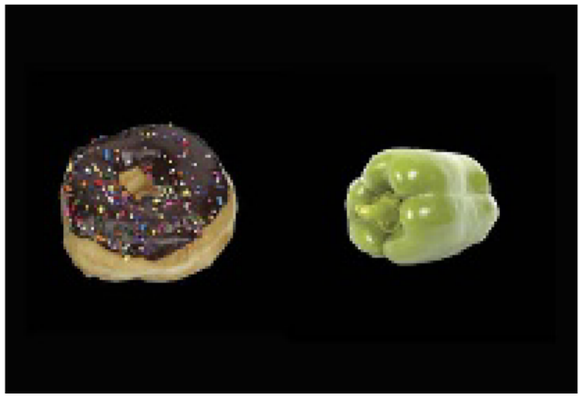


‘Which do you want?’

Tajima 2019

Which do you want?

‘Which do you want?’



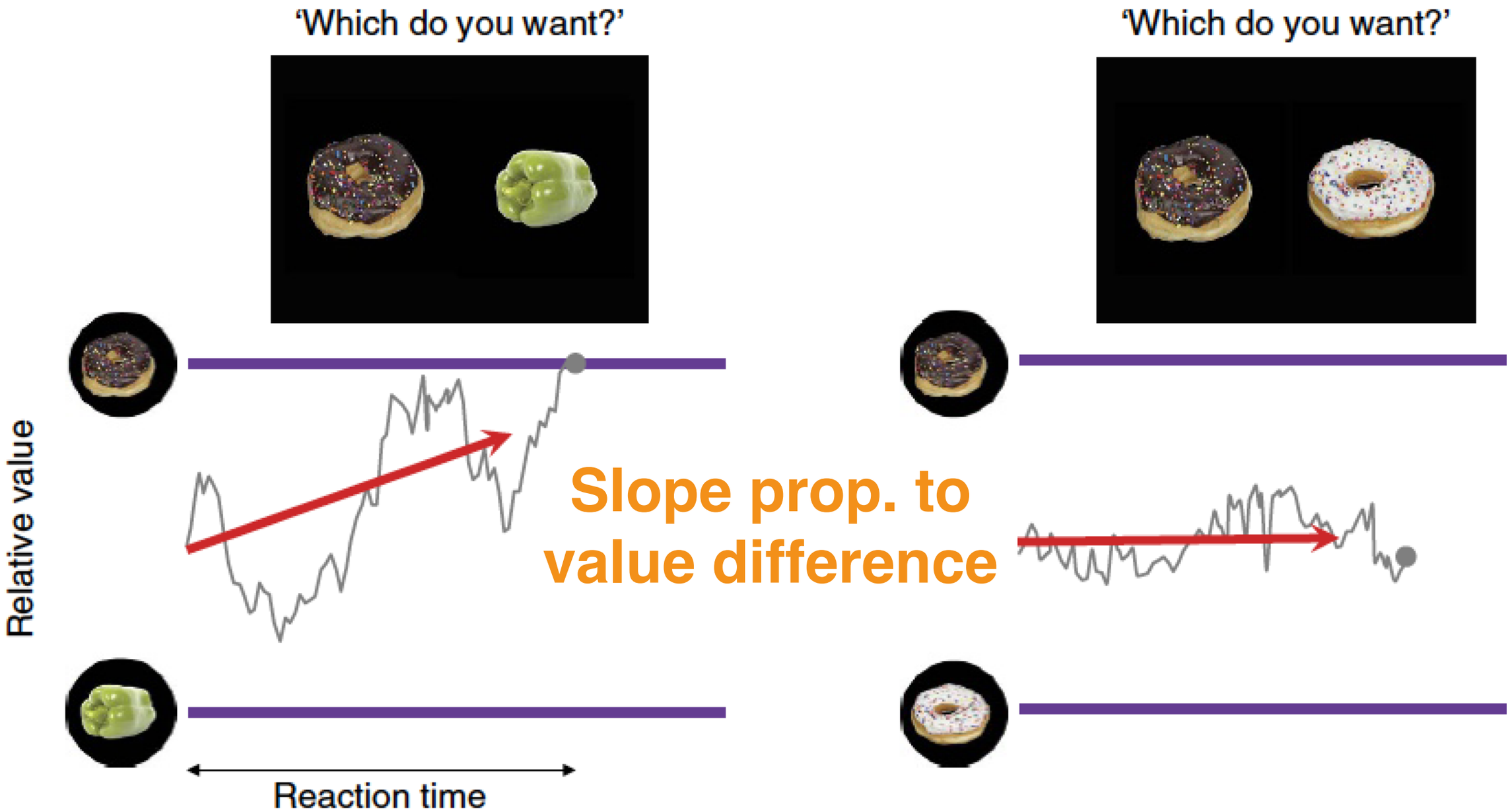
‘Which do you want?’

Large value difference

Small value difference

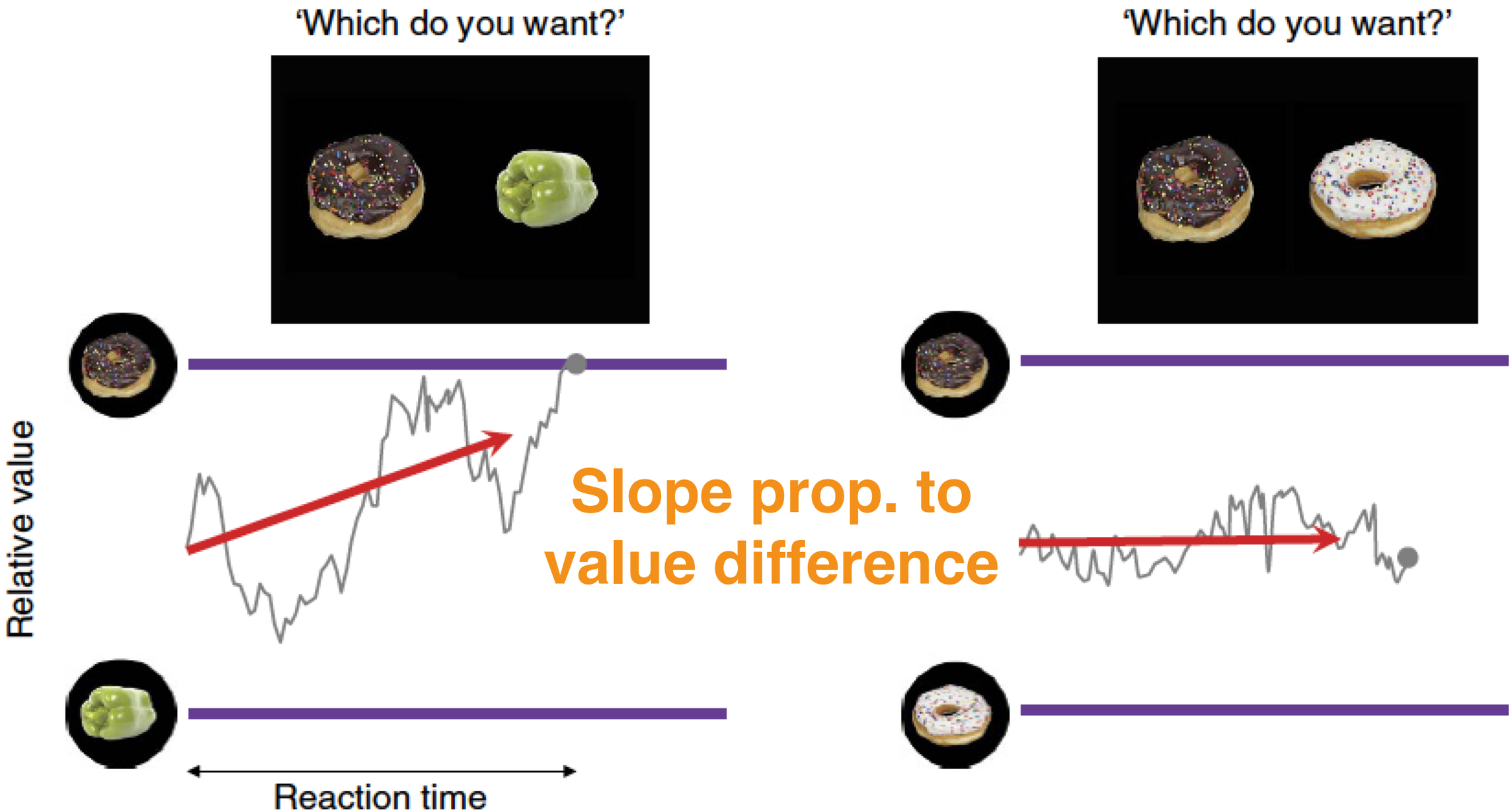
Tajima 2019

Which do you want?



Tajima 2019

Which do you want?

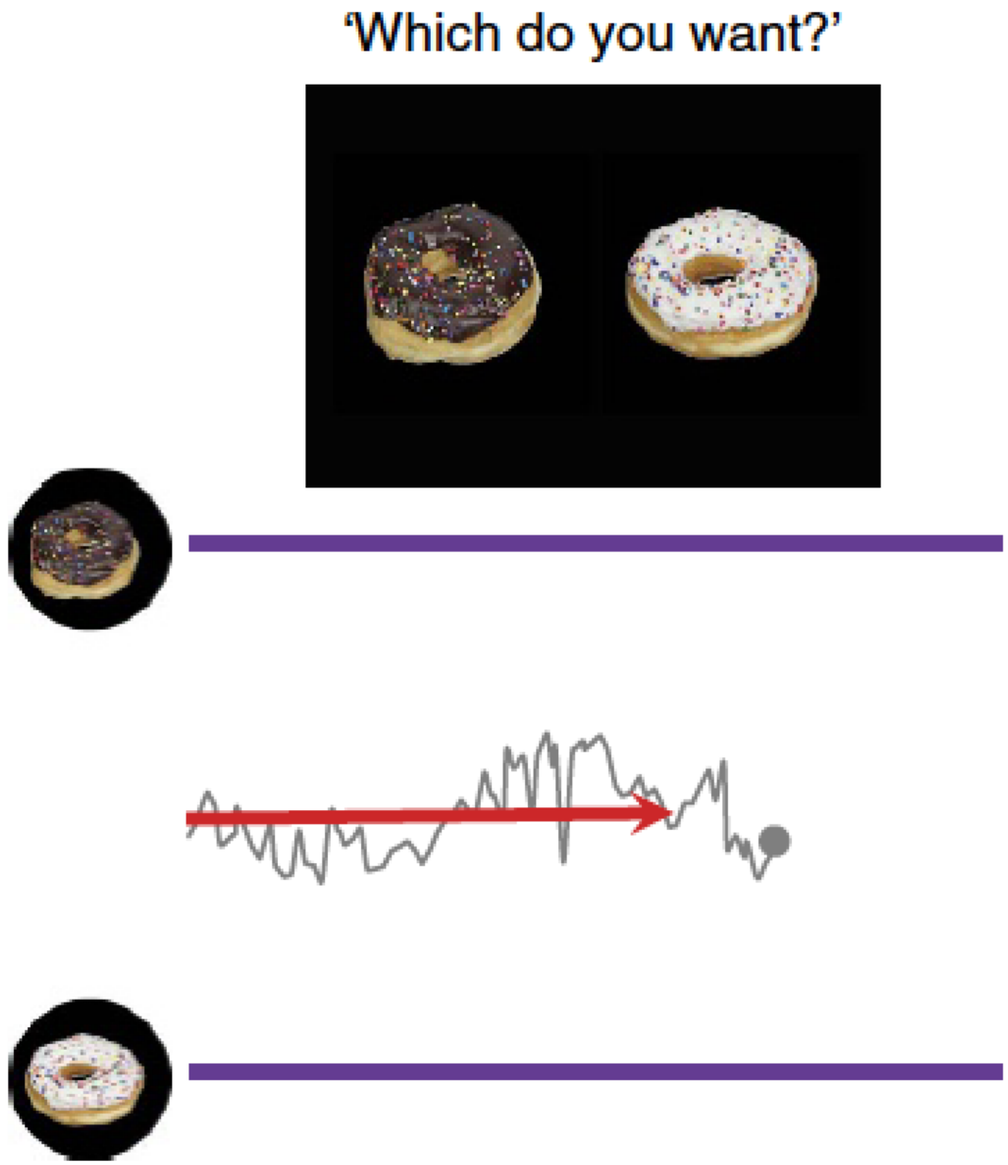


Fits choice & RT data!

Tajima 2019

Which do you want?

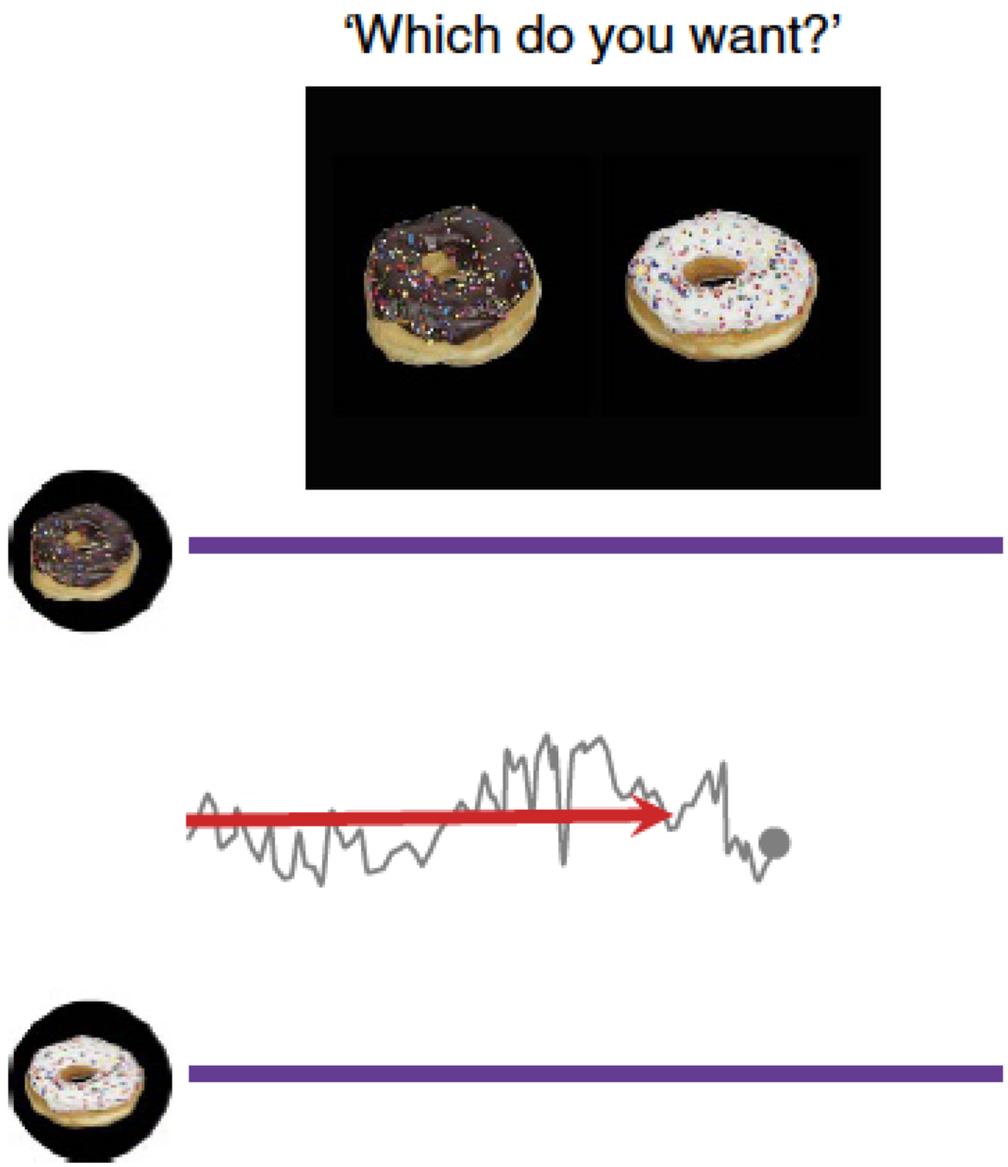
Decision paralysis?



Tajima 2019

Which do you want?

Buridan's Ass
dies of hunger and thirst?



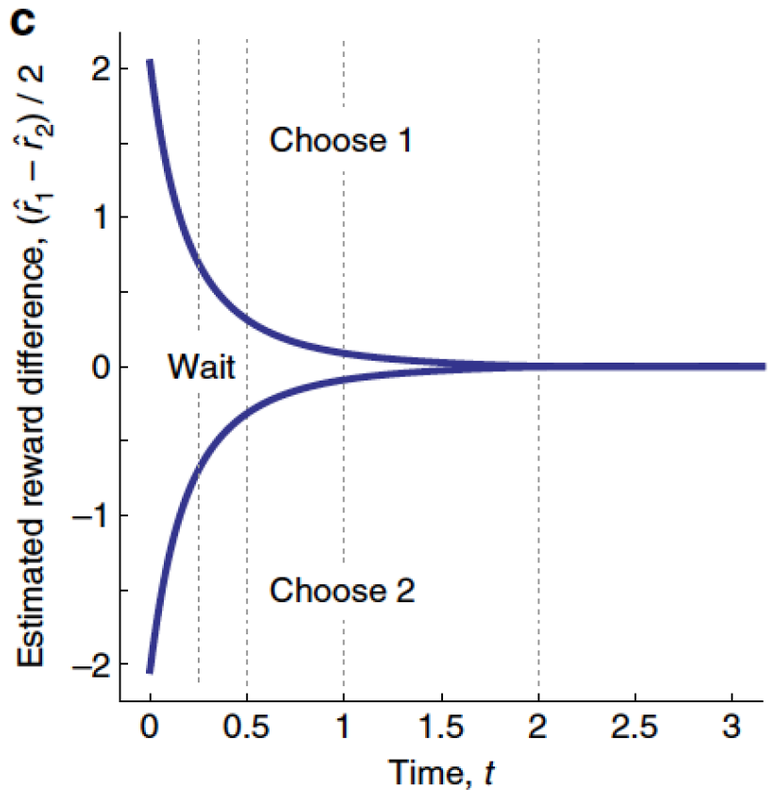
Tajima 2019

Which do you want?

Buridan's Ass dies of hunger and thirst?

Probably not…

Collapsing decision bound to minimize time wasted "accumulating" nothing



Tajima 2019

But… why would the brain need sequential samples in a value based decision making task??????

But… why would the brain need sequential samples in a value based decision making task??????

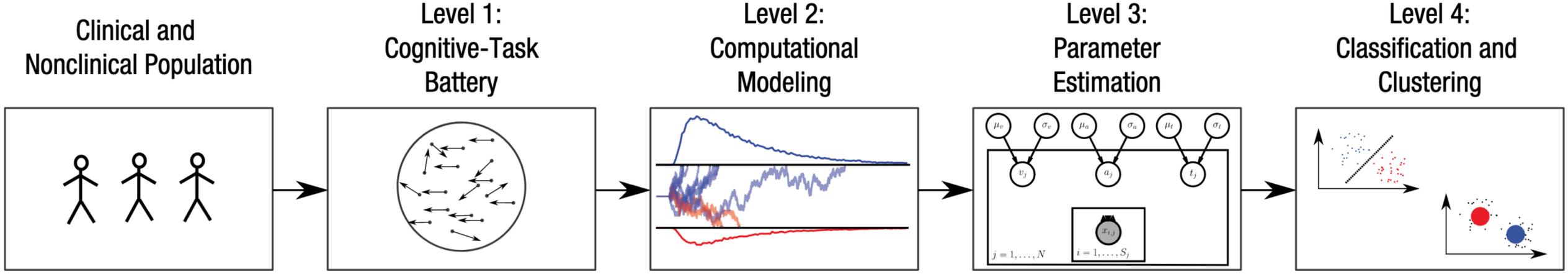
Sampling through attention (attentional DDM)?

Sampling from memory?

Open question…

DDM and computational psychiatry

DDM and computational psychiatry



Wiecki 2015

How do we know whether decisions are from a drift diffusion process?

How do we know whether decisions are from a drift diffusion process?

Unfalsifiability and mutual translatability of major modeling schemes for choice reaction time.

Database: APA PsycArticles Journal Article

Jones, Matt Dzhafarov, Ehtibar N.

Citation

Jones, M., & Dzhafarov, E. N. (2014). Unfalsifiability and mutual translatability of major modeling schemes for choice reaction time. *Psychological Review, 121*(1), 1–32. https://doi.org/10.1037/a0034190

Abstract

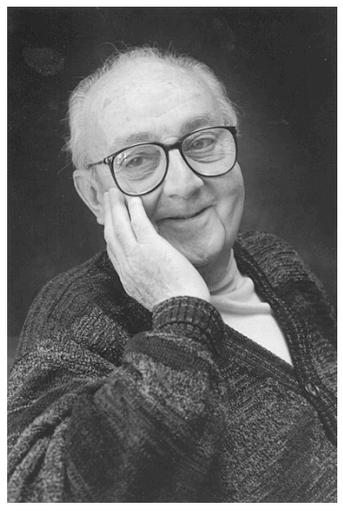
[Correction Notice: An Erratum for this article was reported in Vol 121(1) of *Psychological Review* (see record 2014-03591-005). The link to supplemental material was missing. All versions of this article have been corrected.] Much current research on speeded choice utilizes models in which the response is triggered by a stochastic process crossing a deterministic threshold. This article focuses on 2 such model classes, 1 based on continuous-time diffusion and the other on linear ballistic accumulation (LBA). Both models assume random variability in growth rates and in other model components across trials. We show that if the form of this variability is unconstrained, the models can exactly match any possible pattern of response probabilities and response time distributions. Thus, the explanatory or predictive content of these models is determined not by their structural assumptions but, rather, by distributional assumptions (e.g., Gaussian distributions) that are traditionally regarded as implementation details.

How do we know whether decisions are from a drift diffusion process?

Behavior may not be sufficient…

How do we know whether decisions are from a drift diffusion process?

Behavior may not be sufficient…

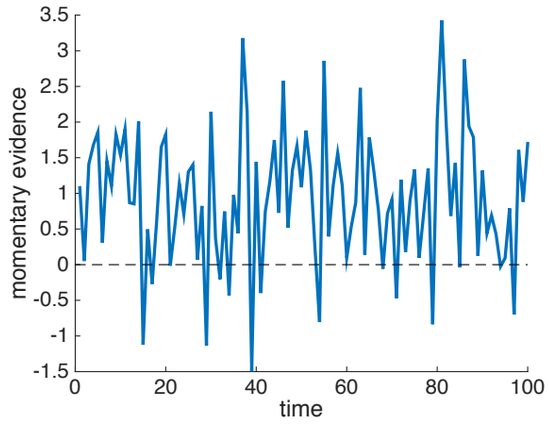


**“All models are wrong,
But some are useful”**

Looking for drift diffusion in the brain

Drift diffusion in the brain

Step 1: compute momentary evidence



Drift diffusion in the brain

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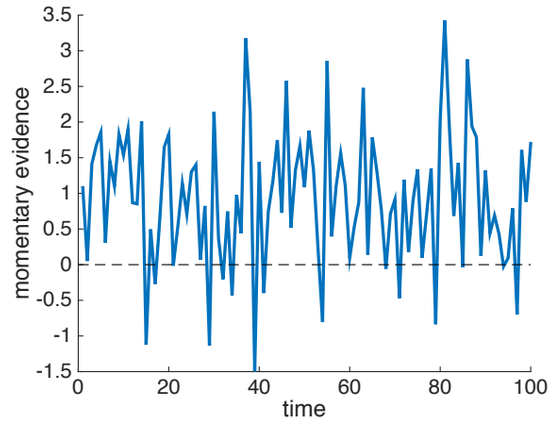
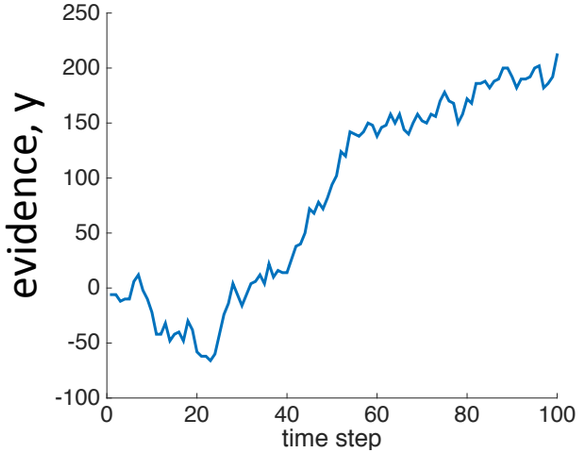


Step 2: integrate evidence over time to make a decision



Drift diffusion in the brain

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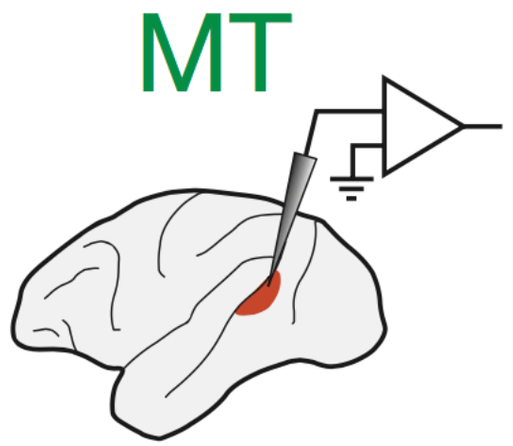

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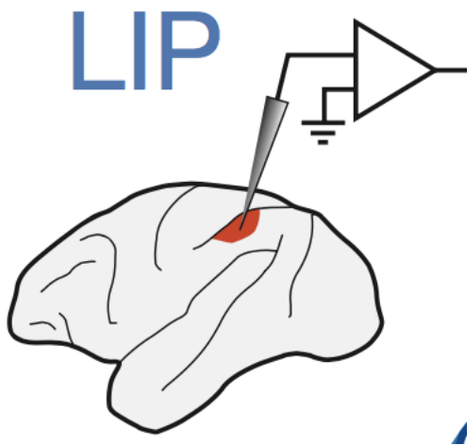
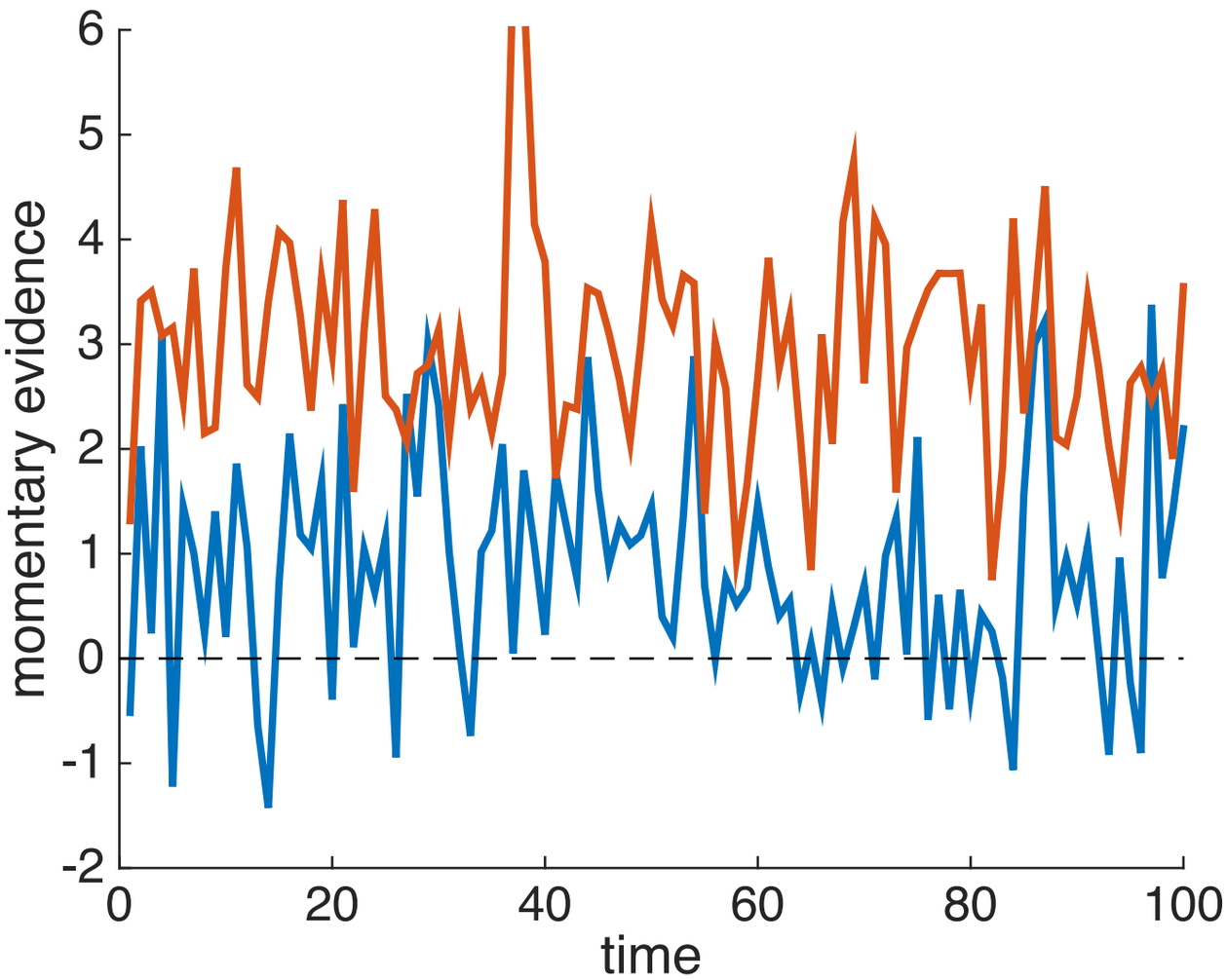
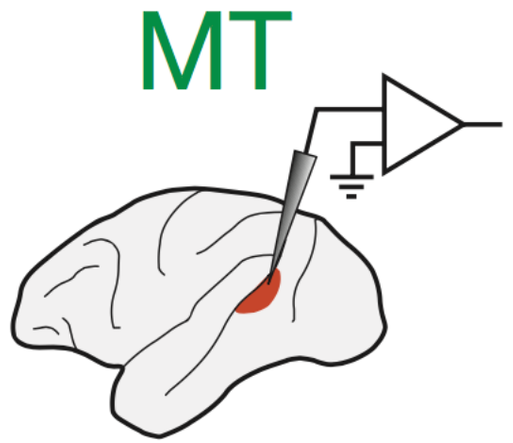


MT: momentary motion evidence





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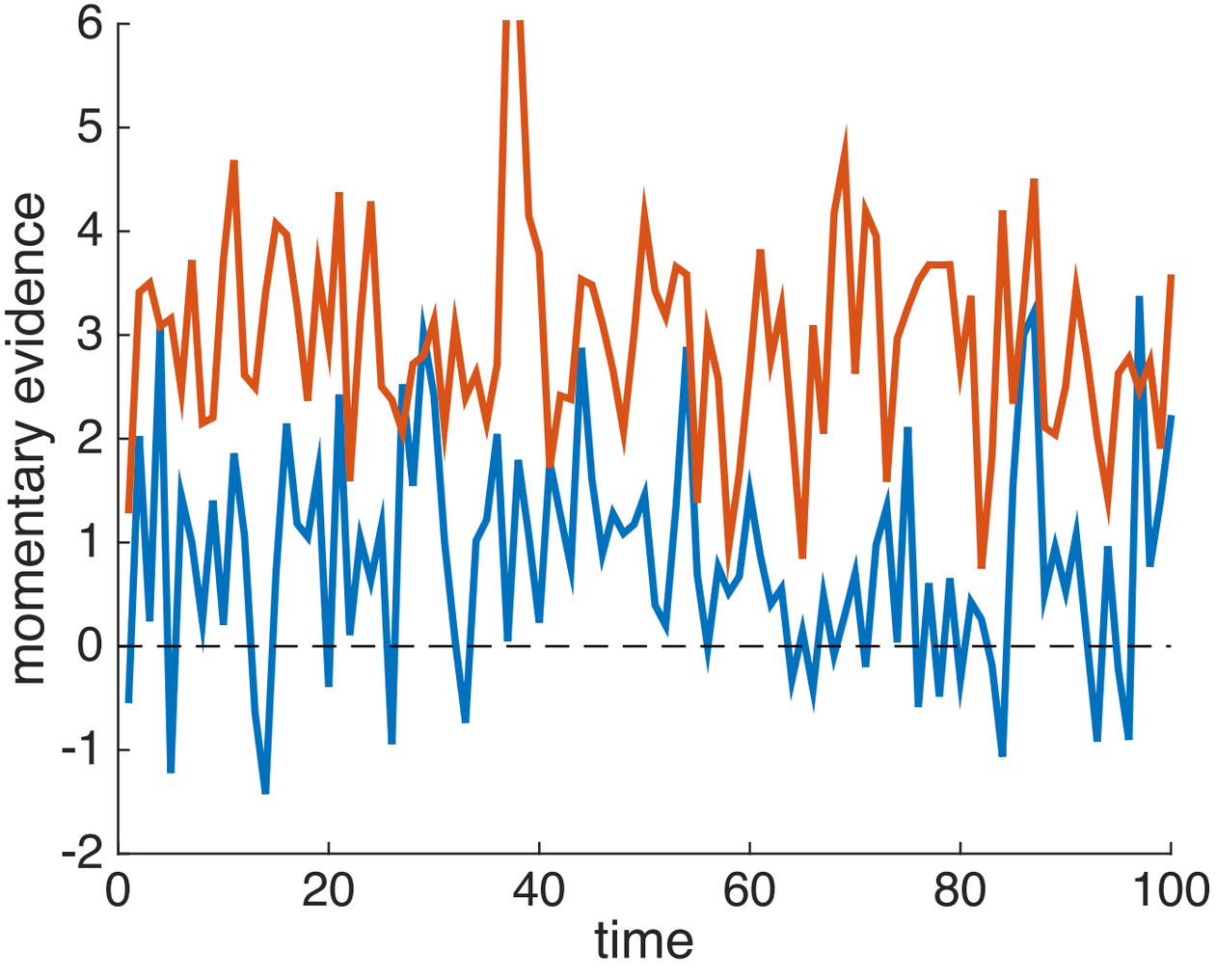
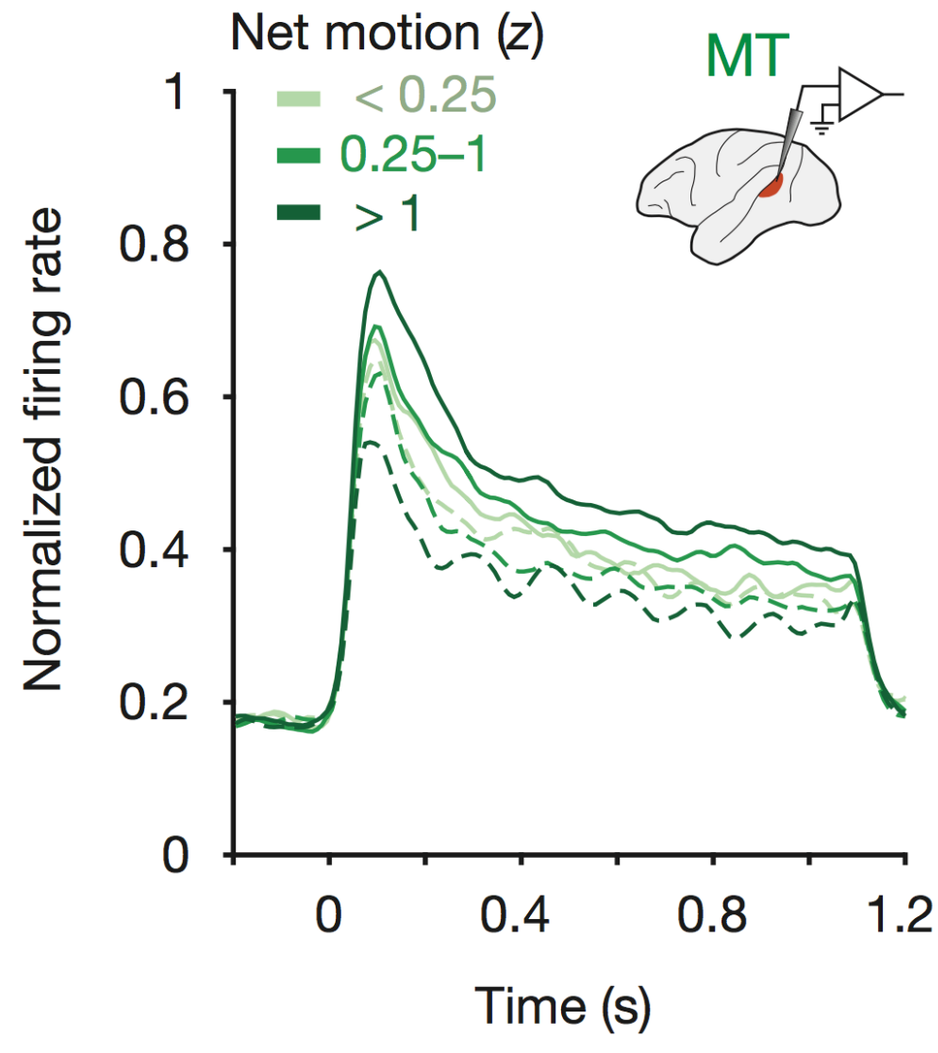




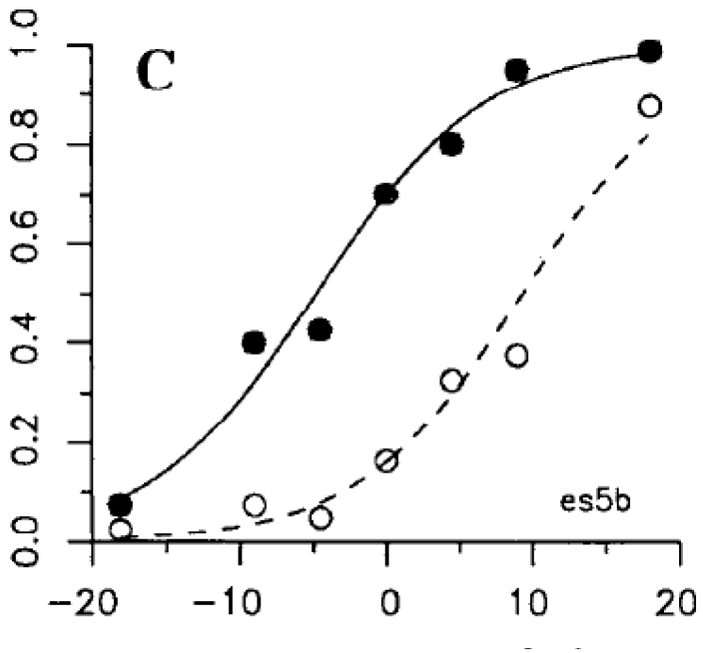
MT: momentary motion evidence

Is activity in MT *sufficient* to drive choice?

MT: momentary motion evidence

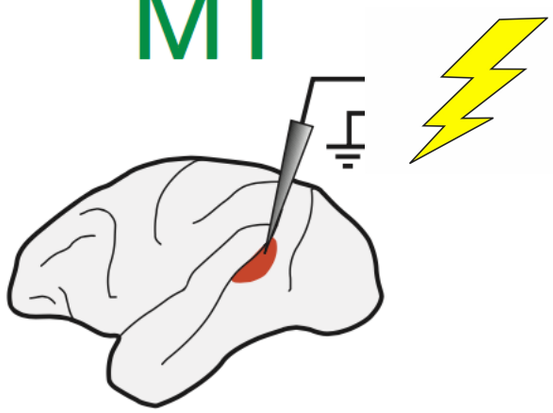
Is activity in MT *sufficient* to drive choice?

Proportion choices "preferred" by MT neuron



YES!!!

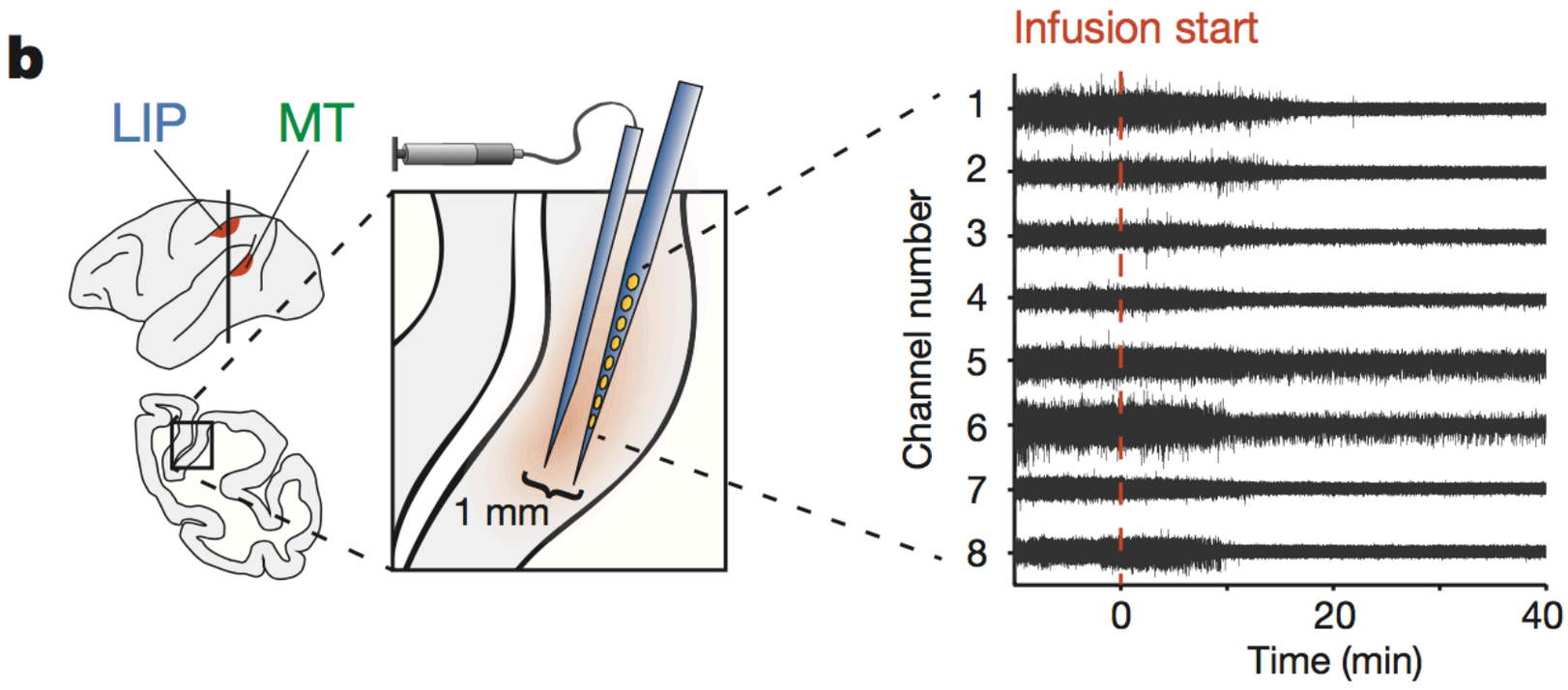
MT



Salzman, 1992

MT: momentary motion evidence

Is activity in MT *necessary* to drive choice?



Katz 2016

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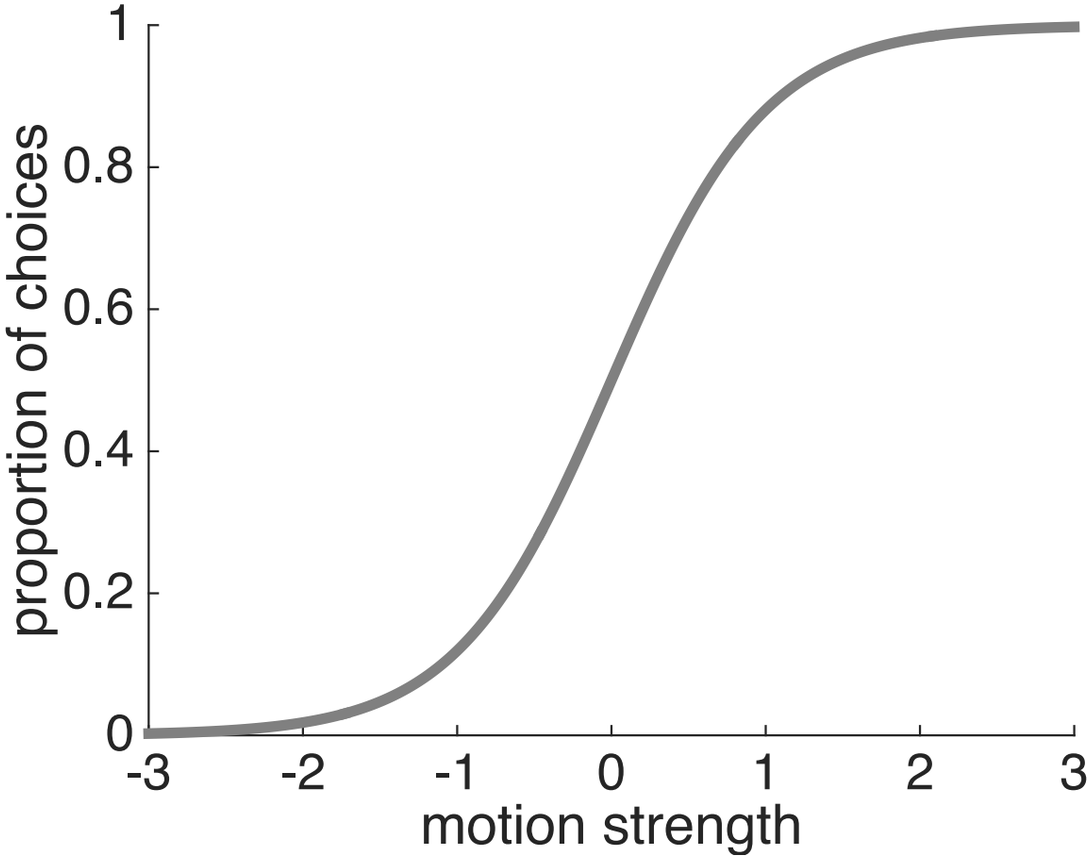

Katz 2016

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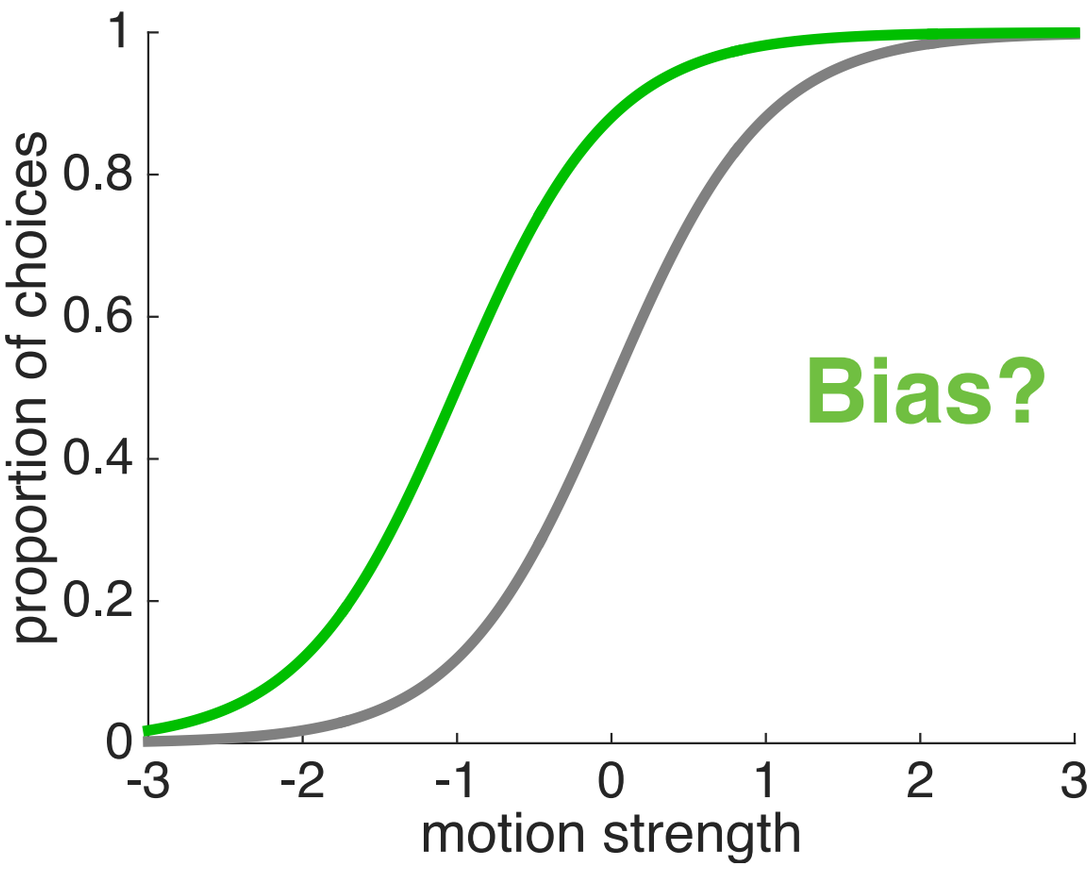

Katz 2016

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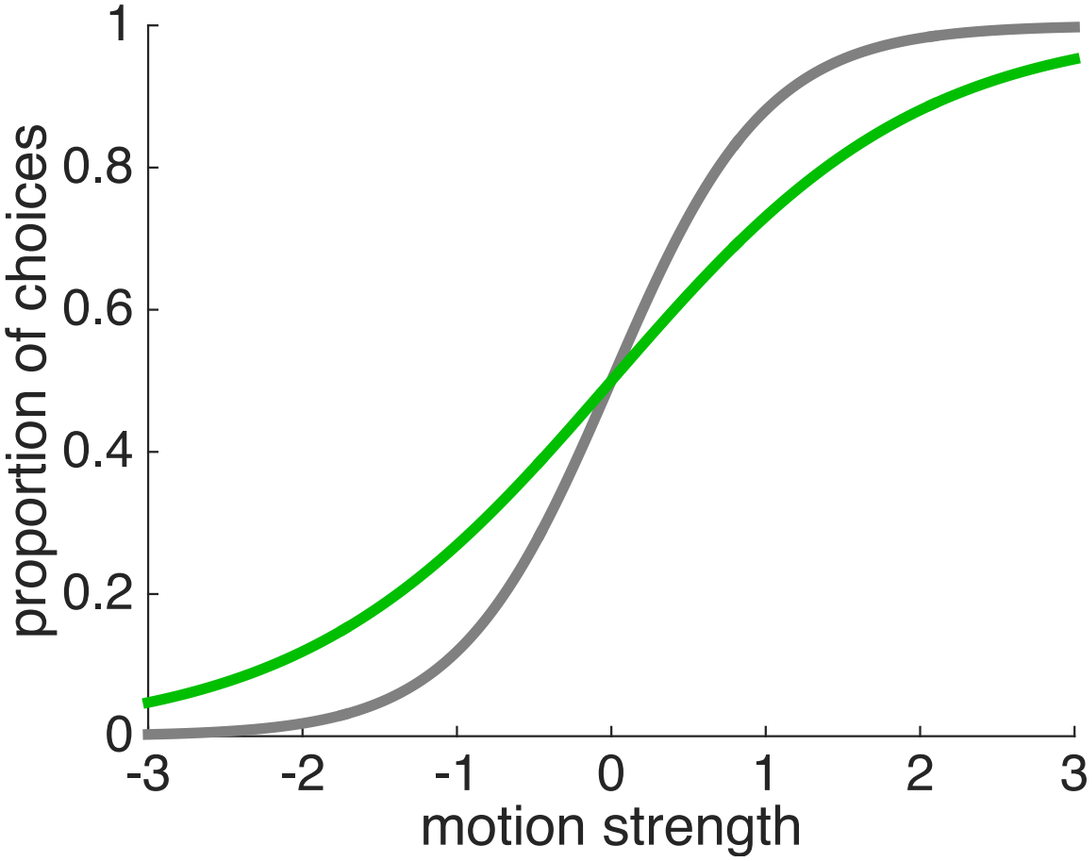
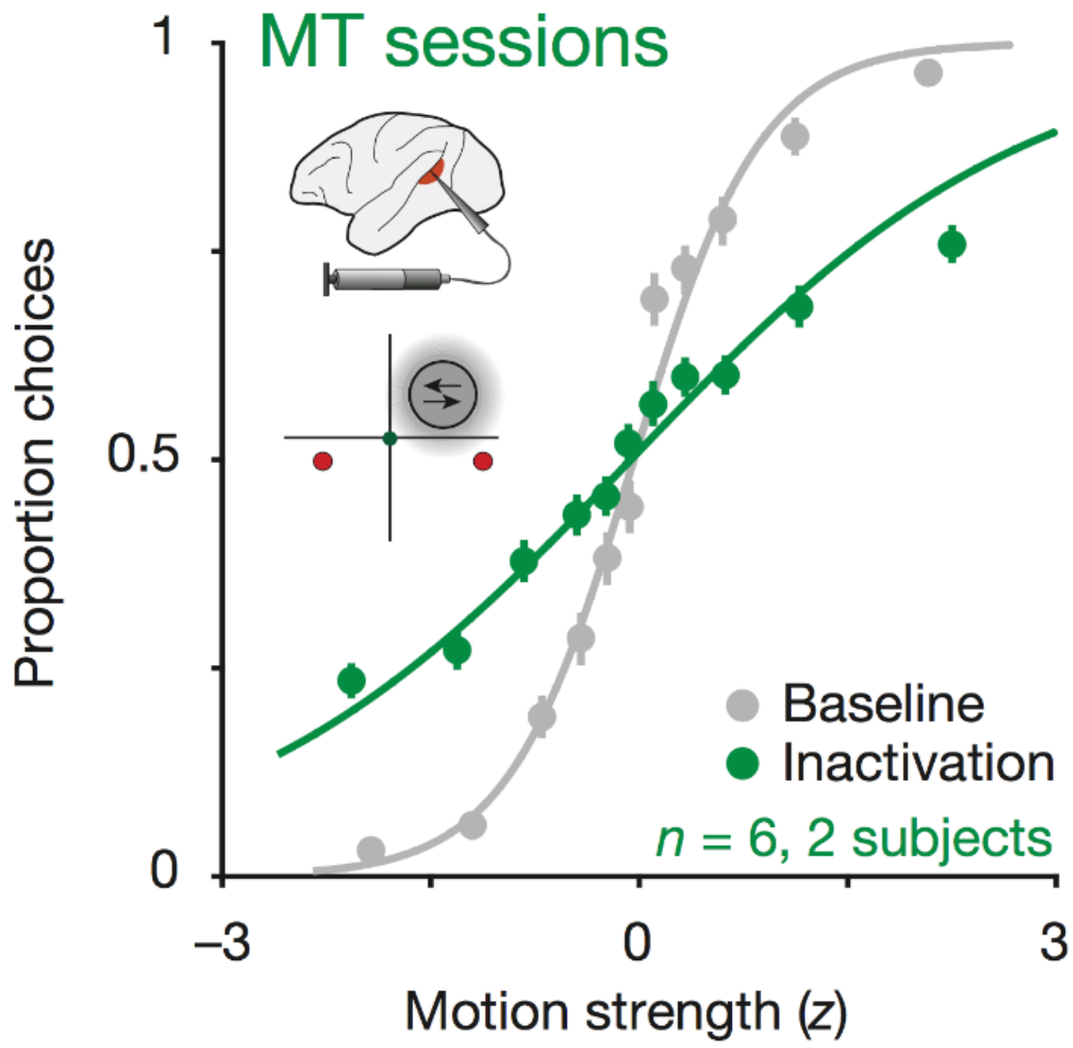


Reduced sensitivity?

Katz 2016

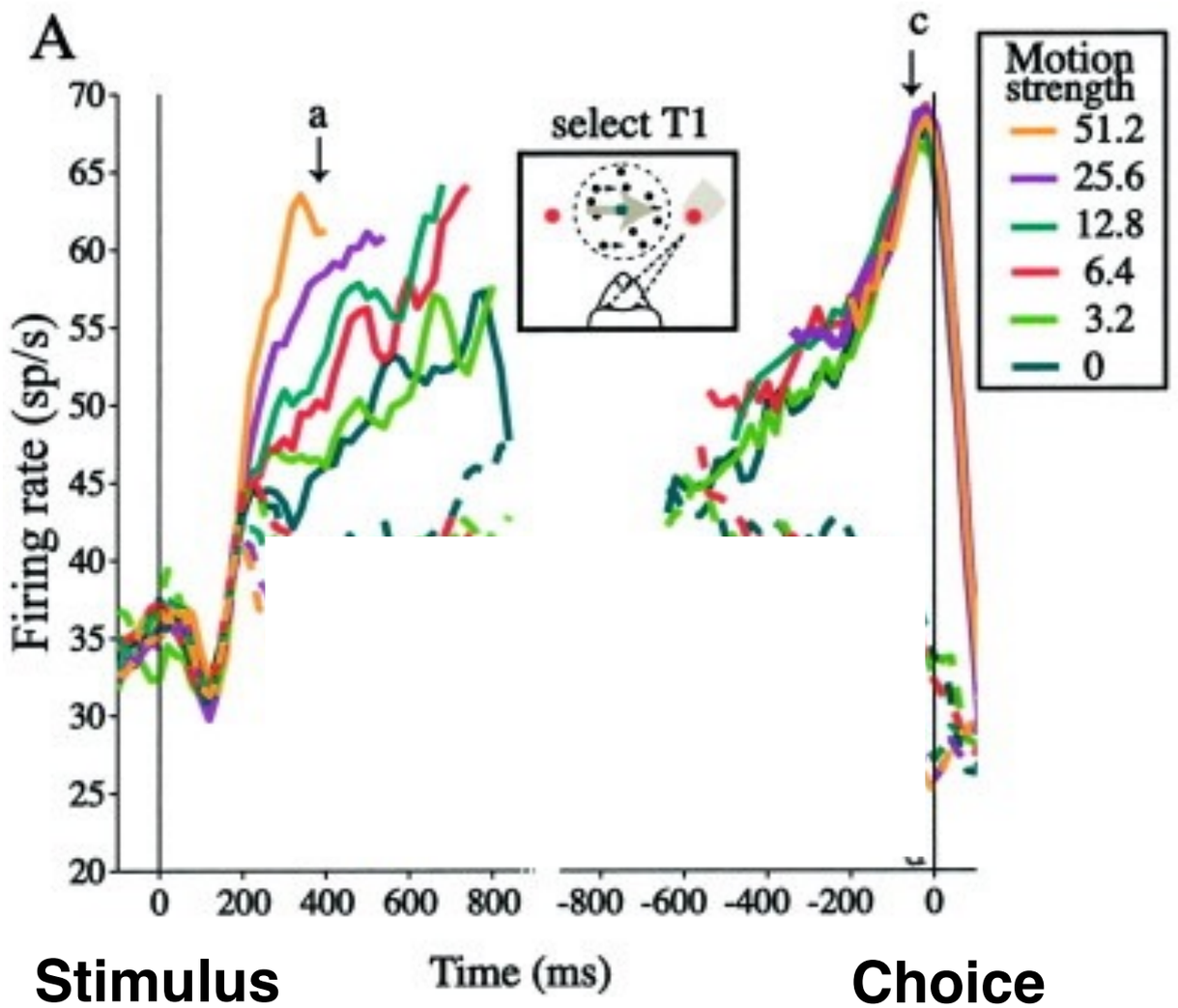
MT: momentary motion evidence

Is activity in MT *necessary* to drive choice?



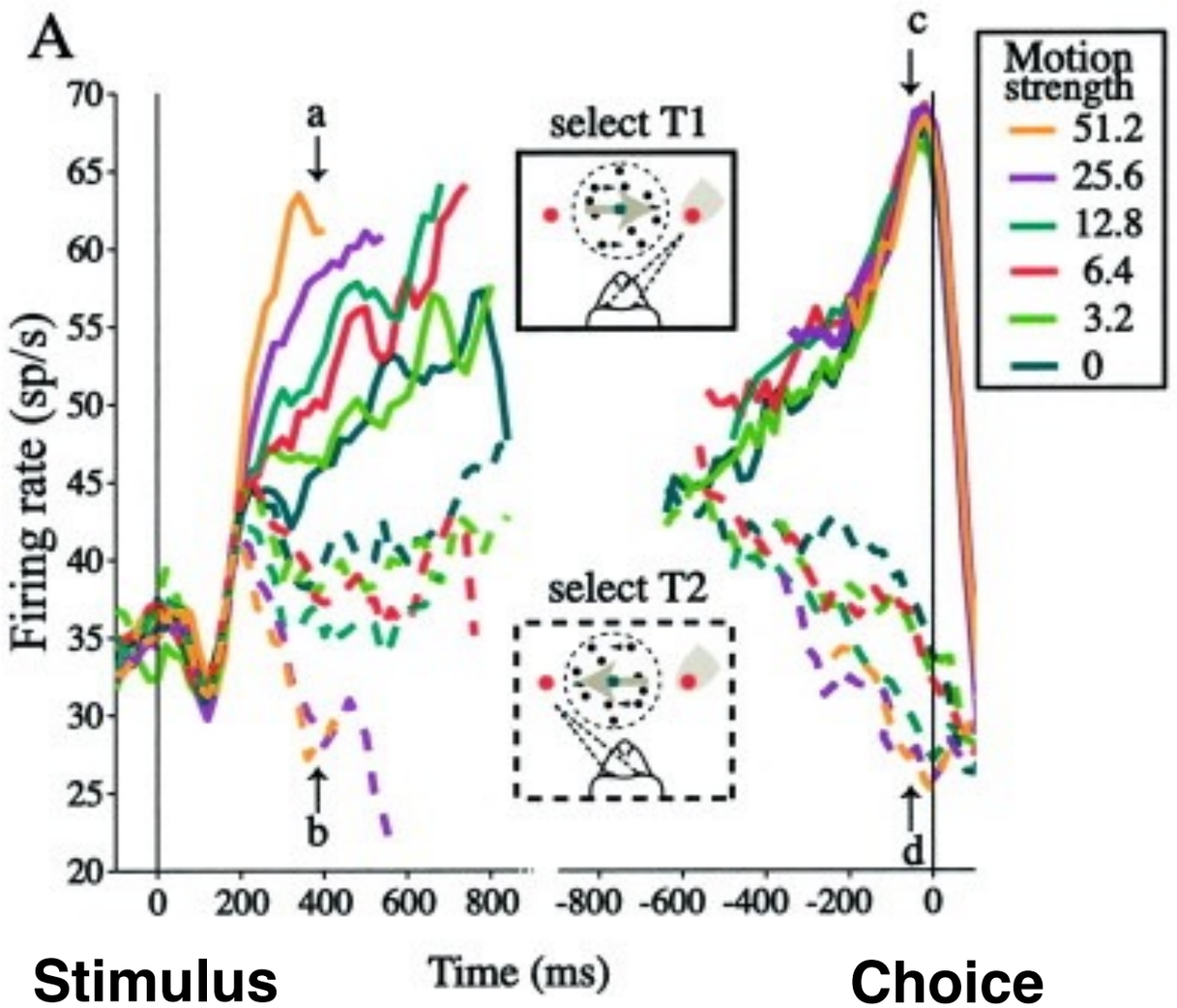
Katz 2016

LIP: Accumulated evidence



Roitman & Shadlen, 2002

LIP: Accumulated evidence



Roitman & Shadlen, 2002

LIP: Accumulated evidence

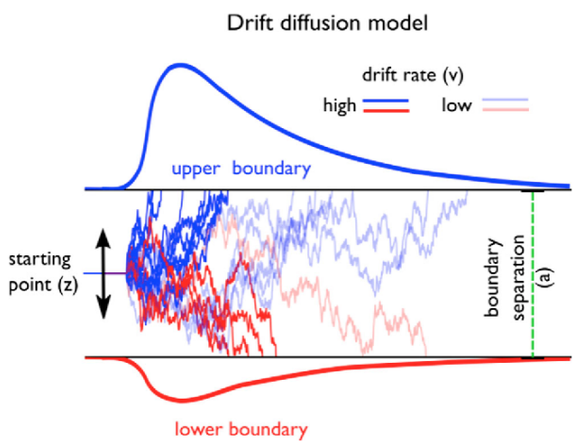
Computational

Evidence accumulation

$$\log\left[\frac{\Pr(m|h_1)}{\Pr(m|h_0)}\right]$$

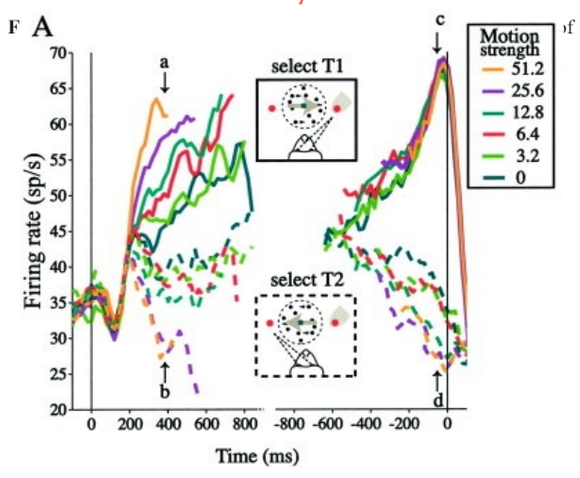
Algorithmic

Drift diffusion to bound



Implementation

Neural responses in LIP/FEF



LIP: Accumulated evidence

Katz 2016

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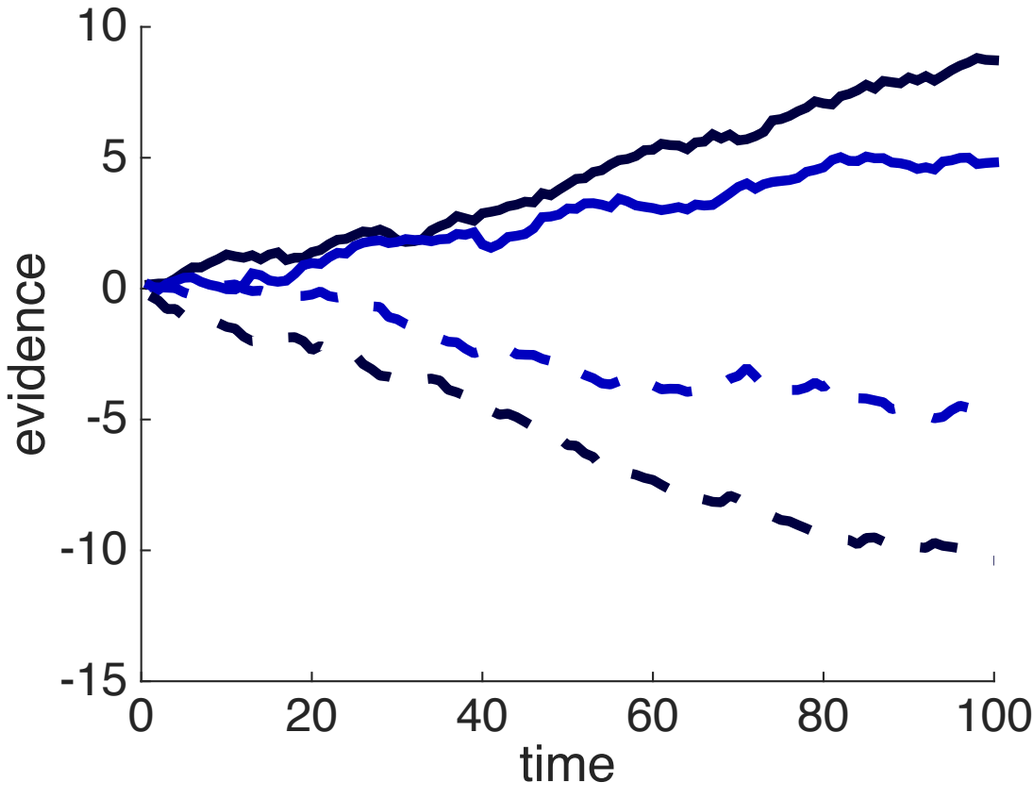
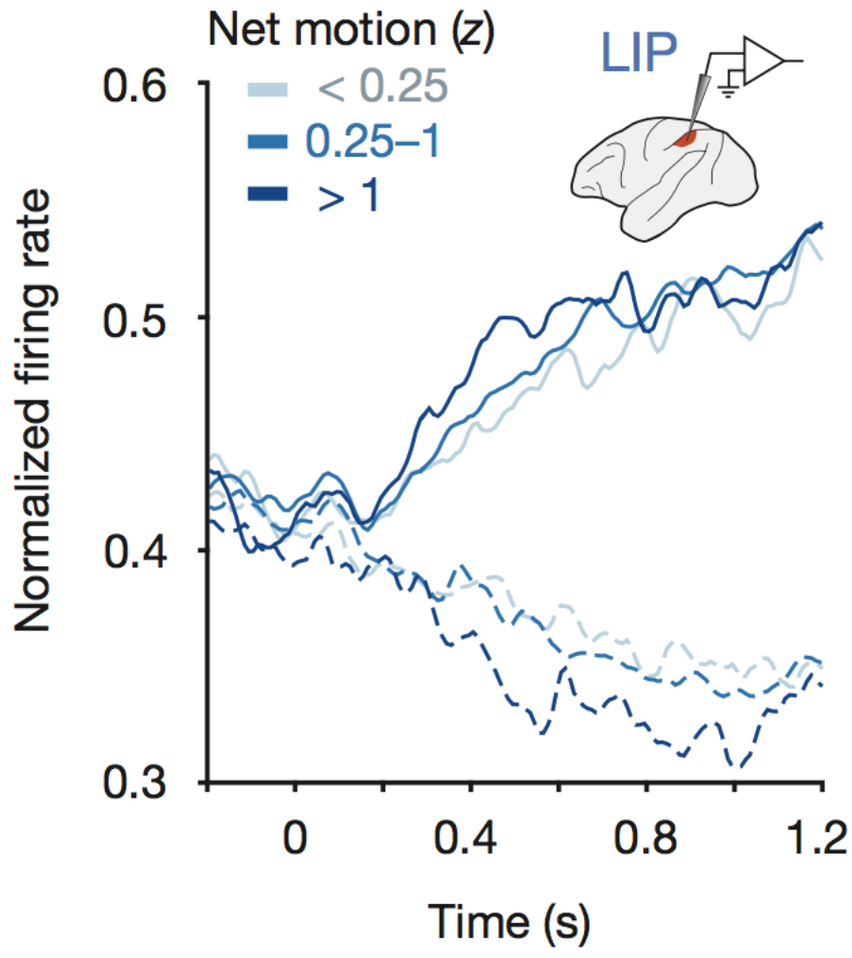
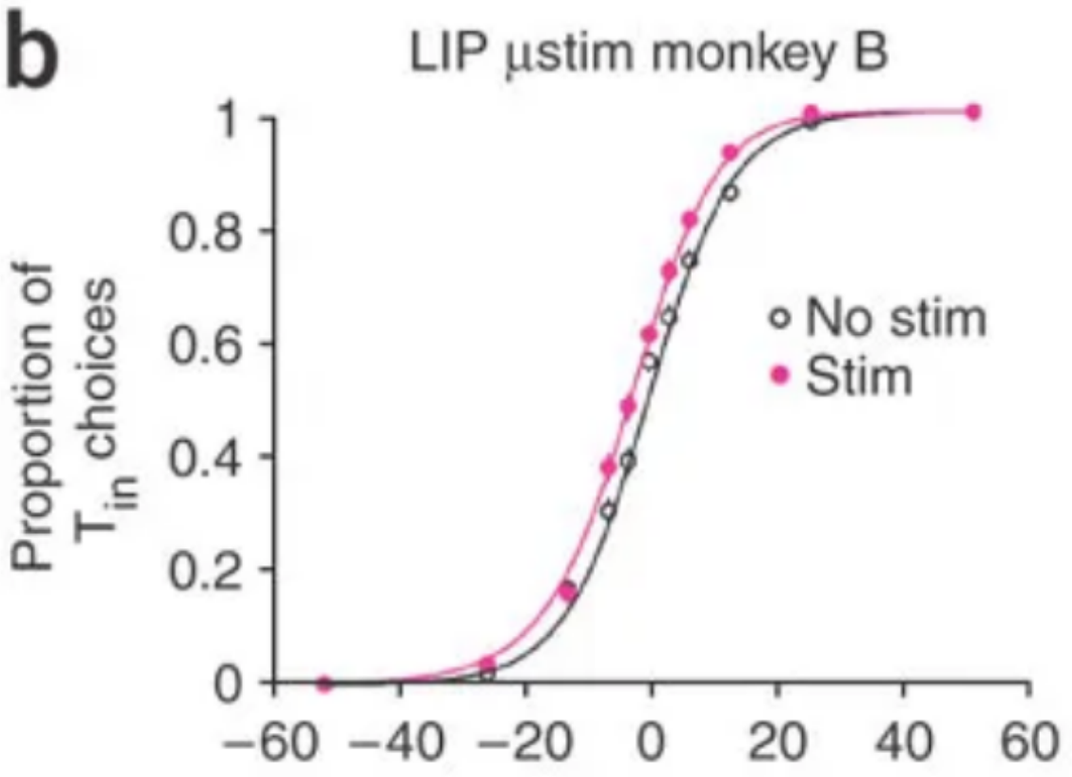




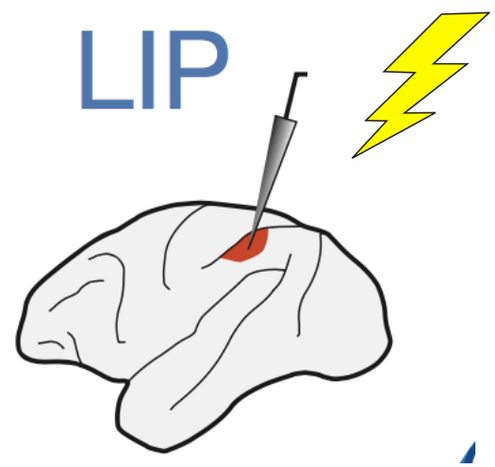
Katz 2016

LIP: Accumulated evidence

Is LIP firing *sufficient* to influence evidence accumulation? **YES (but less impressively)**



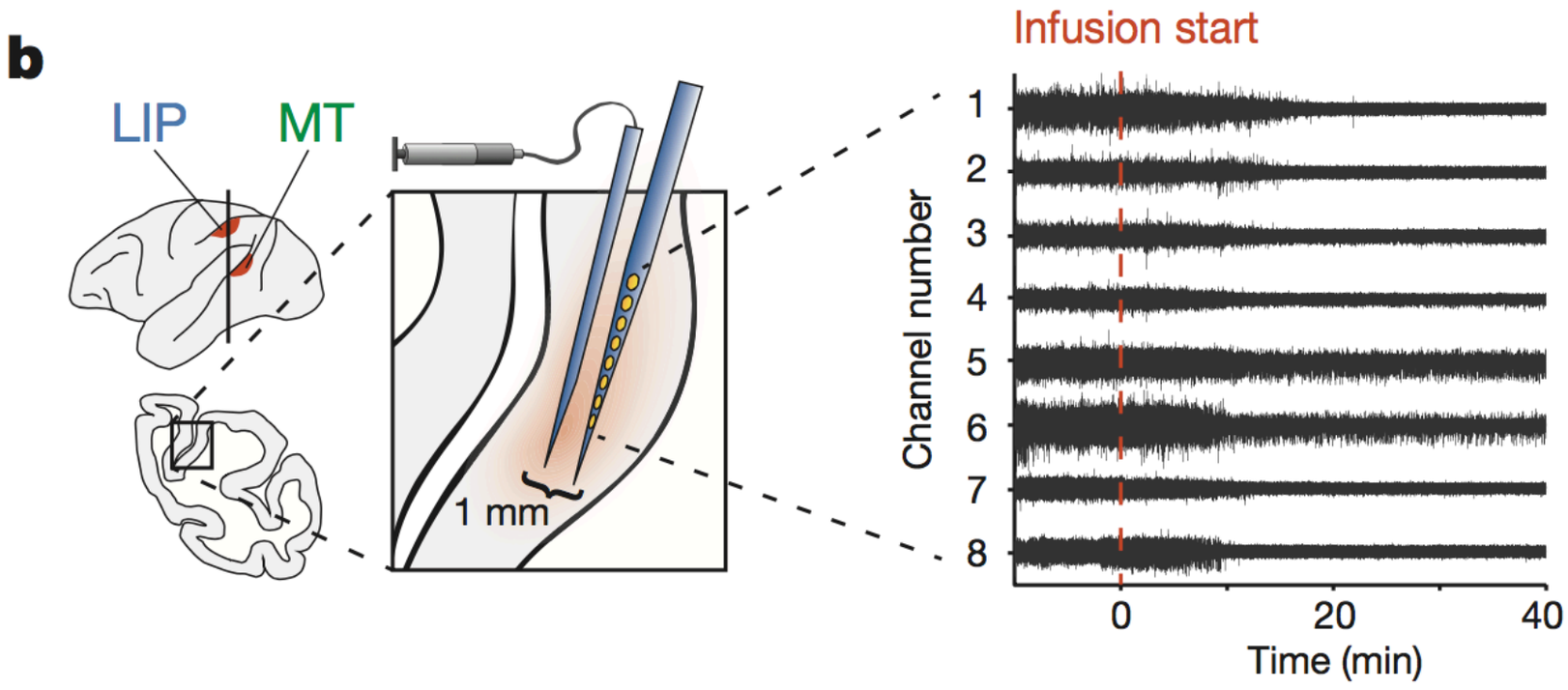
Motion information



Hanks, 2006

LIP: Accumulated evidence

Is LIP firing *necessary* to influence evidence accumulation?

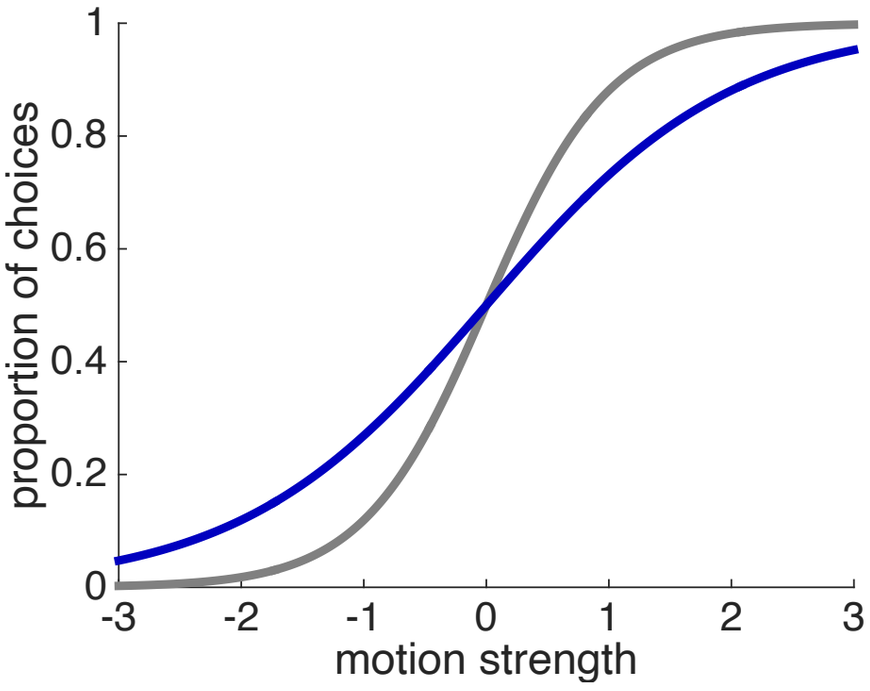


Katz 2016

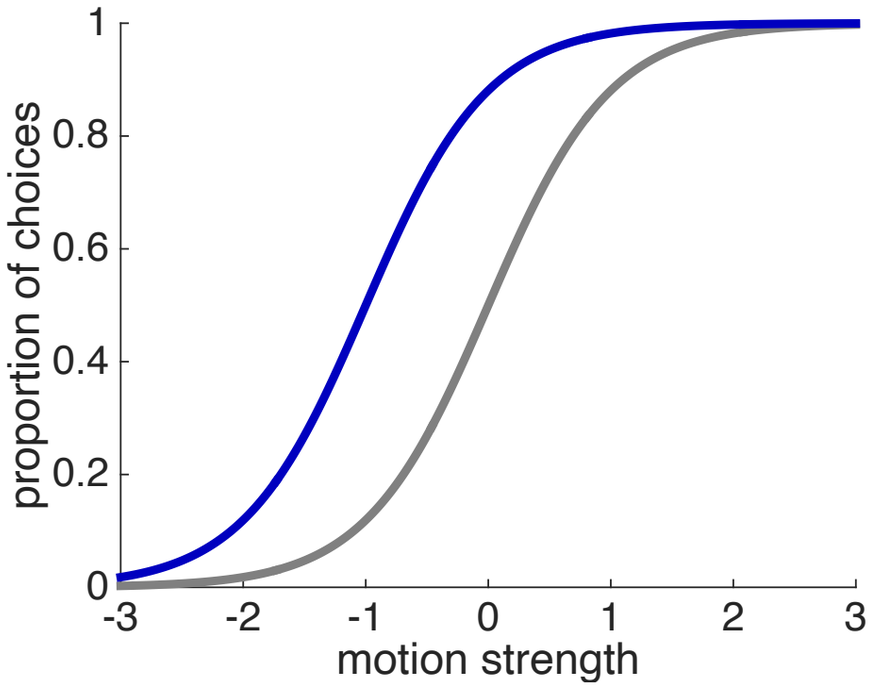
LIP: Accumulated evidence

Is LIP firing *necessary* to influence evidence accumulation?

Sensitivity?



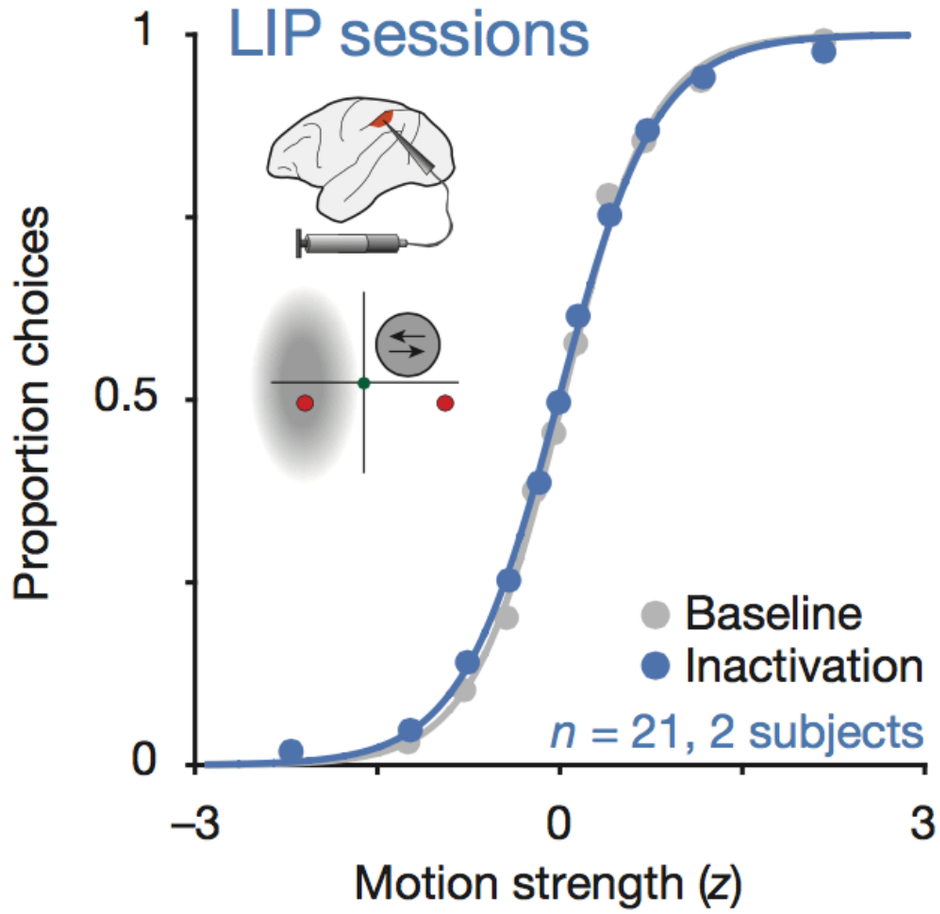
Bias?



LIP: Accumulated evidence

Is LIP firing *necessary* to influence evidence accumulation?

No???



Katz 2016

What is going on?

- Compensation?
- Wrong model?
- Wrong brain area?

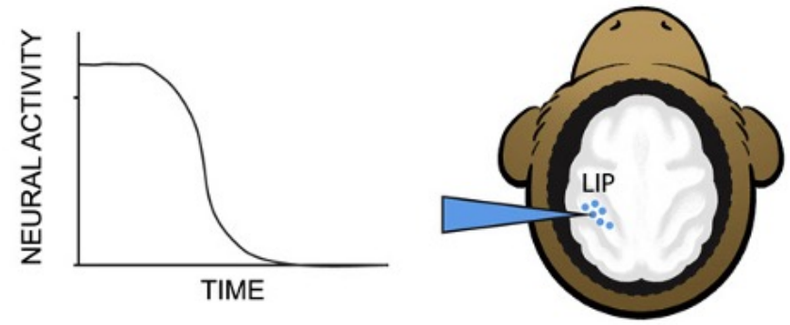
What is going on?

- **Compensation?**
- Wrong model?
- Wrong brain area?

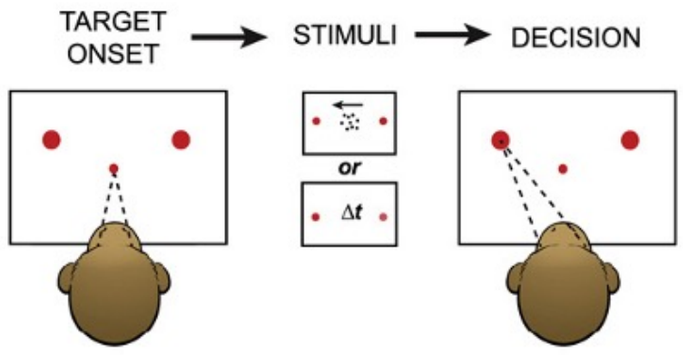
Compensation?

INACTIVATION

pharmacological&
chemogenetic

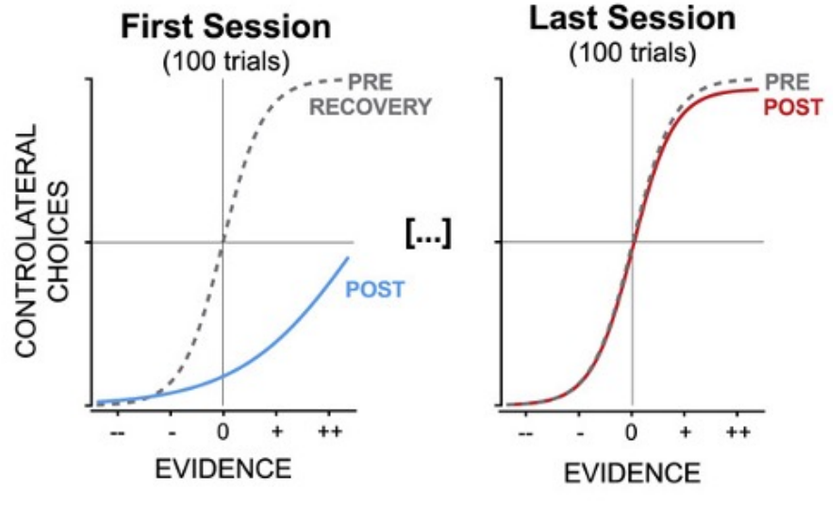


DECISION MAKING



BIAS FOLLOWED BY RECOVERY

within sessions
and across sessions



Jeurissen 2022

	Encodes decision	Discriminates evidence levels	Common activity level at choice	Sufficient	Necessary

Criteria for neural diffusion to bound

Parietal (LIP)

	Encodes decision	Discriminates evidence levels	Common activity level at choice	Sufficient	Necessary
Parietal (LIP)					

	Encodes decision	Discriminates evidence levels	Common activity level at choice	Sufficient	Necessary
Caudate/Striatum	✓	✓	✗	✓	✓
Parietal (LIP)	✓	✓	✓	✓	✗/✓

	Encodes decision	Discriminates evidence levels	Common activity level at choice	Sufficient	Necessary
Prefrontal (FEF/FOF)	✓	✗/✓	✓	✓	✗/✓
Caudate/Striatum	✓	✓	✗	✓	✓
Parietal (LIP)	✓	✓	✓	✓	✗/✓

Drift diffusion in the brain

One of the best examples
of theory shedding light
on computation in the brain

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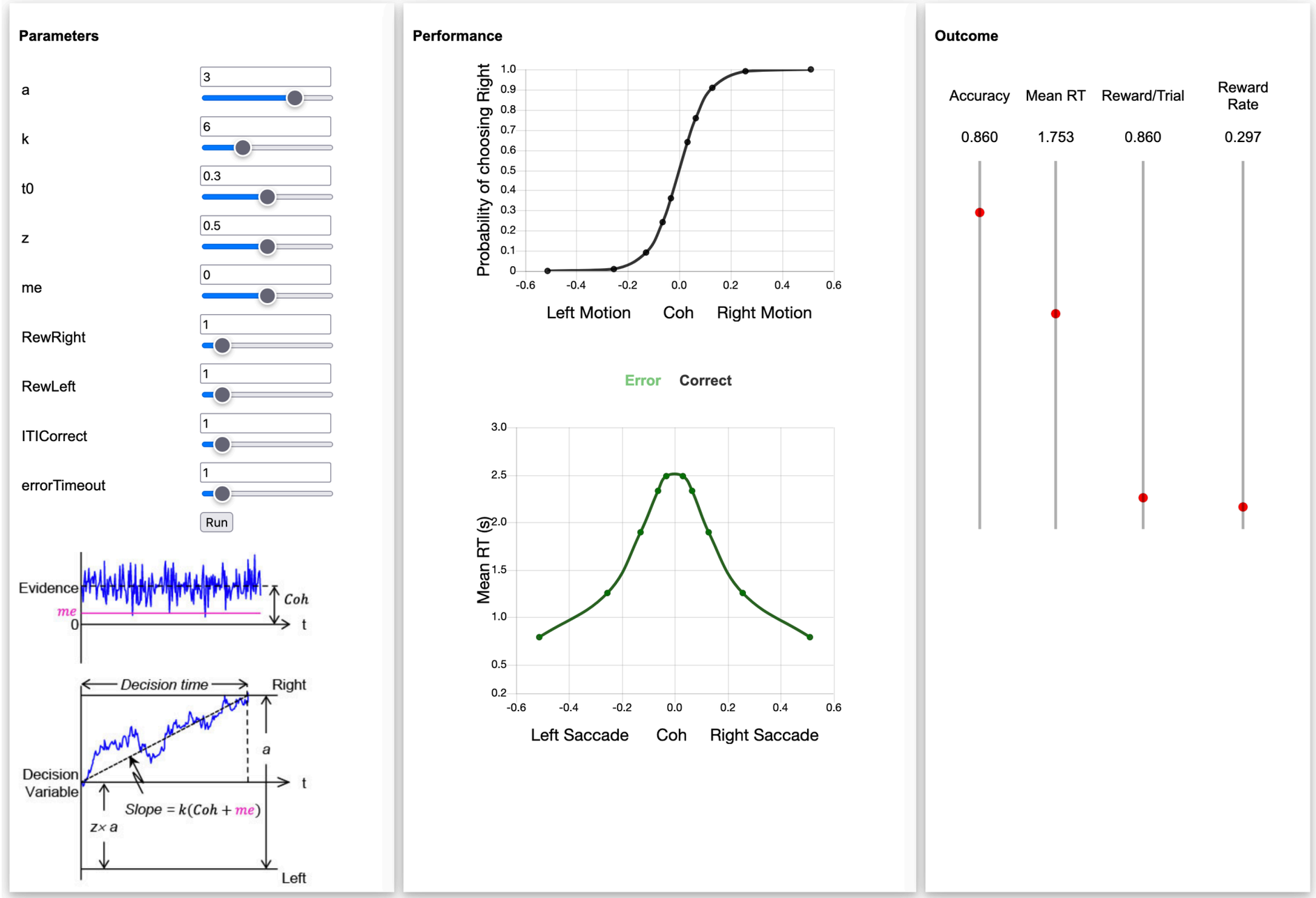
But still more work to be done…

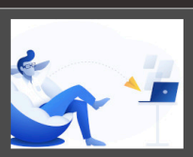
Resources on DDM

Online simulation tools



Long Ding https://www.med.upenn.edu/longding1/javascript/model_bias.html

HDDM

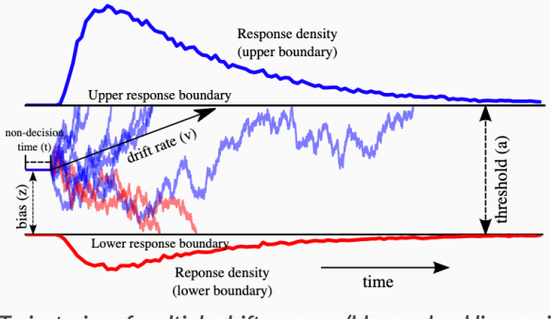


Sequential Sampling Models

Sequential Sampling Models generally fall into one of two classes: (i) diffusion models which assume that *relative* evidence is accumulated over time and (ii) race models which assume independent evidence accumulation and response commitment once the first accumulator crossed a boundary ([LaB62], [Vic70]). Currently, HDDM includes two of the most commonly used SSMs: the drift diffusion model (DDM) ([RR98], [RM08]) belonging to the class of diffusion models and the linear ballistic accumulator (LBA) ([BH08]) belonging to the class of race models.

Drift Diffusion Model

The DDM models decision making in two-choice tasks. Each choice is represented as an upper and lower boundary. A drift-process accumulates evidence over time until it crosses one of the two boundaries and initiates the corresponding response ([RR98], [SR04]). The speed with which the accumulation process approaches one of the two boundaries is called drift-rate *v* and represents the relative evidence for or against a particular response. Because there is noise in the drift process, the time of the boundary crossing and the selected response will vary between trials. The distance between the two boundaries (i.e. threshold *a*) influences how much evidence must be accumulated until a response is executed. A lower threshold makes responding faster in general but increases the influence of noise on decision making and can hence lead to errors or impulsive choice, whereas a higher threshold leads to more cautious responding (slower, more skewed RT distributions, but more accurate). Response time, however, is not solely comprised of the decision making process – perception, movement initiation and execution all take time and are lumped in the DDM by a single non-decision time parameter *t*. The model also allows for a prepotent bias *z* affecting the starting point of the drift process relative to the two boundaries. The termination times of this generative process gives rise to the reaction time distributions of both choices.



Trajectories of multiple drift-process (blue and red lines, middle panel). Evidence is accumulated over time (x-axis) with drift-rate v until one of two boundaries (separated by threshold a) is crossed and a response

https://hddm.readthedocs.io/en/latest/methods.html

Frank, Wiecki, et al…

Questions?