



# Active inference

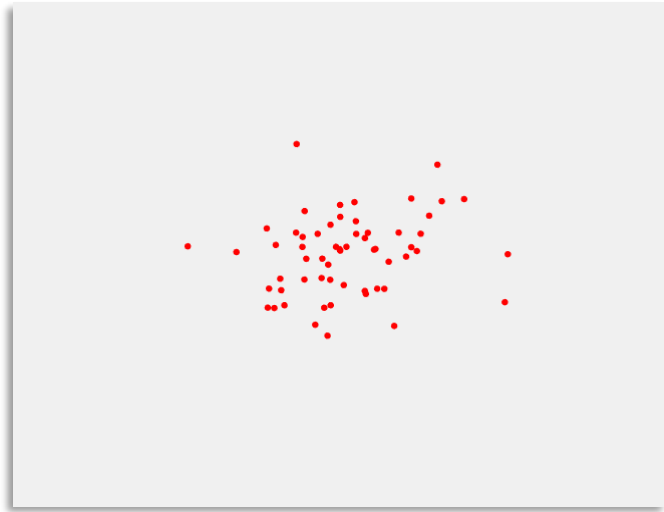
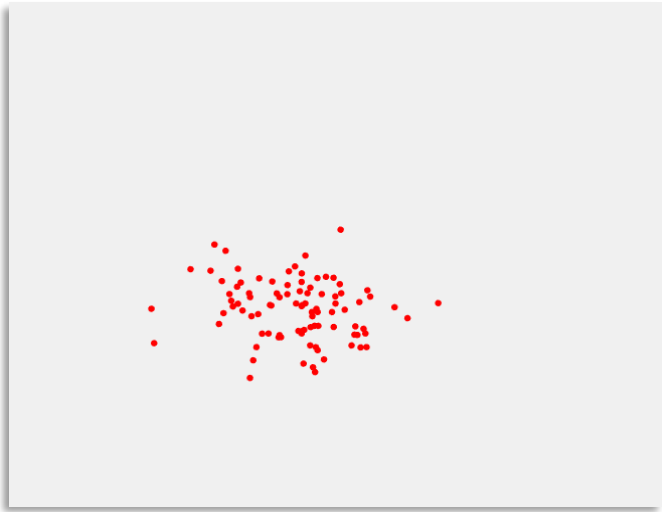
Computational psychiatry course  
2022

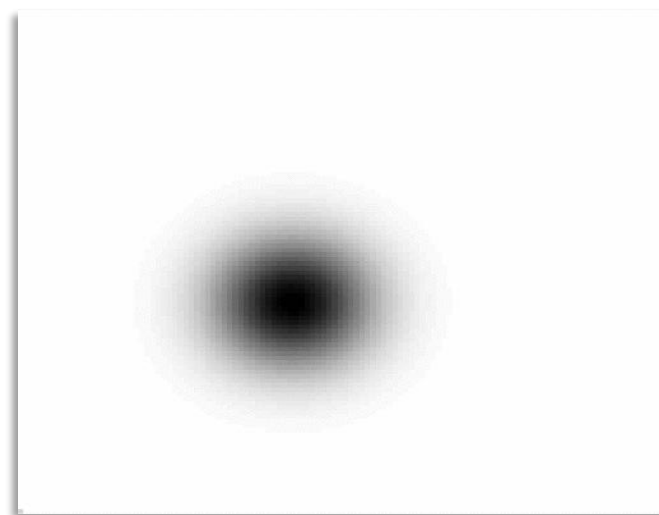
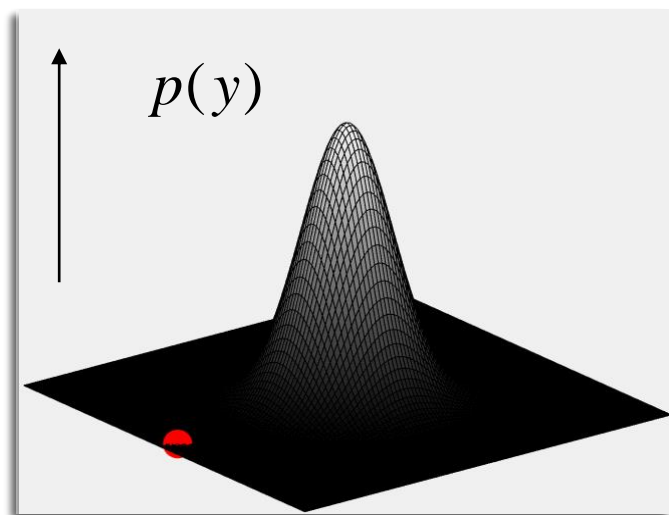


Active inference  
Generative models  
Exploitation  
Exploration  
Movement  
Hierarchy

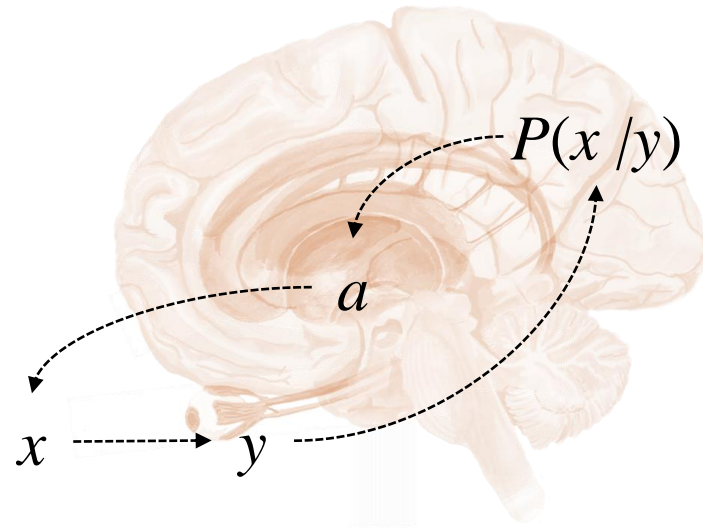
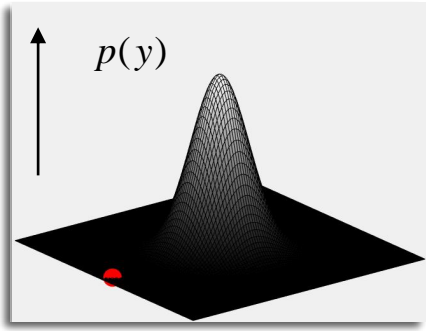


**Active inference**  
Generative models  
Exploitation  
Exploration  
Movement  
Hierarchy





# Self-evidencing – Active Inference



$$P(y | x)P(x) = P(x | y)P(y)$$

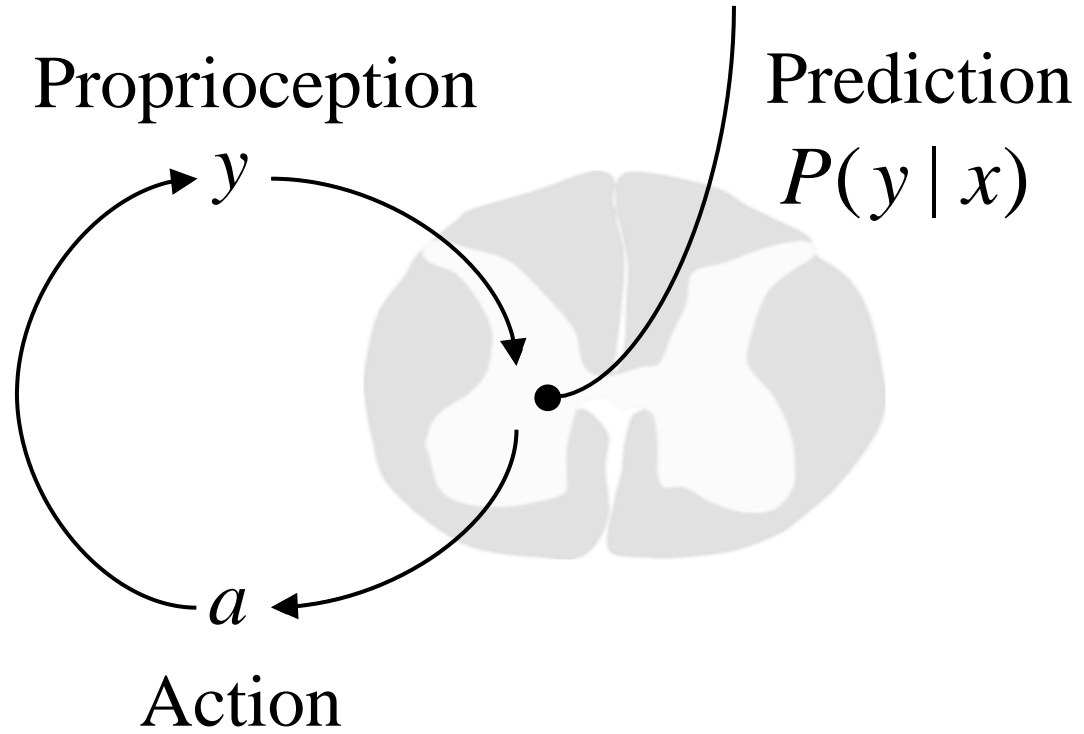
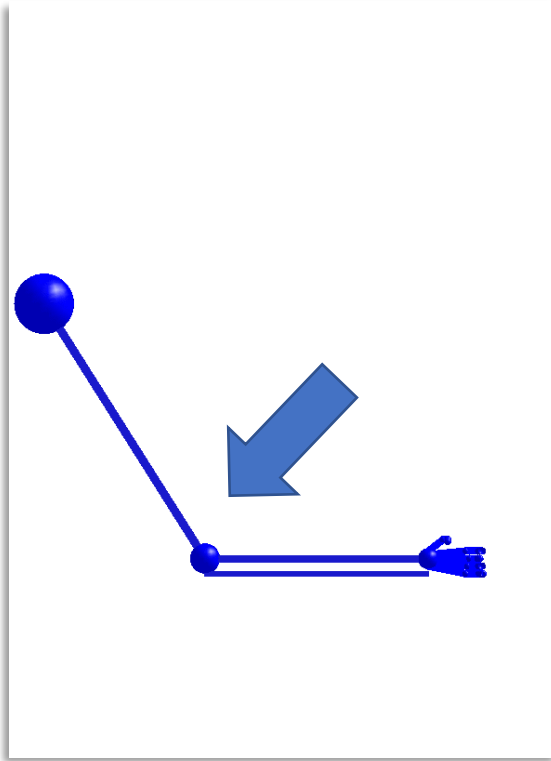
$\underbrace{\hspace{1.5cm}}_{\text{Likelihood}} \underbrace{\hspace{1.5cm}}_{\text{Prior}}$

$\underbrace{\hspace{1.5cm}}_{\text{Posterior}} \underbrace{\hspace{1.5cm}}_{\text{Evidence}}$

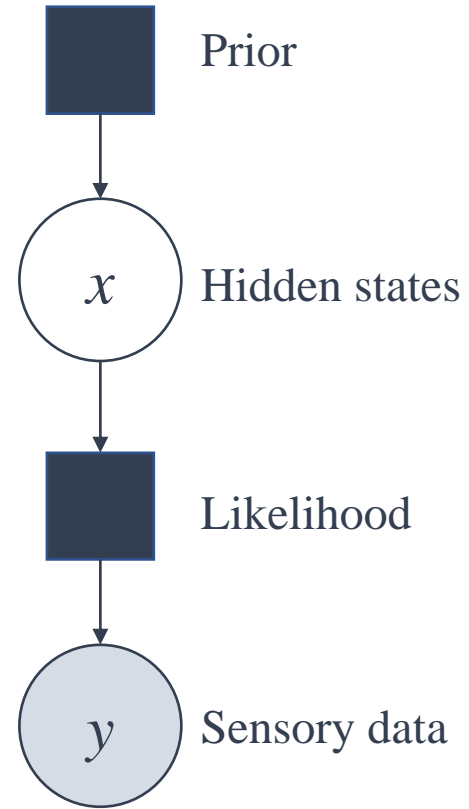
$\underbrace{\hspace{3cm}}_{\text{Generative model}}$

$\underbrace{\hspace{3cm}}_{\text{Inverted model}}$

# Reflexes



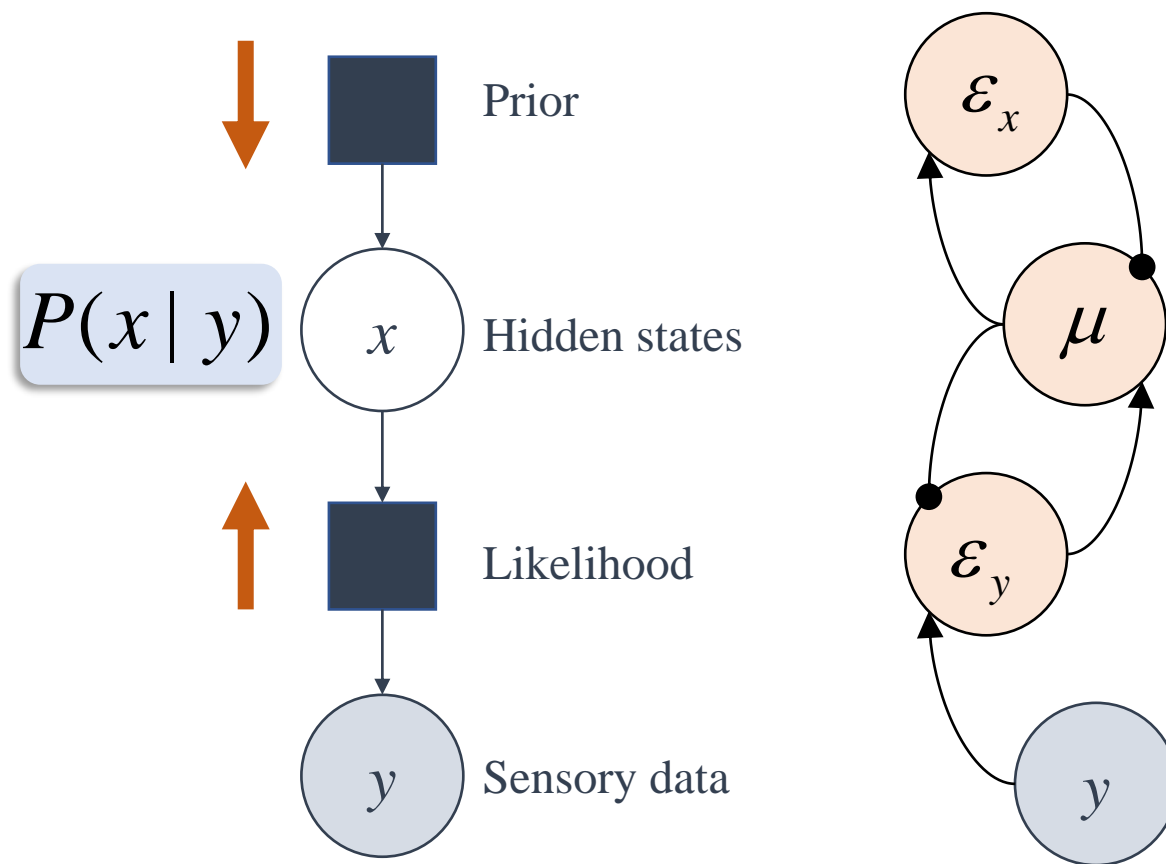
# Models and messages



$$P(x \mid y)P(y) = P(y \mid x)P(x)$$



# Models and messages

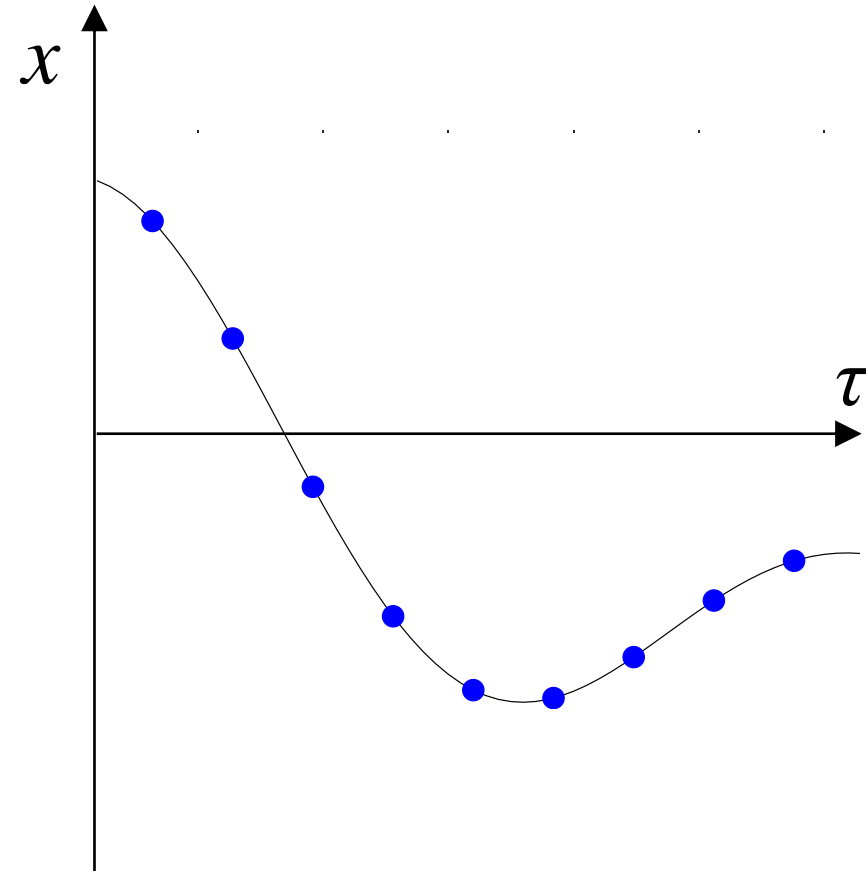
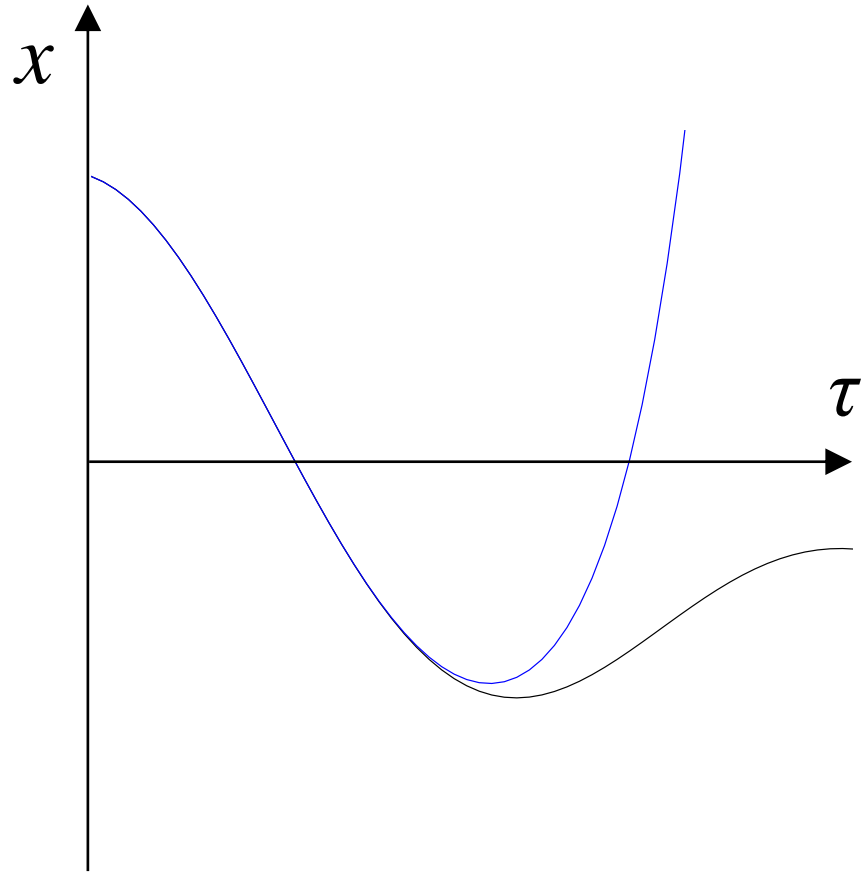


$$P(x | y) \propto P(y | x) P(x)$$

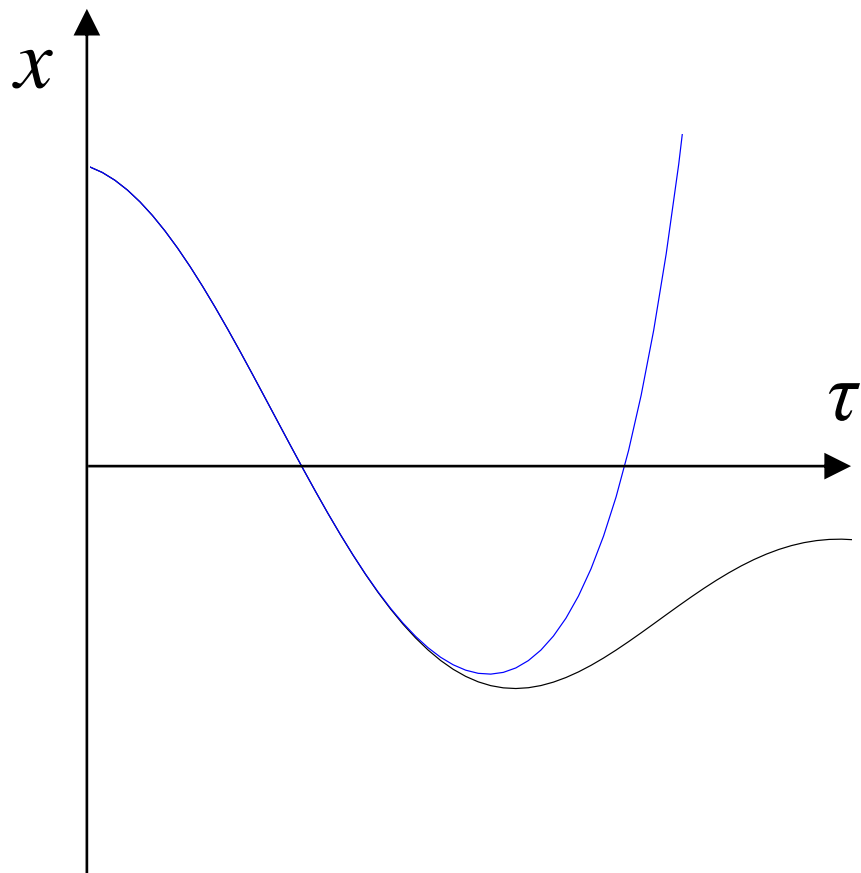


Active inference  
**Generative models**  
Exploitation  
Exploration  
Movement  
Hierarchy

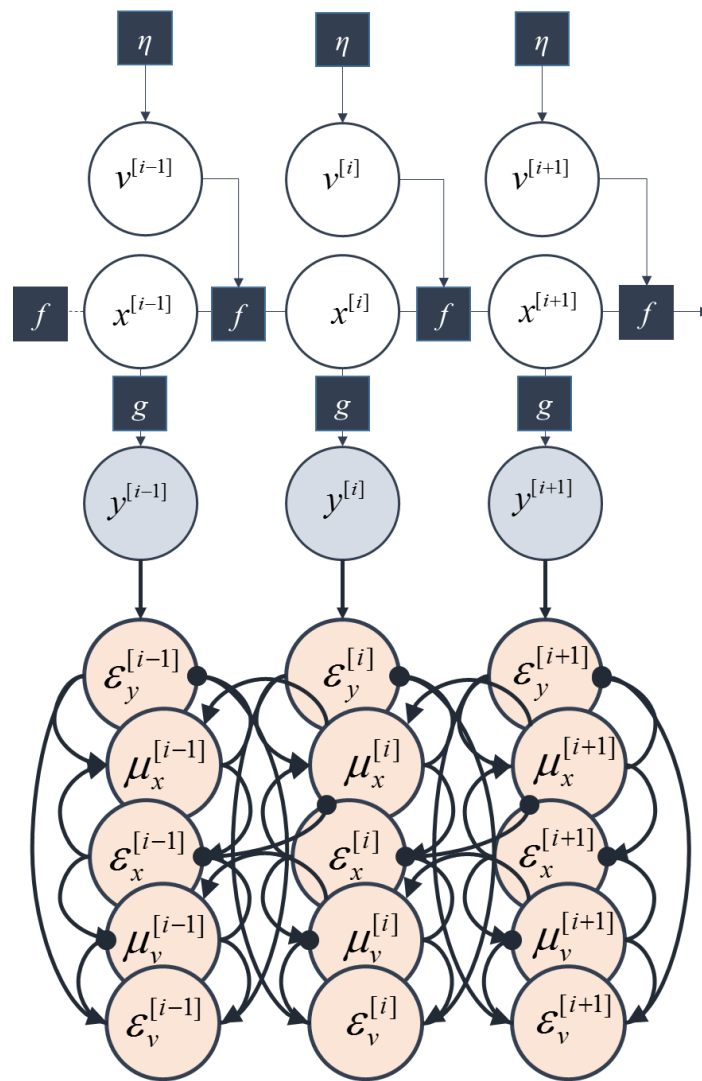
# Representing dynamics in generative models



# Continuous time models



$$x(\tau) \approx x_0 + \tau x'_0 + \frac{1}{2} \tau^2 x''_0 + \frac{1}{6} \tau^3 x'''_0 + \dots$$



## Generative model

$$p(\tilde{y}, \tilde{x}, \tilde{v}) = \prod_i p(v^{[i]}) p(x^{[i+1]} | x^{[i]}, v^{[i]}) p(y^{[i]} | x^{[i]}, v^{[i]})$$

$$p(y^{[i]} | x^{[i]}, v^{[i]}) = \mathcal{N}(g^{[i]}(x^{[i]}, v^{[i]}), \Pi_y^{[i]})$$

$$p(x^{[i+1]} | x^{[i]}, v^{[i]}) = \mathcal{N}(f^{[i]}(x^{[i]}, v^{[i]}), \Pi_x^{[i]})$$

$$p(v^{[i]}) = \mathcal{N}(\eta^{[i]}, \Pi_v^{[i]})$$

## Bayesian message passing

$$\varepsilon_y^{[i]} = y^{[i]} - g^{[i]}(\mu_x^{[i]}, \mu_v^{[i]})$$

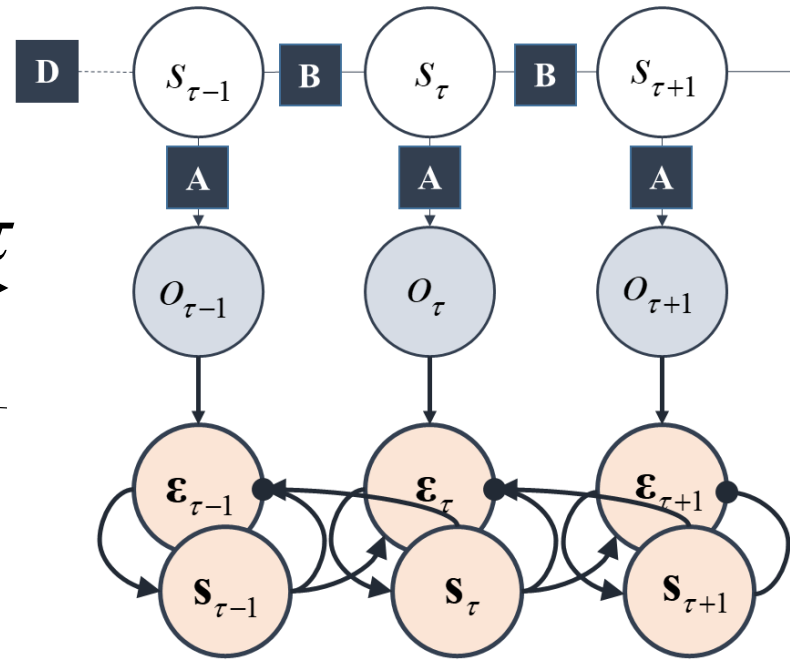
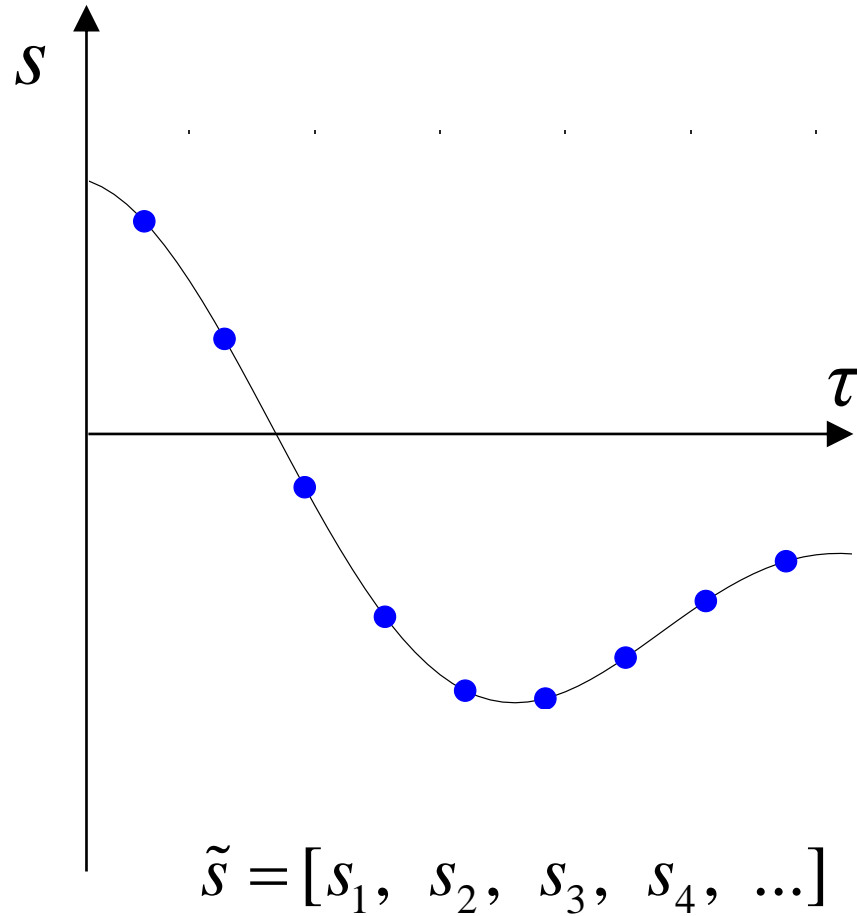
$$\varepsilon_x^{[i]} = \mu_x^{[i+1]} - f^{[i]}(\mu_x^{[i]}, \mu_v^{[i]})$$

$$\varepsilon_v^{[i]} = \mu_v^{[i]} - \eta^{[i]}$$

$$\begin{aligned} \dot{\mu}_x^{[i]} &= \mu_x^{[i+1]} \\ &\quad + \partial_{\mu_x^{[i]}} g^{[i]} \cdot \Pi_y^{[i]} \varepsilon_y^{[i]} - \Pi_x^{[i-1]} \varepsilon_x^{[i-1]} + \partial_{\mu_x^{[i]}} f^{[i]} \cdot \Pi_x^{[i]} \varepsilon_x^{[i]} \end{aligned}$$

$$\begin{aligned} \dot{\mu}_v^{[i]} &= \mu_v^{[i+1]} \\ &\quad + \partial_{\mu_v^{[i]}} g^{[i]} \cdot \Pi_y^{[i]} \varepsilon_y^{[i]} + \partial_{\mu_v^{[i]}} f^{[i]} \cdot \Pi_x^{[i]} \varepsilon_x^{[i]} - \Pi_v^{[i]} \varepsilon_v^{[i]} \end{aligned}$$

# Discrete time models



## Generative model

$$P(\tilde{o}, \tilde{s}) = P(s_1) \prod_{\tau} P(s_{\tau+1} | s_{\tau}) P(o_{\tau} | s_{\tau})$$

$$P(o_{\tau} | s_{\tau}) = \text{Cat}(\mathbf{A})$$

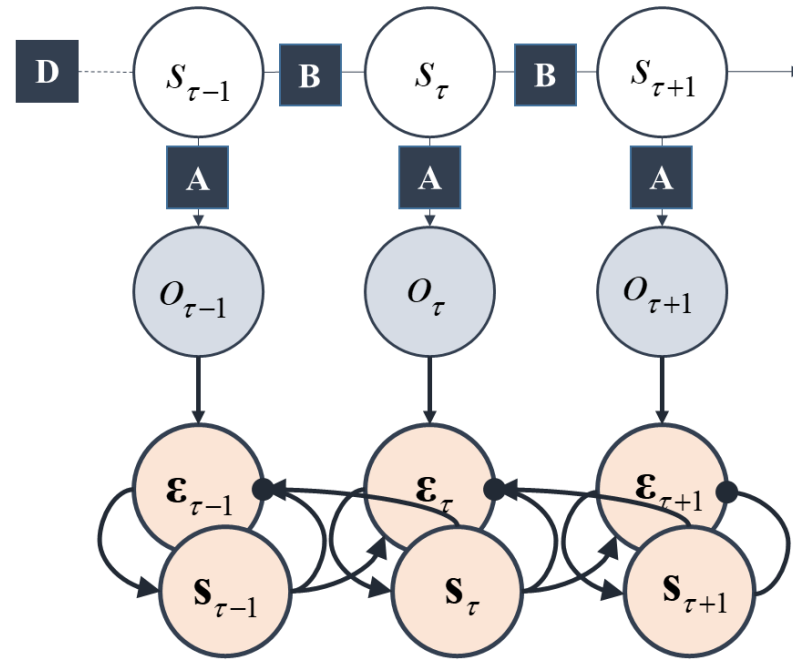
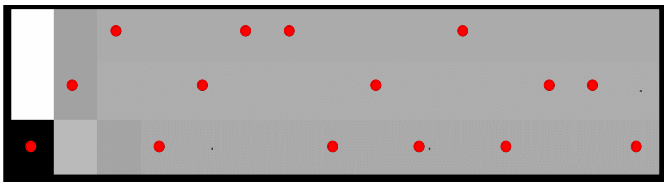
$$P(s_{\tau} | s_{\tau-1}) = \text{Cat}(\mathbf{B})$$

$$P(s_1) = \text{Cat}(\mathbf{D})$$

## Bayesian message passing

$$\mathbf{s}_{\tau} = \sigma(\mathbf{v}_{\tau}); \quad \dot{\mathbf{v}}_{\tau} = \boldsymbol{\varepsilon}_{\tau}$$

$$\boldsymbol{\varepsilon}_{\tau} = \ln \mathbf{A} \cdot o_{\tau} + \frac{1}{2} \ln(\mathbf{B}_{\tau}^{\dagger} \mathbf{s}_{\tau}) + \frac{1}{2} \ln(\mathbf{B}_{\tau+1}^{\dagger} \mathbf{s}_{\tau+1}) - \ln \mathbf{s}_{\tau}$$



### Generative model

$$P(\tilde{o}, \tilde{s}) = P(s_1) \prod_{\tau} P(s_{\tau+1} | s_{\tau}) P(o_{\tau} | s_{\tau})$$

$$P(o_{\tau} | s_{\tau}) = \text{Cat}(\mathbf{A})$$

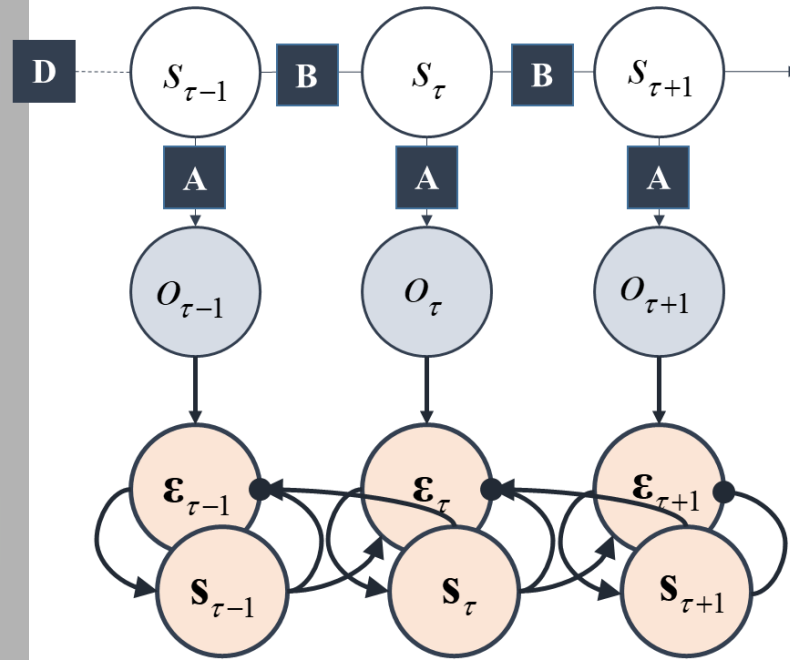
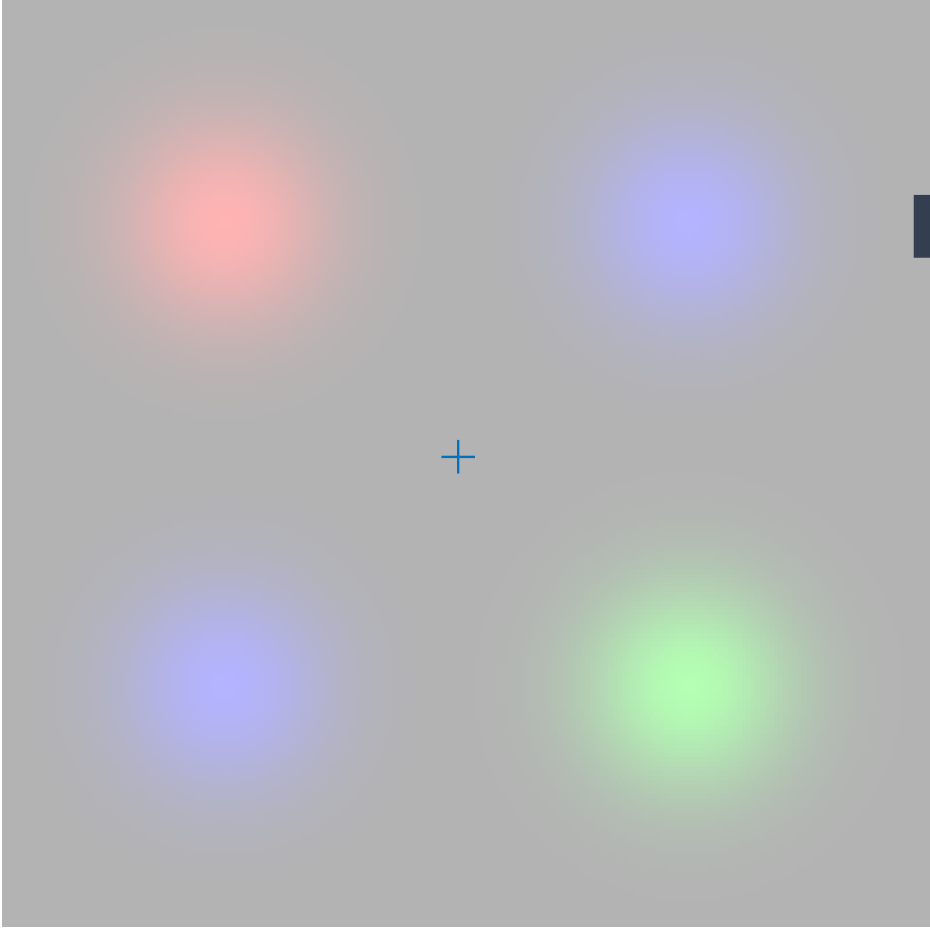
$$P(s_{\tau} | s_{\tau-1}) = \text{Cat}(\mathbf{B})$$

$$P(s_1) = \text{Cat}(\mathbf{D})$$

### Bayesian message passing

$$\mathbf{s}_{\tau} = \sigma(\mathbf{v}_{\tau}); \quad \dot{\mathbf{v}}_{\tau} = \boldsymbol{\epsilon}_{\tau}$$

$$\boldsymbol{\epsilon}_{\tau} = \ln \mathbf{A} \cdot \mathbf{o}_{\tau} + \frac{1}{2} \ln(\mathbf{B}_{\tau}^{\dagger} \mathbf{s}_{\tau}) + \frac{1}{2} \ln(\mathbf{B}_{\tau+1}^{\dagger} \mathbf{s}_{\tau+1}) - \ln \mathbf{s}_{\tau}$$



### Generative model

$$P(\tilde{o}, \tilde{s}) = P(s_1) \prod_{\tau} P(s_{\tau+1} | s_{\tau}) P(o_{\tau} | s_{\tau})$$

$$P(o_{\tau} | s_{\tau}) = \text{Cat}(\mathbf{A})$$

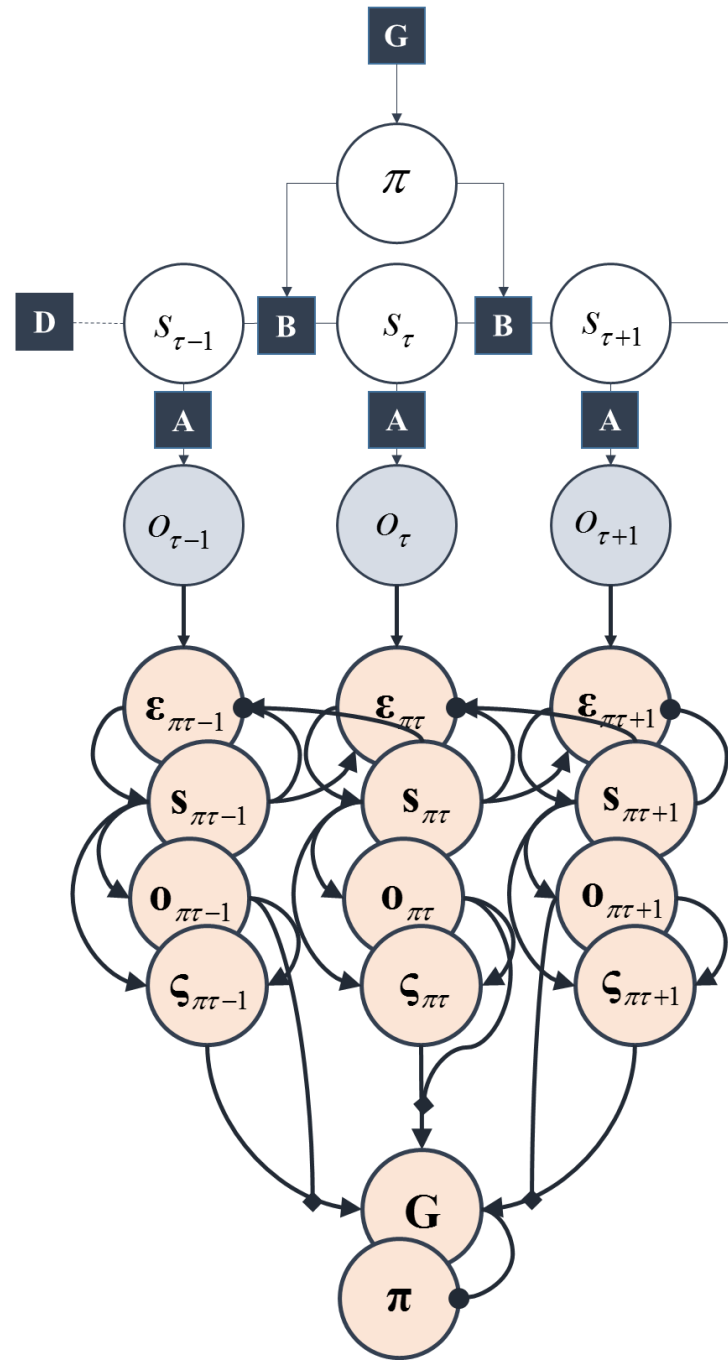
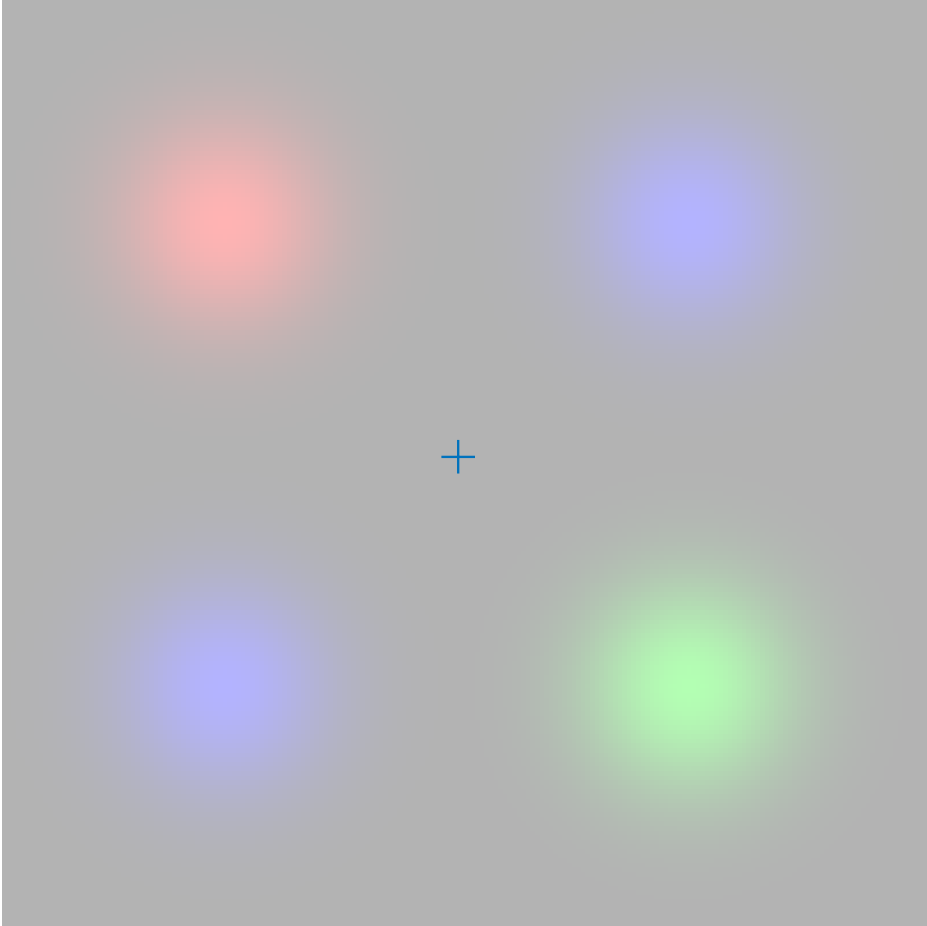
$$P(s_{\tau} | s_{\tau-1}) = \text{Cat}(\mathbf{B})$$

$$P(s_1) = \text{Cat}(\mathbf{D})$$

### Bayesian message passing

$$\mathbf{s}_{\tau} = \sigma(\mathbf{v}_{\tau}); \quad \dot{\mathbf{v}}_{\tau} = \boldsymbol{\varepsilon}_{\tau}$$

$$\boldsymbol{\varepsilon}_{\tau} = \ln \mathbf{A} \cdot o_{\tau} + \frac{1}{2} \ln(\mathbf{B}_{\tau} \mathbf{s}_{\tau}) + \frac{1}{2} \ln(\mathbf{B}_{\tau+1}^{\dagger} \mathbf{s}_{\tau+1}) - \ln \mathbf{s}_{\tau}$$



### Generative model

$$P(\tilde{o}, \tilde{s}, \pi) = P(s_1)P(\pi) \prod_{\tau} P(s_{\tau+1} | s_{\tau}, \pi) P(o_{\tau} | s_{\tau})$$

$$P(o_{\tau} | s_{\tau}) = \text{Cat}(\mathbf{A})$$

$$P(s_{\tau} | s_{\tau-1}, \pi) = \text{Cat}(\mathbf{B}_{\pi\tau})$$

$$P(o_{\tau}) = \text{Cat}(\mathbf{C})$$

$$P(s_1) = \text{Cat}(\mathbf{D})$$

$$P(\pi) = \sigma(-\mathbf{G})$$

### Bayesian message passing

$$\mathbf{s}_{\tau} = \pi \cdot \mathbf{s}_{\pi\tau}$$

$$\mathbf{s}_{\pi\tau} = \sigma(\mathbf{v}_{\pi\tau}); \quad \dot{\mathbf{v}}_{\pi\tau} = \boldsymbol{\varepsilon}_{\pi\tau}$$

$$\boldsymbol{\varepsilon}_{\pi\tau} = \ln \mathbf{A} \cdot \mathbf{o}_{\tau} + \frac{1}{2} \ln(\mathbf{B}_{\pi\tau} \mathbf{s}_{\pi\tau}) + \frac{1}{2} \ln(\mathbf{B}_{\pi\tau+1}^{\dagger} \mathbf{s}_{\pi\tau+1}) - \ln \mathbf{s}_{\pi\tau}$$

$$\mathbf{o}_{\pi\tau} = \mathbf{A} \mathbf{s}_{\pi\tau}$$

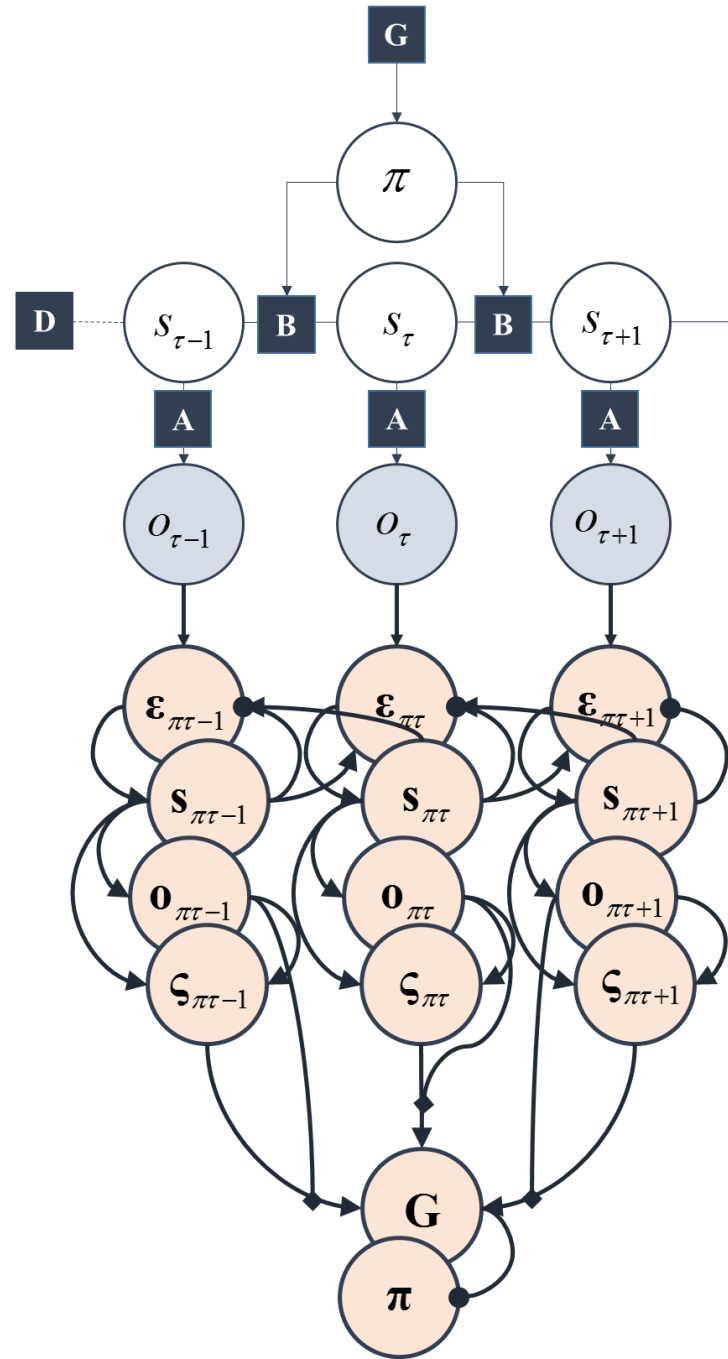
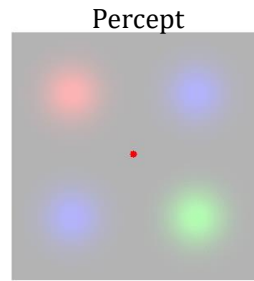
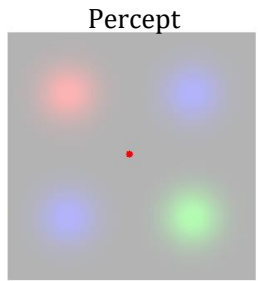
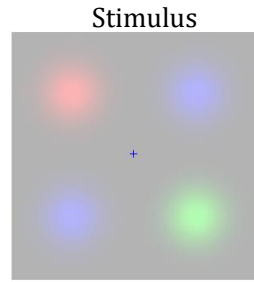
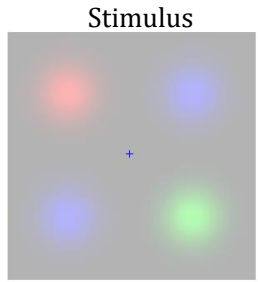
$$\boldsymbol{\varsigma}_{\pi\tau} = \ln \mathbf{o}_{\pi\tau} - \ln \mathbf{C} + \mathbf{H} \cdot \mathbf{s}_{\pi\tau}$$

$$\mathbf{H} = -\text{diag}(\mathbf{A} \cdot \ln \mathbf{A})$$

$$\mathbf{G}_{\pi} = \mathbf{o}_{\pi\tau} \cdot \boldsymbol{\varsigma}_{\pi\tau}$$

$$\pi = \sigma(-\mathbf{G})$$





### Generative model

$$P(\tilde{o}, \tilde{s}, \pi) = P(s_1)P(\pi) \prod_{\tau} P(s_{\tau+1} | s_{\tau}, \pi) P(o_{\tau} | s_{\tau})$$

$$P(o_{\tau} | s_{\tau}) = \text{Cat}(\mathbf{A})$$

$$P(s_{\tau} | s_{\tau-1}, \pi) = \text{Cat}(\mathbf{B}_{\pi\tau})$$

$$P(o_{\tau}) = \text{Cat}(\mathbf{C})$$

$$P(s_1) = \text{Cat}(\mathbf{D})$$

$$P(\pi) = \sigma(-\mathbf{G})$$

### Bayesian message passing

$$\mathbf{s}_{\tau} = \pi \cdot \mathbf{s}_{\pi\tau}$$

$$\mathbf{s}_{\pi\tau} = \sigma(\mathbf{v}_{\pi\tau}); \quad \dot{\mathbf{v}}_{\pi\tau} = \boldsymbol{\varepsilon}_{\pi\tau}$$

$$\boldsymbol{\varepsilon}_{\pi\tau} = \ln \mathbf{A} \cdot \mathbf{o}_{\tau} + \frac{1}{2} \ln(\mathbf{B}_{\pi\tau} \mathbf{s}_{\pi\tau}) + \frac{1}{2} \ln(\mathbf{B}_{\pi\tau+1}^{\dagger} \mathbf{s}_{\pi\tau+1}) - \ln \mathbf{s}_{\pi\tau}$$

$$\mathbf{o}_{\pi\tau} = \mathbf{A} \mathbf{s}_{\pi\tau}$$

$$\boldsymbol{\zeta}_{\pi\tau} = \ln \mathbf{o}_{\pi\tau} - \ln \mathbf{C} + \mathbf{H} \cdot \mathbf{s}_{\pi\tau}$$

$$\mathbf{H} = -\text{diag}(\mathbf{A} \cdot \ln \mathbf{A})$$

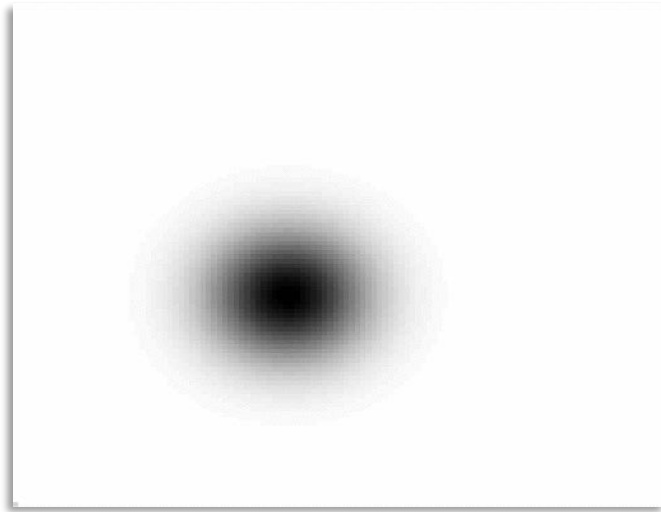
$$\mathbf{G}_{\pi} = \mathbf{o}_{\pi\tau} \cdot \boldsymbol{\zeta}_{\pi\tau}$$

$$\pi = \sigma(-\mathbf{G})$$



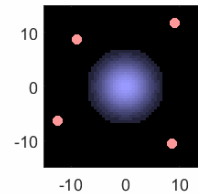
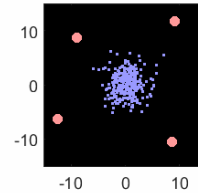
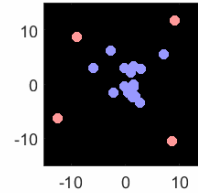
Active inference  
Generative models  
**Exploitation**  
Exploration  
Movement  
Hierarchy

## Goals and steady state



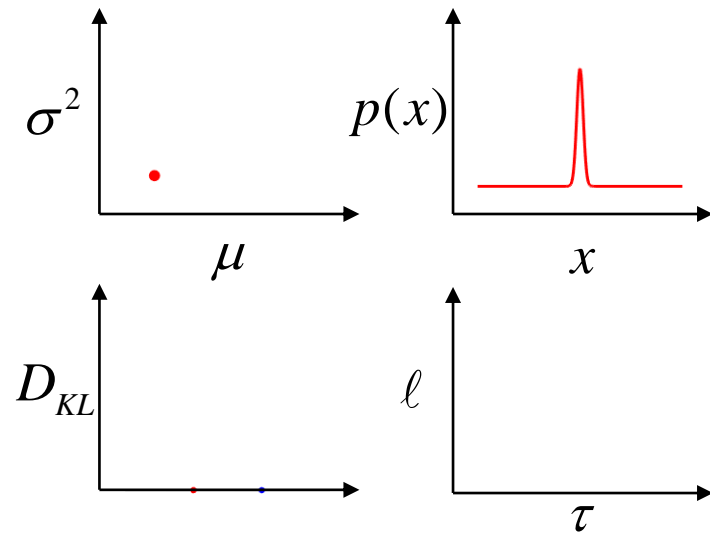
$$D_{KL} [P(o | \pi) || P(o | C)]$$

# Goals and steady state



$$D_{KL} \left[ P(o | \pi) \parallel P(o | C) \right]$$

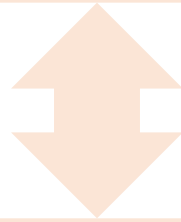
# Goals and steady state



$$D_{KL} \left[ P(o | \pi) \parallel P(o | C) \right]$$

## Goals and steady state

$$-H[P(o|\pi)] - \mathbb{E}_{P(o|\pi)}[\ln P(o|C)]$$



$$D_{KL}[P(o|\pi) \parallel P(o|C)]$$



Active inference  
Generative models  
Exploitation  
**Exploration**  
Movement  
Hierarchy

## Information gain

$$\mathcal{I}(\pi) = D_{KL} \left[ P(o, s | \pi) \parallel P(o | \pi) P(s | \pi) \right]$$

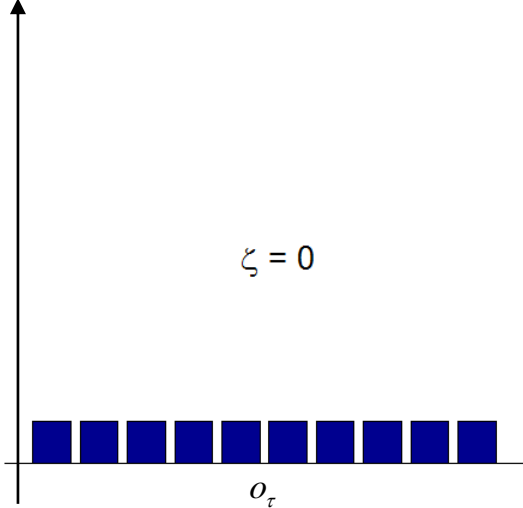
$$= \mathbb{E}_{P(o|\pi)} \left[ D_{KL} \left[ P(s | o, \pi) \parallel P(s | \pi) \right] \right]$$

$$= H \left[ P(o | \pi) \right] - \mathbb{E}_{P(s|\pi)} \left[ H \left[ P(o | s) \right] \right]$$

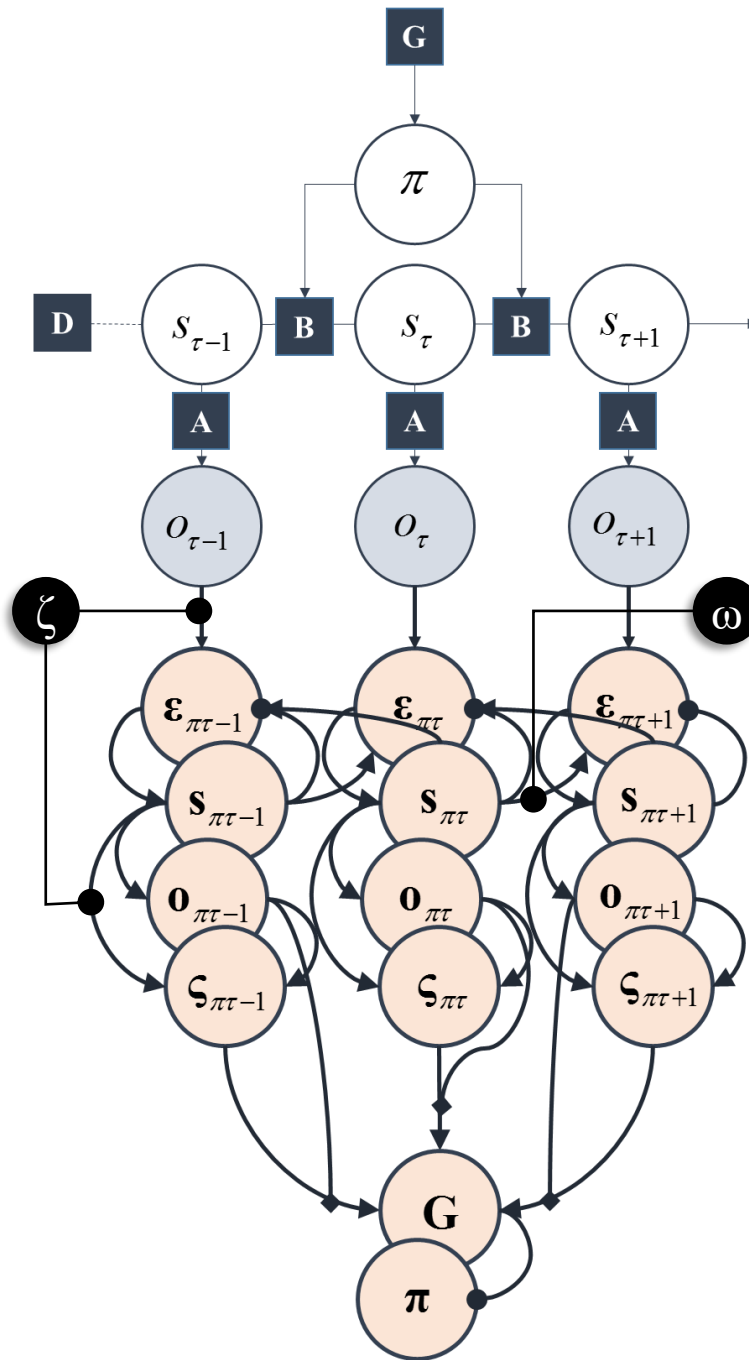
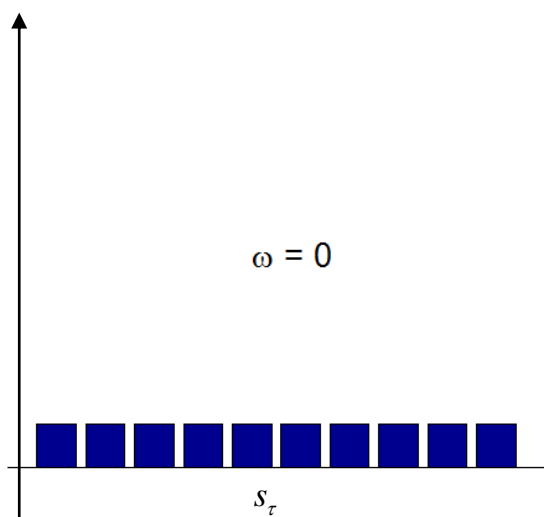
$$P(o, s | \pi) = P(s | o, \pi) P(o | \pi) = P(o | s) P(s | \pi)$$



$$P(o_\tau | s_\tau = i)$$



$$P(s_{\tau+1} | s_\tau = i, \pi)$$



### Generative model

$$P(\tilde{o}, \tilde{s}, \pi) = P(s_1)P(\pi) \prod_{\tau} P(s_{\tau+1} | s_{\tau}, \pi) P(o_{\tau} | s_{\tau})$$

$$P(o_{\tau} | s_{\tau}) = \text{Cat}(\mathbf{A})$$

$$P(s_{\tau} | s_{\tau-1}, \pi) = \text{Cat}(\mathbf{B}_{\pi\tau})$$

$$P(o_{\tau}) = \text{Cat}(\mathbf{C})$$

$$P(s_1) = \text{Cat}(\mathbf{D})$$

$$P(\pi) = \sigma(-\mathbf{G})$$

### Bayesian message passing

$$\mathbf{s}_{\tau} = \pi \cdot \mathbf{s}_{\pi\tau}$$

$$\mathbf{s}_{\pi\tau} = \sigma(\mathbf{v}_{\pi\tau}); \quad \dot{\mathbf{v}}_{\pi\tau} = \mathbf{e}_{\pi\tau}$$

$$\mathbf{e}_{\pi\tau} = \ln \mathbf{A} \cdot \mathbf{o}_{\tau} + \frac{1}{2} \ln(\mathbf{B}_{\pi\tau} \cdot \mathbf{s}_{\pi\tau}) + \frac{1}{2} \ln(\mathbf{B}_{\pi\tau+1}^{\dagger} \cdot \mathbf{s}_{\pi\tau+1}) - \ln \mathbf{s}_{\pi\tau}$$

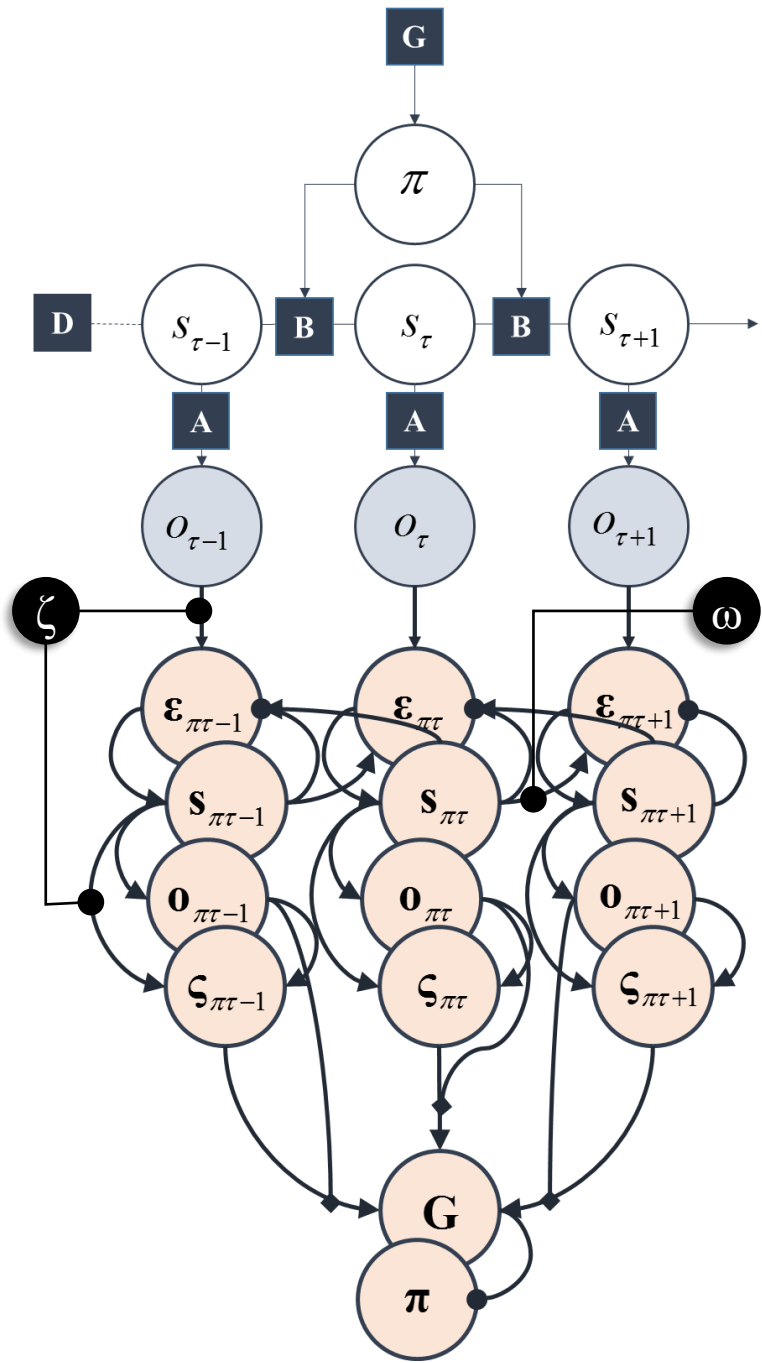
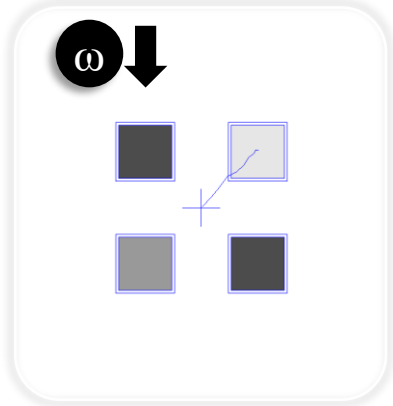
$$\mathbf{o}_{\pi\tau} = \mathbf{A} \mathbf{s}_{\pi\tau}$$

$$\mathbf{z}_{\pi\tau} = \ln \mathbf{o}_{\pi\tau} - \ln \mathbf{C} + \mathbf{H} \cdot \mathbf{s}_{\pi\tau}$$

$$\mathbf{H} = -\text{diag}(\mathbf{A} \cdot \ln \mathbf{A})$$

$$\mathbf{G}_{\pi} = \mathbf{o}_{\pi\tau} \cdot \mathbf{z}_{\pi\tau}$$

$$\pi = \sigma(-\mathbf{G})$$



## Generative model

$$P(\tilde{o}, \tilde{s}, \pi) = P(s_1)P(\pi) \prod_{\tau} P(s_{\tau+1} | s_{\tau}, \pi) P(o_{\tau} | s_{\tau})$$

$$P(o_\tau | s_\tau) = \text{Cat}(\mathbf{A})$$

$$P(s_\tau \mid s_{\tau-1}, \pi) = Cat(\mathbf{B}_{\pi\tau})$$

$$P(o_\tau) = Cat(\mathbf{C})$$

$$P(s_1) = Cat(\mathbf{D})$$

$$P(\pi) = \sigma(-\mathbf{G})$$

## Bayesian message passing

$$\mathbf{S}_\tau = \boldsymbol{\pi} \cdot \mathbf{S}_{\pi\tau}$$

$$\mathbf{s}_{\pi\tau} = \sigma(\mathbf{v}_{\pi\tau}); \dot{\mathbf{v}}_{\pi\tau} = \boldsymbol{\varepsilon}_{\pi\tau}$$

$$\varepsilon_{\pi\tau} = \ln \mathbf{A} \cdot o_{\tau} + \frac{1}{2} \ln(\mathbf{B}_{\pi\tau} \mathbf{s}_{\pi\tau}) + \frac{1}{2} \ln(\mathbf{B}_{\pi\tau+1}^{\dagger} \mathbf{s}_{\pi\tau+1}) - \ln \mathbf{s}_{\pi\tau}$$

$$\mathbf{o}_{\pi\tau} = \mathbf{A}\mathbf{s}_{\pi\tau}$$

$$\mathfrak{s}_{\pi\tau} = \ln \mathbf{o}_{\pi\tau} - \ln \mathbf{C} + \mathbf{H} \cdot \mathbf{s}_{\pi\tau}$$

$$\mathbf{H} = -diag(\mathbf{A} \cdot \ln \mathbf{A})$$

$$\mathbf{G}_{\pi} = \mathbf{0}_{\pi\tau} \cdot \mathbf{S}_{\pi\tau}$$

$$\pi = \sigma(-\mathbf{G})$$

## Expected free energy

Explore

$$H[P(o|\pi)] - \mathbb{E}_{P(s|\pi)}[H[P(o|s)]]$$

$$\mathbb{E}_{P(o|\pi)}[\ln P(o|C)] + H[P(o|\pi)] - \mathbb{E}_{P(s|\pi)}[H[P(o|s)]]$$

$$\mathbb{E}_{P(o|\pi)}[\ln P(o|C)] + H[P(o|\pi)]$$

Exploit

## Expected free energy

$$P(\pi) = \sigma[-G(\pi)]$$

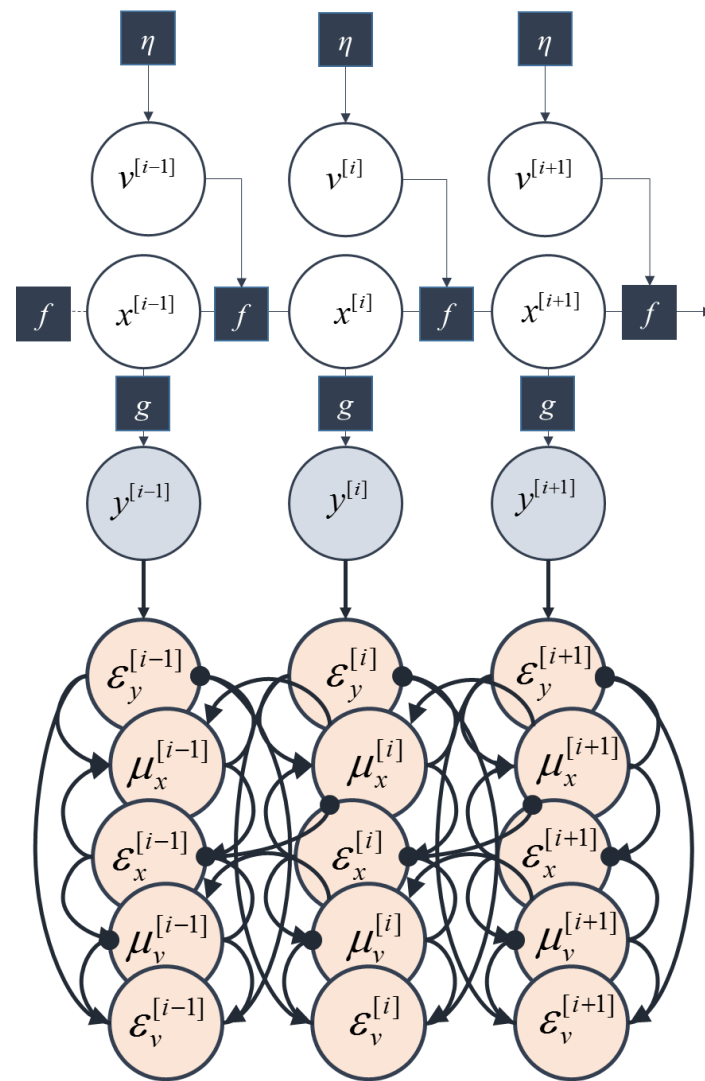
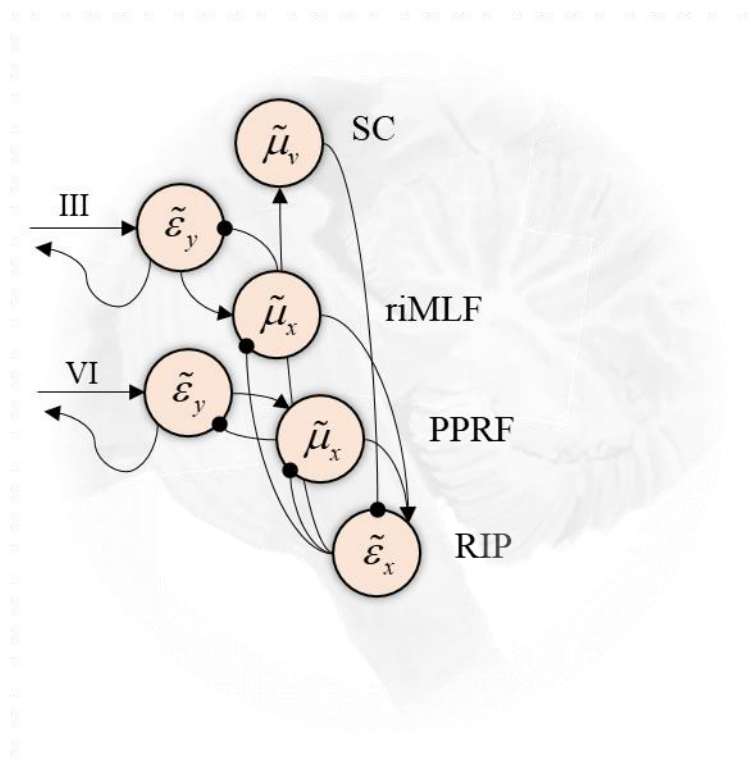
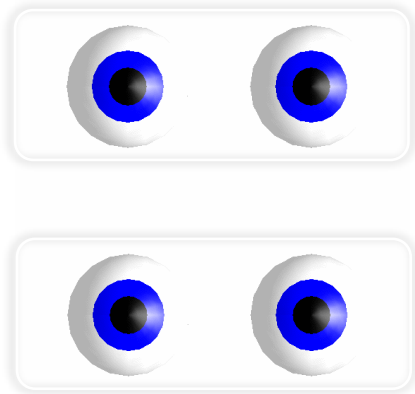
$$-G(\pi) = \mathbb{E}_{P(o|\pi)} [\ln P(o|C)] + H[P(o|\pi)] - \mathbb{E}_{P(s|\pi)} [H[P(o|s)]]$$

$$= \mathbb{E}_{P(o|\pi)} [\ln P(o|C)] + \mathbb{E}_{P(o,s|\pi)} [\ln P(s|\pi, o) - \ln P(s|\pi)]$$

$$-F(\pi) = \ln P(o) + \mathbb{E}_{Q(s|\pi)} [\ln P(s|\pi, o) - \ln Q(s|\pi)]$$



Active inference  
Generative models  
Exploitation  
Exploration  
**Movement**  
Hierarchy



### Generative model

$$p(\tilde{y}, \tilde{x}, \tilde{v}) = \prod_i p(v^{[i]}) p(x^{[i+1]} | x^{[i]}, v^{[i]}) p(y^{[i]} | x^{[i]}, v^{[i]})$$

$$p(y^{[i]} | x^{[i]}, v^{[i]}) = \mathcal{N}(g^{[i]}(x^{[i]}, v^{[i]}), \Pi_y^{[i]})$$

$$p(x^{[i+1]} | x^{[i]}, v^{[i]}) = \mathcal{N}(f^{[i]}(x^{[i]}, v^{[i]}), \Pi_x^{[i]})$$

$$p(v^{[i]}) = \mathcal{N}(\eta^{[i]}, \Pi_v^{[i]})$$

### Bayesian message passing

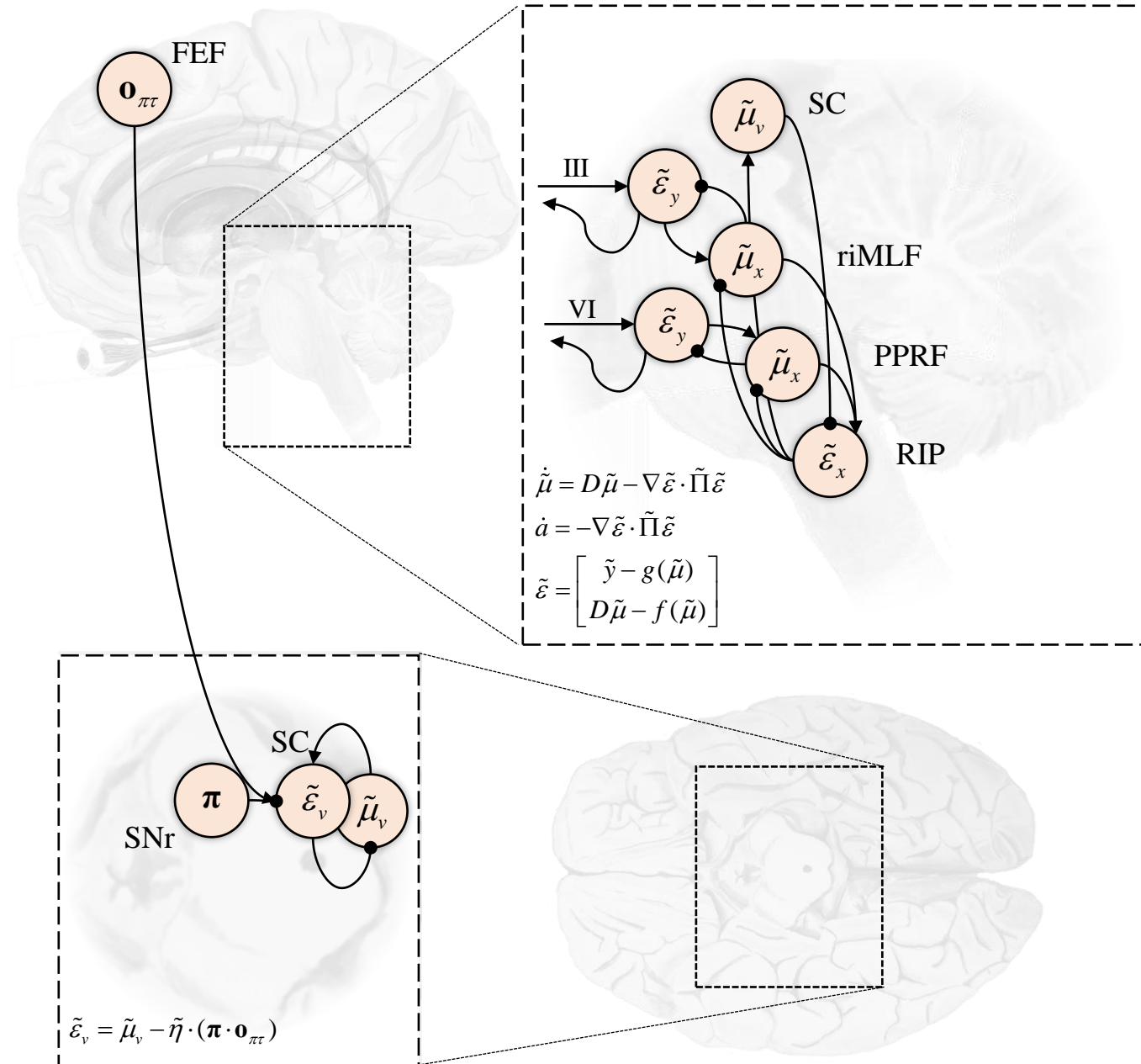
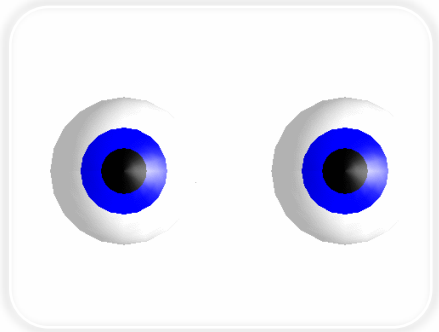
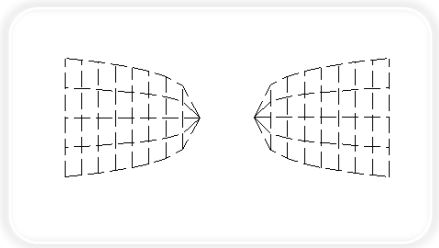
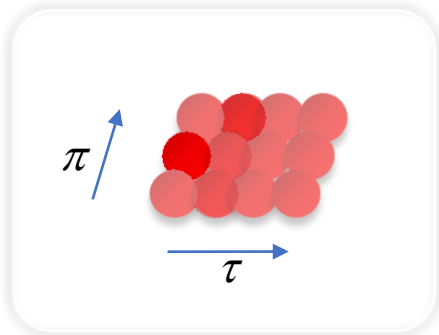
$$\epsilon_y^{[i]} = y^{[i]} - g^{[i]}(\mu_x^{[i]}, \mu_v^{[i]})$$

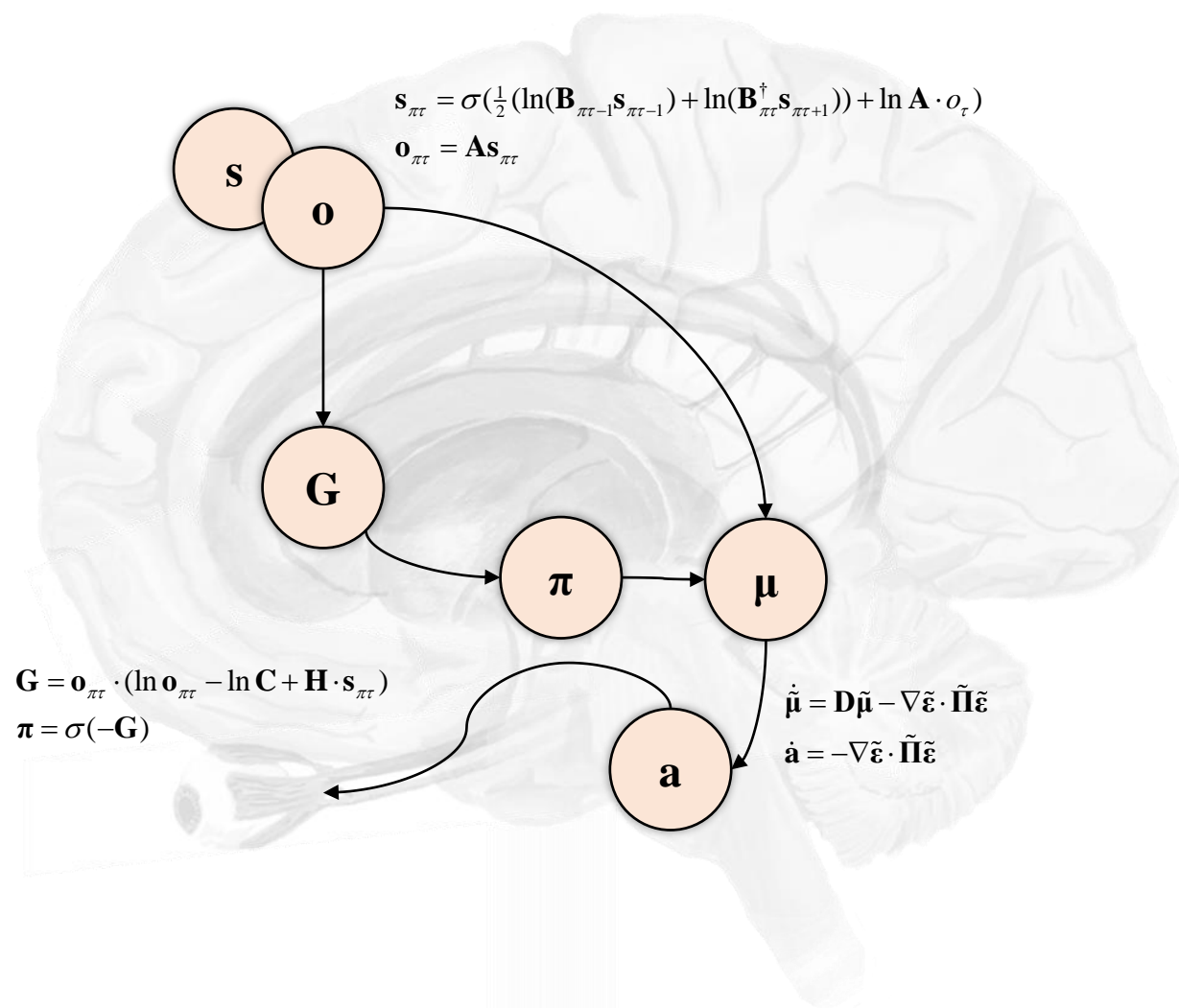
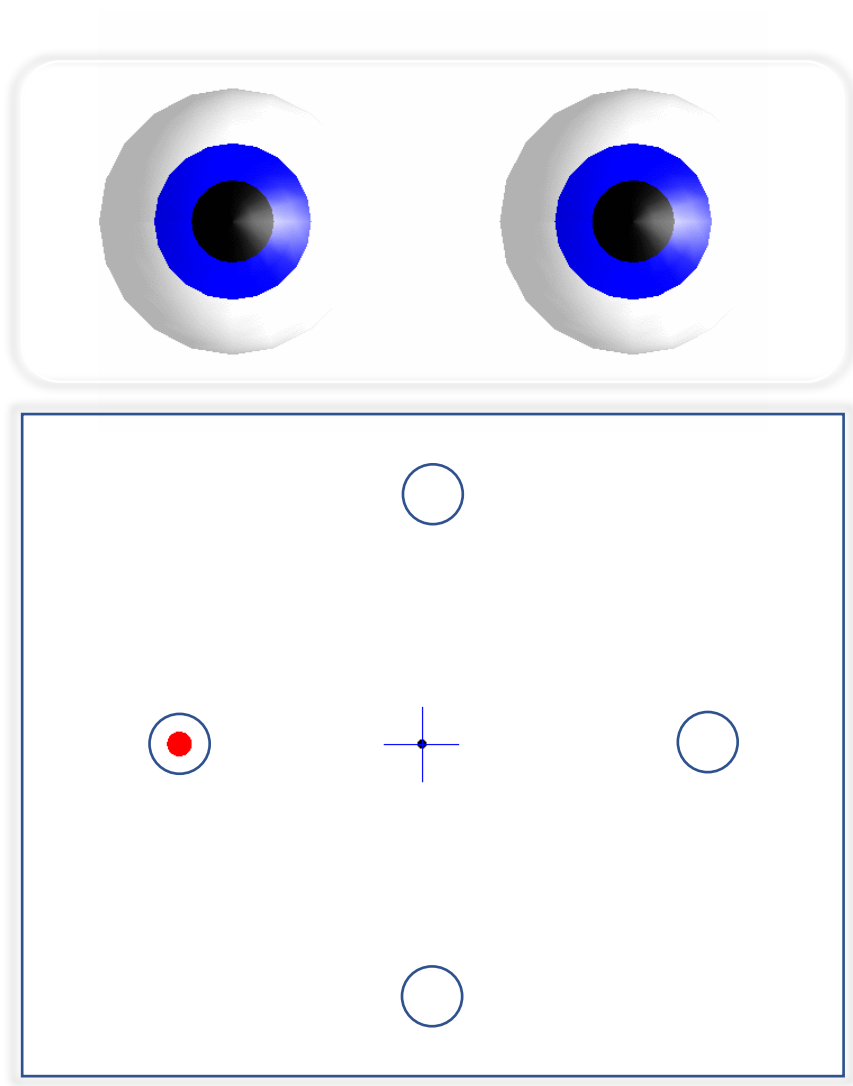
$$\epsilon_x^{[i]} = \mu_x^{[i+1]} - f^{[i]}(\mu_x^{[i]}, \mu_v^{[i]})$$

$$\epsilon_v^{[i]} = \mu_v^{[i]} - \eta^{[i]}$$

$$\begin{aligned} \dot{\mu}_x^{[i]} &= \mu_x^{[i+1]} \\ &\quad + \partial_{\mu_x^{[i]}} g^{[i]} \cdot \Pi_y^{[i]} \epsilon_y^{[i]} - \Pi_x^{[i-1]} \epsilon_x^{[i-1]} + \partial_{\mu_x^{[i]}} f^{[i]} \cdot \Pi_x^{[i]} \epsilon_x^{[i]} \end{aligned}$$

$$\begin{aligned} \dot{\mu}_v^{[i]} &= \mu_v^{[i+1]} \\ &\quad + \partial_{\mu_v^{[i]}} g^{[i]} \cdot \Pi_y^{[i]} \epsilon_y^{[i]} + \partial_{\mu_v^{[i]}} f^{[i]} \cdot \Pi_x^{[i]} \epsilon_x^{[i]} - \Pi_v^{[i]} \epsilon_v^{[i]} \end{aligned}$$

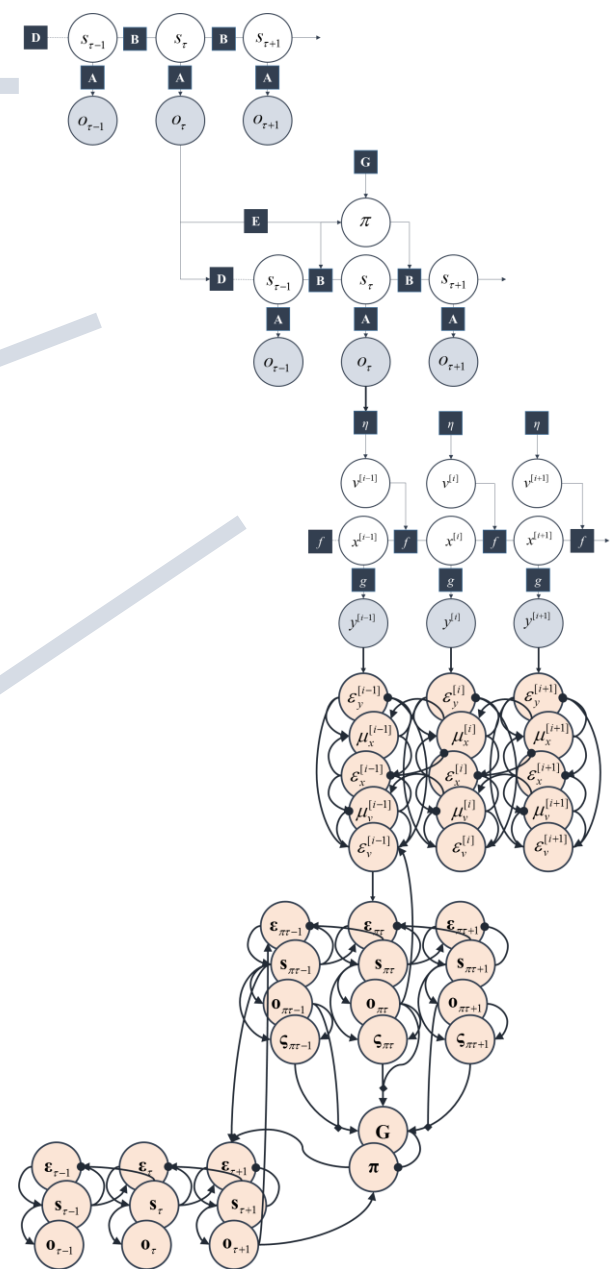
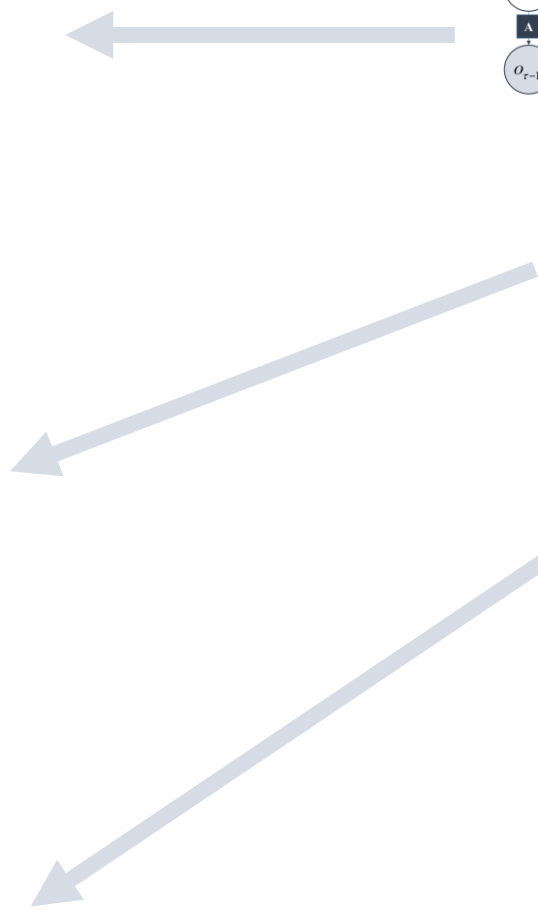
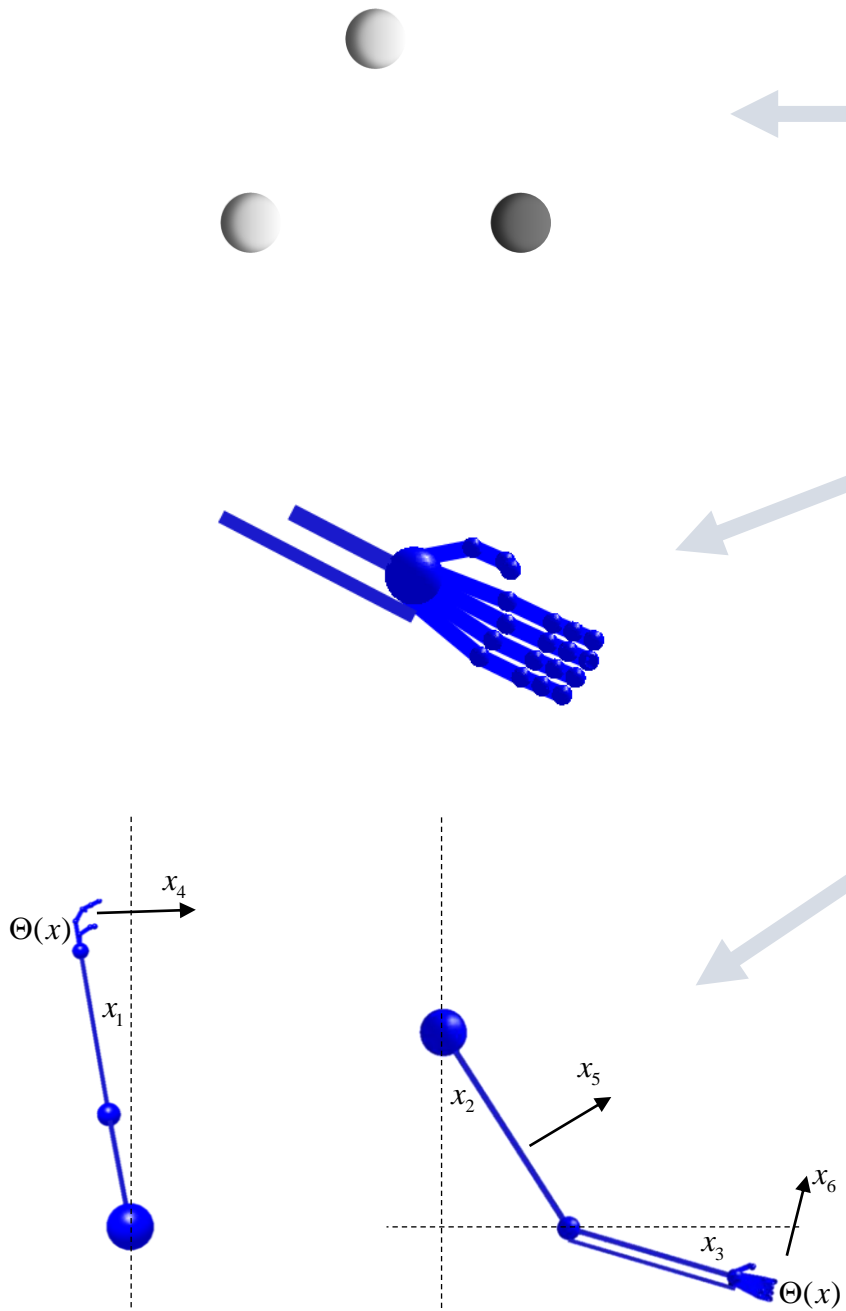


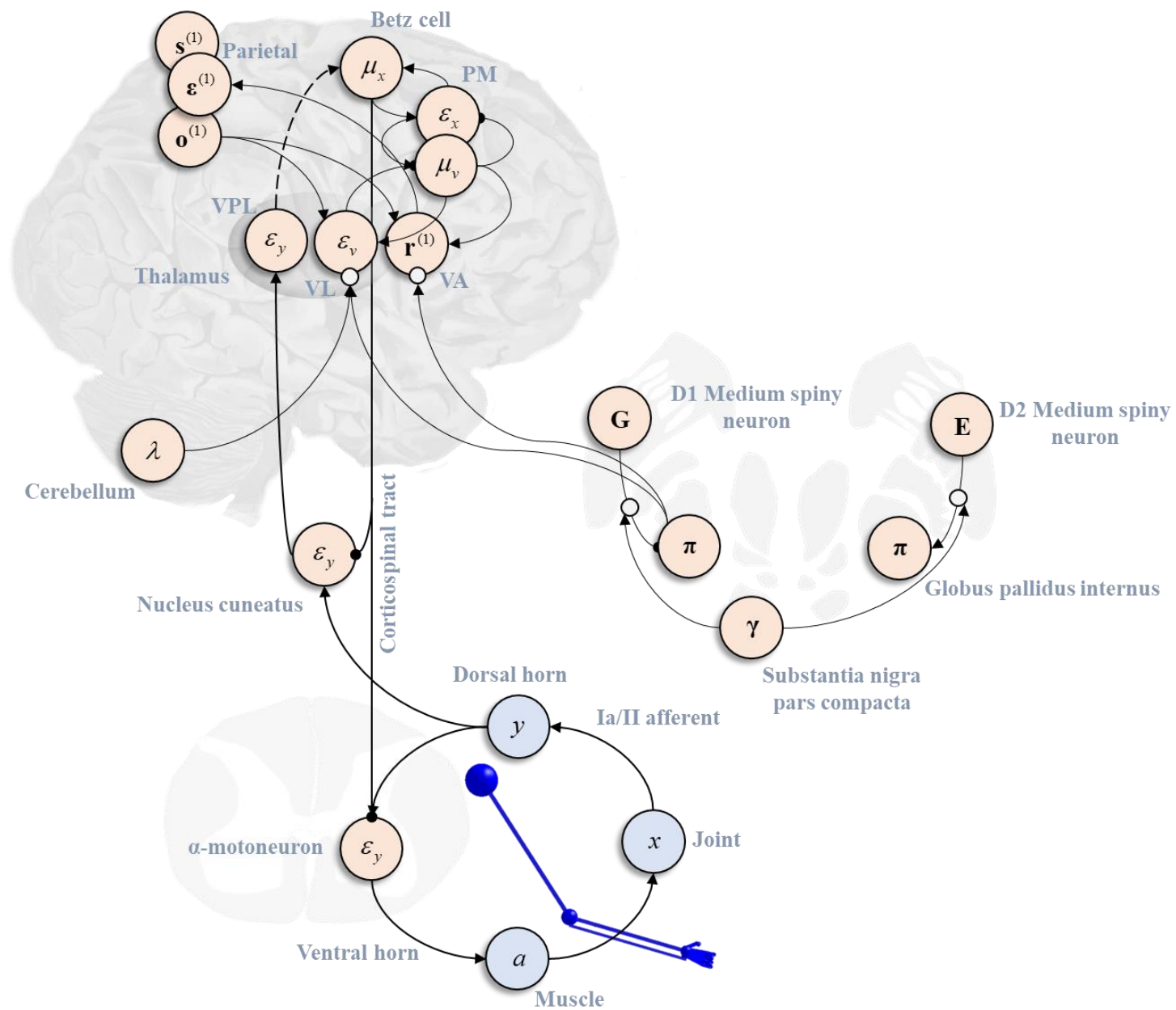
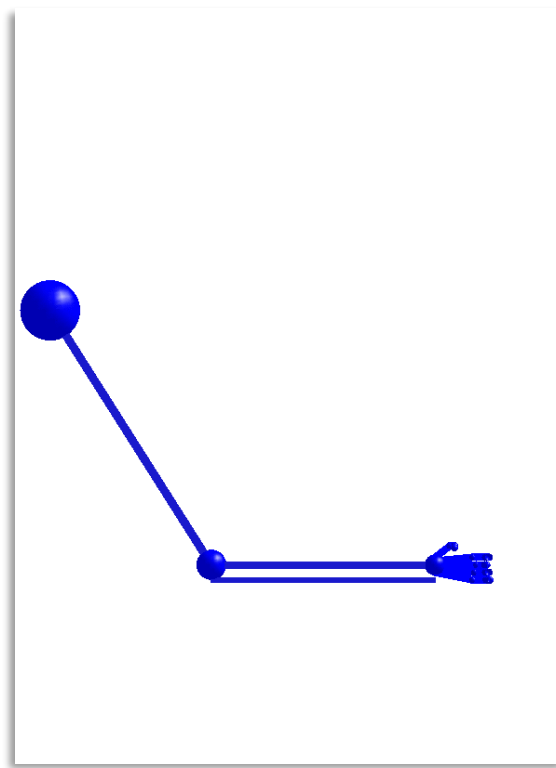


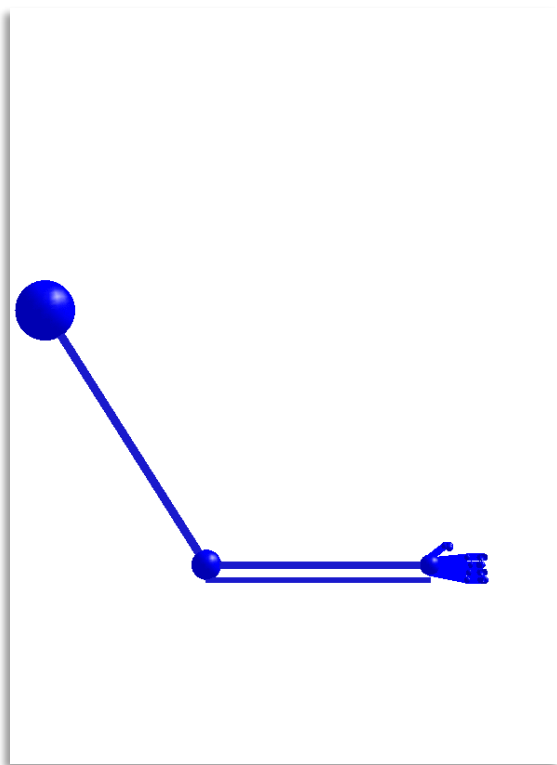


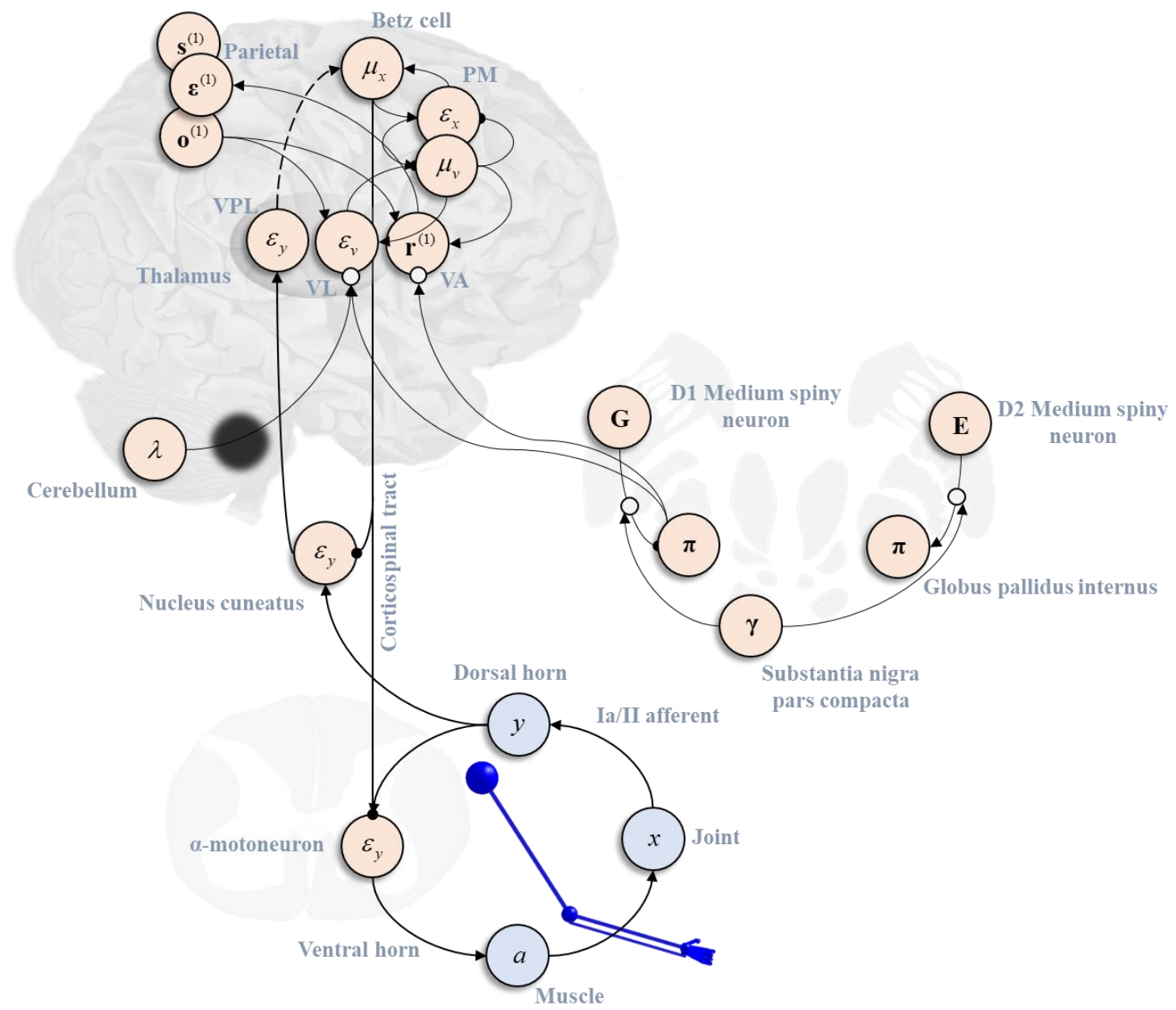
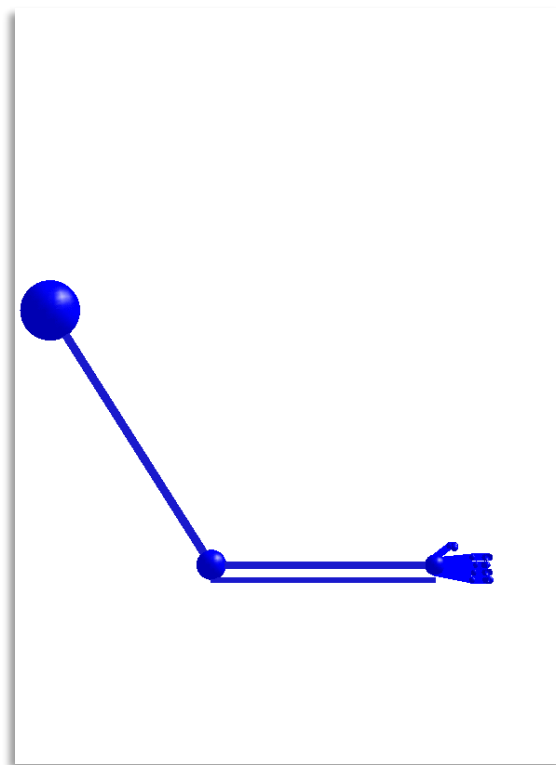


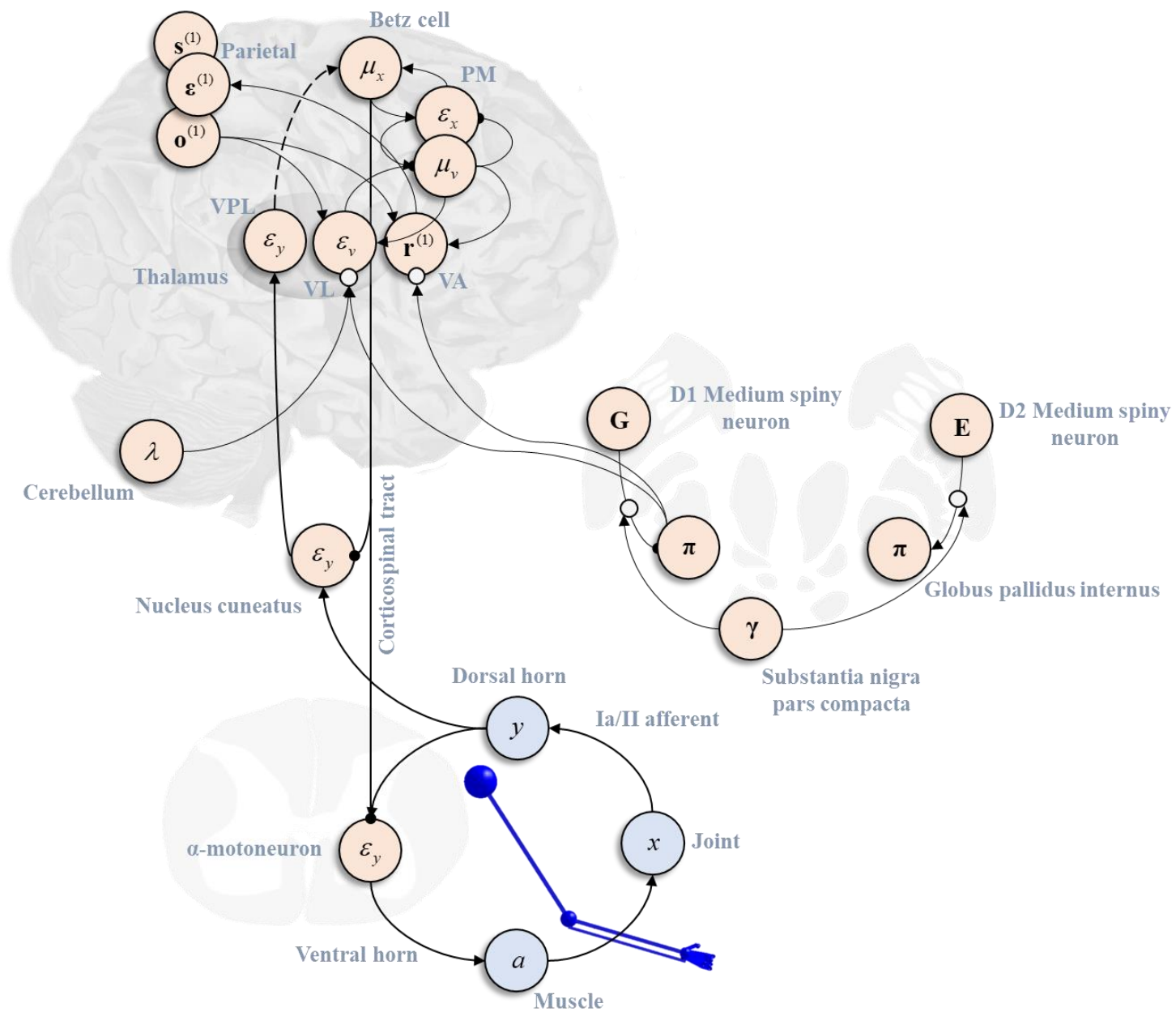
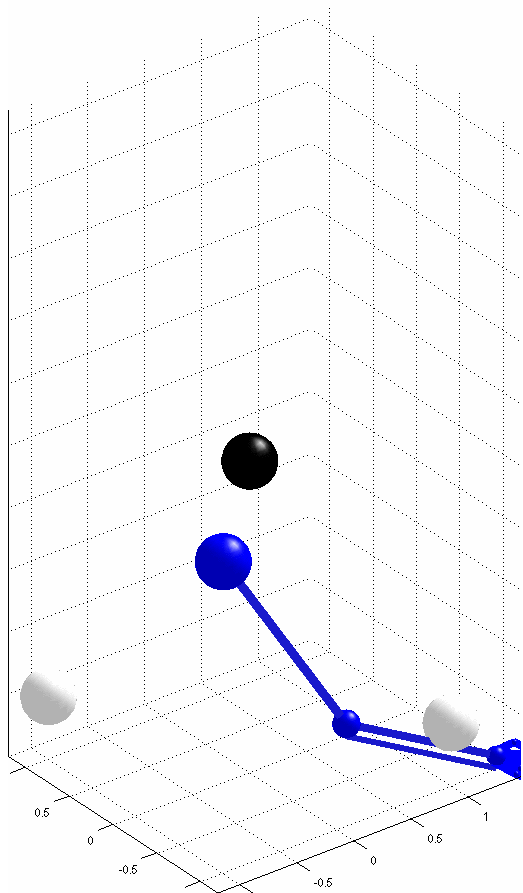
Active inference  
Generative models  
Exploitation  
Exploration  
Movement  
**Hierarchy**



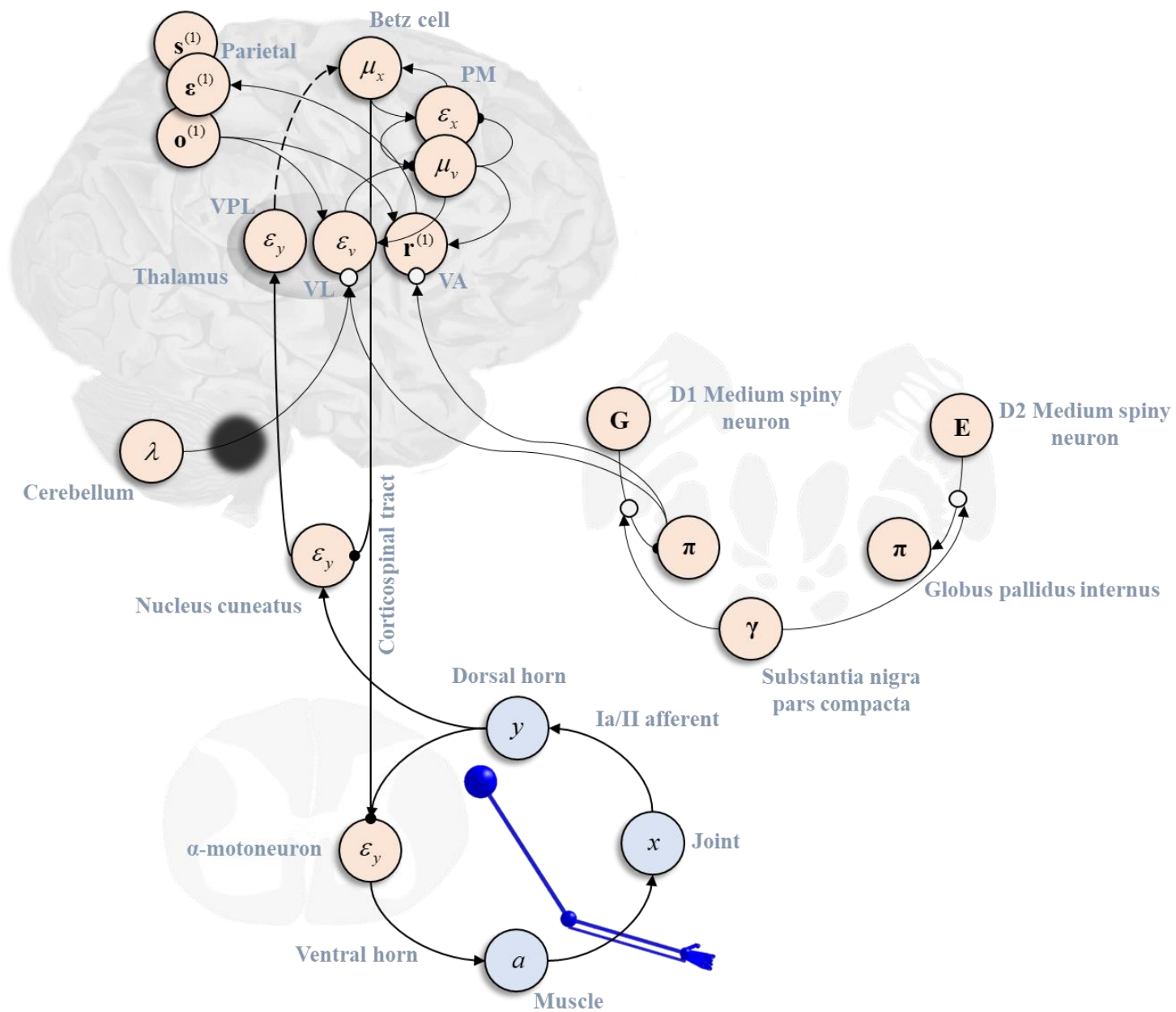
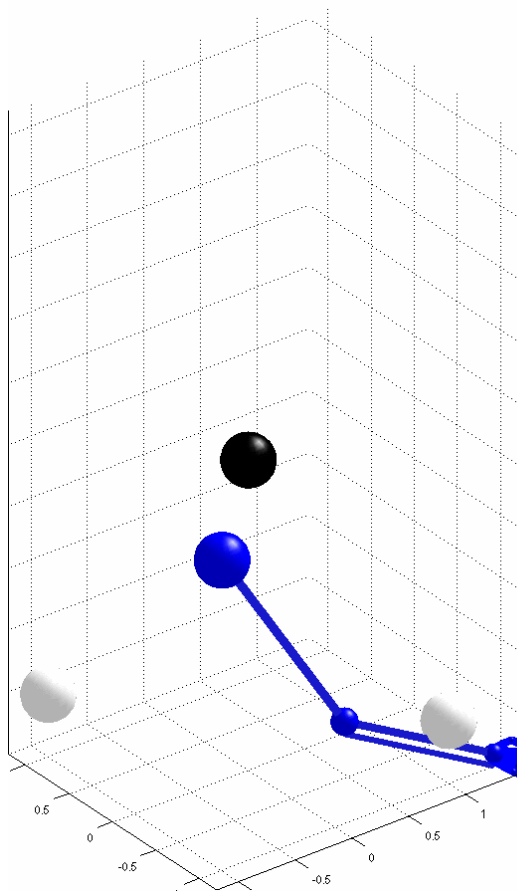


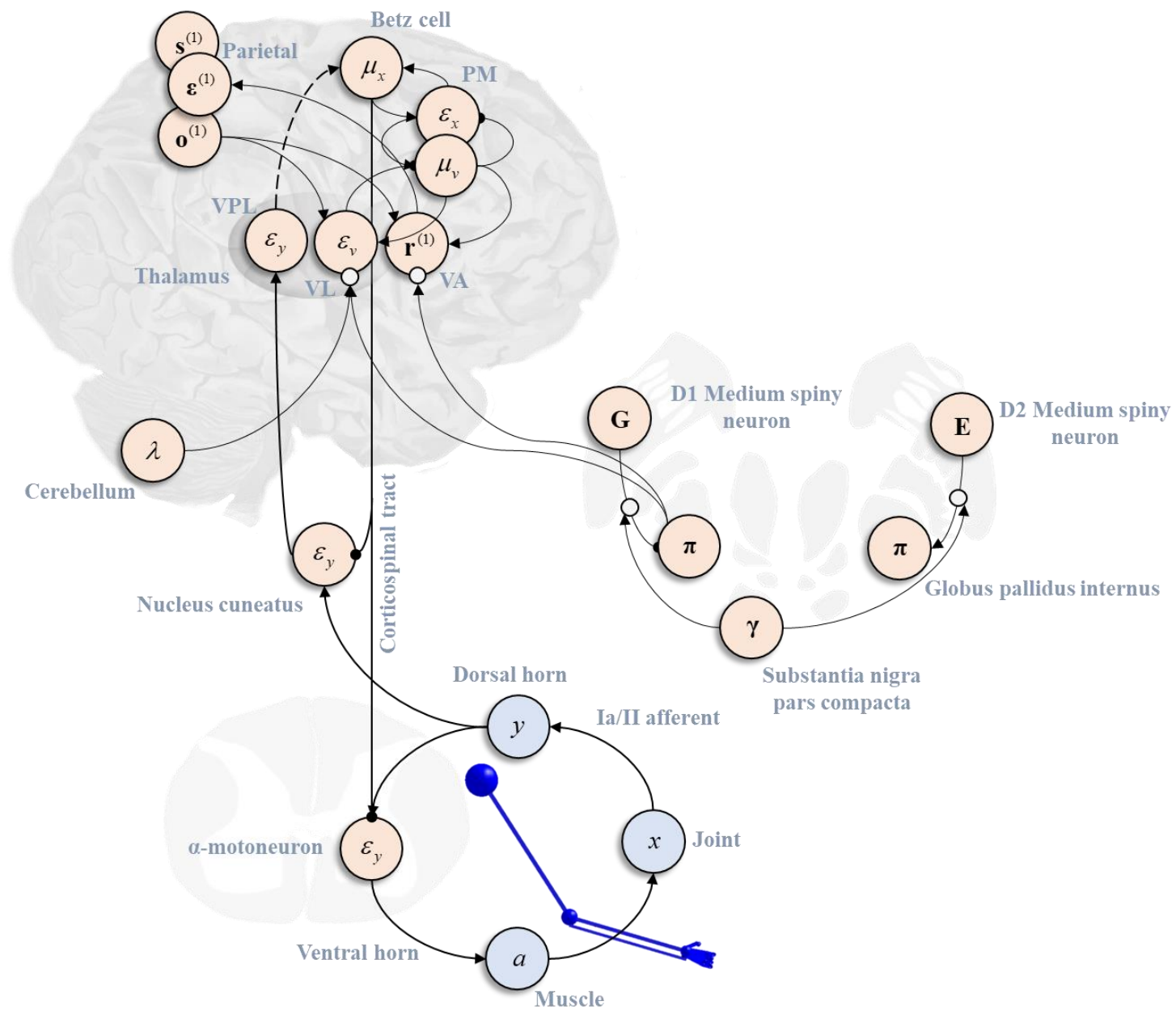




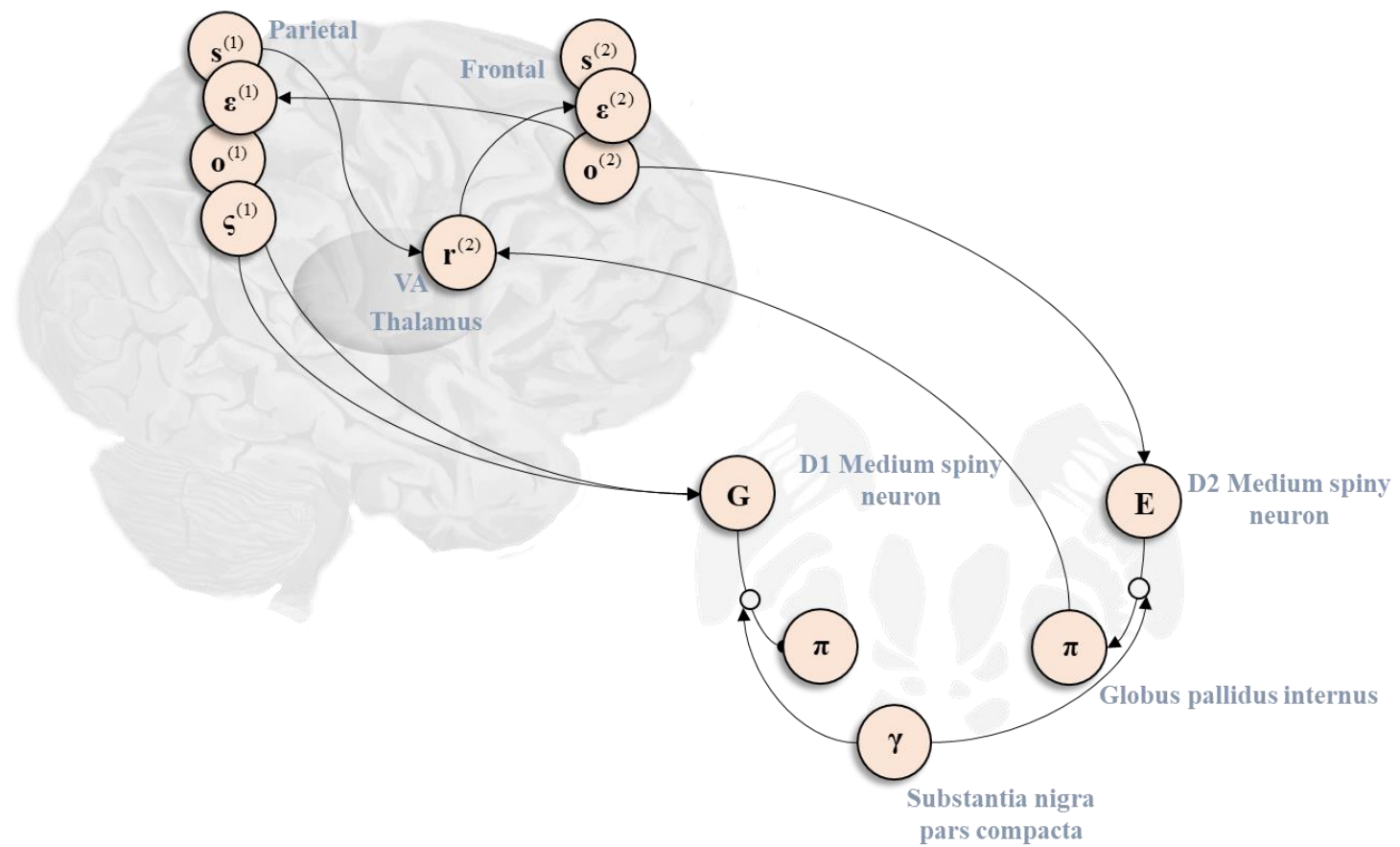


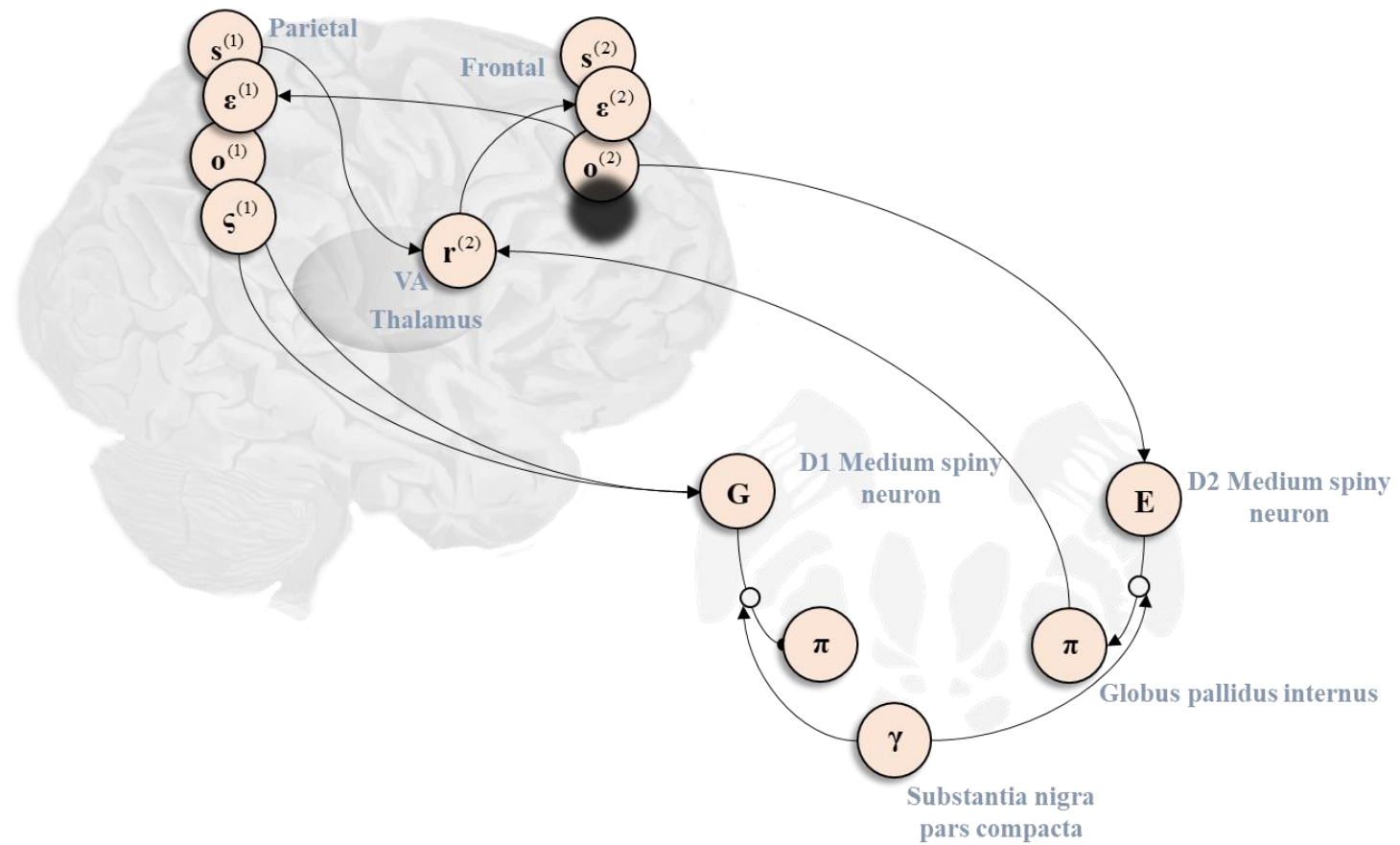
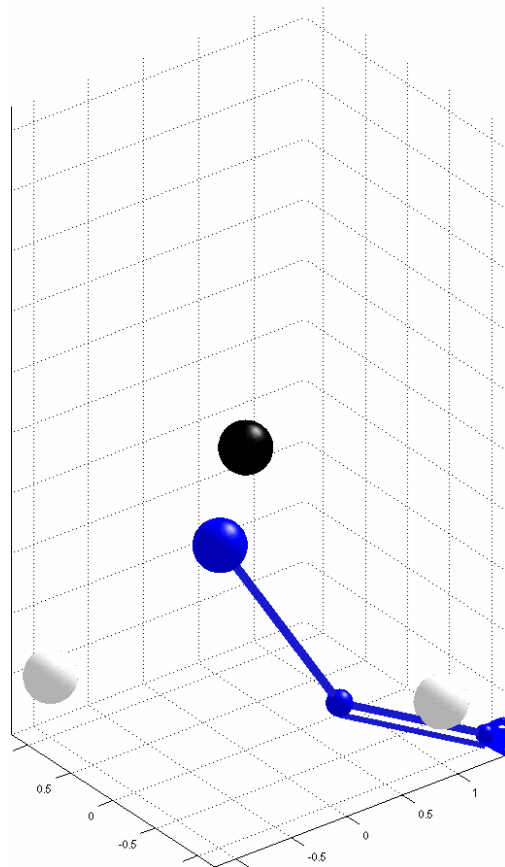


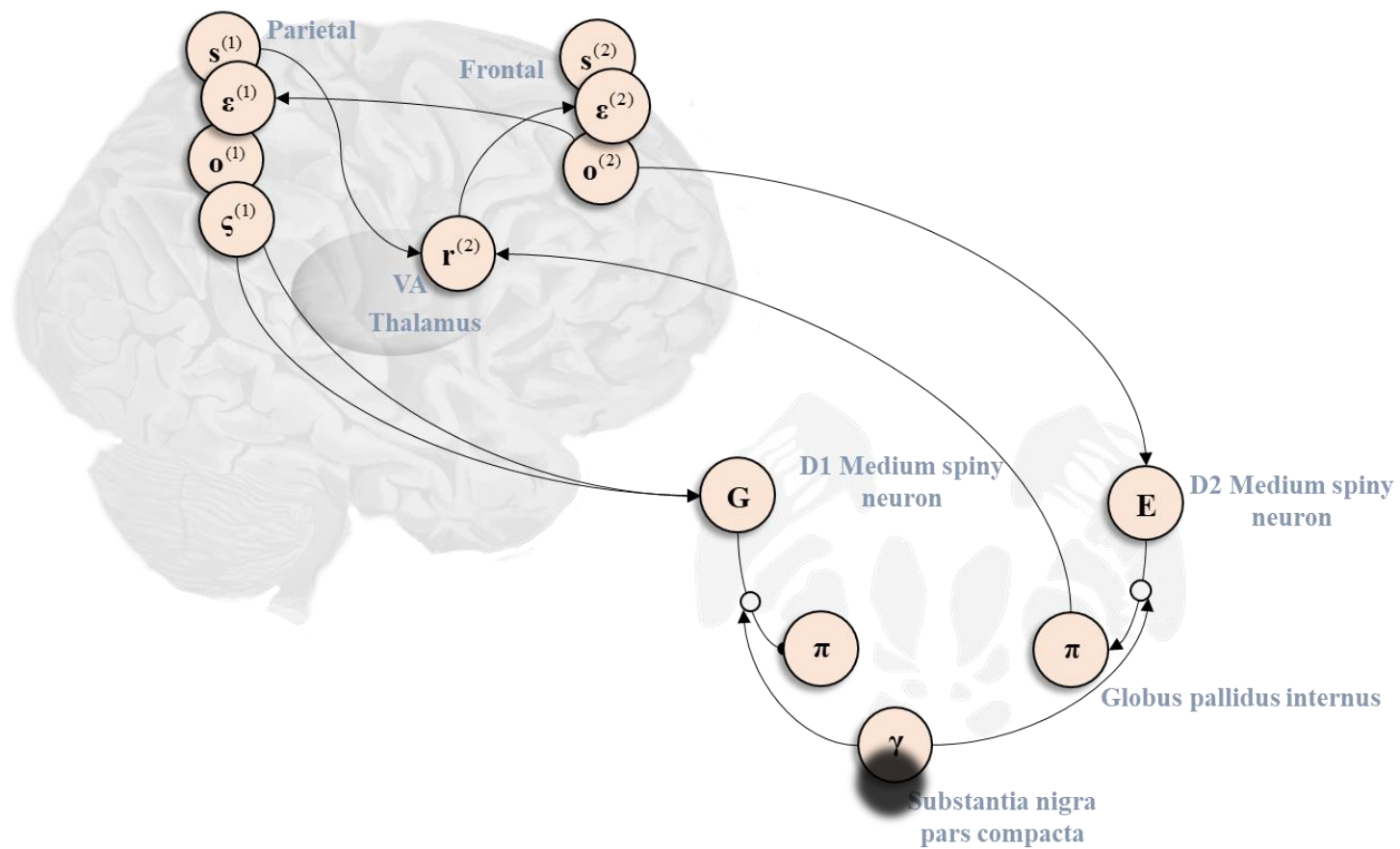
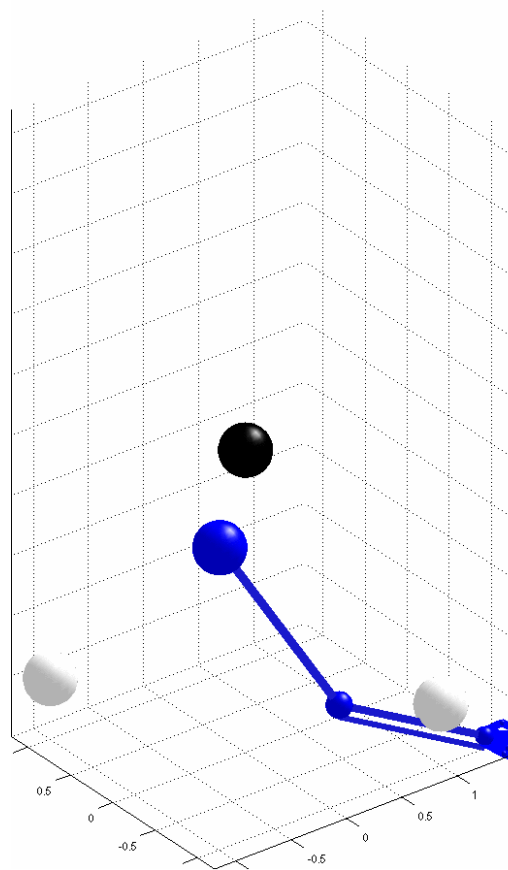




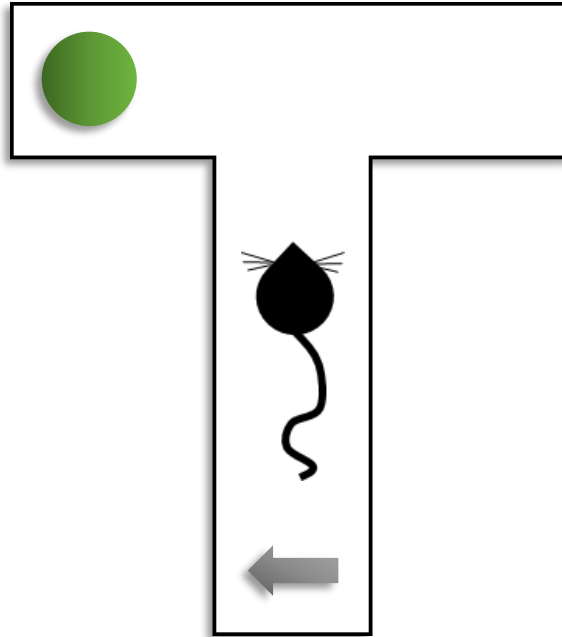














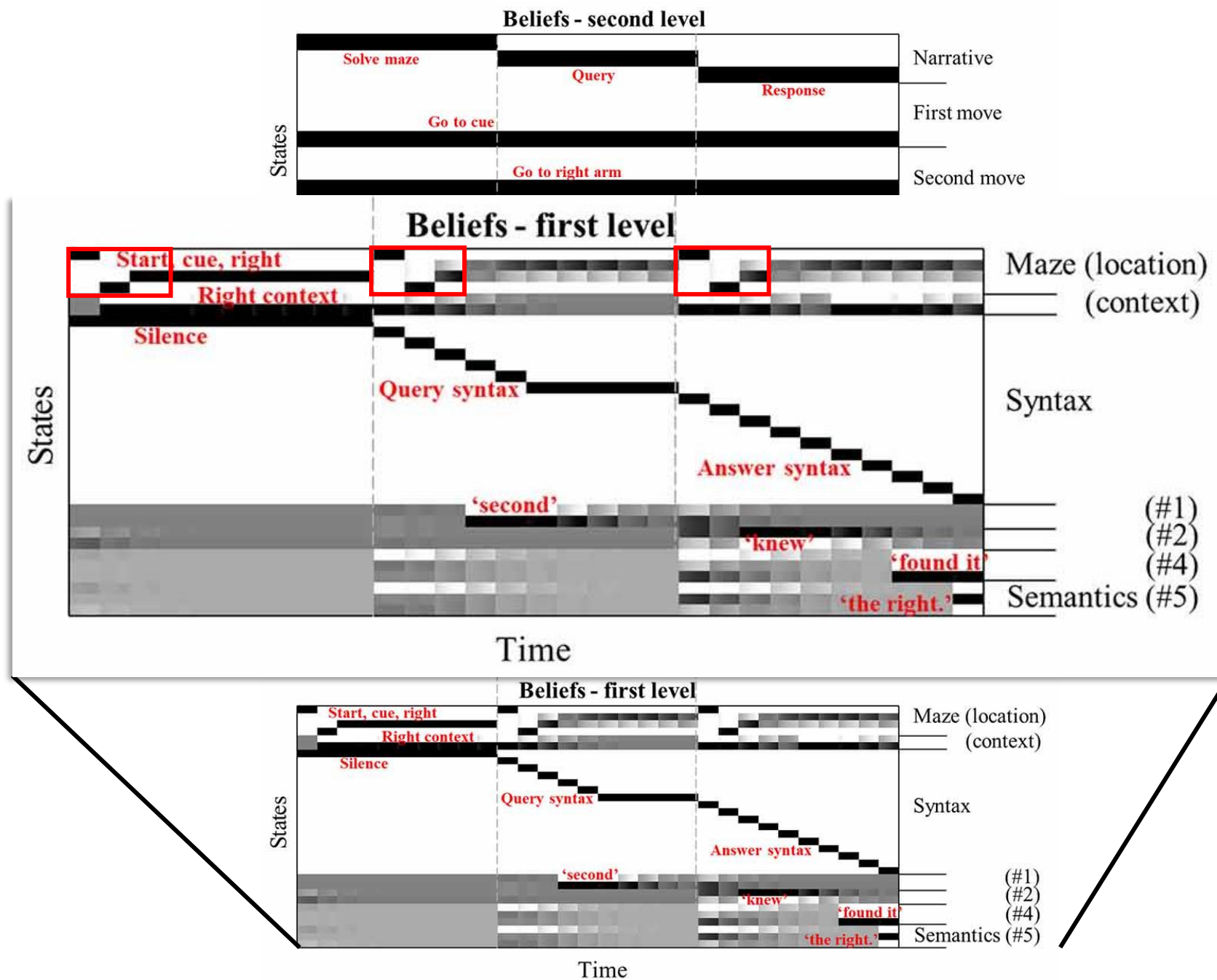
# Navigating a T-maze








-  Inferred location
-  True location
-  Inferred context

-  Inferred location
-  True location
-  Inferred context

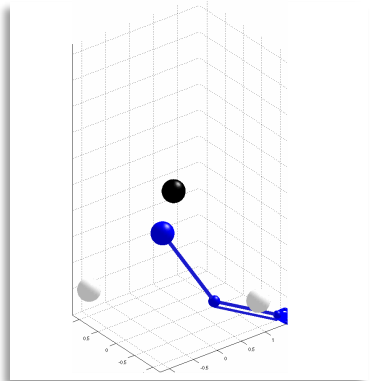
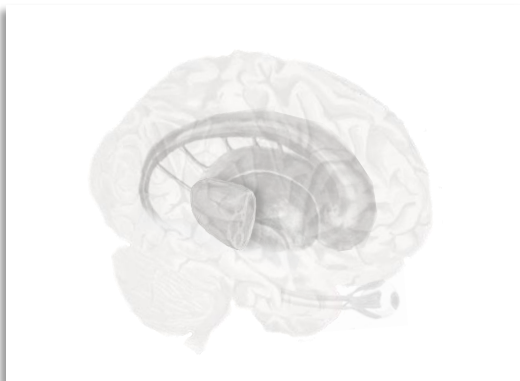
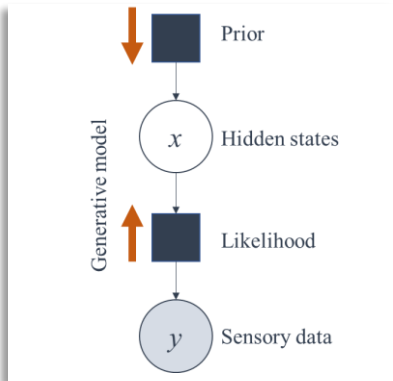
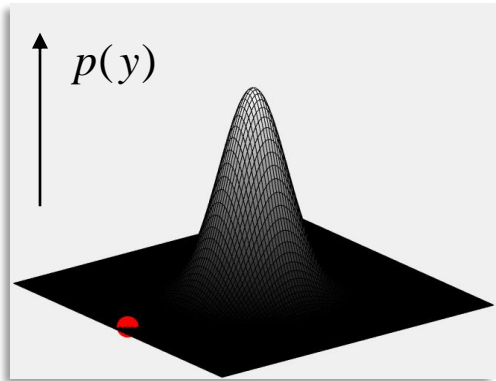
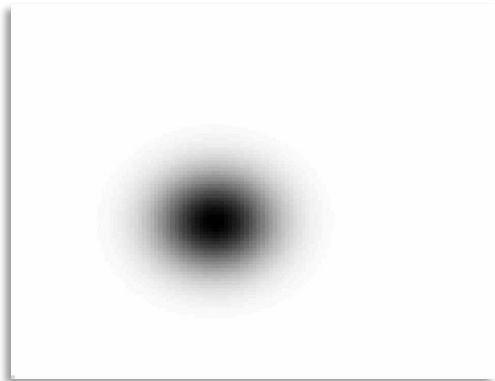




-  Inferred location
-  True location
-  Inferred context

Confabulation







## Thanks

Berk Mirza  
David Benrimoh  
Dimitrije Markovic  
Emma Holmes  
Giovanni Pezzulo  
Jakub Limanowski  
Jakob Hohwy  
Jelle Bruineberg  
Karl Friston  
Lance Da Costa  
Noor Sajid  
Peter Vincent  
Rick Adams  
Stefan Kiebel  
Vishal Rawji  
And many others



Rosetrees Trust

Supporting the best in medical research



UCL



wellcome  
centre  
human  
neuroimaging