



Universität
Zürich ^{UZH}

ETH zürich



Translational Neuromodeling Unit

Predictive Coding

Computational Psychiatry Course Zurich

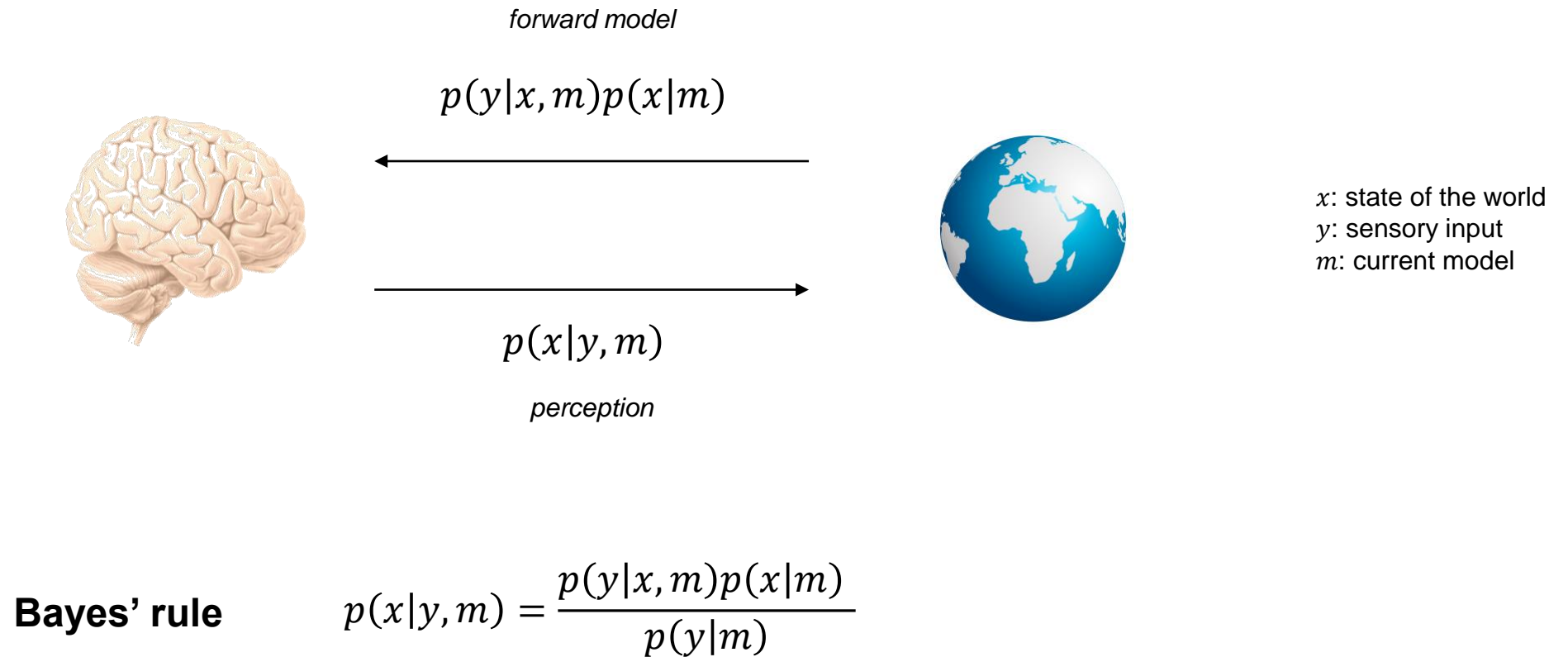
14.09.2022

Alex Hess

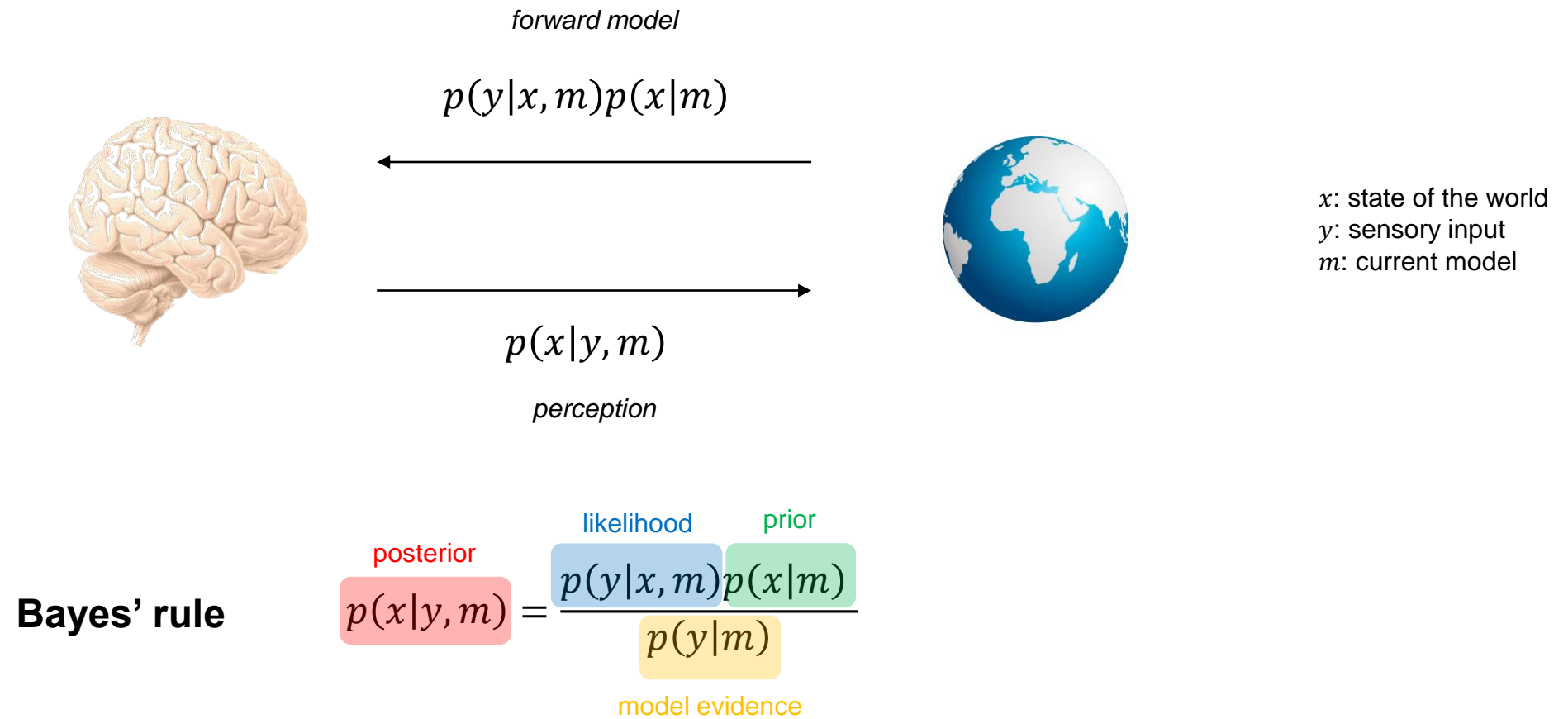
Translational Neuromodeling Unit

University of Zurich & ETH Zurich

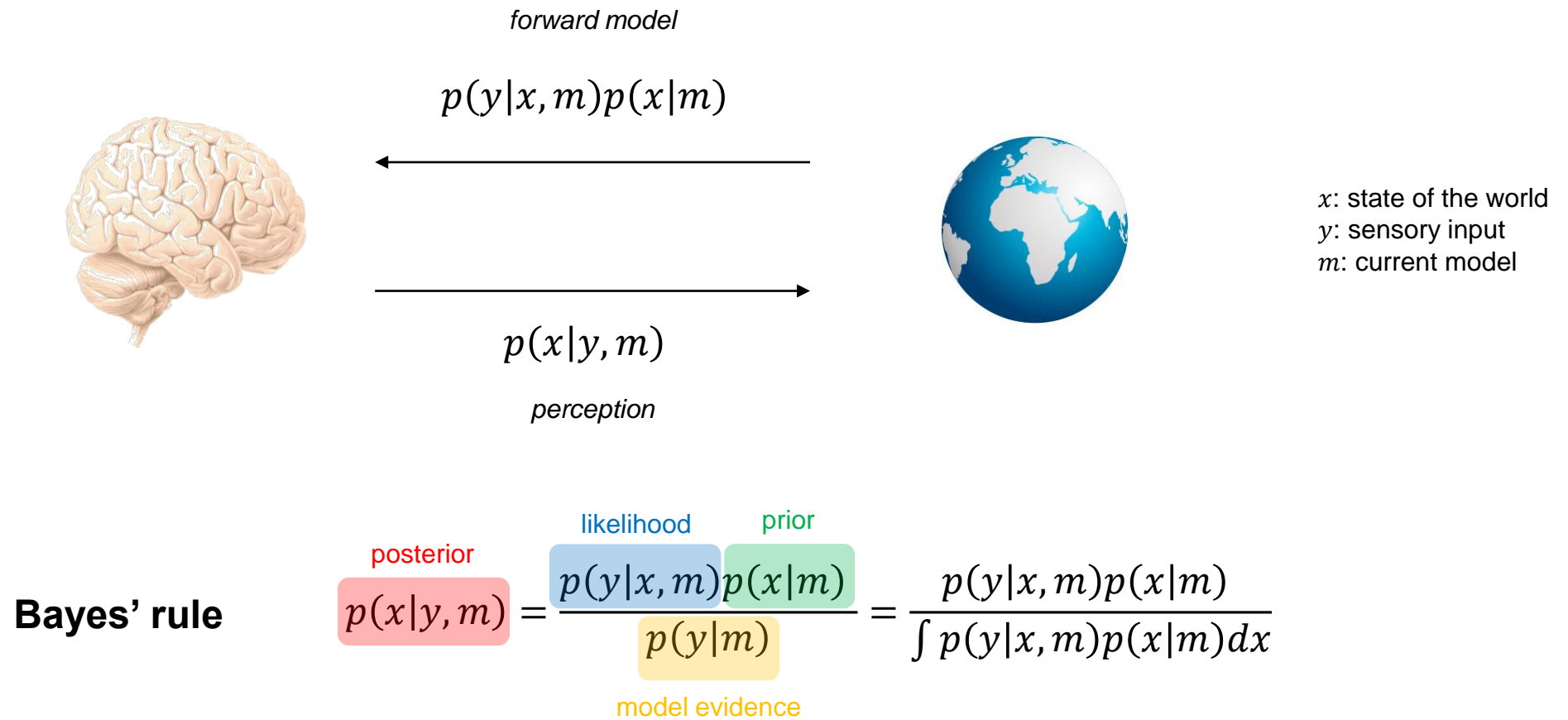
“Bayesian brain” hypothesis



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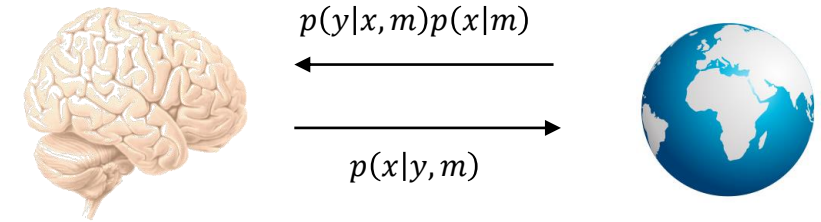


(Bayesian) Predictive Coding

what?

how?

implementation?



Bayes' rule

$$p(x|y, m) = \frac{p(y|x, m)p(x|m)}{p(y|m)}$$
$$= \frac{p(y|x, m)p(x|m)}{\int p(y|x, m)p(x|m)dx}$$

Figure adapted from a slide by Klaas Enno Stephan

(Bayesian) Predictive Coding

Marr's levels of analysis

computational

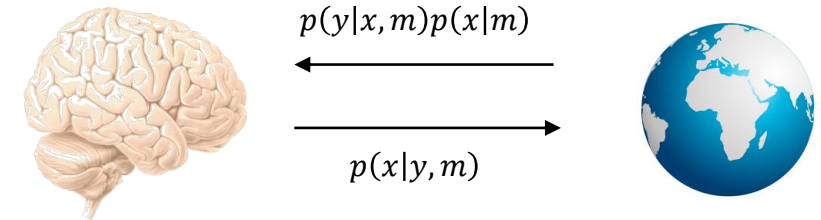
(approximate)
Bayesian inference

algorithmic

predictive coding

implementational

[predictive coding in
the brain]



Bayes' rule

$$p(x|y, m) = \frac{p(y|x, m)p(x|m)}{p(y|m)}$$
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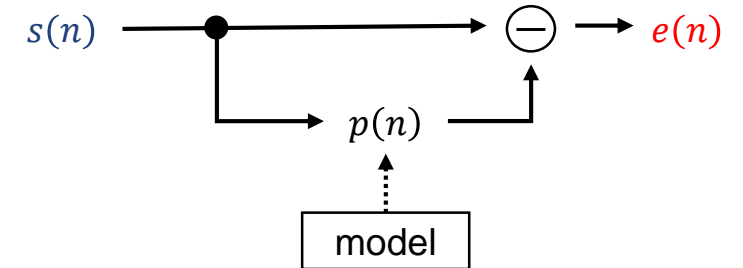
PC in engineering and information theory

Minimum redundancy principle

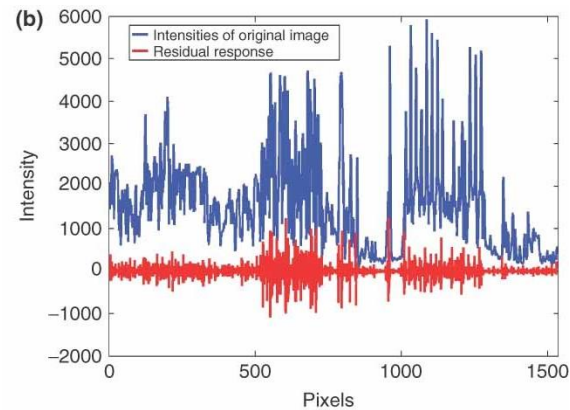
(Barlow, 1961)

- Efficient way to transmit a signal $s(n)$:
 - Model \Rightarrow prediction $p(n)$
 - Residual error $e(n)$
 } reconstruct signal $s(n)$

$$e(n) = s(n) - p(n)$$



Adapted from O'Shaughnessy 1988, *IEEE Potentials*



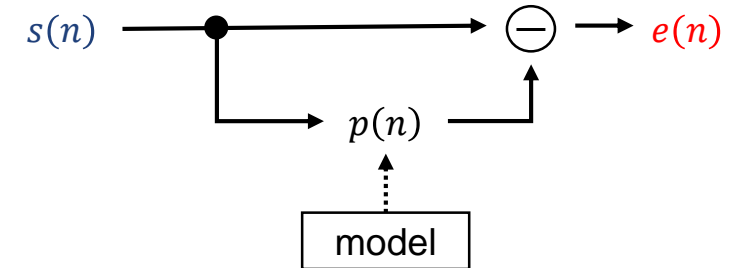
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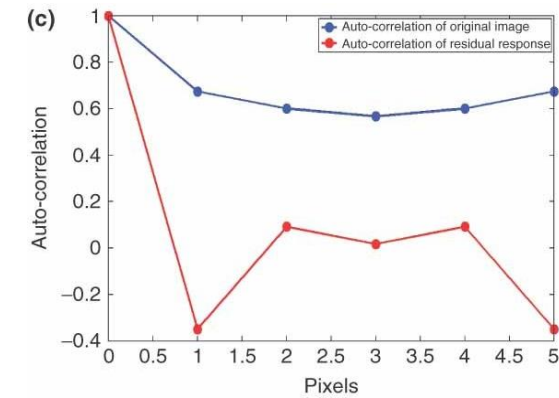
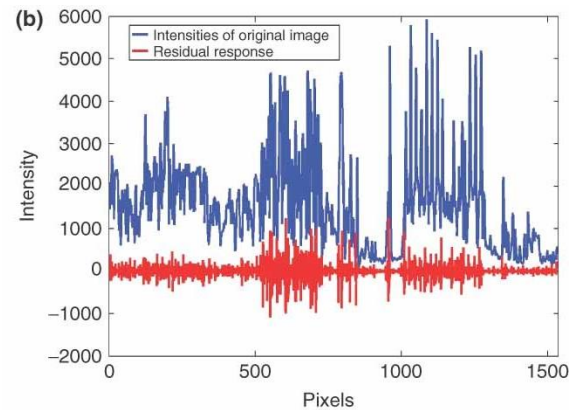
(Barlow, 1961)

- Efficient way to transmit a signal $s(n)$:
 - Model \Rightarrow prediction $p(n)$
 - Residual error $e(n)$ } reconstruct signal $s(n)$
- Decorrelation

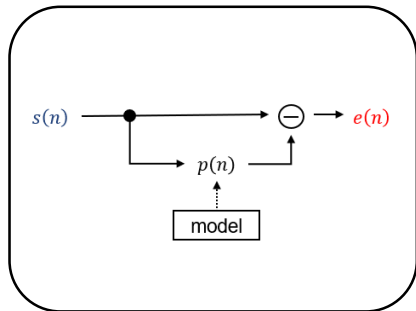
$$e(n) = s(n) - p(n)$$



Adapted from O'Shaughnessy 1988, *IEEE Potentials*

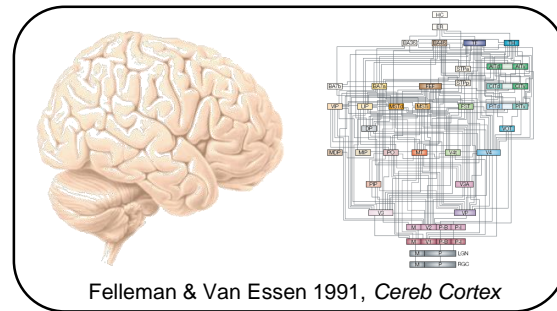


Predictive Coding as neuroscientific theory



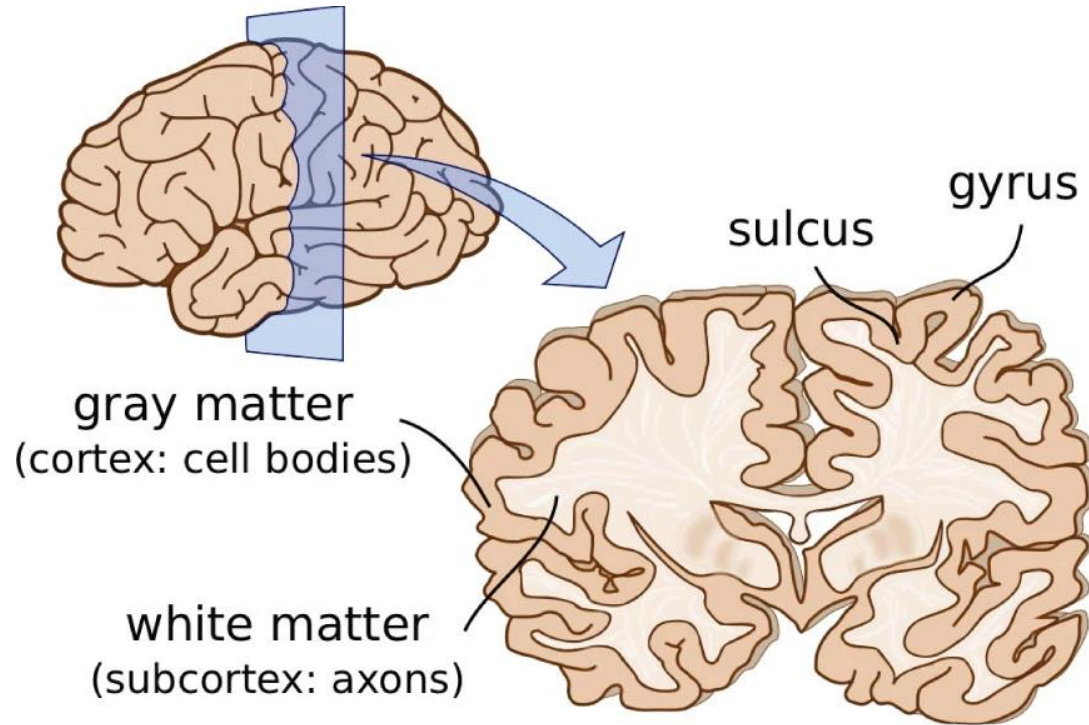
Intellectual antecedents

- Redundancy reduction

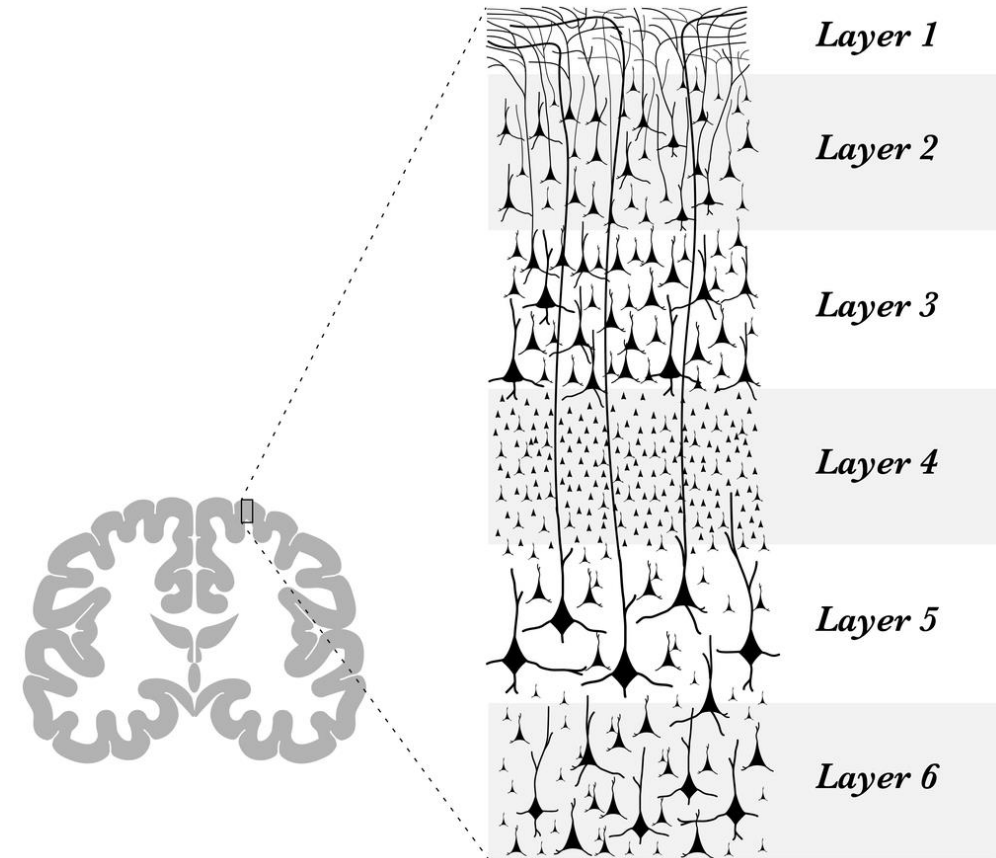


Neuroanatomy

Cerebral Cortex

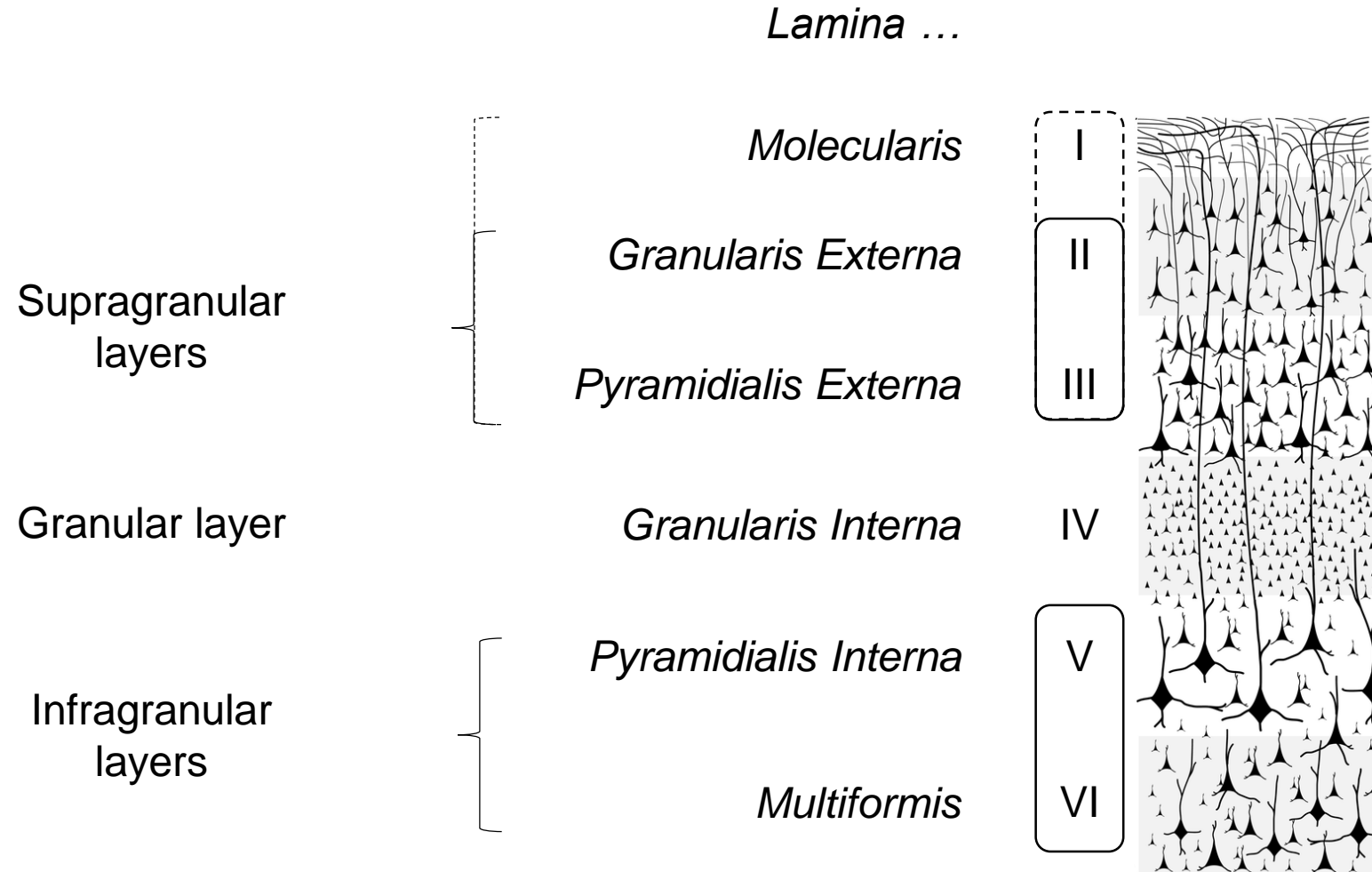


Budday et al. 2014, *Sci Rep*



Barrett 2017

Cell layers of the neocortex



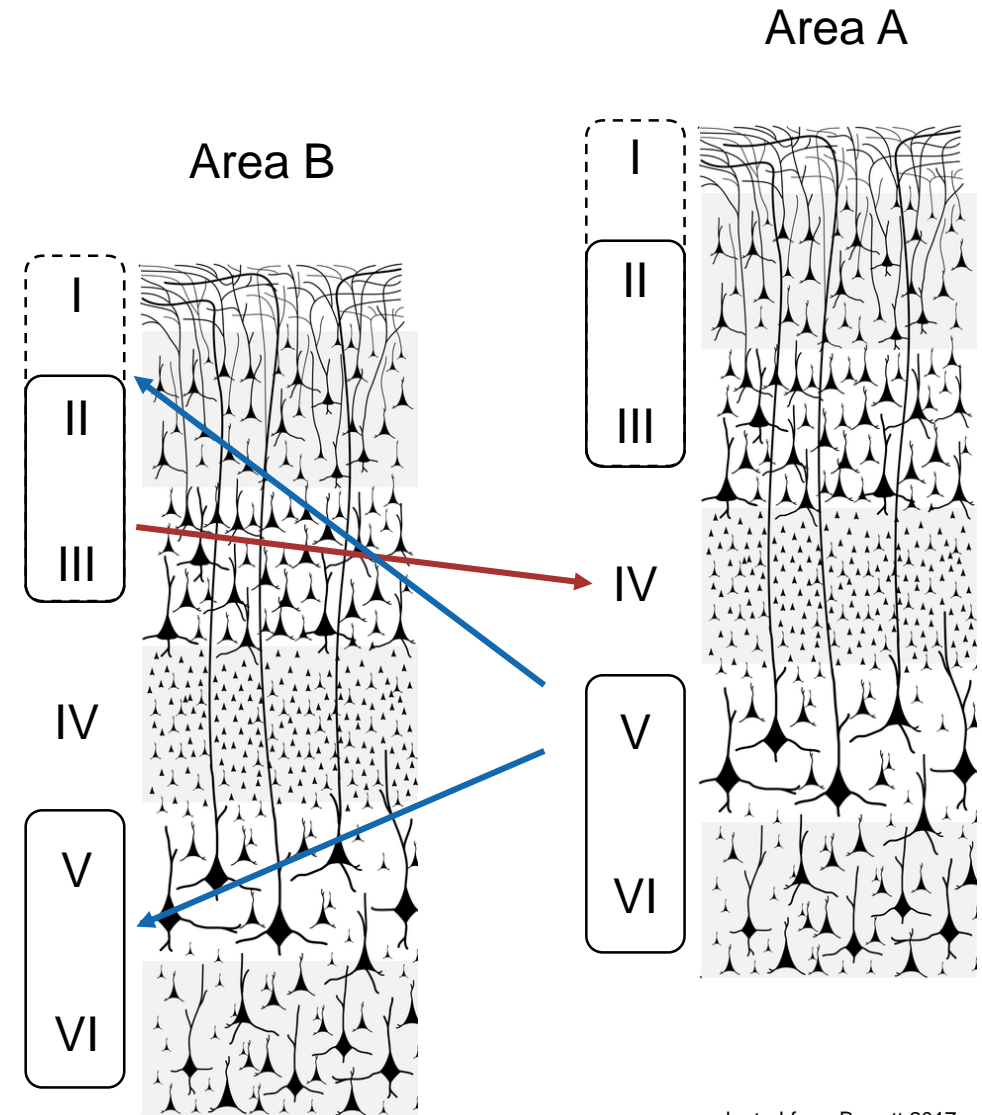
adapted from Barrett 2017

Hierarchical Relationships in the Visual Cortex

Visual cortex of macaque monkeys

(Felleman & Van Essen 1991, *Cereb Cortex*)

- Reciprocity of cortico-cortical connections
- Laminar patterns
 - **Forward connections (ascending pathways):**
 - Origin: superficial pyramidal cells (layers II & III)
 - Termination: granular layer (IV)
 - **Backward connections (descending pathways):**
 - Origin: deep pyramidal cells (layer V)
 - Termination: agranular layers (mainly I & VI)



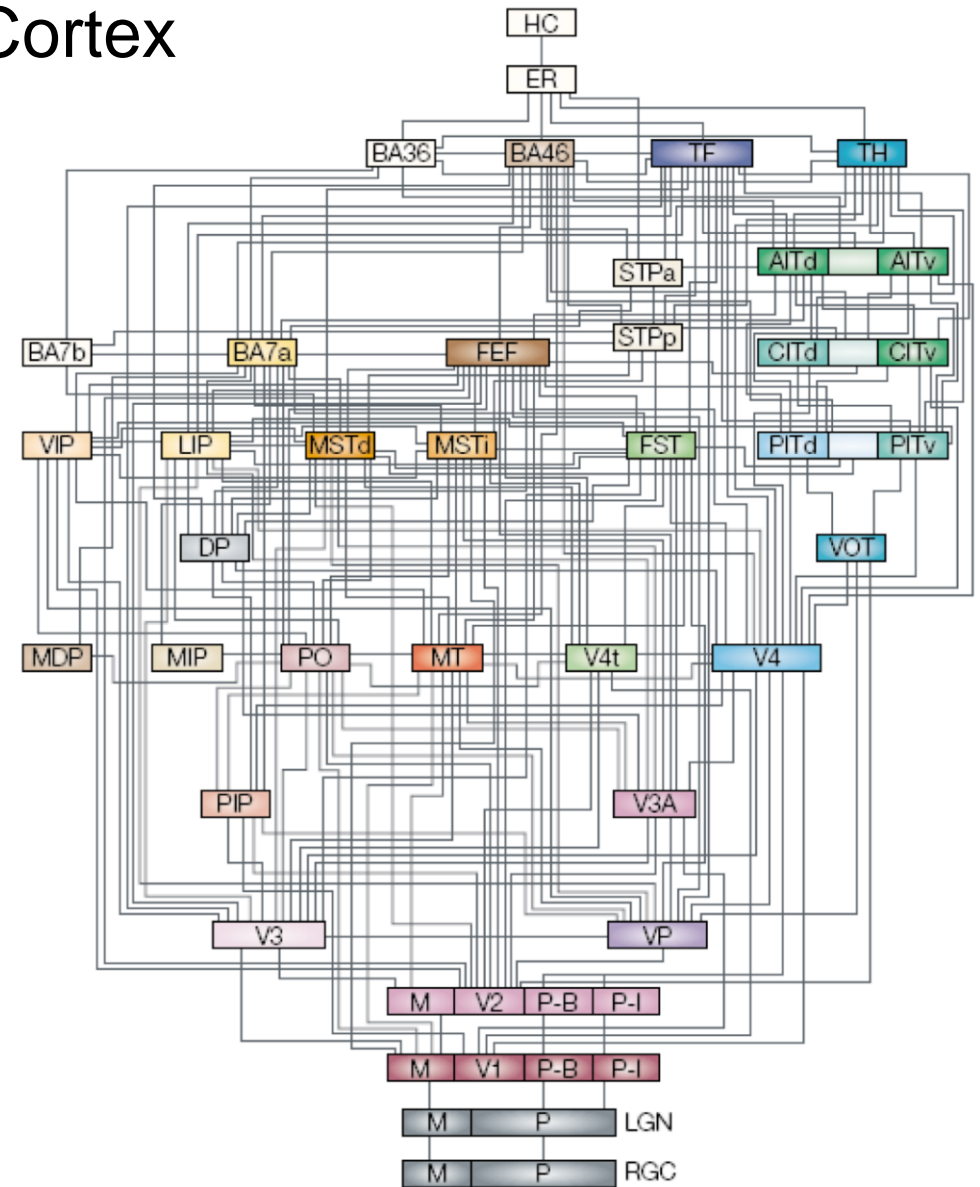
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Hierarchical Relationships in the Visual Cortex

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(Felleman & Van Essen 1991, *Cereb Cortex*)

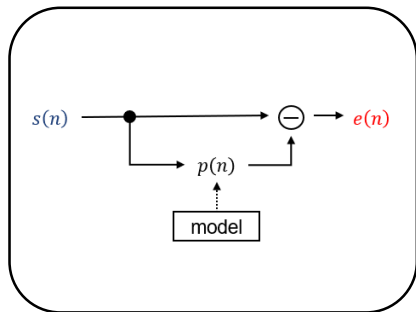
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 - **Backward connections (descending pathways):**
 - Origin: deep pyramidal cells (layer V)
 - Termination: agranular layers (mainly I & VI)
- Identify hierarchy based on laminar patterns of cortical connectivity (forward & backward connections)
- Hierarchical relationships also...
 - In other regions (somatosensory, auditory cortex, etc.)
 - In other species (other primates, cats, rats, etc.)



Felleman & Van Essen 1991, *Cereb Cortex*

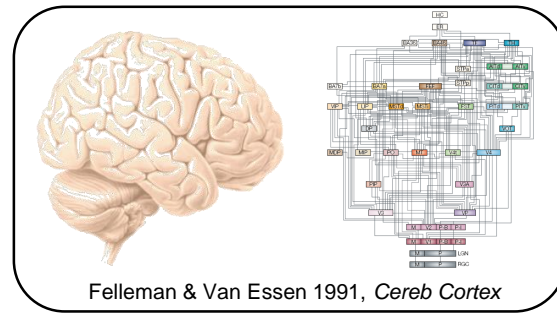
Predictive Coding as neuroscientific theory

On the computational architecture of the neocortex (D. Mumford 1992, *Biol Cybern*)



Intellectual antecedents

- Redundancy reduction

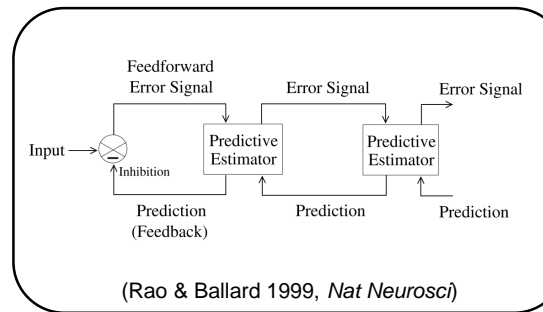


Felleman & Van Essen 1991, *Cereb Cortex*

Neuroanatomy

- Hierarchical organization of cortex

Predictive coding in the visual cortex (Rao & Ballard 1999, *Nat Neurosci*)

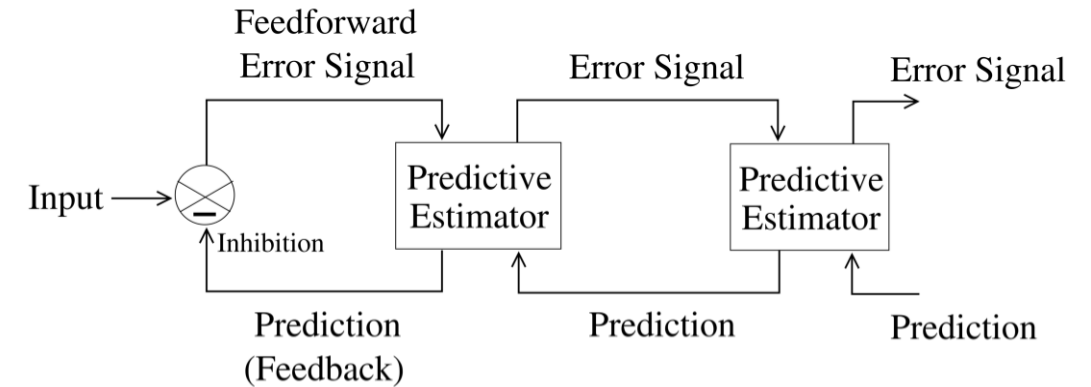


(Rao & Ballard 1999, *Nat Neurosci*)

Hierarchical PC model

Hierarchical predictive coding model

- Hierarchical network
 - Feedback connections: predictions
 - Feedforward connections: error signal
 - Predictive estimator: use error signal to generate next prediction



$$\mathbf{I} = f(\mathbf{U}\mathbf{r}) + \mathbf{n}$$

$$\begin{aligned}\mathbf{r} &= \mathbf{r}^{td} + \mathbf{n}^{td} \\ &= f(\mathbf{U}^h \mathbf{r}^h) + \mathbf{n}^{td}\end{aligned}$$

\mathbf{I} : inputs

\mathbf{r} : causes

\mathbf{U} : weights

f : activation function

\mathbf{n} : noise

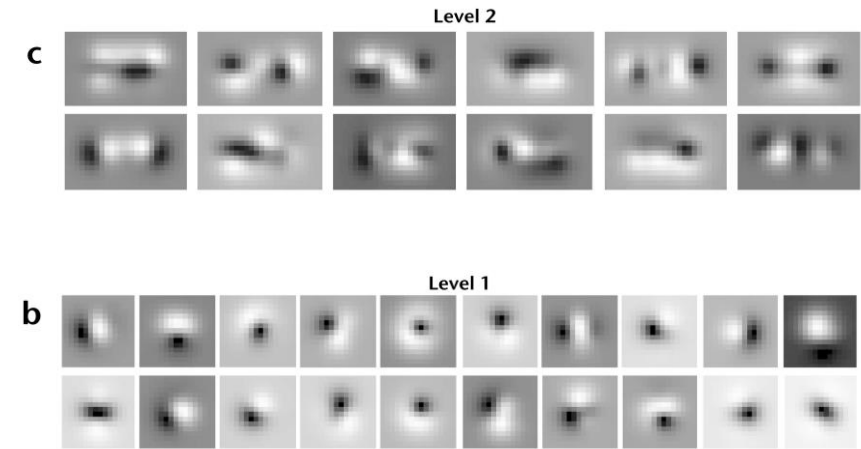
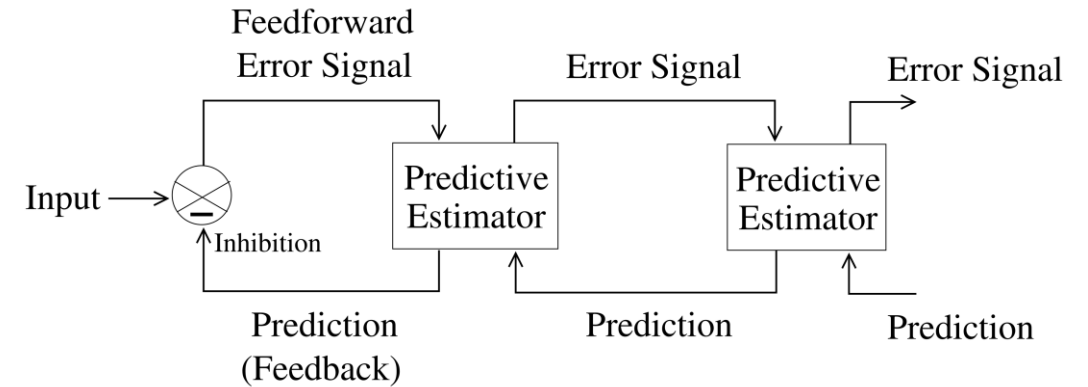
\mathbf{U}^h : higher-level weights

\mathbf{r}^h : higher-level causes

\mathbf{n}^{td} : noise

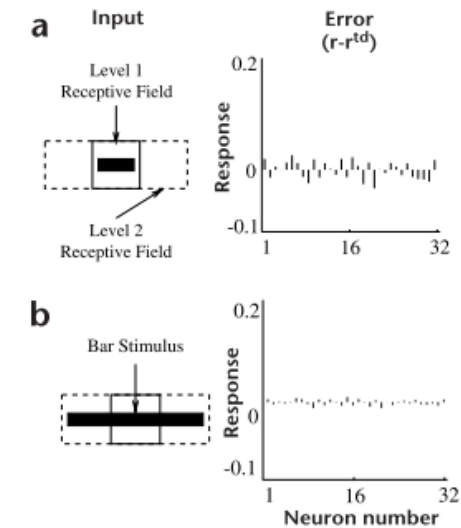
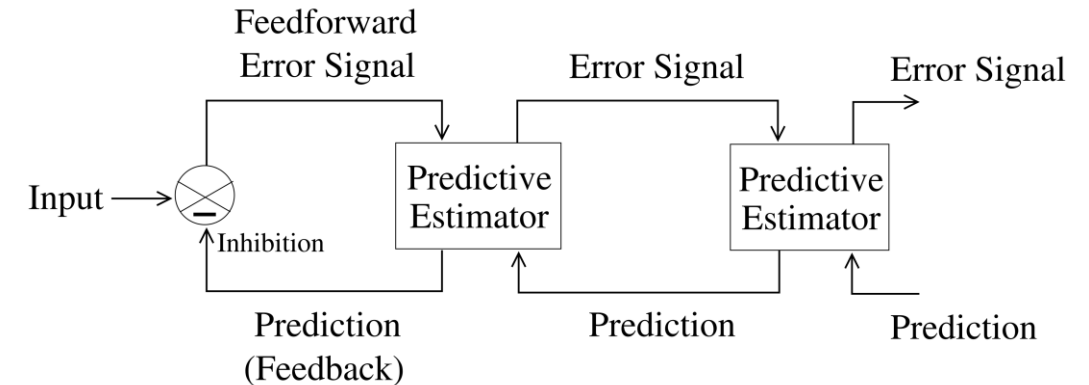
Hierarchical predictive coding model

- Hierarchical network
 - Feedback connections: predictions
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 - Predictive estimator: use error signal to generate next prediction
- Train network on patches of static natural images
 - Learned synaptic weights resemble cell-like receptive fields
 - Receptive field sizes: lower vs. upper levels



Hierarchical predictive coding model

- Hierarchical network
 - Feedback connections: predictions
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 - Predictive estimator: use error signal to generate next prediction
- Train network on patches of static natural images
 - Learned synaptic weights resemble cell-like receptive fields
 - Receptive field sizes: lower vs. upper levels
- Functional explanation for extra-classical receptive field effects:
 - Endstopping: error-detecting model neurons



Rao & Ballard 1999, *Nat Neurosci*

Hierarchical predictive coding model

- Assume probabilistic hierarchical generative model for images
 - Cost function: negative log joint (\Rightarrow MAP estimation)

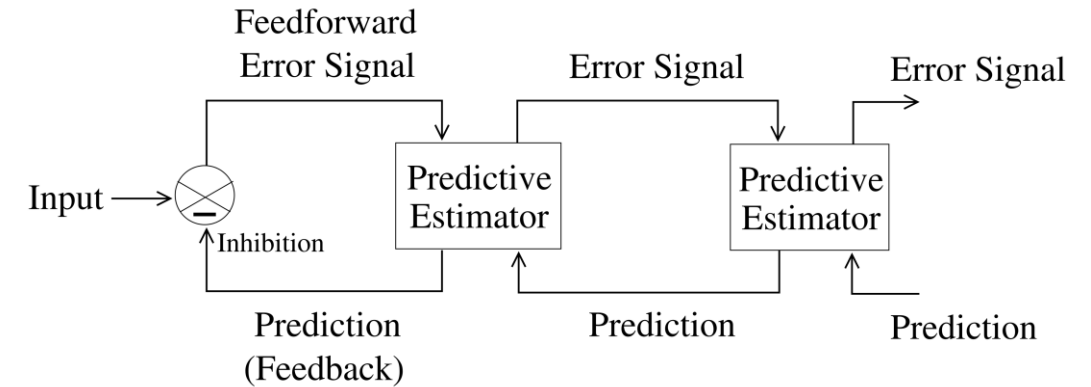
$$\underbrace{\frac{1}{\sigma^2} (\mathbf{I} - f(U\mathbf{r}))^T (\mathbf{I} - f(U\mathbf{r})) + \frac{1}{\sigma_{td}^2} (\mathbf{r} - \mathbf{r}^{td})^T (\mathbf{r} - \mathbf{r}^{td})}_{}$$

$$E = -\log p(\mathbf{I}|\mathbf{r}, U) - \log p(\mathbf{r}) - \log p(U)$$

$$= -\log(p(\mathbf{I}|\mathbf{r}, U) p(\mathbf{r}) p(U))$$

$$\text{posterior} \propto \text{likelihood} * \text{prior}$$

$$p(x|y, m) \propto p(y|x, m) p(x|m)$$



$$\mathbf{I} = f(U\mathbf{r}) + \mathbf{n} \quad \mathbf{r} = \mathbf{r}^{td} + \mathbf{n}^{td}$$

$$\Rightarrow p(\mathbf{I}|\mathbf{r}, U)$$

\mathbf{I} : inputs

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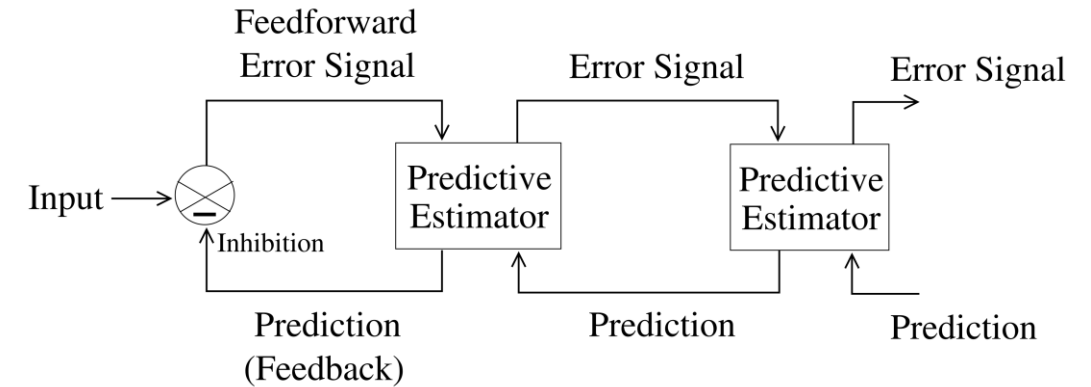
U : weights

f : activation function

\mathbf{n} : noise

Hierarchical predictive coding model

- Assume probabilistic hierarchical generative model for images
 - Cost function: negative log joint (\Rightarrow MAP estimation)
- Network dynamics & synaptic learning rules
 - Error signal weighted by inverse variances (precisions)



$$\mathbf{I} = f(\mathbf{U}\mathbf{r}) + \mathbf{n} \quad \mathbf{r} = \mathbf{r}^{td} + \mathbf{n}^{td}$$

\mathbf{I} : inputs

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\mathbf{U} : weights

f : activation function

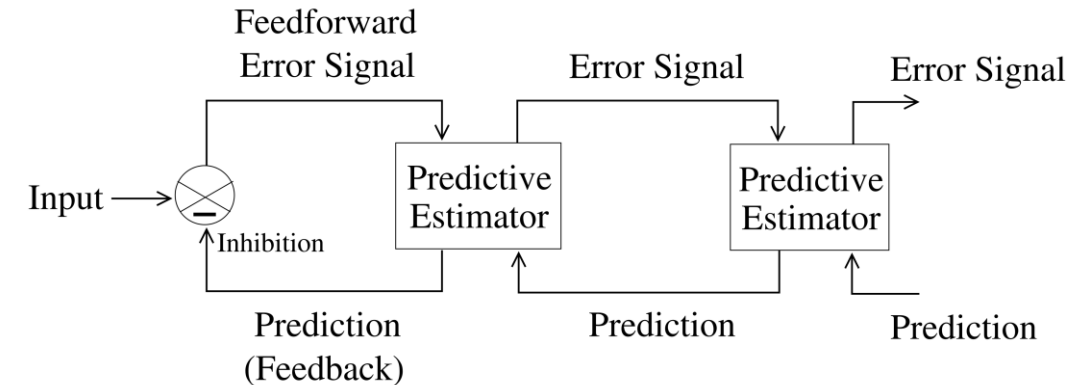
\mathbf{n} : noise

$$\begin{aligned} \frac{d\mathbf{r}}{dt} &= -\frac{k_1}{2} \frac{\partial E}{\partial \mathbf{r}} \\ &= \frac{k_1}{\sigma^2} \mathbf{U}^T \frac{\partial f^T}{\partial \mathbf{U}\mathbf{r}} (\mathbf{I} - f(\mathbf{U}\mathbf{r})) + \frac{k_1}{\sigma_{td}^2} (\mathbf{r}^{td} - \mathbf{r}) - k_1 \alpha \mathbf{r} \end{aligned}$$

$$\frac{d\mathbf{U}}{dt} = -\frac{k_2}{2} \frac{\partial E}{\partial \mathbf{U}} = \frac{k_2}{\sigma^2} \frac{\partial f^T}{\partial \mathbf{U}\mathbf{r}} (\mathbf{I} - f(\mathbf{U}\mathbf{r})) \mathbf{r}^T - \frac{k_2}{2} \lambda \mathbf{U}$$

Hierarchical predictive coding model

- Assume probabilistic hierarchical generative model for images
 - Cost function: negative log joint (\Rightarrow MAP estimation)
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$$\mathbf{I} = f(\mathbf{U}\mathbf{r}) + \mathbf{n} \quad \mathbf{r} = \mathbf{r}^{td} + \mathbf{n}^{td}$$

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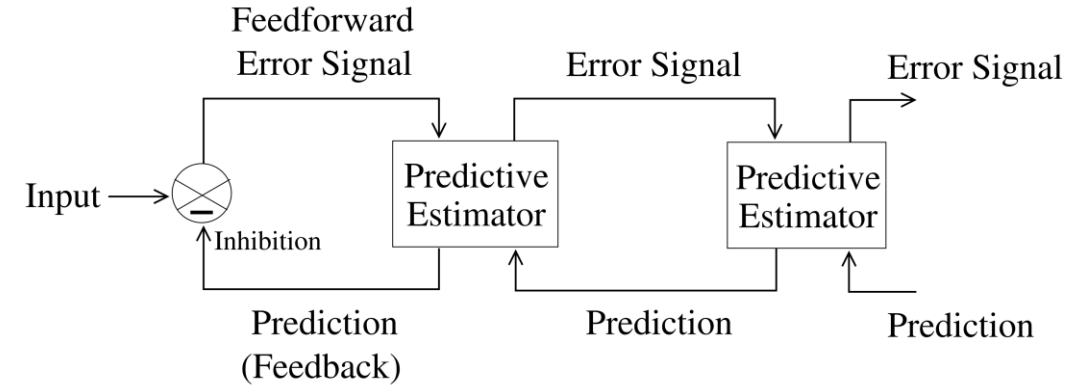
Hierarchical predictive coding model

- Assume probabilistic hierarchical generative model for images
 - Cost function: negative log joint (\Rightarrow MAP estimation)
- Network dynamics & synaptic learning rules
 - **Error signal** weighted by **inverse variances (precisions)**
 - Single cost function accounts for inference (updating \mathbf{r}) & learning (updating \mathbf{U})

$$\Delta \text{belief} \sim \text{precision} \times \text{PE}$$

$$\begin{aligned} \frac{d\mathbf{r}}{dt} &= -\frac{k_1}{2} \frac{\partial E}{\partial \mathbf{r}} \\ &= \frac{k_1}{\sigma^2} \mathbf{U}^T \frac{\partial f^T}{\partial \mathbf{U} \mathbf{r}} (\mathbf{I} - f(\mathbf{U} \mathbf{r})) + \frac{k_1}{\sigma_{td}^2} (\mathbf{r}^{td} - \mathbf{r}) - k_1 \alpha \mathbf{r} \end{aligned}$$

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$$\mathbf{I} = f(\mathbf{U} \mathbf{r}) + \mathbf{n} \quad \mathbf{r} = \mathbf{r}^{td} + \mathbf{n}^{td}$$

\mathbf{I} : inputs

\mathbf{r} : causes

\mathbf{U} : weights

f : activation function

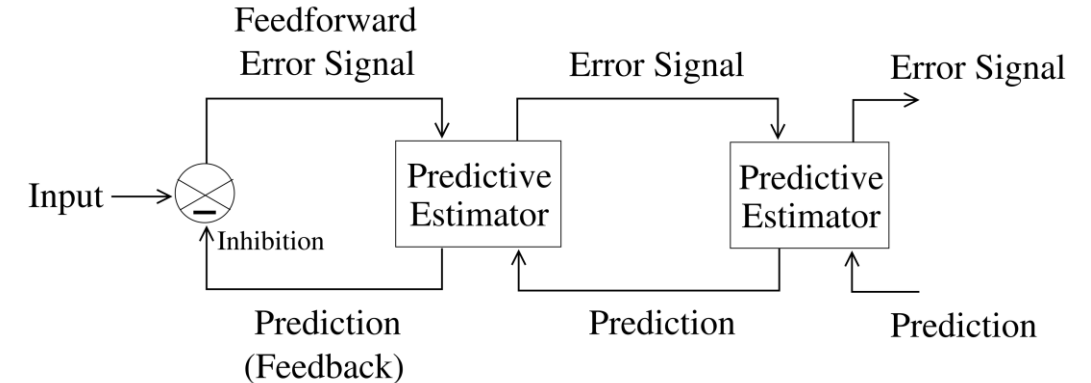
\mathbf{n} : noise

Hierarchical predictive coding model

- Assume probabilistic hierarchical generative model for images
 - Cost function: negative log joint (\Rightarrow MAP estimation)
- Network dynamics & synaptic learning rules
 - Error signal weighted by inverse variances (precisions)
 - Single cost function accounts for inference (updating \mathbf{r}) & learning (updating \mathbf{U})
 - Separation of timescales

$$\begin{aligned}\frac{d\mathbf{r}}{dt} &= -\frac{k_1}{2} \frac{\partial E}{\partial \mathbf{r}} \\ &= \frac{k_1}{\sigma^2} \mathbf{U}^T \frac{\partial f^T}{\partial \mathbf{U} \mathbf{r}} (\mathbf{I} - f(\mathbf{U} \mathbf{r})) + \frac{k_1}{\sigma_{td}^2} (\mathbf{r}^{td} - \mathbf{r}) - k_1 \alpha \mathbf{r}\end{aligned}$$

$$\frac{d\mathbf{U}}{dt} = -\frac{k_2}{2} \frac{\partial E}{\partial \mathbf{U}} = \frac{k_2}{\sigma^2} \frac{\partial f^T}{\partial \mathbf{U} \mathbf{r}} (\mathbf{I} - f(\mathbf{U} \mathbf{r})) \mathbf{r}^T - \frac{k_2}{2} \lambda \mathbf{U}$$



$$\mathbf{I} = f(\mathbf{U} \mathbf{r}) + \mathbf{n} \quad \mathbf{r} = \mathbf{r}^{td} + \mathbf{n}^{td}$$

\mathbf{I} : inputs

\mathbf{r} : causes

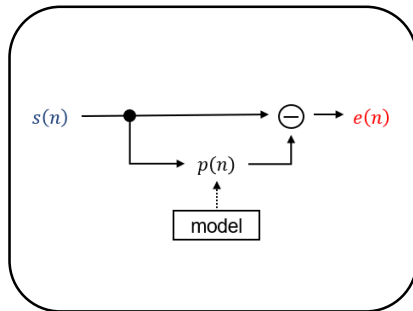
\mathbf{U} : weights

f : activation function

\mathbf{n} : noise

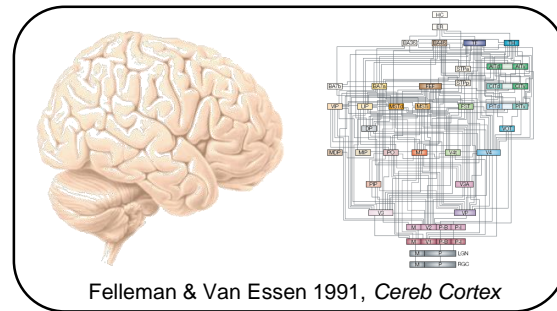
Predictive Coding as neuroscientific theory

On the computational architecture of the neocortex (D. Mumford 1992, *Biol Cybern*)



Intellectual antecedents

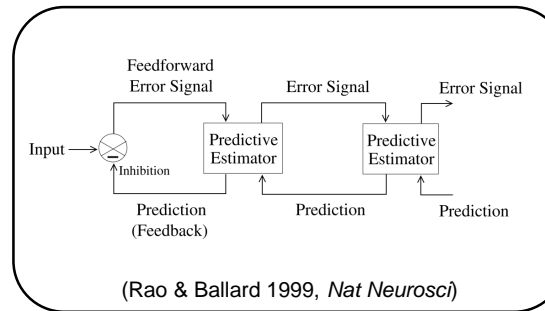
- Redundancy reduction



Neuroanatomy

- Hierarchical organization of cortex

Predictive coding in the visual cortex (Rao & Ballard 1999, *Nat Neurosci*)



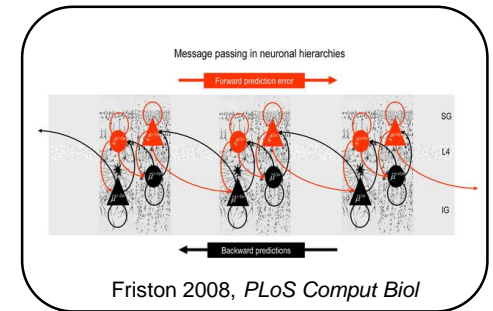
Hierarchical PC model

- Visual cortex
- Point estimate of posterior
- Static representations

Learning and Inference in the Brain (Friston 2003, *Neural Netw*)

A theory of cortical responses (Friston 2005, *Phil Trans Royal Soc B*)

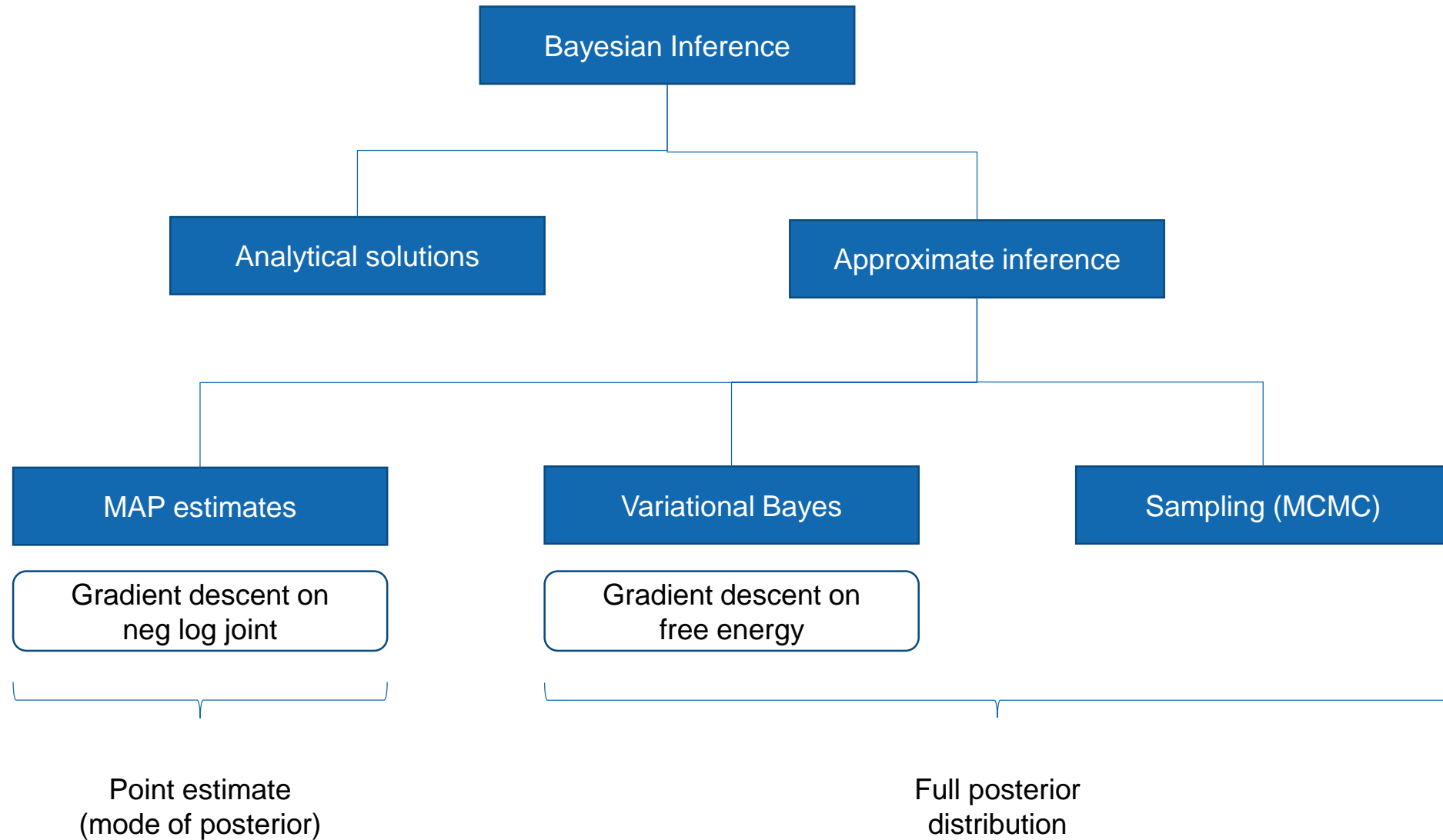
Hierarchical Models in the Brain (Friston 2008, *PLoS Comput Biol*)



PC as variational inference

Recap: Methods for Bayesian inference

Generative Models lecture (Day 2, Klaas Enno Stephan)



Recap: Variational inference

VB & MCMC lecture (Day 2, Lionel Rigoux)

$$p(x|y, m) = \frac{p(y|x, m)p(x|m)}{p(y|m)} \quad p(y|m) = \int p(y|x, m)p(x|m)dx$$

Approximate posterior $q(x|y; \phi)$ e.g. for q Gaussian, $\phi = \{\mu, \Sigma\}$

Find best proxy $q^*(x|y; \phi) = \operatorname{argmin}_{\phi} D_{KL}[q(x|y; \phi) || p(x|y, m)]$

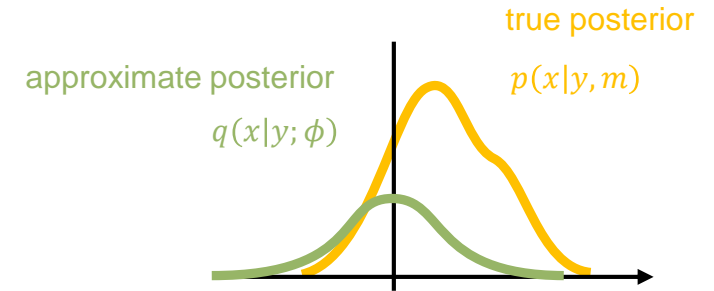


Figure adapted from slide by Yu Yao

Recap: Variational inference

VB & MCMC lecture (Day 2, Lionel Rigoux)

$$p(x|y, m) = \frac{p(y|x, m)p(x|m)}{p(y|m)} \quad p(y|m) = \int p(y|x, m)p(x|m)dx$$

Approximate posterior $q(x|y; \phi)$ e.g. for q Gaussian, $\phi = \{\mu, \Sigma\}$

Find best proxy $q^*(x|y; \phi) = \operatorname{argmin}_{\phi} D_{KL}[q(x|y; \phi) || p(x|y, m)]$

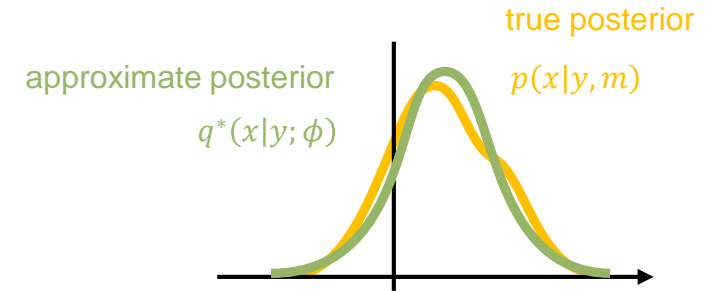
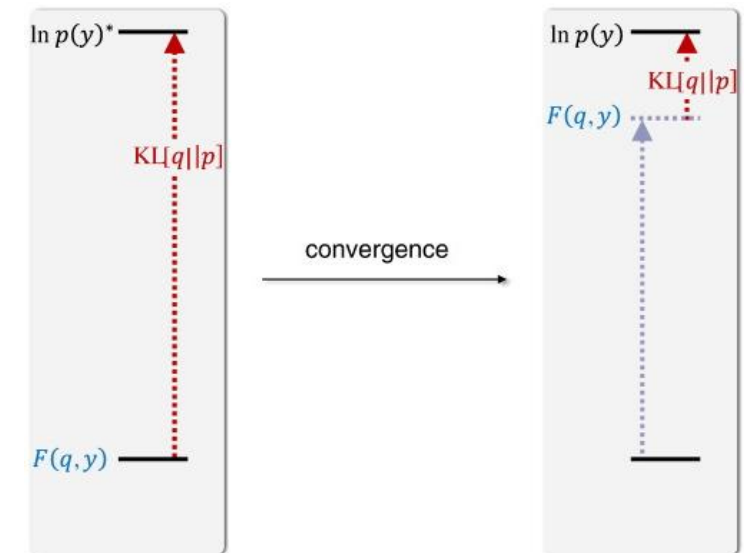


Figure adapted from slide by Yu Yao

$$\begin{aligned} D_{KL}[q(x|y; \phi) || p(x|y, m)] &= \ln p(y|m) - \int q(x|y; \phi) \frac{p(x, y|m)}{q(x|y; \phi)} dx \\ &= \ln p(y|m) - F \end{aligned}$$

$$\ln p(y|m) = D_{KL}[q(x|y; \phi) || p(x|y, m)] + F(q(x|y; \phi), p(x, y|m))$$



Stephan et al. 2017 *NeuroImage*

Predictive coding as variational inference

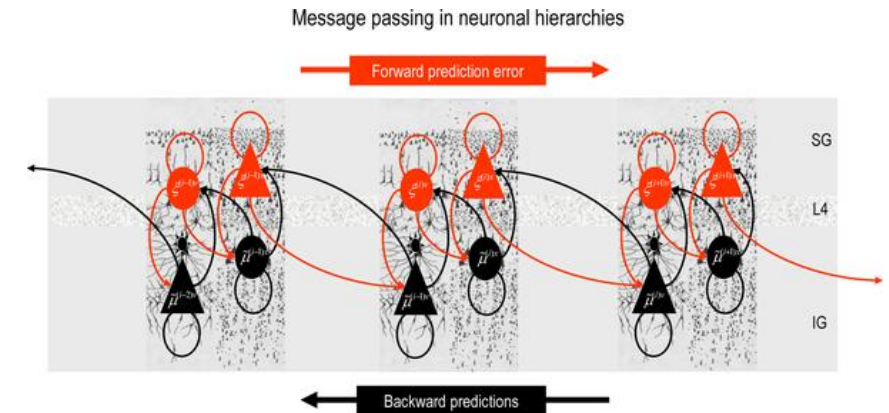
The free energy formulation of predictive coding

(Friston 2003, 2005, 2008)

- Minimal neuronal model
 - PE units (SG layers)
 - Prediction units (IG layers)at each level of the hierarchy
⇒ canonical microcircuit model (Bastos et al. 2012, *Neuron*)
- Model dynamics
 - Differential equations
 - Gradient descent on free energy F
- Importance of precision
- Extension to ...
 - Temporal sequences (dynamic environment)
⇒ minimize free action \bar{F}
 - Action (active inference)

(Friston et al. 2010, *Biol Cybern*; Adams et al. 2013, *Brain Struct Funct*)

Active Inference lecture
(Day 3, Thomas Parr)



Friston 2008, *PLoS Comput Biol*

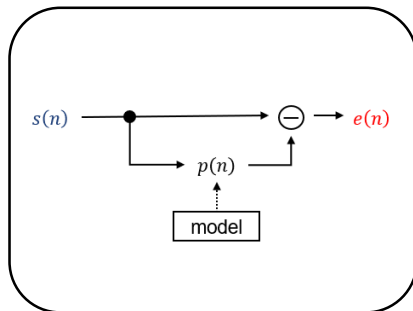
$$F = \int q(x|y; \phi) \frac{p(x, y|m)}{q(x|y; \phi)} dx$$

$$\bar{F} = \int F_t dt$$

Predictive Coding as neuroscientific theory

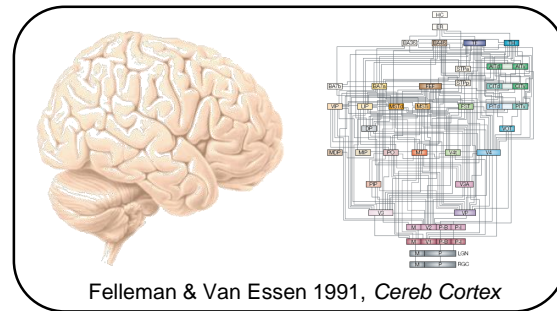
Non-Bayesian

On the computational architecture of the neocortex
(D. Mumford 1992, *Biol Cybern*)



Intellectual antecedents

- Redundancy reduction

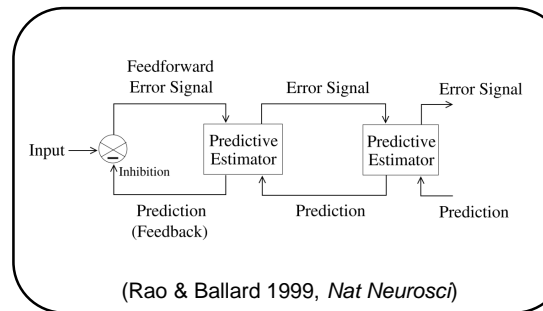


Neuroanatomy

- Hierarchical organization of cortex

PC as approximate Bayesian inference

Predictive coding in the visual cortex
(Rao & Ballard 1999, *Nat Neurosci*)



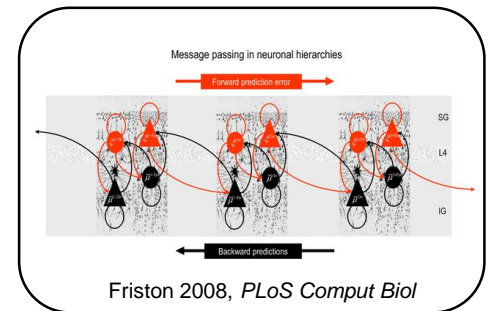
Hierarchical PC model

- Visual cortex
- Point estimate of posterior
- Static representations

Learning and Inference in the Brain
(Friston 2003, *Neural Netw*)

A theory of cortical responses
(Friston 2005, *Phil Trans Royal Soc B*)

Hierarchical Models in the Brain
(Friston 2008, *PLoS Comput Biol*)



PC as variational inference

- Cortical function
- Estimate full posterior
- Dynamic representations

Predictive coding in computational psychiatry



Predictive coding in computational psychiatry

The role of precision

- Finding the right balance
- Disorders of precision?

Schizophrenia lecture
(Day 1, Jakob Siemerikus)

Schizophrenia/Psychosis

(Stephan et al. 2006, *Biol Psychiatry*; Corlett et al. 2011, *NPP*; Adams et al. 2013, *Front Psychiatry*; Friston et al. 2016, *Schizophr Res*; Sterzer et al. 2018, *Biol Psychiatry*)

Autism lecture
(Day 1, Helene Haker Rössler)

Autism Spectrum Disorder

(Pellicano & Burr 2012, *TiCS*; Van de Cruys et al. 2014, *Psychol Rev*; Lawson et al. 2014, *Front Hum Neurosci*; Haker et al. 2016, *Front Psychiatry*; Lawson et al. 2017, *Nat Neurosci*)

From exteroception...

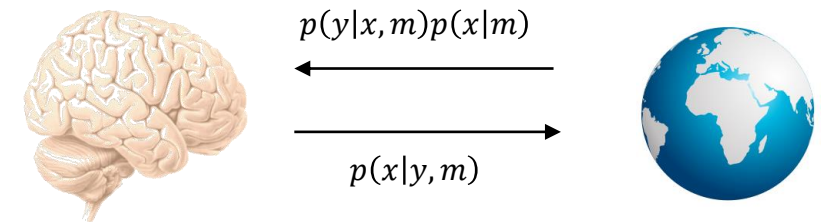


Figure adapted from a slide by Klaas Enno Stephan

Predictive coding in computational psychiatry

The role of precision

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From exteroception to interoception

- Interoceptive predictive coding
(Seth et al. 2012, *Front Psychol*; Seth 2013, *TiCS*; Barrett & Simmons 2015, *Nature Rev Neurosci*;)
- Crucial role in mental health disorders
- **Fatigue & depression** (Stephan et al. 2016, *Front Hum Neurosci*)

Fatigue lecture
(Day 1, Inês Pereira)

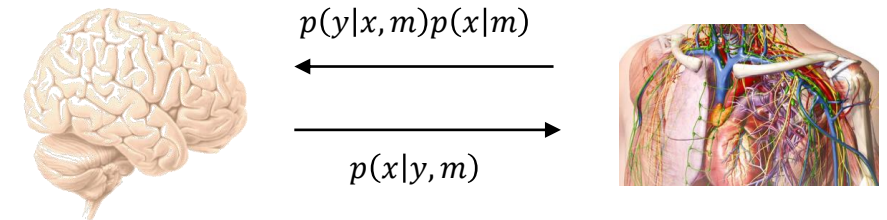


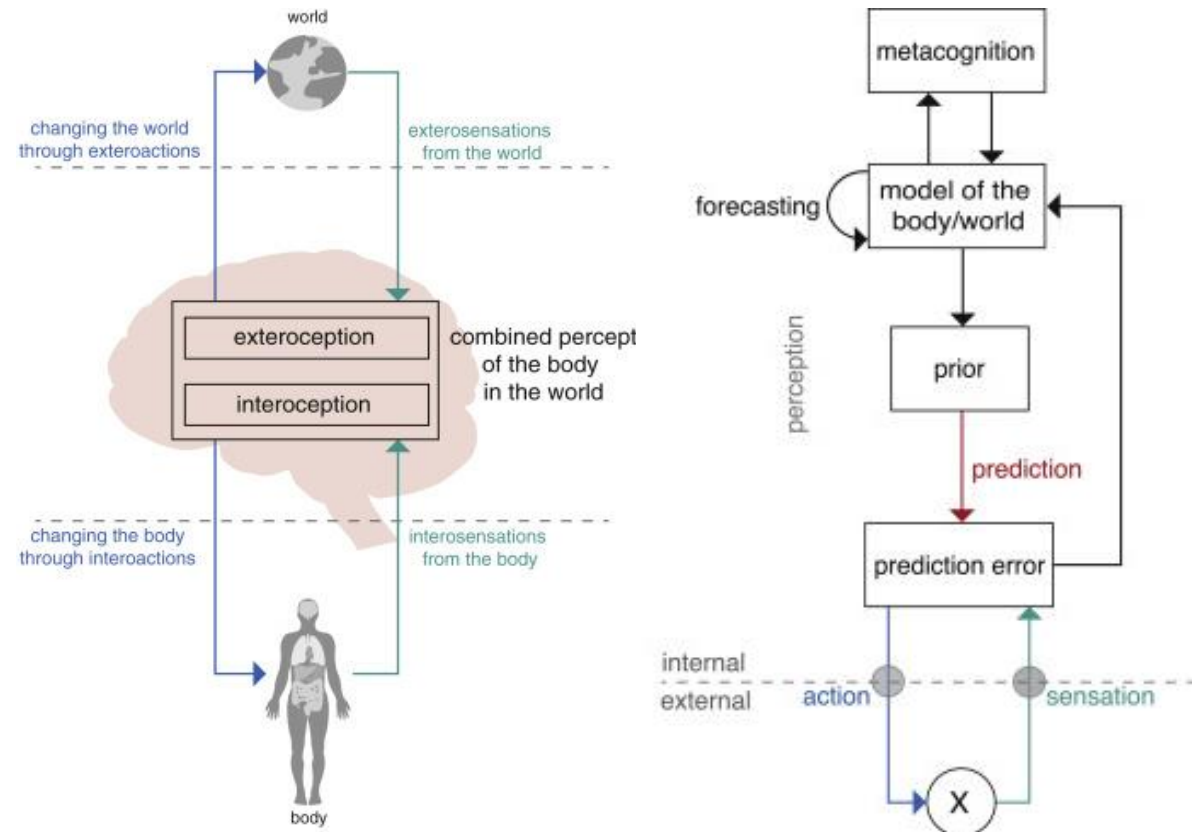
Figure adapted from a slide by Klaas Enno Stephan

Hierarchical Bayesian Inference in Computational Psychiatry

(Petzschner et al. 2017, *Biol Psychiatry*)

- General framework to model adaptive behaviour
- Possible primary disruption at:
 - Sensory inputs (sensations)
 - Inference (perception)
 - Forecasting
 - Control (action)
 - Metacognition
- At any of the above, possible disturbance of:
 - predictions
 - prediction error computation
 - Estimation of precision

⇒ guide differential diagnosis



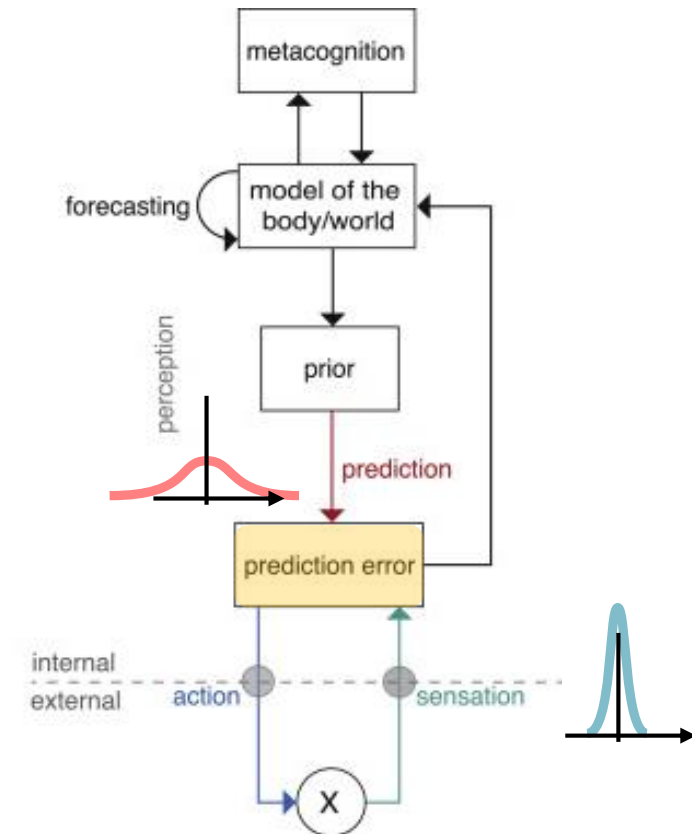
Petzschner et al. 2017, *Biol Psychiatry*

Hierarchical Bayesian Inference in Computational Psychiatry

(Petzschner et al. 2017, *Biol Psychiatry*)

Example: Autism Spectrum Disorder

- Patients: excessive processing of irrelevant details
 - 2 competing explanations
 - Sensory inputs of overwhelming precision
 - Too imprecise higher-order beliefs
- ⇒ large PEs during perception
- Disambiguate 2 hypotheses:
 - Assess individual sensory processing (experiment + model)
 - Detect (sub)groups

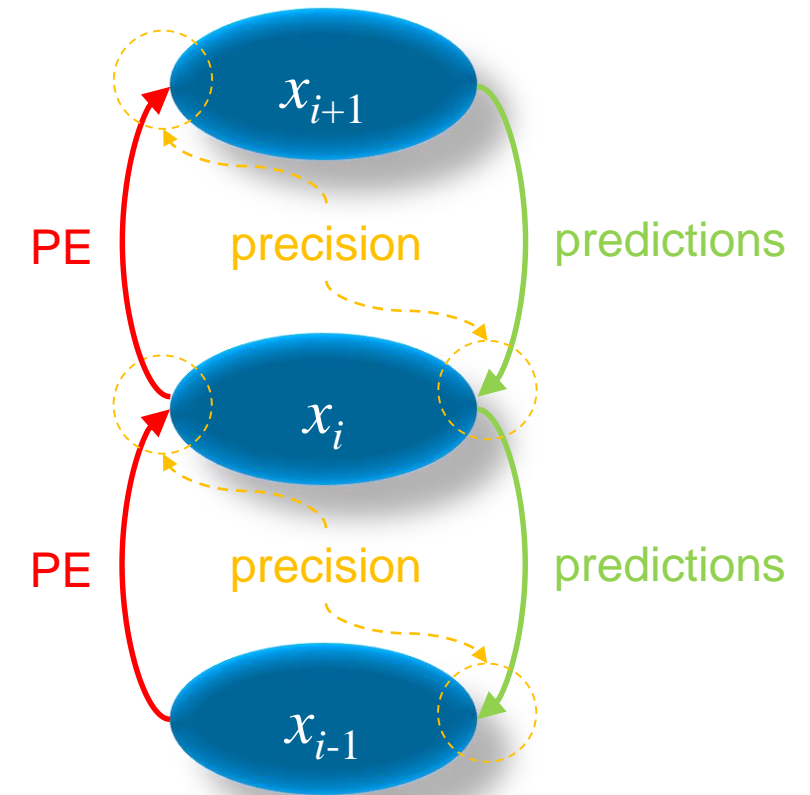


Petzschner et al. 2017, *Biol Psychiatry*

Predictive coding in a nutshell

- Possible way of implementing Hierarchical Bayesian inference in the brain
- Based on
 - Redundancy reduction
 - Hierarchical organization of cortex
- Computational quantities:
 - Each layer makes **predictions** about activity in layer immediately below
 - Predictions are compared with inputs of each layer
 - **Prediction errors (PE)** signalled upwards
 - Relative influence of PEs and predictions is determined by their relative **precision** (certainty)
- Goal of the brain:
 - minimize PE at each level of the hierarchy
- Utility of this framework for Computational Psychiatry & Computational Psychosomatics

$$\Delta \text{belief} \sim \text{precision} \times \text{PE}$$



Adapted from Stephan et al. 2016, *Brain*

Further reading

REVIEWS

Theoretical & experimental review Millidge et al. 2021, *arXiv:2107.12979*

Experimental evidence for PC in the brain Walsh et al. 2020, *Ann N Y Acad Sci*

PC algorithms Spratling et al. 2017, *Brain Cogn*

TUTORIALS

PC as variational inference Bogacz 2017, *J Math Psychol*; Buckley 2017, *J Math Psychol*

OTHER

PC & laminar fMRI Stephan et al. 2019, *NeuroImage*

PC networks and backpropagation of error algorithm Whittington & Bogacz 2017, *Neural Comput*; Song et al. 2020, *Adv Neural Inf Process Syst*

PC, variational autoencoders & normalizing flows Marino 2020, *arXiv:2011.07464*



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Thank you!

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