Drift Diffusion Model

Computational Psychiatry Course Zurich
September 11, 2024
Matt Nassar

Perceptual Decisions

Perceptual Decisions

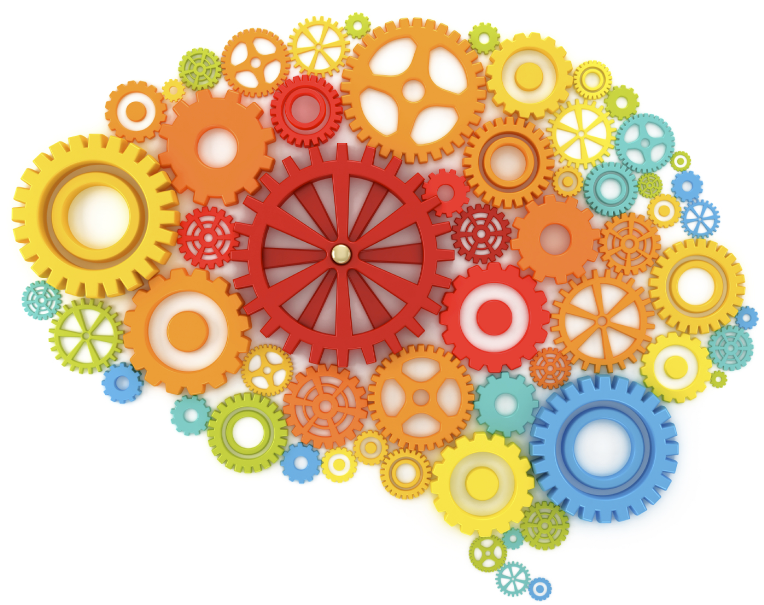
Perceptual Decisions

Perceptual Decisions

Perceptual Decisions

Important for survival!!!

Perceptual Decisions

Input →  → **Output**

Perceptual Decisions

Input →  → **Output**

Total control over input:

Perceptual Decisions

Input →  → **Output**

Total control over input:

Rapid measurement of decision & timing

a,s,d,f

Perceptual Decisions

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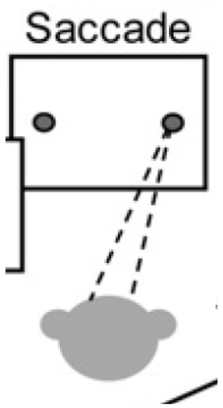


Perceptual Decisions

Paradigms conducive to animal research

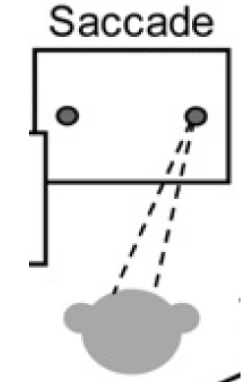
Input →  → **Output**

Total control over input:

Rapid measurement of decision & timing

a,s,d,f

Saccade



Perceptual Decisions

Paradigms conducive to animal research

Record neurons!

Silence neurons!

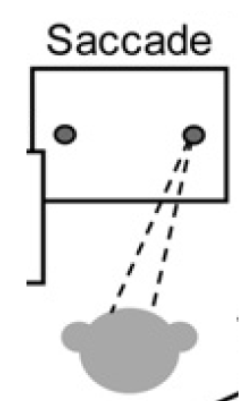
Activate neurons!

Input →  → **Output**

Total control over input:

Rapid measurement of decision & timing

a,s,d,f



Perceptual Decisions

Paradigms conducive to animal research

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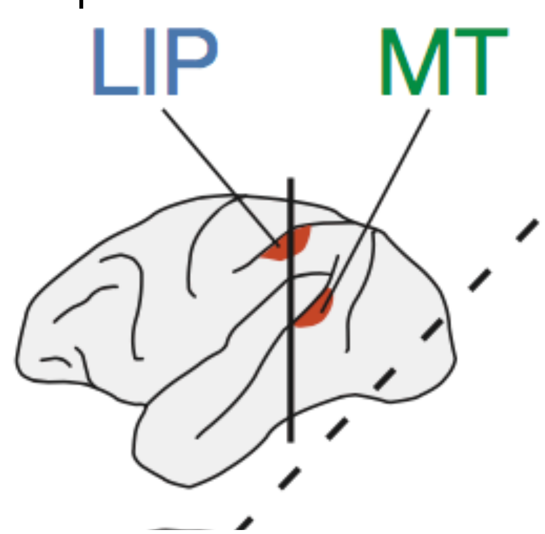
Silence neurons!

Activate neurons!

We have a good idea where to look for sensory inputs and motor outputs

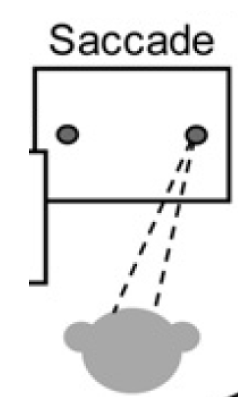
Input → 

→ **Output**

Total control over input:

Rapid measurement of decision & timing

a,s,d,f



Perceptual Decisions

Amenable to good experiments!!!

We have a good idea where to look for sensory inputs and motor outputs

Paradigms conducive to animal research

Record neurons!

Silence neurons!

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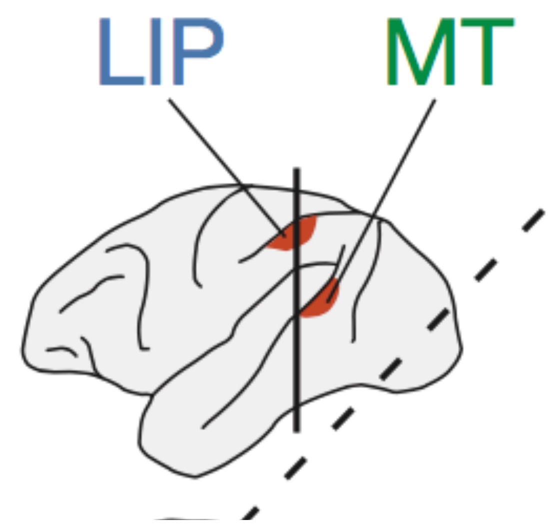
Input

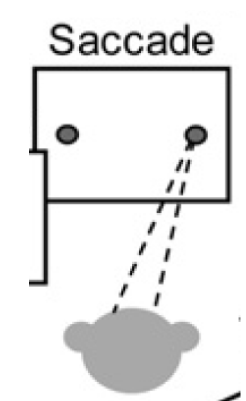


Output

Total control over input:

Rapid measurement of decision & timing

a,s,d,f



Which way are dots moving?

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A simple model of perceptual decision making:

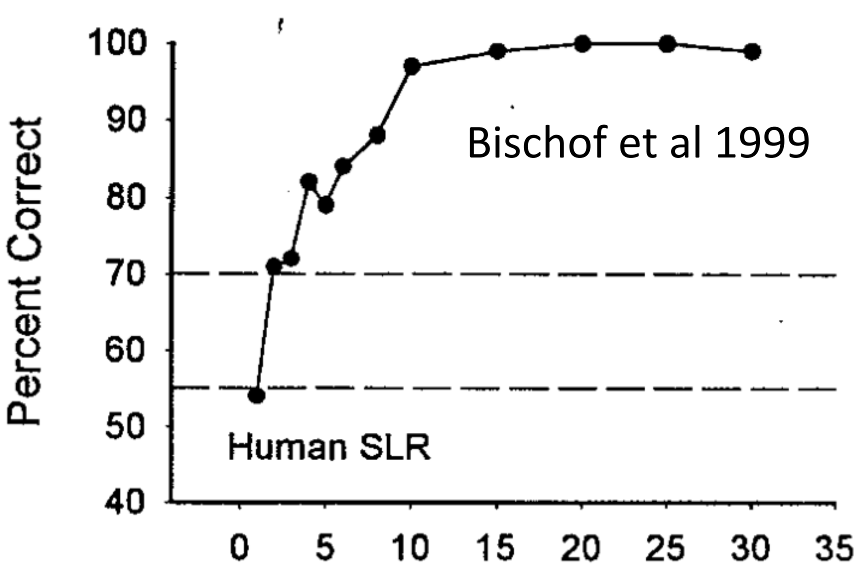
- Observe one frame (timestep) of dot motion

A simple model of perceptual decision making:

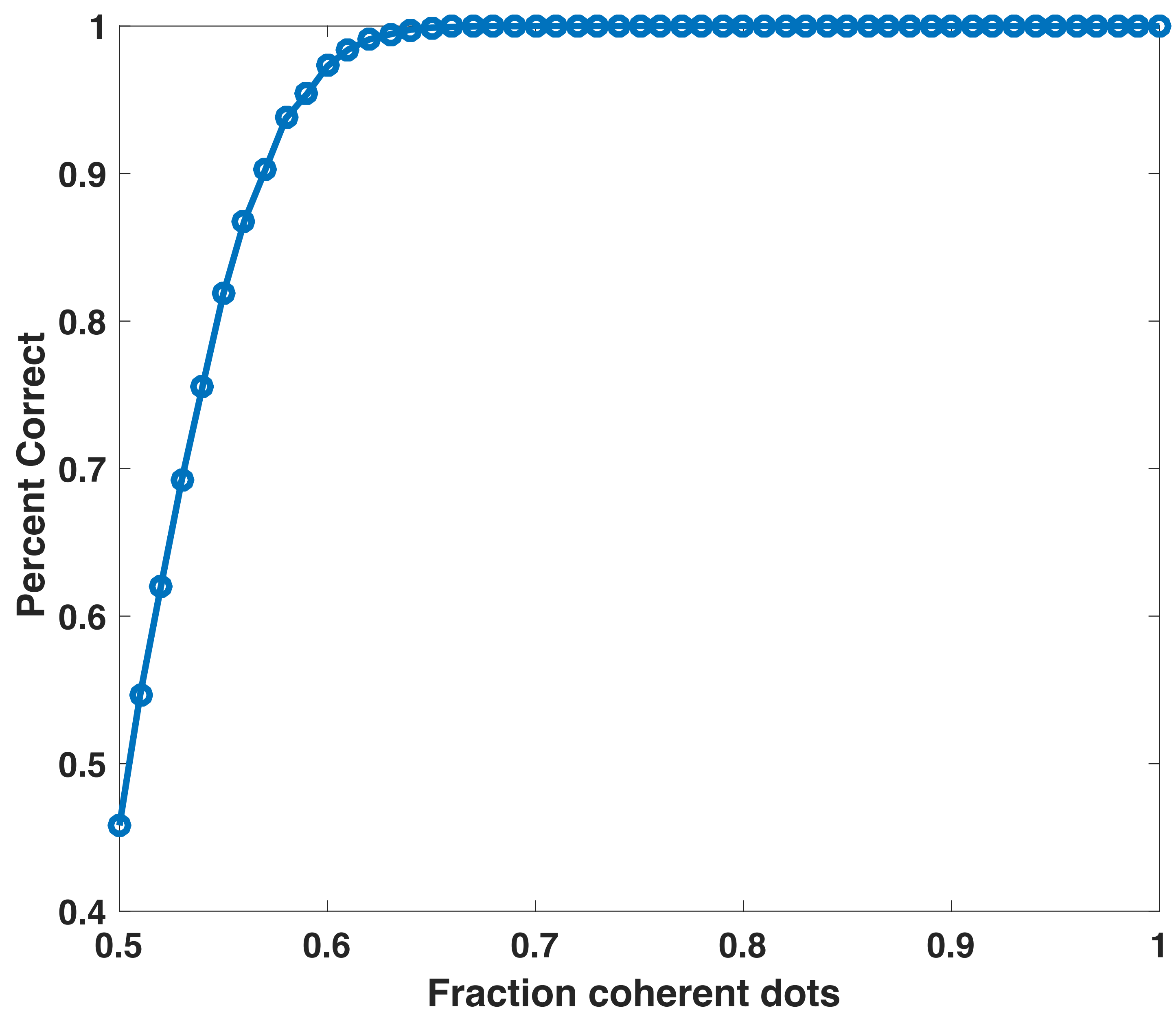
- Observe one frame (timestep) of dot motion
- Count ones that are moving rightward
- If that is greater than 1/2 the dots, say right
- Otherwise say left

Our simple model makes a prediction

One human's performance



"Coherence" (0 for f = 0.5, 100 for f = 1)



Which way are dots moving?

When *should* you stop to make a decision?

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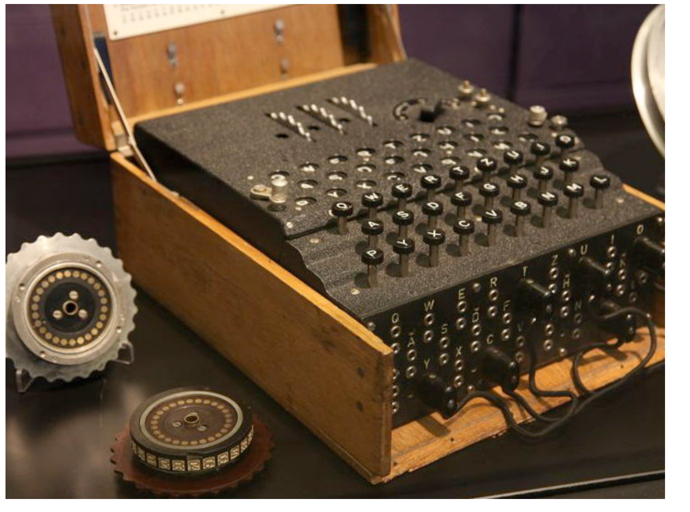
The "enigma"



Weight of evidence =

$$\log\left[\frac{\Pr(m|h_1)}{\Pr(m|h_0)}\right] + \log\left[\frac{\Pr(m|h_1)}{\Pr(m|h_0)}\right] + \ldots$$

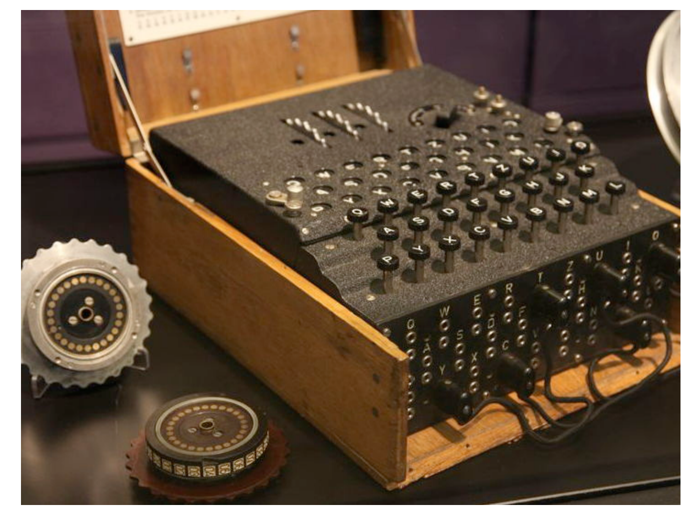
Alan Turing

Accumulate to some log odds ratio

Stop at preferred level of certainty

When *should* you stop to make a decision?

The "enigma"



Key insights:

Evidence can be ***accumulated*** over time

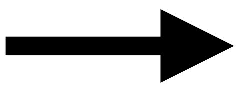
A fixed ***stopping criterion*** on accumulated evidence ensures the shortest time to a given level of decision certainty

Alan Turing

Wald 1947

Dot motion evidence accumulation

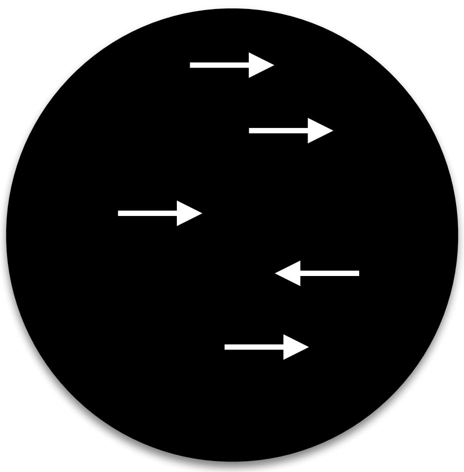
"True" Motion Direction



Dot motion evidence accumulation

"True" Motion Direction

What we see:



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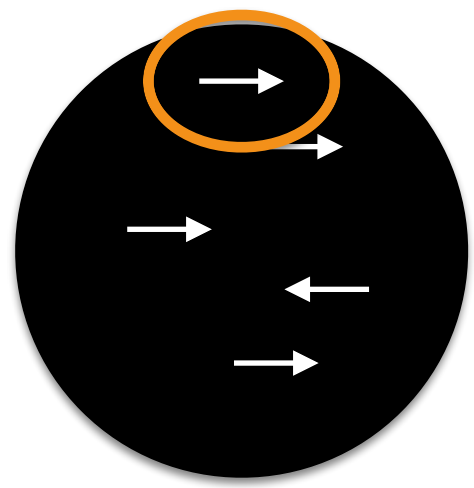

Motion in Each frame Comes from Flip of a weighted coin

Dot motion evidence accumulation

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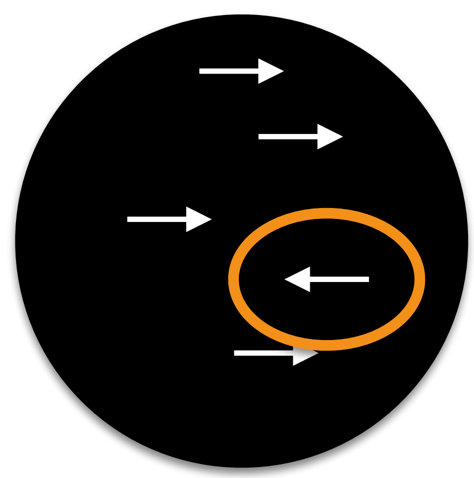

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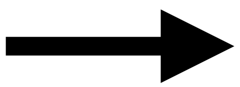

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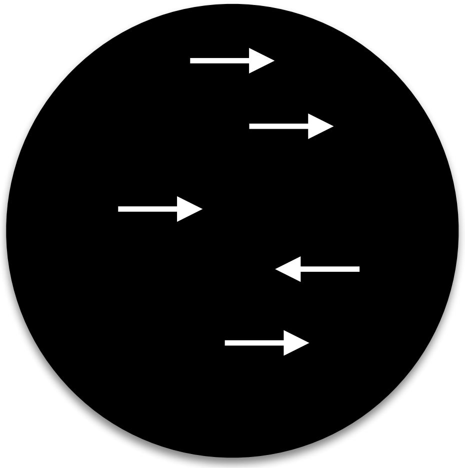

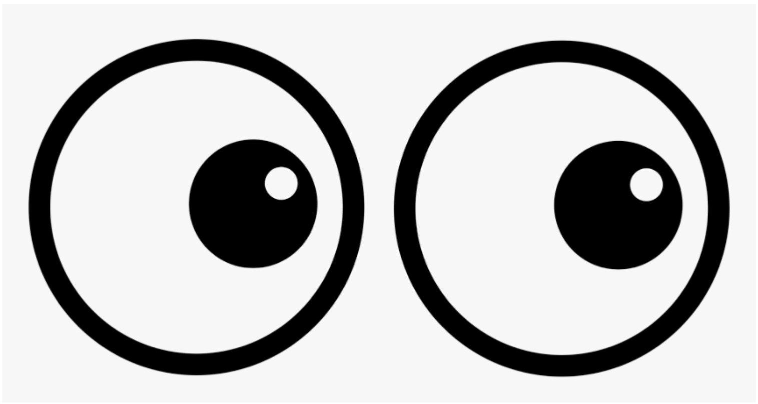

Dot motion evidence accumulation

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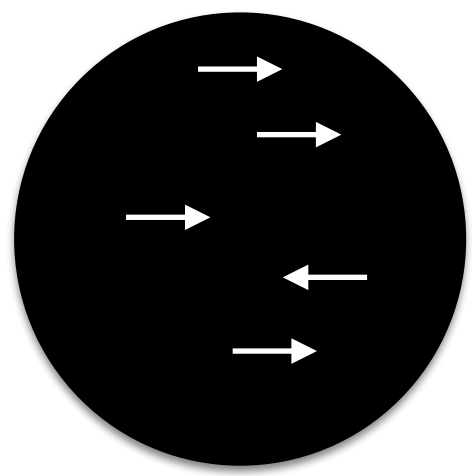

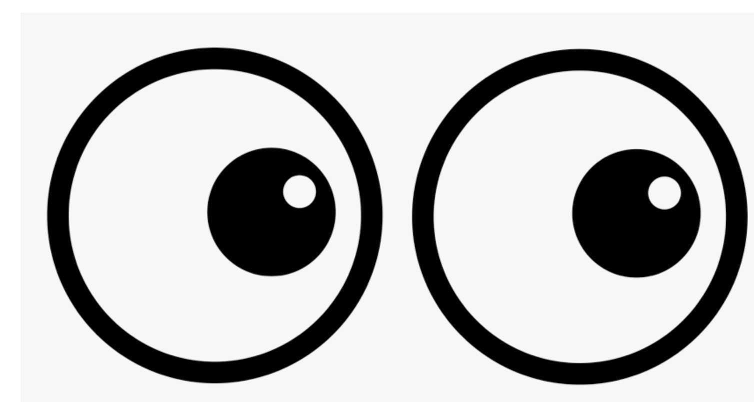

$$p(right|observations) =$$

Dot motion evidence accumulation

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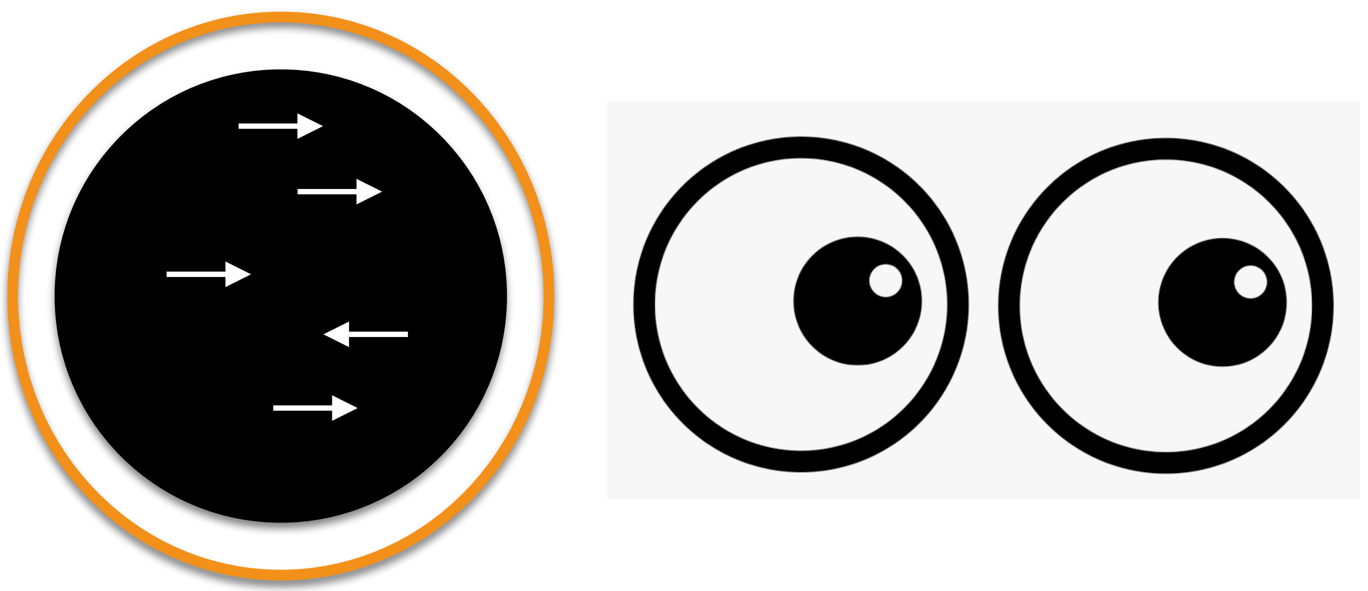
What we see:



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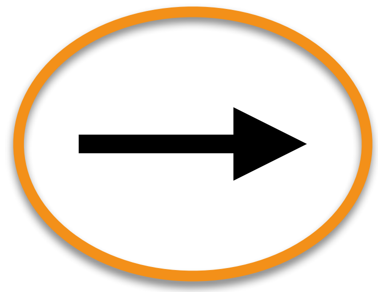

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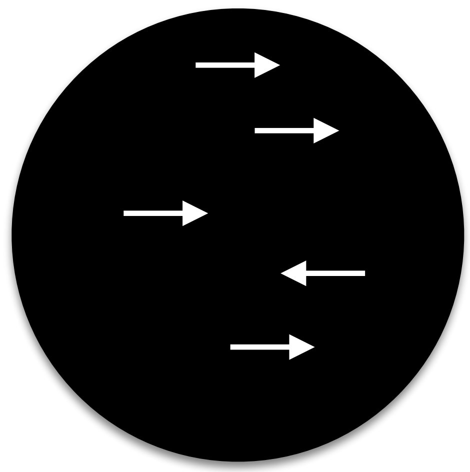

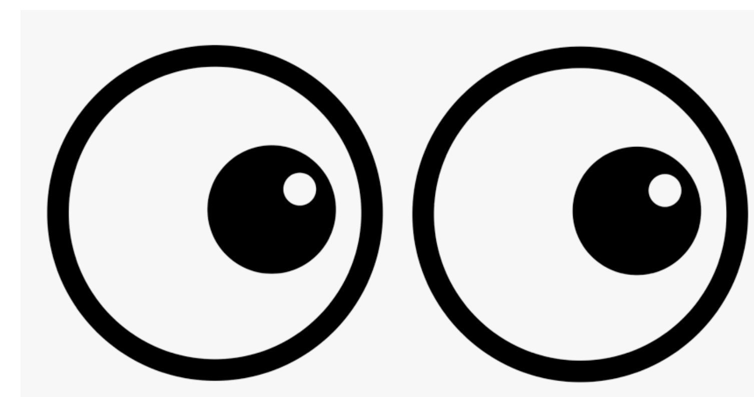

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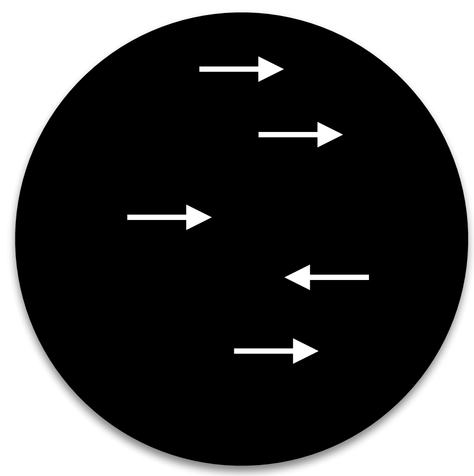
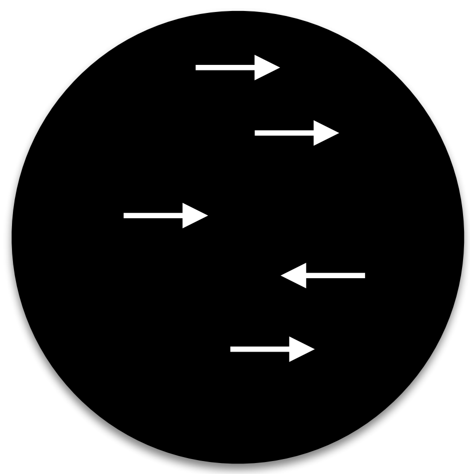


$$p(right|observations) = \frac{p(observations|right)p(right)}{p(observations)}$$

Dot motion evidence accumulation

“True” Motion Direction **?**

What we see:



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Dot motion evidence accumulation

"True" Motion Direction

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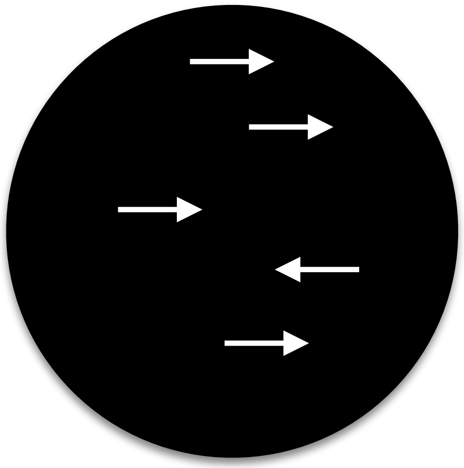

Bayes rule:

$$p(right|observations) = \frac{p(observations|right)p(right)}{p(observations)}$$

Dot motion evidence accumulation

"True" Motion Direction

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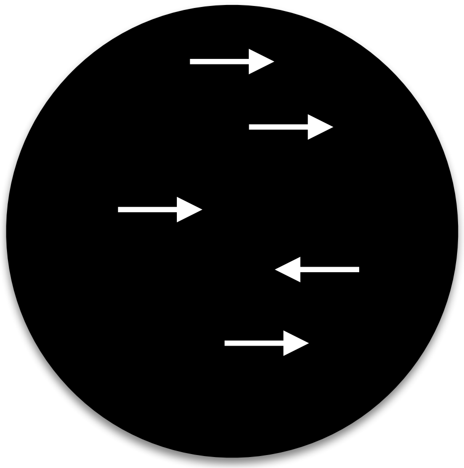

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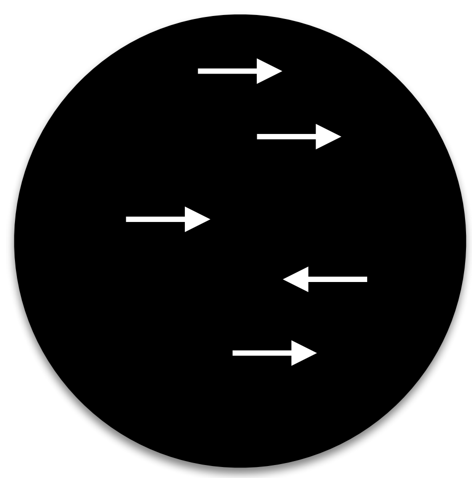

Bayes rule:

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$$p(left|observations) = \frac{p(observations|left)p(left)}{p(observations)}$$

Dot motion evidence accumulation

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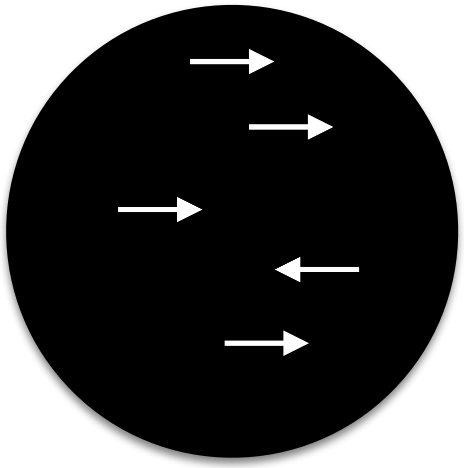

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Dot motion evidence accumulation

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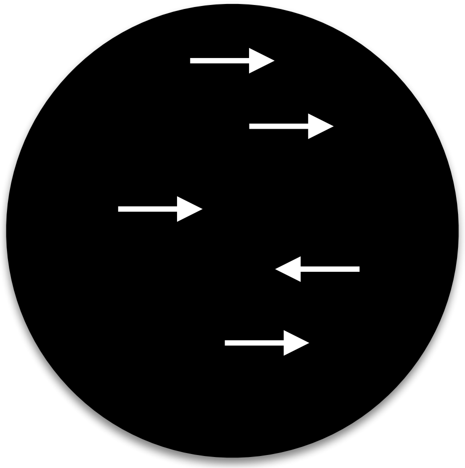

$$\frac{p(right|observations) = \dfrac{p(observations|right)p(right)}{\cancel{p(observations)}}}{p(left|observations) = \dfrac{p(observations|left)p(left)}{\cancel{p(observations)}}}$$

Dot motion evidence accumulation

“True” Motion Direction

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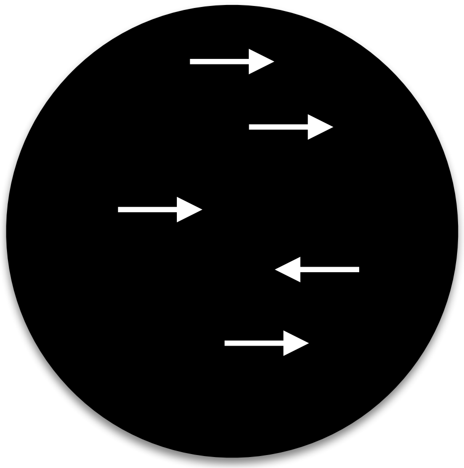

Ratio:

Posterior **Likelihood** **Prior**

$$\frac{p(right|observations)}{p(left|observations)} = \frac{p(observations|right)p(right)}{p(observations|left)p(left)}$$

Dot motion evidence accumulation

"True" Motion Direction

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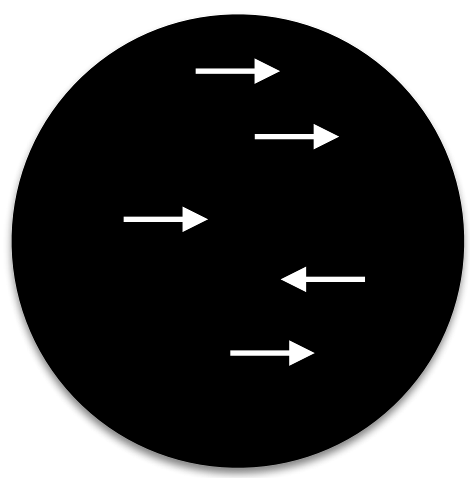

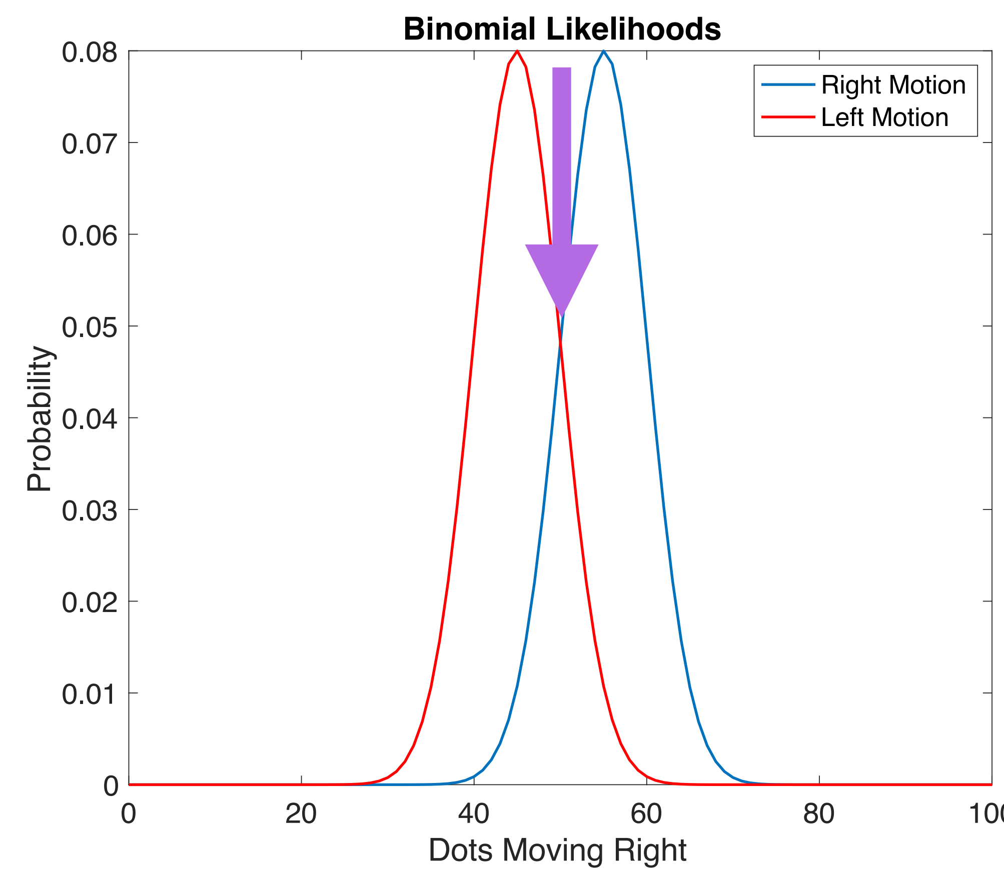

Ratio:

Posterior **Likelihood** **Prior**

$$\frac{p(right|observations)}{p(left|observations)} = \boxed{\frac{p(observations|right)}{p(observations|left)}}\frac{p(right)}{p(left)}$$

Dot motion evidence accumulation

“True” Motion Direction

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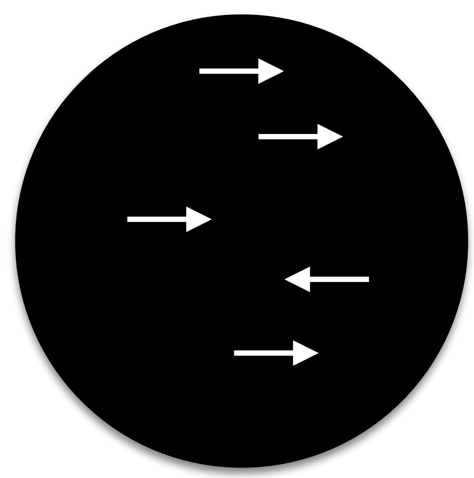

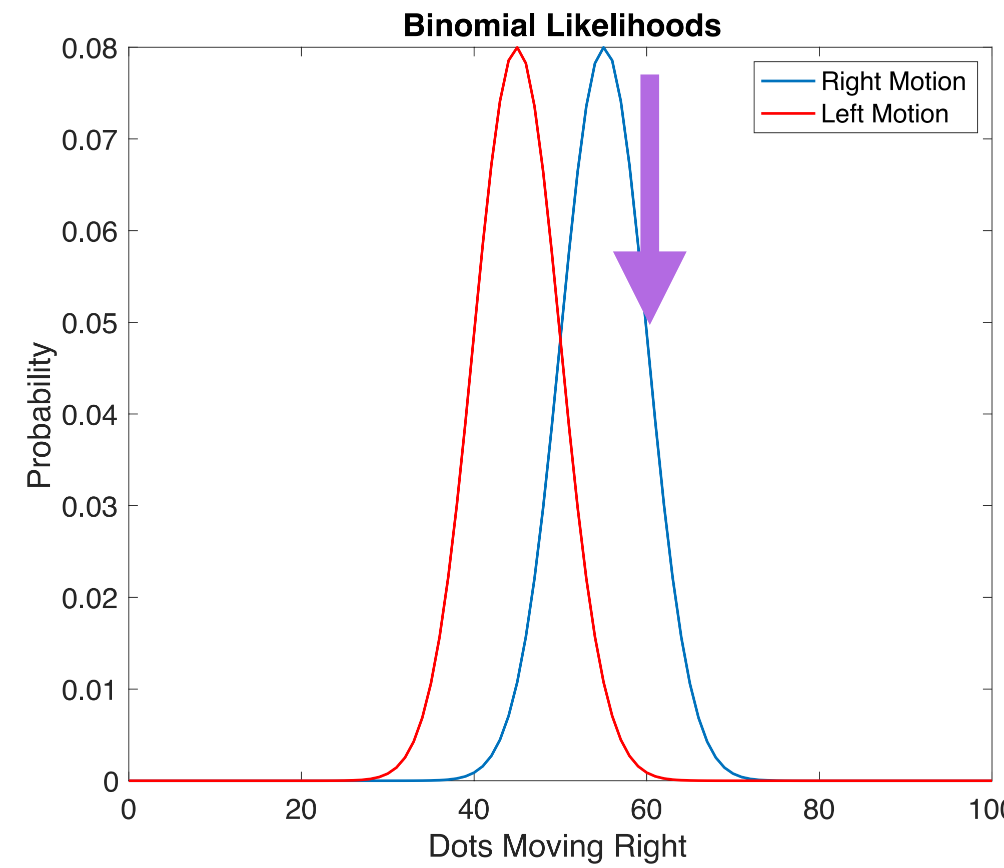

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Dot motion evidence accumulation

"True" Motion Direction

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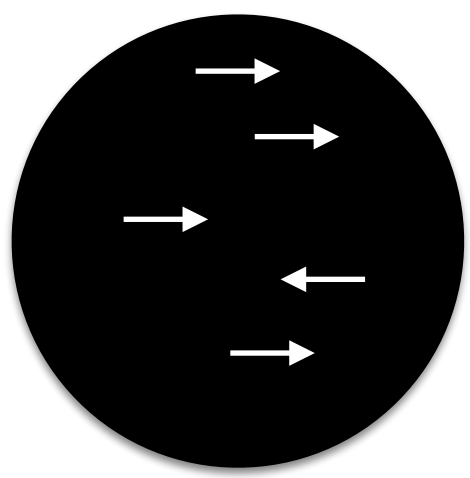

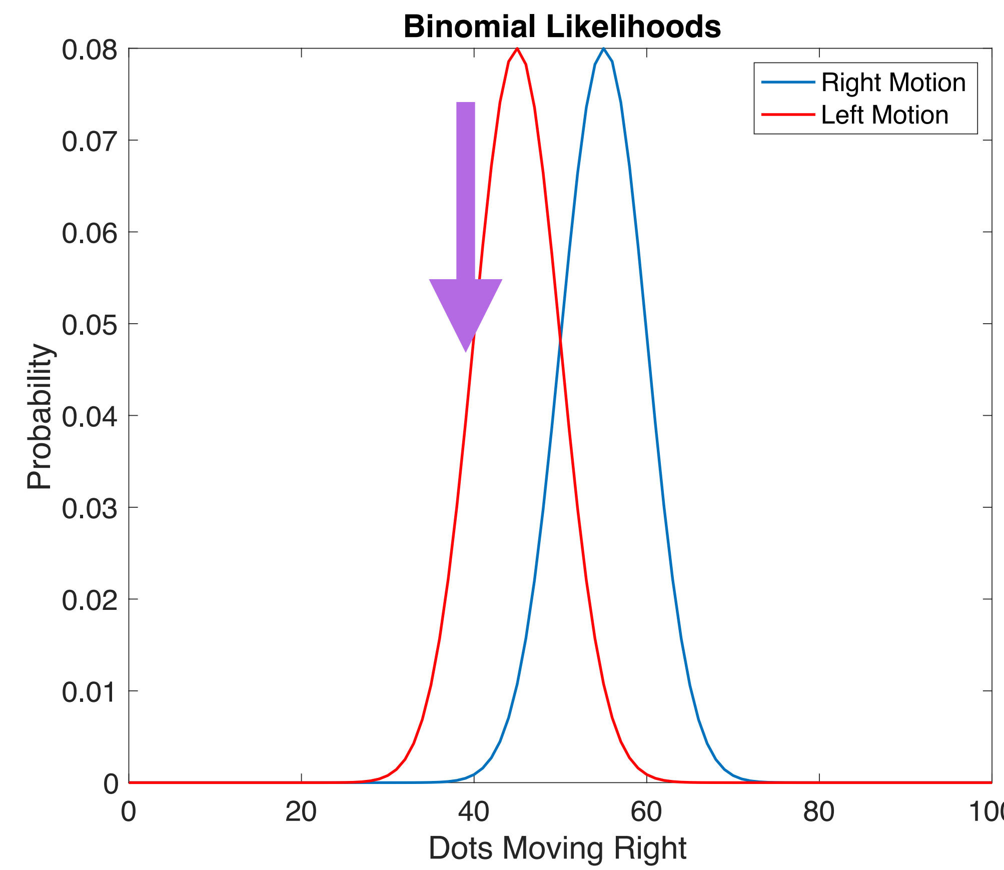

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Posterior **Likelihood** **Prior**

$$\frac{p(right|observations)}{p(left|observations)} = \boxed{\frac{p(observations|right)}{p(observations|left)}}\frac{p(right)}{p(left)}$$

Dot motion evidence accumulation

Log Ratio:

Posterior **Likelihood** **Prior**

$$log\left(\frac{p(right|observations)}{p(left|observations)}\right) = log\left(\frac{p(observations|right)}{p(observations|left)}\right) + log\left(\frac{p(right)}{p(left)}\right)$$

Dot motion evidence accumulation

Log Ratio:

Posterior **Likelihood** **Prior**

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Dot motion evidence accumulation

Log Ratio:

Posterior **Likelihood**

$$log\left(\frac{p(right|observations)}{p(left|observations)}\right) = log\left(\frac{p(observations|right)}{p(observations|left)}\right) + log\left(\frac{p(observations|right)}{p(observations|left)}\right) + \ldots$$

timestep 1 **timestep 2** **…**

Dot motion evidence accumulation

Log Ratio:

Posterior **Likelihood** **Prior**

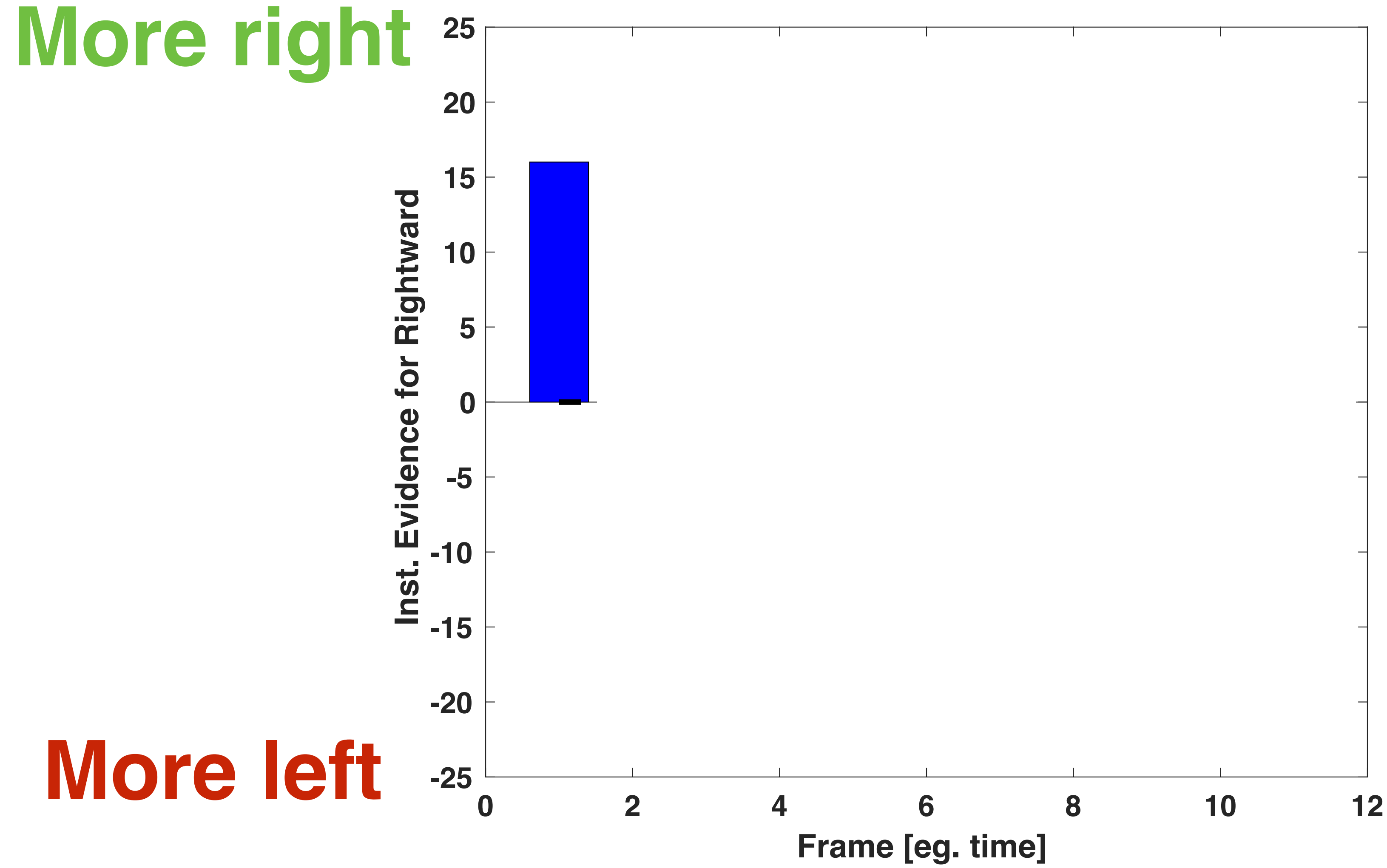
$$\boxed{log\left(\frac{p(right|observations)}{p(left|observations)}\right)} = log\left(\frac{p(observations|right)}{p(observations|left)}\right) + log\left(\frac{p(right)}{p(left)}\right)$$

Dot motion evidence accumulation

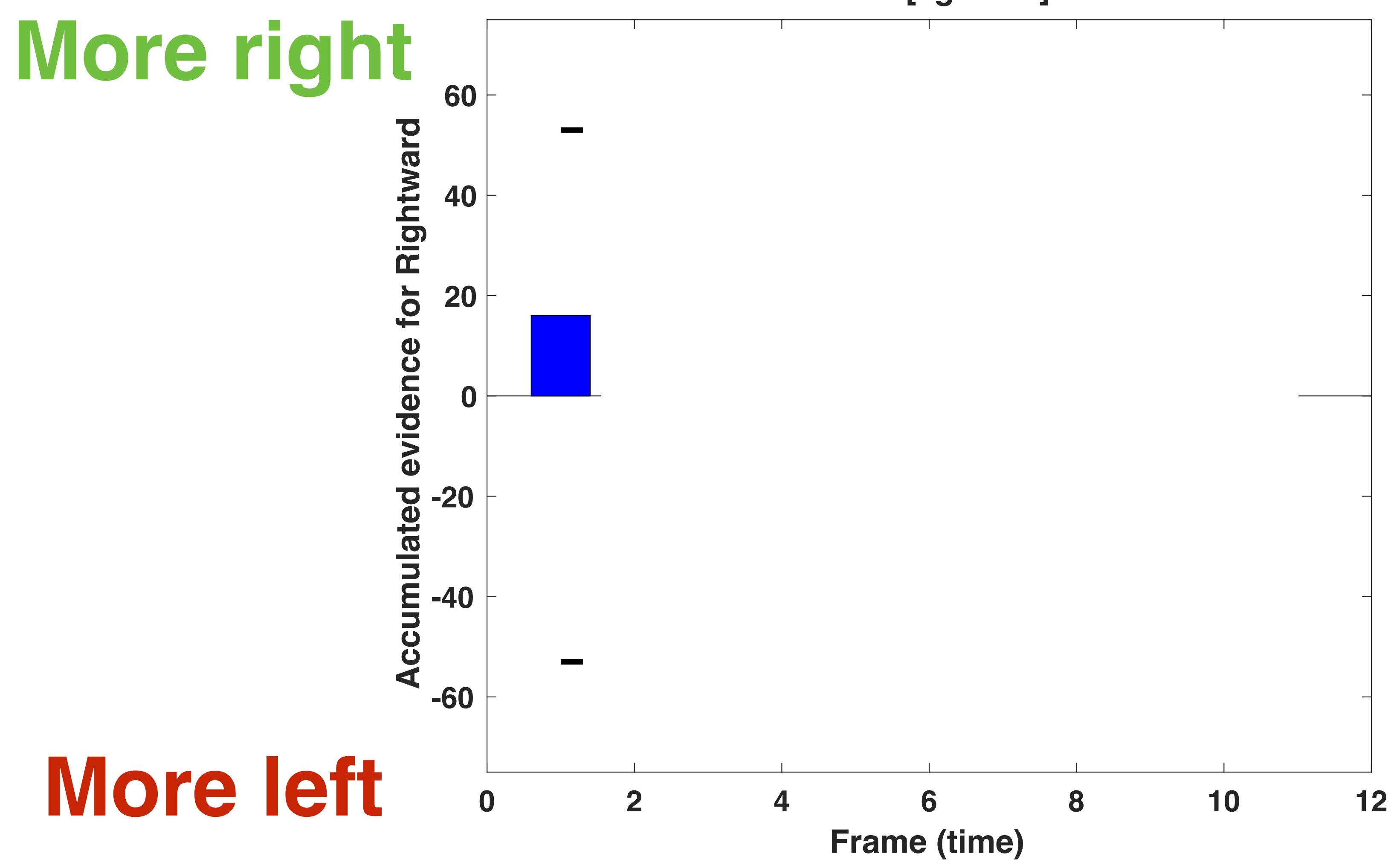
Log Ratio:

$$\underbrace{\boxed{log\left(\frac{p(right|observations)}{p(left|observations)}\right)}}_{\text{Posterior}} = \underbrace{log\left(\frac{p(observations|right)}{p(observations|left)}\right)}_{\text{Likelihood}} + \underbrace{\boxed{log\left(\frac{p(right)}{p(left)}\right)}}_{\text{Prior}}$$

Instantaneous evidence



Accumulated evidence



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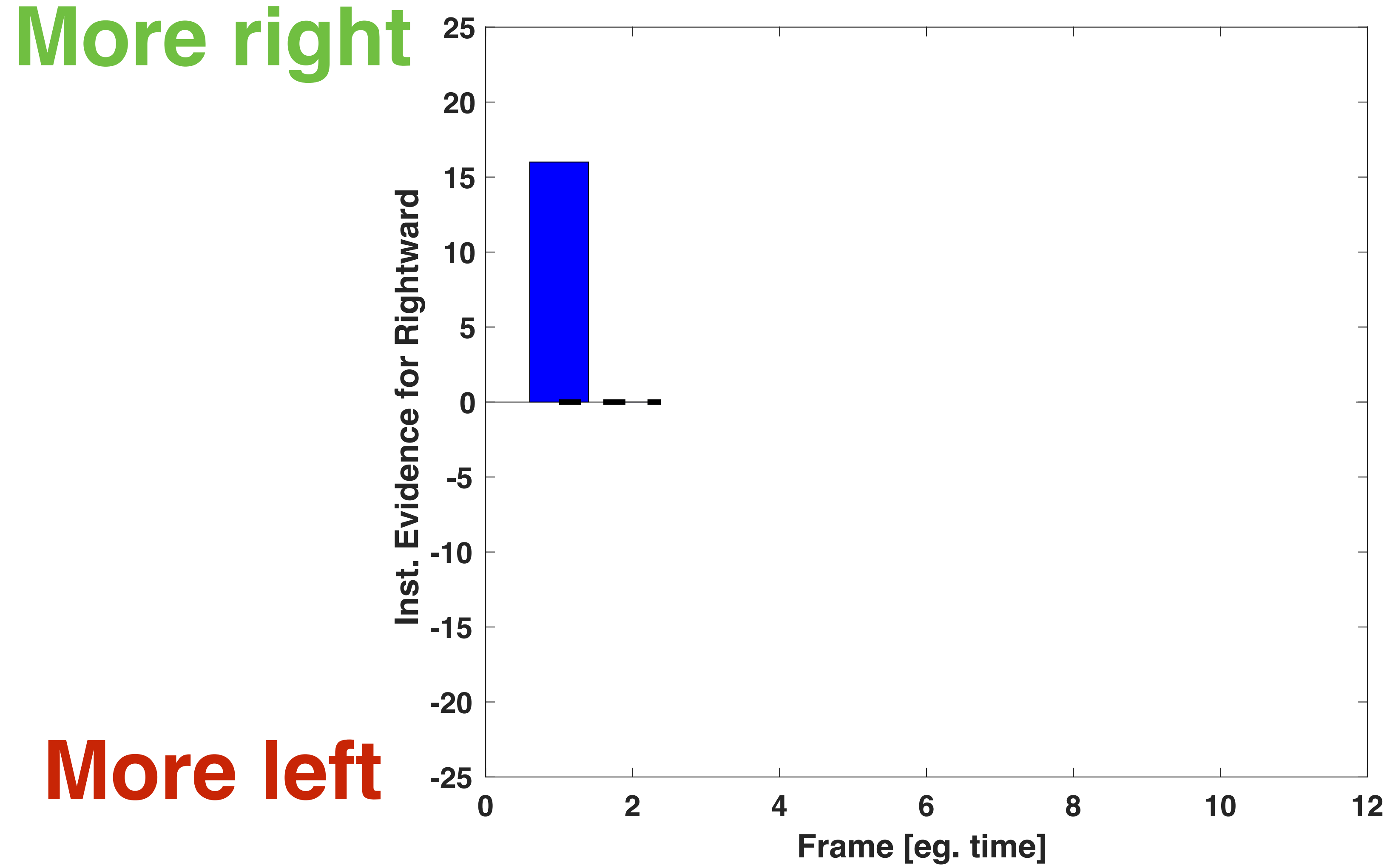

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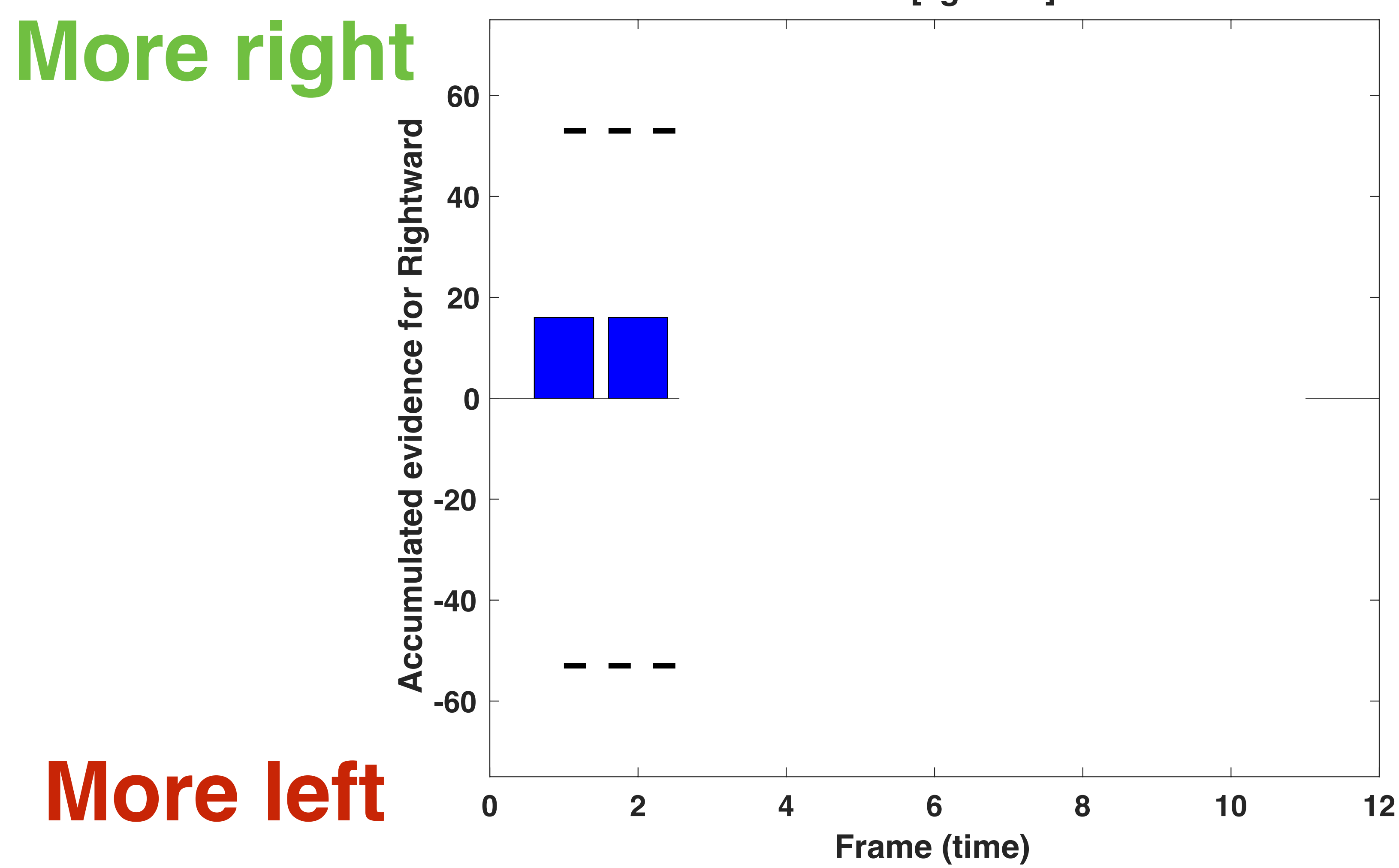

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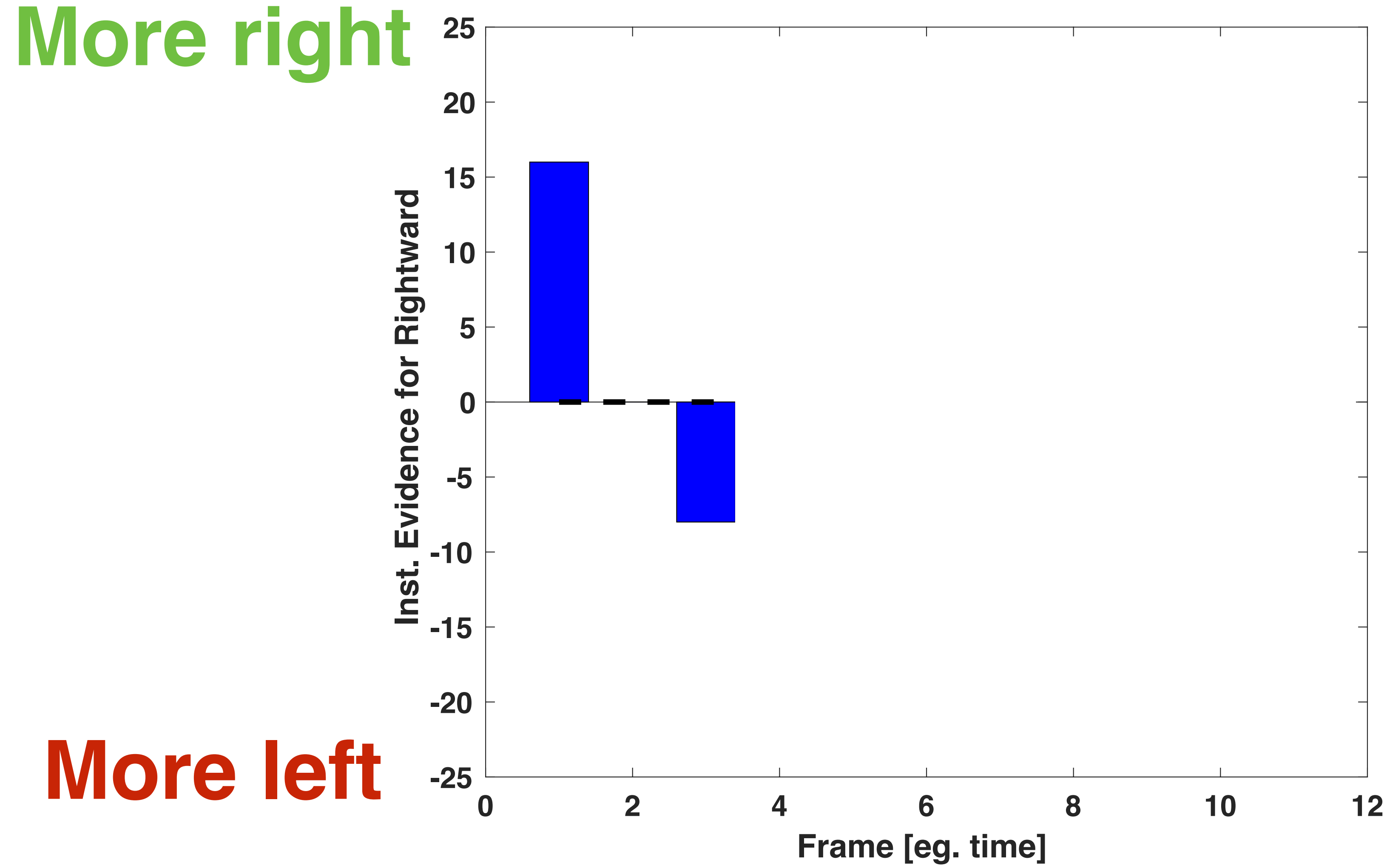

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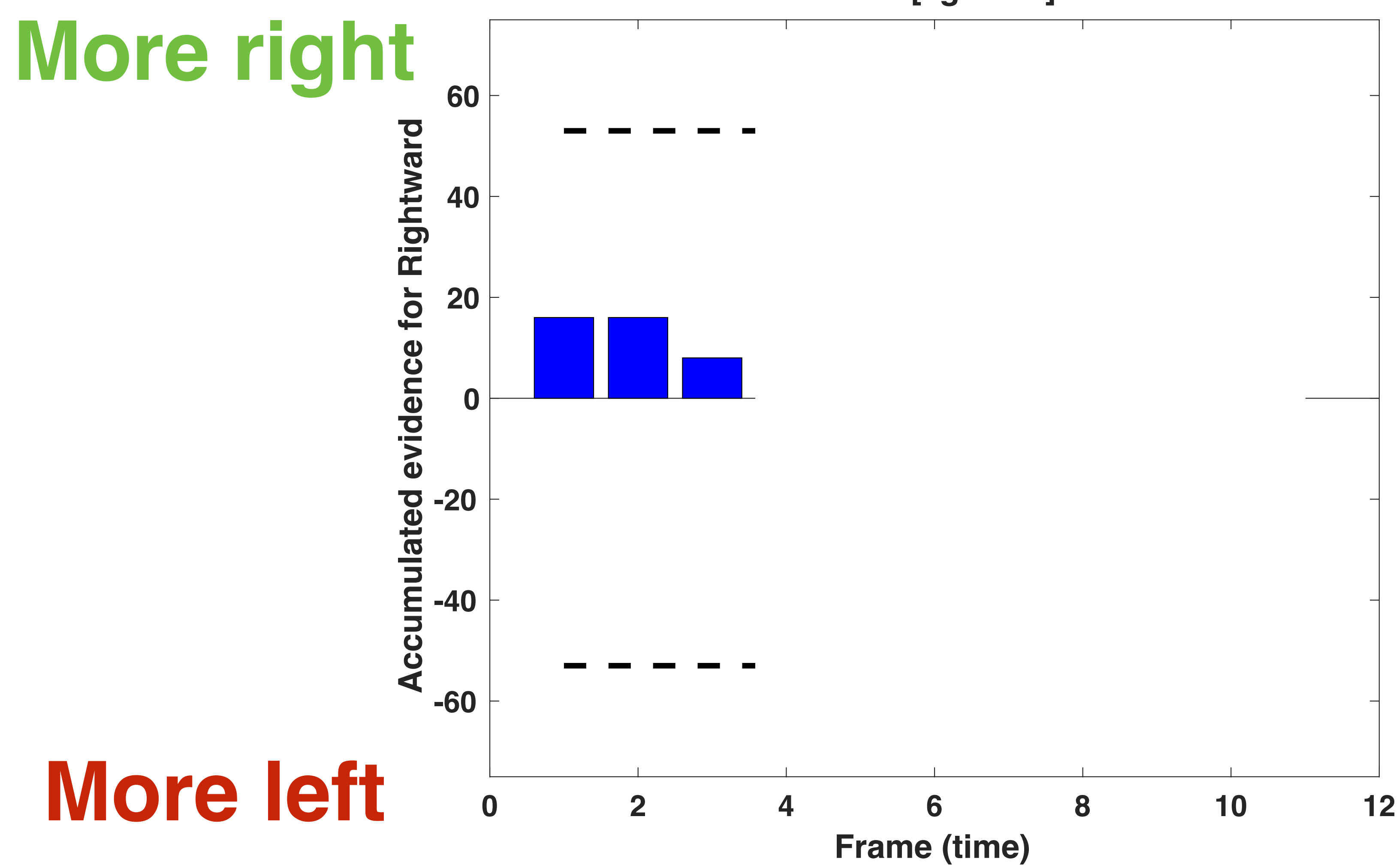
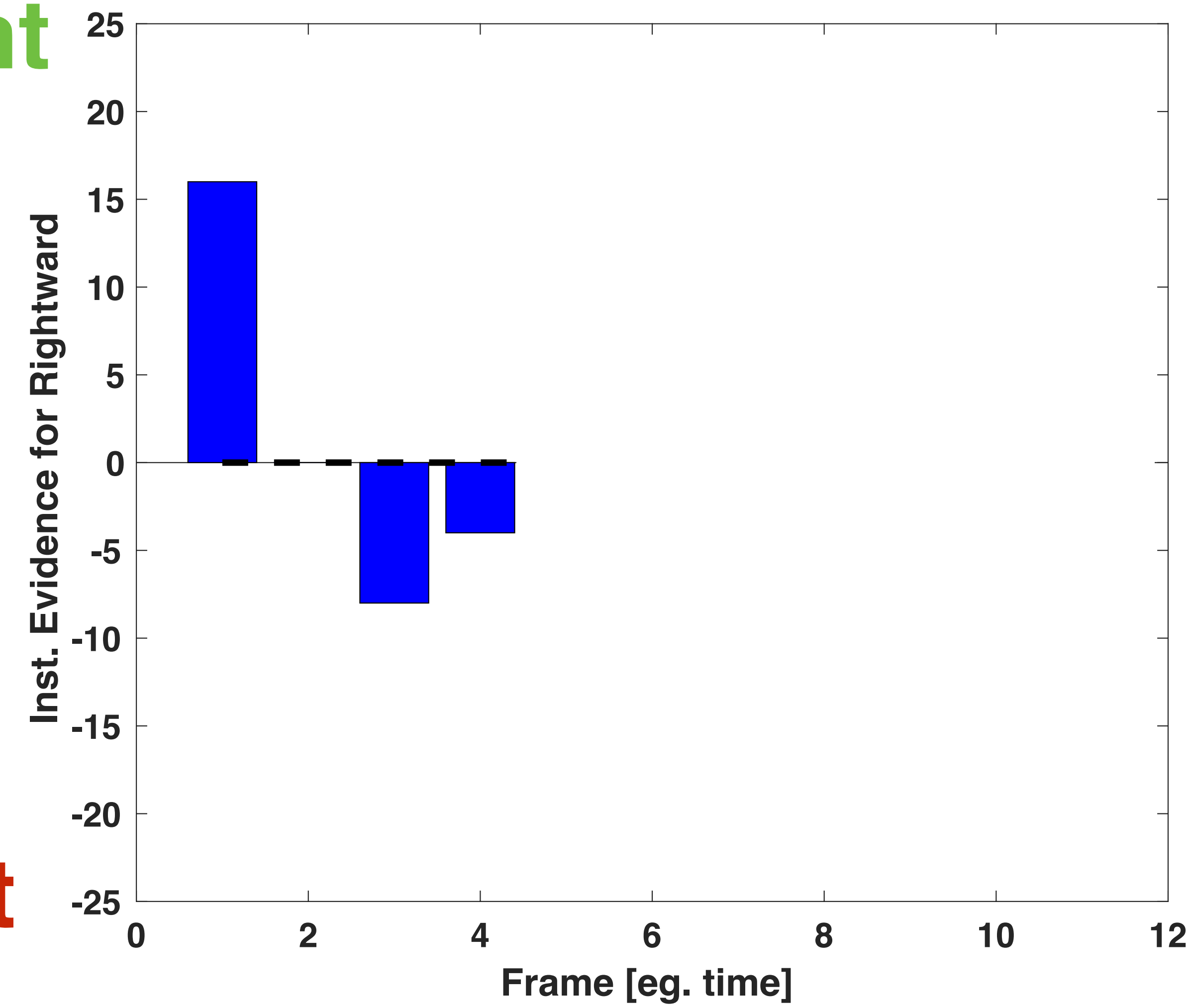


Instantaneous evidence

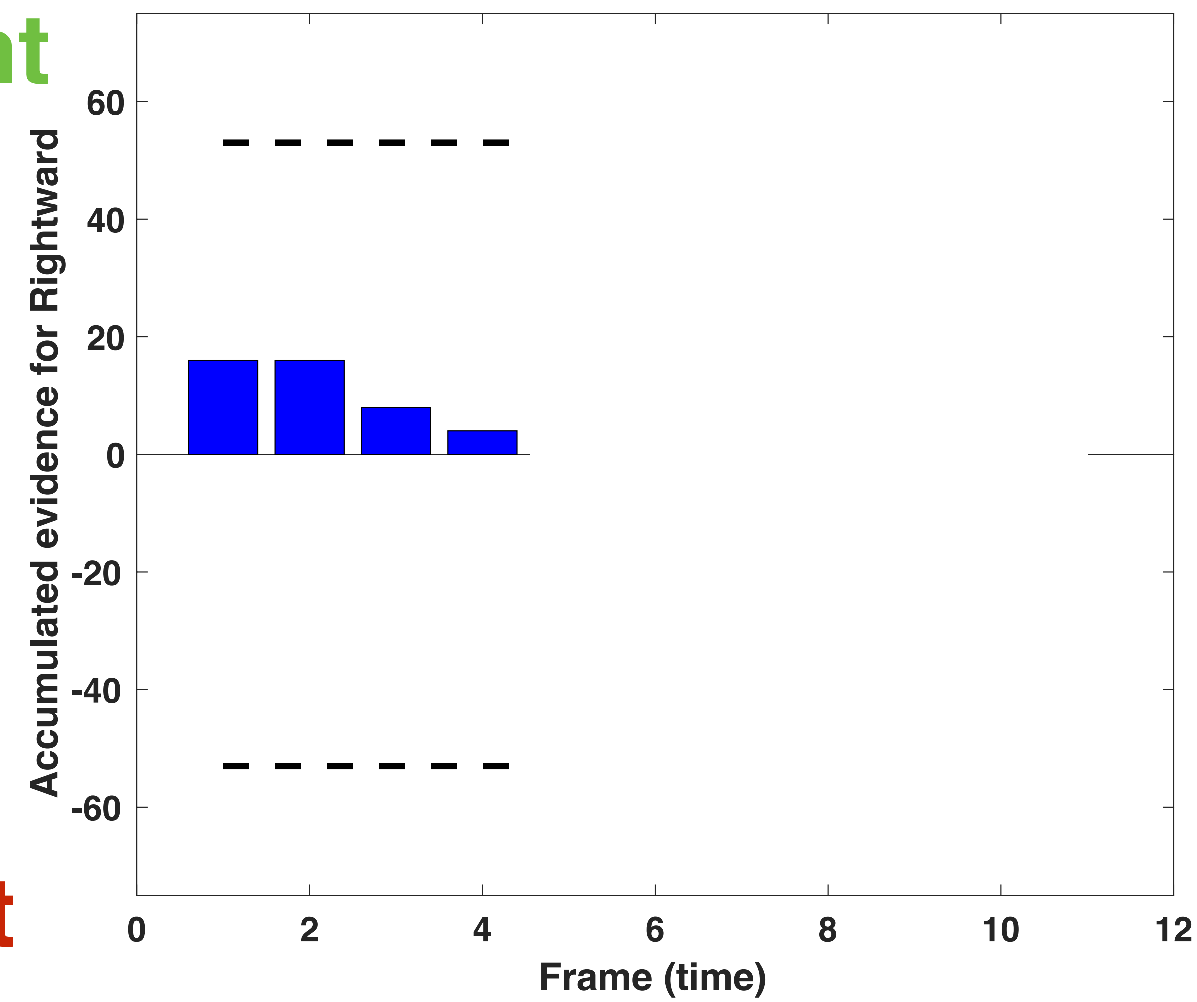
More right



More left

Accumulated evidence

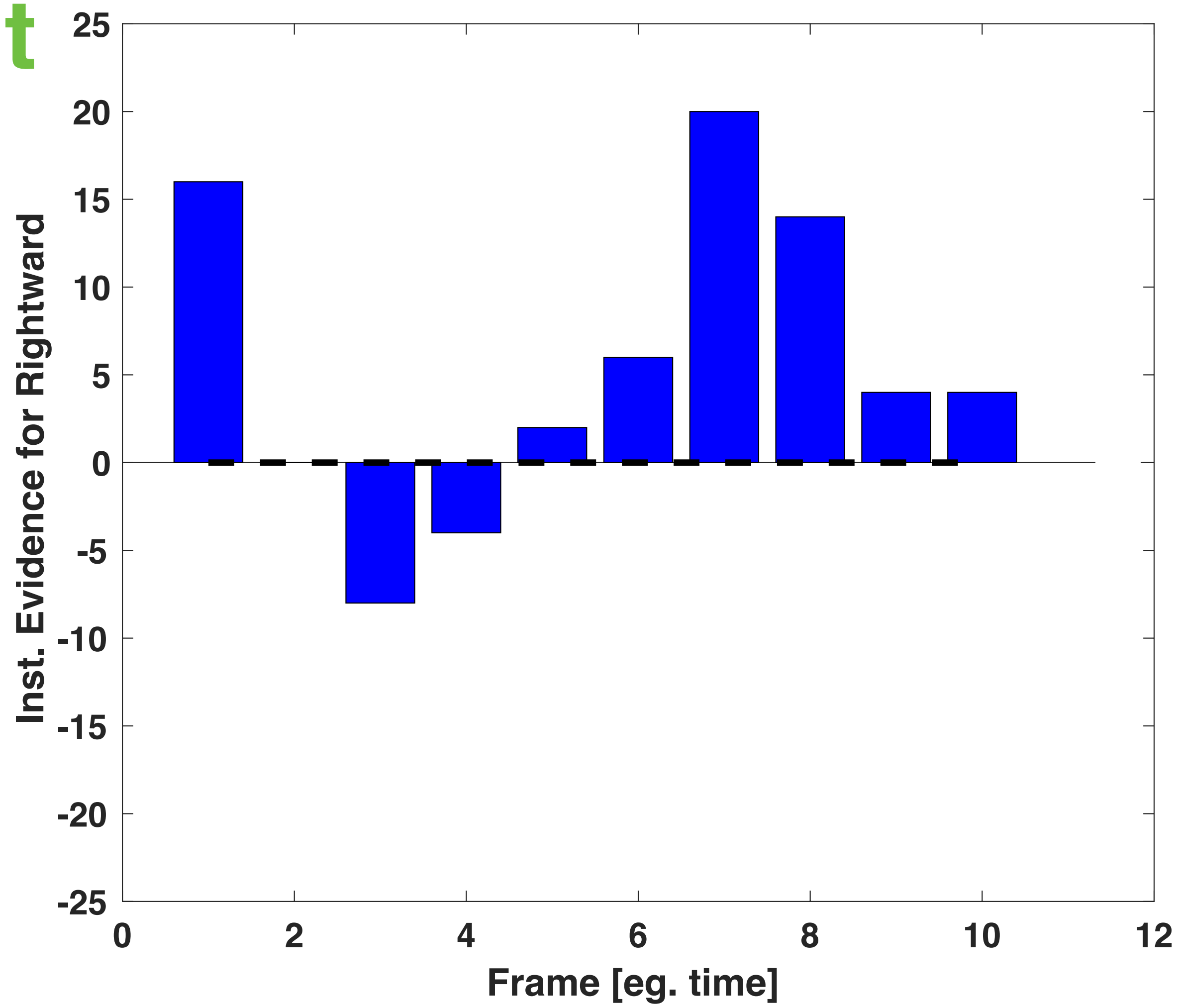
More right



More left

Instantaneous evidence

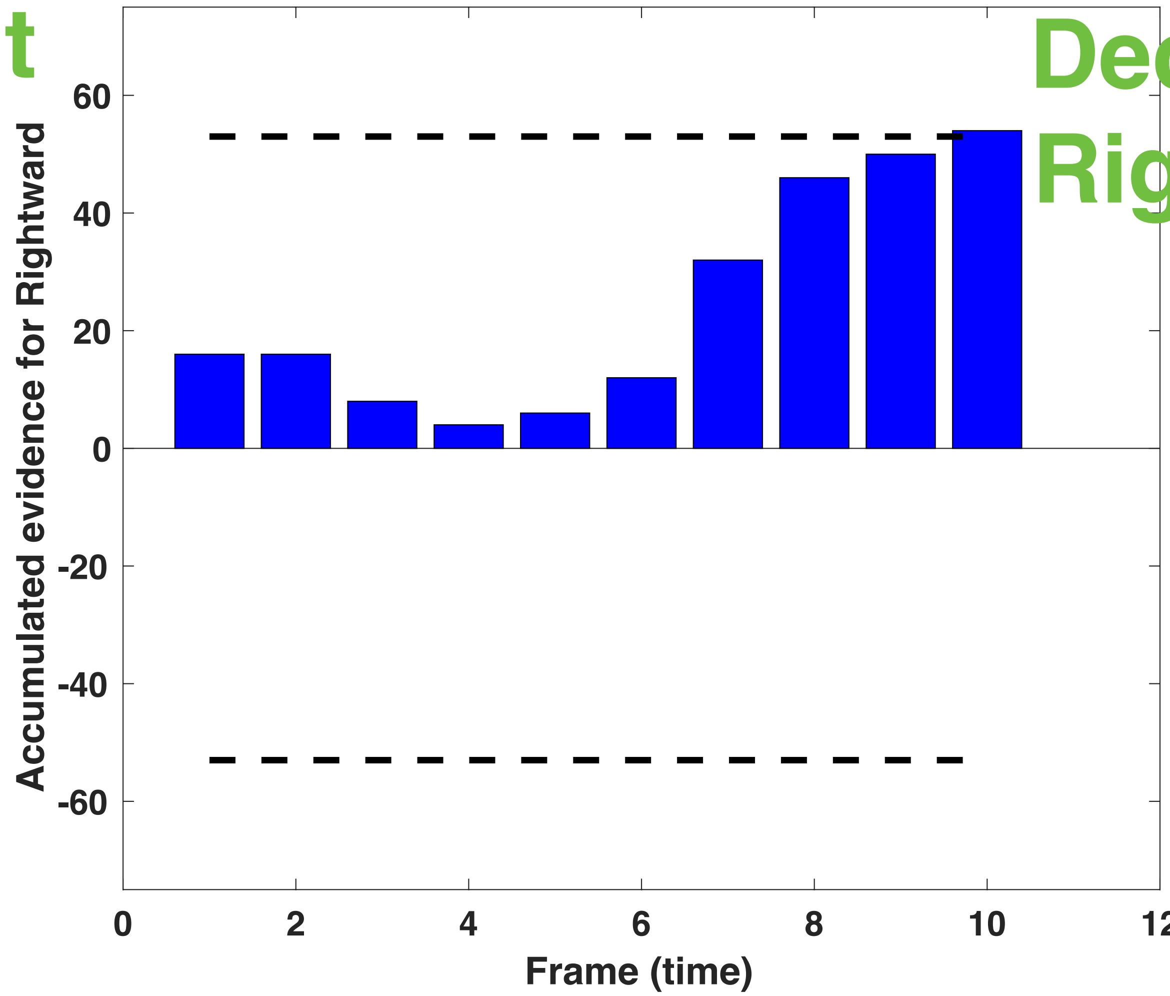
More right



More left

Accumulated evidence

More right

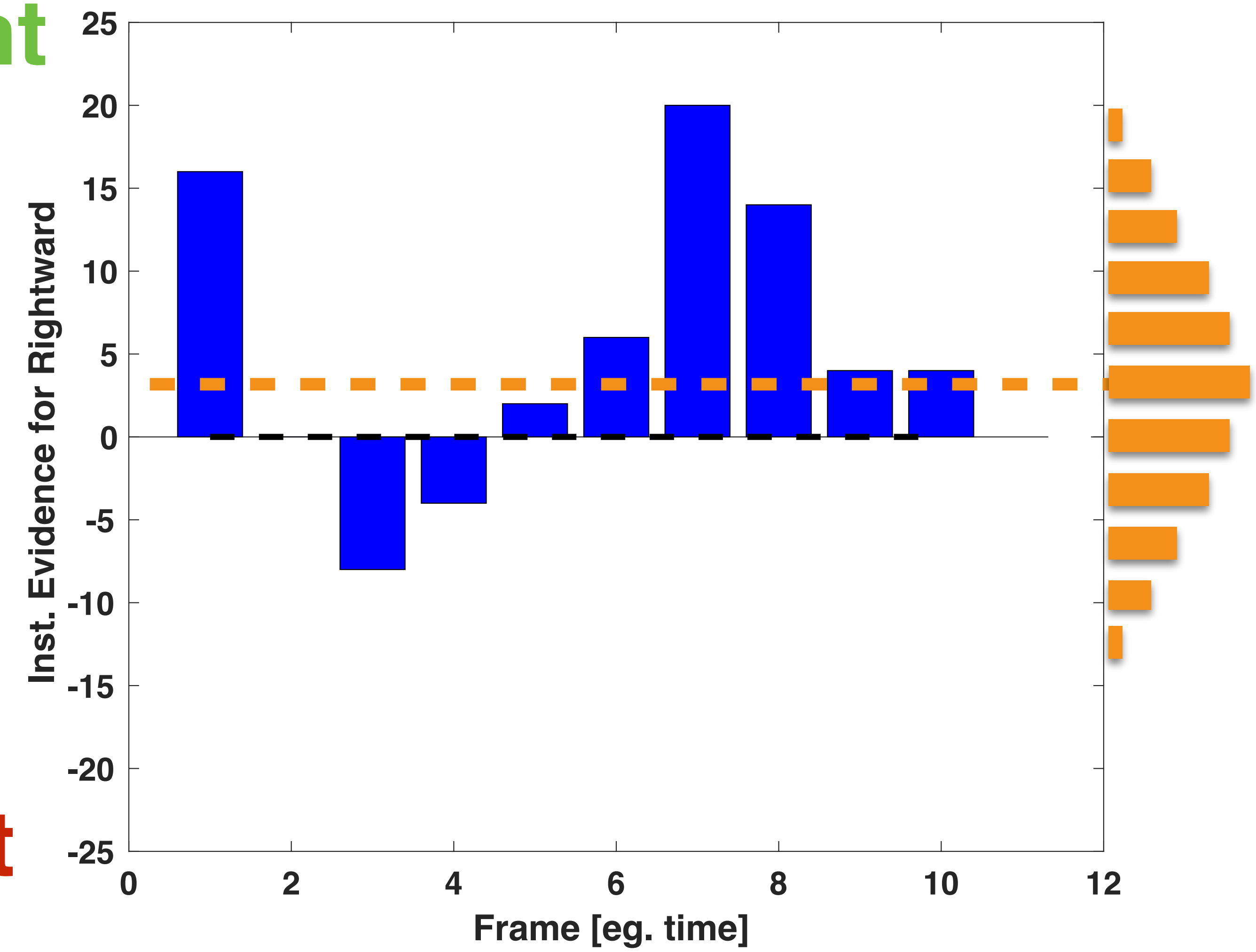


Decide Right!!

More left

More right

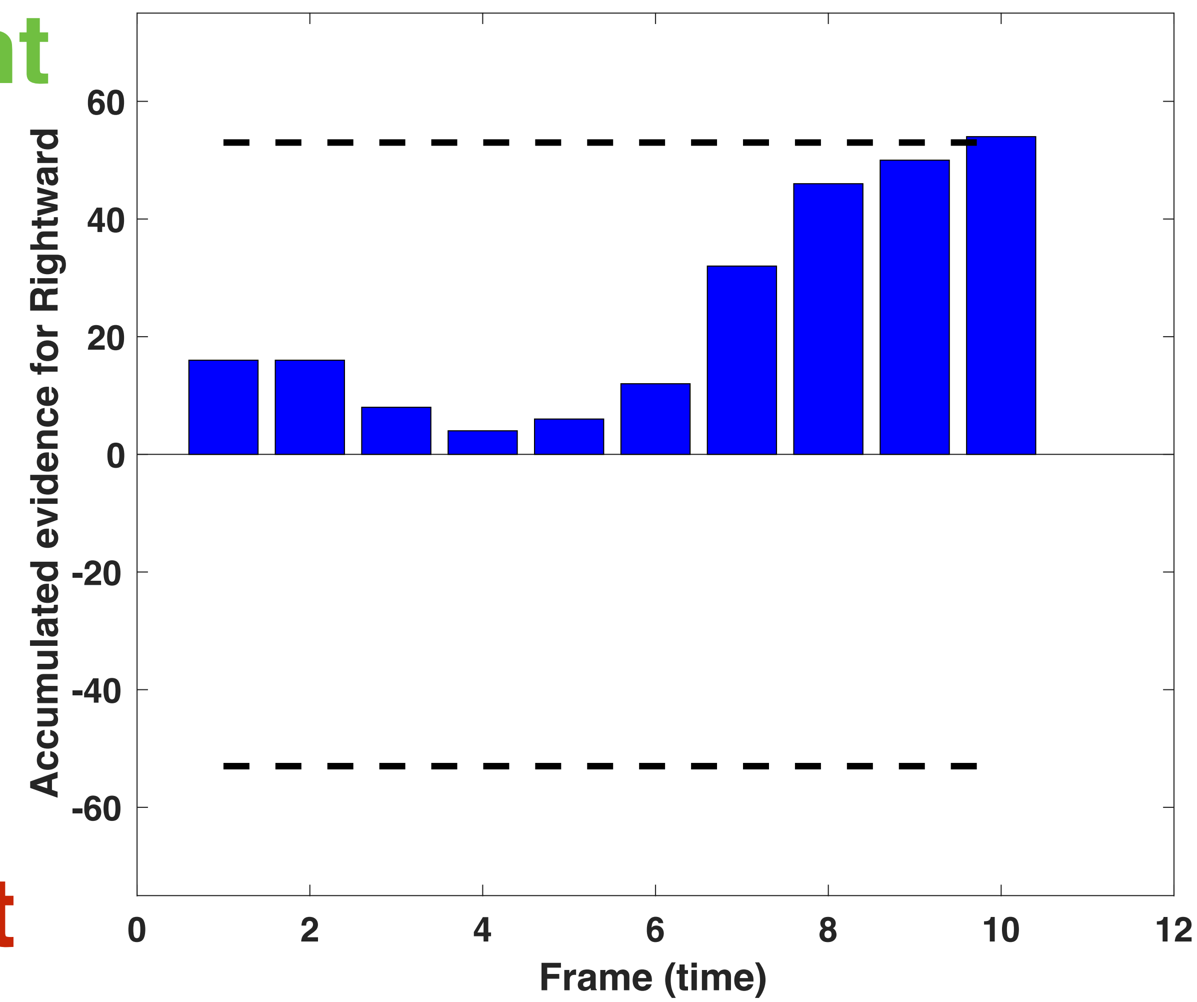
Instantaneous evidence



More left

More right

Accumulated evidence



More left

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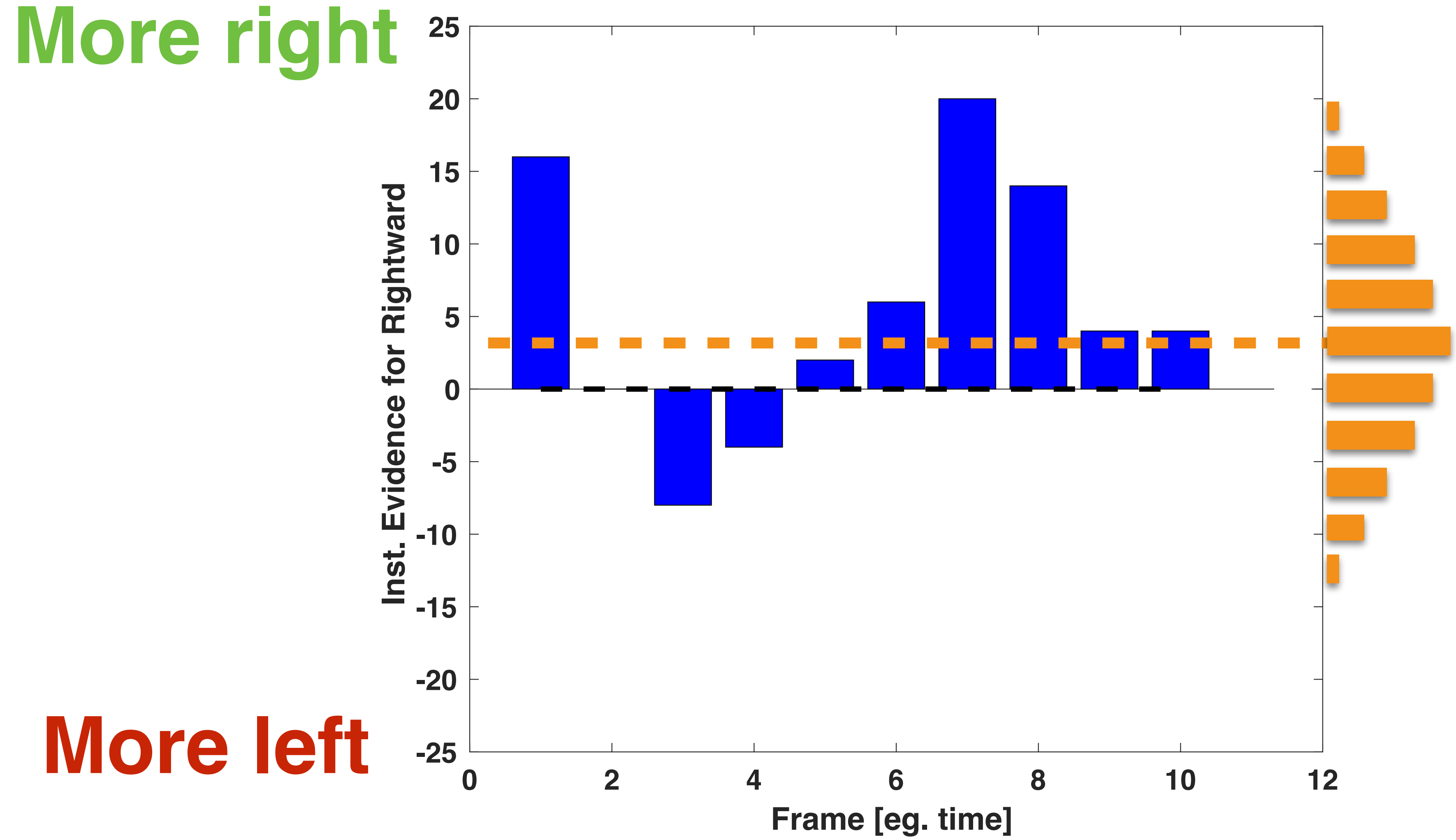
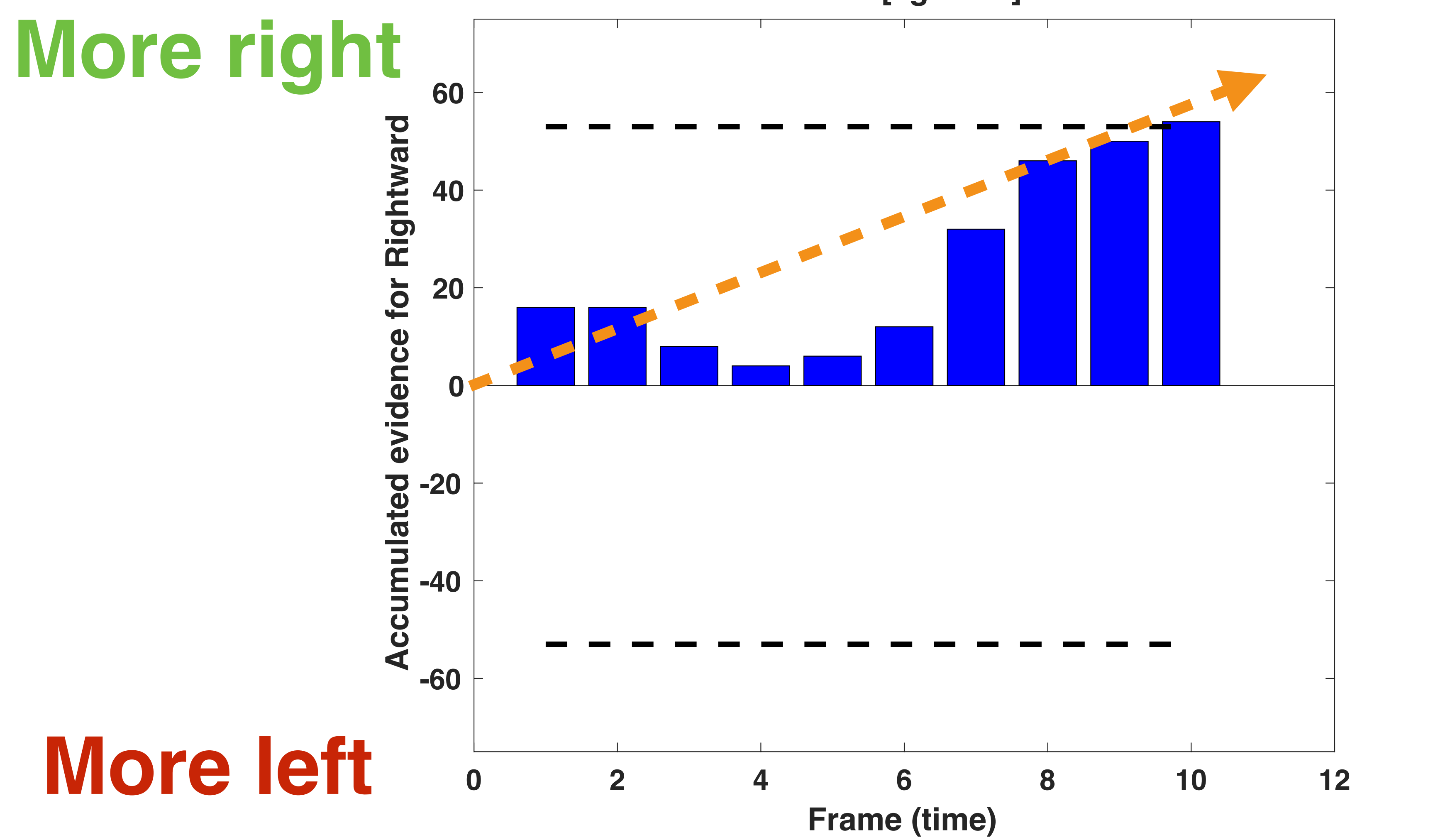


Accumulated evidence



Drift diffusion model:

Drift diffusion model:

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Motivated by the critical dissipative quasi-geostrophic equation, we prove that **drift-diffusion** equations with L^2 initial data and minimal assumptions on the drift are locally Hölder continuous. As an application we show that solutions of the quasi-geostrophic equation with …
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TV Wiecki, I Sofer, MJ Frank - Frontiers in neuroinformatics, 2013 - frontiersin.org
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… A basic goal of decision **neuroscience** and neuroeconomics is to characterize the computations carried out by the brain to make different … Over the last decade, a sizable number of studies have found that standard **drift-diffusion**-models (DDM; Ratcliff, 1978, 2002; Busemeyer …
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MJ Mulder, L Van Maanen, BU Forstmann - **Neuroscience**, 2014 - Elsevier
… (A) **Drift Diffusion** Model … The challenge for cognitive **neuroscience** is to develop clever experimental designs to separate the multicolinearity between these … Crucially, in experimental psychology as well as the cognitive **neurosciences**, the SAT is thought to affect specifically a …
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The **drift diffusion** model can account for the accuracy and reaction time of value-based choices under high and low time pressure
MM Mormann, J Malmaud, A Huth, C Koch… - … and Decision Making, 2010 - papers.ssrn.com
… The **drift diffusion** model (DDM) is one of the corner- stones of modern psychology (Ratcliff … & Smith, 2004; Smith & Ratcliff, 2004) and, increasingly, of behavioral **neuro**- science (Bogacz … An important open problem in behavioral **neuroscience** is how the brain compares values to …
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Drift diffusion model:

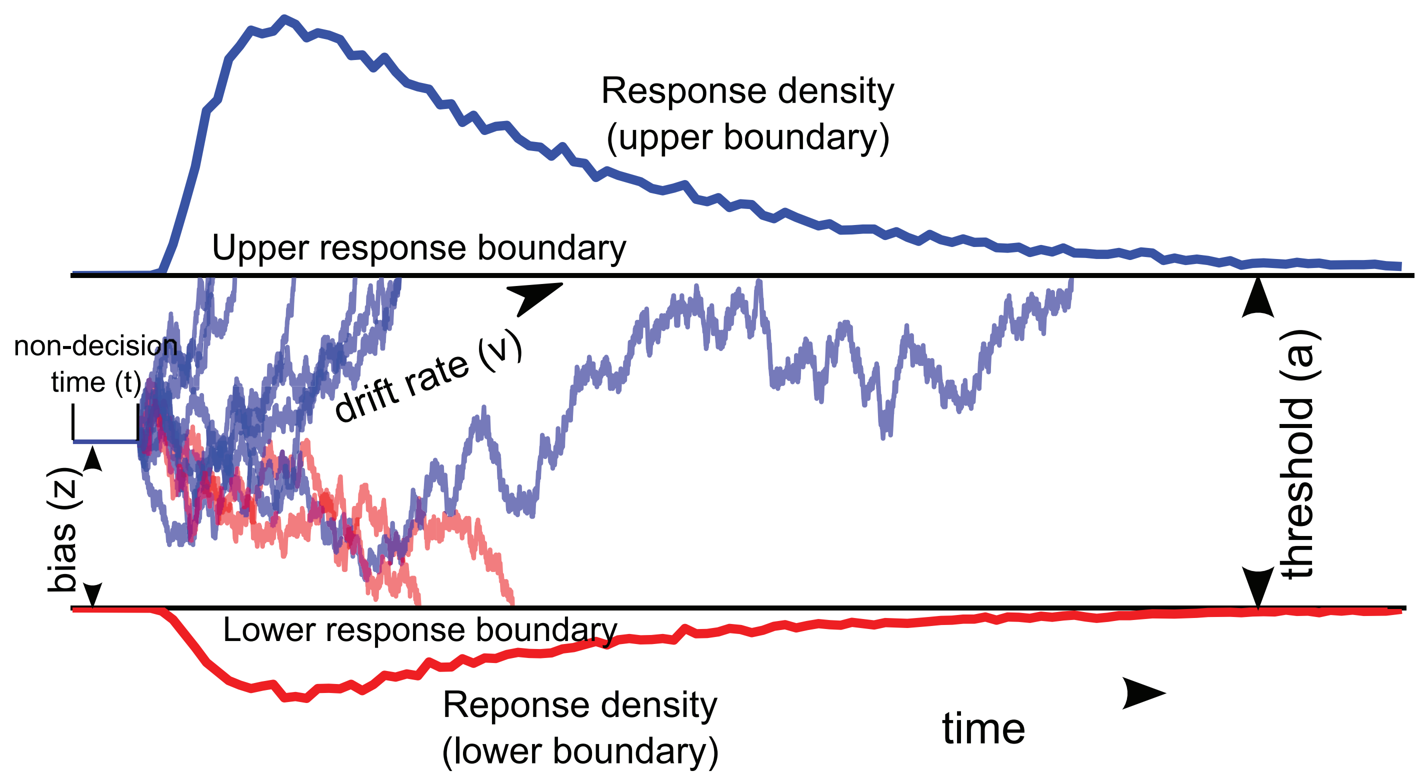
Key advance:

Model both choice AND reaction time!!!

Drift diffusion model:

Key advance:

Model both choice AND reaction time!!!



Wiecki & Frank

Drift diffusion model:

Our simple model with the following changes:

Drift diffusion model:

Our simple model with the following changes:

1) accumulate evidence over time 

Drift diffusion model:

Our simple model with the following changes:

1) accumulate evidence over time
2) make a decision after reaching a criterion

Drift diffusion model:

Our simple model with the following changes:

1) accumulate evidence over time
2) make a decision after reaching a criterion
3) normal distribution of evidence

Drift diffusion model formalism:

Parameters:

A = Drift rate
y0 = Starting point
c = Drift noise (std)
z = Decision threshold

Variables:

y = Accumulated evidence
t = timestep

Drift diffusion model formalism:

Parameters:

A = Drift rate
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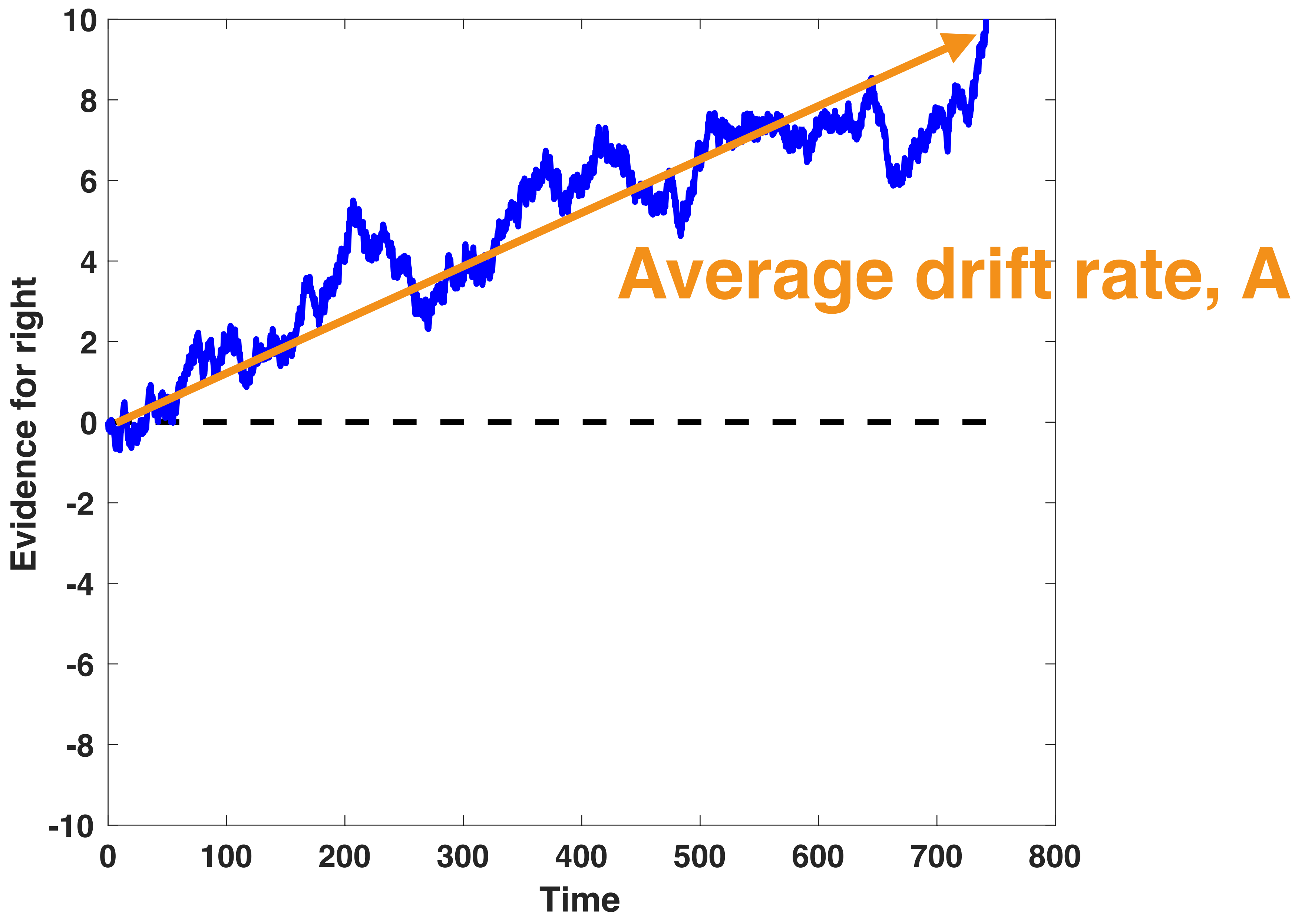
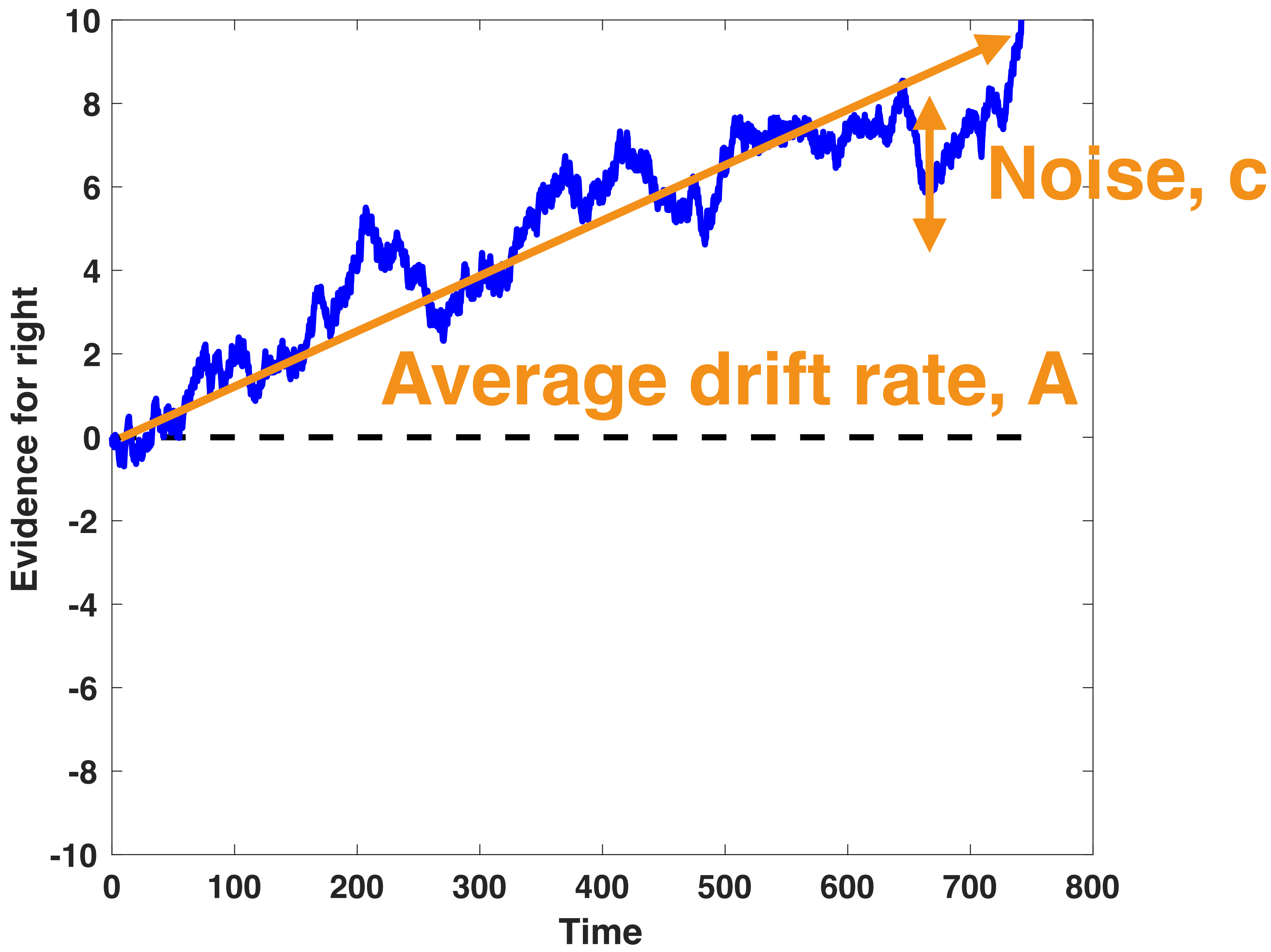
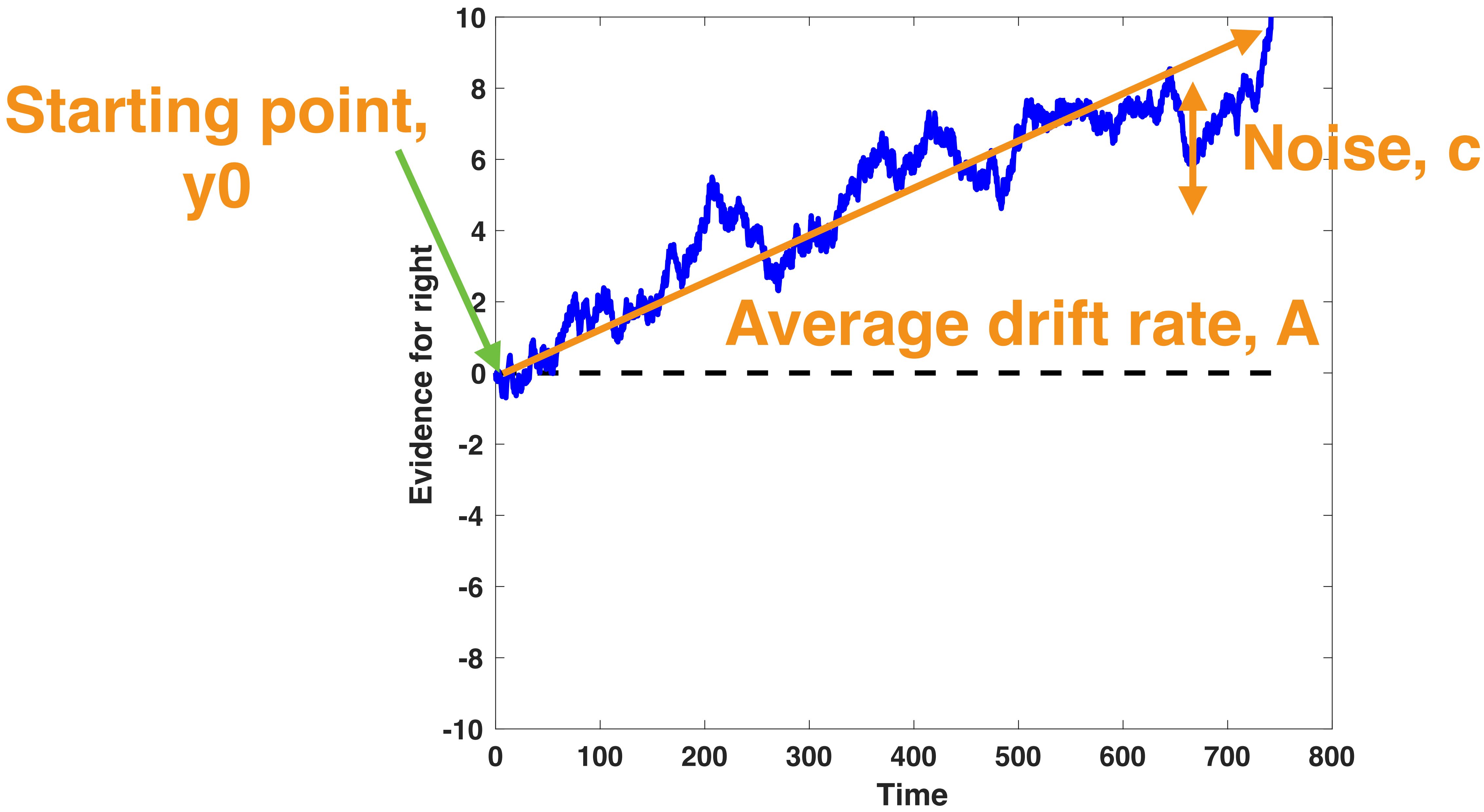
Drift diffusion model:



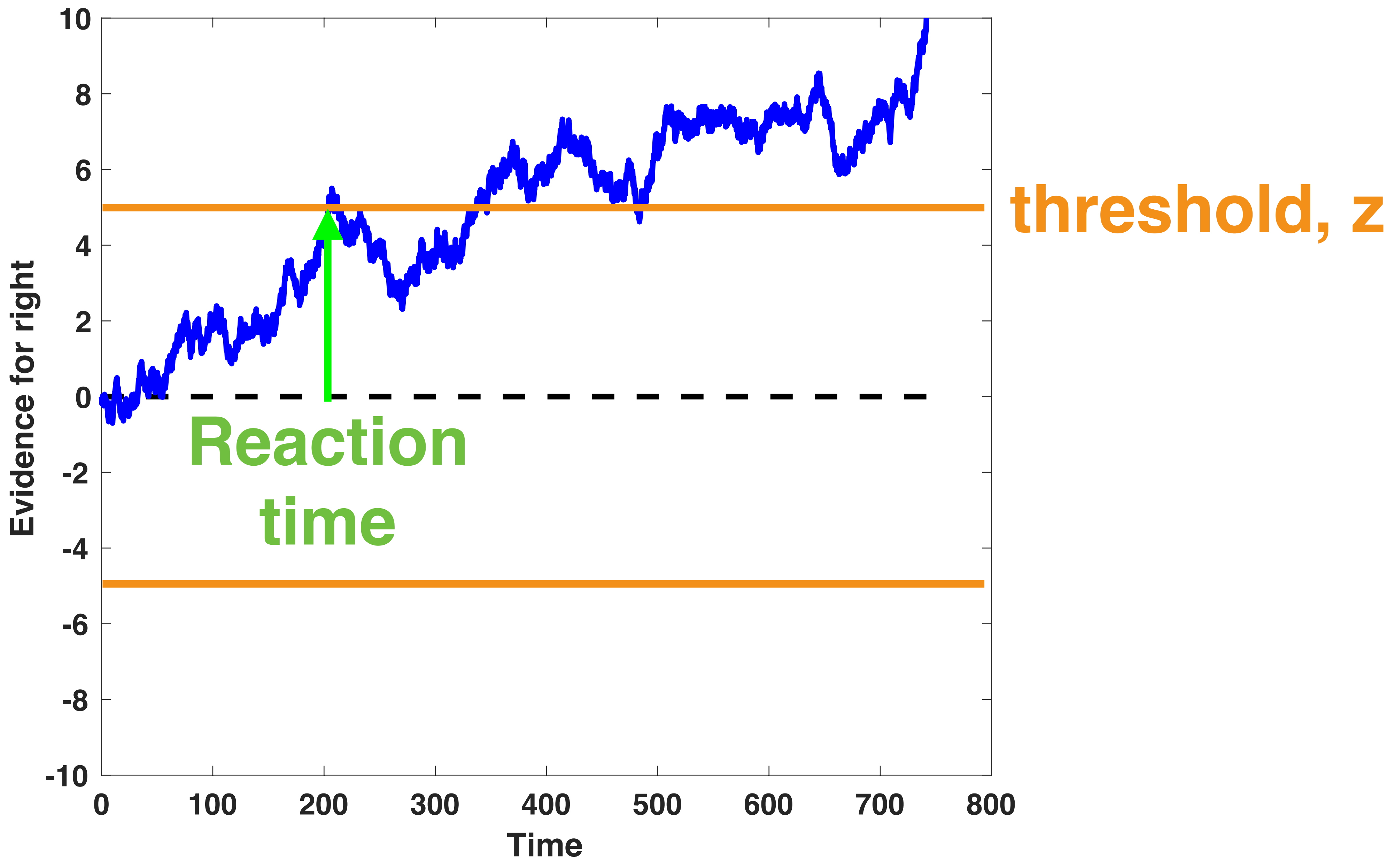
Drift diffusion model:



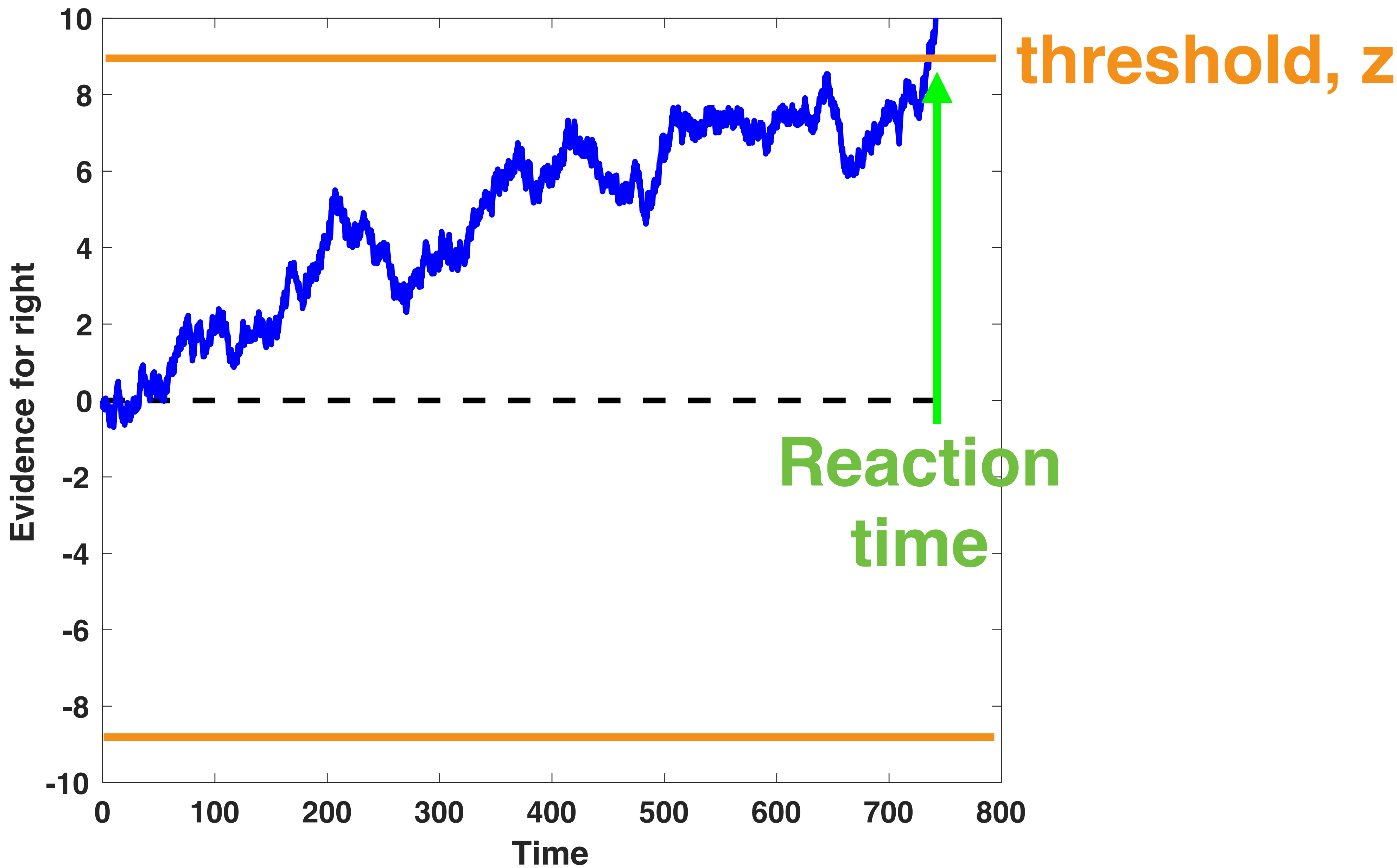
Drift diffusion model:



Drift diffusion model:



Drift diffusion model:



Drift diffusion model formalism:

Parameters:

$\boxed{\begin{array}{l} A = \text{Drift rate} \\ y0 = \text{Starting point} \\ c = \text{Drift noise (std)} \\ z = \text{Decision threshold} \end{array}}$

Variables:

y = Accumulated evidence
t = timestep

Drift diffusion model formalism:

Parameters:

A = Drift rate
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z = Decision threshold

Variables:

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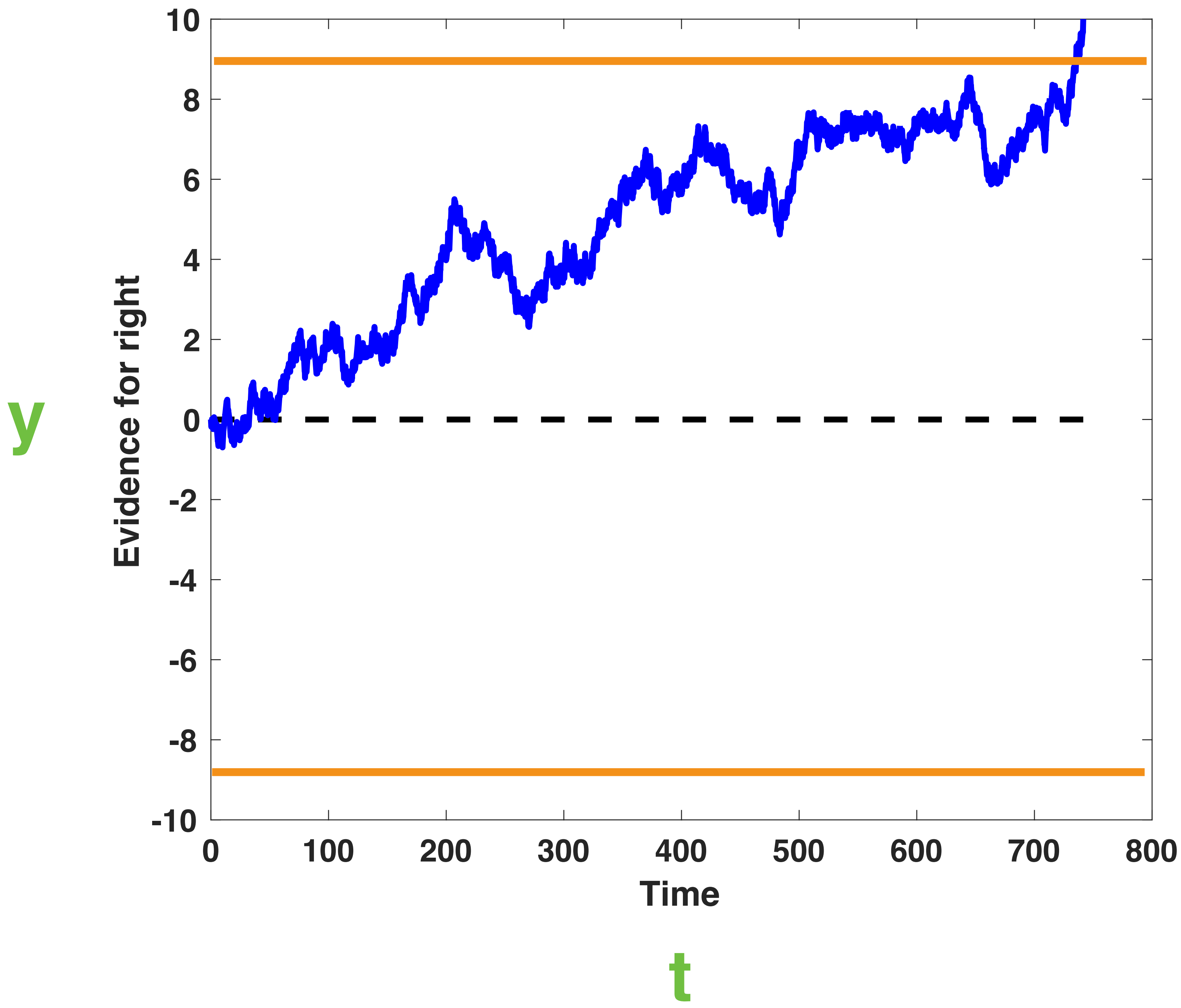
Drift diffusion model:



Drift diffusion model formalism:

Parameters:

A = Drift rate
y0 = Starting point
c   = Drift noise (std)
z   = Decision threshold

Variables:

y   = Accumulated evidence
t   = timestep

Drift diffusion model formalism:

Parameters:

A = Drift rate
y0 = Starting point
c = Drift noise (std)
z = Decision threshold

Additional parameters:

Non-decision time
Start point variability
Drift rate variability

Variables:

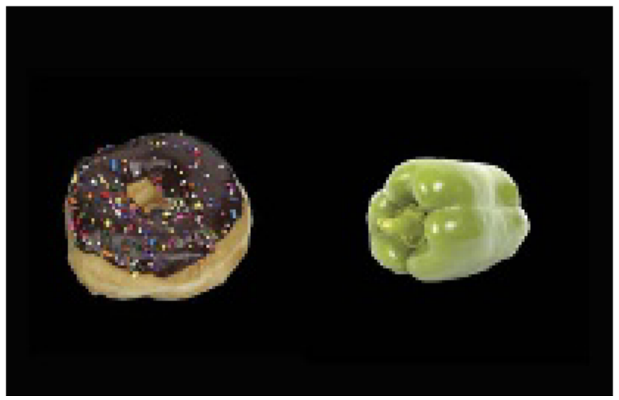
y = Accumulated evidence
t = timestep

Simulation?

Drift diffusion for other sorts of decisions

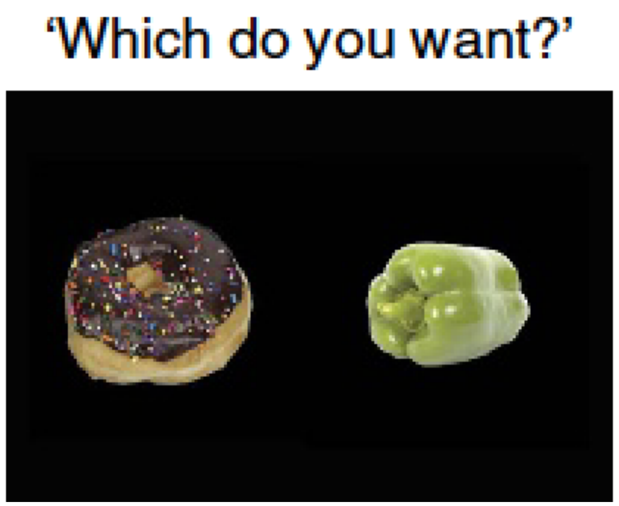
Which do you want?

‘Which do you want?’



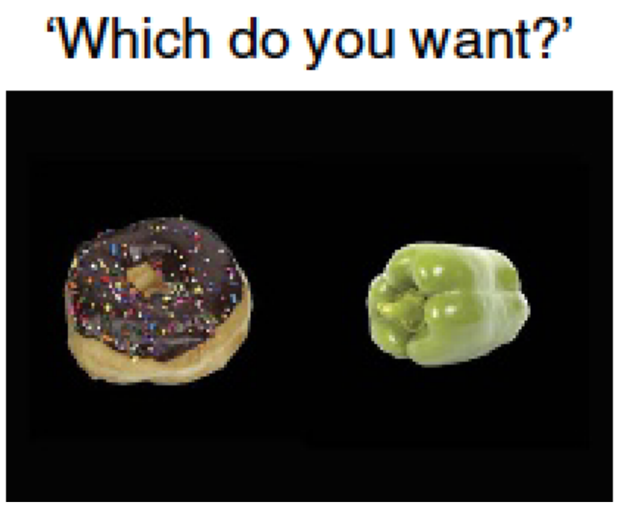
Tajima 2019

Which do you want?



Tajima 2019

Which do you want?

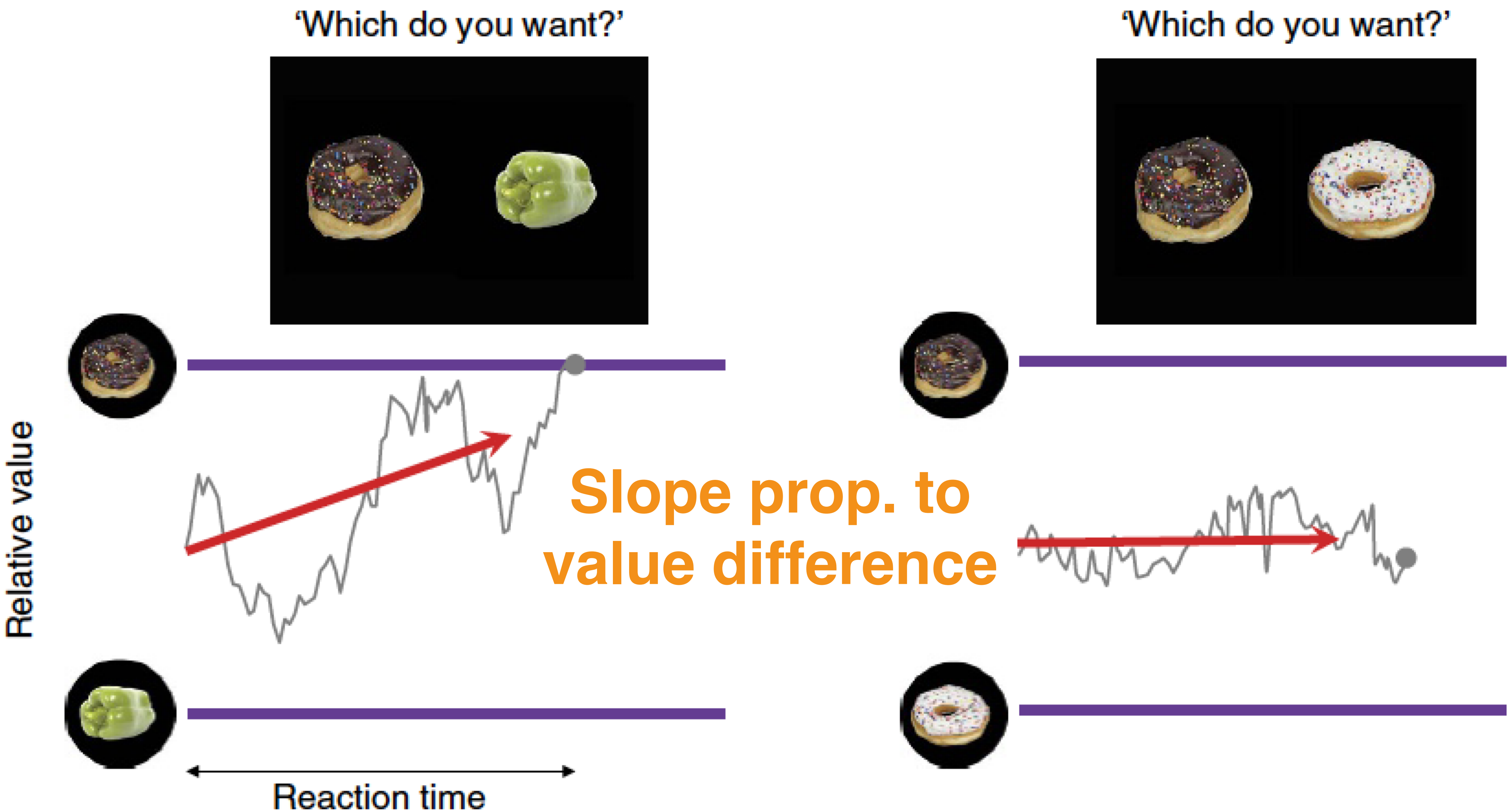


Large value difference

Small value difference

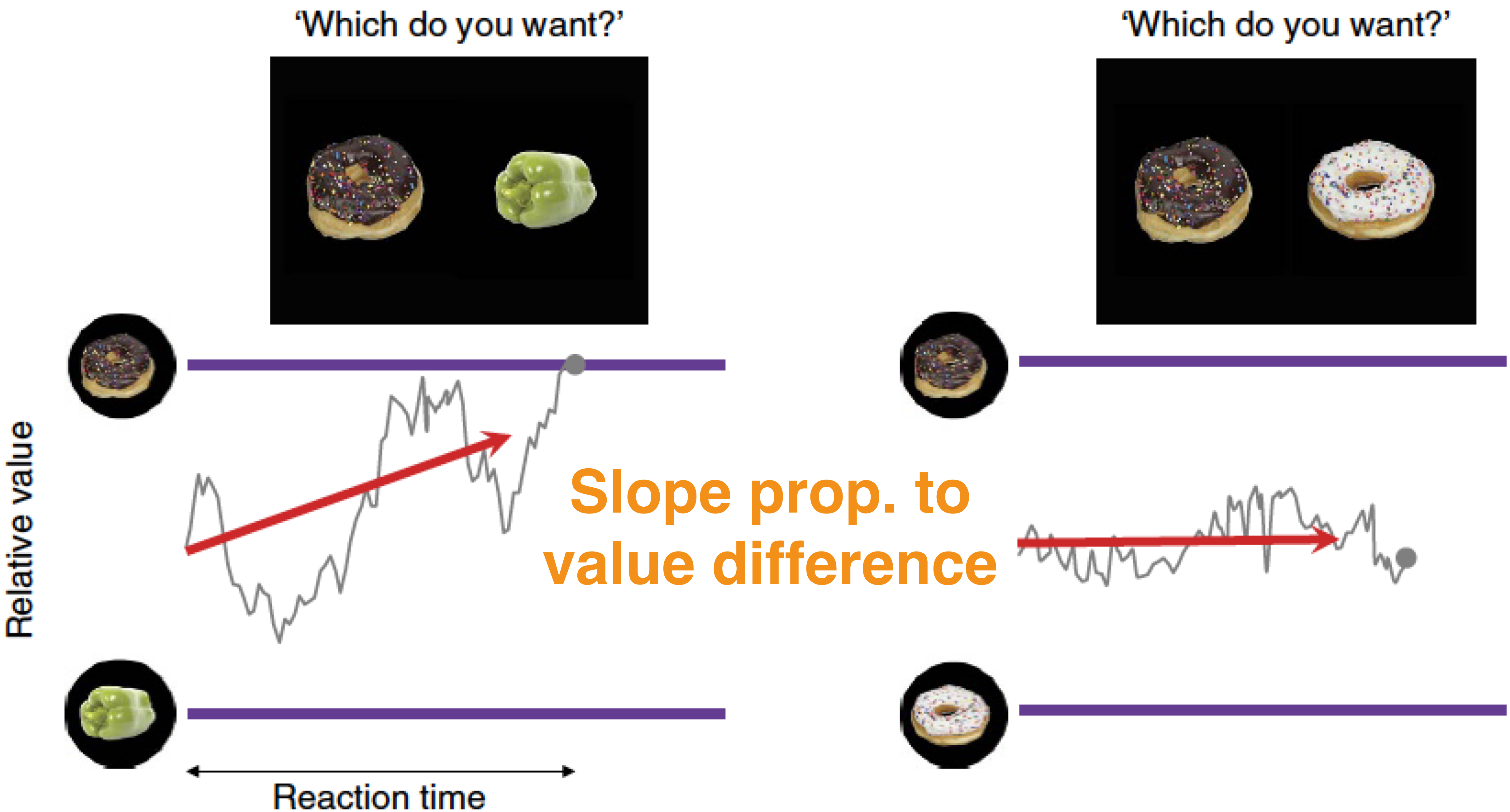
Tajima 2019

Which do you want?



Tajima 2019

Which do you want?



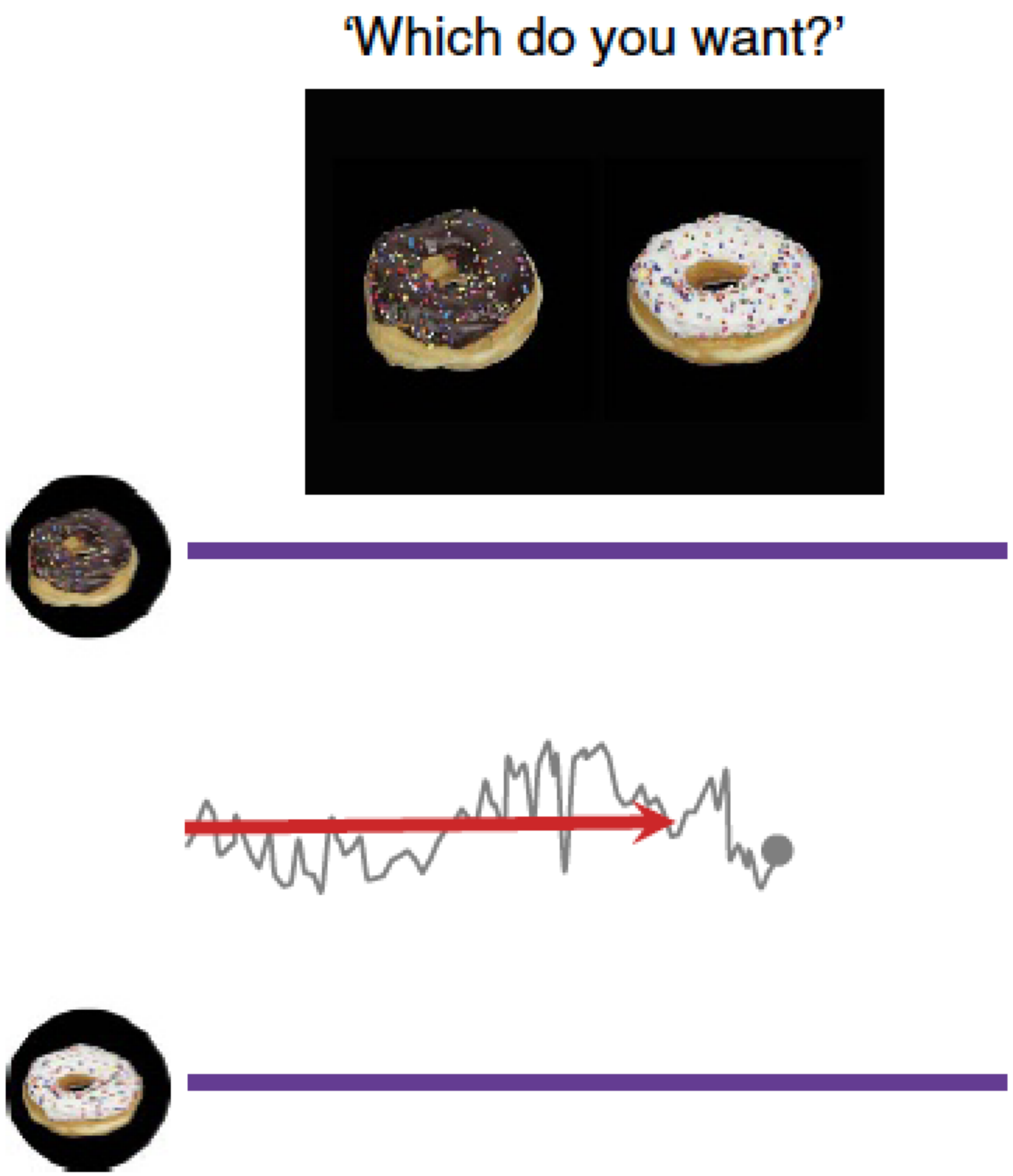
Slope prop. to value difference

Fits choice & RT data!

Tajima 2019

Which do you want?

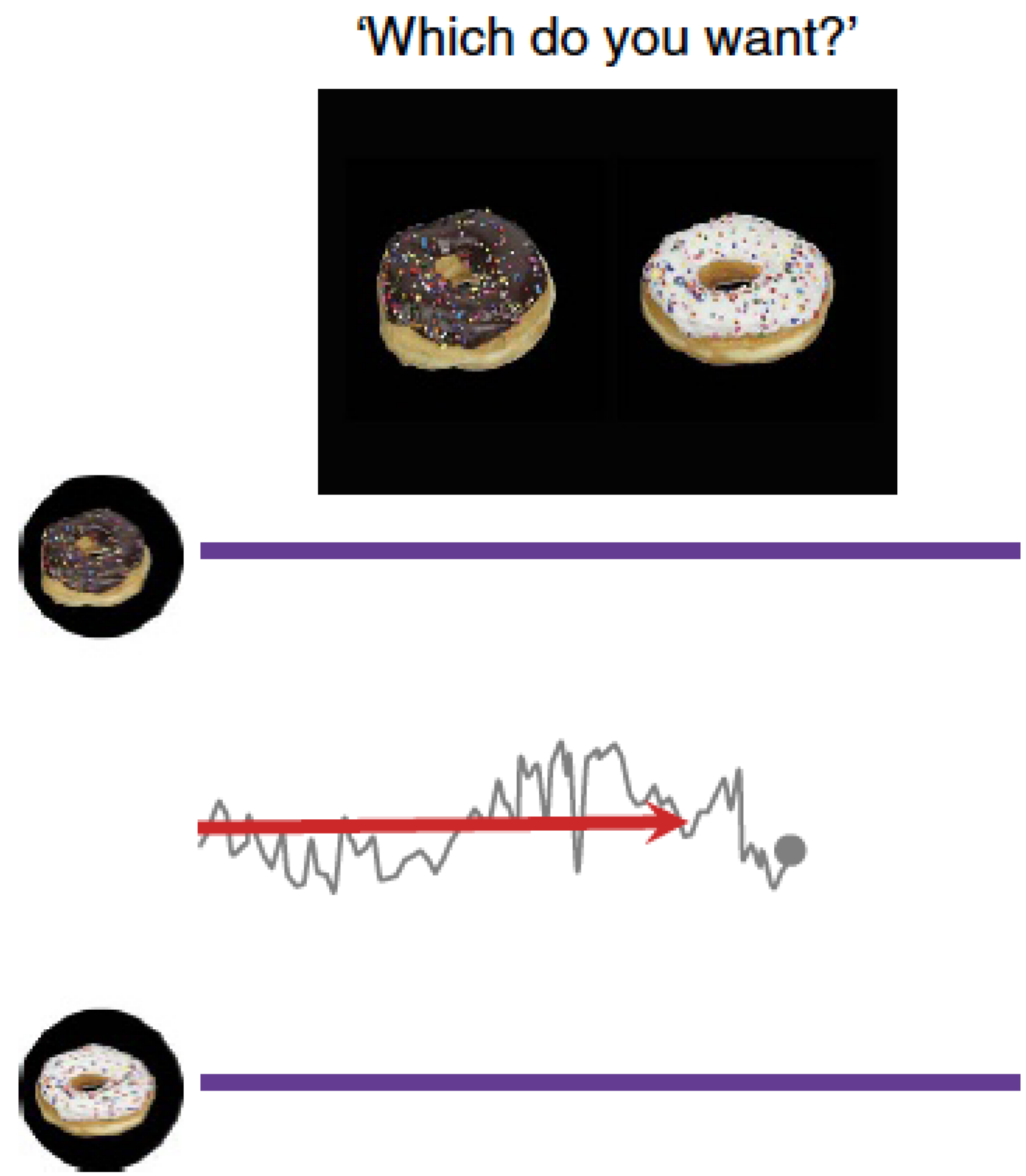
Decision paralysis?



Tajima 2019

Which do you want?

Buridan's Ass
dies of hunger and thirst?



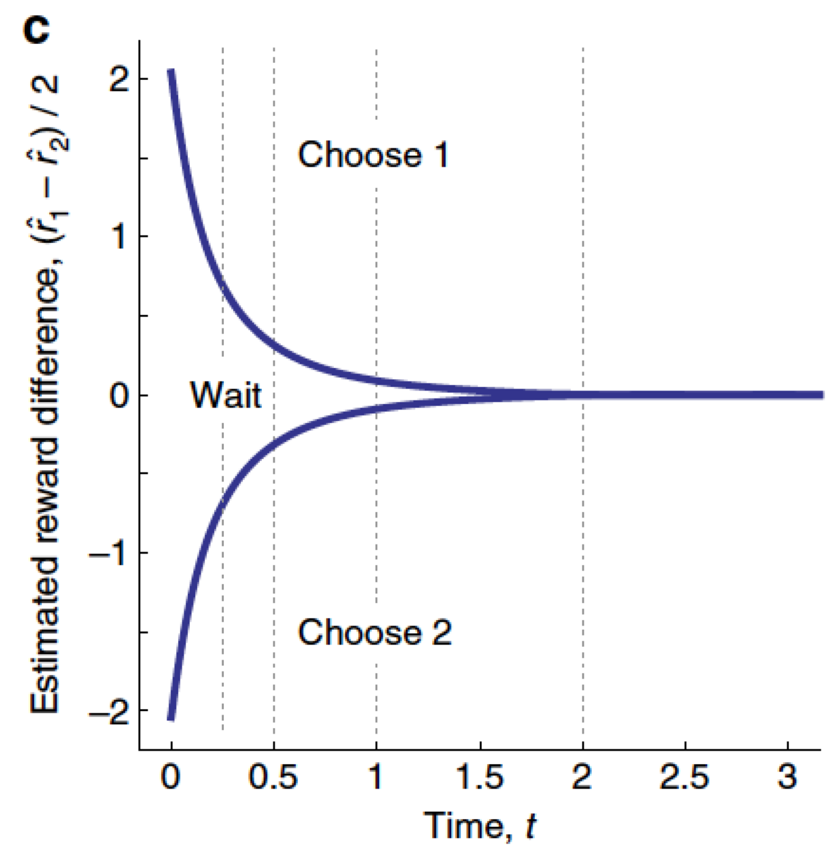
Tajima 2019

Which do you want?

Buridan's Ass
dies of hunger and thirst?

Probably not…

Collapsing decision bound
to minimize time wasted
"accumulating" nothing



Tajima 2019

But… why would the brain need sequential samples in a value based decision making task??????

But… why would the brain need sequential samples in a value based decision making task??????

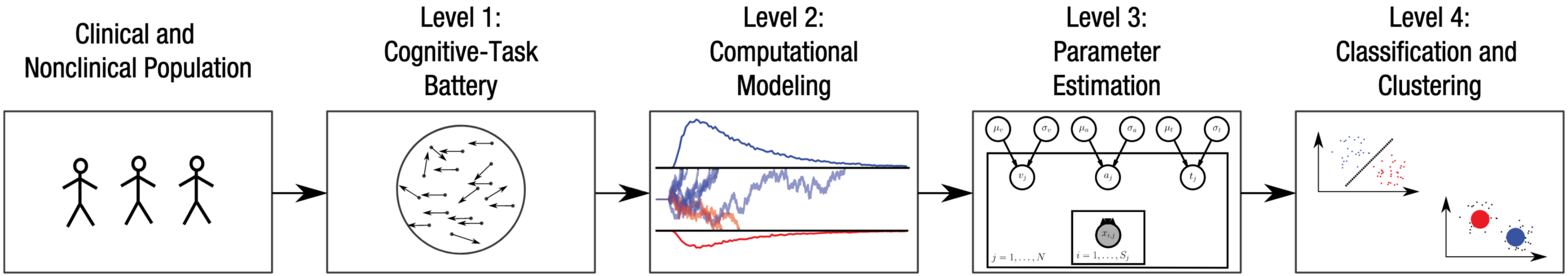
Sampling through attention (attentional DDM)?

Sampling from memory?

Open question…

DDM and computational psychiatry

DDM and computational psychiatry



Wiecki 2015

How do we know whether decisions are from a drift diffusion process?

How do we know whether decisions are from a drift diffusion process?

Unfalsifiability and mutual translatability of major modeling schemes for choice reaction time.

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Jones, Matt Dzhafarov, Ehtibar N.

Citation

Jones, M., & Dzhafarov, E. N. (2014). Unfalsifiability and mutual translatability of major modeling schemes for choice reaction time. *Psychological Review, 121*(1), 1–32. https://doi.org/10.1037/a0034190

Abstract

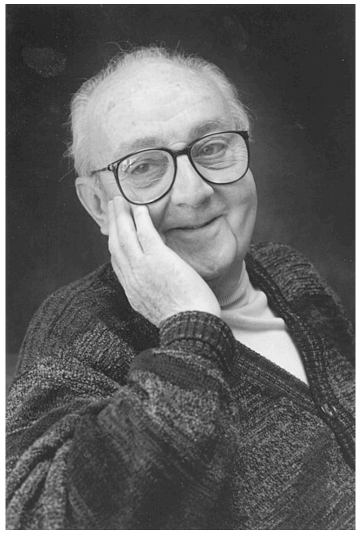
[Correction Notice: An Erratum for this article was reported in Vol 121(1) of *Psychological Review* (see record 2014-03591-005). The link to supplemental material was missing. All versions of this article have been corrected.] Much current research on speeded choice utilizes models in which the response is triggered by a stochastic process crossing a deterministic threshold. This article focuses on 2 such model classes, 1 based on continuous-time diffusion and the other on linear ballistic accumulation (LBA). Both models assume random variability in growth rates and in other model components across trials. We show that if the form of this variability is unconstrained, the models can exactly match any possible pattern of response probabilities and response time distributions. Thus, the explanatory or predictive content of these models is determined not by their structural assumptions but, rather, by distributional assumptions (e.g., Gaussian distributions) that are traditionally regarded as implementation details.

How do we know whether decisions are from a drift diffusion process?

Behavior may not be sufficient…

How do we know whether decisions are from a drift diffusion process?

Behavior may not be sufficient…

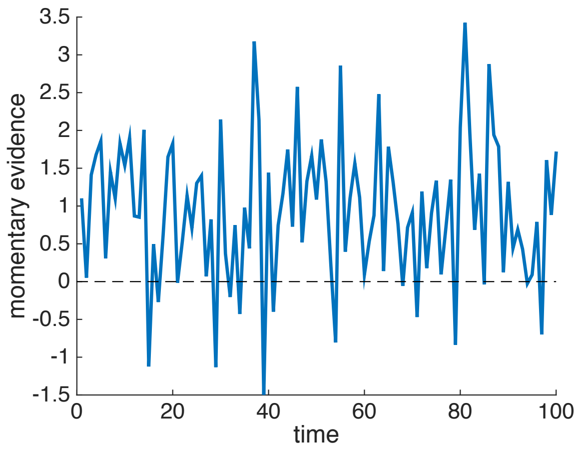


**"All models are wrong,
But some are useful"**

Looking for drift diffusion in the brain

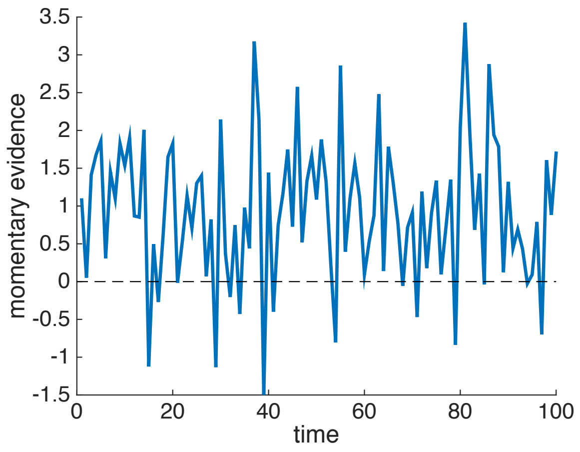
Drift diffusion in the brain

Step 1: compute momentary evidence

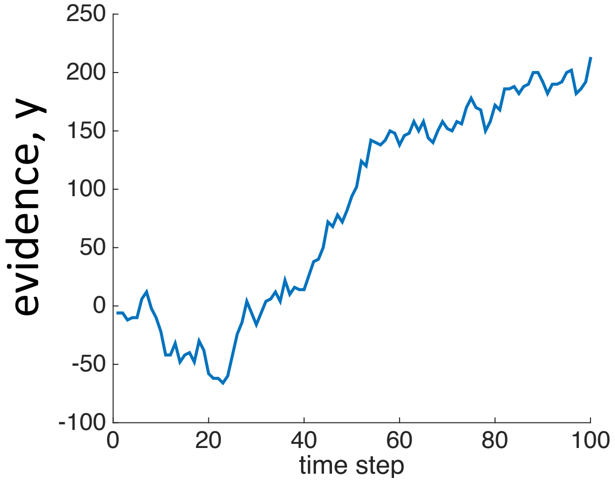


Drift diffusion in the brain

Step 1: compute momentary evidence

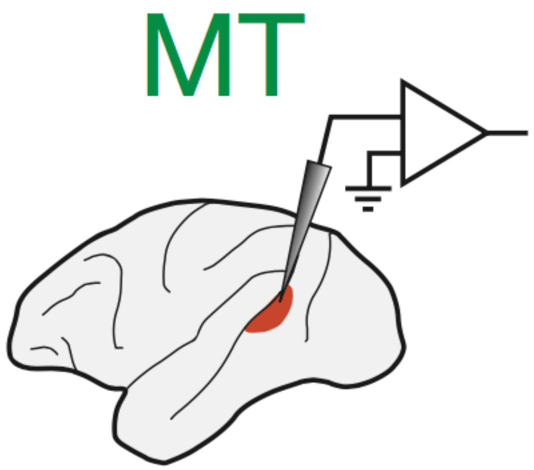


Step 2: integrate evidence over time to make a decision

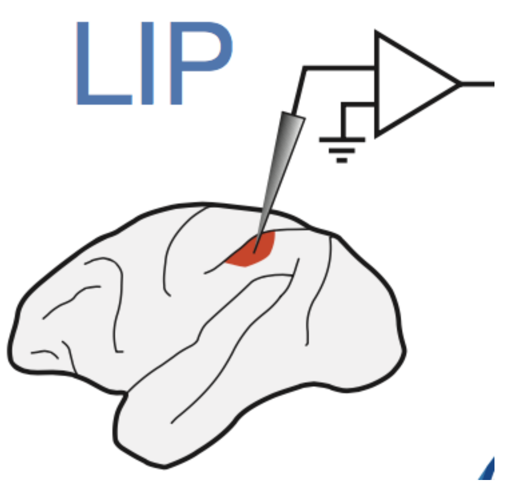


Drift diffusion in the brain

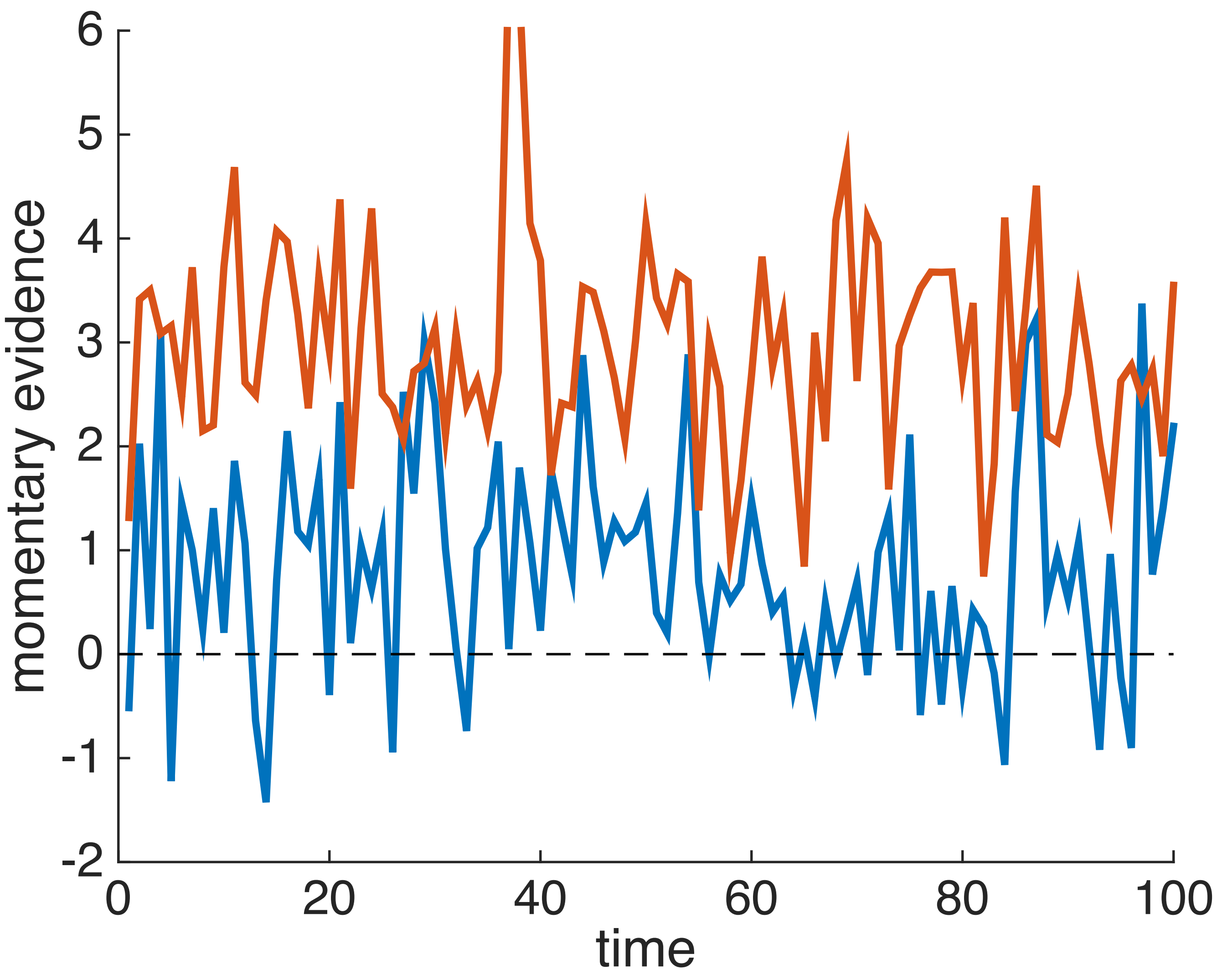
Step 1: compute momentary evidence

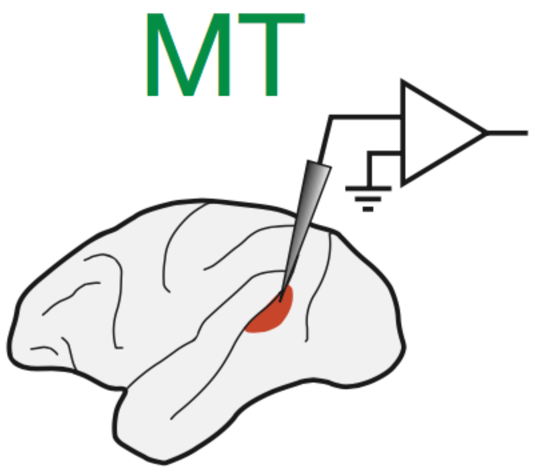


Step 2: integrate evidence over time
to make a decision

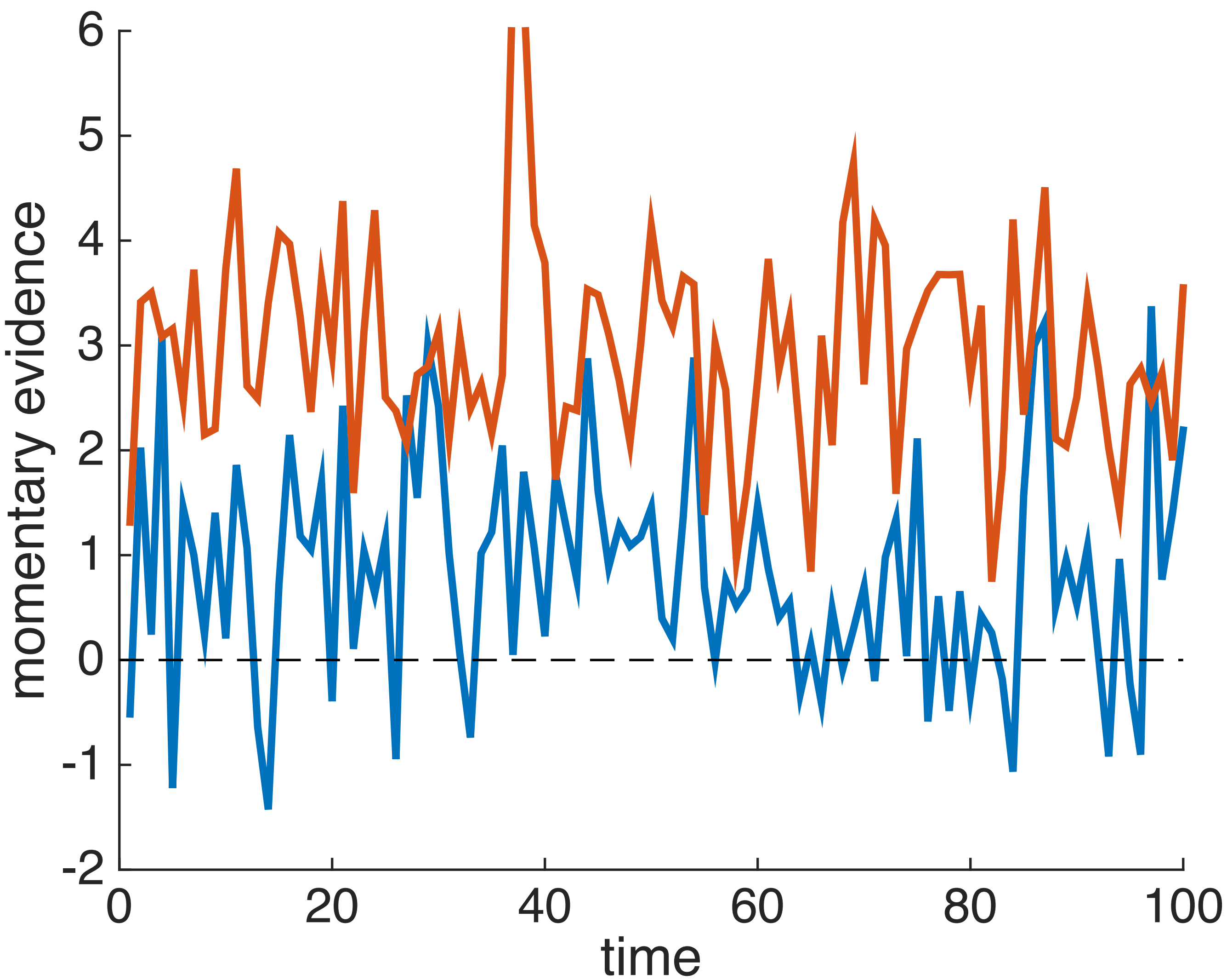


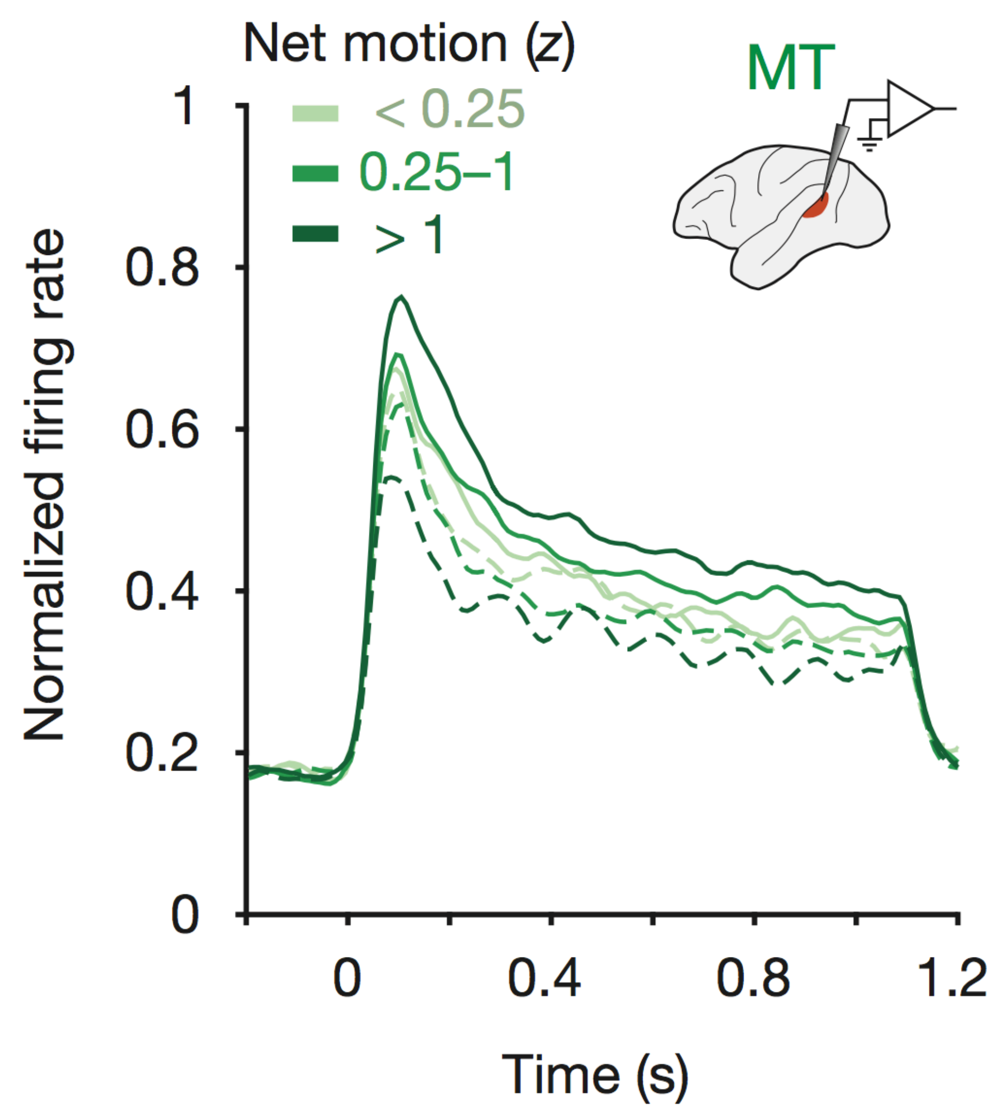
MT: momentary motion evidence





MT: momentary motion evidence



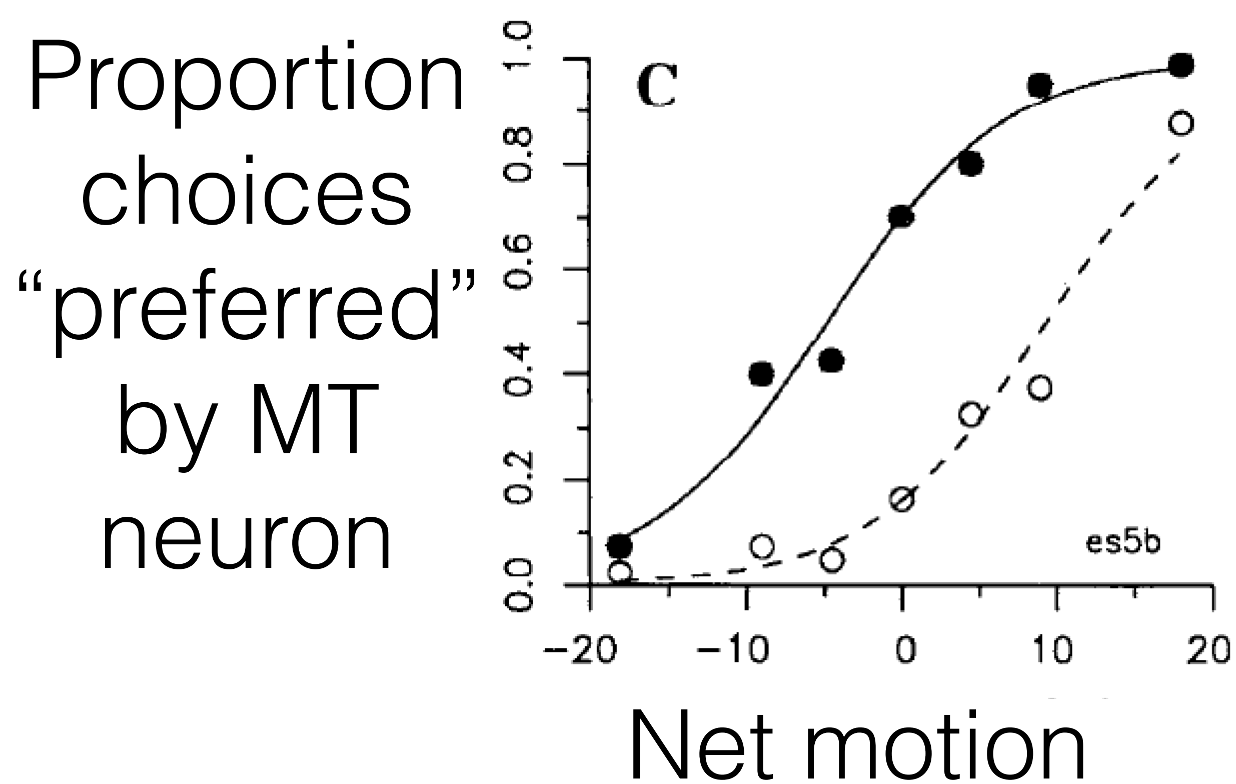


MT: momentary motion evidence

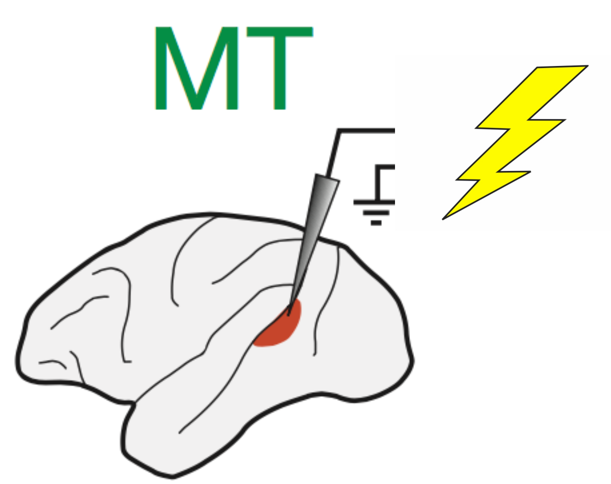
Is activity in MT *sufficient* to drive choice?

MT: momentary motion evidence

Is activity in MT *sufficient* to drive choice?



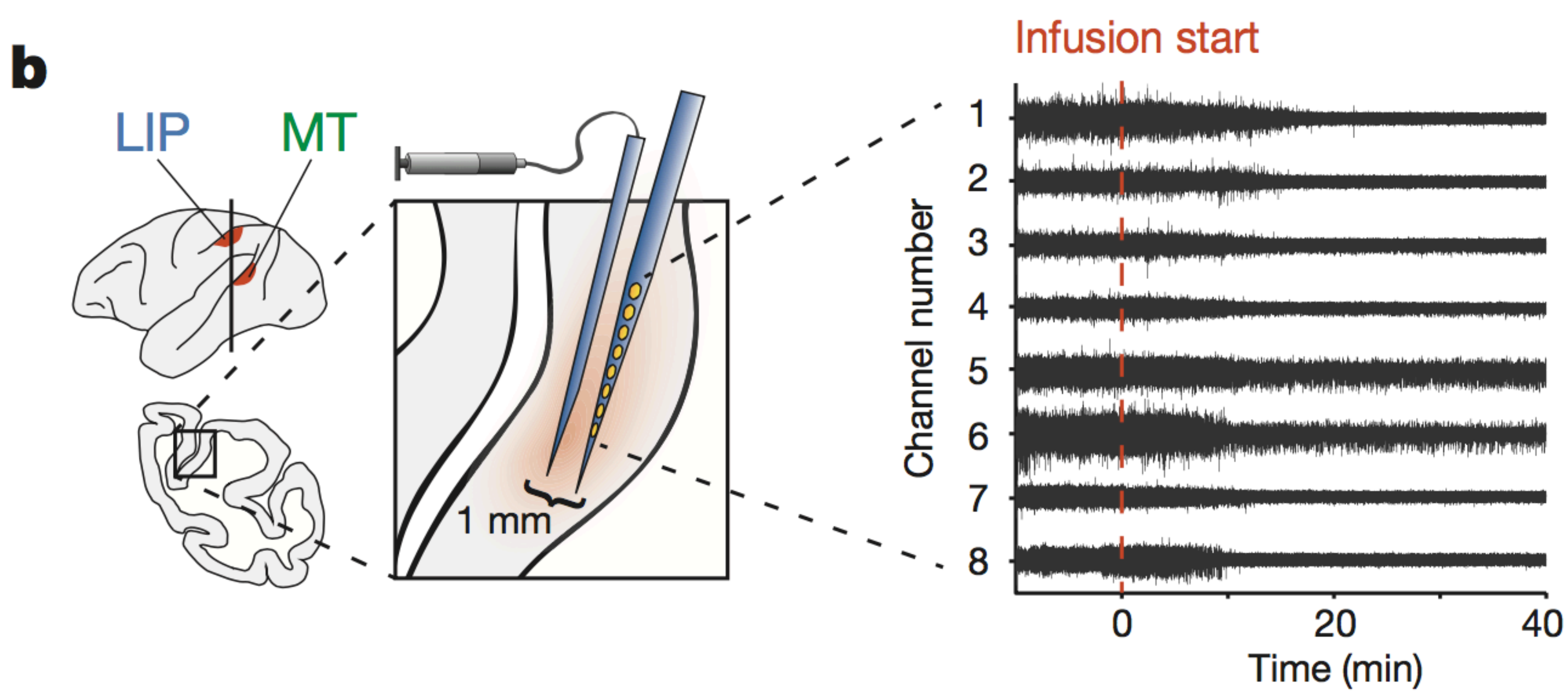
YES!!!



Salzman, 1992

MT: momentary motion evidence

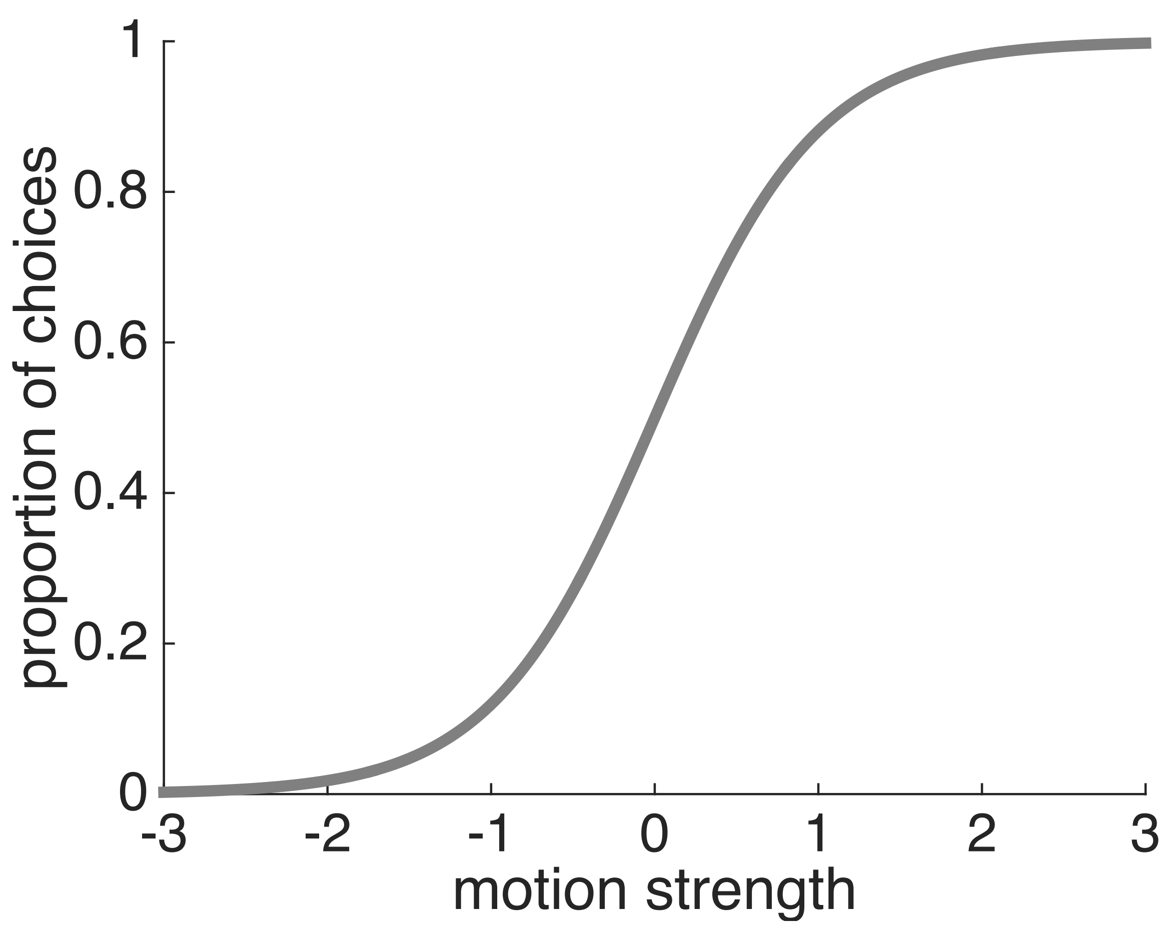
Is activity in MT *necessary* to drive choice?



Katz 2016

MT: momentary motion evidence

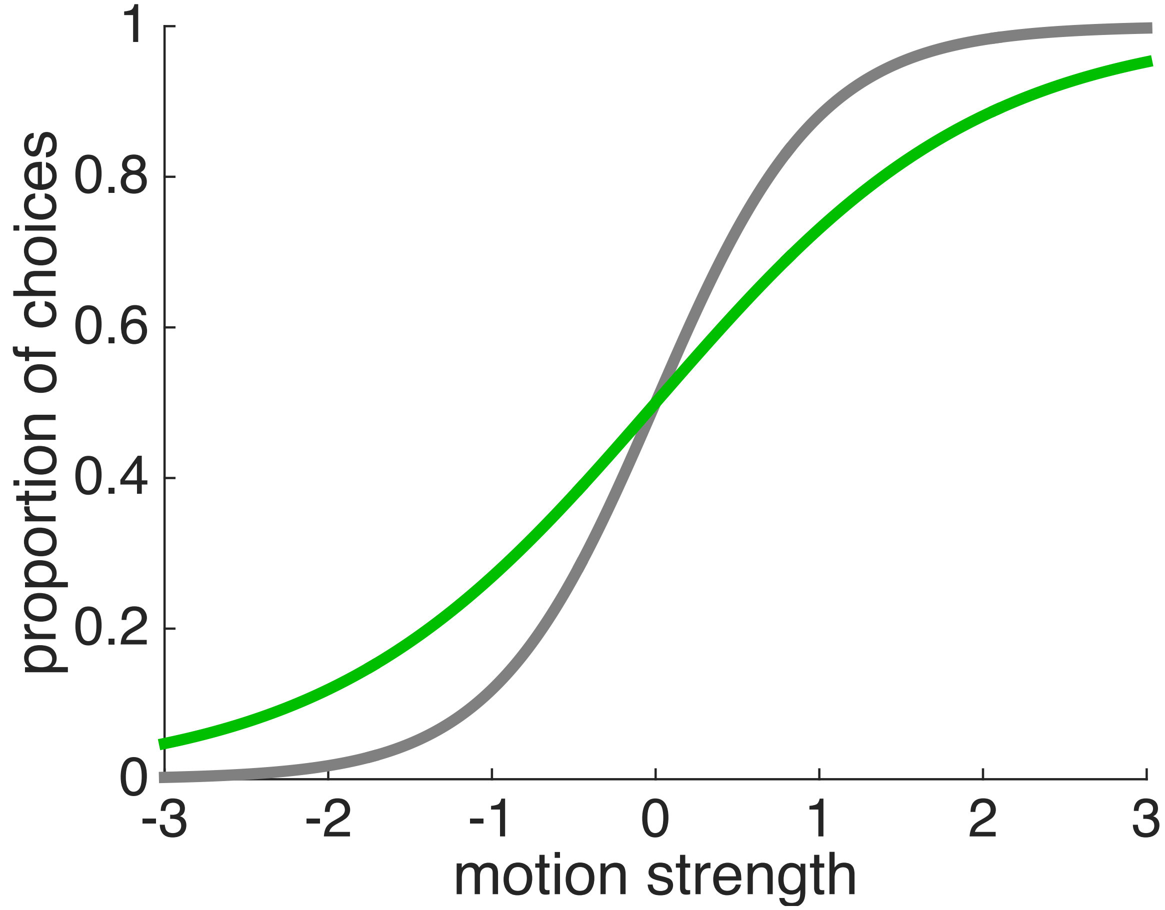
Is activity in MT *necessary* to drive choice?



Katz 2016

MT: momentary motion evidence

Is activity in MT *necessary* to drive choice?

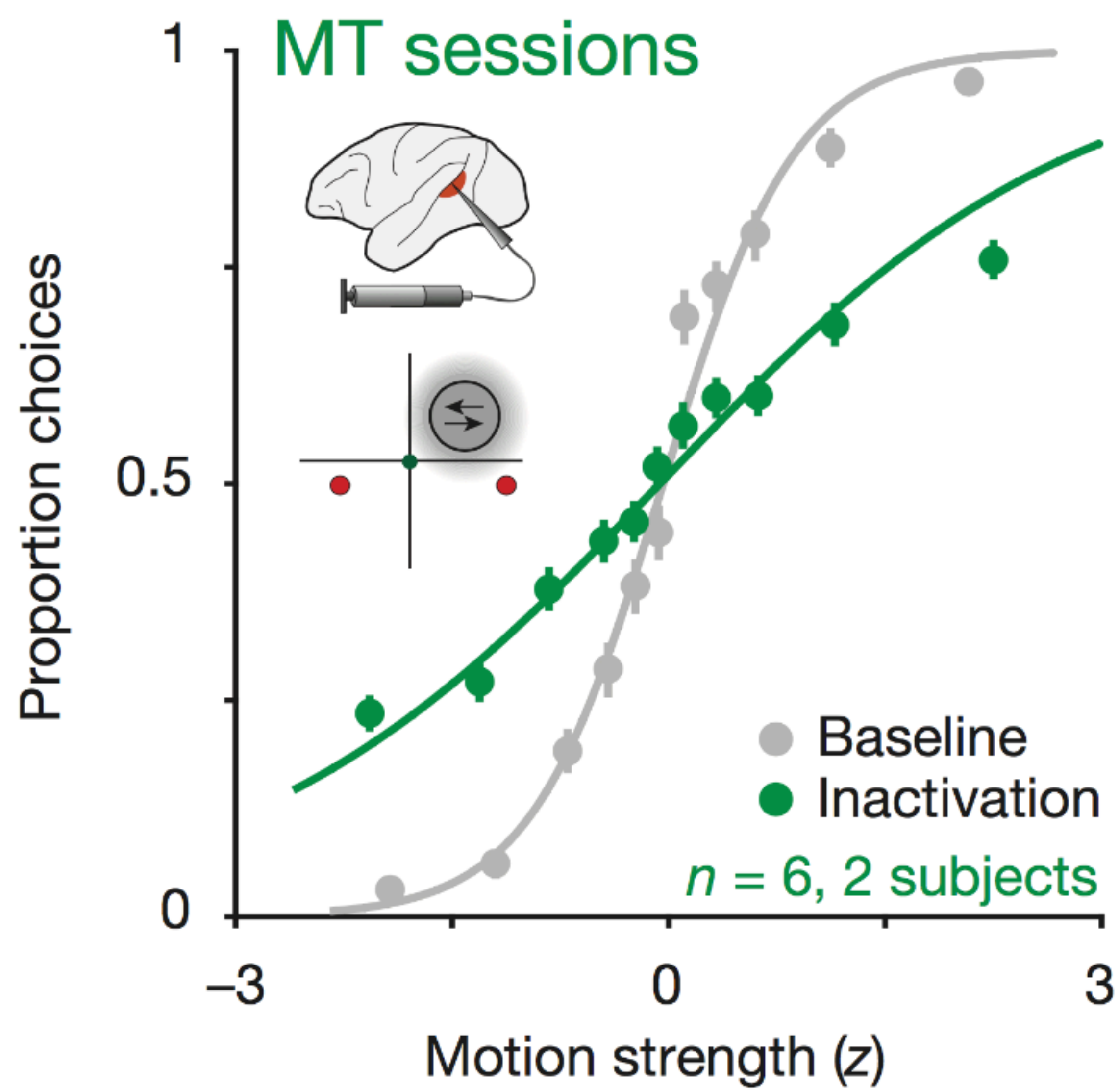


Reduced sensitivity?

Katz 2016

MT: momentary motion evidence

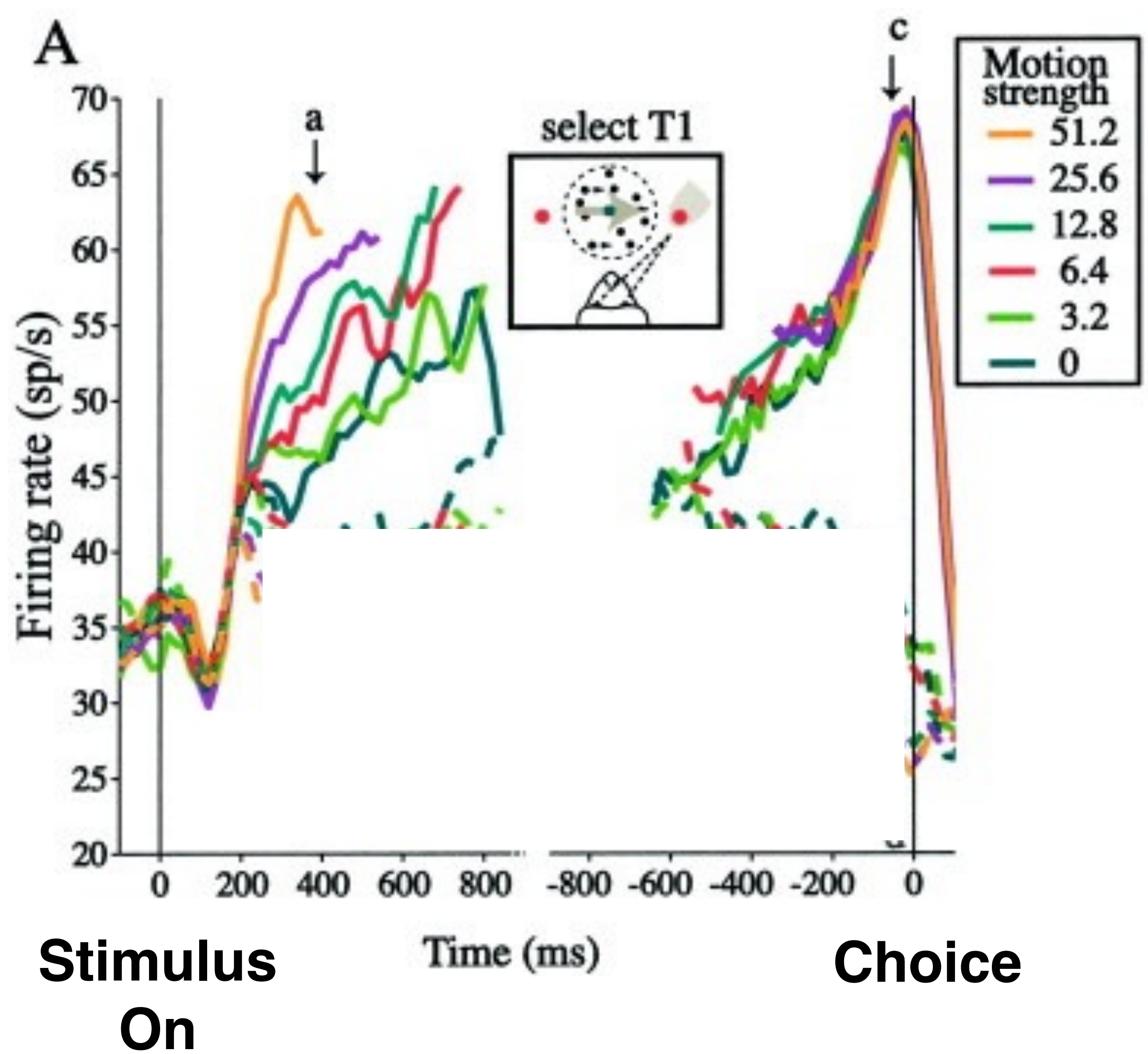
Is activity in MT *necessary* to drive choice?



YES!!!

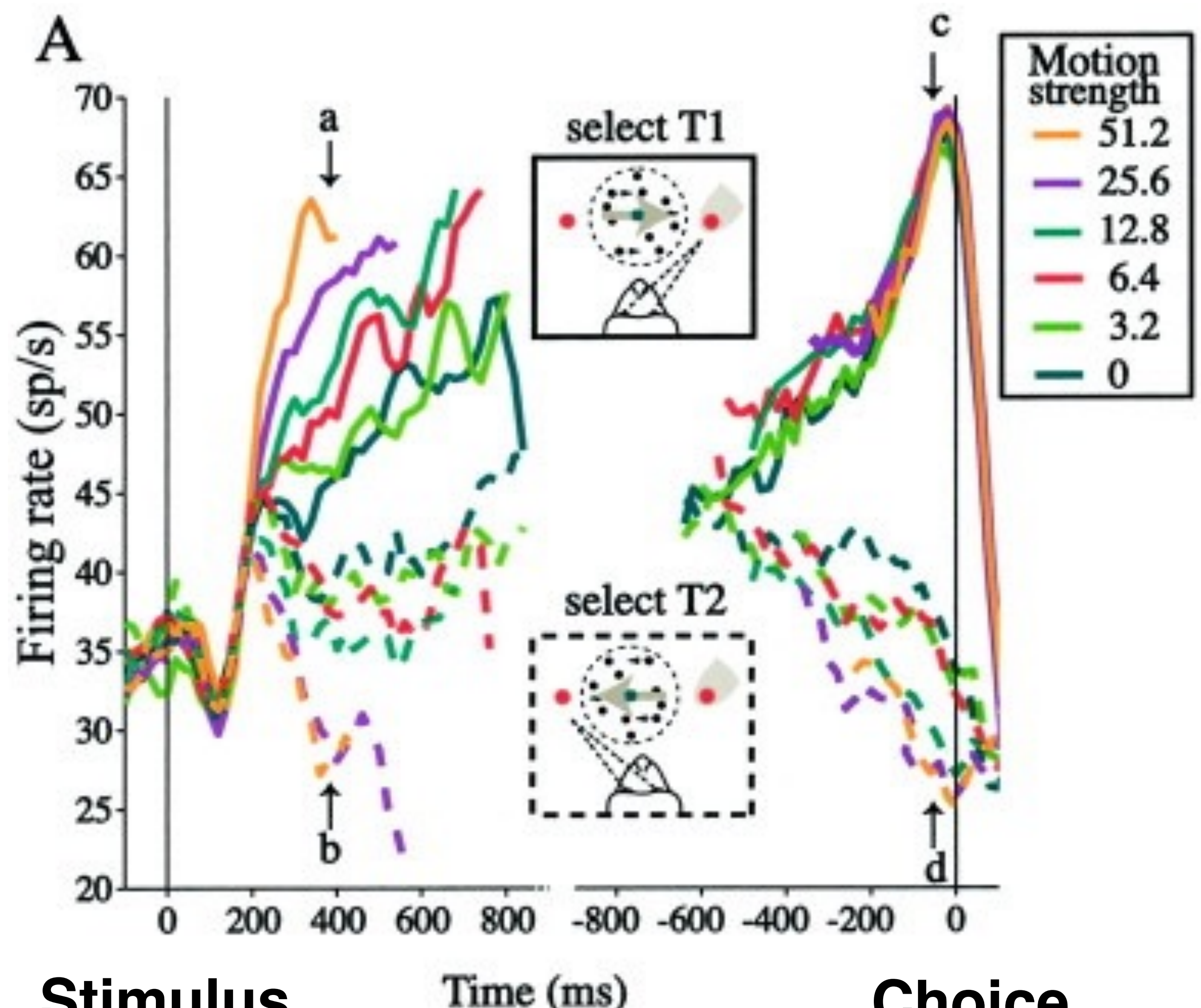
Katz 2016

LIP: Accumulated evidence



Roitman & Shadlen, 2002

LIP: Accumulated evidence



Stimulus On

Choice

Roitman & Shadlen, 2002

LIP: Accumulated evidence

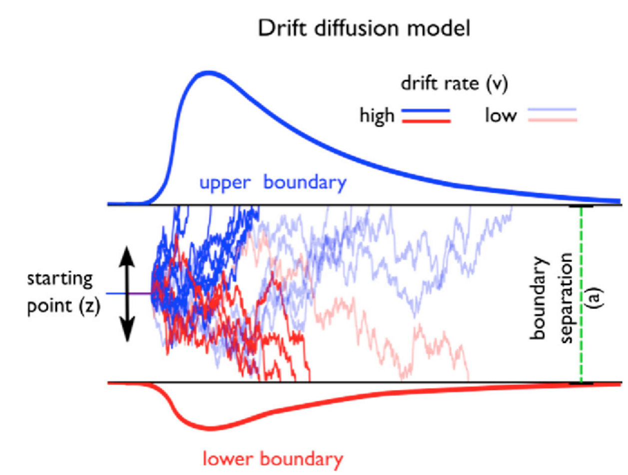
Computational

Evidence accumulation

$$\log \left[\frac{\Pr(m|h_1)}{\Pr(m|h_0)} \right]$$

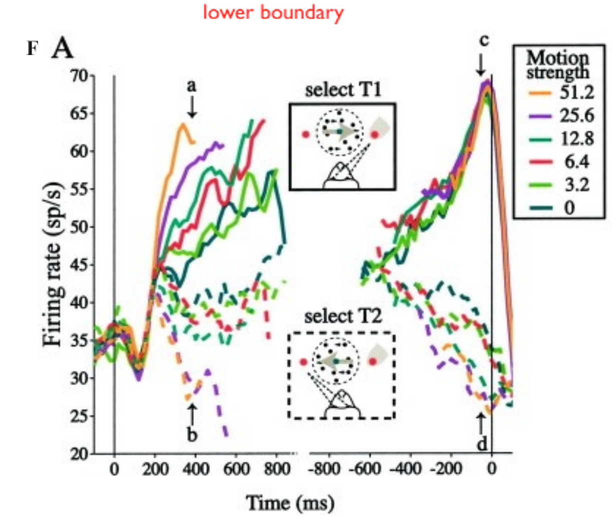
Algorithmic

Drift diffusion to bound



Implementation

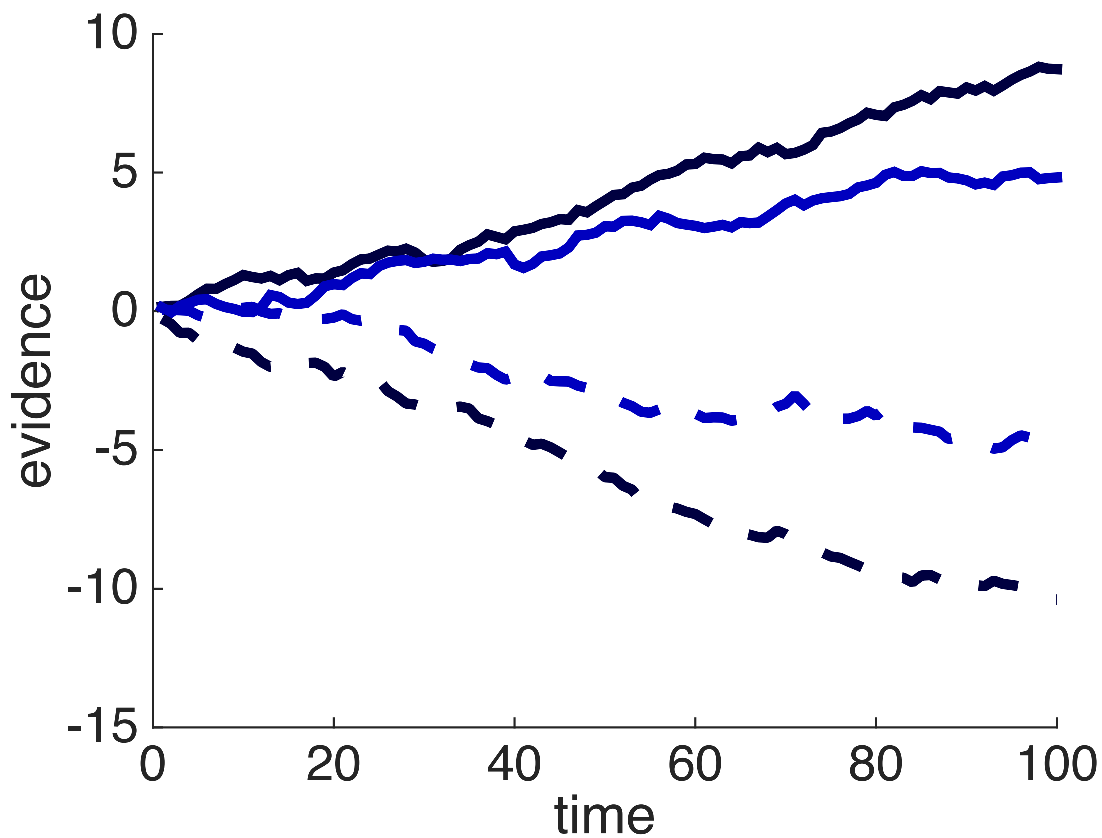
Neural responses in LIP/FEF

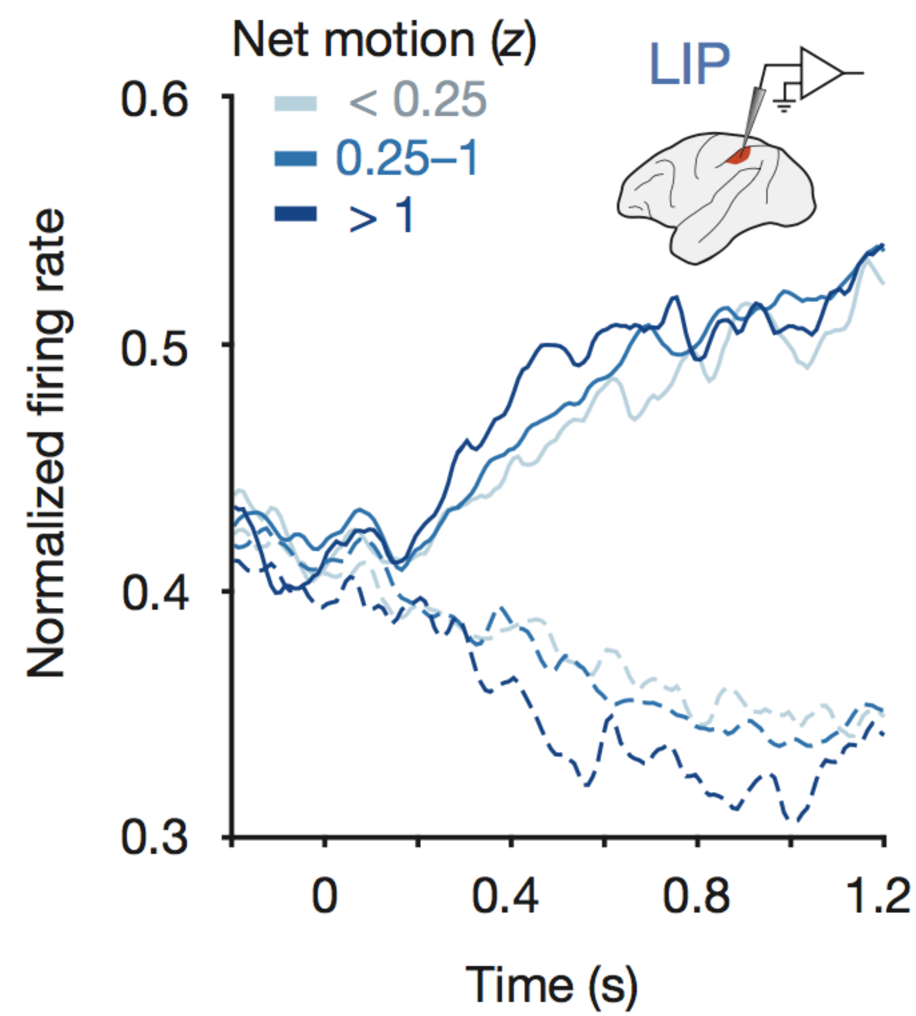


LIP: Accumulated evidence

Katz 2016

LIP: Accumulated evidence



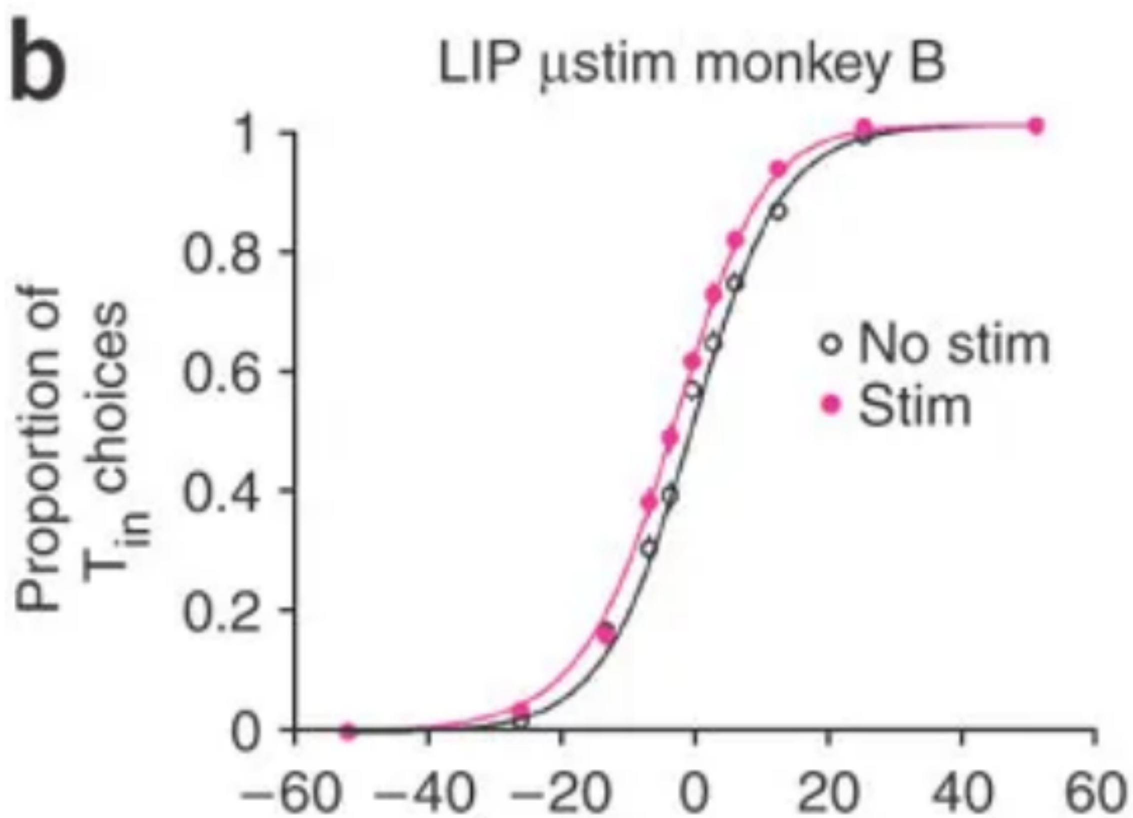


Katz 2016

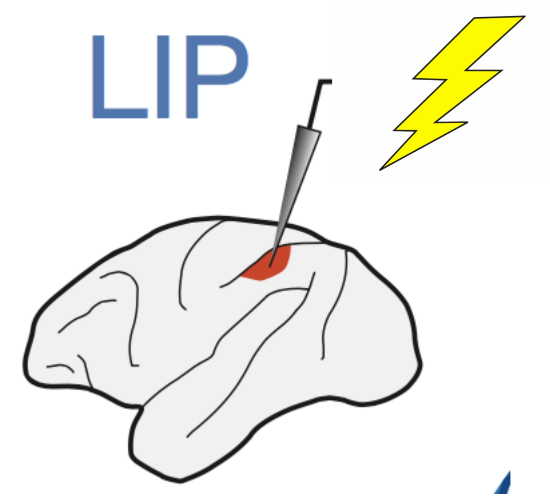
LIP: Accumulated evidence

Is LIP firing *sufficient* to influence evidence accumulation?

YES (but less impressively)



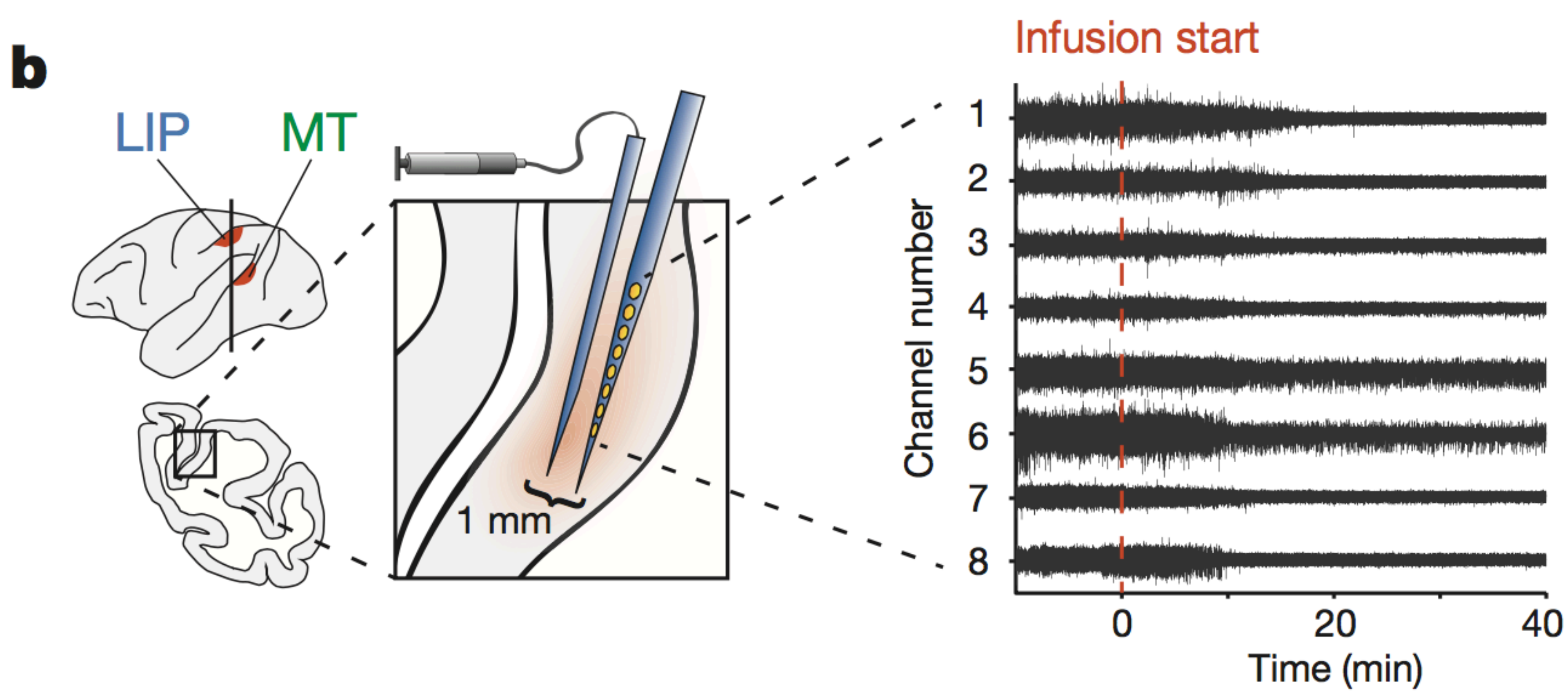
Motion information



Hanks, 2006

LIP: Accumulated evidence

Is LIP firing *necessary* to influence evidence accumulation?

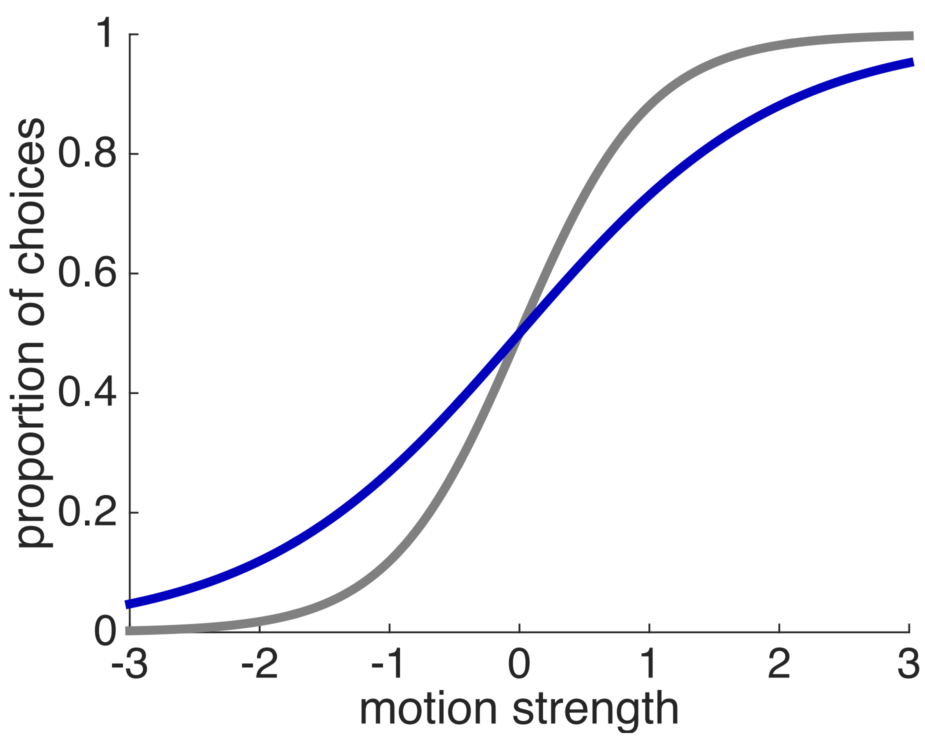


Katz 2016

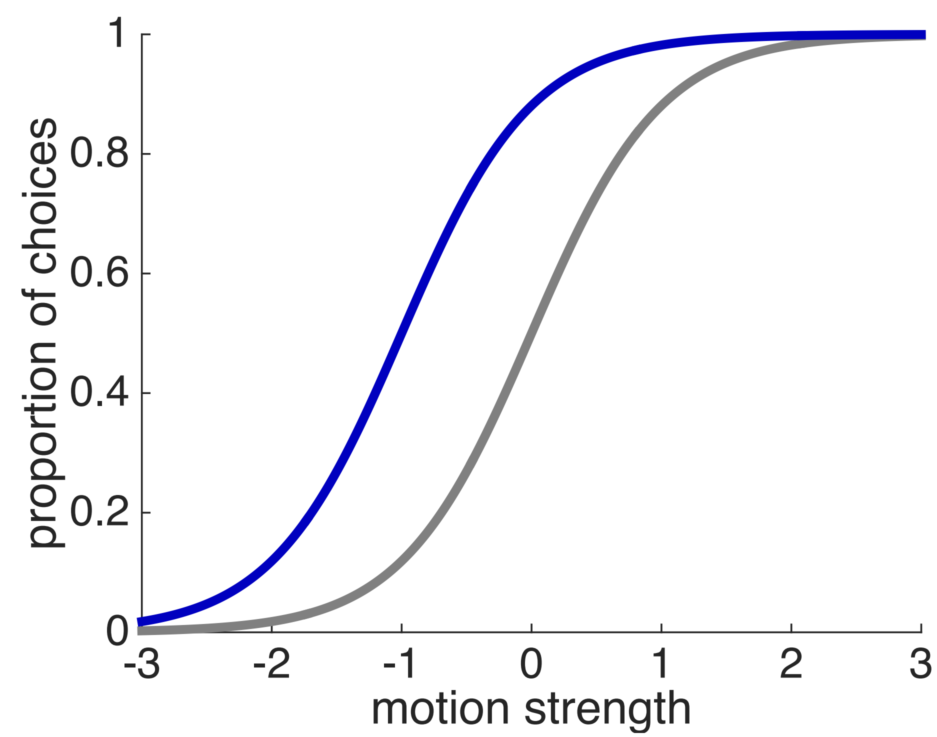
LIP: Accumulated evidence

Is LIP firing *necessary* to influence evidence accumulation?

Sensitivity?



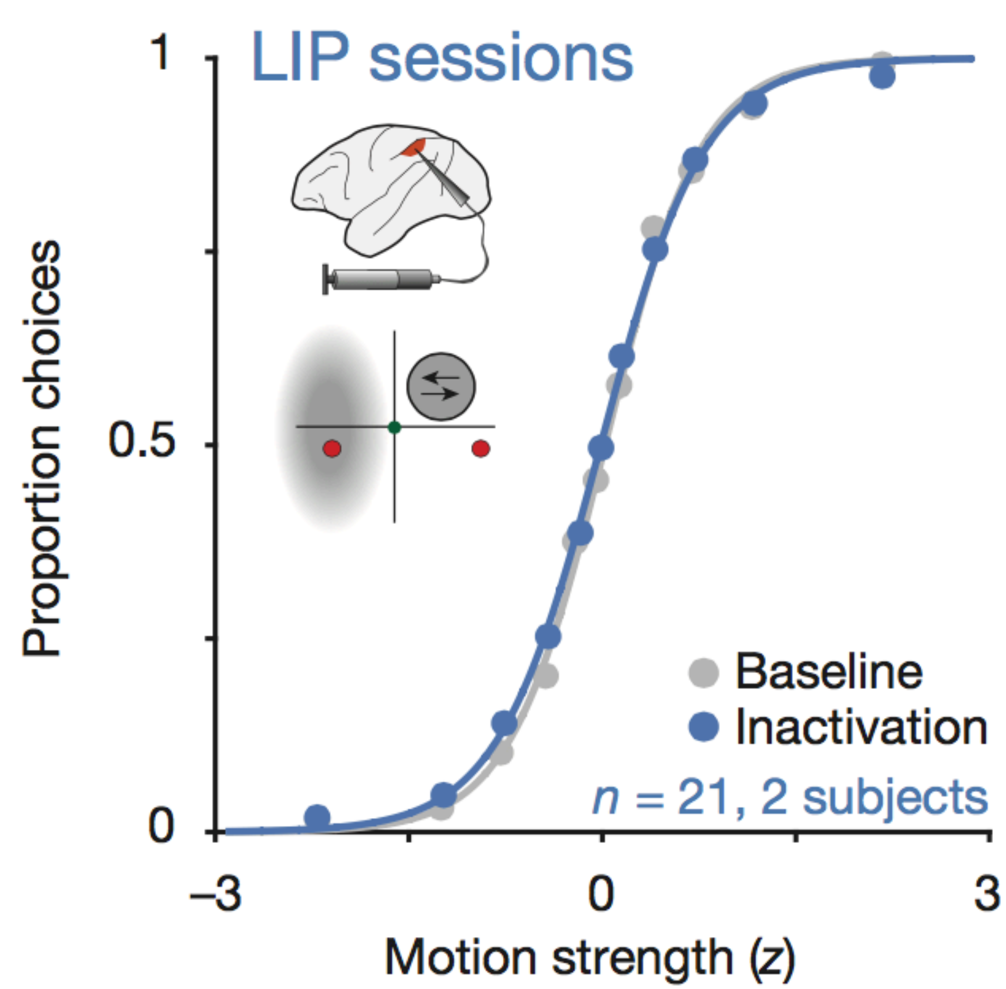
Bias?



LIP: Accumulated evidence

Is LIP firing *necessary* to influence evidence accumulation?

No???



Katz 2016

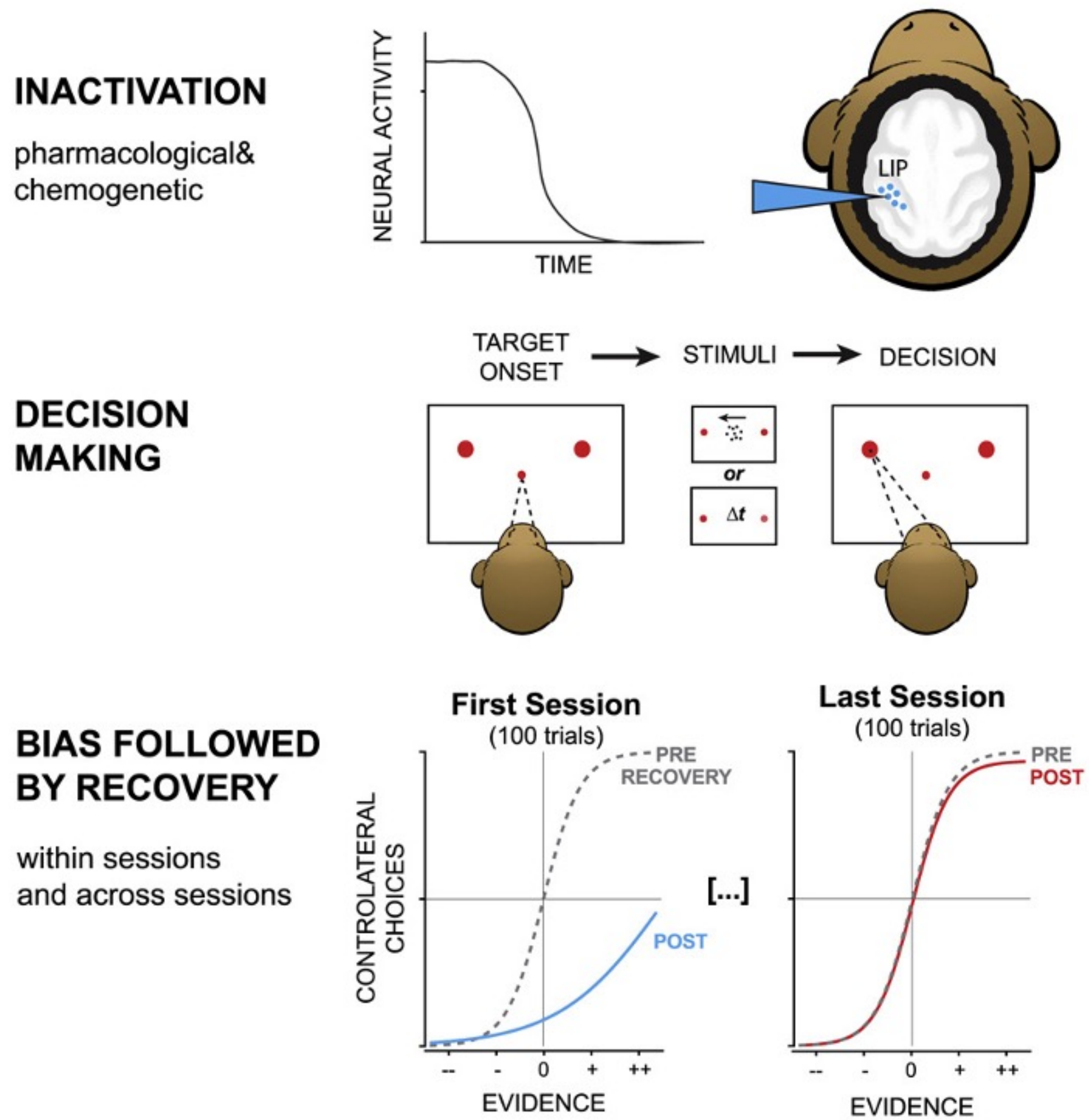
What is going on?

- Compensation?
- Wrong model?
- Wrong brain area?

What is going on?

- **Compensation?**
- Wrong model?
- Wrong brain area?

Compensation?



Jeurissen 2022

Summary

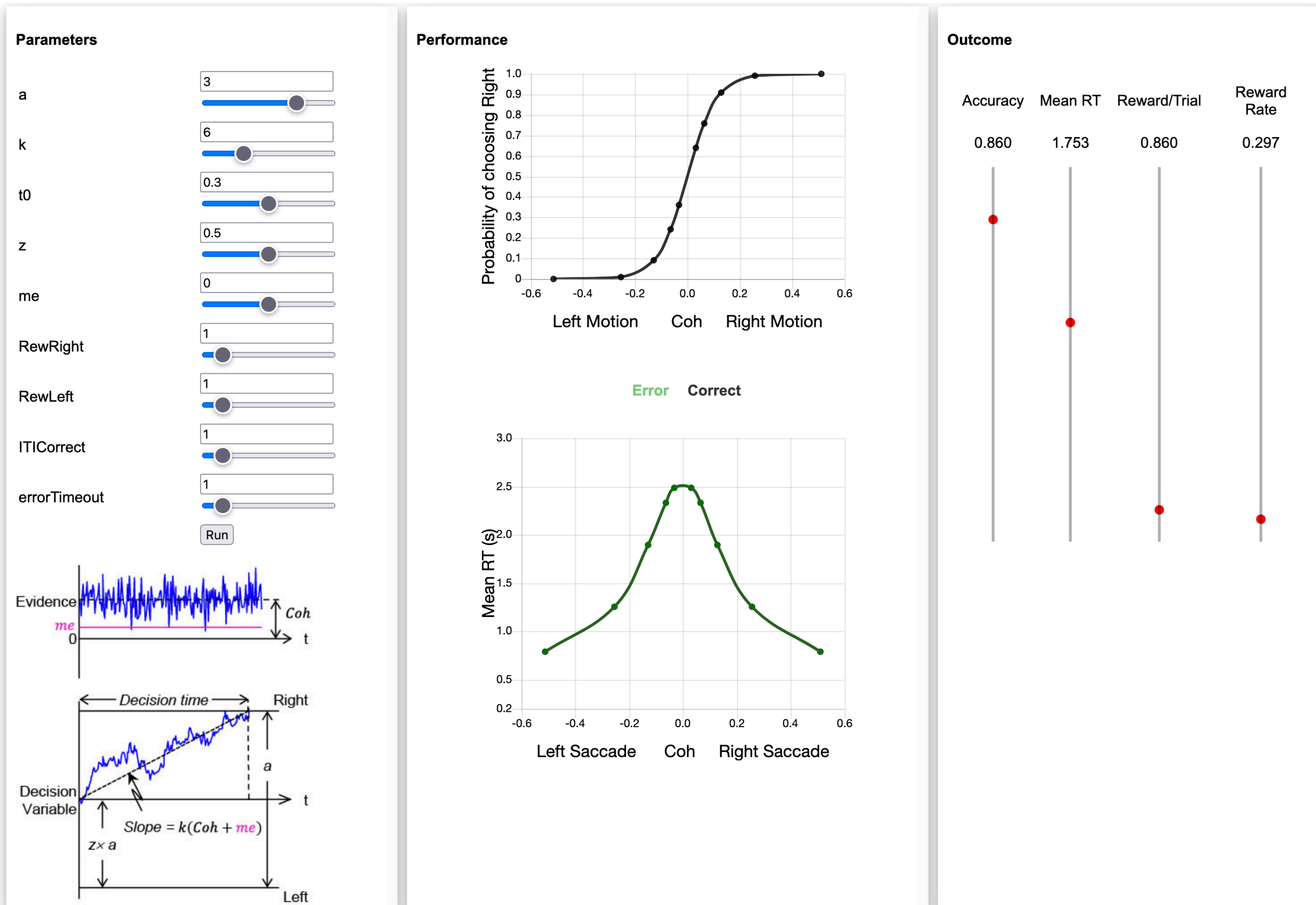
- Accumulation of log-likelihood ratio to a bound is an efficient way to make decisions under uncertainty

- Drift diffusion framework can separate different parameters that affect performance — accumulation rate versus threshold

- It can also account for different “reasons” someone might make an error (too impulsive, or spacing out) through trial-to-trial variability terms

- There is strong evidence for something like a drift diffusion model playing out in the brain as monkeys and rodents do perceptual decision making tasks

Online simulation tools



Long Ding

https://www.med.upenn.edu/longding1/javascript/model_bias.html

HDDM

HDDM

latest

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Drift Diffusion Model
Basic Tutorials
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LAN Extension
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» Sequential Sampling Models

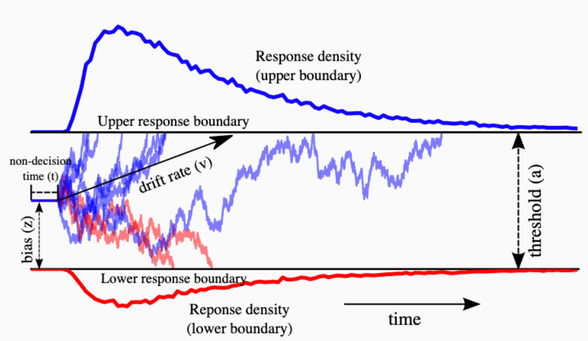
Edit on GitHub

Sequential Sampling Models

Sequential Sampling Models generally fall into one of two classes: (i) diffusion models which assume that *relative* evidence is accumulated over time and (ii) race models which assume independent evidence accumulation and response commitment once the first accumulator crossed a boundary ([LaB62], [Vic70]). Currently, HDDM includes two of the most commonly used SSMs: the drift diffusion model (DDM) ([RR98], [RM08]) belonging to the class of diffusion models and the linear ballistic accumulator (LBA) ([BH08]) belonging to the class of race models.

Drift Diffusion Model

The DDM models decision making in two-choice tasks. Each choice is represented as an upper and lower boundary. A drift-process accumulates evidence over time until it crosses one of the two boundaries and initiates the corresponding response ([RR98], [SR04]). The speed with which the accumulation process approaches one of the two boundaries is called drift-rate *v* and represents the relative evidence for or against a particular response. Because there is noise in the drift process, the time of the boundary crossing and the selected response will vary between trials. The distance between the two boundaries (i.e. threshold *a*) influences how much evidence must be accumulated until a response is executed. A lower threshold makes responding faster in general but increases the influence of noise on decision making and can hence lead to errors or impulsive choice, whereas a higher threshold leads to more cautious responding (slower, more skewed RT distributions, but more accurate). Response time, however, is not solely comprised of the decision making process – perception, movement initiation and execution all take time and are lumped in the DDM by a single non-decision time parameter *t*. The model also allows for a prepotent bias *z* affecting the starting point of the drift process relative to the two boundaries. The termination times of this generative process gives rise to the reaction time distributions of both choices.



Trajectories of multiple drift-process (blue and red lines, middle panel). Evidence is accumulated over time (x-axis) with drift-rate v until one of two boundaries (separated by threshold a) is crossed and a response

https://hddm.readthedocs.io/en/latest/methods.html

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Questions?