

Workshop

MLDS and MLCM: two scaling methods to study stimulus appearance

Guillermo Aguilar

Technische Universität
Berlin

ECVP 2025 Mainz

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Website



Detailed Conference
Program



WLAN: UNI-MAINZ

Identity: **ecvp25**
PW: **#JohannesGutenberg**
System certificates
Domain: **uni-mainz.de**



ecvp2025.uni-mainz.de

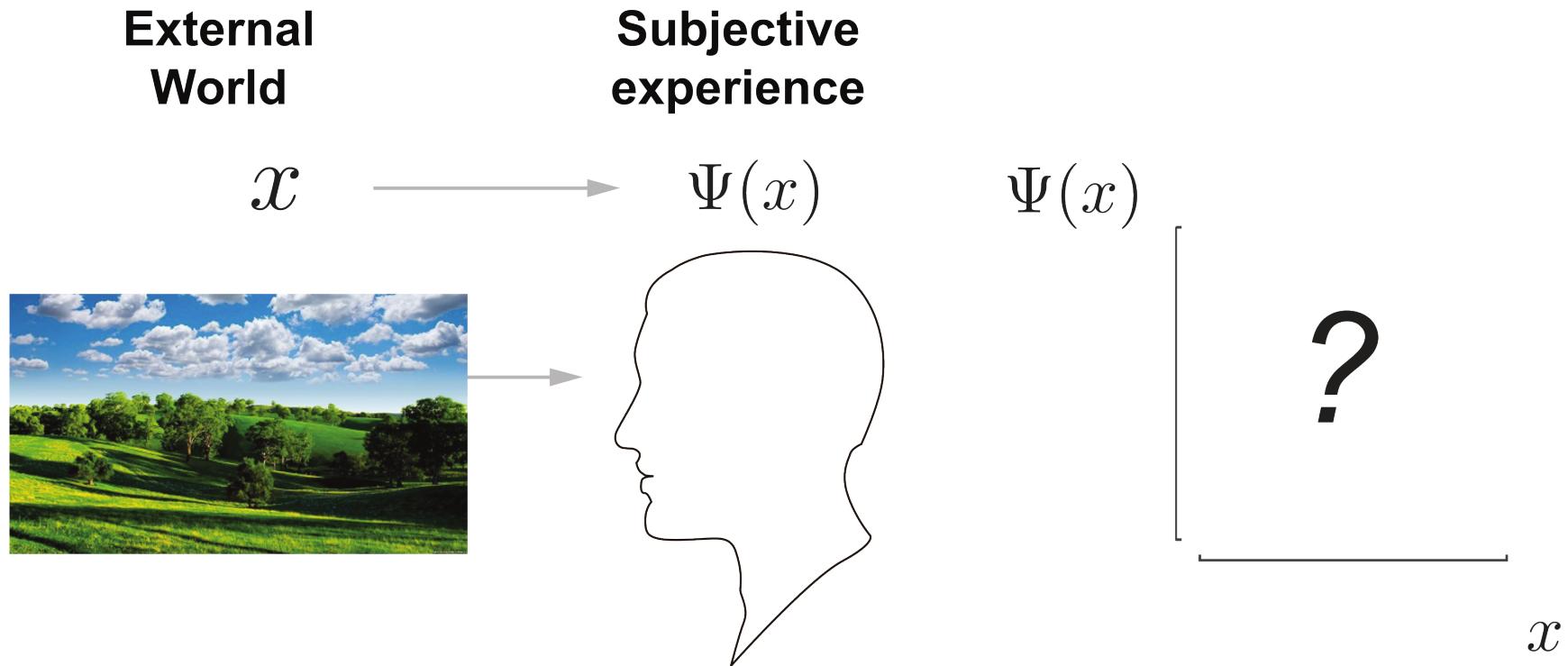


ecvp2025@uni-mainz.de

Contents – 13:30 to 16:00

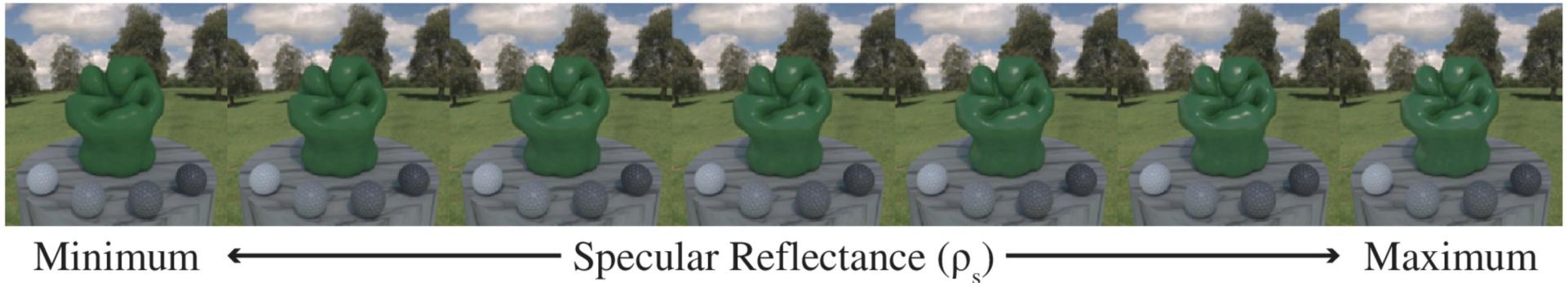
- General introduction to scaling methods
- **Maximum Likelihood Difference Scaling (MLDS)**
 - Theory
 - Exercise: acquire your own data
 - Code: analyze data with the MLDS package
 - Exercise: analyze your data
- **Maximum Likelihood Conjoint Measurement (MLCM)**
 - Theory
 - Code: analyze data with the MLCM package
 - Exercise: acquire and analyze your own data

Intro: psychophysical methods



Intro: psychophysical methods

Example: Glossiness (Cheeseman et al. 2021)

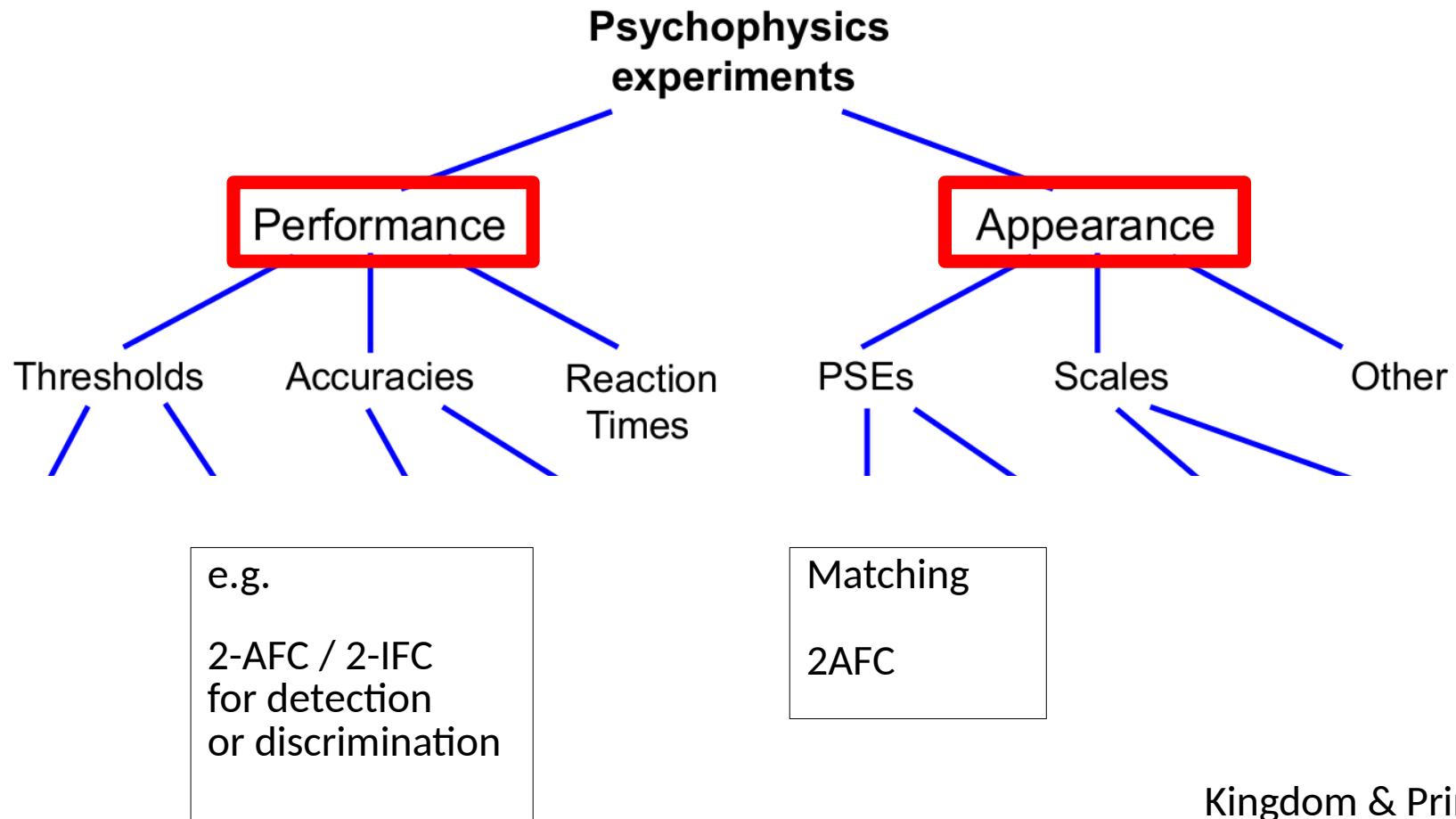


Minimum ← Specular Reflectance (ρ_s) → Maximum

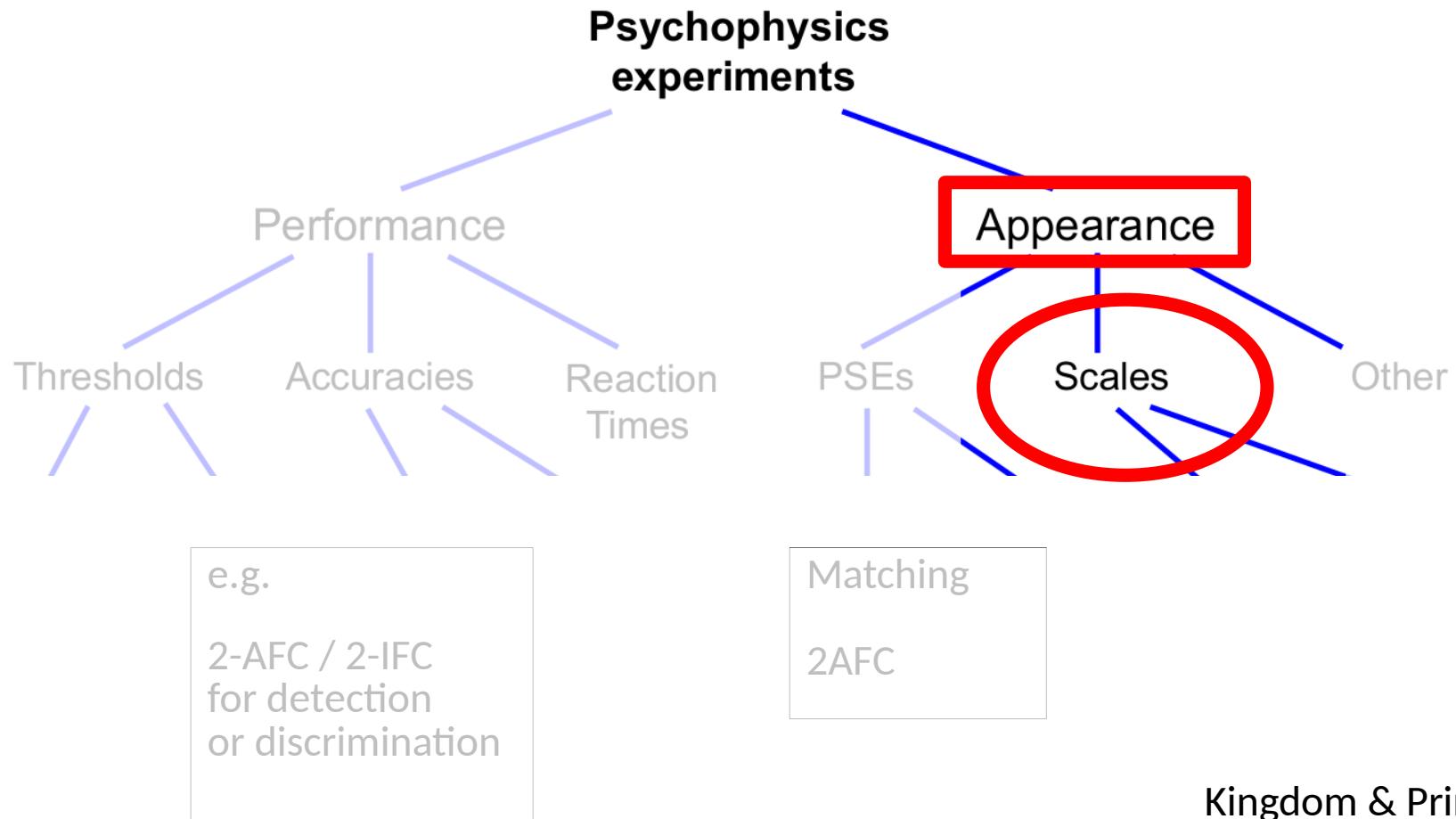
How does specular reflectance map to perceived glossiness?

Physical attribute → Perceptual dimension

Intro: psychophysical methods



Intro: psychophysical methods



Intro: scaling methods

Aim to characterize mapping between stimulus and perceptual dimensions

$$\Psi(x)$$

Perceptual dimension

?

Stimulus dimension

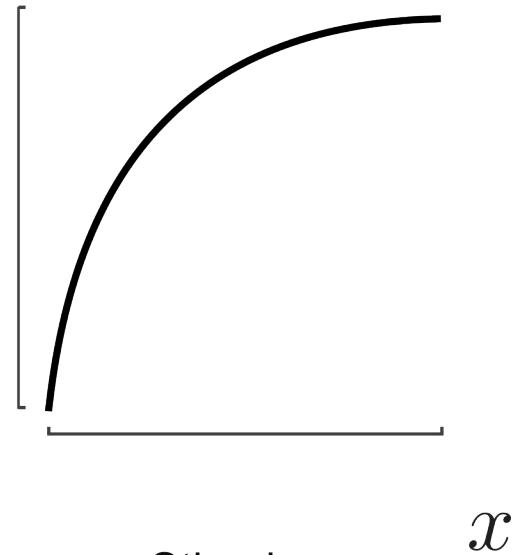
x

Intro: scaling methods

Aim to **characterize mapping between stimulus and perceptual dimensions**

$$\Psi(x)$$

Perceptual dimension



Stimulus dimension

Intro: scaling methods

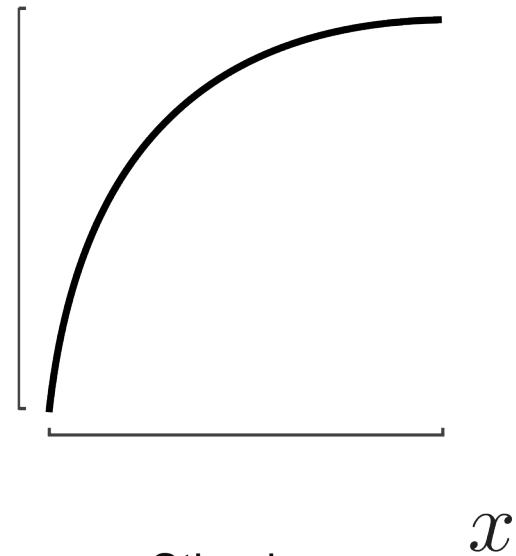
Aim to **characterize mapping between stimulus and perceptual dimensions**

→ scale (y-axis) should represent the magnitude of perceived intensity

- make predictions
- compare with computational models

$$\Psi(x)$$

Perceptual dimension



Stimulus dimension

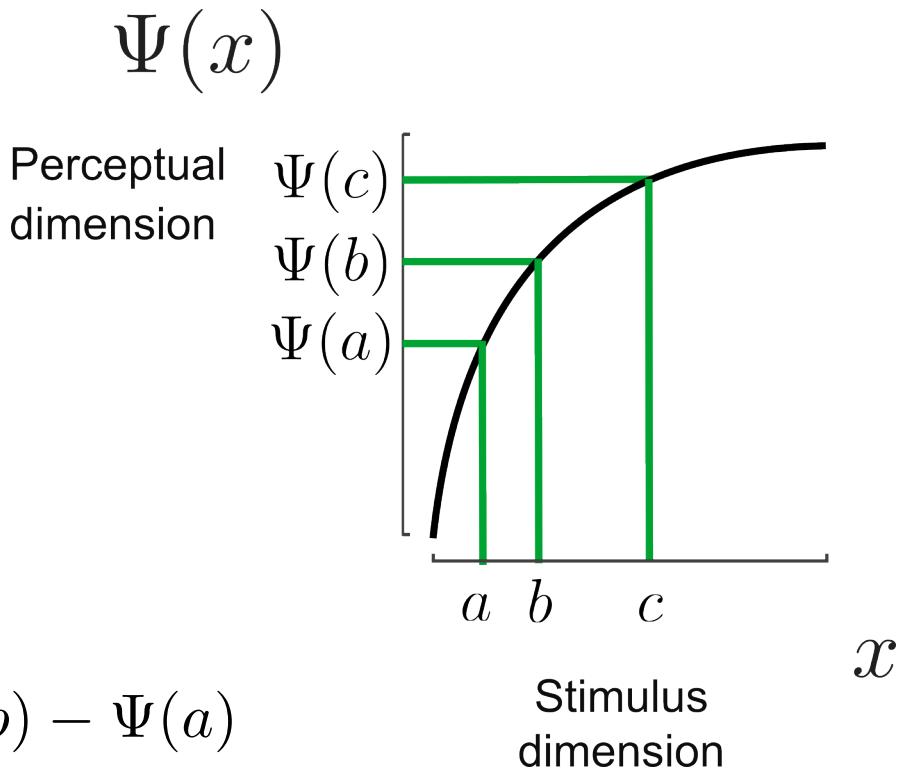
Intro: scaling methods

Aim to **characterize mapping between stimulus and perceptual dimensions**

→ scale (y-axis) should represent the magnitude of perceived intensity

- make predictions
- compare with computational models

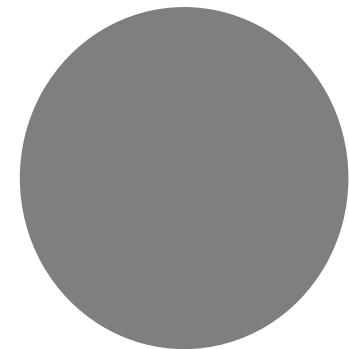
$$\begin{aligned}\Psi(c) - \Psi(b) &\sim \Psi(b) - \Psi(a) \\ (c - b) &>> (b - a)\end{aligned}$$



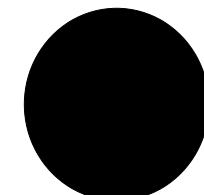
First thought: magnitude estimation

- Task:
“Give a number that reflects the perceived intensity of the stimulus”
- Known to produce unreliable and suffer from confounds
 - Regression effect
→ O's do not use whole range
 - Anchoring effects
→ scale' shape changes depending on the anchor given
- Assumes linear mapping between perception and the “number line”

Stimulus



Anchor



= 0

Maximum Likelihood Difference Scaling (MLDS)

Key features

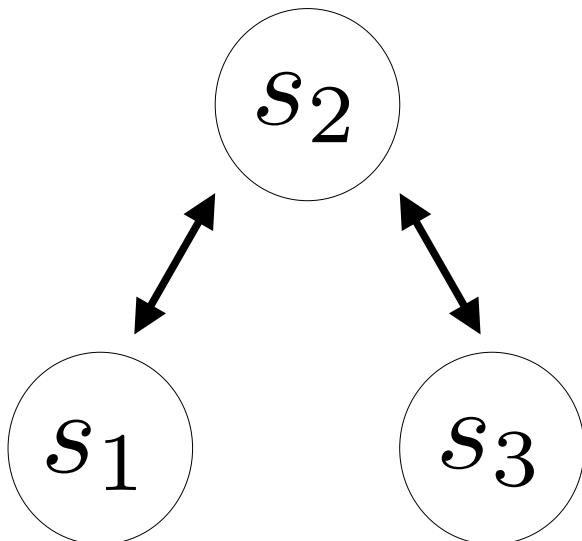
- Scaling method based on judgments of **perceptual differences** (intervals)
- For variations on **one perceptual dimension** (e.g. contrast, brightness, glossiness, image quality)
- **Forced choice** task, producing binary data
- **Explicit observer model**
- Model considers **stochasticity** on the observers' judgments

Maloney & Yang (2003)
Knoblauch & Maloney (2012)

Experimental design in MLDS

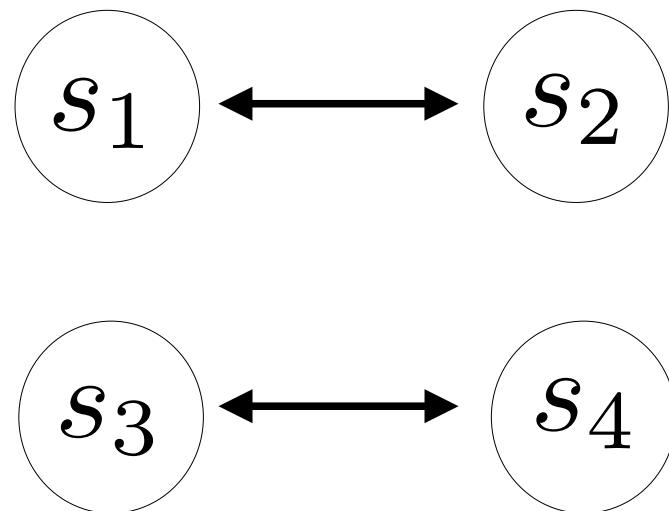
Task: method of triads or quadruples.

Triads



Which stimulus is more different
than the middle one?

Quadruples



Which pair is more different?

Forced-choice judgment

Advantages:

- judgment is a binary decision
 - more easily modeled
 - need of an observer model (more detail later)
- quick trials, only 1 – 2 seconds
 - (vs. several seconds of adjustment in matching tasks)

Disadvantages:

- requires many repetitions



Examples of MLDS usage

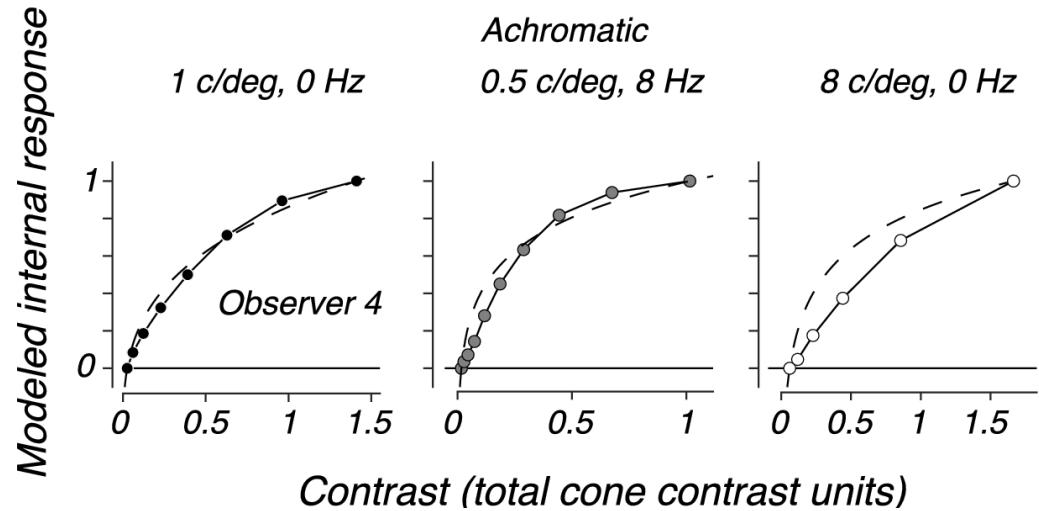
Contrast perception (Shooner & Mullen, 2022)

Experiment 2: Contrast difference scaling

B



"Which of the bottom stimuli differs most from the stimulus on top?"

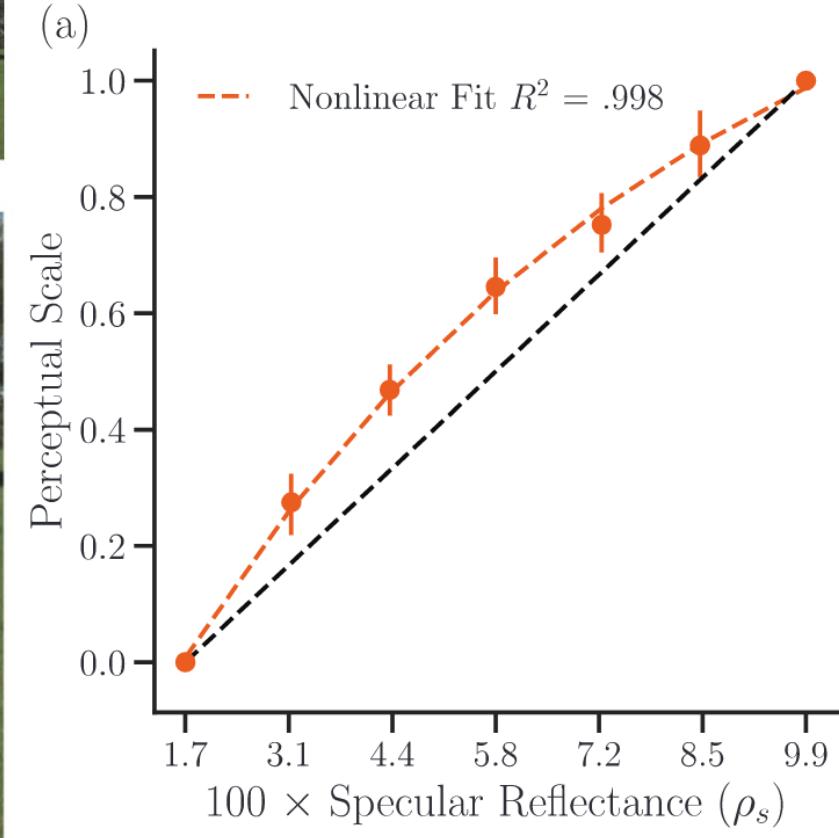


Examples of MLDS usage

Glossiness (Cheeseman et al. 2021)



Minimum ← Specular Reflectance (ρ_s) → Maximum



Examples of MLDS usage

Perception of image degradation due to compression (Charrier et al. 2007)

1:1



6:1



9:1



12:1



15:1



18:1



21:1



24:1



27:1

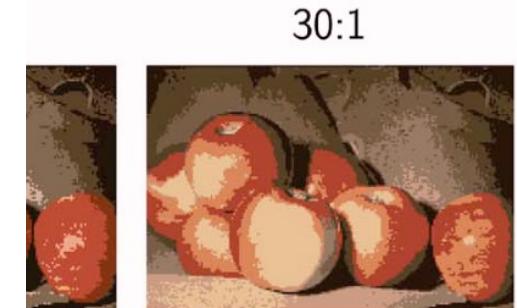
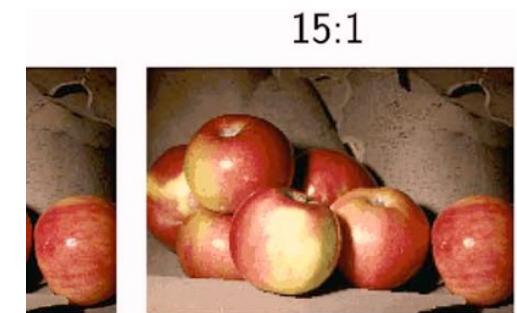
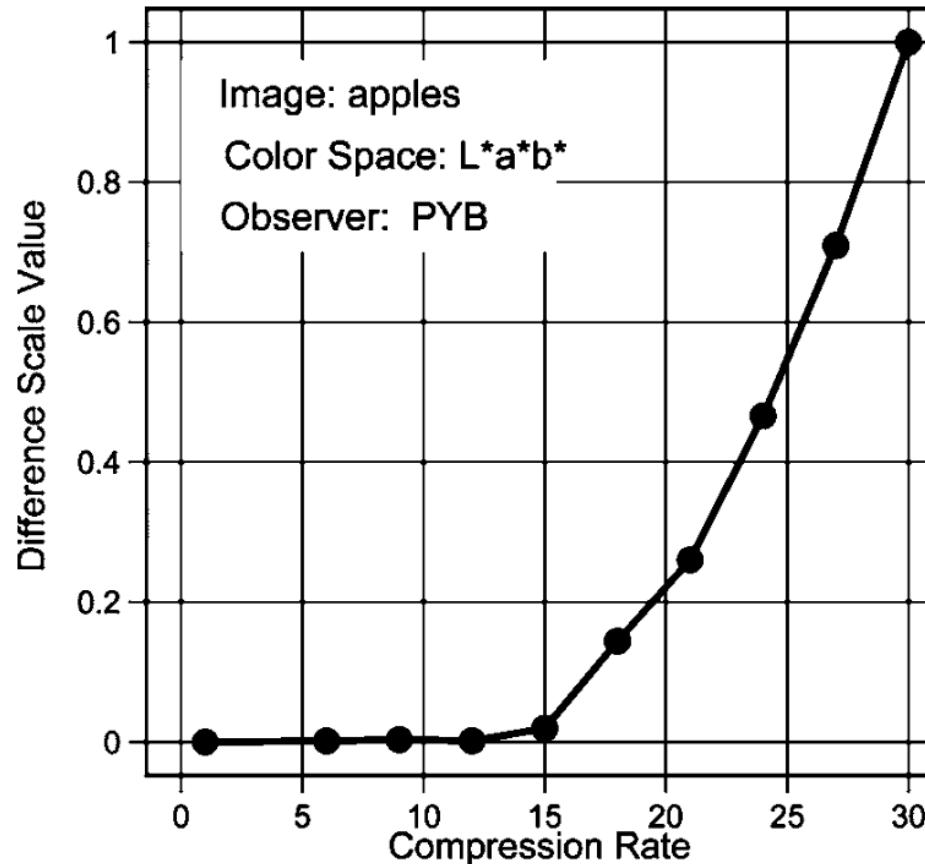
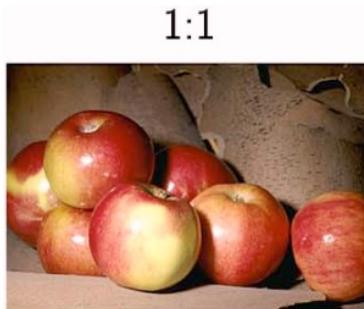


30:1

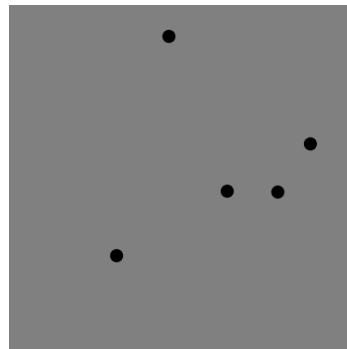


Examples of MLDS usage

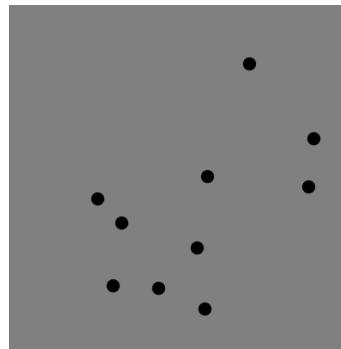
Perception of image degradation due to compression (Charrier et al. 2007)



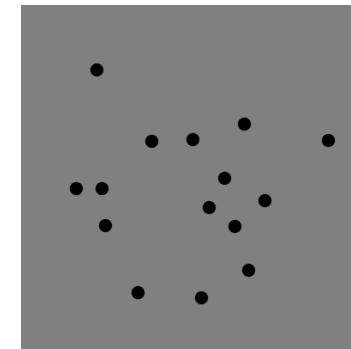
Tutorial today: perception of numerosity



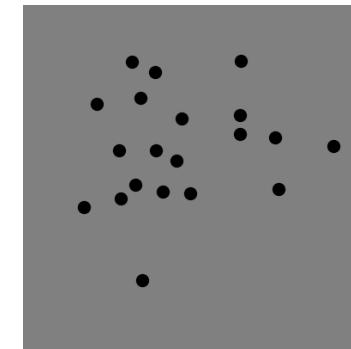
5



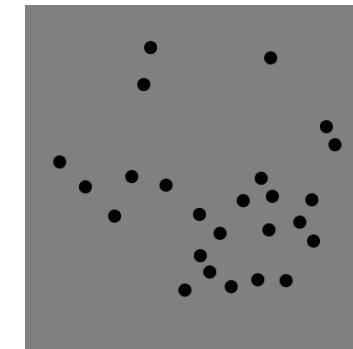
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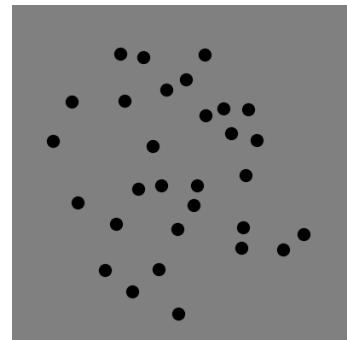
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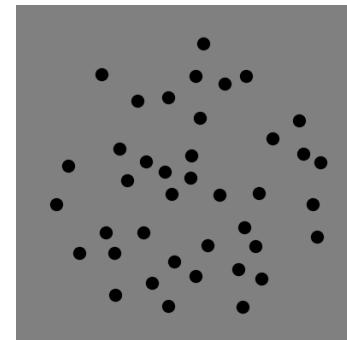
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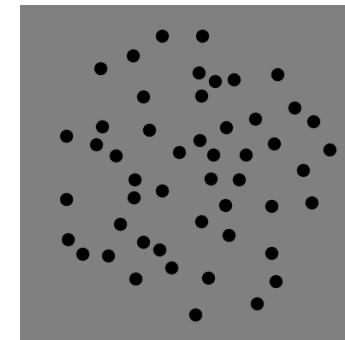
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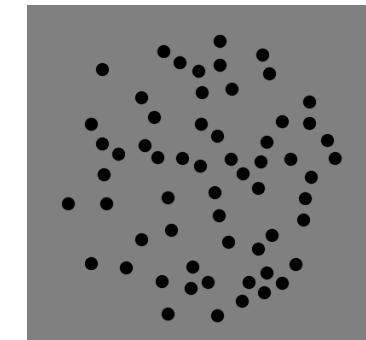
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40

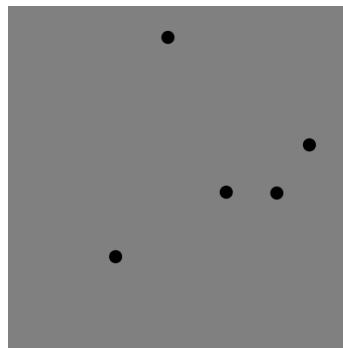


50

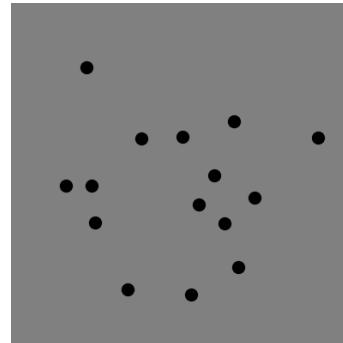


60

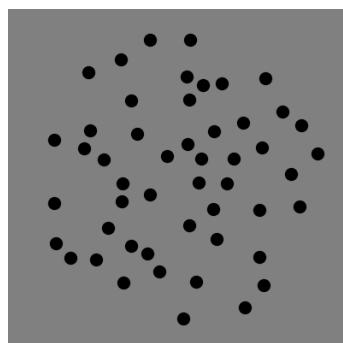
Tutorial today: perception of numerosity



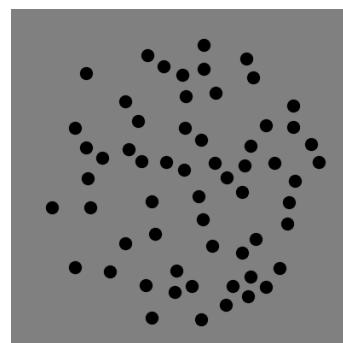
5



15



50



60

Exercise: acquire your own data

Go to

<https://kutt.it/mlds-exp>

and run the MLDS experiment with the method of triads

At the end of the experiment you will get a CSV file.

Keep it.



Observations?

- Mix of difficult and easy trials
- “Odd one out” type of judgment
- Did you use any strategy?

MLDS: details of experimental design

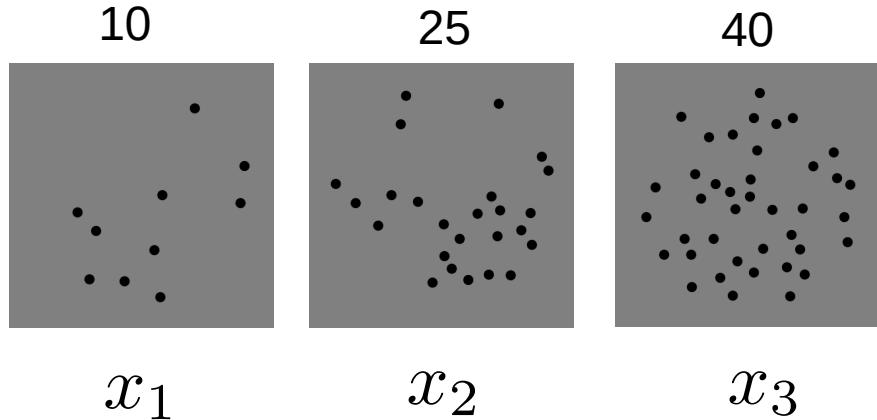
- Triads are always ascending $s_1 > s_2 > s_3$

or descending $s_1 < s_2 < s_3$

- Number of unique triads $\binom{N}{3}$ for N stimuli

- Repeat in block or random sampling

MLDS observer model and estimation

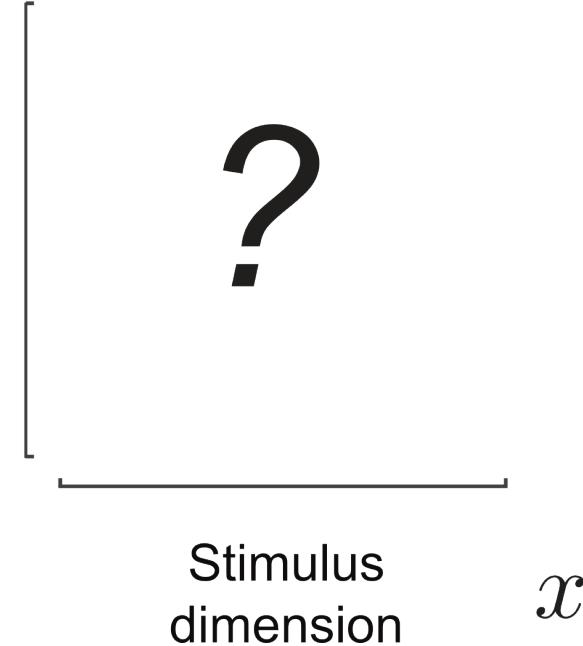


Decision model

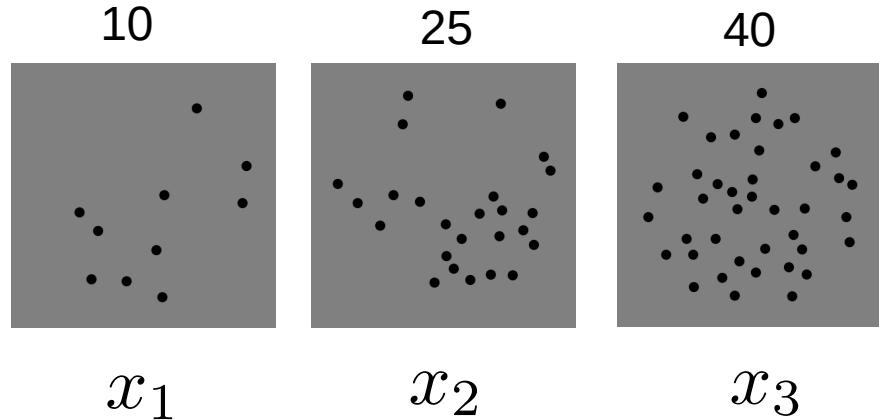
$$\begin{aligned}\Delta &= [\Psi(x_3) - \Psi(x_2)] - [\Psi(x_2) - \Psi(x_1)] + \epsilon \\ &= \Psi(x_3) - 2\Psi(x_2) + \Psi(x_1) + \epsilon \\ \epsilon &\sim N(0, \sigma^2)\end{aligned}$$

if $\Delta > 0 \rightarrow \text{"right"}$
otherwise $\rightarrow \text{"left"}$

$\Psi(x)$
Perceptual dimension



MLDS observer model and estimation

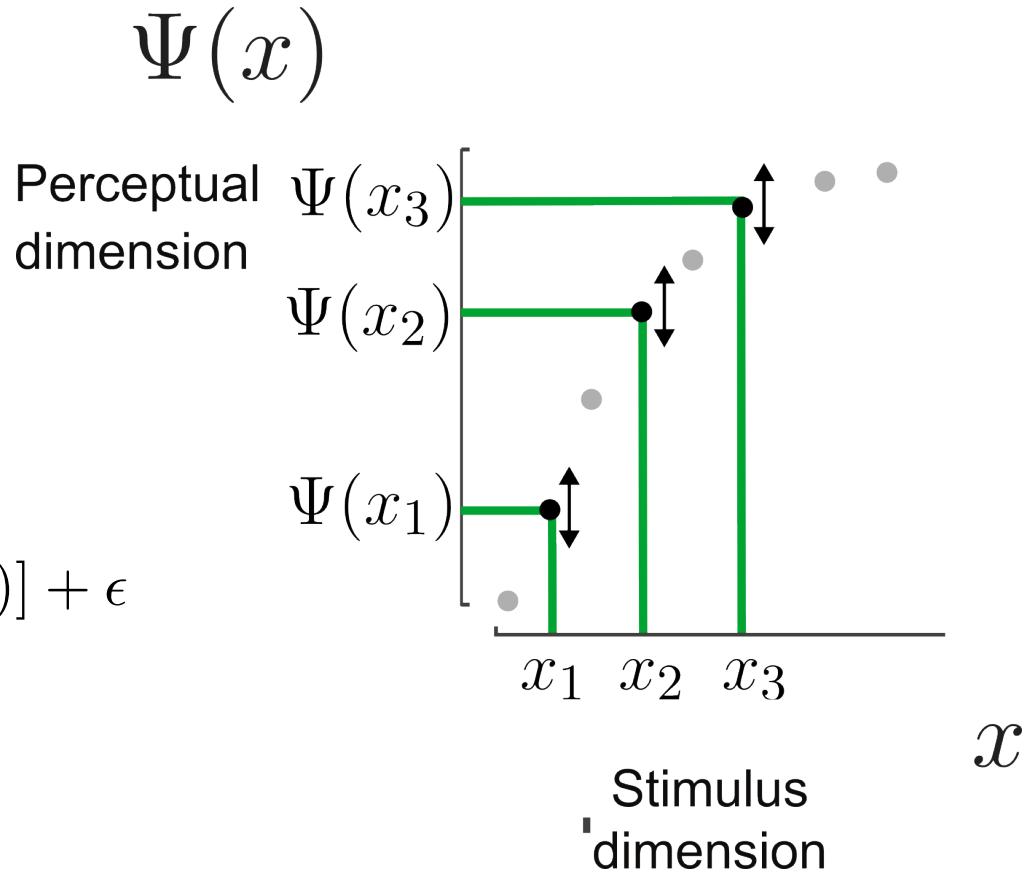


Decision model

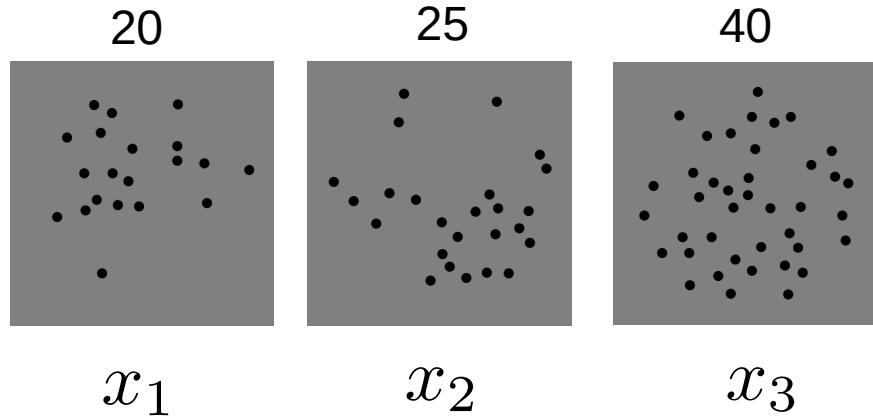
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$$\epsilon \sim N(0, \sigma^2)$$

if $\Delta > 0 \rightarrow \text{"right"}$
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MLDS observer model and estimation

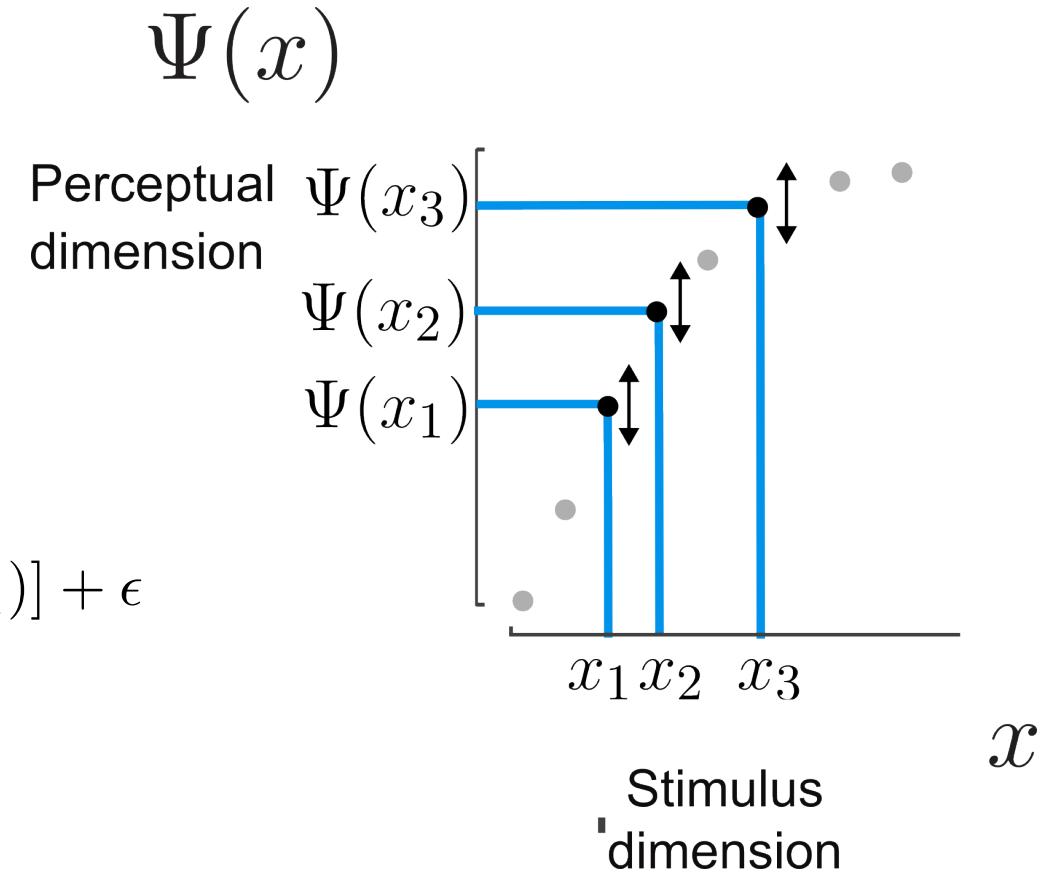


Decision model

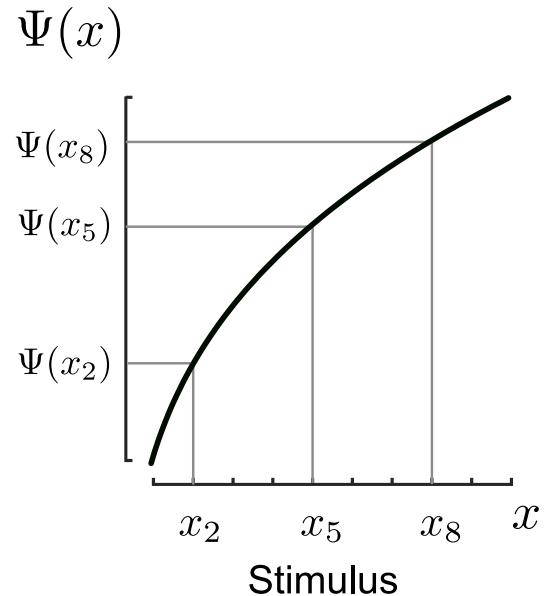
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if $\Delta > 0 \rightarrow \text{"right"}$
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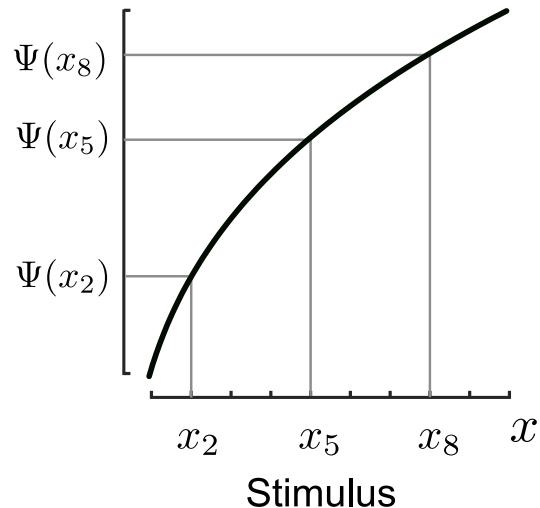
MLDS as a Generalized Linear Model (GLM)



Decision model $\Delta = \Psi(x_8) - 2\Psi(x_5) + \Psi(x_2)$

MLDS as a Generalized Linear Model (GLM)

$$\Psi(x)$$



$$g(\mathbf{E}[Y]) = \mathbf{X} * \boldsymbol{\beta} + \epsilon$$

| |
|---|
| 0 |
| 0 |
| 1 |
| 0 |
| |

| | | | | | | | | | | |
|---|---|----|---|----|----|---|---|----|---|---|
| 0 | 1 | 0 | 0 | -2 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | -2 | 1 | 0 |
| 0 | 1 | 0 | 0 | -2 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | -2 | 0 | 0 | 0 | 0 | 1 |

⋮

⋮

| |
|--------------|
| β_1 |
| β_2 |
| β_3 |
| β_4 |
| β_5 |
| β_6 |
| β_7 |
| β_8 |
| β_9 |
| β_{10} |

Decision model

$$\Delta = \Psi(x_8) - 2\Psi(x_5) + \Psi(x_2)$$

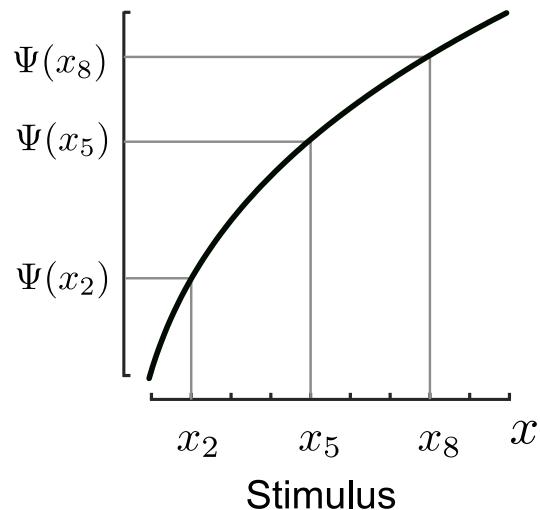
GLM

$$g(E[Y]) = 1 \cdot \beta_8 - 2 \cdot \beta_5 + 1 \cdot \beta_2$$

Wiebel, Aguilar & Maertens (2017)

MLDS as a Generalized Linear Model (GLM)

$$\Psi(x)$$

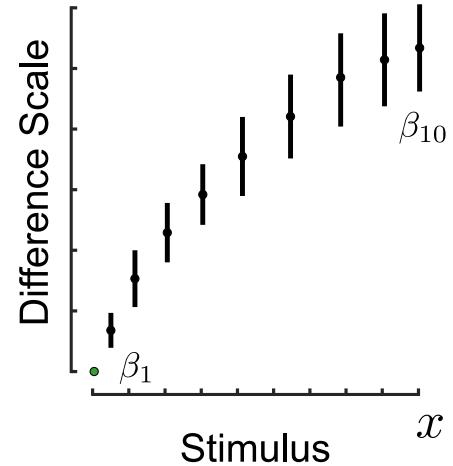


$$g(\mathbf{E}[Y]) = \mathbf{X} * \boldsymbol{\beta} + \epsilon$$

$$\begin{matrix} 0 \\ 0 \\ 1 \\ 0 \\ \vdots \end{matrix}$$

| | | | | | | | | | | |
|---|---|----|---|----|----|---|---|----|---|---|
| 0 | 1 | 0 | 0 | -2 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | -2 | 1 | 0 |
| 0 | 1 | 0 | 0 | -2 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | -2 | 0 | 0 | 0 | 0 | 1 |

$$\begin{matrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \\ \beta_7 \\ \beta_8 \\ \beta_9 \\ \beta_{10} \end{matrix}$$



Decision model

$$\Delta = \Psi(x_8) - 2\Psi(x_5) + \Psi(x_2)$$

GLM

$$g(E[Y]) = 1 \cdot \beta_8 - 2 \cdot \beta_5 + 1 \cdot \beta_2$$

Wiebel, Aguilar & Maertens (2017)

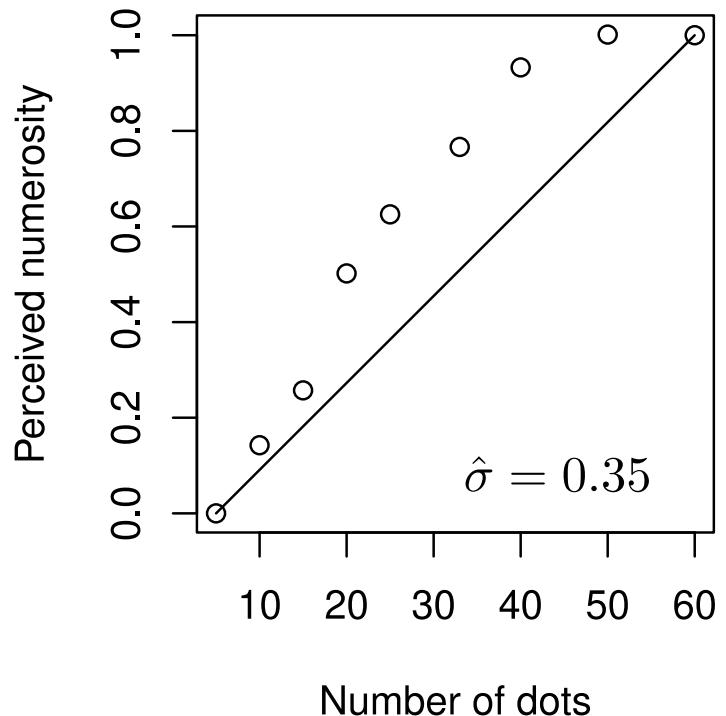
Live coding:

analyzing MLDS data in R

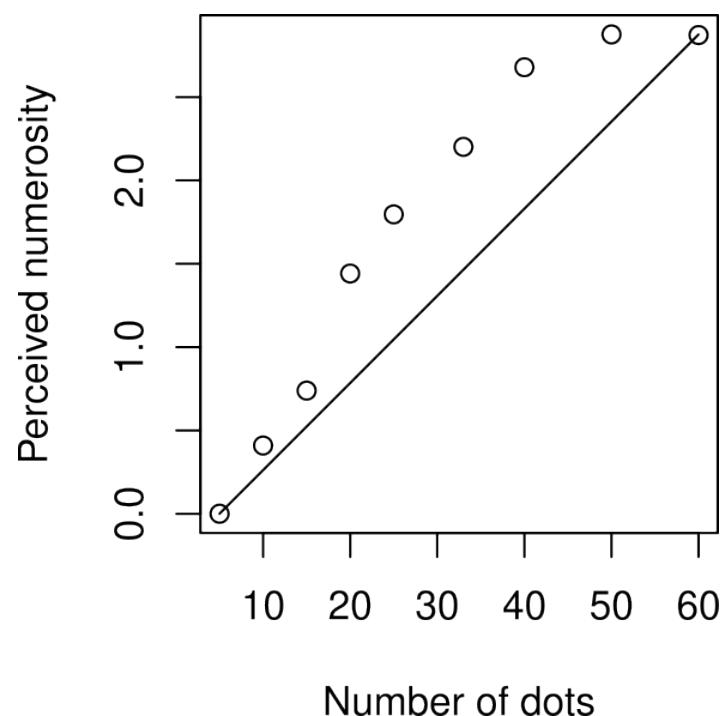
Interpretation of results

Two parametrizations

“standard”: range from 0 to 1



“unconstrained”: range from 0 to $\frac{1}{\hat{\sigma}}$



Exercise: analyze your own data

1) Go to github and download the repository

<https://github.com/computational-psychology/ecvp2025-mlds-mlcm-tutorial>

2) Copy your CSV files to the folder `data/mlds/`

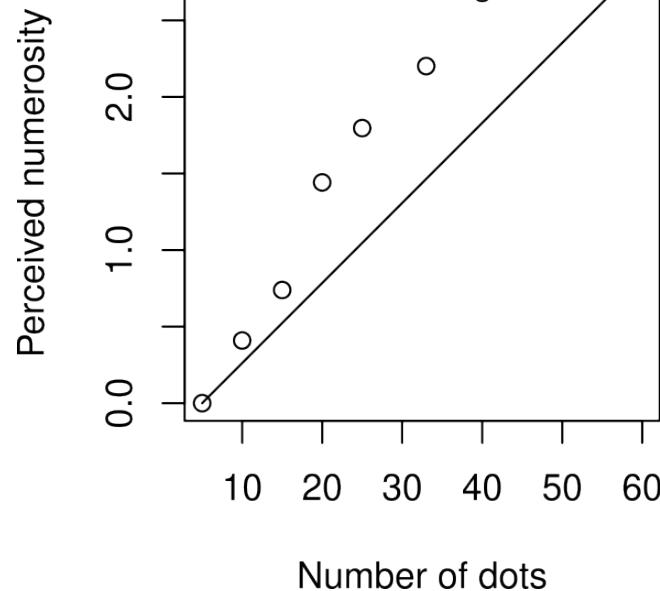
3) In Rstudio, open file: `tutorial_mlds.Rmd`

4) Edit code to match the filename of your data (line 53)

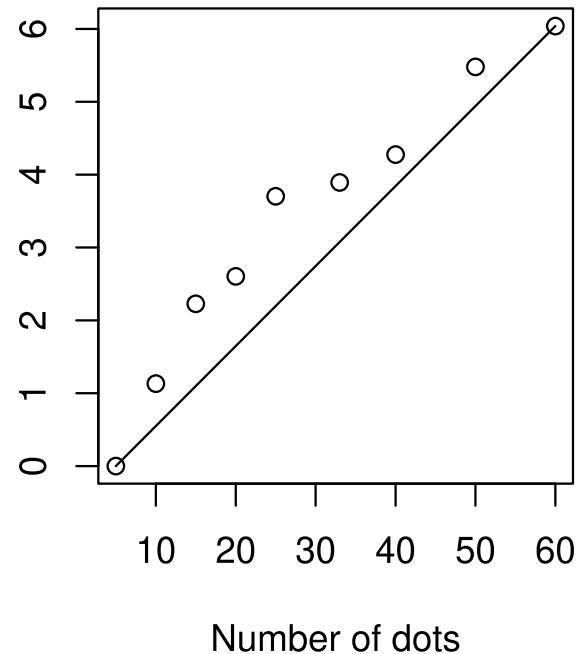
If you run more than once, concatenate your files editing lines 65-72

5) Run all cells (top right, *Run / Run all*)

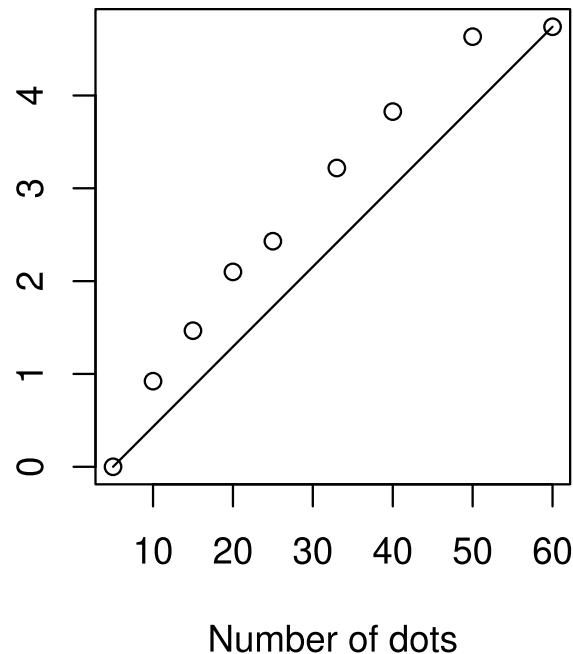
Observer GA



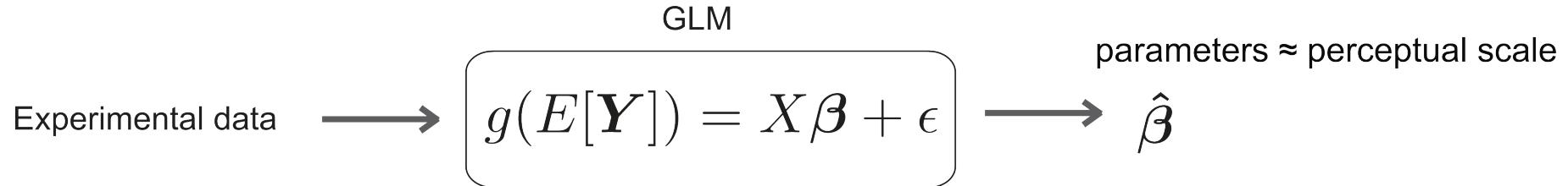
Observer CH



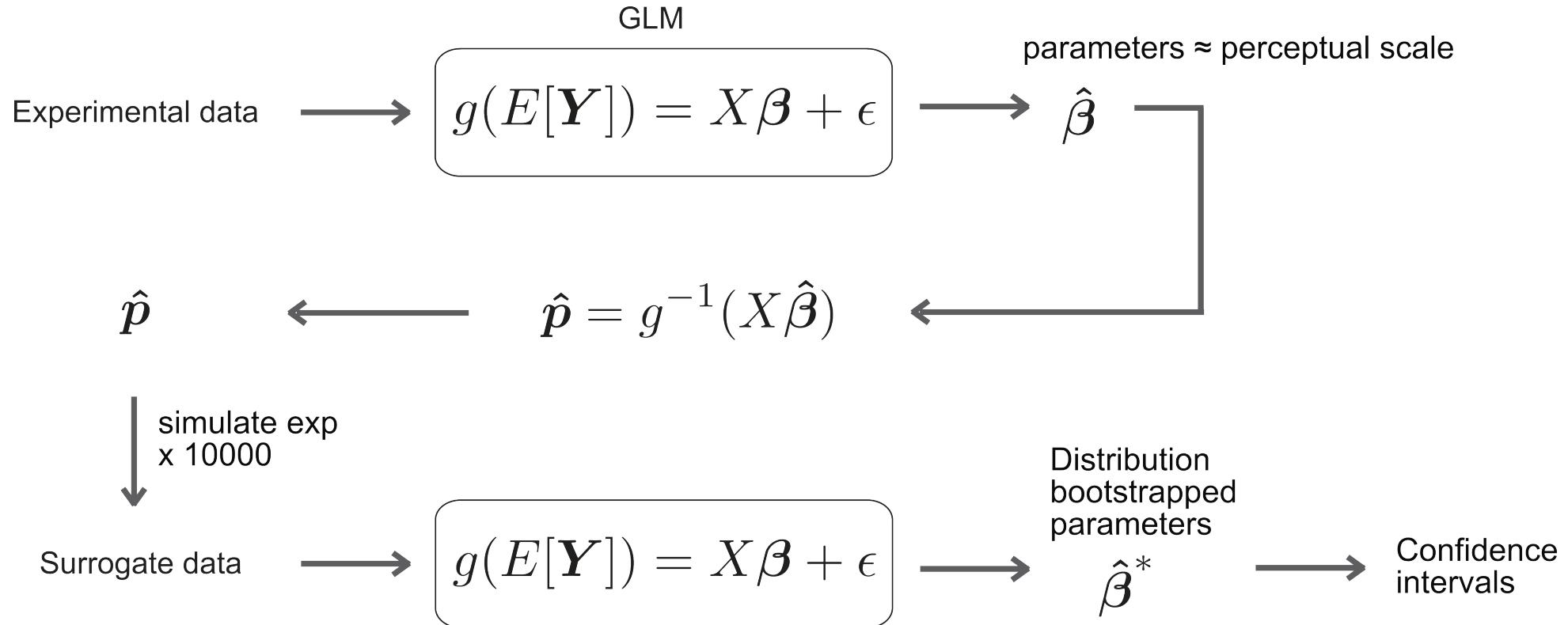
Observer JXV



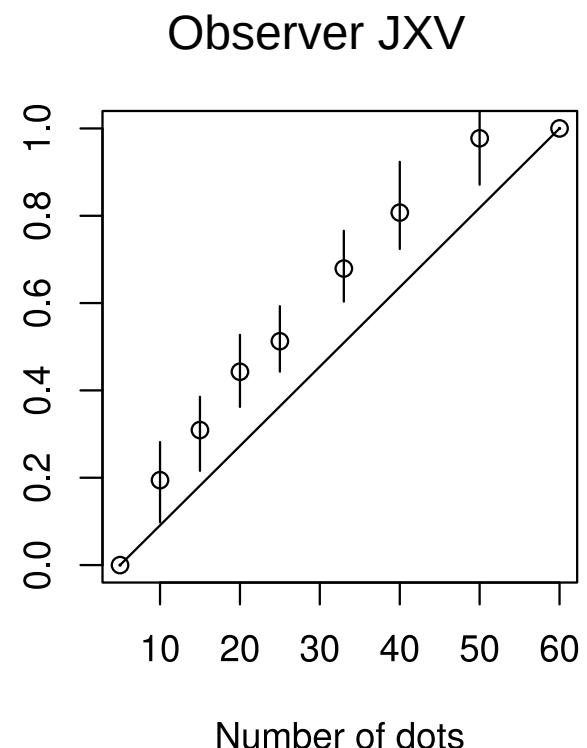
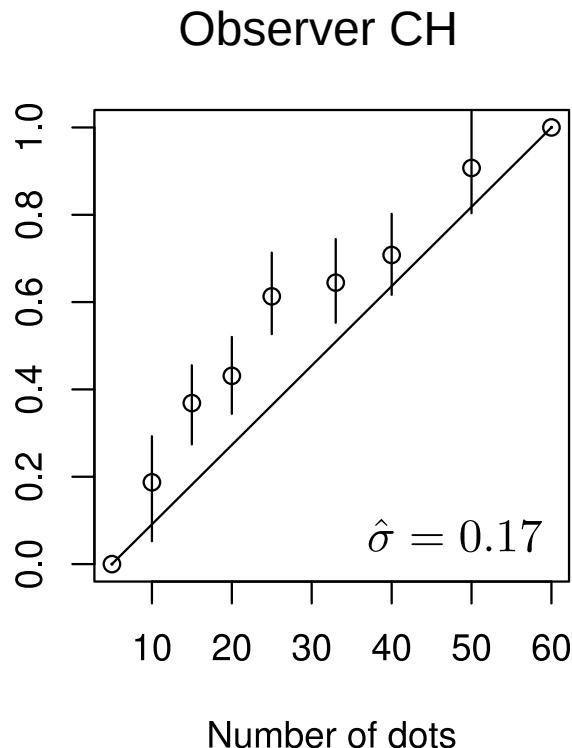
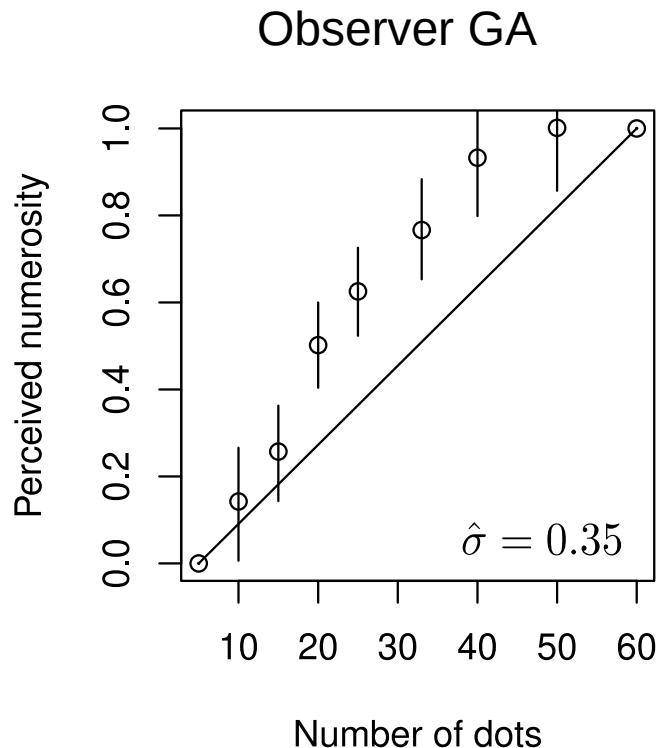
Confidence intervals



Confidence intervals



Confidence intervals



To keep in mind - Model assumptions

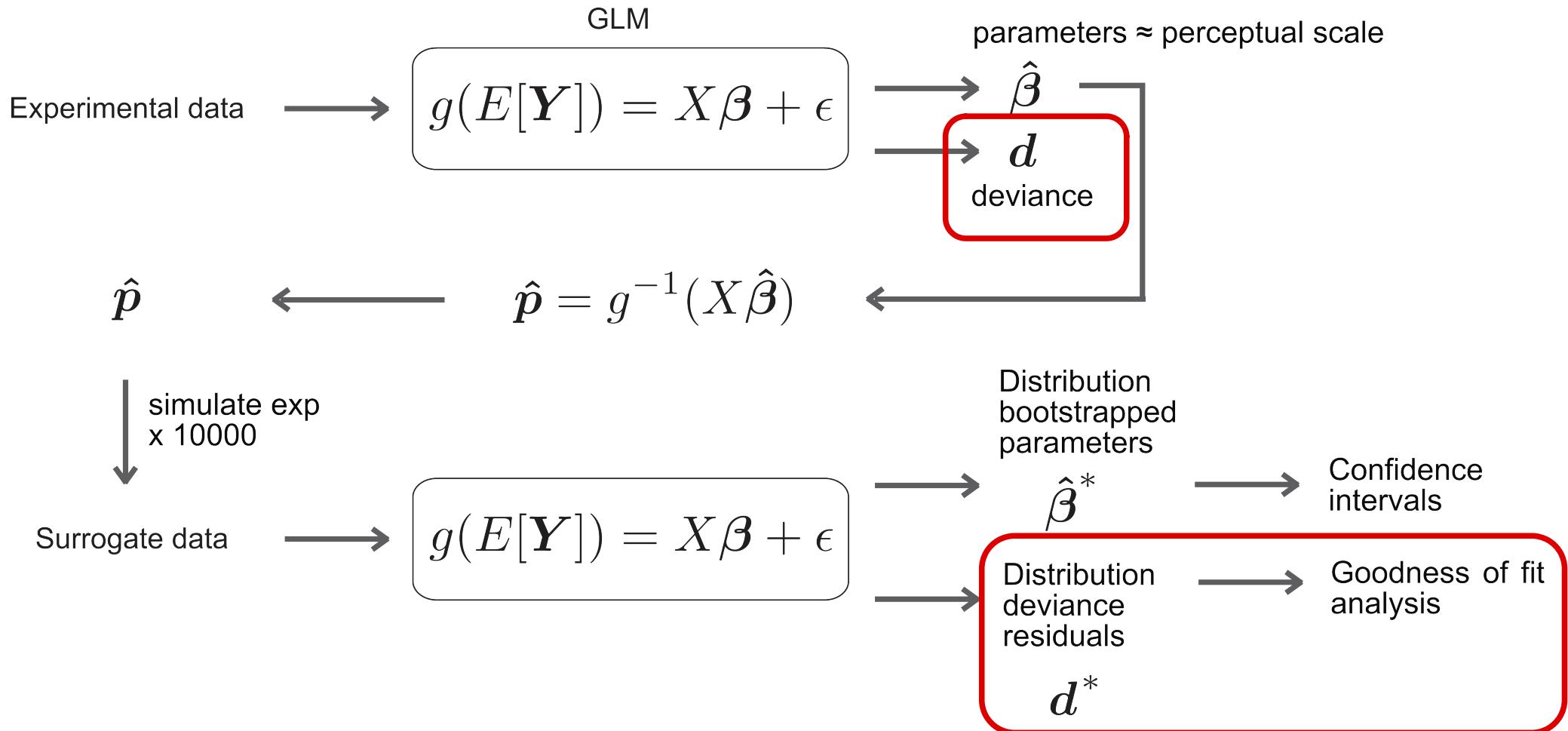
- observer is performing the difference as stated in the decision model
- observer is able to sort stimuli (weak order assumption, Krantz et al.)
- i.i.d. assumptions: each trial is a Bernoulli variable, independent of each other (e.g. no learning)

We can test the integrity of assumptions via simulation

Logic

if the observer model holds,
then experimental data is no different than simulated data created using the
observer model

Testing model assumptions: goodness of fit evaluation



Goodness of fit evaluation - plots

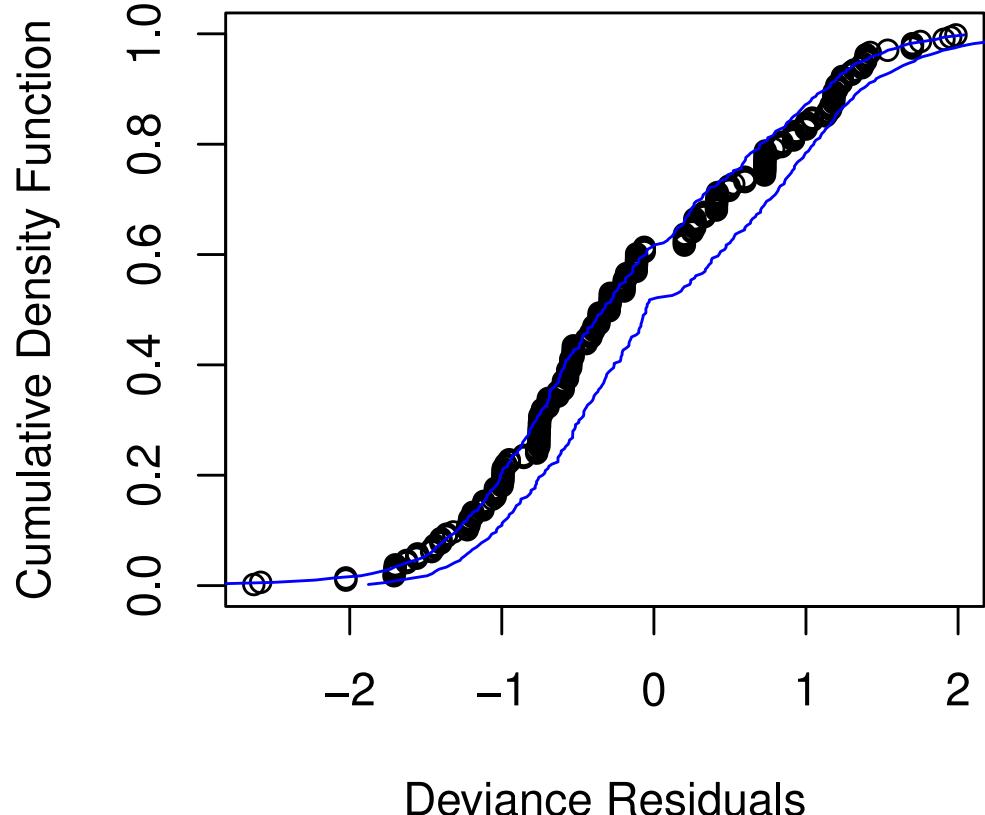
Goodness of fit evaluation - plots

Check 1: (deviance) residuals

- cumulative function of deviance residuals (markers)
- 95% envelope from *simulated* deviance residuals (blue lines)

This envelope represents *expected* range

For an appropriate goodness of fit:
data (markers) should be inside the range



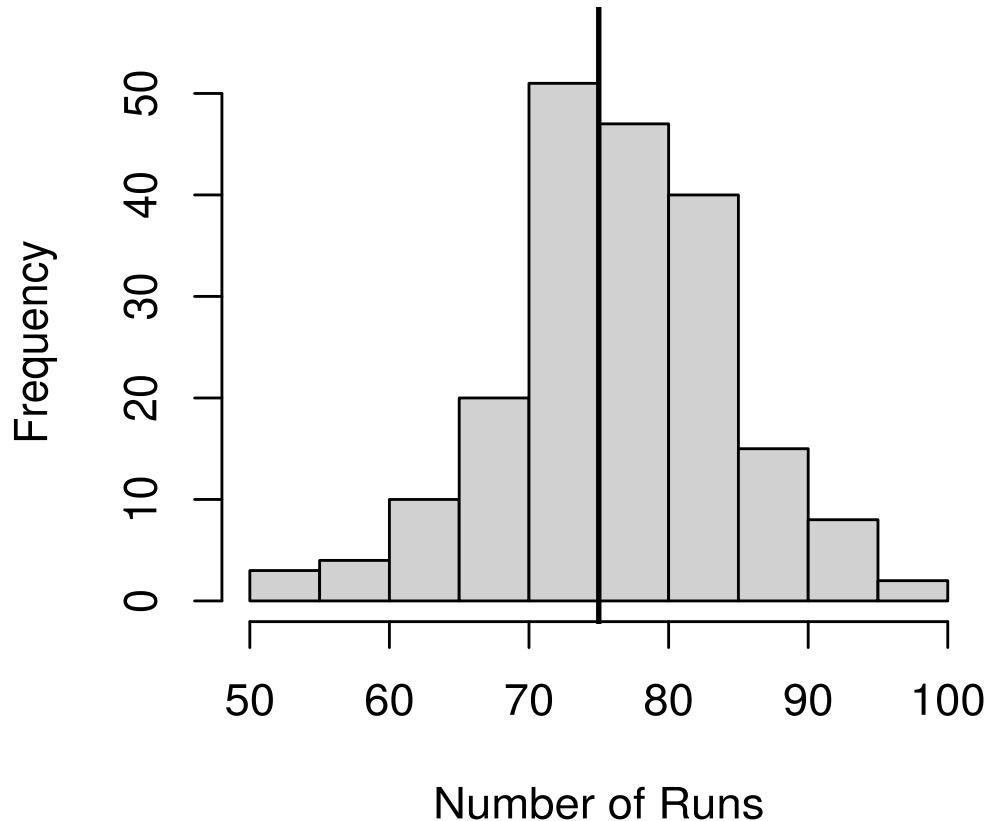
Goodness of fit evaluation - plots

Check 2: distribution of deviance crossings or “runs”

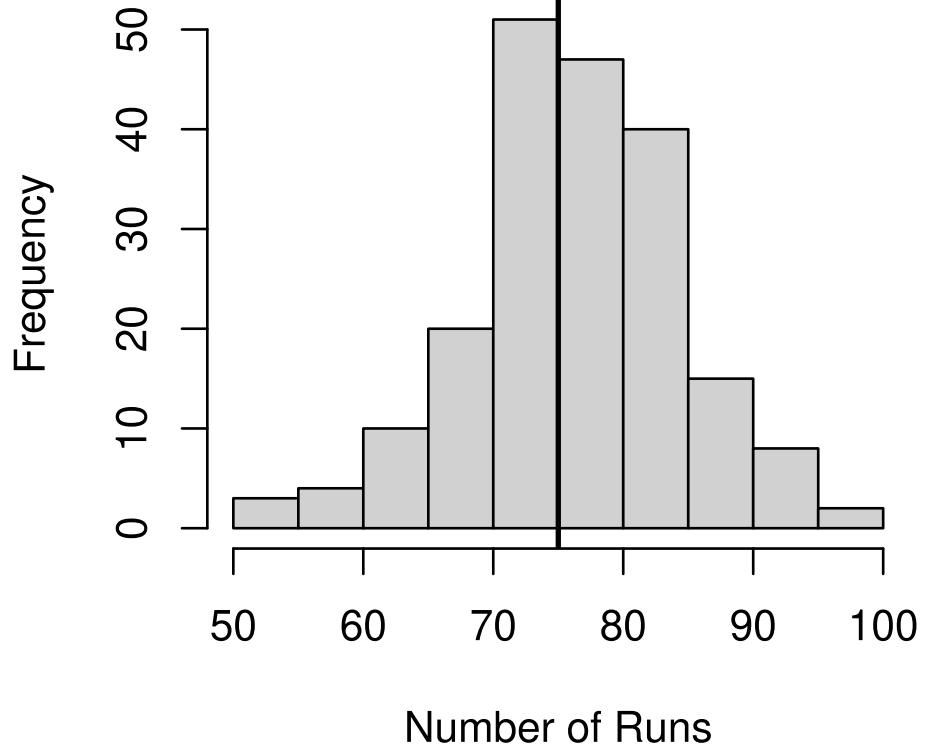
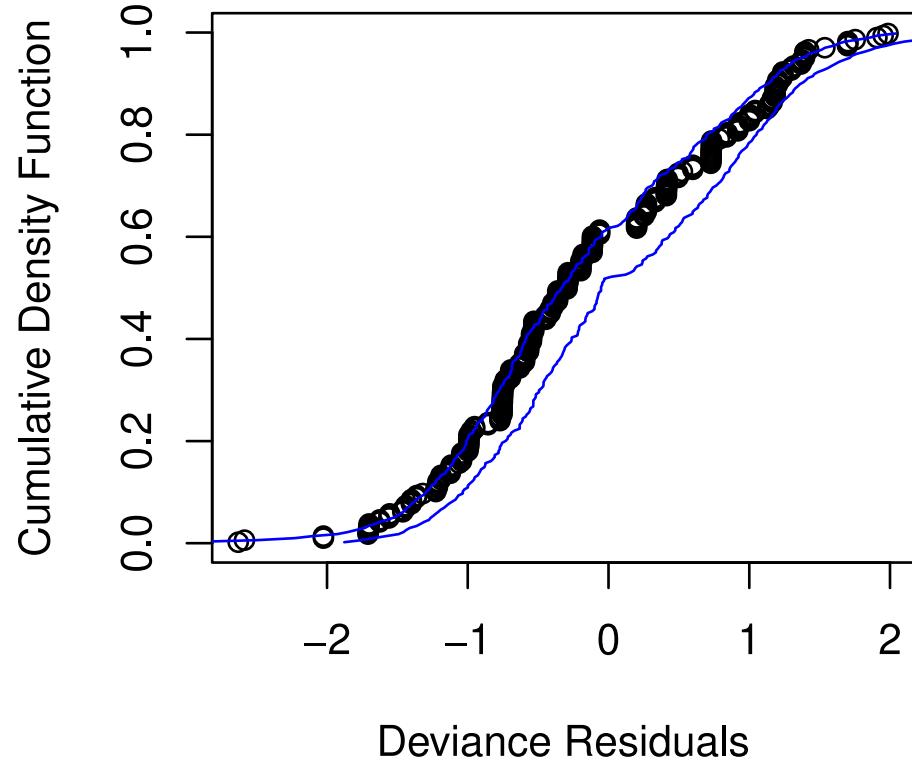
- sort deviance residuals by linear predictor
- count how many have consecutively the same sign (vertical line)
- do the same for the *simulated* deviance residuals, and plot as histogram

For an appropriate goodness of fit:

“Runs” from data (vertical line) should not be sig. different than the distribution



Goodness of fit evaluation - plots

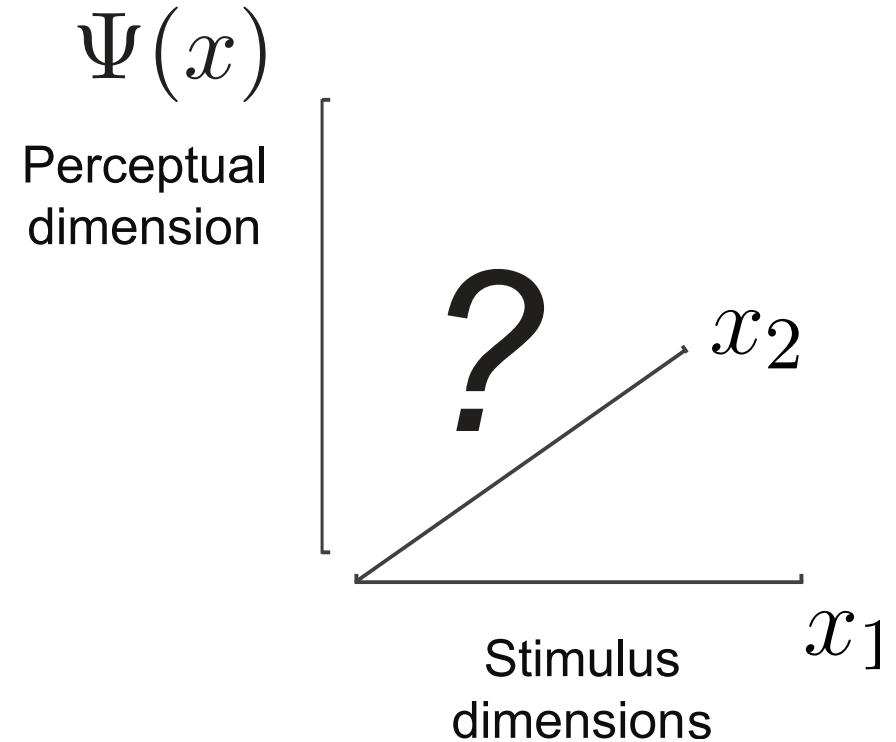


Maximum Likelihood Conjoint Measurement (MLCM)

Motivation for conjoint measurement

What happens when more than 1 stimulus dimensions affect how stimuli look like?

→ conjoint measurement



Motivation for conjoint measurement

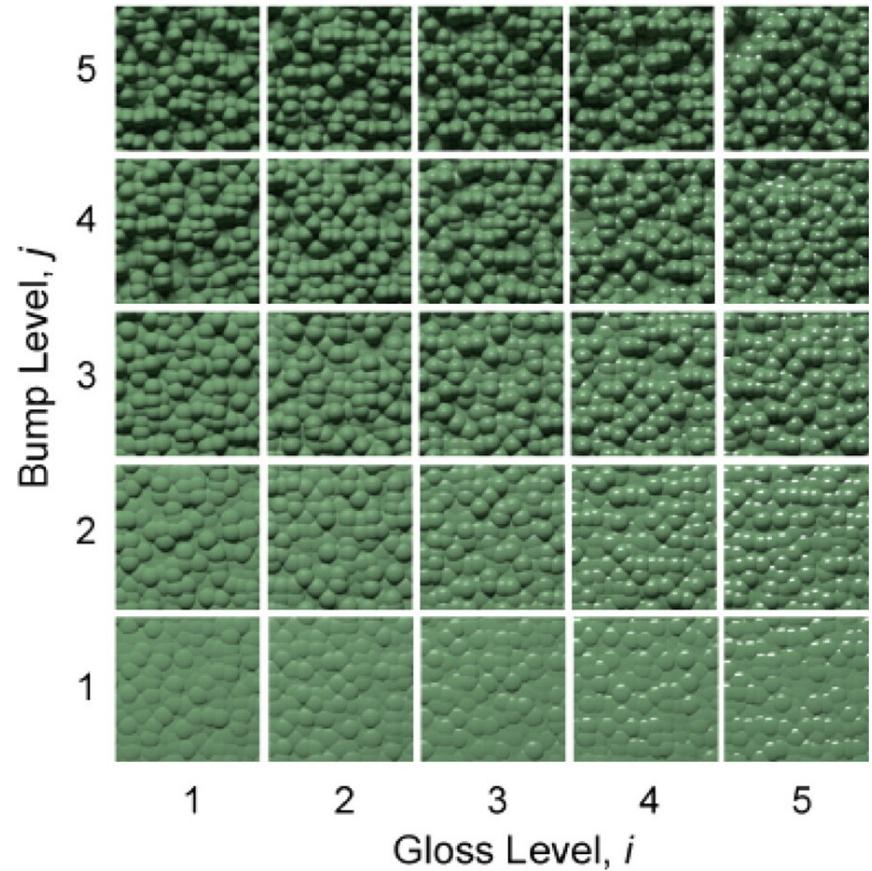
What happens when more than 1 stimulus dimensions affect how stimuli look like?

→ conjoint measurement

Example:

Perceived bumpiness (depth) depends on
- physical bumpiness (depth) and
- gloss

(Ho et al. 2008)



Motivation for conjoint measurement

What happens when more than 1 stimulus dimensions affect how stimuli look like?

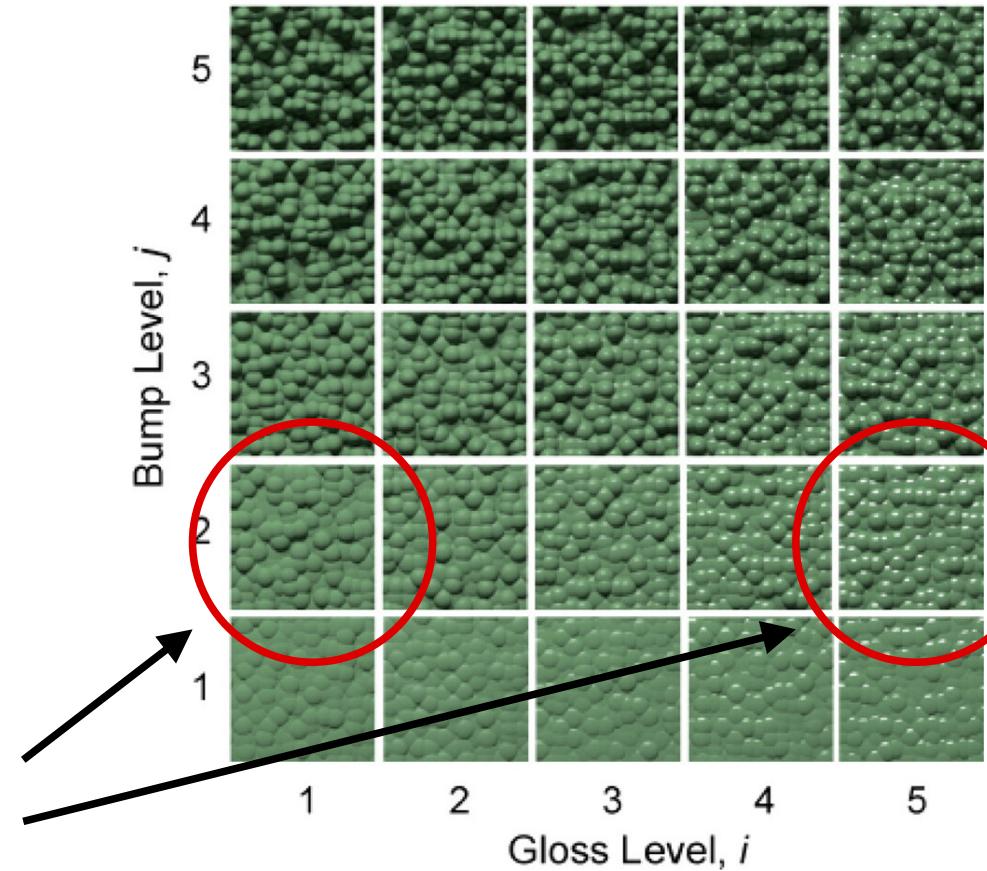
→ conjoint measurement

Example:

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- physical bumpiness (depth) and
- gloss

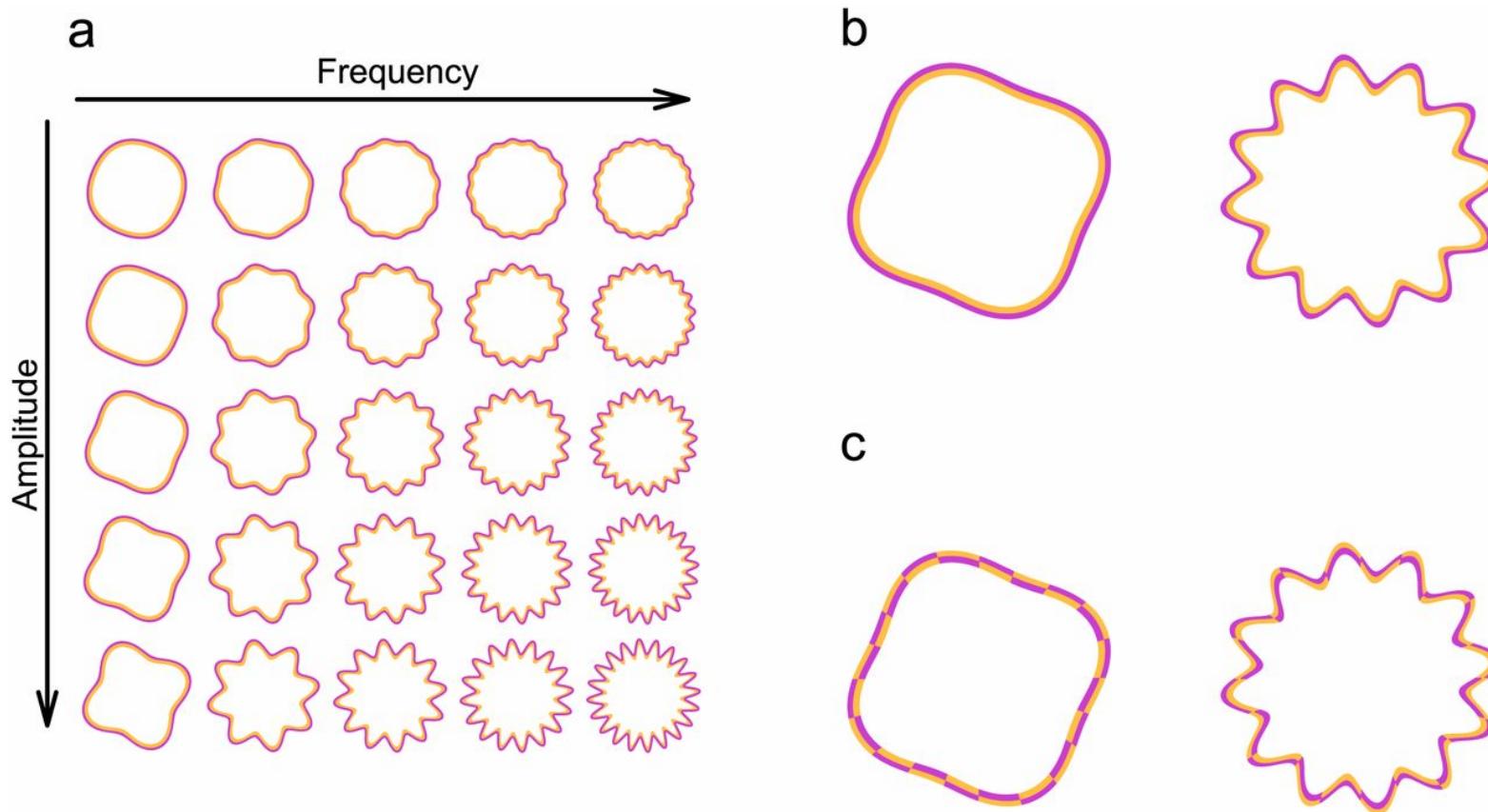
(Ho et al. 2008)

Same physical “bumpiness”, glossier stimulus looks more bumpy than matte one

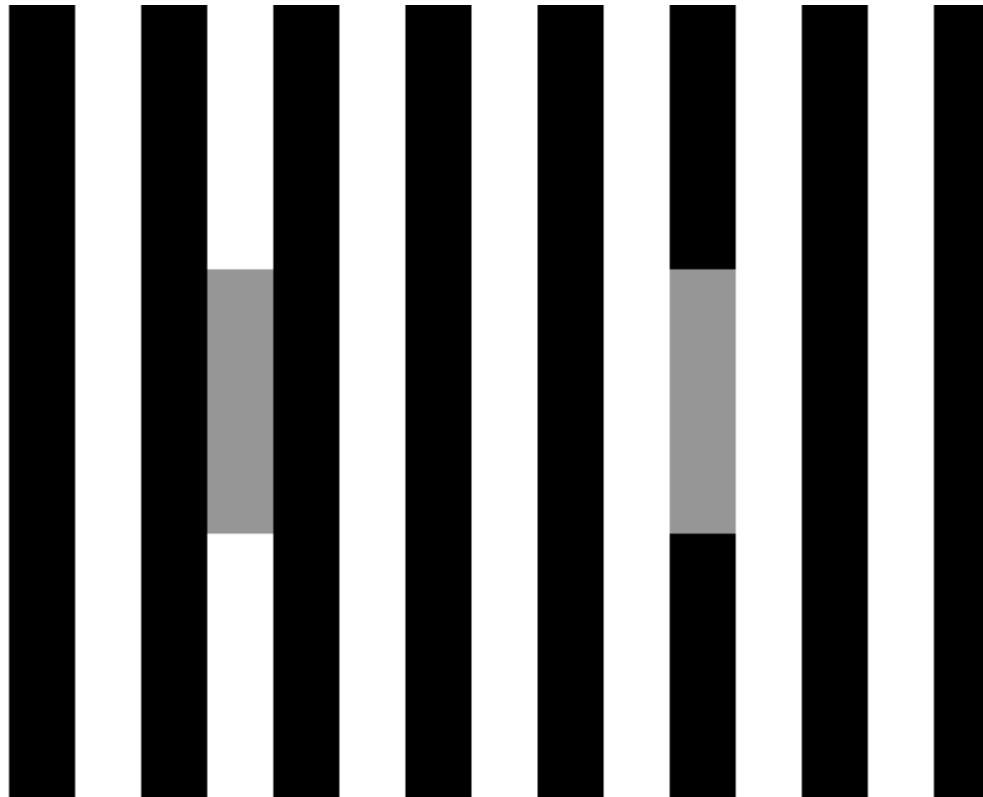


Examples of MLCM usage

Perception of the watercolor effect (Gerardin et al. 2014)

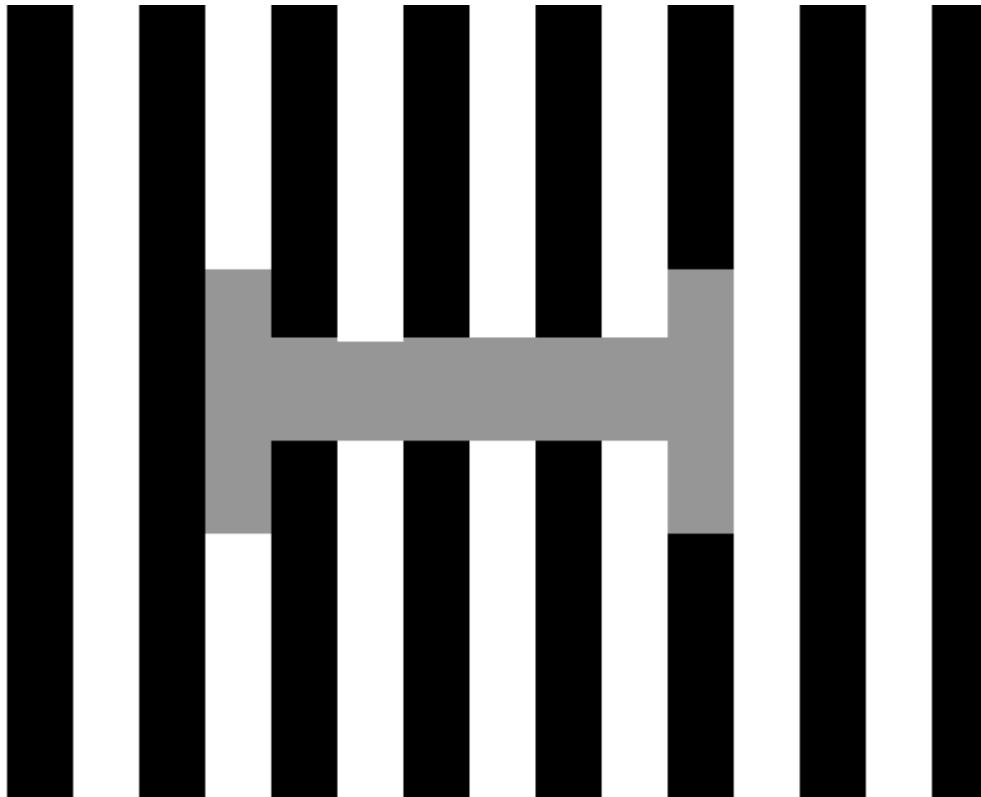


Example today: brightness in White's illusion



White (1979)

Example today: brightness in White's illusion



White (1979)

Target placement

on black

x_1



...

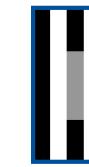
Target luminance



...



...



...



...



...

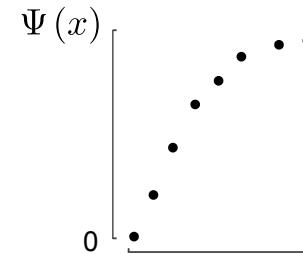
x_{10}

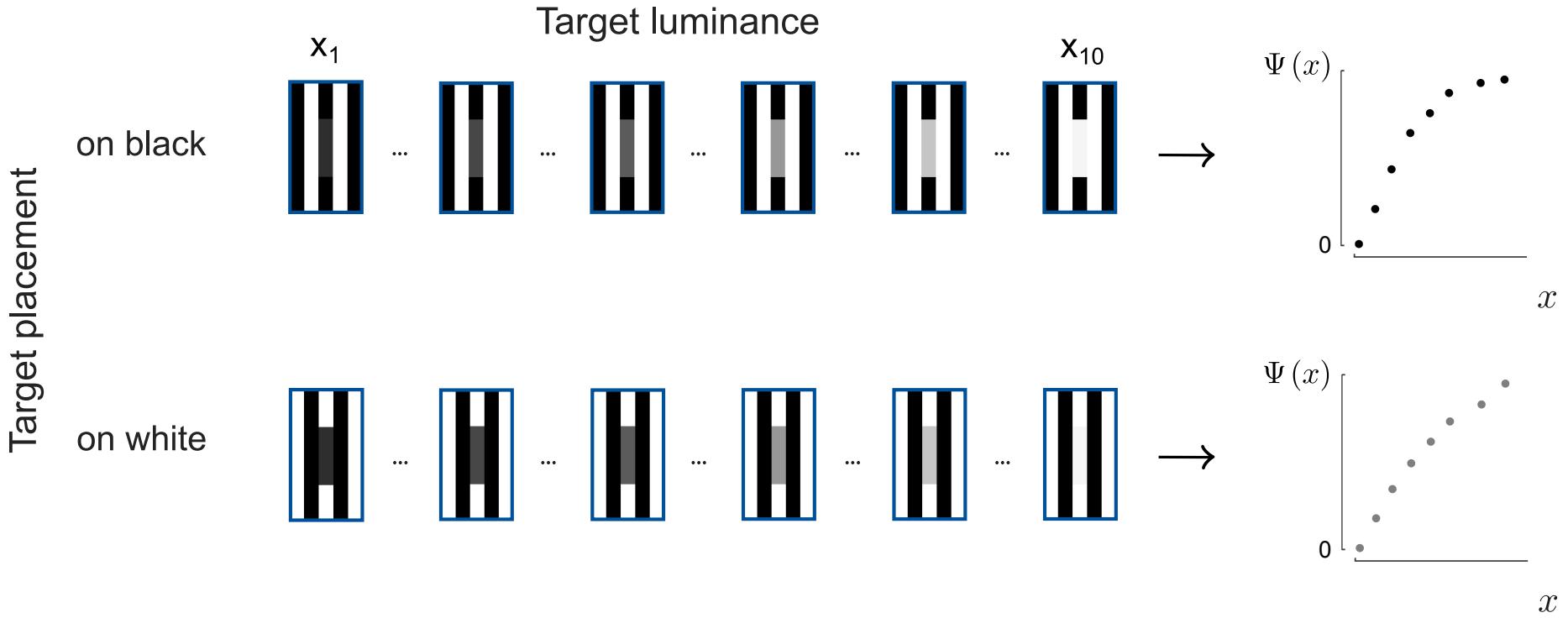


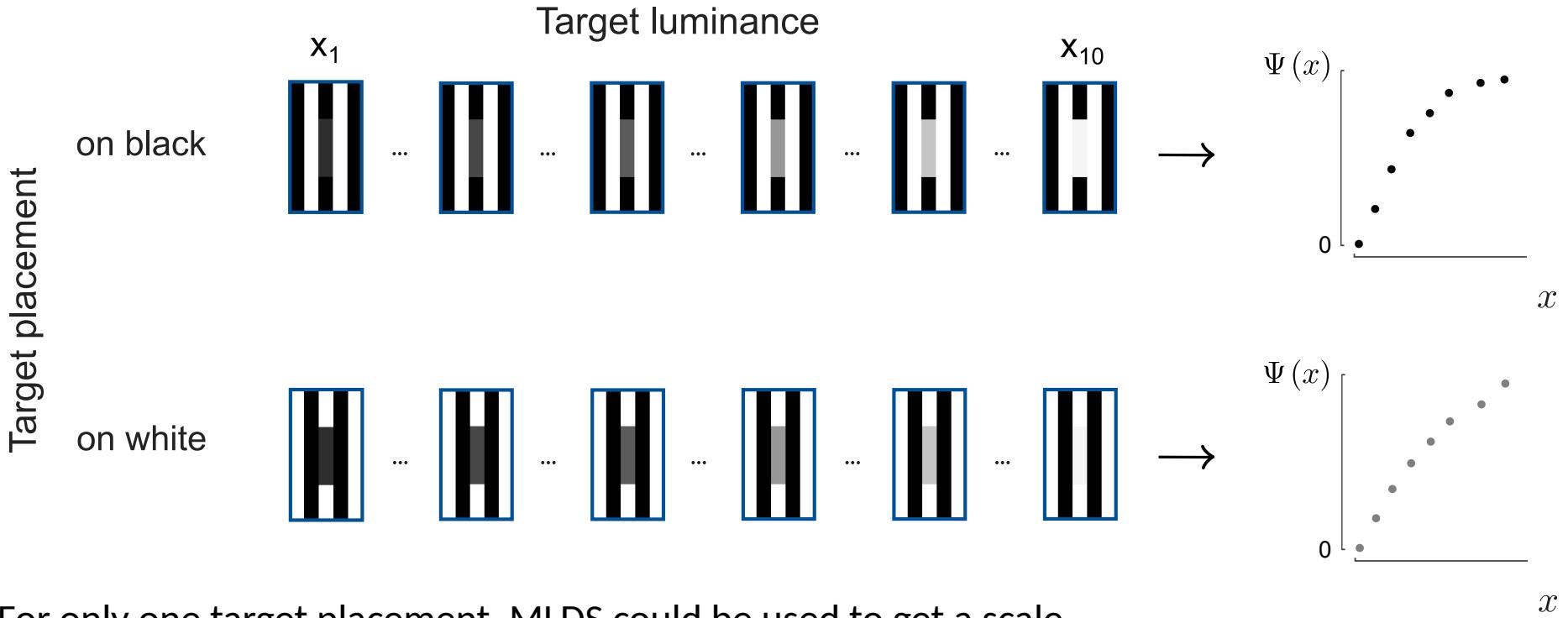
$\Psi(x)$

0

x







For only one target placement, MLDS could be used to get a scale

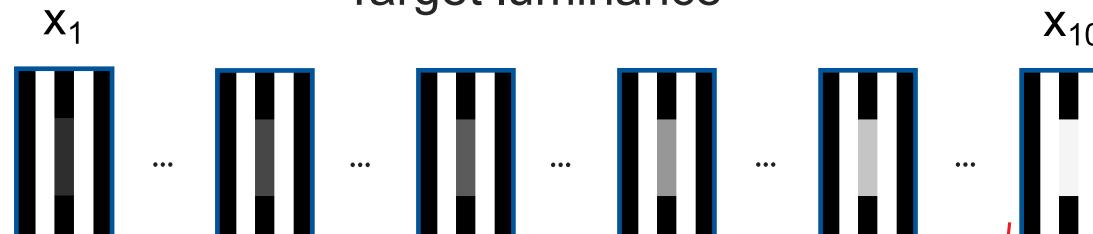
BUT! We would obtain two independent scales, both anchored at zero.

Scales can be scaled and shifted, so we can't relate them meaningfully to each other

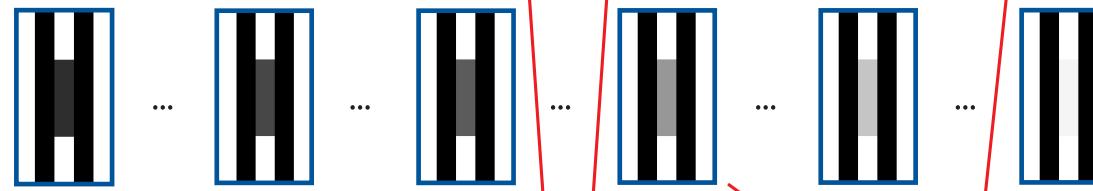
We need to add **target placement** as a **2nd stimulus dimension**
 → **MLCM**

Target placement

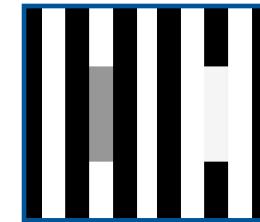
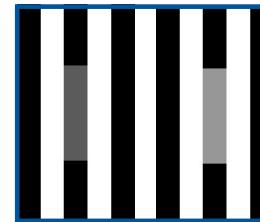
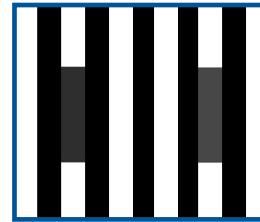
on black



on white



Example
stimuli



$$\Psi^B(x) = ?$$

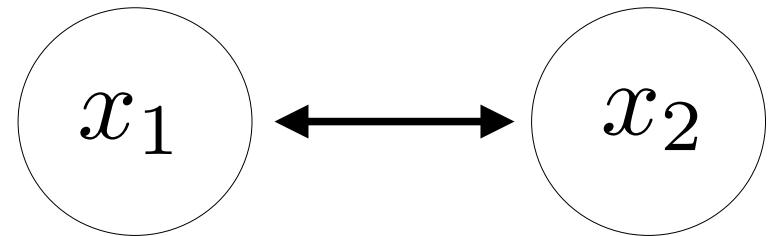
$$\Psi^W(x) = ?$$

Experimental design in MLCM

Stimuli vary in two or more stimulus dimensions

| | | Dim 2 | | | | |
|-------|--------|--------|--------|--------|-----|--------|
| | | S(1,1) | S(1,2) | S(1,3) | ... | S(1,n) |
| | | S(2,1) | S(2,2) | S(2,3) | | |
| Dim 1 | S(3,1) | S(3,2) | ... | | | |
| | ... | | | ... | | |
| | S(m,1) | | | | ... | |

Paired comparisons



Which stimulus is
brighter?
glossier?

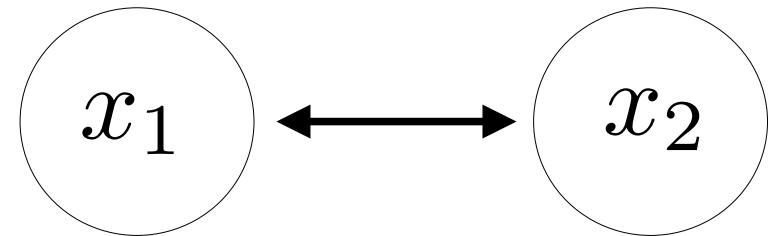
...
<insert your perceptual dimension> ?

Experimental design in MLCM

Stimuli vary in two or more stimulus dimensions

| | | Dim 2 | | | | |
|-------|--------|--------|--------|--------|-----|--------|
| | | S(1,1) | S(1,2) | S(1,3) | ... | S(1,n) |
| | | S(2,1) | S(2,2) | S(2,3) | | |
| Dim 1 | S(3,1) | S(3,2) | ... | | | |
| | ... | | | ... | | |
| | S(m,1) | | | | ... | |

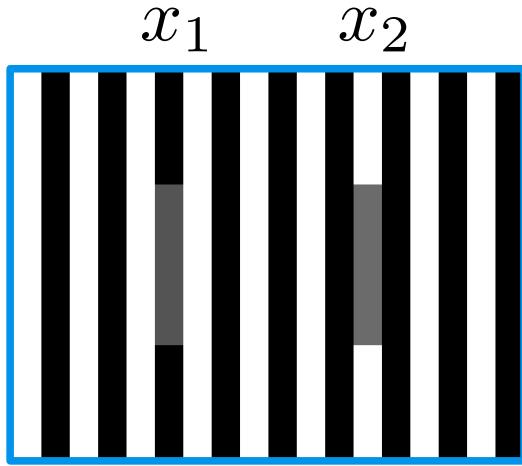
Paired comparisons



Which stimulus is
brighter?
glossier?

...
<insert your perceptual dimension> ?

MLCM observer model(s)

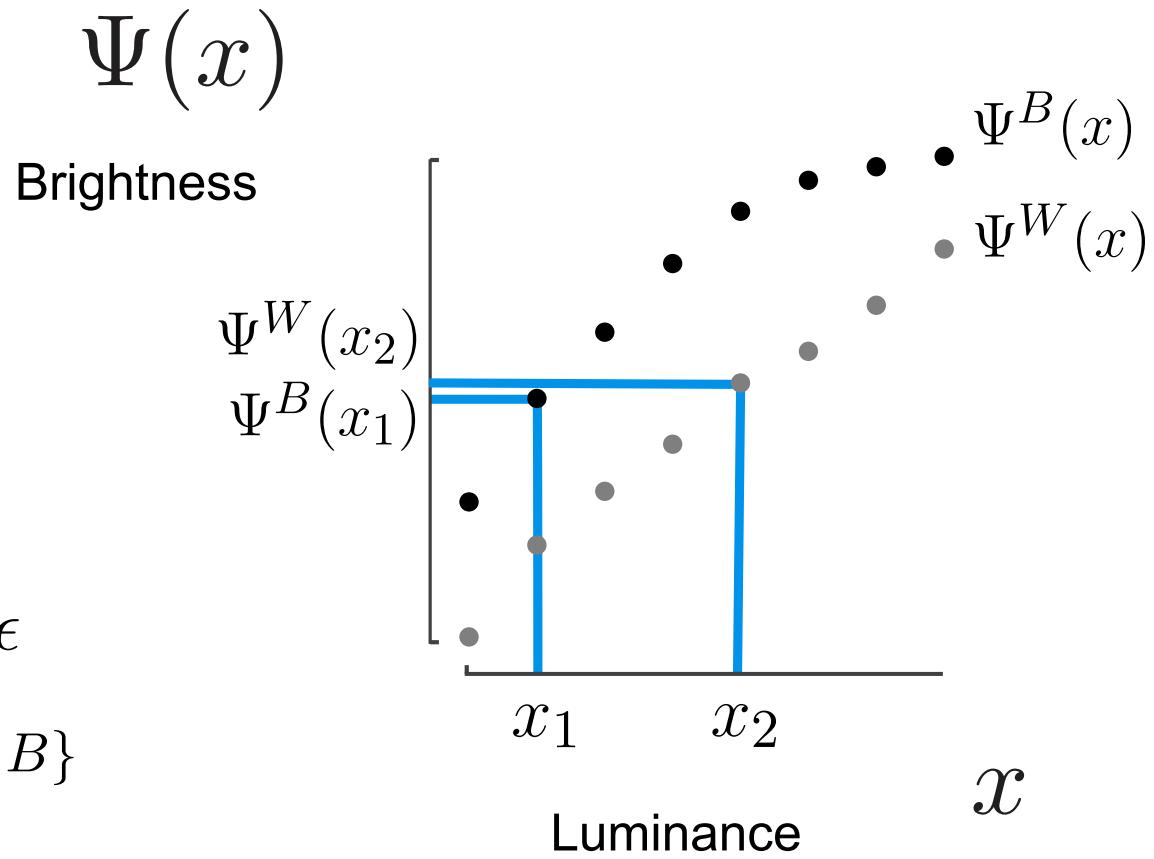


Decision model

$$\Delta = \Psi^{C_2}(x_2) - \Psi^{C_1}(x_1) + \epsilon$$

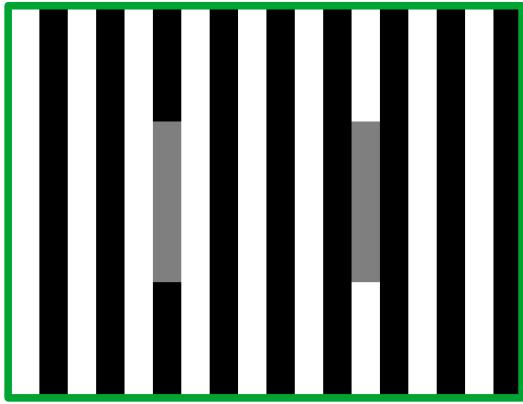
$$\epsilon \sim N(0, \sigma^2) \quad C_1, C_2 \in \{W, B\}$$

if $\Delta > 0 \rightarrow$ “right”
otherwise \rightarrow “left”



MLCM observer model(s)

$$x_1 = x_2$$



Decision model

$$\Delta = \Psi^{C_2}(x_2) - \Psi^{C_1}(x_1) + \epsilon$$

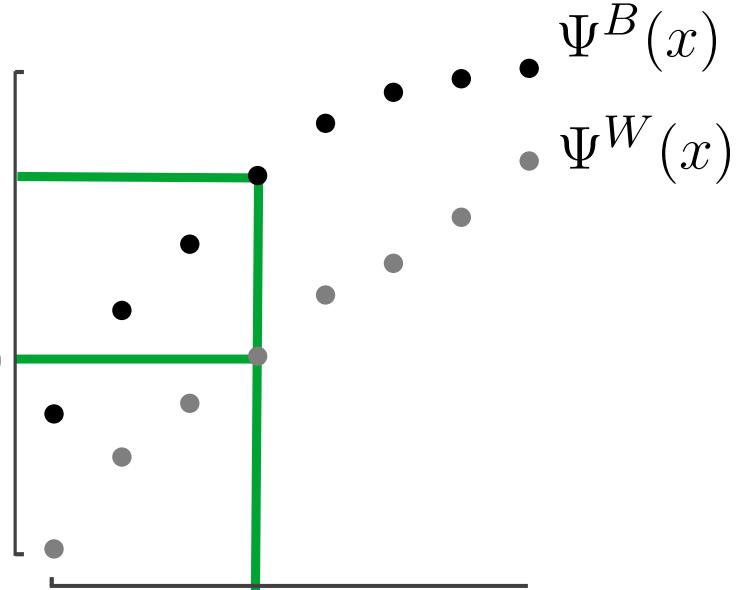
$$\epsilon \sim N(0, \sigma^2) \quad C_1, C_2 \in \{W, B\}$$

if $\Delta > 0 \rightarrow$ “right”
otherwise \rightarrow “left”

$$\Psi(x)$$

Brightness

$$\begin{aligned} \Psi^B(x_1) \\ \Psi^W(x_2) \end{aligned}$$



$$x_1 = x_2$$

Luminance

x

Three observer models in MLCM

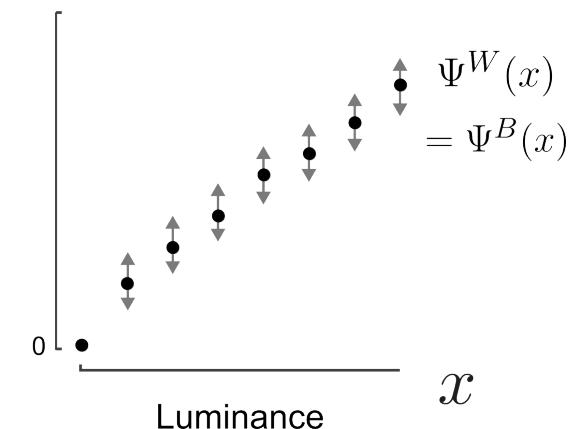
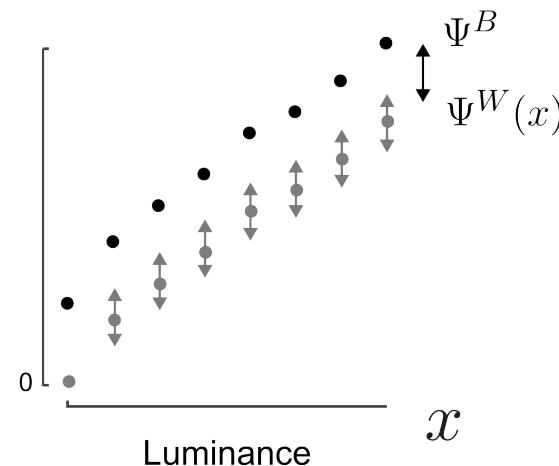
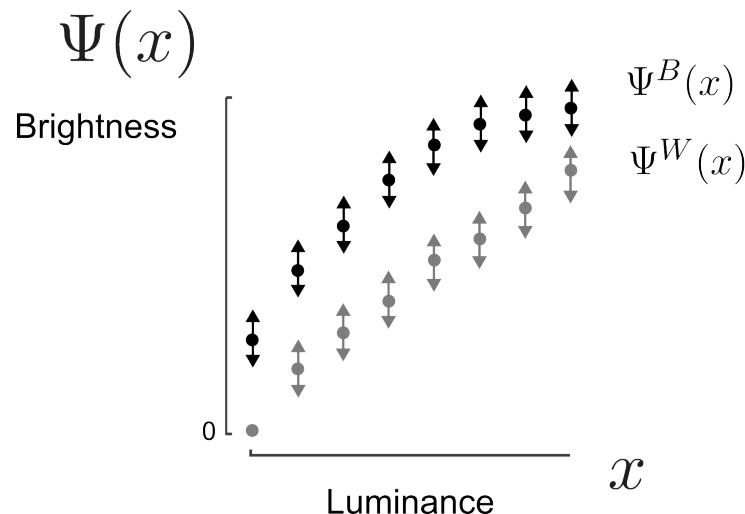
Full
(also called “saturated”)
All scale values fitted

Additive
Values for one dimension fitted,
effect of other dimension is
additive

Independent
Dimensions are
Independent

More general

More constrained



→ Need of statistical model comparison

Exercise: analyze MLCM data

Run the MLCM experiment on White's illusion
(Optional)

Session 1: all comparisons

Session 2: only comparisons across contexts

Copy your CSV files to the folder [data/mlcm/](#)

Analyse MLCM data

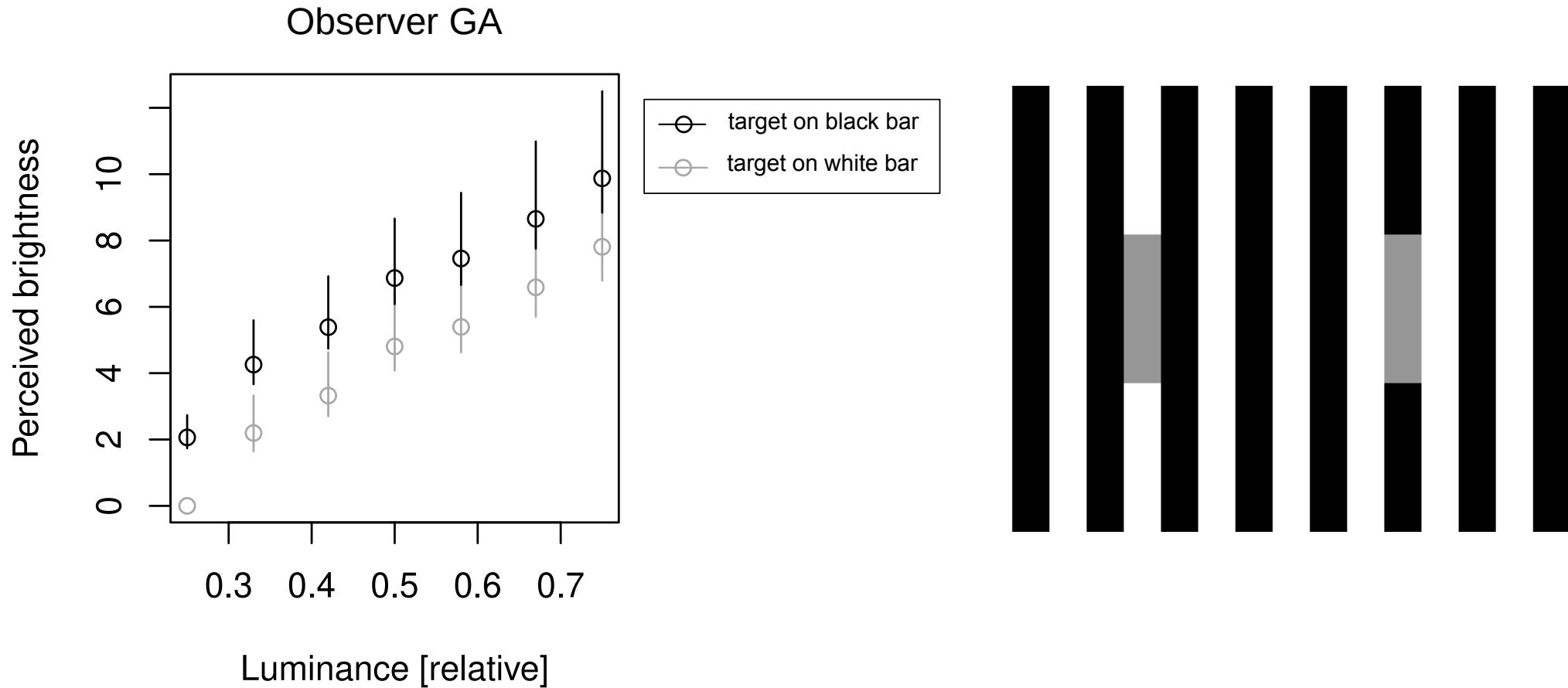
In Rstudio, open file: [tutorial_mlcm.Rmd](#)

Run all cells (top right, *Run / Run all*)

<https://kutt.it/mlcm-exp>

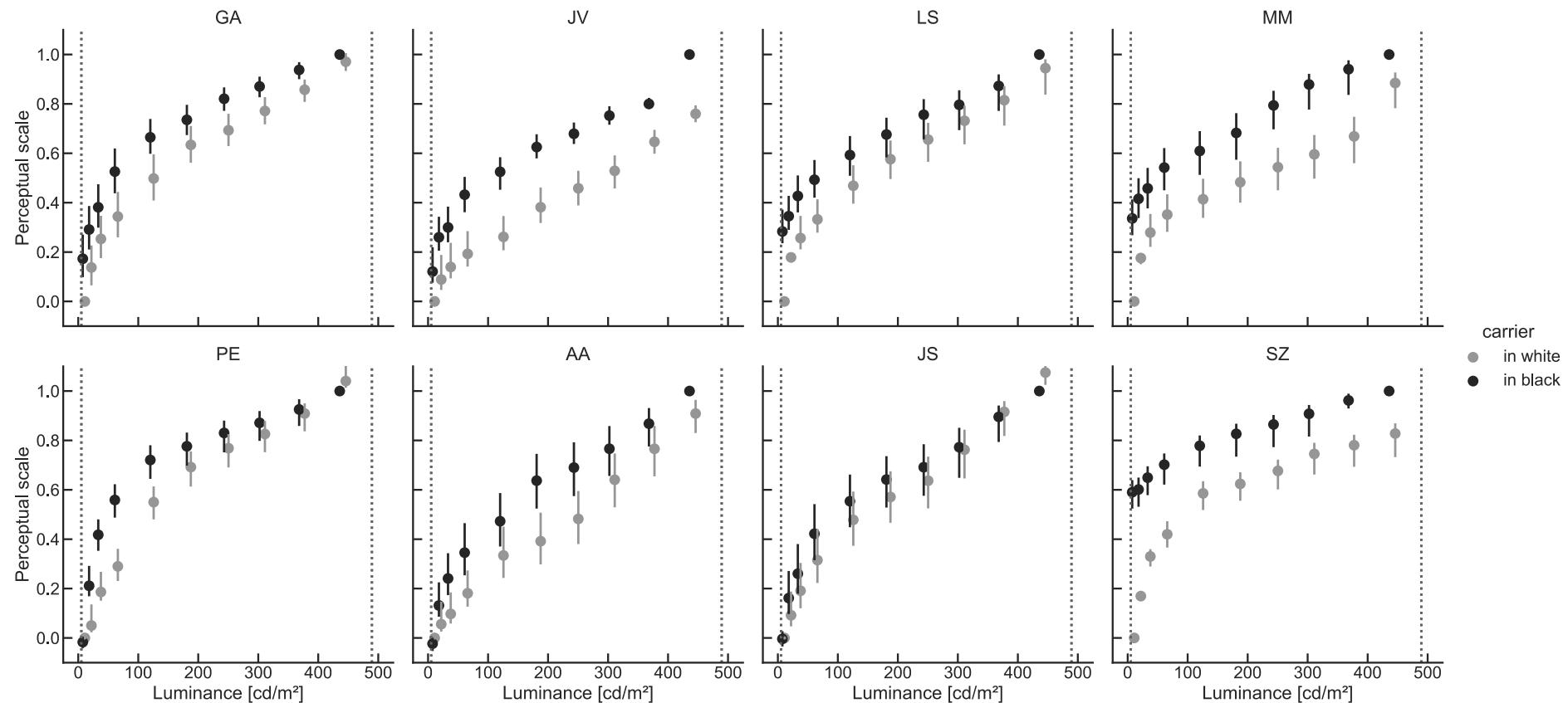


Interpretation of results



Interpretation of results

Scales for White's illusion (Vincent, Maertens & Aguilar 2024)



Confidence intervals and goodness of fit evaluation

Same as for MLDS!

- similar functions in R
 $\text{boot.mlds}() \rightarrow \text{boot.mlcm}()$
- same *binom.diagnostics()* for GoF evaluation
- same logic and interpretation

To keep in mind – Practical considerations

Choice of stimulus spacing

- one should be able to order the stimuli (perceptually)
- that means spacing should be supra-threshold
- but not only easy trials, both methods require “hard” comparisons, for which the choice probability is different than 0 or 1 (always pressing only one option)
- this sometimes requires piloting

Repeats: fixed repeat number vs. random sampling

Summary - MLDS and MLCM

Methods to measure **appearance** of stimuli

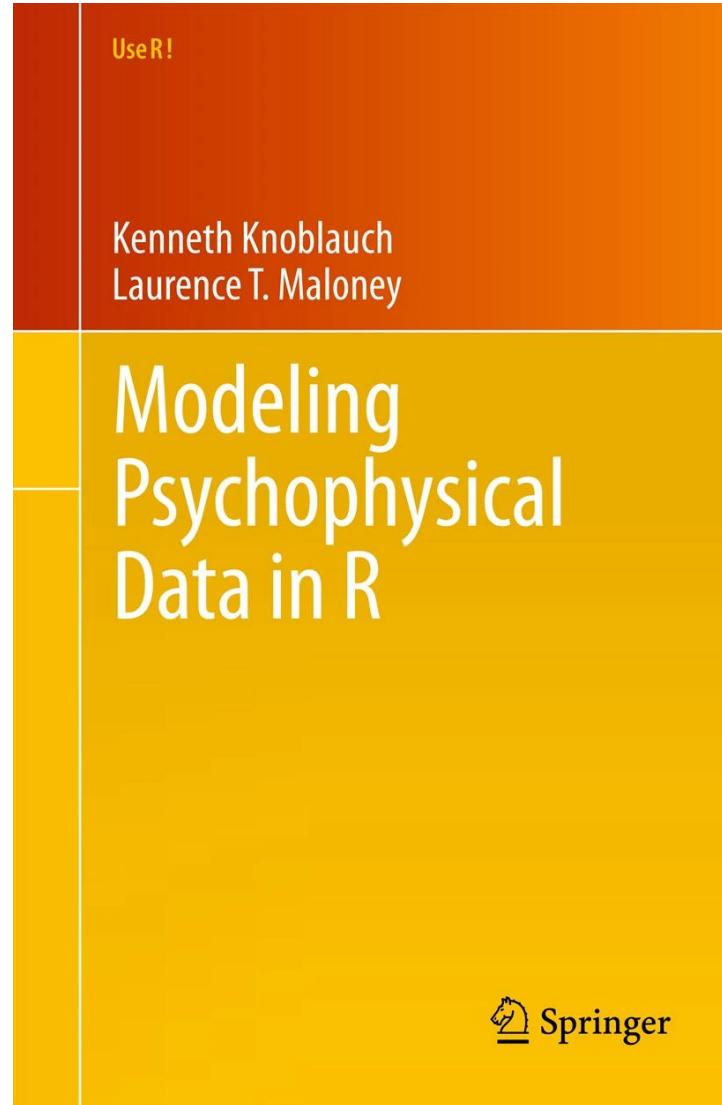
Based on **supra-threshold comparsions**

Outcome: **perceptual scales**

| | MLDS | MLCM |
|----------------|-------------------------------------|------------------------|
| Use case | 1 stimulus dimension | > 1 stimulus dimension |
| | 1 perceptual dimension | |
| Task | Triads or quadruples | Paired comparisons |
| Observer model | Double differencing | Simple differencing |
| Estimation | Maximum likelihood via binomial GLM | |

Thank you

References



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References

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