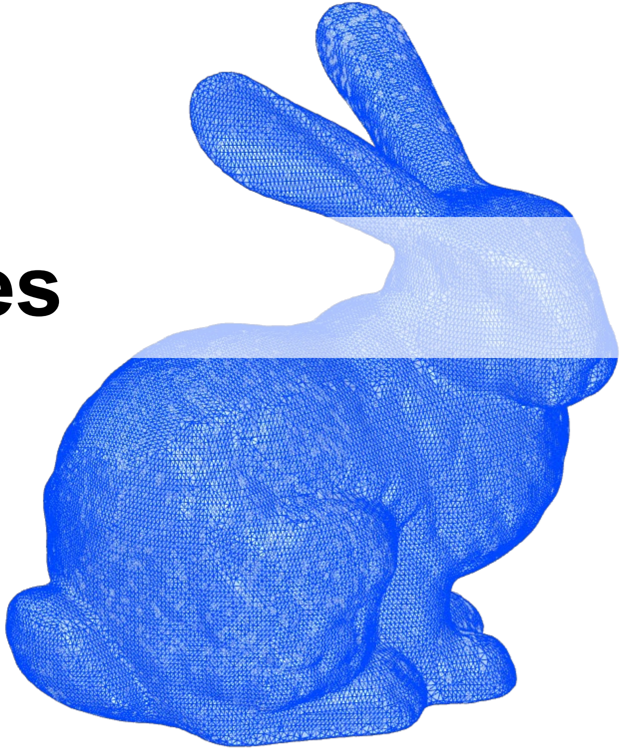
Meshes



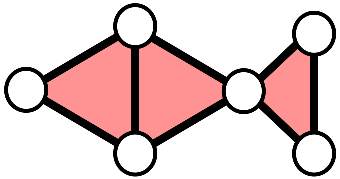
Content

Definition

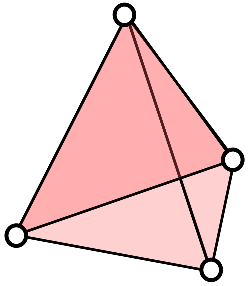
What is a mesh?

A mesh is a collection of **vertices** (points in space) connected by **edges** (line segments) that enclose **faces** (polygonal elements).

$$\mathcal{M} = \{\boldsymbol{\mathcal{V}}, \boldsymbol{\mathcal{E}}, \boldsymbol{\mathcal{F}}\}$$



Together, these may form a discrete representation of a surface.

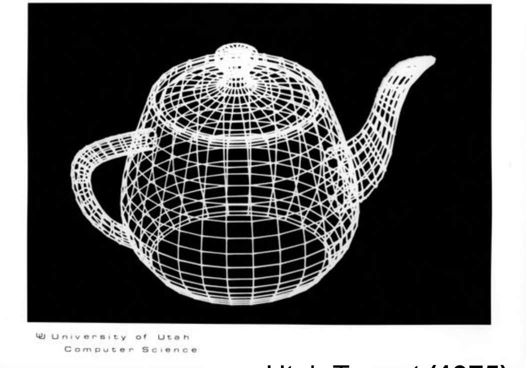


Why are meshes used?

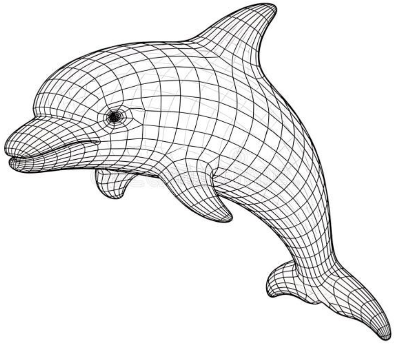
- Simple and efficient to store
- Capable of approximating complex shapes with arbitrary accuracy
- Compatible with most geometric and numerical algorithms

→ Meshes are widely used in computational design, graphics, simulation and fabrication

“humble beginning”



Utah Teapot **(1975)**



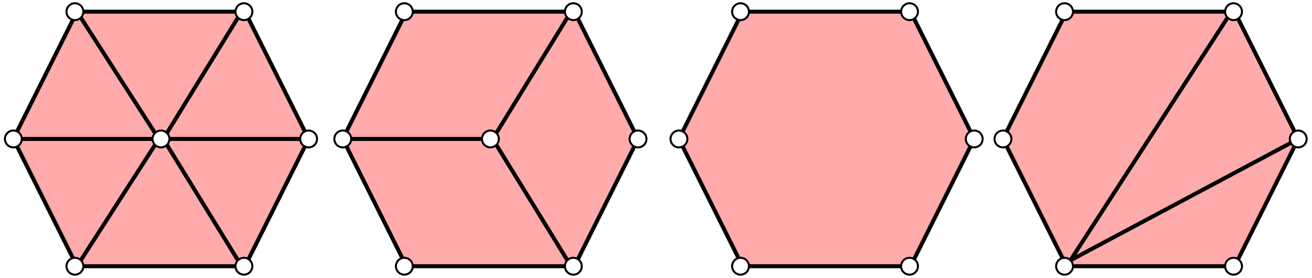
Quick remark on meshes: Hand or glove?

Often, we use surface meshes to represent solid objects (limited model)
Volumetric representations exist as well (increased complexity)

Properties

Types of faces

- **Triangular:** Always planar, guaranteed convex elements
- **Quad:** Useful for subdivision surfaces and simulations
- Rather uncommon formats: **n-gons** (polygons), **hybrid**



Many ways of representing the same surface

Types of faces

- **Triangular:** Always planar, guaranteed convex elements
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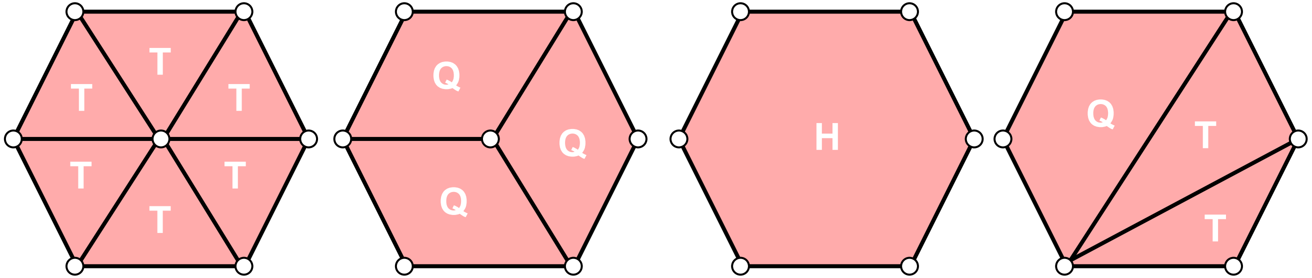
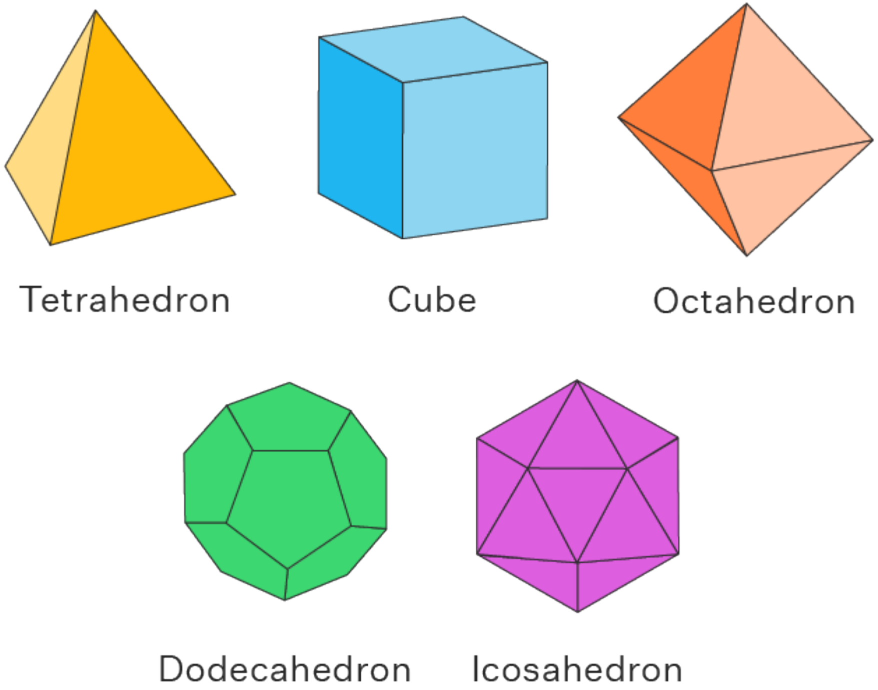
- Rather uncommon formats: **n-gons** (polygons), **hybrid**



Many ways of representing the same surface

Types of faces

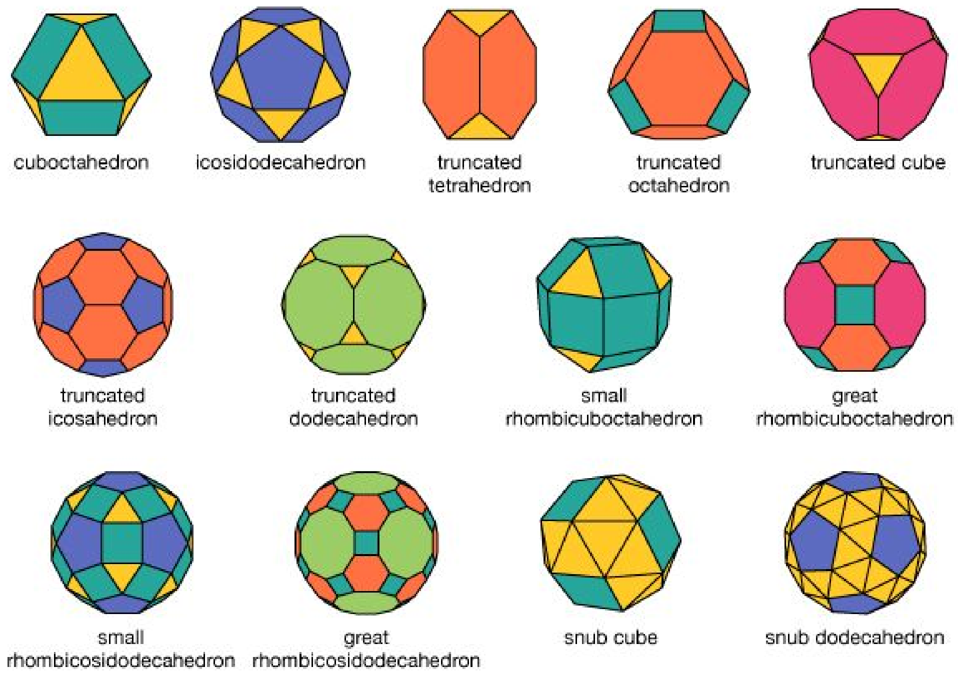
5 Platonic Solids



Platonic solids are convex polyhedra with identical faces.

Types of faces

13 Archimidean Solids



Archimidean solids are convex polyhedra with a single vertex configuration.

Connectivity

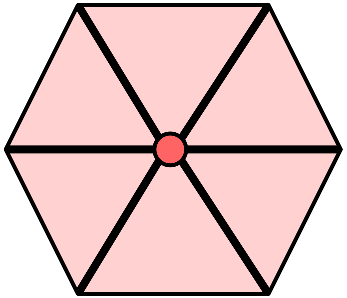
Connectivity describes how the discrete elements of a mesh are topologically related to each other, independent of their geometric embedding.

There are 6 kinds of connectivity:

From/To	***Vertex***	***Edge***	***Face***
Vertex		Vertex-Edge	Vertex-Face
Edge	Edge-Vertex		Edge-Face
Face	Face-Vertex	Face-Edge	

Connectivity

Vertex – Edge:
Edges adjacent to a given vertex



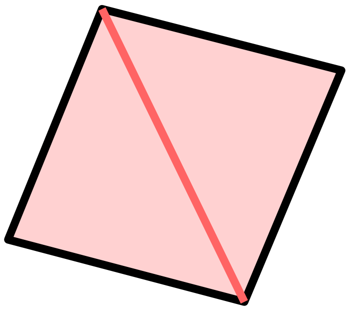
(Irregular structure → Vertex degree)

Edge – Vertex:
Vertices adjacent to a given edge

(All edges have two vertices)

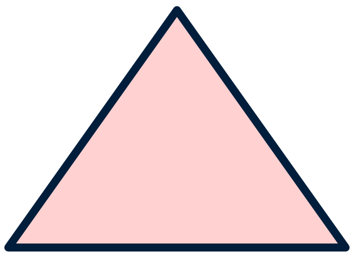
Connectivity

Edge – Face:
Faces adjacent to a given edge



(More details coming in manifolds and boundaries)

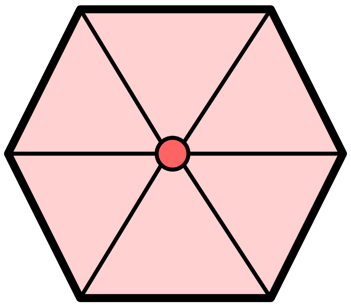
Face – Edge:
Edges adjacent to a given face



(n-gons have n edges)

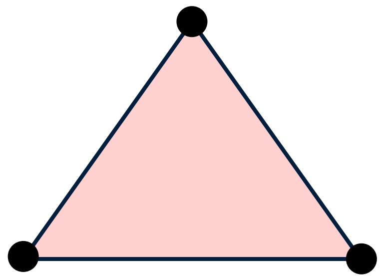
Connectivity

Vertex – Face:
Faces adjacent to a given vertex



(Irregular structure → Vertex degree)

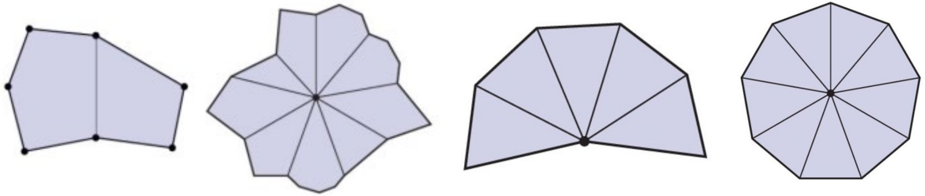
Face – Vertex:
Vertices adjacent to a given face



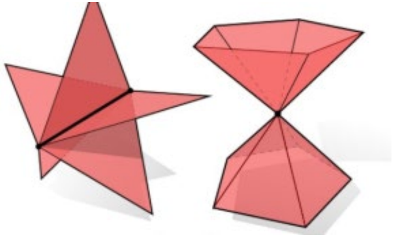
(n-gons have n edges)

Manifold

Conditions:
Each edge is incident to at most two faces **AND** Each vertex neighborhood resembles a disk



Non-manifold examples (aka polygon soups)

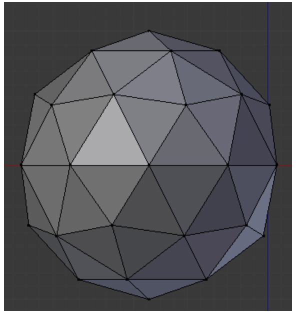


Source: CMU 15-462/662

Boundaries

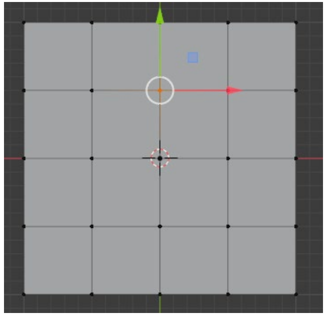
- **Open**: Has edges belonging to only one face
- **Closed**: All edges belong to exactly two faces

Closed



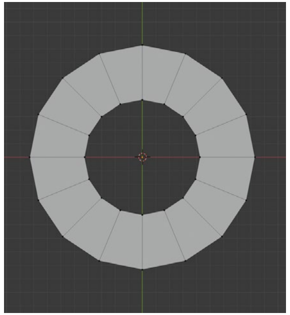
0 boundaries

Open



1 boundary

Open

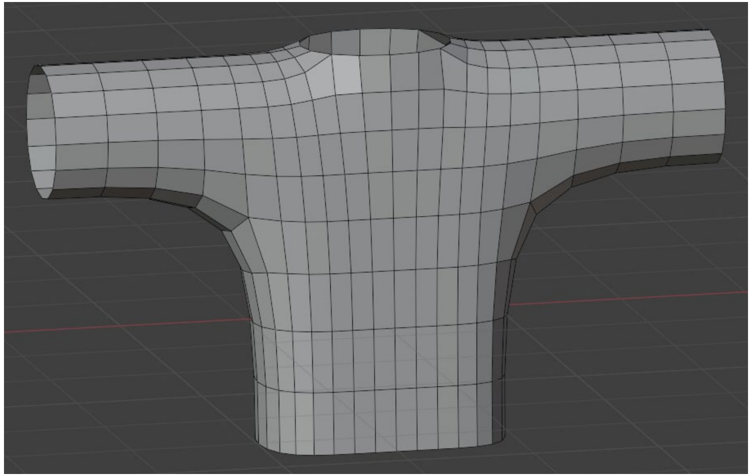


2 boundaries

Only closed meshes may have an inside/outside

Is this mesh closed or open?

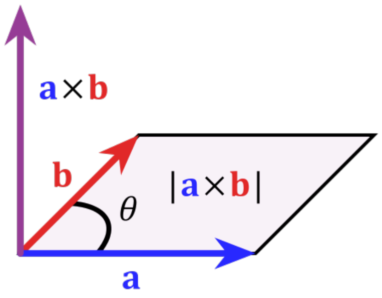
How many boundaries?

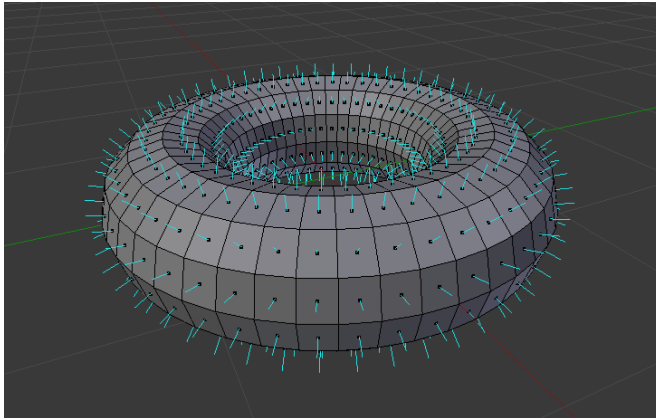


Mesh normal vectors

Surface normal vector: Direction orthogonal to two linearly independent tangent vectors

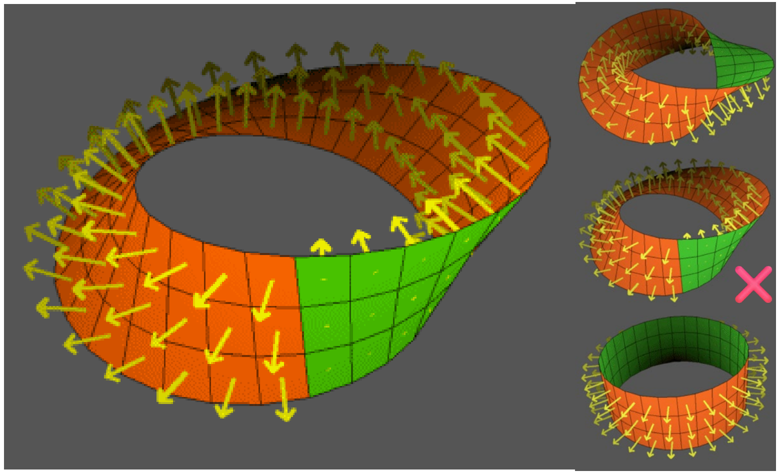
For meshes with planar faces: normal vector constant within face





Orientability

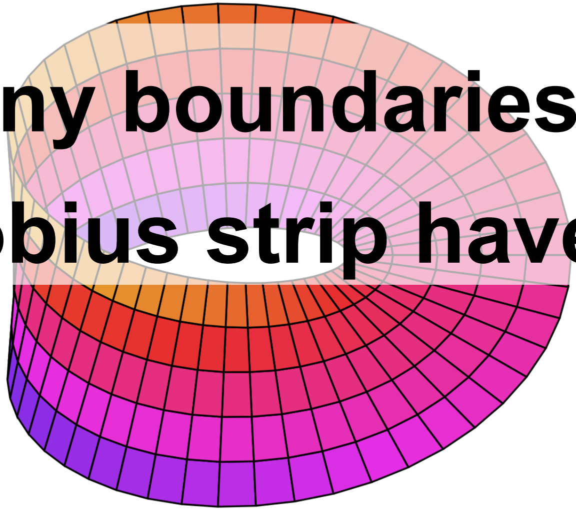
Consistently define a normal direction



Möbius strip

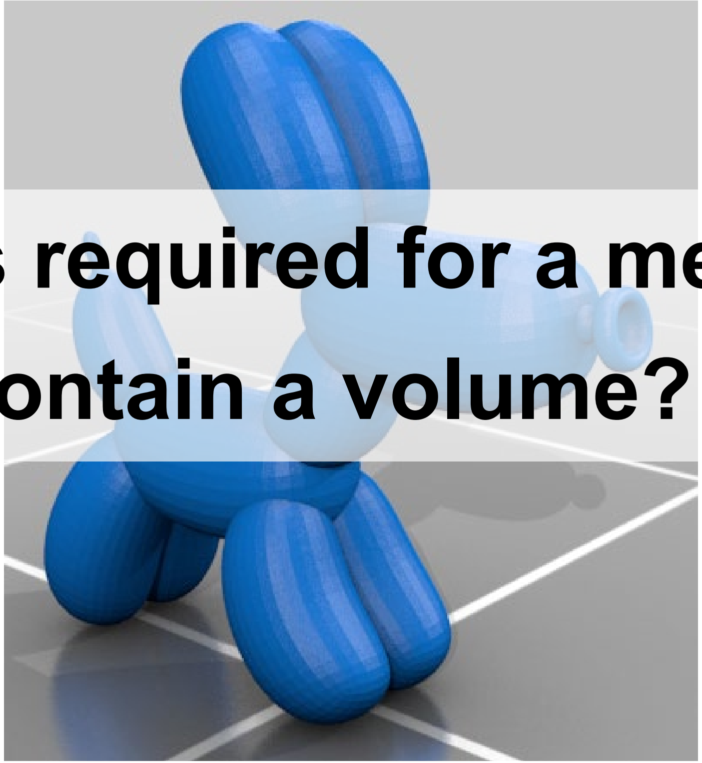
Klein's bottle

How many boundaries does a Möbius strip have?



Möbius strip

What is required for a mesh to contain a volume?



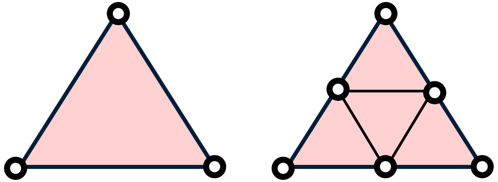
Boundaries?
Manifold?
Orientable?

Euler characteristic

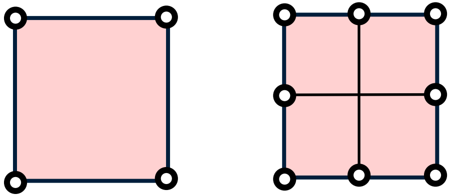
Mesh invariant under local modifications:

$$\boldsymbol{\xi = V - E + F}$$

Fundamental (topological) property of a shape



$\mathbf{3} - 3 + \mathbf{1} = \mathbf{1} = \mathbf{6} - 9 + \mathbf{4}$



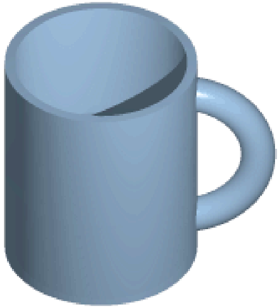
$\mathbf{4} - 4 + \mathbf{1} = \mathbf{1} = \mathbf{9} - 12 + \mathbf{4}$

Genus of a surface

Euler-L'Huilier formula:

$$\xi = V - E + F = 2 \ - 2\ g$$

Invariant under continuous deformations

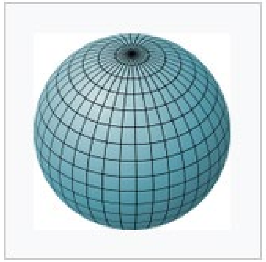


$$g = 1$$

Genus of a surface

Euler-L'Huilier formula:

$$\boldsymbol{\xi = V - E + F = 2 - 2\,g}$$

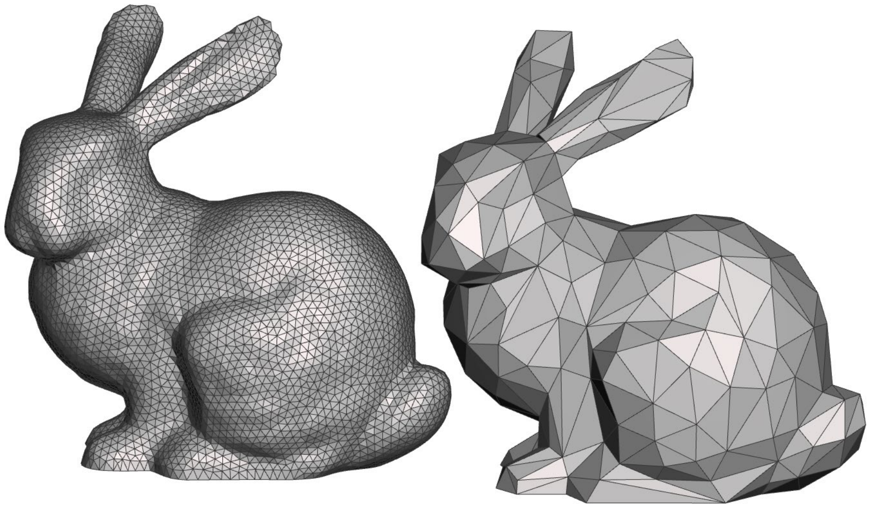
   

genus 0 genus 1 genus 2 genus 3

Operations

Decimation (Simplification)

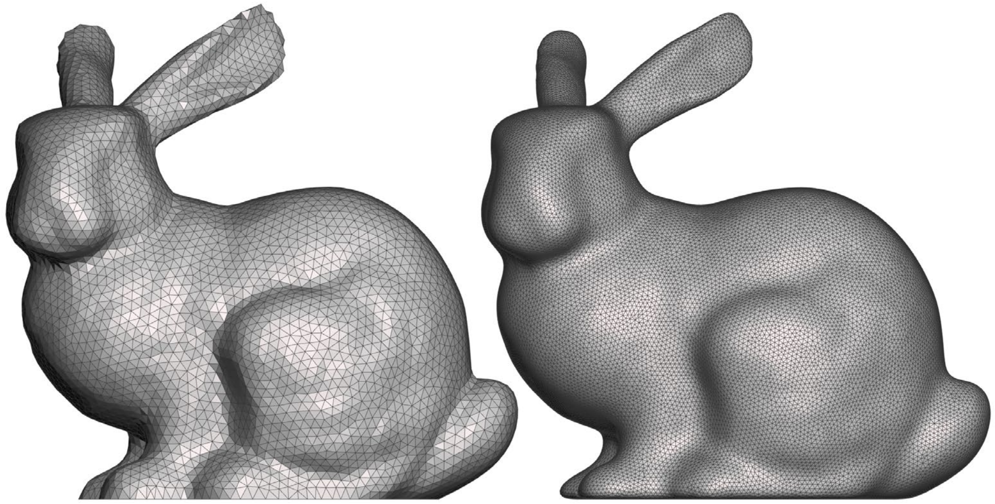
- Reduces the number of faces while approximating the original shape
- Algorithms: Edge Collapse, Vertex-Clustering, Quadratic Error Metrics
- Useful for reducing memory



g3Sharp - gradientspace

Refinement (Subdivision)

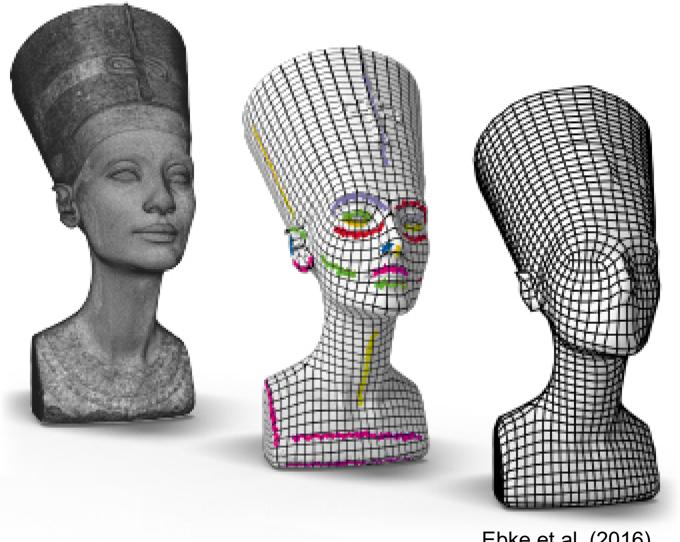
- Increasing the mesh resolution by splitting faces
- Adding capacity to the mesh to represent more complex shapes



g3Sharp - gradientspace

Remeshing

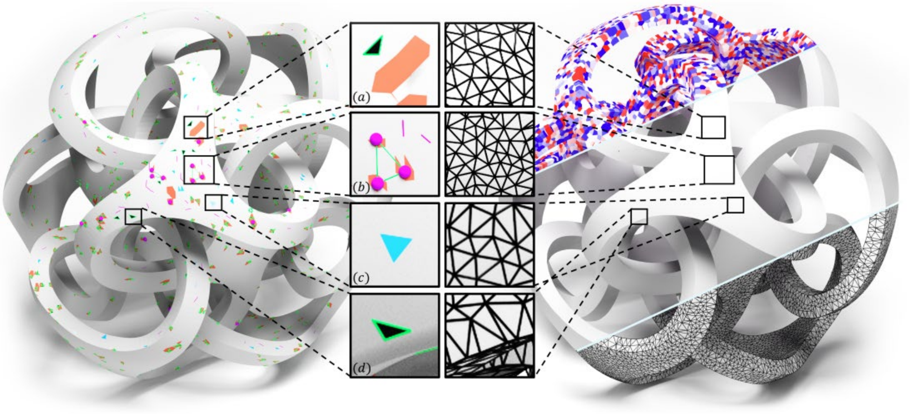
- Adjust mesh quality
- Many possible metrics:
    - Edge length uniformity
    - Face isotropy
    - Alignment with features



Ebke et al. (2016)

Repair

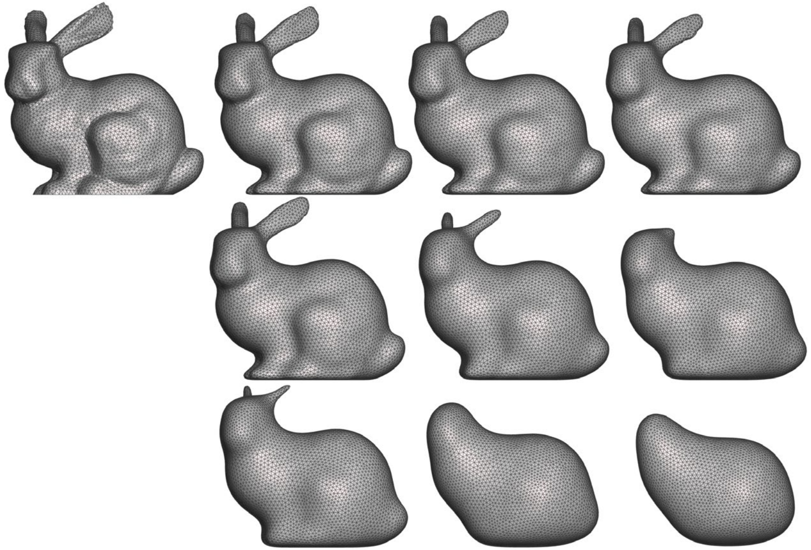
- Fix unintended features of a mesh automatically
- Can be very time-consuming. Often a bottleneck
    a) Self-intersections
    b) Non-manifold vertices
    c) Duplicate faces
    d) Missing faces



Wen et al. (2025)

Fairing / Smoothening

- Attenuates the high-frequency details of a shape

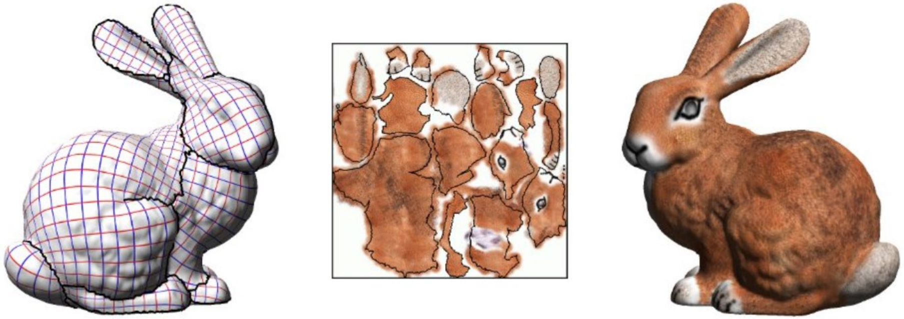


g3Sharp - gradientspace

UV Maps

- Flattening of a mesh (embedding in a lower-dimensional space)
- Apply textures through the inverse map

Parametrized Mesh Atlas Texturized Mesh

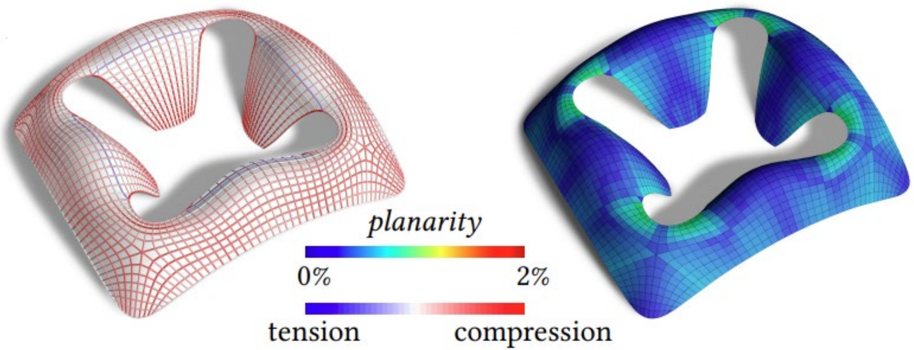


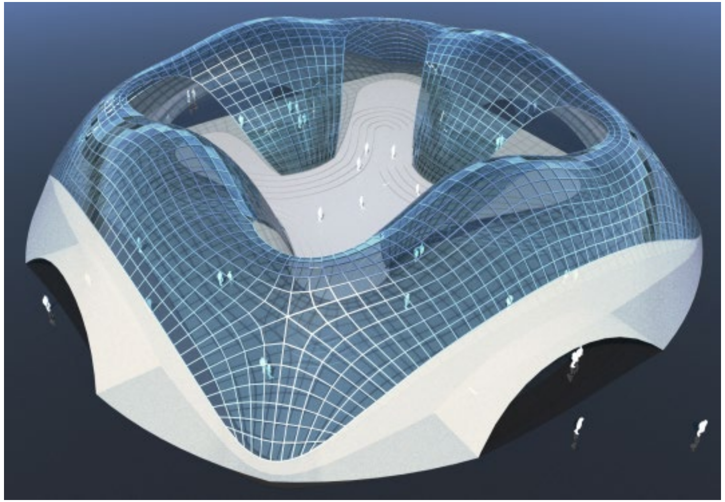
Lévy et. Al. (2002)

Applications

Design and modelling

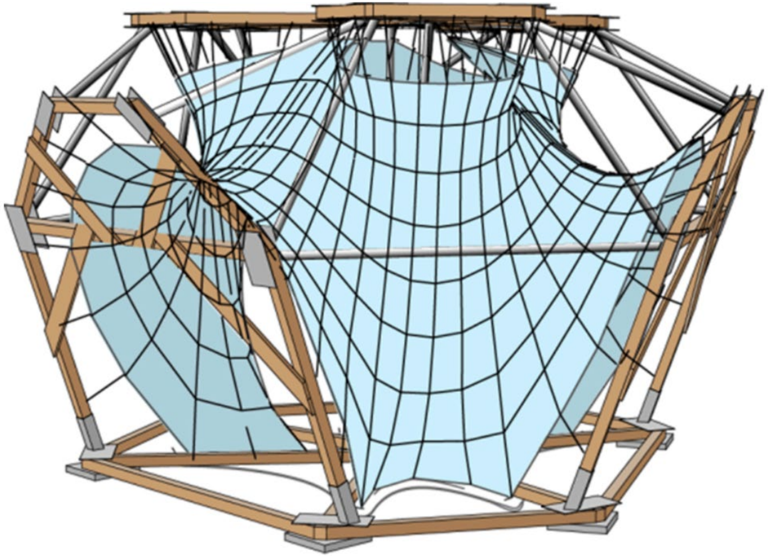
- Complex geometric structures in architecture and design
- Planar quad meshes in architecture are relevant for panel designs





Material Minimizing Forms and Structures (Kilian, 2020)

Design and modelling

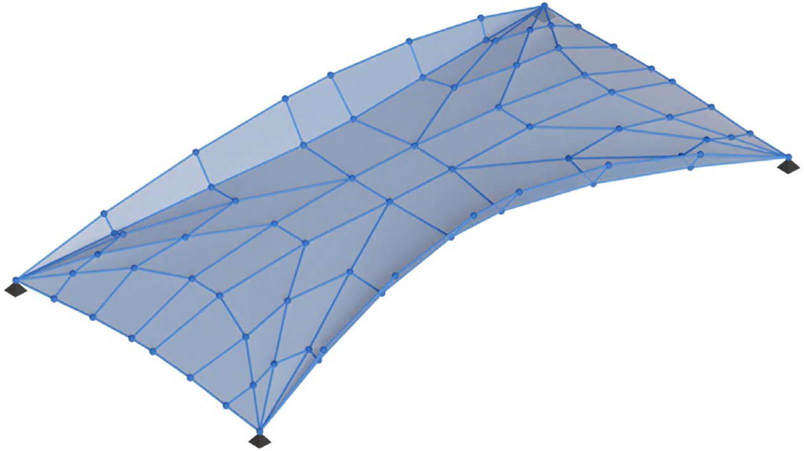


KnitCandela (Popescu, 2020)

Digital fabrication

Knitcrete Bridge (Rennen, 2023)

Digital fabrication

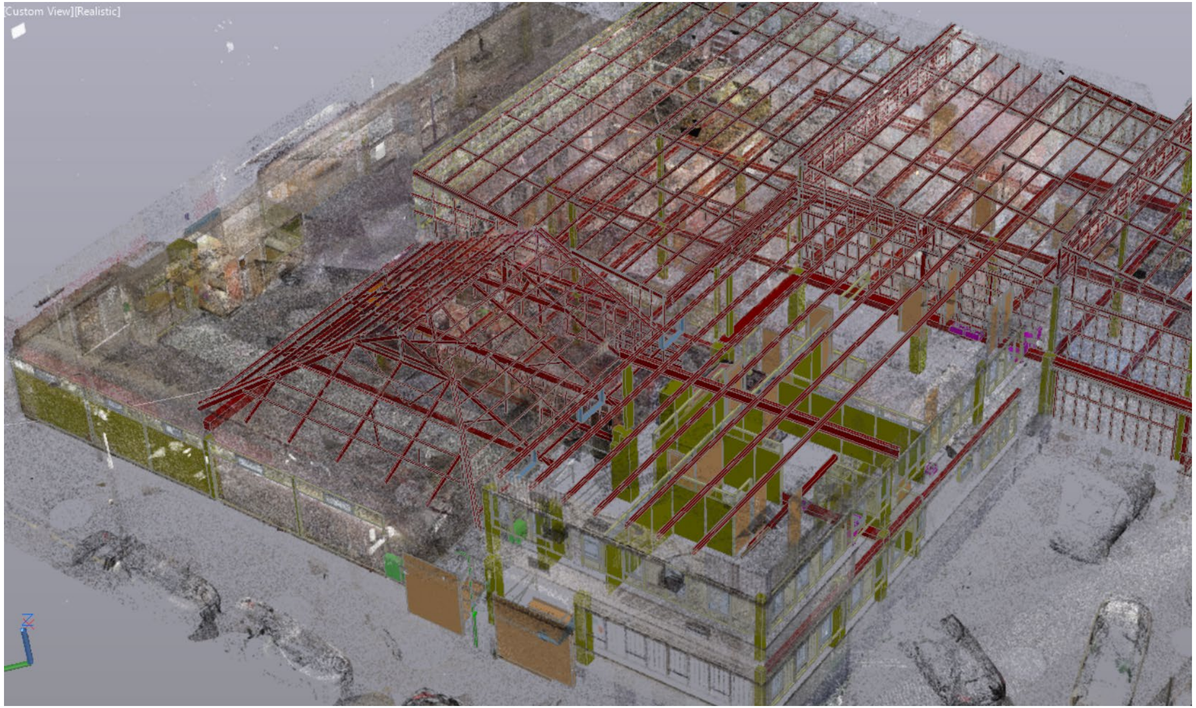


Bridge the gap (Schneider, 2024)

Geometry reconstruction: Heritage restoration

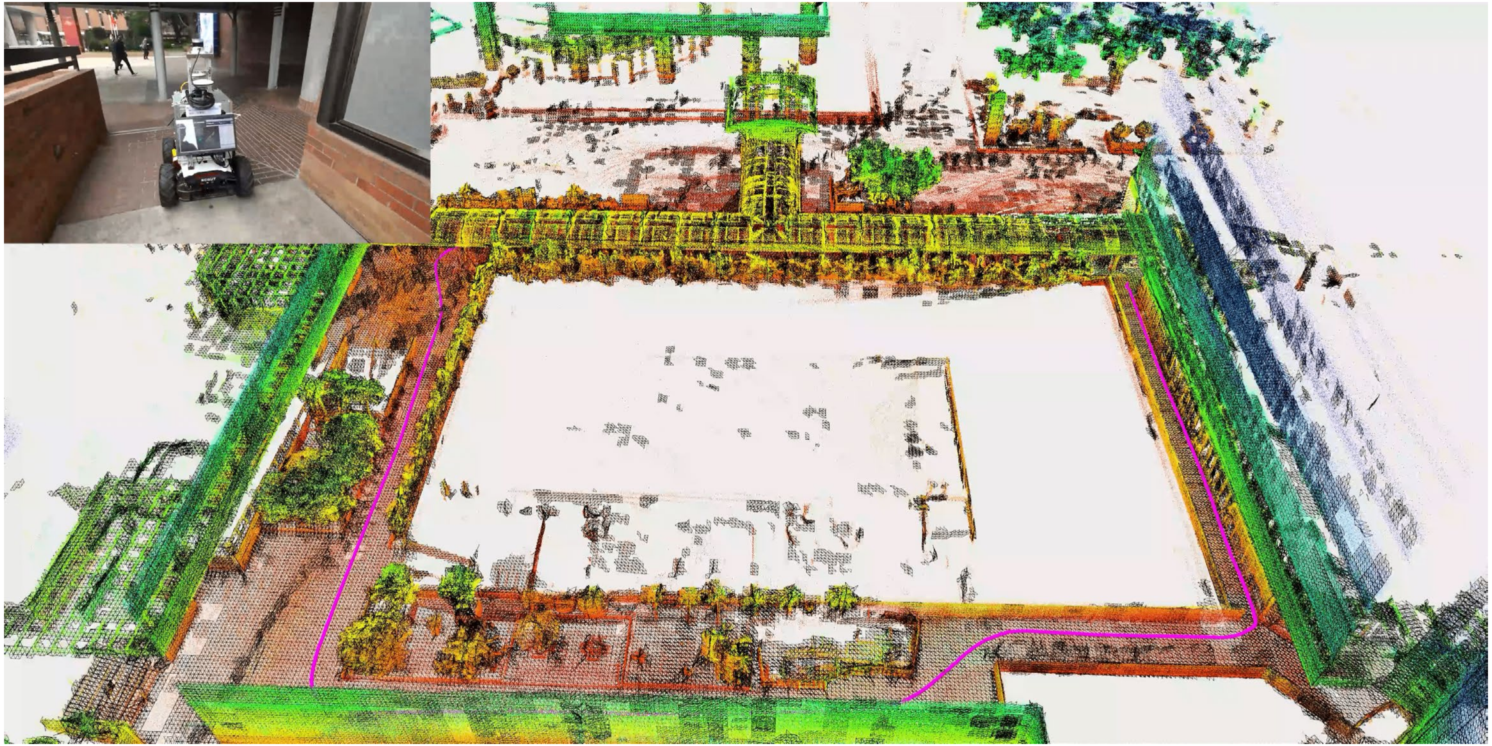
Bent et al. 2022

Geometry reconstruction: Construction renovation

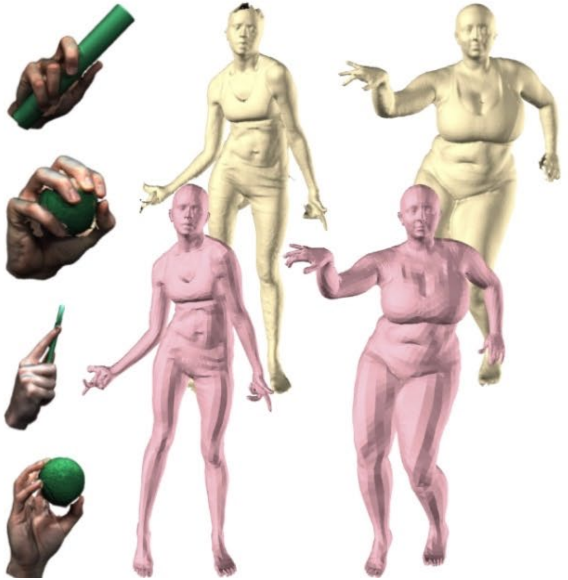


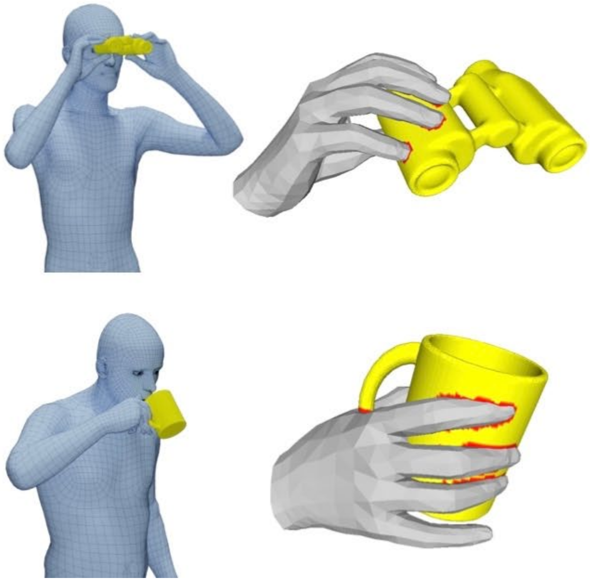
Resource: https://www.recon.nz/

Geometry reconstruction: Mesh-based SLAM



Geometry reconstruction: Object perception

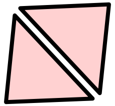




Max Planck Institute for Intelligent Systems

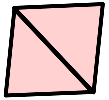
Export formats

- **STL**



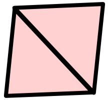
  - Stores only triangles, without explicit adjacency
  - Widely used in 3D printing (many people working hard to change that → 3MF)

- **OBJ**



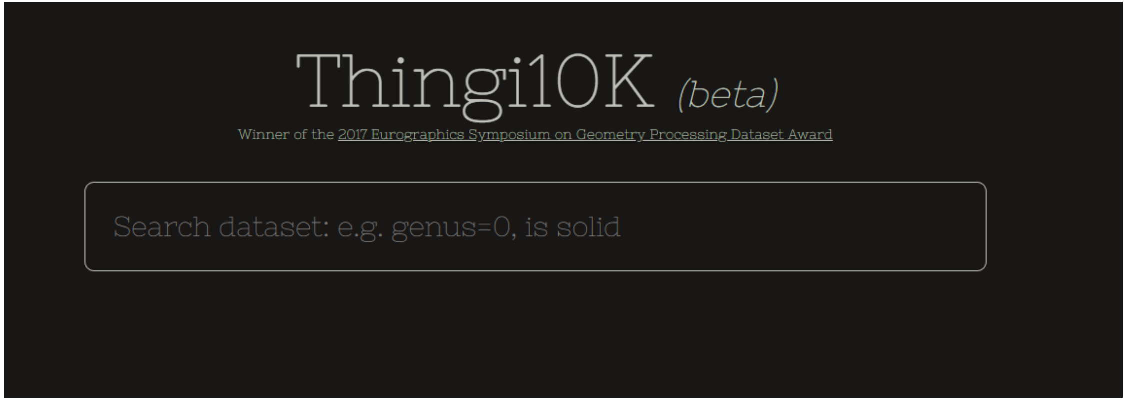
  - Stores vertices and polygonal faces, optional normal and UV coordinates
  - Popular in CAD and graphics

- **PLY:**



  - Flexible format that can store vertex attributes
  - Often used in 3D scanning

Datasets

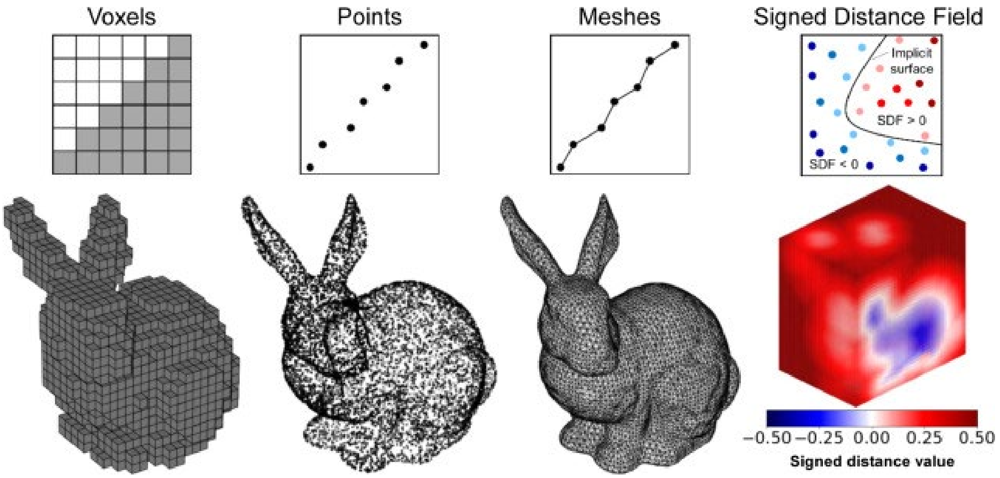


And many other more to test your ideas extensively

Outlook

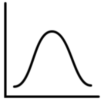
Diversity of shape representations

Each of them contains a particular structure. Different paradigms to describe an object.

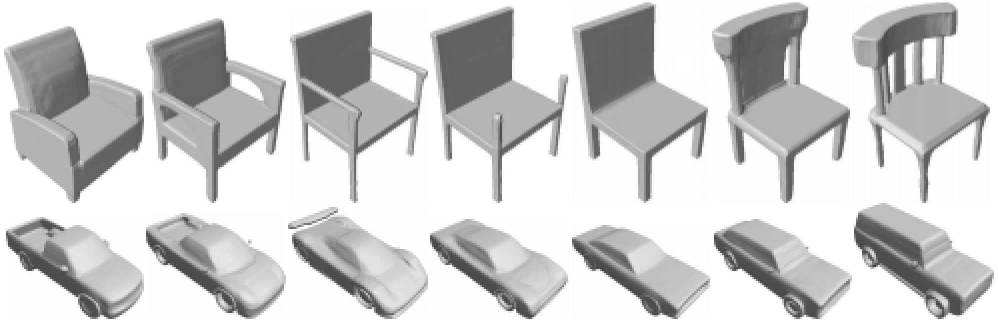


Novel shape representations

No shape representation is perfect → Application-based representations



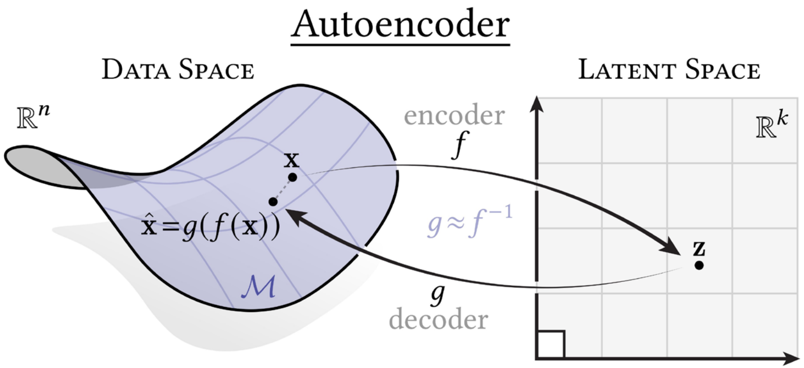
Gaussian Splats



Neural Representations (DeepSDF)

Geometry, After-All

Geometric principles remain an underlying fundamental element



Crane (2025)

Open up your Grasshopper/Rhino environment and try out Rhino meshes!

Look into a RhinoCommon API Mesh class for further mesh properties and functions.

Enjoy creating!