

TITLE

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<http://github.com/kingoslo/attaboy>

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ABSTRACT

This is a report submission for the fourth project of «Computational physics» at the Institute of Physics, University of Oslo, autumn 2016.

For a 2×2 -grid of consisting of spin values ± 1 , it is straightforward to verify that the partition-function is given by

$$Z(\beta) = 12 + 2(e^{-8J\beta} + e^{8J\beta})$$

and thus the n -momentums of energy E and magnetization M are given by

$$\langle E^n \rangle(\beta) = \frac{2}{Z} (8^n e^{-8J\beta} + (-8)^n e^{8J\beta}), \quad \langle M^n \rangle(\beta) = \frac{1}{Z} (4^n e^{8J\beta} + 4(2)^n + 4(-2)^n + (-4)^n e^{8J\beta})$$

respectively. From these we obtain the expressions for expected energy and expected magnitude of magnetization

$$\langle E \rangle(\beta) = \frac{2^4}{Z} (e^{-8J\beta} - e^{8J\beta}) \quad \text{and} \quad \langle |M| \rangle(J\beta) = \frac{2^3}{Z} (2 + e^{8J\beta}).$$

Use these, it is straight forward to compute the heat capacity at constant volume and magnetic susceptibility since they are

$$C_V = \frac{1}{kT^2} \sigma_E^2 = \frac{2^7}{kT^2 Z} \left[(e^{-8J\beta} + e^{8J\beta}) - \frac{2}{Z} (e^{-8J\beta} - e^{8J\beta})^2 \right]$$
$$\chi = \frac{1}{kT} (\langle M^2 \rangle - \langle |M| \rangle^2) = \frac{2^5}{kT Z} \left[(1 + e^{8J\beta}) - \frac{2}{Z} (2 + e^{8J\beta})^2 \right]$$

INTRODUCTION

> I Motivate the reader, the first part of the introduction gives always a motivation and tries to give the overarching ideas

> I What I have done

> I The structure of the report, how it is organized etc

| | exact: | numerical | Error at 10^8 iterations |
|-----------------------|---------------------|-----------------|----------------------------|
| $\langle E \rangle$ | 7.9839301406925038 | 3.9945928 | a |
| $\langle M \rangle$ | 3.9946429309943987 | -7.98379632 | a |
| C_V | 0.12832932745714487 | 0.129378400754 | a |
| χ | 0.01604295806490974 | 0.0162110021882 | a |

METHODS

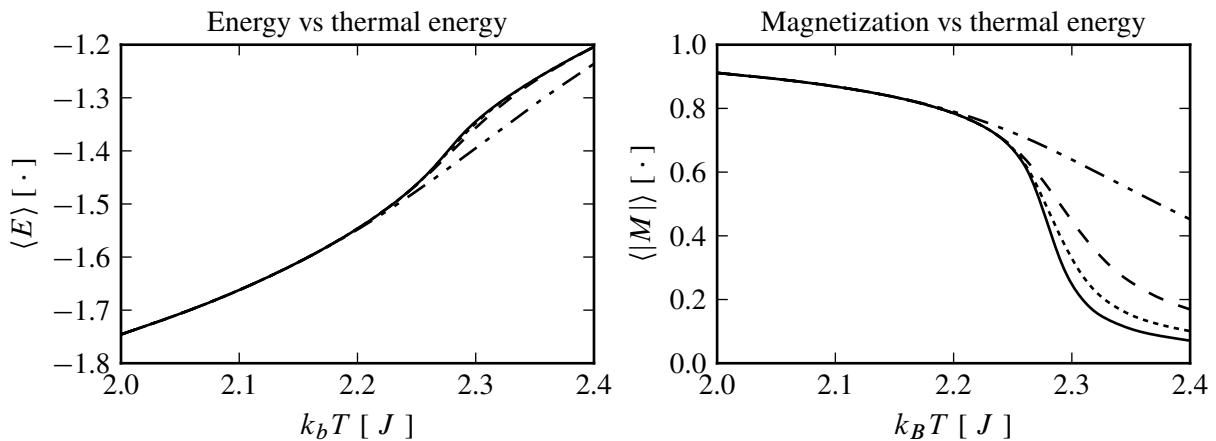
> I Describe the methods and algorithms

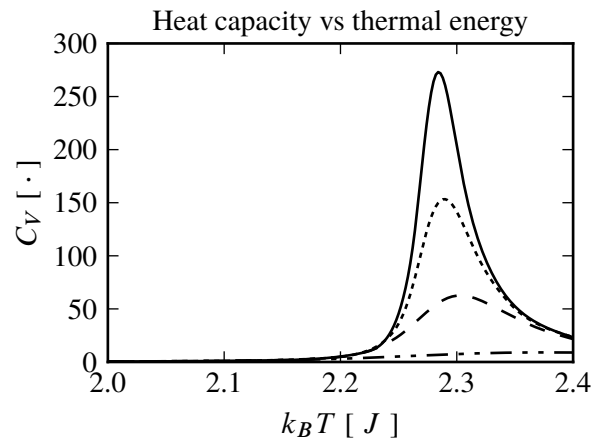
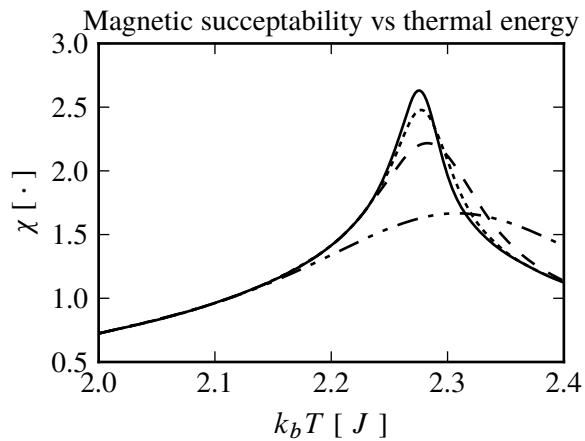
> I You need to explain how you implemented the methods and also say something about the structure of your algorithm and present some parts of your code

> I You should plug in some calculations to demonstrate your code, such as selected runs used to validate and verify your results. The latter is extremely important!! A reader needs to understand that your code reproduces selected benchmarks and reproduces previous results, either numerical and/or well-known closed form expressions.

```
for (int k = 0; k < N; k++) { // sample iterations
    for (int i = 0; i < m; i++) { // loop matrix cols
        for (int j = 0; j < n; j++) { // matrix rows
            int u = rand_int(gen);
            int v = rand_int(gen);
            double dE = 2*A[u][v]*(A[u][mod(v+1,n)] +
                                   A[mod(u+1,m)][v] +
                                   A[u][mod(v-1,n)] +
                                   A[mod(u-1,m)][v]);
            if (exp(-beta*dE) > rand_double(gen)) {
                // selection criterion
                A[u][v] = - A[u][v];
                E += dE;
                M += 2*A[u][v];
            }
        }
    }
}
// sample if we believe we're at equilibrium
if (k > samplepoint) {
    avg[0] += E;
    avg[1] += E*E;
    avg[2] += abs(M);
    avg[3] += M*M;
}
```

RESULTS AND DISCUSSION





I Present your results

> I Give a critical discussion of your work and place it in the correct context.

> I Relate your work to other calculations/studies

> I An eventual reader should be able to reproduce your calculations if she/he wants to do so. All input variables should be properly explained.

> I Make sure that figures and tables should contain enough information in their captions, axis labels etc so that an eventual reader can gain a first impression of your work by studying figures and tables only.

CONCLUSION AND PERSPECTIVES

> I State your main findings and interpretations

> I Try as far as possible to present perspectives for future work

> I Try to discuss the pros and cons of the methods and possible improvements