TITLE

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http://github.com/kingoslo/attaboy

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ABSTRACT

This is a report submission for the fourth project of «Computational physics» at the Institute of Physics, University of Oslo, autumn 2016.

For a 2×2-grid of consisting of spin values ± 1, it is straightforward to verify that the partition-function is given by

$$Z(\beta) = 12 + 2\left(e^{-8J\beta} + e^{8J\beta}\right)$$

and thus the n-momentums of energy E and magnetization M are given by

$$\langle E^n \rangle (\beta) = \frac{2}{Z}\left(8^n e^{-8J\beta} + (-8)^n e^{8J\beta}\right), \quad \langle M^n \rangle (\beta) = \frac{1}{Z}\left(4^n e^{-8J\beta} + 4(2)^n + 4(-2)^n + (-4)^n e^{8J\beta}\right)$$

respectively. Use these, it is straight forward to compute the heat capacity at constant volume and magnetic susceptibility since these are proportional to the variance of E and M. They are

$$C_V = \frac{1}{kT^2}\sigma_E^2 = \frac{128}{ZkT^2}\left(e^{-8J\beta} + e^{8J\beta} - \left(e^{-8J\beta} - e^{8J\beta}\right)^2\right) = \frac{2^8}{ZkT^2}\left(\cosh 8J\beta - 2\sinh^2 8J\beta\right)$$

$$\chi = \frac{1}{kT}\sigma_M^2 = \frac{16}{ZkT}\left(e^{-8J\beta} + 2 + e^{8J\beta} - \left(e^{-8J\beta} - e^{8J\beta}\right)^2\right) = \frac{2^5}{ZkT}\left(1 + \cosh 8J\beta - 2\sinh^2 8J\beta\right)$$

It remains to obtain the expressions for expected energy and mean magnitude of magnetization and expectation they are obtained from the momentum of the associated quantities:

$$\langle E \rangle (\beta) = \frac{2^4}{Z}\left(e^{-8J\beta} + e^{8J\beta}\right) \qquad \text{and} \qquad \langle |M| \rangle (J\beta) = \frac{2^2}{Z}\left(e^{-8J\beta} + 4 + e^{8J\beta}\right).$$