SOME TITLE

Marius Jonsson (Institutt for Vanskelig Fysikk, Oscars gate 19, 0352 OSLO, Norway)

http://github.com/kingoslo/flintstones

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ABSTRACT

This is a report submission for the first project of «Computational physics 2» at the Institute of Physics, University of Oslo, autumn 2016.

INTRODUCTION

A.

The report is structured by «introduction»-, «methods»-, «results and discussion»- and finally a «conclusion and perspectives»-sections.

METHODS

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RESULTS AND DISCUSSION

CONCLUSION AND PERSPECTIVES

APPENDIX

THEOREM 1 (Hastings-Metropolis theorem)**.** *Suppose that $C\pi_i$ is a discrete probability distribution. If q_{ij} is any irreducible transition probability matrix, and X_n is a Markov chain with transition probability matrix*

$$P_{ij} = \begin{cases} \alpha_{ij} q_{ij}, & j \neq i \\ q_{ii} + \sum_{k=0}^{\infty} q_{ik}(1-\alpha_{ik}) & j = i \end{cases}, \qquad \textit{where} \qquad \alpha_{ij} = \min\left(\frac{\pi_j q_{ji}}{\pi_i q_{ij}}, 1\right), \tag{1}$$

then X_n is time reversible with stationary distribution π_i.

Proof. Assume that the hypothesis is true. Then in particular $q_{ij} \neq 0$ for all i, j since q is irreducible. Notice that if

$$\frac{\pi_j q_{ji}}{\pi_i q_{ij}} = 1,$$

then there is nothing to prove since then $\alpha_{ij} = 1$ and $\alpha_{ji} = 1$, and therefore

$$\pi_i P_{ij} = \pi_j P_{ji} \tag{2}$$

is automatic. Hence it suffices to prove (2) for the two cases

$$\frac{\pi_j q_{ji}}{\pi_i q_{ij}} > 1 \qquad \text{and} \qquad \frac{\pi_j q_{ji}}{\pi_i q_{ij}} < 1,$$

separately. Suppose first that $\pi_j q_{ji} > \pi_i q_{ij}$ (†). Write:

$$\pi_i P_{ij} \overset{(1)}{=} \pi_i q_{ij} \alpha_{ij} \overset{(1)(\dagger)}{=} \pi_i q_{ij} \cdot 1 = \pi_i q_{ij} \frac{\alpha_{ji}}{\alpha_{ji}} = \alpha_{ji} \pi_i q_{ij} \frac{1}{\alpha_{ji}} \overset{(\dagger)}{=} \alpha_{ji} \pi_i q_{ij} \frac{\pi_j q_{ji}}{\pi_i q_{ij}} = \alpha_{ji} \pi_j q_{ji} \overset{(1)}{=} \pi_j P_{ji}.$$

In the case that $\pi_j q_{ji} < \pi_i q_{ij}$ (‡), write

$$\pi_i P_{ij} \overset{(1)}{=} \pi_i q_{ij} \alpha_{ij} \overset{(1)(\ddagger)}{=} \pi_i q_{ij} \frac{\pi_j q_{ji}}{\pi_i q_{ij}} = \pi_j q_{ji} = \pi_j q_{ji} \cdot 1 \overset{(1)(\ddagger)}{=} \pi_j q_{ji} \cdot \alpha_{ji} = \pi_j P_{ji},$$

which means X_n is time reversible with stationary probability π_i. ■