A stochastic computational method for bubbly flows with first steps towards representing inception

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Abstract

We present a computational model for bubbly, cavitating flows. For disperse bubbles that are small compared to mixture-averaged length-scales, we use an Euler–Euler sub-grid model that represents the bubbles as evolved statistical quantities. This strategy represents the stochastic features of bubble dynamics observed experimentally. We present numerical experiments that exercise the computational model under varying conditions, including those crafted to investigate cavitation inception.

Keywords: Sub-grid modeling, bubbles, cavitation, inception

Introduction

Cavitating flows arise in many engineering applications: around ship propellers (Cook 1928), in hydraulic machinery (Arndt 1981), and during medical therapies like lithotripsy (Cleveland et al. 2000). In these flows, bubbles incept, oscillate as they carry through the flow, and sometimes collapse. Directly simulating the flow of these dispersions is challenging because the bubble dynamics often occur at small length and time scales.

In the dilute limit, a strategy for overcoming this scale separation recognizes that the dynamics of each bubble are unimportant compared to statistics of the bubble population. Ensemble averaging two-way couples the liquid phase to the statistics of the bubbles (Zhang and Prosperetti 1994). The associated bubble variables become stochastic, Eulerian fields. The population balance equation (PBE) represents the evolution of a probability density according to the dynamic variables of the problem (Randolph and Larson 1987; Ramkrishna 2000). These variables must provide sufficient information to determine each bubble's dynamic and thermodynamic evolution.

The PBE is commonly used to model the particle size distribution, which can depend on particle motion, coagulation, and breakup (Fox 2003). In this case, the relevant dynamic variables are the particle velocities. PBE-based models have successfully modeled flowing soot during combustion processes (Mueller et al. 2009), aerosol sprays (Sibra et al. 2017), and more. For oscillating bubbles, the relevant variables are the bubble radius, bubble radial velocity, and sufficient information regarding the bubble contents (Brennen 1995). These variables can be appended when appropriate to those associated with relative motion, coalescence, and break up (Carrica et al. 1999; Heylmun et al. 2019).

Past sub-grid cavitation models were restricted to static parameters, like the equilibrium bubble radius (Bryngelson et al. 2019; Ando et al. 2011) We interrogated a model based on the population balance formalism and implemented it via quadrature moment methods. The model capabilities are stressed via numerical experiments. A cavitating bubble screen case assesses the quadrature closure method's ability to track the bubble population statistics. A case that induces a vortex is used as a first step toward formulating the model with inception in mind.

Population Balance Modeling Strategy

A PBE-based model represents the evolving distributions of bubble radius, radial velocity, and equilibrium radius. It is written as

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial R}(f\dot{R}) + \frac{\partial}{\partial \dot{R}}(f\ddot{R}) = 0, \tag{1}$$

and governs the conditional PDF $f(R, \dot{R}, R_o)$ in the absence of bubble coalescence or breakup, though it can accommodate these effects if needed.

Implementation via Quadrature-based Methods

Figure 1 summarizes the PBE–quadrature approach. The method of moments represents and evolves the number density function via its moments (Hulburt and Katz 1964). This requires a dynamical, thermodynamic model for the bubbles. The most general form of a spherical bubble model consists of a set of PDEs for the balance of mass, momentum, and energy. Under further assumptions, these can be reduced to a set of ODEs for the bubble radius and radial velocity, with equilibrium radius as a parameter (Plesset and Prosperetti 1977). The set of ODEs for the bubbles determines the moment transport equations (Frenklach and Harris 1987). Quadrature-based moment methods then invert the moment set for a quadrature rule approximating the quantities required to close the moment transport equations (McGraw 1997). With multiple independent variables, conditional quadrature moment methods are computationally preferable (Yuan and Fox 2011). This work inverts the moments for the quadrature moments via the conditional hyperbolic quadrature method of moments, CHyQMOM (Fox et al. 2018; Patel et al. 2019; Fox and Laurent 2021).

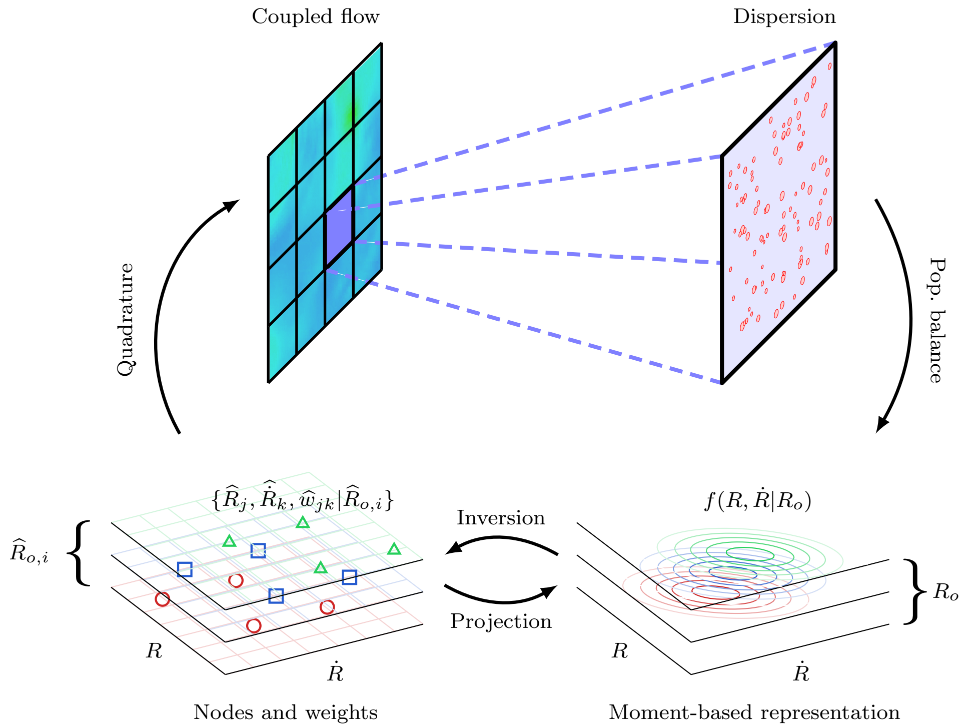

Figure 1: Schematic of the population balance model and quadrature moment method.

Two-way Coupling

Ensemble phase-averaging methods determine the governing flow equations for oscillating bubble populations. An Euler–Euler approach represents the flow, and the sub-grid averaged bubble dynamics. The usual compressible flow equations are altered to have a mixture pressure

$$p = (1-\alpha)p_\ell + \alpha \left(\frac{\overline{R^3 p_{bw}}}{\overline{R^3}} - \rho \frac{\overline{R^3 \dot{R}^2}}{\overline{R^3}} \right), \qquad (2)$$

where ρ and p_ℓ are the suspending liquid density and pressure, α is the void fraction, R are the bubble radii, $\dot{R}$ are their time derivatives, p_{bw} is the bubble wall pressure, and the overbars correspond to moments. The void fraction transports as

$$\frac{\partial \alpha}{\partial t} + \boldsymbol{u} \cdot \nabla \alpha = 3\alpha \frac{\overline{R^2 \dot{R}}}{\overline{R^3}}, \qquad (3)$$

where the right-hand side is associated with bubble growth or shrinkage.

Example Results

We first consider an acoustically excited bubble screen. The screen has an initial void fraction and width and is impinged by a single cycle of a sinusoid with amplitude 0.3 times the ambient pressure. The bubbles cause part of the wave to reflect and part to transmit through the screen.

Figure 2 shows the pressure in the center of the screen. With an increasing initial variance of bubble sizes σ_R, we observe a qualitatively different response in the screen, all of

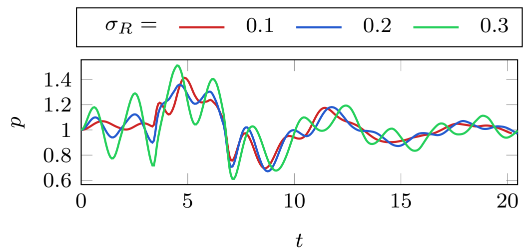

Figure 2: Pressure in the center of an acoustically excited bubble screen of initial void fraction 10^{-4} and different initial variances σ_R of the bubble radius R.

which deviate from the usual $\sigma_R = 0$ treatment and assumption. We also consider cases with commensurate variances in the other bubble variables.

We also explore flow configurations that produce meaningful bubble accelerations and negative pressures, for example, a vortex ring carrying a bubble population. This flow prompts an attempt at modeling cavitation inception via the PBE model of fig. 1.

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