Digital Image Processing

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Contents

Introduction

- Morphology = study of forms.
- Useful for describing image components:
  - Boundaries.
  - Skeletons.
  - Convex hull.
- Also useful for pre- and post-processing:
  - Filtering.
  - Thinning.
  - Pruning.

Set theory

Set theory is fundamental to understand morphology.

- Sets represent objects in an image.
- The set of all white pixels is a complete representation of this image.
- In binary images:
  - sets are members of $\mathcal{Z}^2$ (bi-dimensional space of integers).
  - each element z_i is a bi-dimensional vector of coordinates (x_i, y_i).
- In grayscale images:
  - Set components are in $\mathcal{Z}^3$.

Consider A a set in $\mathcal{Z}^2$.

If $w_i = (x_i, y_i)$ is an element of A, then $w \in A$.

If w is not an element of A, then $w \notin A$.

Also:

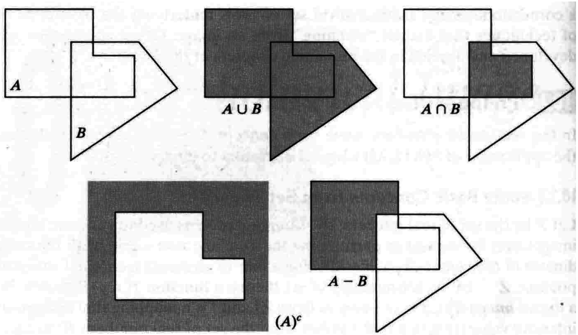
Empty set	$\emptyset$
A subset of B	$A \subseteq B$
Union of A and B	$A \cup B$
Intersection between A and B	$A \cap B$
Disjoint sets	$A \cap B = \emptyset$
Set specification	$B = \{w \vert \text{condition } \}$

Also:

Complement of A	$A^c = \{w \mid w \notin A\}$
Difference between A and B	$A - B = \{w \mid w \in A, w \notin B\}$
Reflection of B	$\hat{B} = \{w \mid w = -b, b \in B\}$
Translation of A by $z = (x, y)$	$(A)_z = \{c \mid c = a + z, a \in A\}$

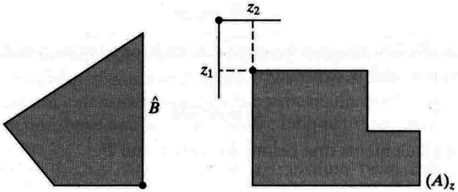
a b c
d e

FIGURE 10.1
(a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.



a b

FIGURE 10.2
(a) Reflection of B. (b) Translation of A by z. The sets A and B are from Fig. 10.1, and the black dot denotes their origin.



Logical operators

Set Operation	MATLAB Expression for Binary Images	Name
$A \cap B$	A & B	AND
$A \cup B$	A | B	OR
A^c	~A	NOT
$A - B$	A & ~B	DIFFERENCE

TABLE 10.1 Using logical expressions in MATLAB to perform set operations on binary images.

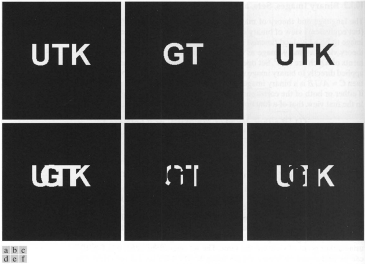


FIGURE 10.3 (a) Binary image A. (b) Binary image B. (c) Complement ~A. (d) Union A | -B. (e) Intersection A & B. (f) Set difference A & ~B.

Dilation

Definition

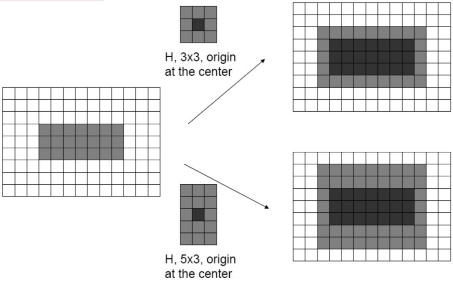
$$A \oplus B = \left\{ z \middle| \left(\hat{B} \right)_z \cap A \neq \emptyset \right\}$$

$$A \oplus B = \left\{ z \middle| [(\hat{B})_z \cap A] \subseteq A \right\}$$

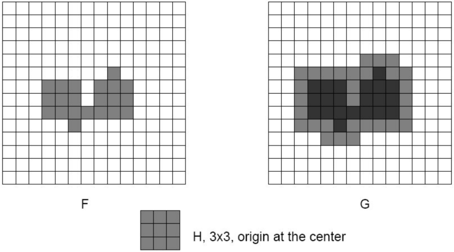
- Reflexion of B followed by a translation of z.
- Result contains all shifts z st. there is at least one intersection between A and B.

- B is the *Structuring Element* (SE).

Example:



Another example:



Properties of dilation

- Associativity

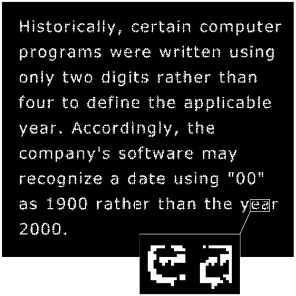
  $A \oplus (B \oplus C) = (A \oplus B) \oplus C.$

- Commutativity $A \oplus B = B \oplus A.$

Application: Gap filling.

a c
b

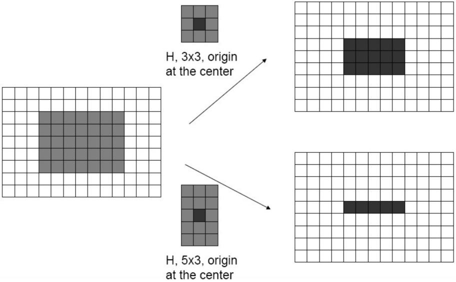
FIGURE 9.7
(a) Sample text of poor resolution with broken characters (see magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.



0	1	0
1	1	1
0	1	0

Erosion

Example:



Definition

$A \ominus B = \{z | (B)_z \subseteq A\}$

$A \ominus B = \{z | (B)_z \cap A^c = \emptyset\}$

- Result contains all shifts z st. B lies inside A.

Properties of erosion

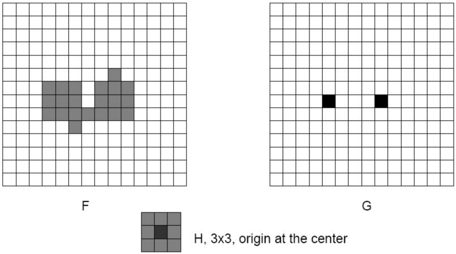
- It is NOT commutative:

$$A \ominus B \neq B \ominus A.$$

- It is NOT associative, but:

$$(A \ominus B) \ominus C = A \ominus (B \ominus C).$$

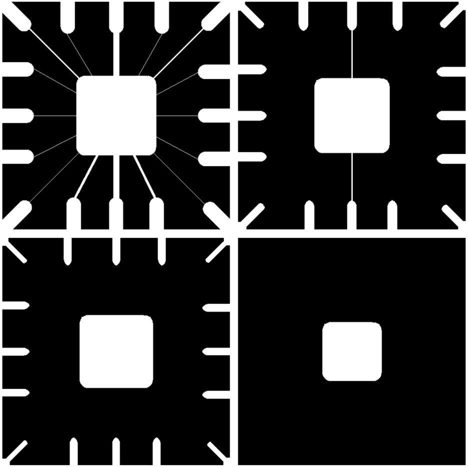
Another example:



Application: Removing image components.

a b
c d

FIGURE 9.5 Using erosion to remove image components. (a) A 486×486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11×11, 15×15, and 45×45, respectively. The elements of the SEs were all 1s.



Dilation and erosion

Duality

Erosion and Dilation are duals of each other wrt. set complementation and reflection:

- $(A \oplus B)^c = A^c \ominus \hat{B}$.
- $(A \ominus B)^c = A^c \oplus \hat{B}$.

Opening

- Erosion followed by a dilation:

$$A \circ B = (A \ominus B) \oplus B$$

- Effect similar to erosion (tends to remove background).
- Less destructive than erosion.
- Determined by a structuring element.

Tends to:

- Preserve foreground regions that:
  - have shape similar to SE;
  - completely contain the SE.
- Removes regions of the object that cannot contain the SE.
- Smooths objects contours.
- Breaks thin connections.
- Removes thin protrusions.

Opening can be expressed as a fitting process:

$$A \circ B = \bigcup \{(B)_z \,|\, (B)_z \subseteq A\} .$$

where $\bigcup\{\cdot\}$ denotes union of all sets inside braces.

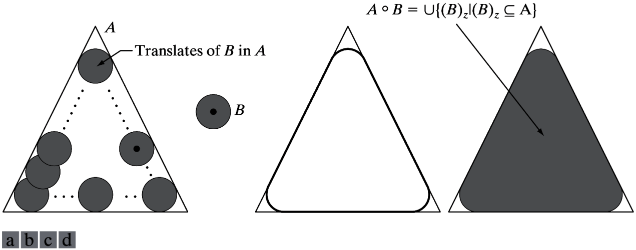
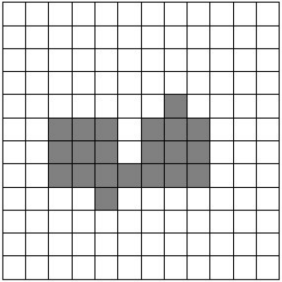
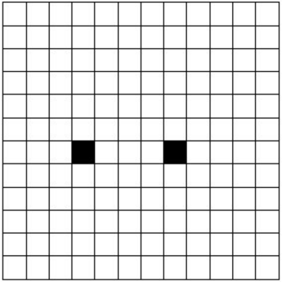


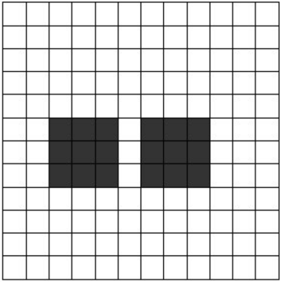
FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (b) Structuring element. (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded). We did not shade A in (a) for clarity.



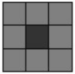
F



$F \ominus H$



$(F \ominus H) \oplus H$



H, 3x3, origin at the center

Closing

- Dilation followed by an erosion:

$$A \bullet B = (A \oplus B) \ominus B$$

- Similar to dilation (tends to enlarge foreground boundaries).
- Less destructive than dilation.
- Determined by a structuring element (SE).

Tends to:

- Preserve background regions that:
  - have similar shape to SE;
  - completely contain the SE.
- Removes regions of the background that cannot contain the SE.
- Smooths objects contours.
- Joins narrow breaks.
- Fills long thing gulfs.
- Fills holes smaller than the SE.

In closing, the ball B is rolled outside the boundary of A:

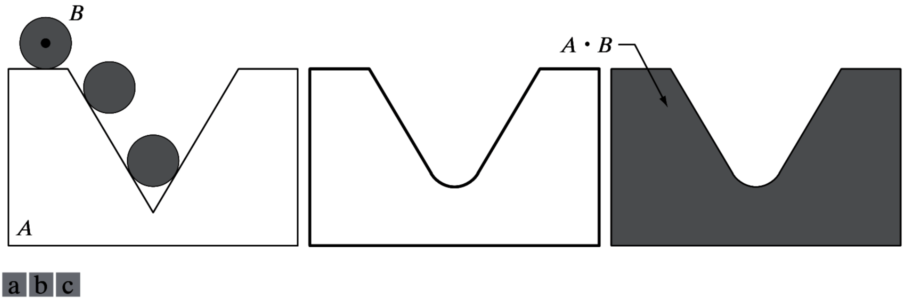
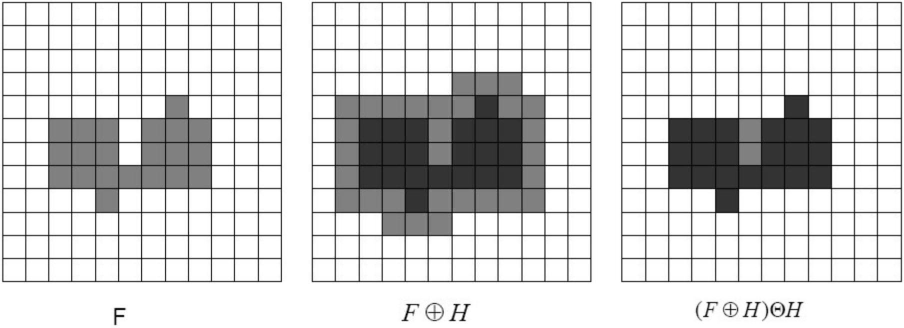
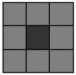


FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade A in (a) for clarity.





H, 3x3, origin at the center

Opening and closing

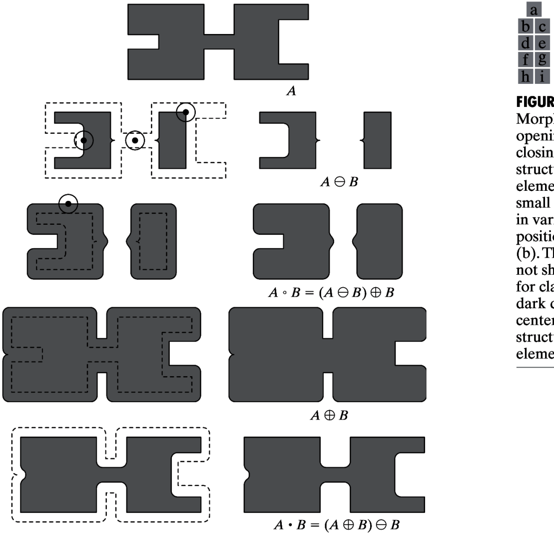


FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.

Duality

Opening and closing are duals of each other wrt. set complementation and reflection:

- $(A \circ B)^c = A^c \bullet \hat{B}$.
- $(A \bullet B)^c = A^c \circ \hat{B}$.

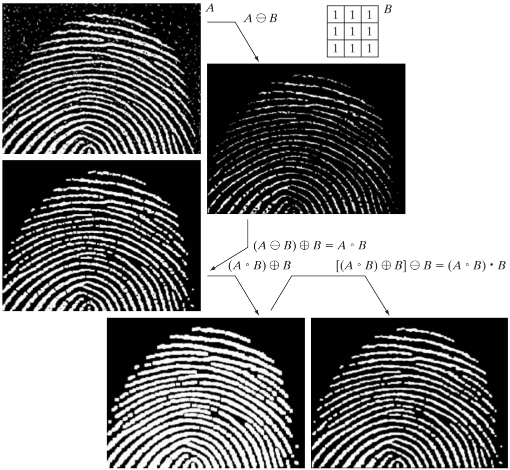
Properties of opening

- $A \circ B$ is a subset of A.
- If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$.
- $(A \circ B) \circ B = A \circ B$.

Properties of closing

- A is a subset of $A \bullet B$.
- If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$.
- $(A \bullet B) \bullet B = A \bullet B$.

Example: Morphological filtering.



a b
d c
e f

FIGURE 9.11 (a) Noisy image. (b) Structuring element. (c) Eroded image. (d) Opening of A. (e) Dilation of the opening. (f) Closing of the opening. (Original image courtesy of the National Institute of Standards and Technology.)

Hit-or-miss transform

The hit-or-miss transform is a basic tool for shape detection.

Definition

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$$

$$A \circledast B = (A \ominus B_1) - \left(A \oplus \hat{B}_2\right)$$

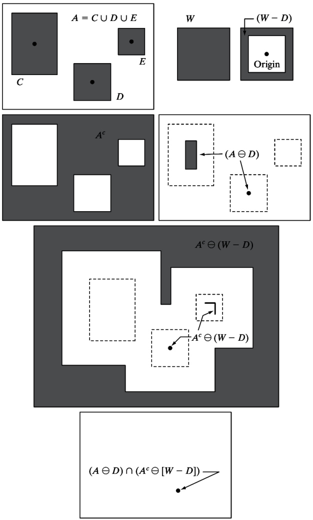


FIGURE 9.12 (a) Set A. (b) A window, W, and the local background of D with respect to W, $(W - D)$. (c) Complement of A. (d) Erosion of A by D. (e) Erosion of A^c by $(W - D)$. (f) Intersection of (d) and (e), showing the location of the origin of D, as desired. The dots indicate the origins of C, D, and E.

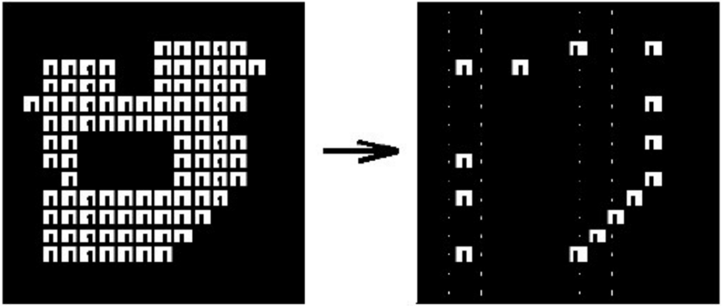
Application: Corner detection.

	1	
0	1	1
0	0	

	1	
1	1	0
	0	0

	0	0
1	1	0
	1	

0	0	
0	1	1
	1	



Boundary extraction

Definition:

$$\beta(A) = A - (A \ominus B)$$

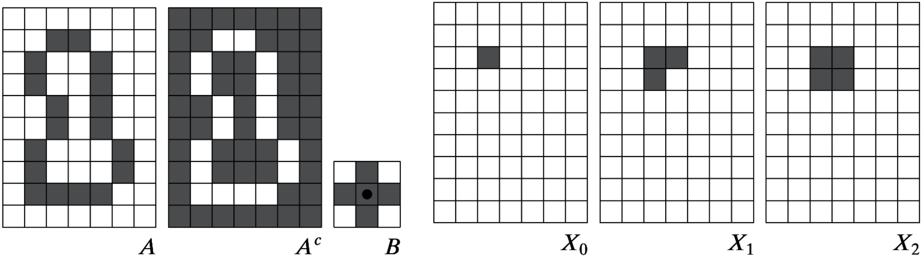
Using $B =$`ones(3)`.

Figure: A

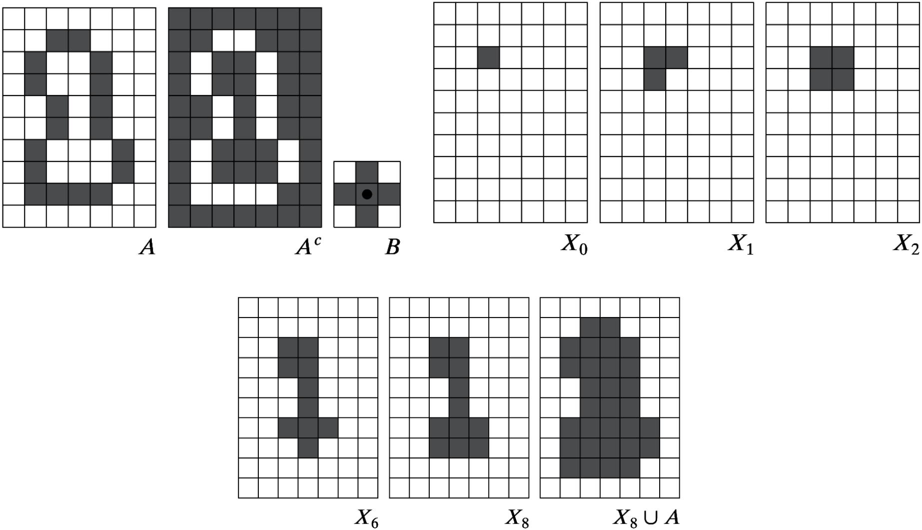
Figure: $A \ominus B$

Figure: β

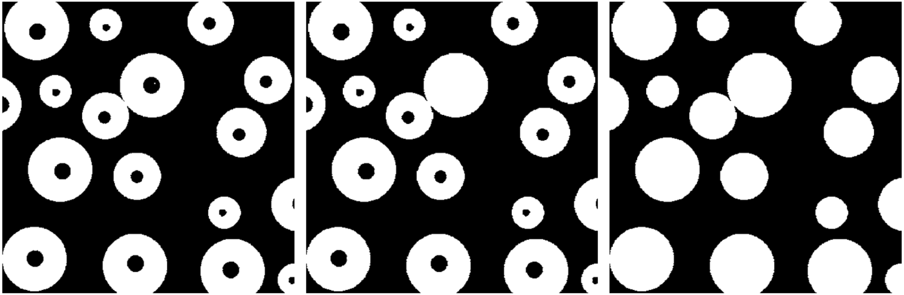
Iterations: $X_k = (X_{k-1} \oplus B) \cap A^c \quad k = 1, 2, \ldots$



Result:

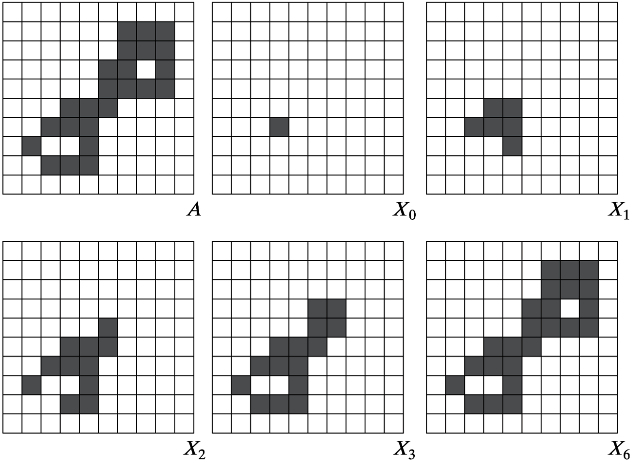


Example: Morphological hole filling:



Extraction of connected components

Iterations $X_k = (X_{k-1} \oplus B) \cap A \quad k = 1, 2, \ldots$



Ex 9.7: Detecting foreign objects in packaged food.

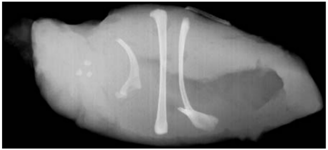


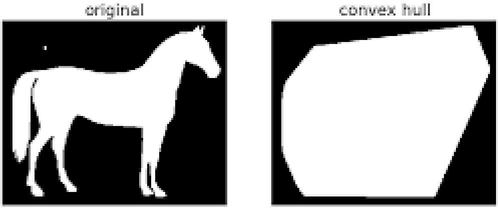
Figure: Chicken breast with bone fragments.

Figure: Threshold and erosion of original.

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

Convex hull

- A set A is *convex* if a line segment joining any two points in A lies entirely within A.
- The convex *hull* H of set S is the smallest convex set containing S.
- The set difference is called the convex deficiency of S.
- Useful for description.



$X_k^i = \left(X_{k-1} \circledast B^i\right) \cup A$

Four SEs B^i.

$i = 1, \ldots 4.$

$k = 1, 2, \ldots$

$X_0^i = A.$

When $X_k^i = X_{k-1}^i$ we let

$D^i = X_k^i.$

Final convex hull:

$C(A) = \bigcup_{i=1}^{4} D^i.$

MATLAB $=$ `convhull`.

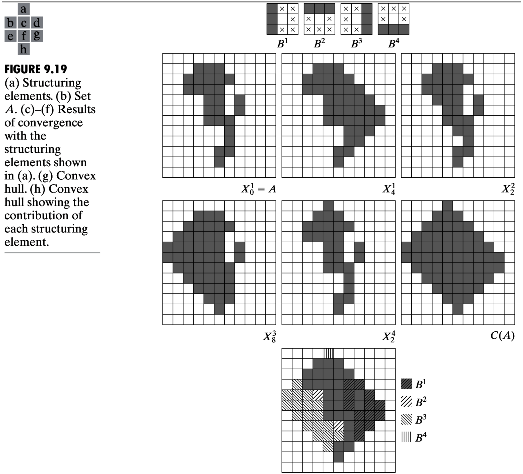


FIGURE 9.19 (a) Structuring elements. (b) Set A. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.

Limit growth beyond the minimum dimensions guaranteeing convexity.

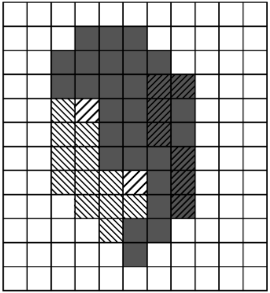


FIGURE 9.20 Result of limiting growth of the convex hull algorithm to the maximum dimensions of the original set of points along the vertical and horizontal directions.

Thinning

Thinning of set A by SE B is defined as

$$A \otimes B = A - (A \circledast B)$$

$$A \otimes B = A \cap (A \circledast B)^c .$$

- No background operation is required (we want only hits).
- Sequence of SEs:

$$\{B\} = \{B^1, B^2, \dots, B^n\} .$$

$$A \otimes \{B\} = ((\dots((A \otimes B^1) \otimes B^2) \dots) \otimes B^n)$$

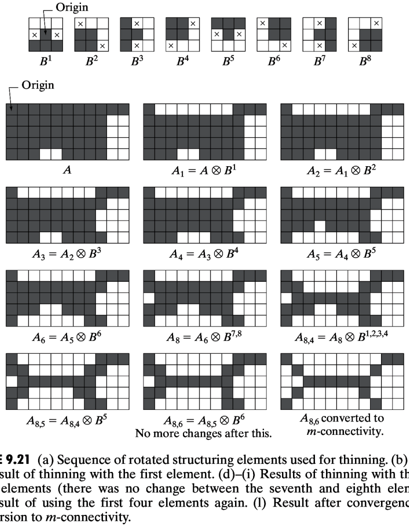


FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first four elements again. (l) Result after convergence. (m) Conversion to m-connectivity.

Thickening

Thickening is defined as $A \odot B = A \cup (A \circledast B)$

$A \odot \{B\} = ((\ldots((A \odot B^1) \odot B^2)\ldots) \odot B^n)$

In practice:

$A \odot \{B\} = (A^c \otimes \{B\})^c$. then post process to remove disconnected points.

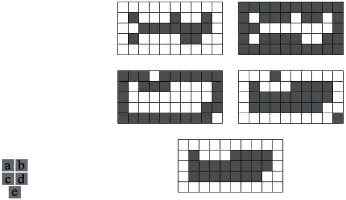


FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

Skeletons

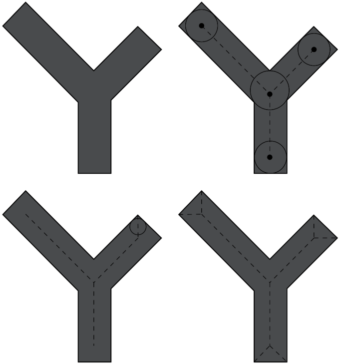


FIGURE 9.23 (a) Set A. (b) Various positions of maximum disks with centers on the skeleton of A. (c) Another maximum disk on a different segment of the skeleton of A. (d) Complete skeleton.

The skeleton can be expressed in terms of erosions and openings

$S(A) = \bigcup_{k=0}^{K} S_k(A)$

where

- $S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$
- $A \ominus kB = ((\ldots((A \ominus B) \ominus B) \ominus \ldots) \ominus B)$ k times.

Also, A can be reconstructed using

$A = \bigcup_{k-0}^{K} (S_k(A) \oplus kB)$

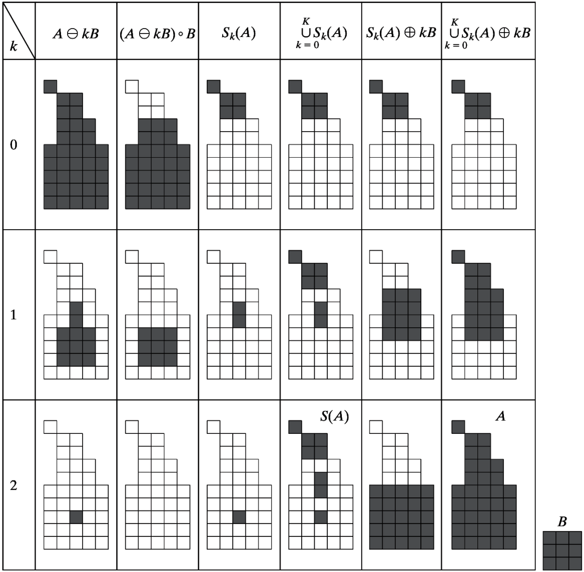


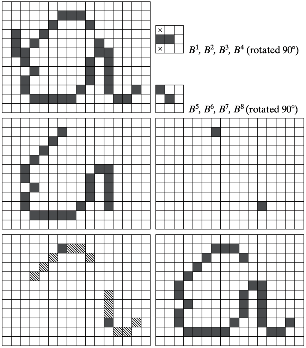
FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

Pruning

- Used for cleaning the result of thinning and skeletons.

Algorithm:

1. Thin $X_1 = A \otimes B$.
2. Find endpoints $X_2 = \bigcup_{k=1}^{8}(X_1 \circledast B^k)$.
3. Fill region $X_3 = (X_2 \oplus H) \cap A$, where $H = \texttt{ones}(3)$.
4. Result $X_4 = X_1 \cup X_3$.



a b
c
d e
f g

FIGURE 9.25 (a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

Morphological reconstruction

Reconstruction is a morphological transformation involving two images and a structuring element:

1. Marker: Starting point for the transformation.
2. Mask: Constrains the transformation.
3. Structuring element: Defines connectivity.

Matlab: `imreconstruct(marker, mask)`.

Algorithm (G is the mask):

1. Initialize h_1 to be the marker image, F s.t. $F \subseteq G$.
2. Create SE $B = $ `ones(3)`.
3. Repeat

$$h_{k+1} = (h_k \oplus B) \cap G$$

   until $h_{k+1} = h_k$.

4. $R_G(F) = h_{k+1}$.

Notice that hole filling $X_k = (X_{k-1} \oplus B) \cap A^c$ is a particular case of image reconstruction.

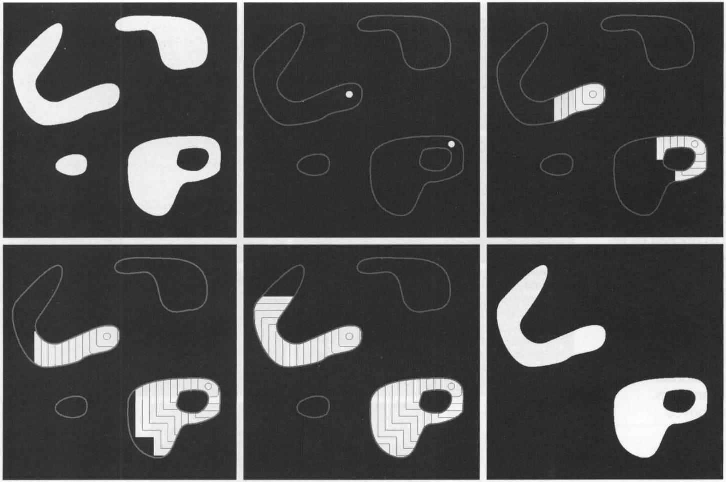


FIGURE 10.21 Morphological reconstruction. (a) Original image (the mask). (b) Marker image. (c)–(e) Intermediate result after 100, 200, and 300 iterations, respectively. (f) Final result. (The outlines of the objects in the mask image are superimposed on (b)–(e) as visual references.)

Opening by reconstruction

Opening:

1. erosion (removes small objects), followed by a;
2. dilation (restores the shapes of remaining objects, depending on SE shape).

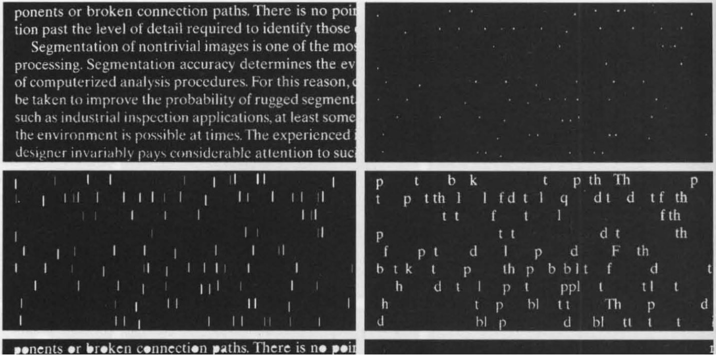
Opening by reconstruction restores the original shapes of the remaining objects.

It is defined as:

$$R_G(G \ominus B).$$

a b
c d
e f
g

FIGURE 10.22 Morphological reconstruction: (a) Original image. (b) Image eroded with vertical line; (c) opened with a vertical line; and (d) opened by reconstruction with a vertical line.



Filling holes

Let:

1. $I(x, y)$ - binary image.
2. Marker image F to be 0 everywhere except on the image border:

$$F(x,y) = \begin{cases} 1 - I(x,y) & \text{if } (x,y) \text{ is on the border of } I \\ 0 & \text{otherwise} \end{cases}$$

Then

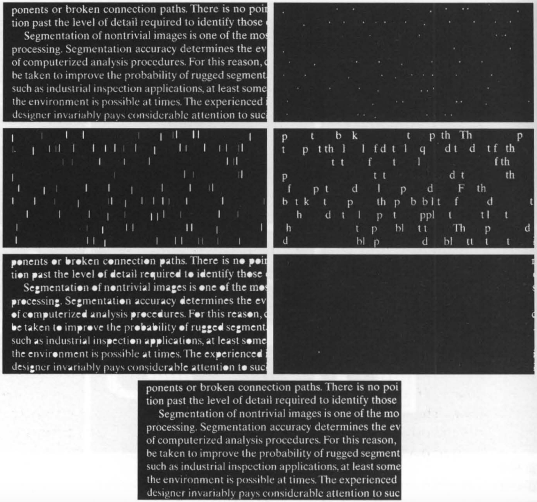
$$H = [R_{I^c}(F)]^c$$

is a binary image equal to I with all holes filled.

Matlab: `g = imfill(f, 'holes')`.

a b
c d
e f
g

FIGURE 10.22 Morphological reconstruction: (a) Original image. (b) Image eroded with vertical line; (c) opened with a vertical line; and (d) opened by reconstruction with a vertical line. (e) Holes filled. (f) Characters touching the border (see right border). (g) Border characters removed.



Clearing border objects

Remove objects touching the border of the image.

Define marker F as

$$F(x,y) = \begin{cases} I(x,y) & \text{if } (x,.y) \text{ is on the border of } I \\ 0 & \text{otherwise} \end{cases}$$

Then

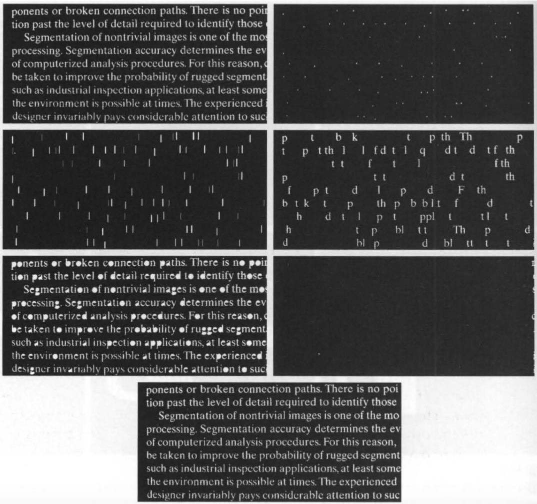
$$H = R_I(F)$$

contains only objects touching the border.

Matlab: `g = imclearborder(f, conn)`.

a b
c d
e f
g

FIGURE 10.22 Morphological reconstruction: (a) Original image. (b) Image eroded with vertical line; (c) opened with a vertical line; and (d) opened by reconstruction with a vertical line. (e) Holes filled. (f) Characters touching the border (see right border). (g) Border characters removed.



Summary of operators

TABLE 9.1 Summary of morphological operations and their properties.

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Translation	$(B)_z = \{w \mid w = b + z, \text{ for } b \in B\}$	Translates the origin of B to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \left\{z \mid (\hat{B}_z) \cap A \neq \varnothing\right\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \left\{z \mid (B)_z \subseteq A\right\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)

TABLE 9.1 *(Continued)*

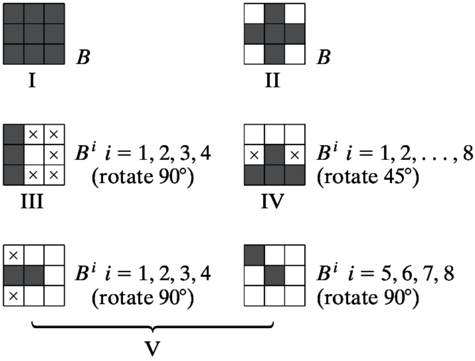
Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)
Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Hole filling	$X_k = (X_{k-1} \oplus B) \cap A^c$; $k = 1, 2, 3, \ldots$	Fills holes in A; $X_0 =$ array of 0s with a 1 in each hole. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A$; $k = 1, 2, 3, \ldots$	Finds connected components in A; $X_0 =$ array of 0s with a 1 in each connected component. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \ldots$; $X_0^i = A$; and $D^i = X_{\text{conv}}^i$	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)

Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2) \ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} = ((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0s and 1s reversed.
Skeletons	$S(A) = \bigcup_{k=0}^{K} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosions of A by B. (I)

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.
Geodesic dilation of size 1	$D_G^{(1)}(F) = (F \oplus B) \cap G$	F and G are called the *marker* and *mask* images, respectively.
Geodesic dilation of size n	$D_G^{(n)}(F) = D_G^{(1)}\left[D_G^{(n-1)}(F)\right]$; $D_G^{(0)}(F) = F$	
Geodesic erosion of size 1	$E_G^{(1)}(F) = (F \ominus B) \cup G$	
Geodesic erosion of size n	$E_G^{(n)}(F) = E_G^{(1)}\left[E_G^{(n-1)}(F)\right]$; $E_G^{(0)}(F) = F$	

Morphological reconstruction by dilation	$R_G^D(F) = D_G^{(k)}(F)$	k is such that $D_G^{(k)}(F) = D_G^{(k+1)}(F)$
Morphological reconstruction by erosion	$R_G^E(F) = E_G^{(k)}(F)$	k is such that $E_G^{(k)}(F) = E_G^{(k+1)}(F)$
Opening by reconstruction	$O_R^{(n)}(F) = R_F^D[(F \ominus nB)]$	$(F \ominus nB)$ indicates n erosions of F by B.
Closing by reconstruction	$C_R^{(n)}(F) = R_F^E[(F \oplus nB)]$	$(F \oplus nB)$ indicates n dilations of F by B.
Hole filling	$H = \left[R_{I^c}^D(F)\right]^c$	H is equal to the input image I, but with all holes filled. See Eq. (9.5-28) for the definition of the marker image F.
Border clearing	$X = I - R_I^D(F)$	X is equal to the input image I, but with all objects that touch (are connected to) the boundary removed. See Eq. (9.5-30) for the definition of the marker image F.

FIGURE 9.33 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the ×'s indicate "don't care" values.



Gray-Scale Morphology

We now deal with digital functions:

- $f(x, y)$ - input image.
- $b(x, y)$ - structuring element (SE).

These functions are discrete:

- x and y are integers.
- f and b assign an intensity value to each pair (x, y).

Gray-scale dilation

Definition of gray-scale dilation:

$$[f \oplus b](x, y) = \max_{(s,t)\in b} \{f(x - s, y - t) + b(s, t)\}$$

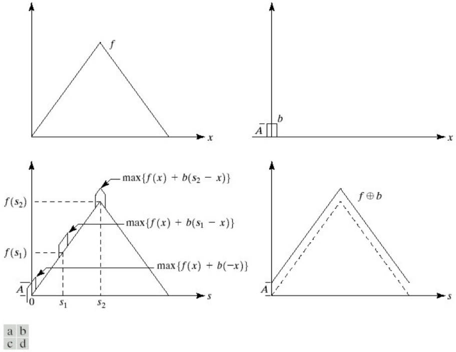


FIGURE 9.27 (a) A simple function. (b) Structuring element of height A. (c) Result of dilation for various positions of sliding b past f. (d) Complete result of dilation (shown solid).

- Similar to convolution.
- max operation substitutes convolution sum.
- Addition substitutes the product in the convolution sum.

General effects:

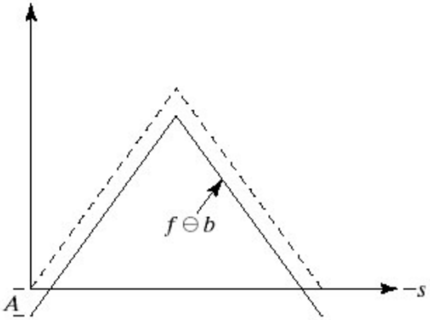
- For positive values of the SE, the resulting image tends to be brighter.
- Dark details are reduced or eliminated, depending on how the values and shapes of the details are related to the SE.

Gray-scale erosion

Definition of gray-scale erosion:

$$[f \ominus b](x, y) = \min_{(s,t)\in b} \{f(x+s, y+t) - b(s,t)\}$$

FIGURE 9.28
Erosion of the function shown in Fig. 9.27(a) by the structuring element shown in Fig. 9.27(b).



- Similar to correlation.

General effects:

- If the SE has positive values, the result tends to be darker.
- Bright details smaller than the SE are reduced, according to the SE's values and shape.

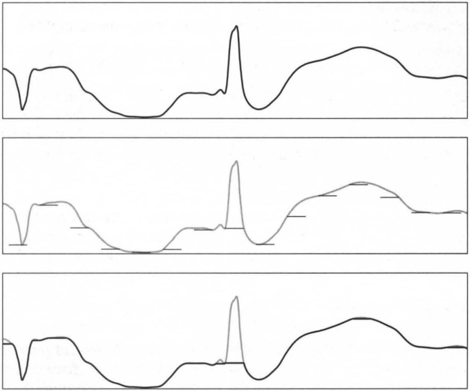
a b c

FIGURE 9.35 (a) A gray-scale X-ray image of size 448 $\times$ 425 pixels. (b) Erosion using a flat disk SE with a radius of two pixels. (c) Dilation using the same SE. (Original image courtesy of Lixi, Inc.)

Gray-scale opening

Defined as

$$f \circ b = (f \ominus b) \oplus b.$$



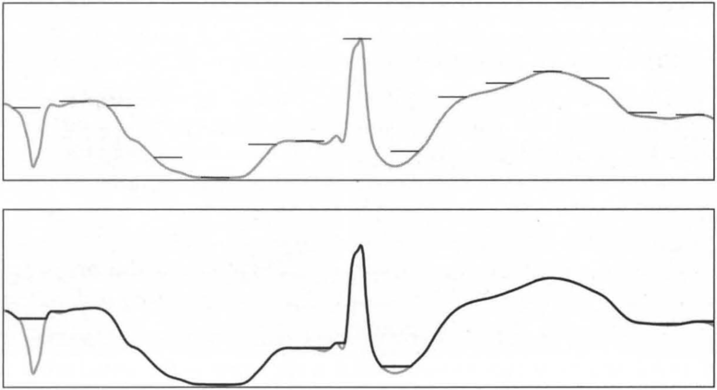
a
b
c
d
e

FIGURE 10.24 Opening and closing in one dimension. (a) Original 1-D signal. (b) Flat structuring element pushed up underneath the signal. (c) Opening. (d) Flat structuring element pushed down along the top of the signal. (e) Closing.

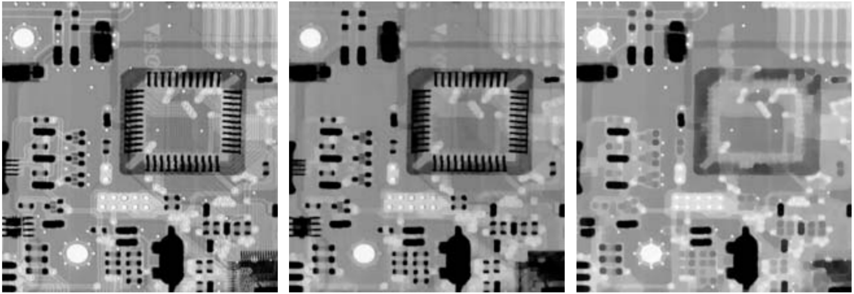
Gray-scale closing

Defined as

$$f \bullet b = (f \oplus b) \ominus b.$$



Gray-scale opening and closing



a b c

FIGURE 9.37 (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Opening using a disk SE with a radius of 3 pixels. (c) Closing using an SE of radius 5.

- Opening suppresses bright details smaller than the structuring element.
- Closing suppresses dark details smaller than the structuring element.
- Both are often combined for smoothing and noise removal.

Morphological smoothing:

- Opening followed by closing, ie., $(f \circ b) \bullet b$.
- Effect: removal of bright and dark artifacts and noise.

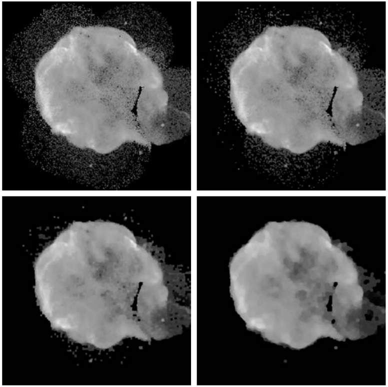
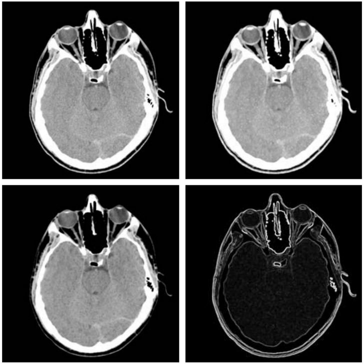


FIGURE 9.38 (a) 566 × 566 image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii, 1, 3, and 5, respectively. (Original image courtesy of NASA.)

Morphological gradient:

- $g = (f \oplus b) - (f \ominus b)$.
- highlights borders.
- less dependent of border direction than Sobel.



a b
c d

FIGURE 9.39 (a) 512 × 512 image of a head CT scan. (b) Dilation. (c) Erosion. (d) Morphological gradient, computed as the difference between (b) and (c). (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)

Top-hat and bottom-hat transforms

Top-hat:

$$T_{\text{hat}}(f) = f - (f \circ b)$$

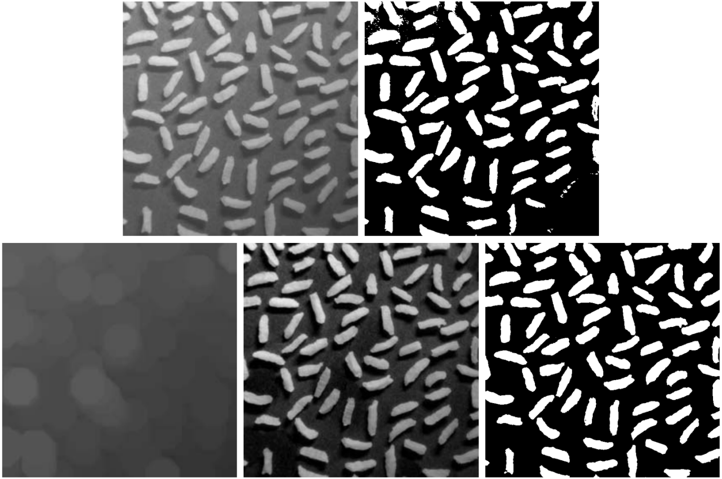
Bottom-hat:

$$B_{\text{hat}}(f) = (f \bullet b) - f$$

Applications:

- Removing objects by using a SE smaller than the object.
- Top-hat: light objects on dark background.
- Bottom-hat: dark objects on bright background.

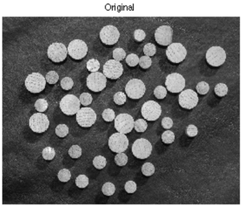
Gray-scale opening and closing

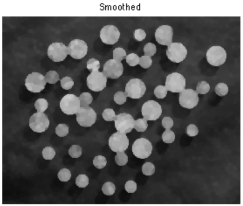


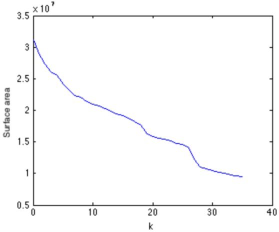
a b
c d e

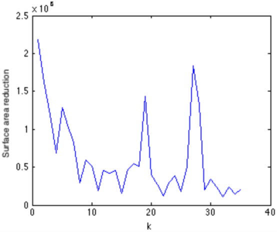
FIGURE 9.40 Using the top-hat transformation for *shading correction*. (a) Original image of size

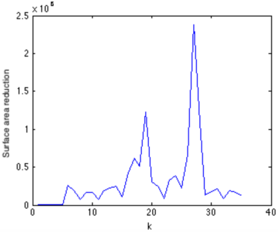
Granulometry



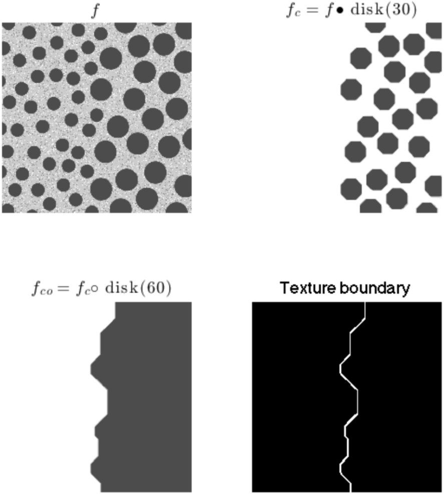








Texture segmentation



Reflections wider than characters

Opening by Reconstruction (OBR)

Top-Hat reconstruction

Final reconstruction result