№1

$X = \{x_i\}$ – выборка сл. величины $N(\sigma, \mu)$

Ф-ия правдоподобия $L = \prod_i p(x_i)$ $\quad p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$l = \ln L = \sum_i \ln(p(x_i)) = \sum_i \left[-\frac{(x_i-\mu)^2}{2\sigma^2} - \ln(\sigma\sqrt{2\pi}) \right] =$

$= -\frac{\sum_i (x_i-\mu)^2}{2\sigma^2} - n\ln(\sigma\sqrt{2\pi})$

$L \to \max \Leftrightarrow l \to \max \Rightarrow l'_\mu = 0, l'_\sigma = 0$

$l'_\mu = -\frac{\sum_i ((x_i-\mu)^2)'_\mu}{2\sigma^2} = \frac{\sum (x_i-\mu)}{\sigma^2} = 0 \Leftrightarrow \sum_i x_i = \sum_i \mu$

$\Rightarrow \sum_i x_i = n\mu \Rightarrow \hat{\mu} = \frac{\sum_i x_i}{n} = \bar{X}$

$\Rightarrow \mu = \bar{X}$ – ср. арифмет. $\Rightarrow \sum_i (x_i - \bar{X})^2 = n \operatorname{var} X$

$l'_\sigma = \left(-\frac{n \operatorname{var} X}{2\sigma^2} - n\ln(\sigma\sqrt{2\pi}) \right)'_\sigma = \frac{n \operatorname{var} X}{\sigma^3} - \frac{n}{\sigma} = 0 \Rightarrow \frac{n}{\sigma} = \frac{n \operatorname{var} X}{\sigma^3}$

$\Rightarrow \hat{\sigma}^2 = \operatorname{var} X \Rightarrow \hat{\sigma} = \sqrt{\operatorname{var} X}$ – среднеквадратичное отклонение

Получили: $\hat{\mu} = \bar{X} = \frac{\sum_i x_i}{n}$, $\hat{\sigma} = \sqrt{\operatorname{var} X} = \sqrt{\frac{\sum_i (x_i - \bar{X})^2}{n}}$