№2 Параболическая регрессия

$$\hat{y} = \hat{a}x^2 + \hat{b}x + \hat{c} \qquad \{(x_i, y_i)\}_{i=1}^{N}$$

$$L = \sum_{i=1}^{N} (\hat{y} - y_i)^2 = \sum_{i=1}^{N} (\hat{a}x_i^2 + \hat{b}x_i + \hat{c} - y_i)^2$$

$$\frac{\partial L}{\partial \hat{a}} = \sum_{i=1}^{N} 2x_i^2(\hat{a}x_i^2 + \hat{b}x_i + \hat{c} - y_i) = 2\hat{a}\sum x_i^4 + 2\hat{b}\sum x_i^3 + 2\hat{c}\sum x_i^2 - 2\sum y_i x_i^2 =$$
$$= 2\hat{a}N\langle x_i^4\rangle + 2\hat{b}N\langle x_i^3\rangle + 2\hat{c}\langle x_i^2\rangle - 2N\langle y_i x_i^2\rangle = 0$$

$$\Rightarrow \hat{a}\langle x_i^4\rangle + \hat{b}\langle x_i^3\rangle + \hat{c}\langle x_i^2\rangle = \langle y_i x_i^2\rangle$$

$$\frac{\partial L}{\partial \hat{b}} = \sum_{i=1}^{N} 2x_i(\hat{a}x_i^2 + \hat{b}x_i + \hat{c} - y_i) = 0$$

$$\Rightarrow \hat{a}\langle x_i^3\rangle + \hat{b}\langle x_i^2\rangle + \hat{c}\langle x_i\rangle = \langle y_i x_i\rangle$$

$$\frac{\partial L}{\partial \hat{c}} = \sum_{i=1}^{N} 2(\hat{a}x_i^2 + \hat{b}x_i + \hat{c} - y_i) = 0$$

$$\Rightarrow \hat{a}\langle x_i^2\rangle + \hat{b}\langle x_i\rangle + \hat{c} = \langle y_i\rangle$$

$$\begin{cases} \hat{a}\langle x_i^4\rangle + \hat{b}\langle x_i^3\rangle + \hat{c}\langle x_i^2\rangle = \langle y_i x_i^2\rangle \\ \hat{a}\langle x_i^3\rangle + \hat{b}\langle x_i^2\rangle + \hat{c}\langle x_i\rangle = \langle y_i x_i\rangle \\ \hat{a}\langle x_i^2\rangle + \hat{b}\langle x_i\rangle + \hat{c} = \langle y_i\rangle \end{cases}$$

$$A = \begin{pmatrix} \langle x_i^4\rangle & \langle x_i^3\rangle & \langle x_i^2\rangle \\ \langle x_i^3\rangle & \langle x_i^2\rangle & \langle x_i\rangle \\ \langle x_i^2\rangle & \langle x_i\rangle & 1 \end{pmatrix} \qquad \bar{y} = \begin{pmatrix} \langle y_i x_i^2\rangle \\ \langle y_i x_i\rangle \\ \langle y_i\rangle \end{pmatrix}$$

$$A\hat{w} = \bar{y}$$