№2 Гиперболическая задача

$\hat{a}y^2 - \hat{b}x^2 = 1 \qquad f(\hat{a}, \hat{b}, x, y) = \hat{a}x^2 - \hat{b}y^2 - 1$

$$L = \sum_{i=1}^{N} \left(f(x_i, y_i) - x_i - y_i\right)^2 = \sum_{i=1}^{N} \left(\hat{a}x_i^2 - \hat{b}y_i^2 - 1 - x_i - y_i\right)^2$$

$$\frac{\partial L}{\partial \hat{a}} = \sum 2x_i^2 (\hat{a}x_i^2 - \hat{b}y_i^2 - 1 - x_i - y_i) = 2\sum (\hat{a}x_i^4 - \hat{b}x_i^2y_i^2 - x_i - x_i^2 - x_iy_i) = 0$$

$$\Rightarrow \hat{a}\langle x_i^4\rangle - \hat{b}\langle x_i^2y_i^2\rangle = \langle x_i\rangle + \langle x_i^2\rangle + \langle x_iy_i\rangle$$

$$\frac{\partial L}{\partial \hat{b}} = -2\sum (\hat{a}x_i^2y_i^2 - \hat{b}y_i^4 - y_i - x_iy_i - y_i^2) = 0$$

$$\Rightarrow \hat{a}\langle x_i^2y_i^2\rangle - \hat{b}\langle y_i^4\rangle = \langle y_i\rangle + \langle y_i^2\rangle + \langle x_iy_i\rangle$$

$$\begin{cases} \hat{a}\langle x_i^4\rangle - \hat{b}\langle x_i^2y_i^2\rangle = \langle x_i\rangle + \langle x_i^2\rangle + \langle x_iy_i\rangle \\ \hat{a}\langle x_i^2y_i^2\rangle - \hat{b}\langle y_i^4\rangle = \langle y_i\rangle + \langle y_i^2\rangle + \langle x_iy_i\rangle \end{cases}$$

$$A = \begin{pmatrix} \langle x_i^4\rangle & -\langle x_i^2y_i^2\rangle \\ \langle x_i^2y_i^2\rangle & -\langle y_i^4\rangle \end{pmatrix} \qquad \bar{y} = \begin{pmatrix} \langle x_i\rangle + \langle x_i^2\rangle + \langle x_iy_i\rangle \\ \langle y_i\rangle + \langle y_i^2\rangle + \langle x_iy_i\rangle \end{pmatrix}$$

$$Aw = \bar{y}$$