

$$f'(x_0) \approx \frac{1}{12h} (f_{i-2} - 8f_{i-1} + 8f_{i+1} - f_{i+2}) = -\frac{f_{i+2} - f_{i-2}}{4h} \cdot \frac{1}{3} + \frac{f_{i+1} - f_{i-1}}{2h} \cdot \frac{4}{3}$$

$$\left| f'(x_0) - \frac{f(x_0+h) - f(x_0-h)}{2h} \cdot \frac{4}{3} + \frac{f(x_0+2h) - f(x_0-2h)}{4h} \cdot \frac{1}{3} \right| =$$

$$= \left| f'(x_0) - \frac{4}{3} \left( f'(x_0) + \frac{f'''(x_0)}{6} h^2 + \frac{2f^{(5)}(\xi_1)}{120 \cdot 2h} h^5 \right) + \frac{1}{3} \left( f'(x_0) + \frac{2f'''(x_0)}{6 \cdot 4h} \cdot 8h^3 + \frac{2f^{(5)}(\xi_2)}{120 \cdot 4h} \cdot 32h^5 \right) \right| =$$

$$= \left| -\frac{4}{3} \frac{f^{(5)}(\xi_1)}{120} h^4 + \frac{16}{3} \frac{f^{(5)}(\xi_2)}{120} h^4 \right| \leq \frac{20}{3} \frac{M_5}{120} h^4 = \frac{M_5}{18} h^4 = \varepsilon_{\text{method}}$$

$$\varepsilon_{\text{comp}} = \frac{1}{12h} (\Delta f + 8\Delta f + 8\Delta f + \Delta f) = \frac{18\Delta f}{12h}$$

$$\varepsilon_{\text{total}} = \frac{18\Delta f}{12h} + \frac{M_5 h^4}{18}, \quad (\varepsilon_{\text{total}})'_h = \frac{2M_5}{9} h^3 - \frac{18\Delta f}{12h^2} = 0$$

$$h^5 = \frac{9 \cdot 18\Delta f}{2 \cdot 12 M_5} = \frac{27\Delta f}{4 M_5} \Rightarrow h^* \approx \frac{3}{2} \cdot \sqrt[5]{\frac{\Delta f}{M_5}}$$