Chapter 14

Exercise 14.1

For X1:

Eigenvalues are: 0.873 and 0.127

Eigenvalues are: $[0.526, 0.850]$

and $[-0.850, 0.526]$

For X2:

Eigenvalues are: 0.397 and 0.103.

Eigenvalues are: $[0.615, 0.788]$

and $[-0.788, 0.615]$

Exercise 14.2

* features $f2$ and $f5$ have the largest correlation:

$$0.85.$$

$$\frac{0.75 + 0.37 + 0.30}{3} = 0.473 \text{ is the average of the absolute}$$

correlations between $f2$ and the other 3 features.

$$\frac{0.37 + 0.21 + 0.52}{3} = 0.373 \text{ is the average of the absolute}$$

correlations between $f5$ and the other 3 features.

$\therefore 0.473 > 0.373$

$\therefore$ We remove $f2$ from the feature set.

14.3 :

For the first part:

$$\text{accuracy rate} = \frac{984}{1000} = 0.984.$$

$$\text{Recall} = \frac{2}{10} = 0.2,$$

$$\text{Precision} = \frac{2}{10} = 0.2,$$

$$\text{F-score} = \frac{2 \times 0.2 \times 0.2}{0.2 + 0.2} = 0.2,$$

$$\text{FP rate} = \frac{8}{990} = 0.008.$$

$$\text{True negative rate} = \frac{982}{982 + 8} = 0.992.$$

For the second part:

$$\text{accuracy rate} = \frac{992}{1000} = 0.992.$$

$$\text{Recall} = \frac{10}{10} = 1,$$

$$\text{Precision} = \frac{10}{18} \approx 0.556 \approx 0.56,$$

$$\text{F-score} = \frac{2 \times 1 \times 0.556}{1 + 0.556} \approx 0.71,$$

$$\text{FP rate} = \frac{8}{990} = 0.008.$$

$$\text{True negative rate} = \frac{982}{982 + 8} = 0.992.$$

14.4 :

$$\text{accuracy rate} = \frac{975}{1000} = 0.975.$$

$$\text{Recall} = \frac{500}{510} \approx 0.980 \approx 0.98,$$

$$\text{Precision} = \frac{500}{515} \approx 0.971 \approx 0.97,$$

$$\text{F-score} = \frac{2 \times 0.980 \times 0.971}{0.980 + 0.971} \approx 0.98,$$

$$\text{FP rate} = \frac{15}{490} \approx 0.031.$$

$$\text{True negative rate} = \frac{475}{490} \approx 0.969.$$

14.5 :
First part:

$$\text{accuracy rate} = \frac{875}{1000} = 0.875.$$

$$\text{Recall} = \frac{20}{100} = 0.2,$$

$$\text{Precision} = \frac{20}{65} \approx 0.308 \approx 0.31,$$

$$\text{F-score} = \frac{2 \times 0.2 \times 0.308}{0.2 + 0.308} \approx 0.24,$$

$$\text{FP rate} = \frac{45}{900} = 0.05.$$

$$\text{True negative rate} = \frac{855}{900} = 0.95.$$

Second part:

$$\text{accuracy rate} = \frac{1000}{1000} = 1.$$

$$\text{Recall} = \frac{100}{100} = 1,$$

$$\text{Precision} = \frac{100}{100} = 1,$$

$$\text{F-score} = \frac{2 \times 1 \times 1}{1 + 1} = 1,$$

$$\text{FP rate} = \frac{0}{900} = 0.$$

$$\text{True negative rate} = \frac{900}{900} = 1$$

Exercise 14.6

$$X^TX = \begin{bmatrix} 4 & 10 \\ 10 & 30 \end{bmatrix}$$

$$(X^TX)^{+} = \frac{1}{20}\begin{bmatrix} 30 & -10 \\ -10 & 4 \end{bmatrix} = \begin{bmatrix} 1.5 & -0.5 \\ -0.5 & 0.2 \end{bmatrix}$$

$$X^TY = \begin{bmatrix} 9 \\ 19 \end{bmatrix}$$

$$\hat{a} = (X^TX)^{-1}X^TY = \begin{bmatrix} 4 \\ -0.7 \end{bmatrix}$$

For $\lambda = 0$, $\quad B(\tilde{a}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$Var(\hat{a}) = \sigma^2\begin{bmatrix} 1.5 & -0.5 \\ -0.5 & 0.2 \end{bmatrix}$$

For $\lambda = 10$

$$X^TX + \lambda I' = \begin{bmatrix} 4 & 10 \\ 10 & 40 \end{bmatrix}$$

$$(X^TX + \lambda I')^{+} \doteq \begin{bmatrix} 0.667 & -0.167 \\ -0.167 & 0.067 \end{bmatrix}$$

$$X^TY = \begin{bmatrix} 9 \\ 19 \end{bmatrix}$$

$$\tilde{a}_R = \begin{bmatrix} 0.667 & -0.167 \\ -0.167 & 0.067 \end{bmatrix}\begin{bmatrix} 9 \\ 19 \end{bmatrix} \doteq \begin{bmatrix} 2.83 \\ -0.23 \end{bmatrix}$$

$$B(\tilde{a}_R) = \begin{bmatrix} 0 & 1.67 \\ 0 & -0.67 \end{bmatrix}a \qquad \text{if } a = \hat{a}, \text{ then } B(\tilde{a}_R) = \begin{bmatrix} -1.2 \\ 0.5 \end{bmatrix}$$

$$Var(\tilde{a}_R) = \sigma^2\begin{bmatrix} 0.39 & -0.055 \\ -0.055 & 0.022 \end{bmatrix}$$