Problems of Chapter 12

12.1

See Table S.6.

Table S.6 Answers to Exercise 12.1

Question number	Mean	Median	Mode
(1)	6.25	6	6
(2)	$\frac{23}{6}$	4	0 and 5
(3)	5.1	5.5	5.5
(4)	5.9	5.9	5.8 and 6.0
(5)	8	9	10

12.2

(1) 2.82
(2) 3.76
(3) 2.47
(4) 0.19
(5) 3.16
(6) 0.3

12.3

(1) $cov(x_1, x_2) = -0.125$ and $r(x_1, x_2) = -0.20$.
(2) $cov(x_1, x_3) = -0.01125$ and $r(x_1, x_3) = -0.19$.
(3) $cov(x_2, x_3) = -0.225$ and $r(x_2, x_3) = -0.24$.
(4) $cov(x_1, x_2) = 0.6$, $cov(x_1, x_3) = 0.055$, $cov(x_2, x_3) = 0.9375$, and $r(x_1, x_2) = 0.98$, $r(x_1, x_3) = 0.95$, $r(x_2, x_3) = 0.99$.

12.4

Class A and Class B have the same relative dispersion: 20%.

12.5

For the first dataset:

1. mode = 87,
2. median = 86.5,
3. IQR = 13,
4. 50 and 110 are outliers.

For the second dataset:

1. mode = 55,
2. median = 54,
3. IQR = 10,
4. 30, 72 and 80 are outliers.

Exercise 12.6

$\mu_{\bar{X}} = \mu = 60$

$\sigma_{\bar{X}} = \frac{10}{\sqrt{100}} = 1$

From $P(|\mu - \bar{X}| < 0.3)$

We have $P(59.7 < \bar{X} < 60.3)$

$= \Phi\left(\frac{60.3-60}{1}\right) - \Phi\left(\frac{59.7-60}{1}\right)$

$= \Phi(+0.3) - \Phi(-0.3)$

$= 0.6179 - 0.3821 = 0.2358$

Exercise 12.7

$$\mu_{\bar{x}} = \mu = 57$$

$$\sigma_{\bar{x}} = \frac{12}{\sqrt{64}} = \frac{3}{2}$$

$$P(56<\bar{x}<58) = \Phi\left(\frac{58-57}{\frac{3}{2}}\right) - \Phi\left(\frac{56-57}{\frac{3}{2}}\right)$$

$$= \Phi\left(\frac{2}{3}\right) - \Phi\left(-\frac{2}{3}\right)$$

$$= 0.7486 - 0.2514 = 0.4972$$

Exercise 12.8

$\mu = 4.58$, $\sigma = 0.3$, $n = 50$

$$\bar{x} = \frac{230}{50} = 4.6$$

$$\sigma_{\bar{x}} = \frac{0.3}{\sqrt{50}}\sqrt{\frac{400-50}{400-1}} \doteq 0.03974$$

$$z = \frac{4.6-4.58}{0.03974} \approx 0.5033$$

$$P(\bar{x}>4.6) = 1-(\text{area from } [z=-\infty - z=0.5033])$$

$$= 1-0.6915 = 0.3085.$$

Exercise 12.9

$$\mu_p = 0.02$$

$$\sigma_p = \sqrt{\frac{0.02\times 0.98}{400}} = 0.007$$

$$z = \frac{0.03-0.02}{0.007} \doteq 1.429$$

$$P(z<1.429) = 92.36\%$$

Exercise 12.10

$\mu_p = 0.25$

$\sigma_p = \sqrt{\frac{0.25 \times 0.75}{80}} = 0.048$

(1) $z = \frac{0.24 - 0.25}{0.048} = -0.21$

$P(z > -0.21) = 1 - 0.4168 = 0.5832$

(2) $z = \frac{0.3 - 0.25}{0.048} = 1.04$

$P(z < 1.04) = 0.8508$

Exercise 12.11

(1) $t = 1.753$

(2) $t = 2.602$

Exercise 12.12

(1) $\chi^2 = 21.03$

(2) $\chi^2 = 26.22$

Exercise 12.13

$\mu = 1486.4$

$\sigma \doteq 98.53$

Exercise 12.14

For CI: 99%

$z_{99\%} = 2.5758$

$[163 - 2.5758 \times \frac{8}{\sqrt{100}}, 163 + 2.5758 \frac{8}{\sqrt{100}}]$

$= [160.939, 165.061]$

For CI: 95%

$z_{97.5} = 1.96$

$[163 - 1.96 \times \frac{8}{\sqrt{100}}, 163 + 1.96 \times \frac{8}{\sqrt{100}}]$

$= [161.432, 164.568]$

Exercise 12.15

95% CI.

$$\left[0.59-1.96\sqrt{\frac{0.59\times0.41}{200}},\quad 0.59+1.96\sqrt{\frac{0.59\times0.41}{200}}\right]$$

$$=[0.522,\ 0.658]$$

99%.

$$\left[0.59-2.5758\sqrt{\frac{0.59\times0.41}{200}},\quad 0.59+2.5758\sqrt{\frac{0.59\times0.4}{200}}\right]$$

$$=[0.500,\ 0.680$$

Exercise 12.16

For CI: 95%

$$\chi^2_{0.025,24}\approx 39.364,\quad \chi^2_{0.975,24}\approx 12.401$$

$$\frac{(25-1)\times 6.4^2}{39.364}\le\sigma^2\le\frac{(25-1)\times 6.4^2}{12.401}$$

$$24.973\le\sigma^2\le 79.271$$

$$4.997\le\sigma\le 8.903$$

For CI: 90%

$$\chi^2_{0.05,24}\approx 36.415,\quad \chi^2_{0.95}\approx 13.848$$

$$26.99\le\sigma^2\le 70.99$$

$$5.196\le\sigma\le 8.425$$

Exercise 12.17

$$\left[0.82-1.96\sqrt{\frac{0.82\times0.18}{5000}},\quad 0.82+1.96\sqrt{\frac{0.82\times0.18}{5000}}\right]$$

$$=[0.809,\quad 0.831]$$

Exercise 12.18

$s=0.07$

$df=19$

$\chi^2_{0.05}=30.144,\quad \chi^2_{0.95}=10.117$

$$\frac{19\times0.07^2}{30.144}\le\sigma^2\le\frac{19\times0.07^2}{10.117}$$

$$0.0031\le\sigma^2\le0.0092$$

$$0.056<\sigma\le0.096$$

Exercise 12.19

$$\left[22.9-\frac{2.58\times3.2}{\sqrt{2000}},\quad 22.9+\frac{2.58\times3.2}{\sqrt{2000}}\right]$$

$$=[22.7,\quad 23.1]$$

Exercise 12.20

$\overline{X}=920.$

$\sigma_S=\dfrac{\sigma}{\sqrt{N}}=\dfrac{100}{\sqrt{16}}=25$

$t=\dfrac{920-1000}{25}=-3.2$

Two critical values for which 2.5% of the area lies in each tail of the t-distribution with $16-1=15$ degrees of freedom are $[-2.131,\ 2.131]$. Since -3.2 is not covered in $[-2.131, 2.131]$, we rejected H_0, that is, H_0: $\mu=1000$ h and this product type is non-defective, at the 5% significance level.

Exercise 12.21

$H_0: = 235g$

$H_1: \neq 235g$

$\bar{X} = 233, \quad \sigma_s = \sqrt{7} = 2.646$

$$t = \frac{233-235}{\frac{2.646}{\sqrt{5}}} = -1.690$$

1 tailed test $\therefore$ 5% below for df=4 corresponds the critical point: -2.132

$\because -1.690 > -2.132$

$\therefore$ do not reject Ho at ~~95%~~ 5% significance level.

Exercise 12.22

$H_0: = 23$

$H_1: \neq 23$

$\bar{X} = 22, \quad \sigma_s = 2.256$

$$t = \frac{22-23}{\frac{2.256}{\sqrt{12}}} = -1.535$$

2 tailed test. The critical value for 5% above/below with df=11 is ± 1.796

$\because -1.535 \in [-1.796, 1.796]$

$\therefore$ We do not reject Ho at the 10% significance level.

Exercise 12.23

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

We do a two-tailed test.

$n_1=10,\ \bar{x}_1=18.5,\ s_1=2,\ \bar{x}_2=17.5,\ s_2=3.5,\ n_2=20$

numerator: (10-1)x2^2+(20-1)x3.5^2

$$\sigma = \sqrt{\frac{\cancel{10\times4+20\times3.5^2}}{10+20-2}} \doteq \cancel{3.19}\ 3.1$$

$$t = \frac{18.5-17.5}{3.1\cancel{3.19}\sqrt{\frac{1}{10}+\frac{1}{20}}} = \cancel{0.81}\ 0.83$$

Two critical values for which 0.5% of the area lies in each tail of the t distribution with 28 degrees of freedom are -2.763 and 2.763. We do not reject H_0 at the 1% significance level.

Exercise 12.24

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

We do a 2-tailed test.

$n_1=8,\ \bar{x}_1=55,\ s_1=5$

$n_2=12,\ \bar{x}_2=50,\ s_2=4$

(8-1)x5^2+(12-1)x4^2

$$\sigma = \sqrt{\frac{\cancel{8\times5^2+12\times4^2}}{8+12-2}} = \cancel{4.67}\ 4.4$$

$$t = \frac{55-50}{4.4\cancel{4.67}\sqrt{\frac{1}{8}+\frac{1}{12}}} = \cancel{2.35}\ 2.48$$

$df=18$. The critical value is 2.101.

$\because$ 2.48 $\cancel{2.35} > 2.101$

$\therefore$ we reject H_0 at 5% significance level.

Exercise 12.25

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

do a 2-tails test at 1%.

$n_1 = 10, \quad \bar{X}_1 = 3.5. \quad S_1 = 0.5$

$n_2 = 15 \quad \bar{X}_2 = 3.2 \quad S_2 = 0.3$

$$\sigma = \sqrt{\frac{\cancel{10 \times 0.5^2 + 15 \times 0.3^2}\ (10-1)\text{x}0.5\text{^}2+(15-1)\text{x}0.3\text{^}2}{10+15-2}} = \cancel{0.409}\ 0.4$$

$$t = \frac{3.5 - 3.2}{0.4\text{-}\cancel{0.409}\sqrt{\frac{1}{10} + \frac{1}{15}}} = \cancel{1.797}\ 1.88, \quad df = 23.$$

The critical value is 2.807

1.88

$\because \cancel{1.797} < 2.807$

$\therefore$ Do not reject H_0 at 1% significance level.

Exercise 12.26

Z_i	1	2	3	4	5	6
Obs	90	145	35	50	15	25
Exp	110	90	70	50	30	10

$$\chi^2 = \frac{(90-110)^2}{110} + \frac{(145-90)^2}{90} + \frac{(35-70)^2}{70} + \frac{(50-50)^2}{50} + \frac{(15-30)^2}{30} + \frac{(25-10)^2}{10}$$

$\doteq 84.75$

$df = 6 - 1 = 5$

$\chi^2_{5\%, 5} = 11.070$

$\because 84.75 > 11.070$

$\therefore$ We reject H_0 at 5% significance level.

Exercise 12.27

Expected: $\frac{240}{8} = 30$, $df = 8-1 = 7$

$\chi^2 = \frac{1}{30}[36+16+25+25+36+16+36+36]$

$= 7.53.$

$\chi^2_{0.05,7} = 14.07$

$\because 7.53 < 14.07$

$\therefore$ We do not reject Ho at 5% significance level.

Exercise 12.28.

$df = 6-1 = 5$

$\chi^2_{5\%,5} = 11.07$

$\chi^2 = \frac{25}{15} + \frac{25}{20} + \frac{16}{25} + \frac{36}{20} + \frac{16}{15} + \frac{16}{5} = 9.62$

$\because 9.62 < 11.07$

$\therefore$ We do not reject Ho at 5% significance level.

Exercise 12.29

Expected. frequencies

	1st	2i	2ii	3	pass	Fail
Uni1	16	25	28	17	12.5	5.5
Uni2	16	25	28	17	12.5	5.5

$\chi^2 = \frac{16}{16} \times 2 + 0 + \frac{1}{28} \times 2 + \frac{9}{17} \times 2 + \frac{2.25}{12.5} \times 2 + \frac{0.25}{5.5} \times 2$

$= 3.581$ $df = (6-1) \times (2 \times 1) = 5$

From table. $\chi^2_{5\%,5} = 11.07$

$\because 3.51 < 11.07$

$\therefore$ Do not reject Ho at 5% significance level.

Exercise 12.30

Expected frequencies

91	9
91	9

$df = (2-1)\times(2-1) = 1$

$$\chi^2 = 2\left[\frac{16}{91} + \frac{16}{9}\right] = 3.907$$

From table: $\chi^2_{0.05,1} = 3.841$

$\because 3.907 > 3.841$

$\therefore$ We reject the H_0 at 5% significance level.

Exercise 12.31

Expected frequencies

	D	P	Fail
F	10	60	10
M	5	30	5

$df = (3-1)(2-1) = 2$

$$\chi^2 = \frac{1}{10} + \frac{16}{60} + \frac{9}{10} + \frac{1}{5} + \frac{16}{30} + \frac{9}{5} \doteq 3.8$$

From table, $\chi^2_{5\%,2} = 5.991$

$\because \chi^2 = 3.8 < 5.991$

$\therefore$ Do not reject H_0 at 5% significance level.

Exercise 12.32.

Expected frequencies

	A	B	C	D
Girls	22.1	54.1	54.1	14.7
Boys	22.9	55.9	55.9	15.3

$df = (2-1)\times(4-1) = 3$

$$\chi^2 = \frac{(20-22.1)^2}{22.1} + \frac{(25-22.9)^2}{22.9} + \frac{(50-54.1)^2}{54.1} + \frac{(60-55.9)^2}{55.9} + \frac{(60-54.1)^2}{54.1} + \frac{(50-55.9)^2}{55.9} + \frac{(15-14.7)^2}{14.7} + \frac{(15-15.3)^2}{15.3}$$

$= 2.282$

From table, $\chi^2_{0.05,3} = 7.815$

$\because \chi^2 = 2.282 < 7.815$

$\therefore$ Do not reject H_0 at 5% significance level.