Chapter 4

Exercise 4.1

(1) $A = \begin{bmatrix} 3 & 1 \\ 3 & 5 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 3-\lambda & 1 \\ 3 & 5-\lambda \end{bmatrix}$$

$$|A - \lambda I| = (3-\lambda)(5-\lambda) - 3 = \lambda^2 - 8\lambda + 12 = 0$$

$$(\lambda - 6)(\lambda - 2) = 0$$

$$\lambda_1 = 6, \quad \lambda_2 = 2$$

for $\lambda_1 = 6$

$$M = \begin{bmatrix} -3 & 1 \\ 3 & -1 \end{bmatrix}$$

$$Mu_1 = \begin{bmatrix} -3 & 1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix} = 0$$

$$\left.\begin{matrix} -3u_{11} + u_{12} = 0 \\ 3u_{11} - u_{12} = 0 \end{matrix}\right. \Rightarrow 3u_{11} = u_{12}$$

$$\sqrt{u_{11}^2 + 9u_{11}^2} = 1$$

$$u_1 = \begin{cases} u_{11} = \frac{1}{\sqrt{10}} \approx 0.316 \\ u_{12} = \frac{3}{\sqrt{10}} \approx 0.948 \end{cases}$$

for $\lambda_2 = 2$

$$M = \begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}$$

$$Mu_2 = \begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} u_{21} \\ u_{22} \end{bmatrix} = 0$$

$$\left.\begin{matrix} u_{21} + u_{22} = 0 \\ 3u_{21} + 3u_{22} = 0 \end{matrix}\right\} \Rightarrow u_{21} = -u_{22}$$

$$\sqrt{u_{21}^2 + u_{21}^2} = 1$$

$$u_{22} \begin{cases} u_{21} = \frac{1}{\sqrt{2}} \approx 0.707 \\ u_{22} = -\frac{1}{\sqrt{2}} \approx -0.707 \end{cases}$$

(2) $B = \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix}$

$$B - \lambda I = \begin{bmatrix} 2-\lambda & 1 \\ 4 & 5-\lambda \end{bmatrix}$$

$$|B - \lambda I| = (2-\lambda)(5-\lambda) - 4 = \lambda^2 - 7\lambda + 6 = 0$$

$$(\lambda - 6)(\lambda - 1) = 0$$

$$\lambda_1 = 6, \ \lambda_2 = 1$$

For $\lambda_1 = 6$

$$M = \begin{bmatrix} -4 & 1 \\ 4 & -1 \end{bmatrix}$$

$$Mu_1 = \begin{bmatrix} -4 & 1 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix} = 0$$

$$\left.\begin{aligned} -4u_{11} + u_{12} &= 0 \\ 4u_{11} - u_{12} &= 0 \end{aligned}\right\} \Rightarrow 4u_{11} = u_{12}$$

$$\sqrt{u_{11}^2 + 16u_{11}^2} = 1$$

$$u_1 = \begin{cases} u_{11} = \frac{1}{\sqrt{17}} \approx 0.243 \\ u_{12} = \frac{4}{\sqrt{17}} \approx 0.970 \end{cases}$$

For $\lambda_2 = 1$

$$M = \begin{bmatrix} 1 & 1 \\ 4 & 4 \end{bmatrix}$$

$$Mu_2 = \begin{bmatrix} 1 & 1 \\ 4 & 4 \end{bmatrix} \begin{bmatrix} u_{21} \\ u_{22} \end{bmatrix} = 0$$

$$u_{21} + u_{22} = 0$$

$$u_{21} = -u_{22}$$

$$\sqrt{u_{21}^2 + (-u_{21})^2} = 1$$

$$u_2 = \begin{cases} u_{21} = \frac{1}{\sqrt{2}} = 0.707 \\ u_{22} = -\frac{1}{\sqrt{2}} = -0.707 \end{cases}$$

(5) $$E = \begin{bmatrix} -1 & 2 & 2 \\ 2 & 2 & 2 \\ -3 & +6 & -6 \end{bmatrix}$$

$$E - \lambda I = \begin{bmatrix} -1-\lambda & 2 & 2 \\ 2 & 2-\lambda & 2 \\ -3 & +6 & -6-\lambda \end{bmatrix} = (-1-\lambda)(2-\lambda)(-6-\lambda) - 12 + 24 + 6(2-\lambda) + 12(1+\lambda) + 4(-6-\lambda)$$

$$= -5\lambda^2 - \lambda^3 + 18\lambda + 72$$

$$\begin{array}{ccccc} -1-\lambda & 2 & 2 & -1-\lambda & 2 \\ 2 & 2-\lambda & 2 & 2 & 2-\lambda \\ -3 & +6 & -6-\lambda & -3 & +6 \end{array}$$

set

$$\lambda^3 + 5\lambda^2 - 18\lambda - 72 = 0$$

$$(\lambda+6)(\lambda+3)(\lambda-4) = 0$$

$$\lambda_1 = -6, \quad \lambda_2 = -3, \quad \lambda_3 = 4$$

For $\lambda_1 = 6$

$$M = \begin{bmatrix} -1+6 & 2 & 2 \\ 2 & 2+6 & 2 \\ -3 & 6 & -6+6 \end{bmatrix} = \begin{bmatrix} -7, & 2, & 2 \\ 2, & -4, & 2 \\ -3, & 6, & -12 \end{bmatrix}$$

$$Mu_1 = \begin{bmatrix} 5, & 2, & 2 \\ 2 & 8 & 2 \\ -3, & 6, & 0 \end{bmatrix} \begin{bmatrix} u_{11} \\ u_{12} \\ u_{13} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

For $\lambda_2 = -3$

$$Mu_2 \begin{bmatrix} 2 & 2 & 2 \\ 2 & 5 & 2 \\ -3 & 6 & -3 \end{bmatrix} \begin{bmatrix} u_{21} \\ u_{22} \\ u_{23} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

For $\lambda_3 = 4$

$$Mu_3 = \begin{bmatrix} -5 & 2 & 2 \\ 2 & -2 & 2 \\ -3 & 6 & -10 \end{bmatrix} \begin{bmatrix} u_{31} \\ u_{32} \\ u_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

① $5u_{11}+2u_{12}+2u_{13}=0$

② $2u_{11}+8u_{12}+2u_{13}=0$

③ $-3u_{11}+6u_{12}+0=0$

② − ① = ③

$-3u_{11}+6u_{12}=0$

$u_{11}=2u_{12}$ ④

subsititute ④ to ①

$10u_{12}+2u_{12}+2u_{13}=0$

$12u_{12}+2u_{13}=0$

$6u_{12}=-u_{13}$

possible vector is

$(2,\ 1,\ -6)$

unit vector:

$$\left(\frac{2}{\sqrt{41}},\ \frac{1}{\sqrt{41}},\ \frac{-6}{\sqrt{41}}\right)$$

$u_{21}+u_{22}+u_{23}=0$ ①

$2u_{21}+5u_{22}+2u_{23}=0$ ②

$-u_{21}+2u_{22}-u_{23}=0$ ③

① + ③:

$3u_{22}=0$

② − 2×①

$3u_{22}=0$

$u_{22}=0$

$u_{21}=-u_{23}$.

$\sqrt{u_{21}^2+u_{23}^2}=1$

$$\begin{cases} u_{21}=\frac{1}{\sqrt{2}} \\ u_{22}=0 \\ u_{23}=-\frac{1}{\sqrt{2}} \end{cases}$$

unit vector

$$\left(\frac{1}{\sqrt{2}},\ 0,\ -\frac{1}{\sqrt{2}}\right)$$

① $-5u_{31}+2u_{32}+2u_{33}=0$

② $2u_{31}-2u_{32}+2u_{33}=0$

③ $-3u_{31}+6u_{32}-10u_{33}=0$

① + ②

$-3u_{31}+4u_{33}=0$

$u_{31}=\frac{4}{3}u_{33}$

① − ②

$-7u_{31}+4u_{32}=0$

$u_{31}=\frac{4}{7}u_{32}$

possible vector:

$$\left[\frac{4}{7},\ 1,\ \frac{3}{7}\right]$$

unit vector: $\left(\frac{4}{\sqrt{74}},\ \frac{7}{\sqrt{74}},\ \frac{3}{\sqrt{74}}\right)$

Exercise 4.2.

$$D=\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$D-\lambda I=\begin{bmatrix} 0-\lambda & 2 \\ -2 & 0-\lambda \end{bmatrix}$$

$$|D-\lambda I|=(-\lambda)(-\lambda)+4=0$$

$$\lambda^2=-4$$

We have complex eigenvalues.

Complex eigenvalues of a matrix with non-zero eigenvectors are beyond the scope of this book.

Exercise 4.1

③ $C = \begin{bmatrix} -4 & -3 \\ 2 & 3 \end{bmatrix}$

$$C - \lambda I = \begin{bmatrix} -4-\lambda & -3 \\ 2 & 3-\lambda \end{bmatrix}$$

$$|C - \lambda I| = (-4-\lambda)(3-\lambda) + 6$$

$$= -12 - 3\lambda + 4\lambda + \lambda^2 + 6$$

$$= \lambda^2 \cancel{+6} + \lambda - 6$$

set $\lambda^2 + \lambda - 6 = 0$

$$(\lambda + 3)(\lambda - 2) = 0$$

$$\lambda_1 = -3 \quad \lambda_2 = 2$$

For $\lambda_1 = -3$

$$m = \begin{bmatrix} -1 & -3 \\ 2 & 6 \end{bmatrix}$$

$$m u_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -3 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-u_{11} - 3u_{12} = 0$$

$$u_{11} = -3u_{12}$$

$$\sqrt{u_{11}^2 + u_{12}^2} = 1$$

$$\sqrt{10u_{12}^2} = 1$$

$$u_1 = \begin{bmatrix} \frac{3}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} \end{bmatrix}$$

For $\lambda_2 = +2$

$$m = \begin{bmatrix} -6 & -3 \\ 2 & 1 \end{bmatrix}$$

$$m u_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -6 & -3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} u_{21} \\ u_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2u_{21} = -u_{22}$$

$$\sqrt{u_{21}^2 + u_{22}^2} = 1$$

$$\sqrt{5u_{21}^2} = 1$$

$$u_2 = \begin{bmatrix} \frac{1}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} \end{bmatrix}$$

④ $D=\begin{bmatrix} 1 & -3 \\ 2 & 6 \end{bmatrix}$

$$D-\lambda I=\begin{bmatrix} 1-\lambda & -3 \\ 2 & 6-\lambda \end{bmatrix}$$

$$|D-\lambda I| = (1-\lambda)(6-\lambda)+6$$
$$= 6-6\lambda-\lambda+\lambda^2+6$$
$$= \lambda^2-7\lambda+12=0$$
$$(\lambda-4)(\lambda-3)=0$$

$\lambda_1=4$, $\lambda_2=3$

For $\lambda_1=4$

$$M=\begin{bmatrix} -3 & -3 \\ 2 & 2 \end{bmatrix}$$

$$MU_1=\underline{0}$$

$$\begin{bmatrix} -3 & -3 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$u_{11}=-u_{12}$$

$$\sqrt{u_{11}^2+u_{12}^2}=1$$

$$\sqrt{2u_{11}^2}=1$$

$$u_1=\begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$

For $\lambda_2=3$

$$M=\begin{bmatrix} -2 & -3 \\ 2 & 3 \end{bmatrix}$$

$$MU_2=0$$

$$\begin{bmatrix} -2 & -3 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} u_{21} \\ u_{22} \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2u_{21}=-3u_{22}$$
$$u_{21}=-\frac{3}{2}u_{22}$$

$$\sqrt{u_{21}^2+u_{22}^2}=1$$

$$\sqrt{\left(\frac{9}{4}+1\right)u_{22}^2}=1$$

$$u_2=\begin{bmatrix} -\frac{3}{\sqrt{13}} \\ \frac{2}{\sqrt{13}} \end{bmatrix}$$

Exercise 4.3.

① $U = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{2}} \\ \frac{3}{\sqrt{10}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$

$$U^{-1} = -\frac{\sqrt{20}}{4}\begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix} = -\frac{\sqrt{5}}{2}\begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{5}}{2\sqrt{2}} & \frac{\sqrt{5}}{2\sqrt{2}} \\ \frac{3}{2\sqrt{2}} & \frac{-1}{2\sqrt{2}} \end{bmatrix}$$

$$D = U^{-1} A U = \begin{bmatrix} 6 & 0 \\ 0 & 2 \end{bmatrix}$$

② $A = \begin{bmatrix} 3 & 2 \\ 2 & 6 \end{bmatrix}$

$$|A - \lambda| = (3-\lambda)(6-\lambda) - 4 = \lambda^2 - 9\lambda + 14 = 0$$

$$(\lambda - 7)(\lambda - 2) = 0$$

$$\lambda_1 = 7 \quad \lambda_2 = 2$$

For $\lambda_1 = 7$

$$\begin{bmatrix} 3-7 & 2 \\ 2 & 6-7 \end{bmatrix}\begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix} = 0$$

$$-4u_{11} + 2u_{12} = 0$$

$$2u_{11} - u_{12} = 0$$

$$2u_{11} = u_{12}$$

$$\sqrt{4u_{11}^2 + u_{11}^2} = 1$$

$$\begin{cases} u_{11} = \frac{1}{\sqrt{5}} \\ u_{12} = \frac{2}{\sqrt{5}} \end{cases}$$

For $\lambda_2 = 2$

$$\begin{bmatrix} 3-2 & 2 \\ 2 & 6-2 \end{bmatrix}\begin{bmatrix} u_{21} \\ u_{22} \end{bmatrix} = 0$$

$$u_{21} + 2u_{22} = 0$$

$$2u_{21} + 4u_{22} = 0$$

$$u_{21} = -2u_{22}$$

$$\sqrt{4u_{22}^2 + u_{22}^2} = 1$$

$$\begin{cases} u_{21} = -\frac{2}{\sqrt{5}} \\ u_{22} = \frac{1}{\sqrt{5}} \end{cases}$$

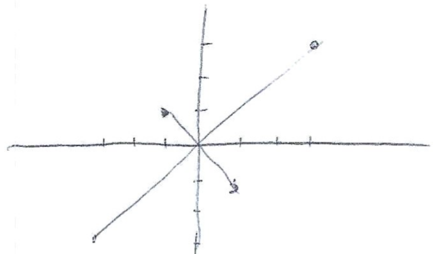
$$U = \begin{bmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix}$$

$$U^{-1} = \begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix}$$

$$D = U^{-1} A U = \begin{bmatrix} 7 & 0 \\ 0 & 2 \end{bmatrix}$$

Exercise 4.4

$$Y = \begin{bmatrix} 3 & 3 \\ 0 & 0 \\ -3 & -3 \\ -1 & 1 \\ 1 & -1 \end{bmatrix}$$



① mean: $\frac{3+0-3-1+1}{5} = 0$ $\qquad \frac{3+0-3+1-1}{5} = 0$

② std: $\sqrt{\frac{3^2+(-3)^2+(-1)^2+1^2}{5-1}}$ $\qquad \sqrt{\frac{3^2+(-3)^2+1^2+(-1)^2}{5-1}}$

$= \sqrt{\frac{20}{4}}$ $\qquad = \sqrt{\frac{20}{4}}$

$= \sqrt{5}$ $\qquad = \sqrt{5}$

③ $\mathrm{cov}(Y_1, Y_2) = \frac{3\times3+(-3)(-3)+(-1)\times1+1\times(-1)}{5-1} = \frac{16}{4} = 4$

④ $\Sigma = \mathrm{cov}(Y) = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$

Exercise 4.4

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$$\Sigma-\lambda I=\begin{bmatrix}5-\lambda & 4\\ 4 & 5-\lambda\end{bmatrix}$$

$$(5-\lambda)(5-\lambda)-16=25-10\lambda+\lambda^2-16=0$$

$$\lambda^2-10\lambda+9=0$$

$$(\lambda-9)(\lambda-1)=0$$

$$\lambda_1=9 \qquad \lambda_2=1$$

For $\lambda_1=9$

$$M=\begin{bmatrix}-4 & 4\\ 4 & -4\end{bmatrix}$$

$$MU_1=\begin{bmatrix}-4 & 4\\ 4 & -4\end{bmatrix}\begin{bmatrix}u_{11}\\ u_{12}\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{cases}-4u_{11}+4u_{12}=0\\ 4u_{11}-4u_{12}=0\end{cases} \Rightarrow u_{11}=u_{12}$$

$$\sqrt{u_{11}^2+u_{11}^2}=1$$

$$u_1=\begin{cases}u_{11}=\frac{1}{\sqrt2}\\ u_{12}=\frac{1}{\sqrt2}\end{cases}$$

For $\lambda_2=1$

$$M=\begin{bmatrix}4 & 4\\ 4 & 4\end{bmatrix}$$

$$Mu_2=\begin{bmatrix}4 & 4\\ 4 & 4\end{bmatrix}\begin{bmatrix}u_{21}\\ u_{22}\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$4u_{21}+4u_{22}=0$$

$$\Rightarrow u_{21}=-u_{22}$$

$$\sqrt{u_{21}^2+(-u_{21})^2}=1$$

$$u_2\begin{cases}u_{21}=\frac{1}{\sqrt2}\\ u_{22}=-\frac{1}{\sqrt2}\end{cases}$$

6 $\frac{9}{10}\times100\%=90\%$ for the first PC.

$\frac{1}{10}\times100\%=10\%$ for the second PC.

7 Projection_PC1 $=[3,3]\times\begin{bmatrix}\frac{1}{\sqrt2}\\ \frac{1}{\sqrt2}\end{bmatrix}=\frac{6}{\sqrt2}=3\sqrt2$

Projection_PC2 $=[3,3]\times\begin{bmatrix}\frac{1}{\sqrt2}\\ -\frac{1}{\sqrt2}\end{bmatrix}=0$

Exercise 4.5 (2)

$$Y = \begin{bmatrix} 3 & 3 \\ 0 & 0 \\ -3 & -3 \\ -1 & 1 \\ 1 & -1 \end{bmatrix}_{5\times 2}$$

① $$Y^T = \begin{bmatrix} 3 & 0 & -3 & -1 & 1 \\ 3 & 0 & -3 & 1 & -1 \end{bmatrix}_{2\times 5}$$

$$Y^T Y = \begin{bmatrix} 20 & 16 \\ 16 & 20 \end{bmatrix}$$

$$\left| \begin{bmatrix} 20-\lambda & 16 \\ 16 & 20-\lambda \end{bmatrix} \right| = (20-\lambda)(20-\lambda) - 256 = 0$$

$$\lambda^2 - 40\lambda + 144 = 0$$

$$(\lambda - 4)(\lambda - 36) = 0$$

$$\lambda_1 = 4 \quad \lambda_2 = 36 \quad s_1 = \sqrt{36} = 6$$

$$s_2 = \sqrt{4} = 2$$

$$S = \begin{bmatrix} 6 & 0 \\ 0 & 2 \end{bmatrix}$$

For $\lambda_1 = 4$

$$M = \begin{bmatrix} 16 & 16 \\ 16 & 16 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$

For $\lambda_2 = 36$

$$M = \begin{bmatrix} -16 & 16 \\ 16 & -16 \end{bmatrix}$$

$$V_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \qquad V = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$U_1 = \frac{1}{s_1} Y V_1 = \frac{1}{6} \begin{bmatrix} 3 & 3 \\ 0 & 0 \\ -3 & -3 \\ -1 & 1 \\ 1 & -1 \end{bmatrix}_{5\times 2} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}_{2\times 1} = \frac{1}{6} \begin{bmatrix} \frac{6}{\sqrt{2}} \\ 0 \\ -\frac{6}{\sqrt{2}} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \\ 0 \\ 0 \end{bmatrix}$$

$$U_2 = \frac{1}{s_2} Y V_2 = \frac{1}{2} \begin{bmatrix} 3 & 3 \\ 0 & 0 \\ -3 & -3 \\ -1 & 1 \\ 1 & -1 \end{bmatrix}_{5\times 2} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

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$$Y=\begin{bmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} 6 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}^T$$

Exercise 4.6

For $k=20$ compression ratio $= \dfrac{668\times 640}{20(668+1+640)} = 16.3$

For $k=5$

compression ratio $= \dfrac{668\times 640}{5(668+1+640)} = 65.32$

Exercise 4.7

$$\lambda_1 = \frac{6^2}{5-1} = 9$$

$$\lambda_2 = \frac{2^2}{5-1} = 1$$

These are the same as those obtained in Exercise 4.4