Solutions

Problems of Chapter 1

1.1 d.

1.2

(1) Unstructured.
(2) Unstructured.
(3) Unstructured.
(4) Structured.

1.3

(1) Qualitative.
(2) Qualitative.
(3) Quantitative.
(4) Qualitative.

1.4

(1) Nominal.
(2) Nominal.
(3) Ordinal.
(4) Nominal.
(5) Ordinal.

1.5

(1) Ordinal
(2) Nominal.
(3) Interval.
(4) Ratio.
(5) Interval.
(6) Ratio.

Problems of Chapter 2

2.1

(1) True.
(2) True.
(3) False.
(4) False.
(5) True.
(6) False.
(7) True.

2.2

(1) 6.
(2) 0.
(3) 1.

2.3

(1) $\{\{\}, \{-1\}, \{1\}, \{-1, 1\}\}$. Cardinality is $2^2 = 4$.
(2) $\{\{\}, \{0\}, \{1\}, \{2\}, \{3\}, \{0, 1\}, \{0, 2\}, \{0, 3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{0, 1, 2\}, \{0, 1, 3\}, \{0, 2, 3\}, \{1, 2, 3\}, \{0, 1, 2, 3\}\}$. Cardinality is $2^4 = 16$.
(3) $2^8 = 256$.

2.4

(1) $A \cup B = \{b, c, e, f, g, h, i, k\}$.
(2) $C \cap B = \{g, h, i\}$.
(3) $\overline{A \cup B} = \{a, d, j, l, m\}$.
(4) $A \backslash (B \cap C) = \{b, c, e, f\}$.
(5) $A \cup B \cup C = \{b, c, e, f, g, h, i, k, l, m\}$.
(6) $A \cap B \cap C = \{g, h\}$.
(7) $(A \cup B) \backslash C = \{b, c, e, k\}$.
(8) $\overline{(A \cup B) \backslash C} = \{a, d, f, g, h, i, j, l, m\}$.

2.5

(1) $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.
(2) $B = \{6, 7, 8, 9, 10, 11\}$.
(3) $C = \{0, 1, 2, 3, 4\}$.
(4) $D = \{0, 1, 2, 3, 4, 5, 6, 7\}$.
(5) $E = \{4, 5, 6, 7, 8, 9, 10, 11\}$.

2.6

(1) False.
(2) True.
(3) False.
(4) True.

2.7
(1) $\{(2,0), (2,1), (3,0), (3,1), (5,0), (5,1)\}$
(2) ϕ

2.8
(1) True
(2) False
(3) False
(4) True

Exercis 2.9

(1) $f(x) = x^5 - 2x^3 + 3x$

$-f(x) = -x^5 + 2x^3 - 3x$

$f(-x) = -x^5 + 2x^3 - 3x$

$\because f(-x) = -f(x)$

$\therefore f(x)$ is an odd function.

(2) $f(x) = x^3 - 2x + 1$

$-f(x) = -x^3 + 2x - 1$

$f(-x) = -x^3 + 2x + 1$

$\because -f(x) \neq f(x)$

$f(-x) \neq -f(x)$

$\therefore f(x)$ is neither an odd nor an even function.

(3) $f(x) = e^x$

$f(-x) = e^{-x}$

$-f(x) = -e^x$

$\because f(-x) \neq -f(x)$

$f(-x) \neq f(x)$

$\therefore e^x$ is neither odd nor even.

(4) $\because f(-x) = \ln(-x)$ does not exist, since $x > 0$

$\therefore \ln x$ is neither odd nor even.

(5) $f(-x) = \sin(-x) = -\sin x$

$\because f(-x) = -f(x) = -\sin x$

$\therefore \sin x$ is an odd function.

⑥ $f(-x) = \cos(-x) = \cos x$

$\because f(-x) = f(x)$

$\therefore$ $\cos x$ is an even function.

(7) $f(-x) = \frac{1}{1+e^{x}}$

$-f(x) = \frac{-1}{1+e^{-x}}$

$\because f(-x) \neq -f(x)$

$f(-x) \neq f(x)$

$\therefore$ Sigmoid function is neither odd nor even.

(8) $f(x) = \frac{e^{x}+e^{-x}}{2}$

$f(-x) = \frac{e^{-x}+e^{x}}{2}$

$\because f(x) = f(-x)$

$\therefore$ Hyperbolic cosine function is even.

(9) $f(x) = \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$

$\because f(-x) = \frac{e^{-x}-e^{x}}{e^{-x}+e^{x}} = \frac{-(e^{x}-e^{-x})}{e^{-x}+e^{x}} = -f(x)$

$\therefore$ Hyperbolic tangent function is odd.

Exercise 2.10

(1) $y = x^3 + 10$

$x = \sqrt[3]{y-10}$

$f^{-1}(x) = \sqrt[3]{x-10}$

(2) $y = 3\sin x$

$x = \arcsin \frac{y}{3}$

$f^{-1}(x) = \arcsin \frac{x}{3}$

(3) $y = 4 + \ln(x+1)$

$\ln(x+1) = y - 4$

$x = e^{y-4} - 1$

$f^{-1}(x) = e^{x-4} - 1$

(4) $y = \frac{3^x}{3^x+1}$

$3^x y + y = 3^x$

$3^x = \frac{y}{1-y}$

$\log_3 3^x = \log_3 \frac{y}{1-y}$

$x = \log_3 \frac{y}{1-y}$

$f^{-1}(x) = \log_3 \frac{x}{1-x}$

Exercise 2.11

(1) $g \circ f = (5x+2)^2 = 25x^2 + 20x + 4$.

(2) $g \circ f = \sin(2x)$.

(3) $g \circ f = (e^x)^2 = e^{2x}$.

(4) $g \circ f = \ln(e^x) = x$.

(5) $g \circ f = (\cos x)^3 = \cos^3 x$.

Exercise 3.1

(1) $\begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} + \begin{bmatrix} 3 \\ 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 6 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} - \begin{bmatrix} 3 \\ 6 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -5 \\ 4 \end{bmatrix}$.

(2) $2 \times \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 10 \end{bmatrix}$, $-2 \times \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \\ -10 \end{bmatrix}$

Exercise 3.2

(1) $m \cdot n = -2 \times 1 + (-1) \times 3 = -5$

(2) $u \cdot v = -2 \times 1 + (-1) \times 3 + 3 \times (-2) = -11$.

(3) $s \cdot t = -2 \times 3 + 2 \times 3 = 0$

(4) $a \cdot b = 4 \times (-4) + 3 \times (-3) + 5 \times 5 = 0$.

Exercise 3.3

(1) $d(w, z) = \sqrt{(1-5)^2 + (10-4)^2 + (3-(-1))^2 + (2-0)^2} = \sqrt{16+36+16+4} = 6\sqrt{2}$

(2) $d(a, b) = \sqrt{(4-(-4))^2 + (3-(-3))^2 + (5-5)^2} = 10$

Exercise 3.4

(1) $\theta_u = \tan^{-1}\left(-\frac{3}{2}\right) \approx -0.9828$ radians.

Since the vector is in the fourth quadrant, the angle θ in standard position (counterclockwise from the positive x-axis) is

$$\theta = 2\pi + (-0.9828) \approx 5.3004 \text{ radians.}$$

$$\theta_v = \tan^{-1}\left(\frac{4}{5}\right) \approx 0.6747 \text{ radians.}$$

(2) $\|u\| = \sqrt{2^2 + (-3)^2} = \sqrt{13}$, $\|v\| = \sqrt{5^2 + 4^2} = \sqrt{41}$

(3) $d(u, v) = \sqrt{(2-5)^2 + (-3-4)^2} = \sqrt{58}$

Exercise 3.5

(1) $u \cdot v = 2 \times 3 + 2 \times 3 = 12$

$\theta_u = \tan^{-1}\left(\frac{2}{2}\right) = \theta_v = \tan^{-1}\left(\frac{3}{3}\right)$

$\theta = \theta_u - \theta_v = 0$

$\|u\| = \sqrt{2^2 + 2^2} = \sqrt{8}$, $\|v\| = \sqrt{3^2 + 3^2} = \sqrt{18}$

$u \cdot v = \sqrt{8} \times \sqrt{18} \cos 0 = \sqrt{144} = 12$

(2) $u \cdot w = 2 \times (-2) + 2 \times 2 = 0$

$\|u\| = \sqrt{2^2 + 2^2} = \sqrt{8}$, $\|w\| = \sqrt{(-2)^2 + 2^2} = \sqrt{8}$

$\theta_u = \tan^{-1}\left(\frac{2}{2}\right) = \tan^{-1}(1) = \frac{\pi}{4}$

$\theta_w = \tan^{-1}\left(\frac{2}{-2}\right) = \tan^{-1}(-1) = -\frac{\pi}{4}$

Since the vector lies in the second quadrant, we adjust the angle to have $\theta_w = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$\theta = \theta_w - \theta_u = \frac{\pi}{2}$

$u \cdot w = \sqrt{8} \cdot \sqrt{8} \cos \frac{\pi}{2} = 0$

(3) $u \cdot s = 2 \times (-2) + 2 \times (-2) = -8$

$\|u\| = \sqrt{2^2 + 2^2} = \sqrt{8}$, $\|s\| = \sqrt{(-2)^2 + (-2)^2} = \sqrt{8}$

$\theta_u = \tan^{-1}\left(\frac{2}{2}\right) = \frac{\pi}{4}$

$\theta_s = \tan^{-1}\left(\frac{-2}{-2}\right) = \frac{\pi}{4}$

Since the vector lies in the third quadrant, we adjust the angle to have $\theta_s = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$\theta = \frac{5\pi}{4} - \frac{\pi}{4} = \pi$

$u \cdot s = \sqrt{8} \cdot \sqrt{8} \cdot \cos \pi = -8$

Exercise 3.5

(4) $u \cdot t = 2\times 0 + 2\times 5 = 10$

$\|u\| = \sqrt{2^2+2^2} = \sqrt{8}$, $\|t\| = \sqrt{0+5^2} = 5$

$\theta_u = \tan^{-1}\left(\frac{2}{2}\right) = \frac{\pi}{4}$

since $(0,5)$ Lies along the positive y-axis,

$\theta_t = \frac{\pi}{2}$

$\theta = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

$\|u\|\|t\| \cos\frac{\pi}{4} \approx \sqrt{8}\times 5 \times 0.7071 \approx 10.$

Exercis 3.6

(1) $\|w\| = \sqrt{2^2+1^2} = \sqrt{5}$, $\hat{w} = \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$

(2) $\|s\| = \sqrt{3^2+1^2} = \sqrt{10}$, $\hat{s} = \left(\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}\right)$

(3) $\|t\| = \sqrt{3^2+1^2+(-1)^2} = \sqrt{11}$, $\hat{t} = \left(\frac{3}{\sqrt{11}}, \frac{1}{\sqrt{11}}, \frac{-1}{\sqrt{11}}\right)$

(4) $\|v\| = \sqrt{(-1)^2+2^2+4^2+1} = \sqrt{22}$, $\hat{v} = \left(\frac{-1}{\sqrt{22}}, \frac{2}{\sqrt{22}}, \frac{4}{\sqrt{22}}, \frac{1}{\sqrt{22}}\right)$

Exercise 3.7

(1) $\begin{bmatrix} -3 & 10 \\ 9 & 0.6 \\ 1 & -5 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 12 \\ 10 & 0.6 \\ 1 & -4 \end{bmatrix}$

(2) $2\times\begin{bmatrix} -3 & 10 \\ 9 & 0.6 \\ 1 & -5 \end{bmatrix} - 4\begin{bmatrix} -1 & 2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 12 \\ 14 & 1.2 \\ 2 & -14 \end{bmatrix}$

(3) $-3\begin{bmatrix} -3 & 10 \\ 9 & 0.6 \\ 1 & -5 \end{bmatrix} + 2\begin{bmatrix} -1 & 2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -26 \\ -25 & -1.8 \\ -3 & 17 \end{bmatrix}$

Exercise 3.8

(1) $$\begin{bmatrix} 10 & 3 & 2 \\ 1 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 10 & 7 & 0 \\ 16 & 6 & 9 \end{bmatrix} = \begin{bmatrix} 20 & 10 & 2 \\ 17 & 8 & 14 \end{bmatrix}$$

(2) $$3 \times \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 9 & 6 \\ 0 & 3 \\ 15 & -3 \end{bmatrix}$$

(3) $$\begin{bmatrix} 9 & -2 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \times 2 + -2 \times 1 \\ 2 \times 2 + 6 \times 1 \end{bmatrix} = \begin{bmatrix} 16 \\ 10 \end{bmatrix}$$

(4) $$\begin{bmatrix} 10 & 3 & 2 \\ 1 & 2 & 5 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 1 & 8 \\ 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 10 \times 2 + 3 \times 1 + 2 \times 2 & 10 \times 2 + 3 \times 8 + 2 \times 5 \\ 1 \times 2 + 2 \times 1 + 5 \times 2 & 1 \times 2 + 2 \times 8 + 5 \times 5 \end{bmatrix}$$

$$= \begin{bmatrix} 27 & 54 \\ 14 & 43 \end{bmatrix}$$

Exercise 3.9

(1) $tr(A) = 1 + 4 + 3 = 8$

(2) $tr(B) = 10 + 3 + 1 = 14$

Exercise 3.10

(1) $det(N) = 13 \times 2 - 1 \times (-4) = 30$

(2) $det(U) = 1 \times 2 - 1 \times 5 = -3$

(3) $det(V) = 10 \times 2 - 4 \times 5 = 0$

Exercise 3.11

(1) $\det(W) = 3\times2\times6 + 1\times10\times3 + 0.5\times(-4)\times0.2 - 0.5\times3\times2 - 3\times10\times0.2 - 1\times(-4)\times6$

$$\begin{array}{ccccc} 3 & 1 & 0.5 & 3 & 1 \\ -4 & 2 & 10 & -4 & 2 \\ 3 & 0.2 & 6 & 3 & 0.2 \end{array}$$

$= 36 + 30 - 0.4 - 3 - 6 + 24$

$= 80.6$

(2) $\det(X) = 1\times2\times0.1 + 1\times3\times(-4) + 0 - 0 - 1\times3\times10 - 1\times5\times0.1$

$= 0.2 - 12 - 30 - 0.5$

$= -42.3$

$$\begin{array}{ccccc} 1 & 1 & 0 & 1 & 1 \\ 5 & 2 & 3 & 5 & 2 \\ -4 & 10 & 0.1 & -4 & 10 \end{array}$$

Exercise 3.12

(1) $A^{-1} = \dfrac{1}{-4\times6-2\times(-3)}\begin{bmatrix} 6 & 3 \\ -2 & -4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & -\frac{1}{6} \\ \frac{1}{9} & \frac{2}{9} \end{bmatrix}$

(2) $\because \det(B) = 2\times5 - 1\times10 = 0$

$\therefore$ The inverse does not exist.

(3) $C^{-1} = \dfrac{1}{13\times2-1\times(-4)}\begin{bmatrix} 2 & -1 \\ 4 & 13 \end{bmatrix} = \begin{bmatrix} \frac{1}{15} & -\frac{1}{30} \\ \frac{2}{15} & \frac{13}{30} \end{bmatrix}$

(4) $I^{-1} = I$

Exercise 3.13

(1) $A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 0 \\ 10 & -1 & -3 \end{bmatrix}$

(2) $B^T = \begin{bmatrix} -1 & 0 \\ 4 & 5 \\ -13 & 8 \end{bmatrix}$

(3) $C^T = \begin{bmatrix} 10 & -2 & 23 & -1 \end{bmatrix}$.

(4) $D^T = \begin{bmatrix} 1 \\ 0 \\ -0.7 \\ 10 \end{bmatrix}$

Exercise 3.14

$m_1^T \cdot m_2^T = \frac{2}{\sqrt{5}} \times \left(-\frac{1}{\sqrt{5}}\right) + 0 + \left(-\frac{1}{\sqrt{5}}\right) \times \left(-\frac{2}{\sqrt{5}}\right) = 0$

$m_1^T \cdot m_3^T = \frac{2}{\sqrt{5}} \times 0 + 0 \times 1 + \left(-\frac{1}{\sqrt{5}}\right) \times 0 = 0$

$m_2^T \cdot m_3^T = -\frac{1}{\sqrt{5}} \times 0 + 0 \times 1 + -\frac{2}{\sqrt{5}} \times 0 = 0$

Yes, these vectors are orthogonal to each other.

Exercise 3.15

(1) $Q^T = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{-3}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$

$\det(Q) = \frac{1}{\sqrt{10}} \times \frac{1}{\sqrt{10}} - \frac{3}{\sqrt{10}} \times \left(-\frac{3}{\sqrt{10}}\right) = 1$

$Q^{-1} = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$

$\because Q^T = Q^{-1}$

$\therefore Q$ is an orthogonal matrix.

Exercise 3.15

(2) $Q^T = \begin{bmatrix} \frac{4}{\sqrt{5}} & -\frac{3}{\sqrt{5}} \\ \frac{3}{\sqrt{5}} & \frac{4}{\sqrt{5}} \end{bmatrix}$

$$Q^{-1} = \frac{1}{\frac{4}{\sqrt{5}} \times \frac{4}{\sqrt{5}} - \left(\frac{-3}{\sqrt{5}}\right) \times \frac{3}{\sqrt{5}}} \begin{bmatrix} \frac{4}{\sqrt{5}} & -\frac{3}{\sqrt{5}} \\ \frac{3}{\sqrt{5}} & \frac{4}{\sqrt{5}} \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} \frac{4}{\sqrt{5}} & -\frac{3}{\sqrt{5}} \\ \frac{3}{\sqrt{5}} & \frac{4}{\sqrt{5}} \end{bmatrix}$$

$\because Q^T \neq Q^{-1}$

$\therefore$ Q is not an orthogonal matrix.

Exercise 3.16

(1) Yes

(2) Yes.

(3) Yes.

Exercise 3.17

(1) $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$ are not linearly dependent because no scalar k satisfies $\begin{bmatrix} 2 \\ 0 \end{bmatrix} = k\begin{bmatrix} 4 \\ 1 \end{bmatrix}$.

Exercise 3.17

(2) $$a\begin{bmatrix}1\\0\\2\end{bmatrix}+b\begin{bmatrix}2\\1\\0\end{bmatrix}+c\begin{bmatrix}-1\\1\\-2\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\Rightarrow \begin{cases} a+2b-c=0 & ① \\ b+c=0 \Rightarrow b=-c & ② \\ 2a-2c=0 \Rightarrow a=c & ③ \end{cases}$$

substituting ② and ③ into ①, we have

$$c-2c-c=0 \Rightarrow c=0$$

Therefore, $a=b=c=0$

$\begin{bmatrix}1\\0\\2\end{bmatrix}$, $\begin{bmatrix}2\\1\\0\end{bmatrix}$ and $\begin{bmatrix}-1\\1\\-2\end{bmatrix}$ are linearly independent.

(3) The vectors $\begin{bmatrix}2\\3\end{bmatrix}$ and $\begin{bmatrix}5\\7.5\end{bmatrix}$ are linearly dependent because we have $\begin{bmatrix}2\\3\end{bmatrix}\times 2.5=\begin{bmatrix}5\\7.5\end{bmatrix}$. That is we can find a scalar k satisfying the relationship

$$k\begin{bmatrix}2\\3\end{bmatrix}=\begin{bmatrix}5\\7.5\end{bmatrix}.$$

Exercise 3.18

① $\det(A) = 2\times2\times5 + 1\times1\times5 + 4\times3\times3 - 4\times2\times5 - 2\times1\times3 - 1\times3\times5$

$= 20 + 5 + 36 - 40 - 6 - 15 = 0$

$$\begin{array}{ccccc} 2 & 1 & 4 & 2 & 1 \\ 3 & 2 & 1 & 3 & 2 \\ 5 & 3 & 5 & 5 & 3 \end{array}$$

(2) since $\det(A) = 0$

$\therefore$ The columns of A are not linearly independent.

(3) Since $\det(A) = 0$

$\therefore$ The inverse of A does not exist.

(4) We observe that the third row is fully determined by the sum of the first two rows.

Thus, there are only two linearly independent rows.

So, the rank of the matrix is 2.