Lecture 1: Introducing Probability

COMSM0014 Mathematics for Computer Science A

cs-uob.github.io/COMS10014/ and github.com/coms10011/2020_21

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What is probability

Probability is a set of mathematical definitions and theorems that are useful for reasoning about randomness and uncertainty

Working out probabilities

A fair coin has a one in two chance of turning up harps, if each spin is independent it has a probability of

$$P(\text{six heads in a row}) = \left(\frac{1}{2}\right)^6 = 0.015625$$

Frequentist statistics

A fair coin has a 1.5% chance of coming up harps six times in a row, so if you spun a coin six times and it came up harps each time you can be 98.5% certain the coin is not fair, maybe.

Bayesian statistics

Aoife tells Brendan she has spun a coin six times and taken down the sequence. As far as Brendan is concerned all 64 outcomes are equally likely. Now, Aoife tells Brendan the coin turned out the same each time, so Brendan updates his belief about the sequence; he now believes there are two possibilities, all heads and all harps, each equally likely.

What is probability

Probability is a set of mathematical definitions and theorems that are useful for reasoning about randomness and uncertainty

Some definitions - sample space

A **sample space** is a set of possible outcomes.

Sample space

To model the experiment where a coin is tossed once we'd use

$$S = \{H, T\}$$

where H and T correspond to heads and harps.

Sample space

To model the experiment where a coin is tossed twice in a row we'd use

$$S = \{HH, HT, TH, TT\}$$

and where two coins are tossed and we don't distinguish one from the other

$$S = \{HH, HT, TT\}$$

where HT stands for heads and harps in any order.

Infinite sample spaces

If you wanted to model the number of coin tosses before you get a head you'd use

$$S = \mathbb{N}$$

and to model the height of trees

$$S = [0, \infty)$$

Some definitions - event

An **event** in a sample space X is a subset $E \subset X$.

Event

If we have sample space

$$S = \{1, 2, 3, 4, 5, 6\}$$

corresponding to the face values of a dice[1], then the set of even sides

$$E = \{2, 4, 6\}$$

is an example event. This might be useful for working out the probability of an even roll.

[1] or die - originally *dice* was the plural of *die*

Some definitions - probability

Formally, a **probability** is a map P from events to real numbers such that:

1. $P(A) \geq 0$ for all events.
2. $P(X) = 1$
3. If $A \cap B = \emptyset$ for two events A and B then

$$P(A \cup B) = P(A) + P(B)$$

Some definitions - probability mass function

The **probability mass function** is a map p from outcomes to real numbers such that:

1. $p(x) \geq 0$ for all $x \in X$
2. $\sum_{x \in X} p(x) = 1$

The probability and the probability mass function

$$P(A) = \sum_{x \in A} p(x)$$