

# Lecture 15: Continuous distributions

COMS10014 Mathematics for Computer Science A

`cs-uob.github.io/COMS10014/` and `github.com/coms10011/2020_21`

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# Discrete versus continuous

- ▶ Discrete:  $X = \{1, 2\}$ ,  $X = \{A, B, C, D\}$ ,  $X = \mathbb{N}$
- ▶ Continuous:  $X = [0, 1]$ ,  $X = [0, \infty)$ ,  $X = \mathbb{R}$

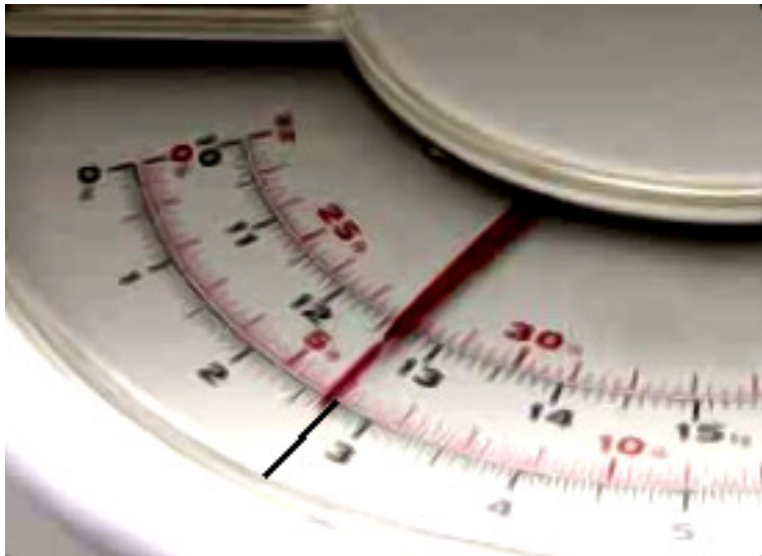
## Weight of a newborn baby

Say  $W$  with  $\langle W \rangle = 2.75\text{kg}$

## Finding an average sized baby



Finding an average sized baby



# Probabilities

We don't mean 'what is the probability of a baby weighing 2.75kg', we mean, what is the probability of a baby in a small range around 2.75kg:

$$P(W \in [2.745, 2.755])$$

# Cumulative and density

We describe the probability distribution with a **distribution function** or **cumulative**:

$$F(x) = P(X < x)$$

or a **density function**:

$$f(x) = \frac{dF}{dx}$$

with

$$F(x) = \int_{-\infty}^x f(y) dy$$

## Cumulative and density

$$F(x) = P(X < x)$$

with

$$F(x) = \int_{-\infty}^x f(y) dy$$

so

$$\lim_{x \rightarrow \infty} F(x) = 1$$

or

$$\int_{-\infty}^{\infty} f(y) dy = 1$$



# Probabilities

$$P(x \in [x_1, x_2]) = P(x \leq x_2) - P(x < x_1)$$

For a continuous variable we don't have to be careful about the distinction between  $x < x_2$  and  $x \leq x_2$ . Hence

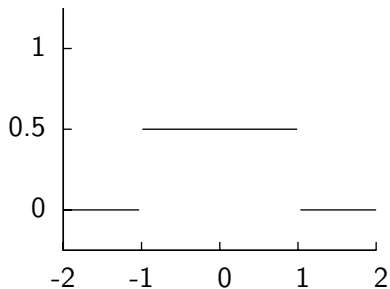
$$P(x \in [x_1, x_2]) = F(x_2) - F(x_1)$$

or

$$P(x \in [x_1, x_2]) = \int_{x_1}^{x_2} f(y) dy$$

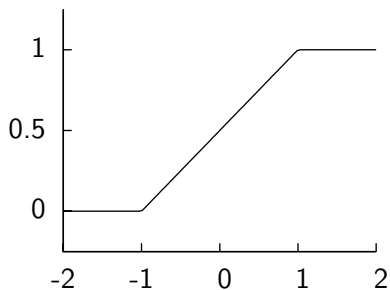
## Constant density

$$f(x) = \begin{cases} \frac{1}{2} & x \in [-1, 1] \\ 0 & \text{otherwise} \end{cases}$$



## Constant density

$$F(x) = \begin{cases} 0 & x < -1 \\ \frac{x+1}{2} & x \in [-1, 1] \\ 1 & x > 1 \end{cases}$$



# Properties

$F(x) \geq F(y)$  if  $x > y$  so it is monotonically increasing function so

$$f(x) = \frac{dF(x)}{dx} \geq 0$$

# Properties

Clearly

$$P(x \in [a, b]) = \int_a^b f(y) dy \leq 1$$

but  $f(x)$  can be greater than one. For example

$$f(x) = \begin{cases} 4 & x \in [0, 1/4] \\ 0 & \text{otherwise} \end{cases}$$

## Example with density greater than one

$$f(x) = \begin{cases} 4 & x \in [0, 1/4] \\ 0 & \text{otherwise} \end{cases}$$

