Lecture 8: Conditional Independence

COMS10014 Mathematics for Computer Science A

cs-uob.github.io/COMS10014/ and github.com/coms10011/2020_21

November 2020

Independent Events

Two events A and B are **independent** iff

$$P(A|B) = P(A)$$

Independent Events

Recall

$$P(A \cap B) = P(A|B)P(B)$$

so A and B are **independent** iff

$$P(A \cap B) = P(A)P(B)$$

Snakes and Ladders or Moksha Patam

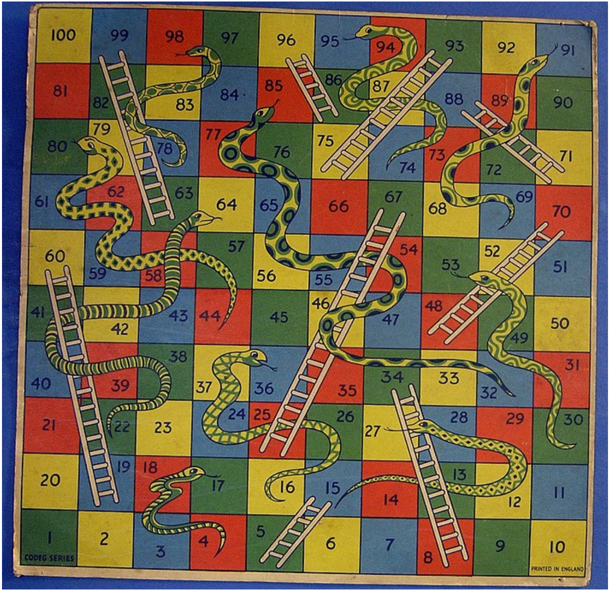
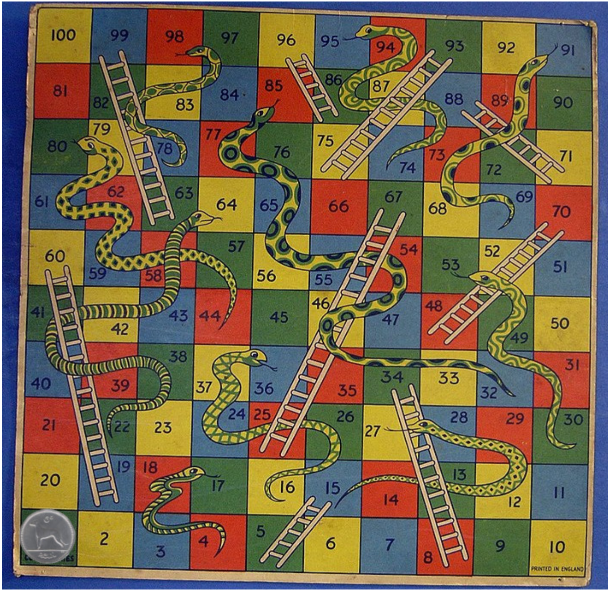


Image from wikipedia.

start at 1



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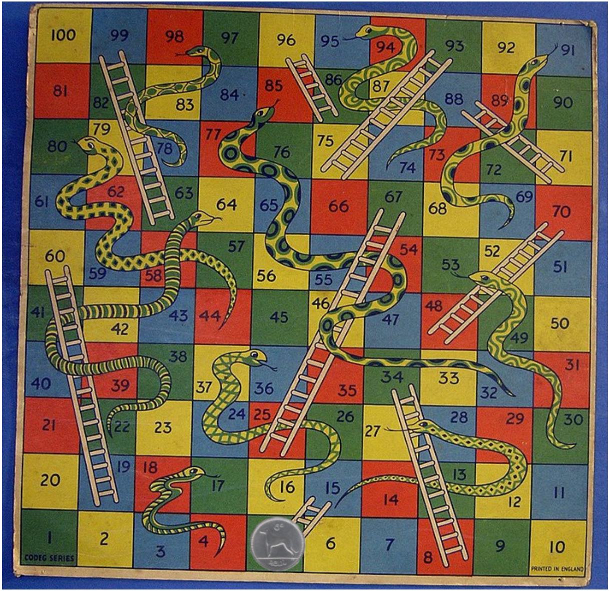
1→5 (roll 4)



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1→15 (up the ladder)

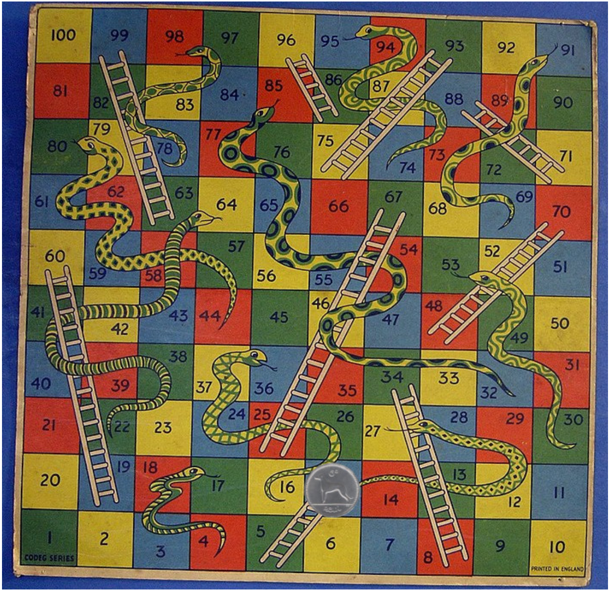

Image from wikipedia.

1→15→19 (roll another 4)

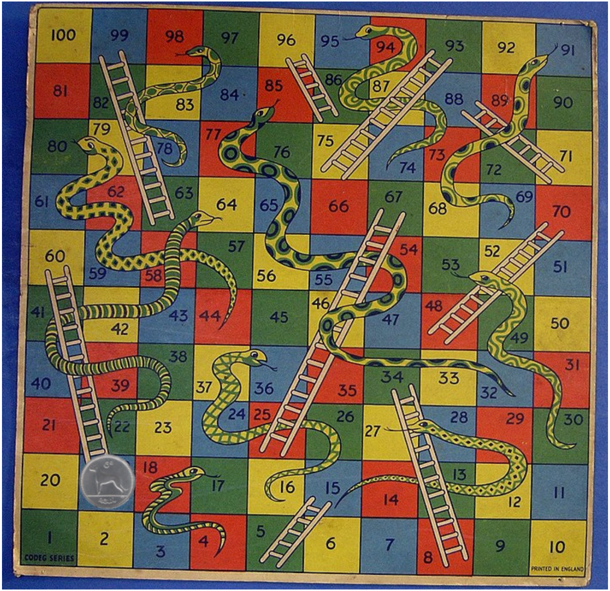

Image from wikipedia.

1→15→19→60 (up the ladder)

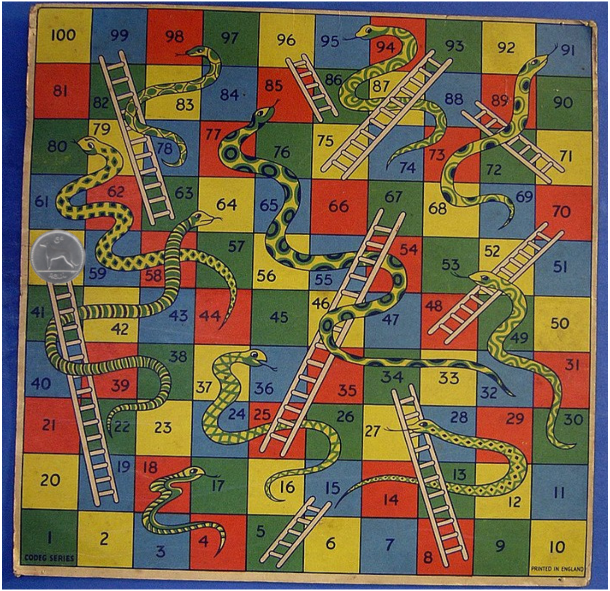

Image from wikipedia.

1→15→19→60→63 (roll a 3)

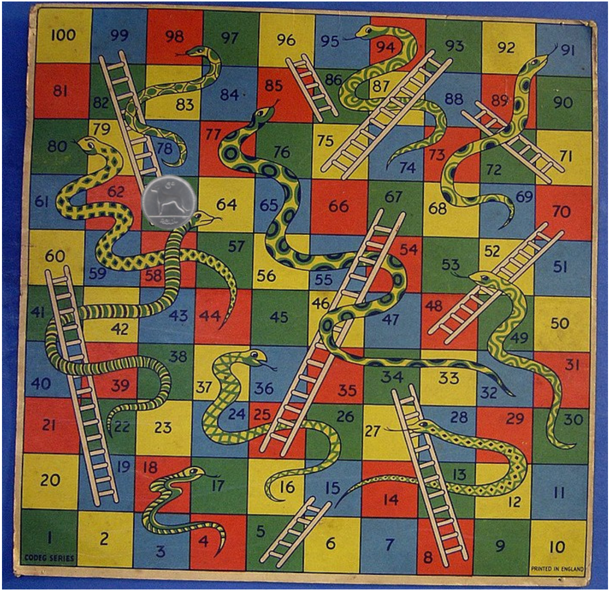

Image from wikipedia.

1→15→19→60→63→99 (up the ladder)

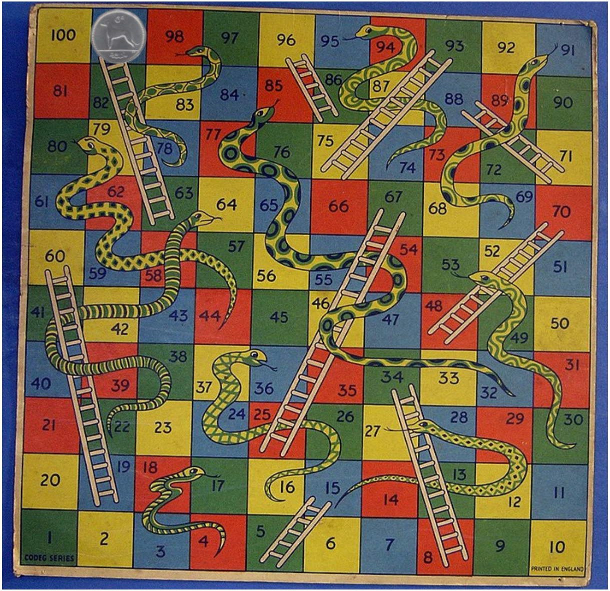

Image from wikipedia.

(4,4,3)

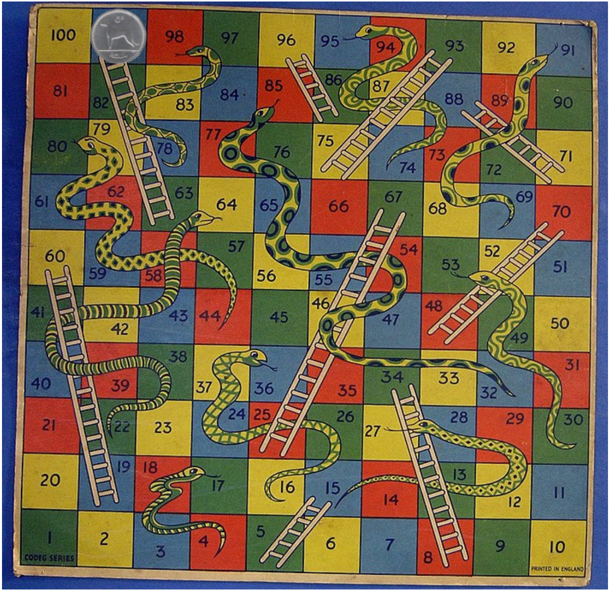

Image from wikipedia.

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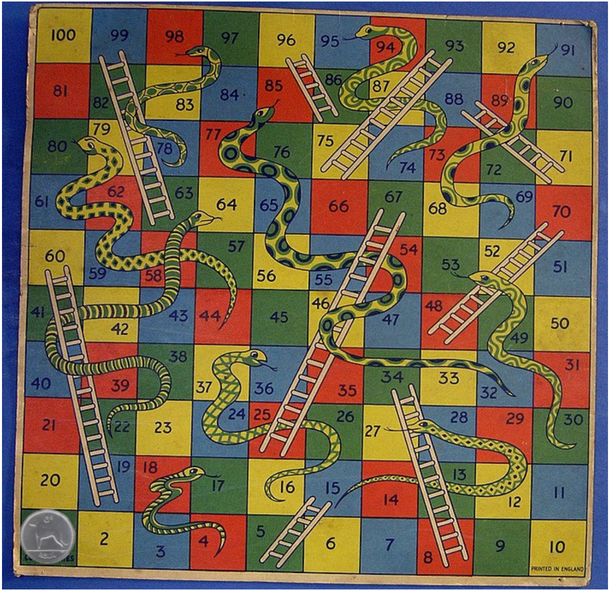


Image from wikipedia.

(8)

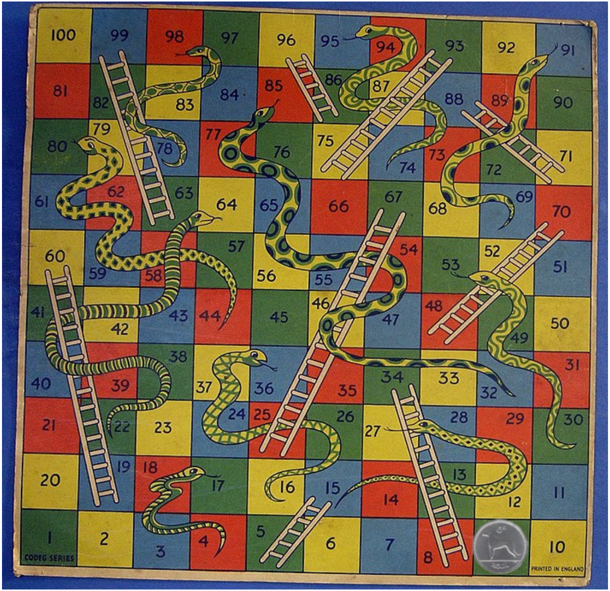


Image from wikipedia.

(8,10)

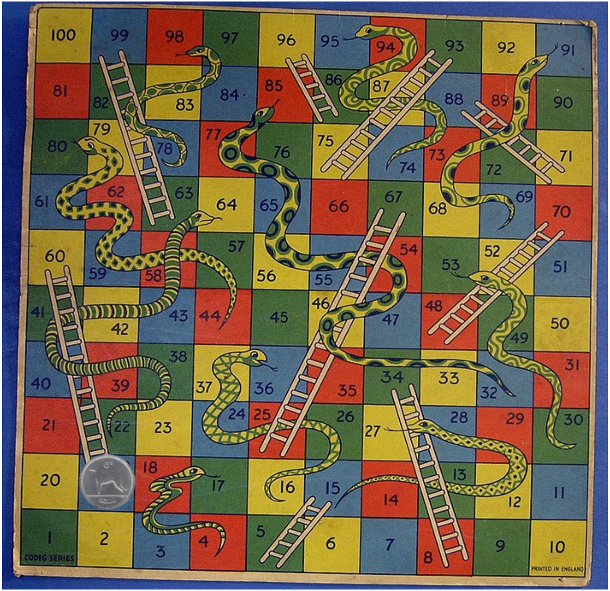


Image from wikipedia.

(8,10)

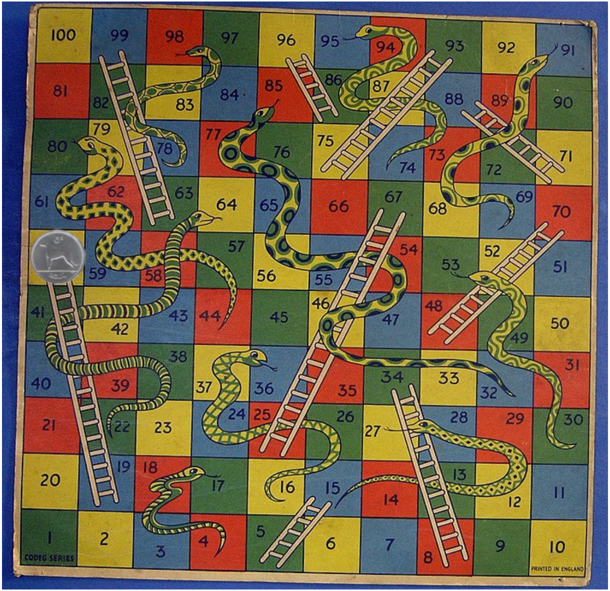


Image from wikipedia.

(8,10,3)

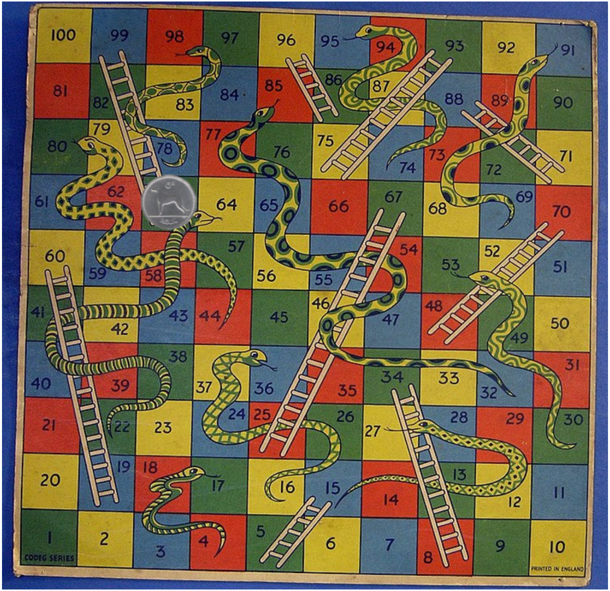


Image from wikipedia.

(8,10,3)

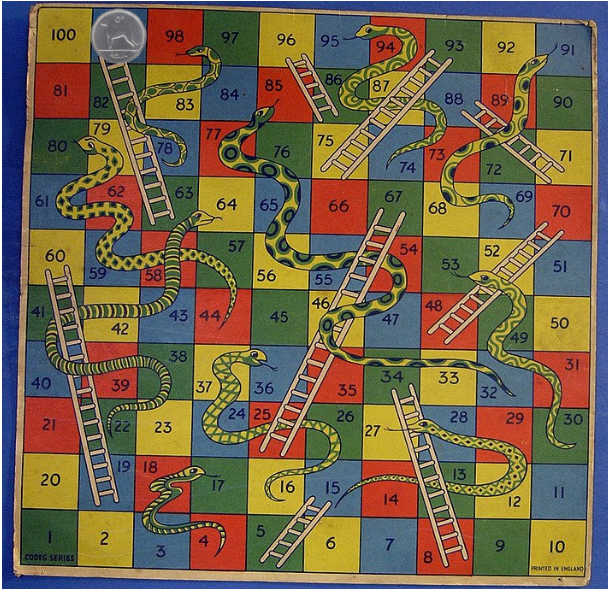


Image from wikipedia.

Probability

- Xn, at place n end of the first move,
- Yn, at place n end of the second move,
- Zn, at place n end of the third move,

after doing all the snakes and ladders stuff.

So $Z99$ is the event of getting to 99 at the end of the third move.

Probability

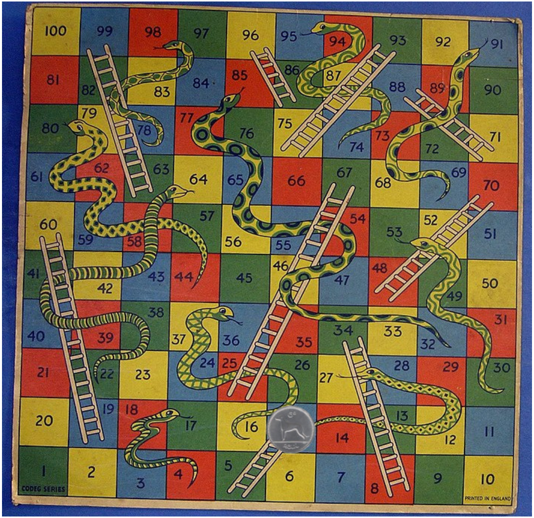
$$P(Z99) = 0.0031$$

You can work this out by adding

$$\begin{aligned} P(Z99) &= P(Z99|Y60 \cap X15)P(Y60|X15)P(X15) \\ &\quad + P(Z99|Y60 \cap X13)P(Y60|X13)P(X13) + \ldots \end{aligned}$$

Conditional probability - rolling a four

$$P(Z99|X15) = 0.0046$$

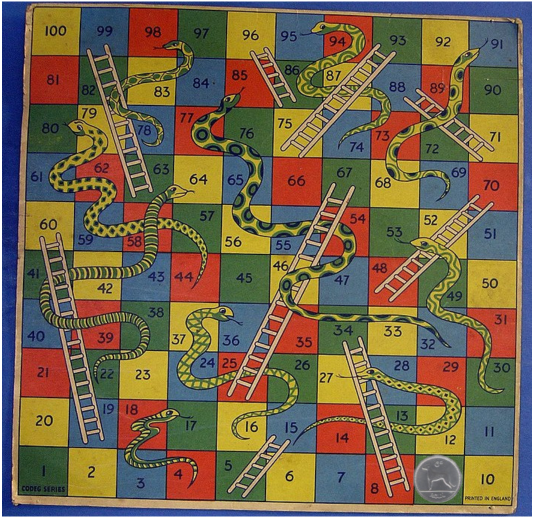


Probability

$$P(Z99) \neq P(Z99|X15)$$

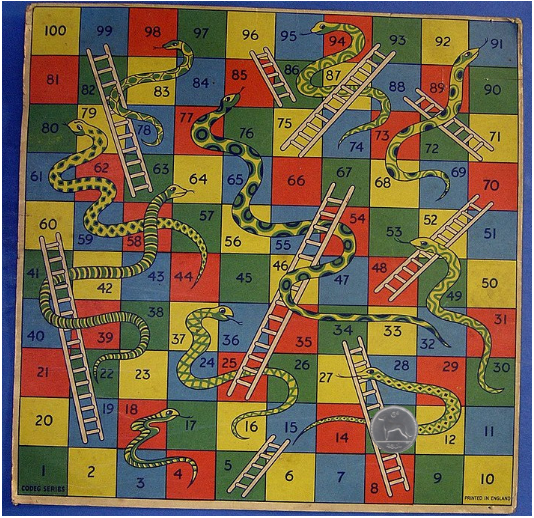
Conditional probability - rolling an eight

$$P(Z99|X9) = 0.0046$$



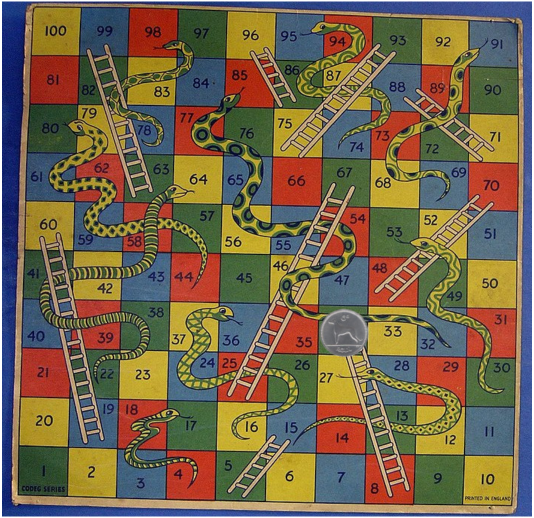
Conditional probability - rolling a 12

$$P(Z99|X13) = 0.0077$$



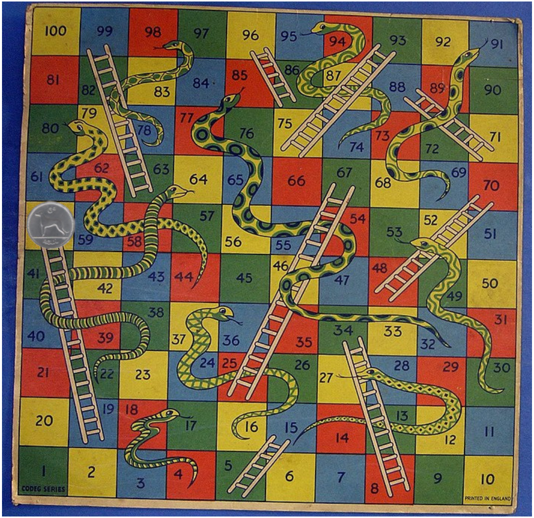
Conditional probability - rolling a seven

$$P(Z99|X34) = 0$$



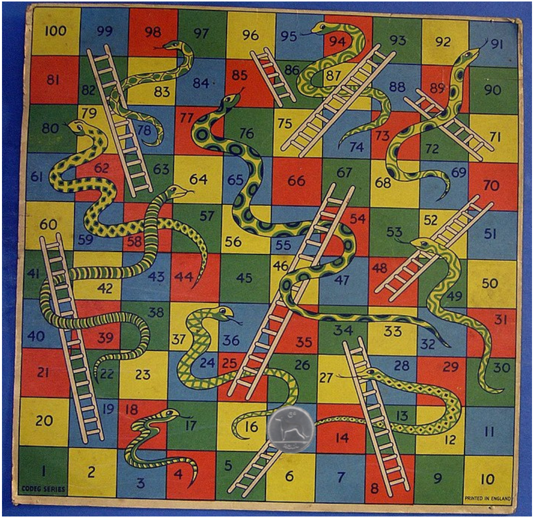
Conditional probability - getting to 60

$$P(Z99|Y60) = 0.0556$$



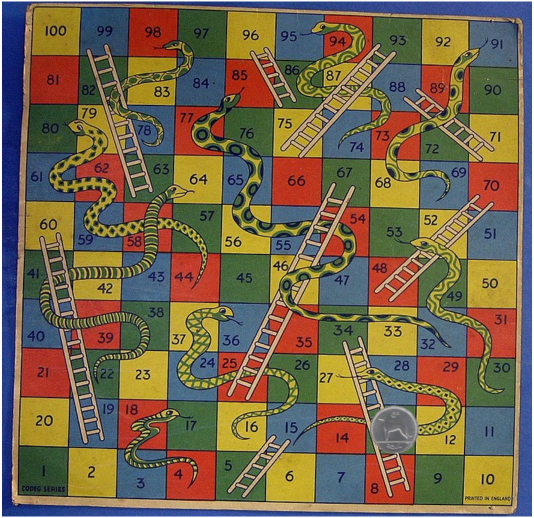
Conditional probability - X no longer matters

$$P(Z99|Y60) = P(Z99|Y60 \cap X15)$$



Conditional probability - X no longer matters

$$P(Z99|Y60) = P(Z99|Y60 \cap X10)$$



Conditional independence

X and Z are conditionally independent:

$$P(Xn_1 \cap Zn_3 | Yn_2) = P(Xn_1 | Yn_2) P(Zn_3 | Yn_2)$$

Conditional independence

A and C are **conditionally independent** given B if

$$P(A \cap C|B) = P(A|B)P(C|B)$$