Lecture 15: Continuous distributions

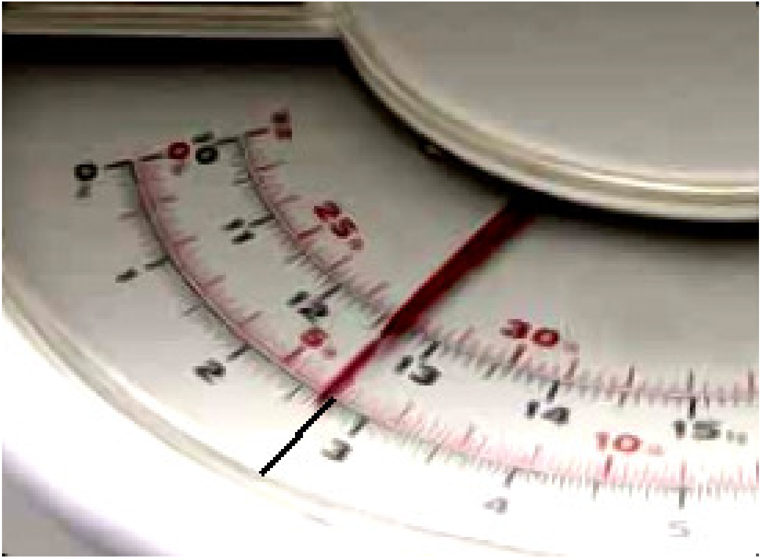
COMS10014 Mathematics for Computer Science A

`cs-uob.github.io/COMS10014/ and github.com/coms10011/2020_21`

December 2020

Finding an average sized baby

Finding an average sized baby



Probabilities

We don't mean 'what is the probability of a baby weighing 2.75kg', we mean, what is the probability of a baby in a small range around 2.75kg:

$$P(W \in [2.745, 2.755])$$

Cumulative and density

We describe the probability distribution with a **distribution function** or **cumulative**:

$$F(x) = P(X < x)$$

or a **density function**:

$$f(x) = \frac{dF}{dx}$$

with

$$F(x) = \int_{-\infty}^{x} f(y)dy$$

Cumulative and density

$$F(x) = P(X < x)$$

with

$$F(x) = \int_{-\infty}^{x} f(y)dy$$

so

$$\lim_{x \to \infty} F(x) = 1$$

or

$$\int_{-\infty}^{\infty} f(y)dy = 1$$

Probabilities

$$P(x \in [x_1, x_2]) = P(x \leq x_2) - P(x < x_1)$$

For a continuous variable we don't have to be careful about the distinction between $x < x_2$ and $x \leq x_2$. Hence

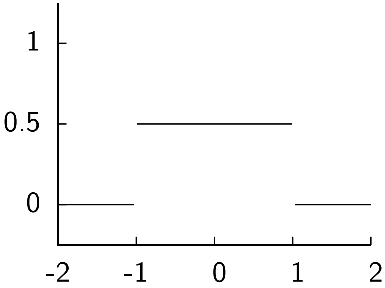
$$P(x \in [x_1, x_2]) = F(x_2) - F(x_1)$$

or

$$P(x \in [x_1, x_2]) = \int_{x_1}^{x_2} f(y)dy$$

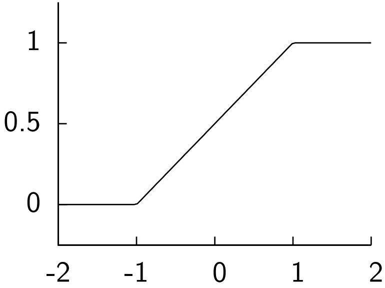
Constant density

$$f(x) = \begin{cases} \frac{1}{2} & x \in [-1, 1] \\ 0 & \text{otherwise} \end{cases}$$



Constant density

$$F(x) = \begin{cases} 0 & x < -1 \\ \frac{x+1}{2} & x \in [-1, 1] \\ 1 & x > 1 \end{cases}$$



Properties

$F(x) \geq F(y)$ if $x > y$ so it is monotonically increasing function so

$$p(x) = \frac{dF(x)}{dx} \geq 0$$

Properties

Clearly

$$P(x \in [a,b]) = \int_a^b f(y)dy \leq 1$$

but $p(x)$ can be greater than one. For example

$$f(x) = \begin{cases} 4 & x \in [0, 1/4] \\ 0 & \text{otherwise} \end{cases}$$

Properties

$$f(x) = \begin{cases} 4 & x \in [0, 1/4] \\ 0 & \text{otherwise} \end{cases}$$