COMS10013 - Analysis - WS1 outline solutions

These are outline solutions to the main questions in worksheet 1, solutions to the other questions will also appear.

1. Differentiate the following functions with respect to x: mostly this will mean using the chain rule.

   a) $3x^2$: just use the polynomial rule to get 6.

   b) $(x+2)^2$: et $u = x+2$ with $du/dx = 1$ to get $2(x+2)$, don't square it out to get x^2+4x+4, differentiating this will of course give the correct answer $2x+4 = 2(x+2)$ but is more work.

   c) ae^{cx} where a and c are constants: use $u = cx$ with $du/dx = c$ to get $ac \exp cx$; rather than do a u substitution, most people just remember this one.

   d) $\exp x^2$: this is a bit harder, $u = x^2$ gives $du/dx = 2x$ and $d \exp u/du = \exp u = \exp x^2$ so the answer is $2x \exp x^2$.

   e) $\sin^2 x + \cos^2 x$: this is a trick question, $\sin^2 x + \cos^2 x = 1$ by Pythagorous's theorem, so the answer is zero. However, to do it the long way, $d \sin^2 x/dx = du^2/du\, du/dx$ where $u = \sin x$ so
   $$\frac{d \sin^2 x}{dx} = 2 \sin x \cos x \tag{1}$$
   The same line of thought gives
   $$\frac{d \cos^2 x}{dx} = -2 \sin x \cos x \tag{2}$$
   and adding them gives zero, as expected.

   f) $\cos^2 x - \sin^2 x$: From our discussion above this is $-4 \sin x \cos x$. Another way to do this would be to remember that $\cos 2x = \cos^2 x - \sin^2 x$.

   g) $\exp 1/x$: usual thing now, $u = 1/x$ so $du/dx = -1/x^2$ using $1/x = x^{-1}$ and the usual rule for powers, hence
   $$\frac{d}{dx} e^{1/x} = -\frac{1}{x^2} e^{1/x} \tag{3}$$

2. Find the local minima and maxima of $y = x^5 - 3x^2 + 6$: so
   $$\frac{dy}{dx} = 5x^4 - 6x = x(5x^3 - 6) \tag{4}$$
   So, note for next year, make these numbers a bit more conventient, but basically there are critical points at $x = 0$ and x at the cube root of $6/5$, the second derivative is
   $$\frac{d^2y}{dx^2} = 20x^3 - 6 \tag{5}$$
   At $x = 0$ this is -6 and so that's a maximum, at $x = \sqrt[3]{6/5}$ this is
   $$\left.\frac{d^2y}{dx^2}\right|_{x=\sqrt[3]{6/5}} = 20 \times \frac{6}{5} - 6 = 18 \tag{6}$$
   so this is a minimum.

3. Find the partial derivatives of $z(x, y) = 5x^2y + 2x \sin y$. This is easier than you'd think, for x derivative think of y as constant and visa versa:

$$\frac{\partial z}{\partial x} = 10xy + 2 \sin y \tag{7}$$

and

$$\frac{\partial z}{\partial y} = 5x^2 + 2x \cos y \tag{8}$$

4. Find the gradient of $z(x, y) = (x + y^2)^2$. So first, using the chain rule

$$z_x = 2(x + y^2) \tag{9}$$

and

$$z_y = 4y(x + y^2) \tag{10}$$

Putting them together gives

$$\nabla(z) = [2(x + y^2), 4y(x + y^2)] \tag{11}$$