COMS10013 - Analysis - WS1

These worksheet are partly, well mostly, taken from worksheets prepared by Chloe Martindale.

Useful facts

- polynomials: $dx^n/dx = nx^{n-1}$

- special function: $d\sin x/dx = \cos x$, $d\cos x/dx = -\sin x$, $d\exp x/dx = \exp x$, $d\log x/dx = 1/x$.

- product rule:
$$\frac{d}{dx}uv = \frac{du}{dx}v + u\frac{dv}{dx}$$

- quotient rule:
$$\frac{d}{dx}\frac{u}{v} = \frac{\frac{du}{dx}v - u\frac{dv}{dx}}{v^2}$$

- chain rule:
$$\frac{d}{dx}u(v(x)) = \frac{du}{dv}\frac{dv}{dx}$$

- reminder regarding exponentials and logs: $\exp\log x = x$ and $\log a^b = b\log a$.

- gradients for $f(x,y)$; $\nabla f = (f_x, f_y)$ where $f_x = \partial f/\partial x$.

- the Hessian
$$H(f) = \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix}$$

- the determinant of a matrix is equal the multiple of its eigenvalues, the trace is the sum.

Questions

These are the questions you should make sure you work on in the workshop.

1. Differentiate the following functions with respect to x;

   a) $3x^2$

   b) $(x+2)^2$

   c) ae^{cx} where a and c are constants.

   d) $\exp x^2$

   e) $\sin^2 x + \cos^2 x$

   f) $\cos^2 x - \sin^2 x$

   g) $\exp 1/x$

2. Find the local minima and maxima of $y = x^5 - 3x^2 + 6$.

3. Find the partial derivatives of $z(x,y) = 5x^2y + 2x\sin y$.

4. Find the gradient of $z(x,y) = (x + y^2)^2$.

Extra questions

These are extra questions you might attempt in the workshop or at a later time.

1. Differentiate x^x with respect to x.

2. The function $z(x,y) = x^2 + y^2 + 2x - 3y$ has a global minimum. Find this by taking the gradient and searching for the point where the gradient is zero.

3. Check that this point you found really is a minimum by computing the Hessian of the function at this point, and checking that it is positive definite, that is, all eigenvalues are positive.