COMS10013 - Analysis - WS1 - moreSolutions

Extra questions

These are extra questions you might attempt in the workshop or at a later time.

1. Differentiate x^x with respect to x. **Solution**: so this is hard unless you know or spot the trick, which is to know that $\log a^b = b \log a$, and that $\exp \log a = a$. In this case we do

$$x^x = \exp \log x^x = \exp (x \log x) \tag{1}$$

and now we have something we can differentiate, in this case using the chain rule and

$$\frac{d}{dx} x \log x = \log x + 1 \tag{2}$$

This means

$$\frac{dx^x}{dx} = (1 + \log x) x^x \tag{3}$$

2. The function $z(x, y) = x^2 + y^2 + 2x - 3y$ has a global minimum. Find this by taking the gradient and searching for the point where the gradient is zero. **Solution**: well the gradient is

$$\text{grad}(z) = (2x + 2, 2y - 3) \tag{4}$$

so that's equal to $(0, 0)$ when $x = -1$ and $y = 3/2$.

3. Check that this point you found really is a minimum by computing the Hessian of the function at this point, and checking that it is positive definite, that is, all eigenvalues are positive. **Solution**: so the Hessian is

$$H = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \tag{5}$$

so, trivially, the eigenvalues are both two and this is a positive definite Hessian.