COMS10013 - exam questions - 25 points

All sections are worth five points; outline solutions are just outlines, you should give more workings!

Part 1

This question is about the definition and properties of differenciation. In one sentence describe in words what df/dt means [1 points]. Give the formal definition of the limit [1 point] and of df/dt [1 point] and use this to argue [2 points] that

$$\frac{d(f+g)}{dt} = \frac{df}{dt} + \frac{dg}{dt}$$

Part 2

This question is about actually taking derivatives. Using the quotient rule work out [1 point]

$$\frac{d\tan x}{dx}$$

Using the chain rule work out [2 points]

$$\frac{d\tan\sin x}{dx}$$

Given the Taylor expansion of the exponential

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

show that [2 points]

$$\frac{de^x}{dx} = e^x$$

Part 3

This question is about partial derivatives. What is the gradient [1 point] of

$$z = xy\sin xy$$

Find the Hessian [2 point], show $(0,0)$ is an extremum [1 point] and indicate whether it is a maximum, minimum or saddle point [1 point].

Part 4

This question is about complex numbers. What are the solutions of [1 point]

$$z^2 + 2z + 2 = 0$$

What are the solutions of [1 point]

$$z^2 + 2z + 5 = 0$$

In each case what is the relationship between the two solutions as complex numbers? [2 points] How many solutions do you expect for [1 point]

$$z^6 - 1 = 0$$

What are these solutions? [1 point]

Part 5

This question is about differential equations. Solve the following [a-c 1 point each, d 2 points] where the dot means the derivative with respect to time.

(a) $\dot{y}(t) - y(t) = 0$ with initial condition $y(0) = 1$.

(b) $\dot{y}(t) + 3y(t) = 0$ with initial condition $y(3) = 3$.

(c) $\dot{y}y(t) = 0$ with initial condition $y(5) = 2$.

(d) $\dot{y} = ry(1 - y)$ with $y(0) = y_0$.