Worksheet

This is a computational worksheet on the integrate and fire neuron; it is not for marking or exam but if you work on it it will help you understand integrate and fire neurons. I am happy to look at attempts or to talk you through the challenges, just send me an email `conor.houghton@bristol.ac.uk`.

Solving differential equations with Euler's method

1. Solve numerically in Python using Euler's method the differential equation

$$\frac{df}{dt} = f^2 - 3f + e^{-t} \tag{1}$$

on the interval $[0, 3]$ with time step $\delta t = 0.01$ and graph the solution, taking care to label the axes. Although Python has good libraries for solving differential equations numerically it would be useful educationally not to use them for these question.

2. For the problem above try solve with $\delta t = 0.01$, 0.1, 0.5 and one. Plot all the curves on one graph. What is a good value of δt for this equation.

Integrate and fire neurons

1. Simulate an integrate and fire model with the following parameters for 1 s: $\tau_m = 10$ms, $E_L = V_r = -70$ mV, $V_t = -40$ mV, $R_m = 10$ MΩ, $I_e = 3.1$ nA. Use Euler's method with timestep $\delta t = 1$ ms. Here E_L is the leak potential, V_r is the reset voltage, V_t is the threshold, R_m is the resistance and τ_m is the membrane time constant. Plot the voltage as a function of time. For simplicity assume that the neuron does not have a refractory period after producing a spike. [20% of marks]. You do not need to plot spikes - once membrane potential exceeds threshold, simply set the membrane potential to V_r.

2. Compute analytically the minimum current I_e required for the neuron with the above parameters to produce an action potential.

3. Simulate the neuron for 1 s for the input current with amplitude I_e which is 0.1 [nA] lower than the minimum current computed above, and plot the voltage as a functions of time.

4. Simulate the neuron for 1s for currents ranging from 2 [nA] to 5 [nA] in steps of 0.1 [nA]. For each amplitude of current count the number of spikes produced, that is the firing rate. Plot the firing rate as the function of the input current. It is possible to calculate this curve analytically; you might find it interesting to try.

5. Simulate two neurons which have synaptic connections between each other, that is the first neuron projects to the second, and the second neuron projects to the first. Both model neurons should have the same parameters: $\tau_m = 20$ ms, $E_L = -70$ mV $V_r = -80$ mV $V_t = -54$ mV $R_m I_e = 18$ mV and their synapses should also have the same parameters: $R_m \bar{g}_s = 0.15$, $P = 0.5$, $\tau_s = 10$ ms. For simplicity take the synaptic conductance to satisfy

$$\tau_s \frac{dg_s}{dt} = -g_s \tag{2}$$

with a spike arriving causing g_s to increase by $\bar{g}_s P$. Simulate two cases: a) assuming that the synapses are excitatory with $E_s = 0$ mV, and b) assuming that the synapses are inhibitory with $E_s = -80$ mV. For each simulation set the initial membrane potentials of the neurons V to different values chosen randomly from between V_r and V_t and simulate one second of activity.

6. In many real neurons the firing rate falls off after the first few spikes. This can be simulated with a slow potassium current. For the neuron described in the first question add a slow potassium current. This current should have reversal potential $E_K = -80$ mV, its conductance should increase by 0.005 $(\mathrm{M}\Omega)^{-1}$ every time there is a spike, otherwise it should decay towards zero with time constant $\tau = 200$ ms. Plot the voltage of this neuron for one second.