**UNIVERSITY OF BRISTOL**

**August / September 2019 Examination Period**

**FACULTY OF ENGINEERING**

**Third Year / M Level Examination for the Degree of**
**Bachelor of Science / Master of Engineering / Masters of Science**

**COMS 30127-R / COMSM 2127-R**
**Computational Neuroscience**

**TIME ALLOWED:**
**2 hours**

# Answers to COMS 30127-R / COMSM 2127-R: Computational Neuroscience

**Intended Learning Outcomes:**

## Section A: short questions - answer all questions

**Q1**. This is a picture of Tan's brain showing a lesion to Broca's area. What were the consequences of this lesion to Tan.

**Solution:** Tan was unable to say anything apart from the word 'Tan'; he was able to understand language and express emotion through intonation.

*A very old looking photograph of a brain, it has a rough hole about where your temple is.*

**Q2**. What are the advantages and disadvantages of Electroencephalography (EEG) as a tool to study neuroscience.

**Solution:** EEG is useful because it allows fine temporal resolution measurements on humans using an non-invasive technique; however the spatial resolution is very poor and the recordings are very noisy.

**Q3**. The Euler approximation used to solve differential equations has errors of order $O(\delta t^2)$, where $\delta t$ is the time step; why is that?

**Solution:** The Euler approximation is derived from the Taylor expansion:

$$f(t+\delta) = f(t) + \delta t \frac{df}{dt} + O(\delta t^2) \tag{1}$$

but does not include the $O(\delta t^2)$ terms.

**Q4**. The differential equation

$$\tau \frac{df}{dt} = c - f$$

with $c$ a constant is in equilibrium at $f = c$; without solving the differential equation describe how we know the equilibrium is stable.

**Solution:** If $f > c$ the derivative is negative and so $f$ falls towards $c$, if $f < c$ the derivative is positive and $f$ grows towards $c$.

**Q5**. Some models of single neurons are simpler than others (for example McCulloch-Pitts versus Hodgkin-Huxley). Name two benefits that simple neuron models have compared with more detailed neuron models.

**Solution:** Any two of: Fewer parameters to constrain to data; Faster to simulate on a computer; Easier to analyse mathematically; More generically applicable to various parts of the brain. [1 mark for each correct answer, up to two].

**Q6**. Which subregion of the hippocampus is thought to perform pattern completion? And which other subregion of the hippocampus is thought to be crucial for pattern separation?

**Solution:** Pattern completion is CA3, pattern separation is Dentate Gyrus (DG). [1 mark each correct answer]

**Q7**. Give the expression for the energy of a Hopfield network as a function of its activity state and synaptic weights. How does the energy relate to pattern completion?

**Solution:** The energy is given by $E = -\frac{1}{2}\sum_{ij} w_{ij}x_i x_j$. The dynamics evolve to minimise the energy, the local minima correspond to completed patterns.

**Q8**. What is an attractor network?

**Solution:** It is a recurrent neural network model where the internal dynamics evolve to eventually settle into some stable pattern. [2 marks]

**Q9**. The receptive field of neurons in primary visual cortex can often be approximated by a Gabor function (see picture). What feature of visual scenes would a neuron with this receptive field be sensitive to?

*A bright oval, running from SW to NE, is flanked by two darker ovals*

**Solution:** Oriented edges [2 marks].

**Q10**. Draw the typical f-I curve for a leaky integrate-and-fire neuron model.

**Solution:** Plot with frequency (f) on y-axis and input current (i) on x-axis. Curve should be zero for small positive values of i, then above some threshold the firing rate begins to rise monotonically, without a discontinuity from zero firing rate. [2 marks, 1 for reasonable attempt.]

**Q11**. What are the two main timescales of synaptic plasticity? Give the respective timescales at which each type of plasticity operates.

**Solution:** Short and long-term synaptic plasticity [1 mark], up to a tens of seconds for the first and from tens of minutes upwards for the second [1 mark]

**Q12**. Draw the basic pair-wise STDP learning curve, with the respective axes.

**Solution:** The answer should have a curve that looks similar to the figure below (Dan, Yang and Poo Neuron 2004).

*This shows the STDP graph, it is desribed in the 2018 summer paper.*

**Q13**. Briefly describe the phenomenon of memory savings and its synaptic theory.

**Solution:** Memory savings is the phenomenon of quick relearning after forgetting of a particular task [1 mark]. The synaptic theory proposes that previous learning experiences (or memories) are encoded in the postsynaptic components of synapses, while forgetting occurs at the presynapse. The postsynapse acts as a memory of previous learning and enables quick relearning [1 mark].

**Q14**. Name an ion which is found in greater concentration outside a neuron than inside.

**Solution:** Sodium, calcium and chlorine are also acceptable.

**Q15**. According to the Hodgkin-Huxley equation the conductance of the potassium gate is proportional to $n^4$. Give the differential equation that describes how $n$ evolves.

**Solution:** This can either be

$$\frac{dn}{dt} = \alpha(1 - n) - \beta n$$

or

$$\tau \frac{dn}{dt} = n_\infty - n$$

# Section B: long questions - answer two questions

**Q1**. This question is about mathematics.

(a) Show that the solution to the differential equation

$$\tau \frac{df}{dt} = g(t) - f$$

is

$$f(t) = \frac{1}{\tau} e^{-t/\tau} \int_0^t e^{s/\tau} g(s) ds + f(0) e^{-t/\tau}$$

*[4 marks]*

(b) Explain how this solution can be described as a filter of the input $g(t)$ and what this tells us about $f(t)$. *[4 marks]*

(c) If

$$g(t) = e^{-t/\tau}$$

what is the solution? *[4 marks]*

(d) Solve the differential equation for $g(t) = c$ a constant. *[4 marks]*

(e) If $c = 1$ and $f(0) = -1$ write an expression for the time until $f = 0$. *[4 marks]*

**Solution:** a) As described in the notes: multiply across by an integrating factor [1 mark]

$$\tau e^{t/\tau} \frac{df}{dt} + e^{t/\tau} f = g(t) e^{t/\tau}$$

rewrite the left hand side as a derivative [2 marks]

$$\tau \frac{d}{dt} \left( e^{t/\tau} f \right) = g(t) e^{t/tau}$$

and then integrate both sides and use the fundamental theorem of calculus to get the solution [1 mark]
b) As described in the notes: the integral can be rewritten using a change of variables $s = u - t$ [2 marks] to give

$$f(t) = \frac{1}{\tau} \int_0^t e^{-u/\tau} g(t-u) du + f(0) e^{-t\tau}$$

so $f(t)$ is a exponentially discounted average of the recent history of $g(t)$ [2 marks].
c) In this case the integral becomes [2 marks]

$$e^{-t/\tau} \int_0^t e^{s/\tau} g(s) ds = e^{-t/\tau} \int_0^t e^{s/tau} e^{-s/tau} ds$$

so the integrant is just one, giving $t$ [1 mark] and so [1 mark]

$$f(t) = \left[\frac{t}{\tau} + f(0)\right] e^{-t/\tau}$$

d) If $g(t) = c$ then the integral becomes [2 marks]

$$\int_0^t e^{s/\tau} ds = \tau \left(e^{t/\tau} - 1\right)$$

and so the solution is [2 marks]

$$f(t) = c + [f(0) - c]e^{-t\tau}$$

e) So we have [2 marks]

$$0 = 1 - 2e^{-t\tau}$$

or [2 marks]

$$t = \tau \ln 2$$

**Q2**. This question is about Perceptrons.

(a) The Perceptron can be considered as a linear classifier. Describe what this means. *[3 marks]*

(b) Draw some example datapoints on a 2D plot in a way that makes it impossible for a Perceptron to separate. *[4 marks]*

(c) Is Perceptron learning supervised or unsupervised? *[2 marks]*

(d) What is the Perceptron learning rule? Give the mathematical rule for updating the weights. *[5 marks]*

(e) Assume a Perceptron initially has thee input weights: $\mathbf{w} = (-1, 1, 2)$. There are also two training input patterns $p_1 = (0, 1, 1)$, $p_2 = (1, 0, 1)$ with associated target labels $d_1 = 1$ and $d_2 = -1$. Perform the weight update rule sequentially for each input pattern, and report the weight vector at each step. Assume the learning rate $\eta = 0.1$. *[6 marks]*

**Solution:** a) Linear classifier means that the decision boundary is a straight line, a linear function of the input values. Values either side of the line are classified as +1 or $-1$ respectively [3 marks, partial marks for an attempt].
b) Plot should have two types of datapoint (corresponding to two classes), and should not be linearly separable. [4 marks]
c) Supervised [2 marks].
d) $w_i(t+1) = w_i(t) + \eta(d - y)x_i$ [5 marks, partial marks for something close.]
e) $\mathbf{w}(t = 1) = \{-1, 2, 1\}$ and $\mathbf{w}(t = 2) = \{-1.2, 2, 0.8\}$. Note first step involves no weight changes. [6 marks, 3 for each part.]

**Q3**. This question is part about Oja's rule and part about sparse coding.

(a) Give Oja's learning rule. *[3 marks]*

(b) Give the expression to which the weight converges for the above rule and explain its meaning. *[5 marks]*

(c) What does sparse coding mean? *[2 marks]*

(d) Name a potential benefit of a sparse coding scheme for the brain. *[3 marks]*

(e) Sparse code features are typically learned in a supervised setting by minimising a loss or error function. Write down an example loss function that could lead to sparse features. *[3 marks]*

(f) Draw an example of a typical 2D basis function learned from a sparse coding rule trained on natural images, as in Olshausen and Field, 1996. *[4 marks]*

**Solution:** a) $\Delta w = \eta(xy - y^2 w)$ [3 marks]
b) By setting $\Delta w = 0$ and given that $y = wx$ we find that $0 = Cw - w^T Cww$, where $C = xx^T$ [3 marks], which corresponds to the covariance matrix of the inputs and allows the network to extract the principal components of its input [3 marks; 5 marks in total].
c) It implies that only a small subset of neurons in a population are active at any one time, each neuron tends to code for a different small feature of the stimulus. [2 marks]
d) Any one of: efficient energy consumption; enhanced discriminability between population activity patterns; regularisation for learning, sensory signals may be inherently sparse [3 marks].
e) One example is

$$E = \sum_i (I_i - \tilde{I}_i(\mathbf{a}))^2 + \lambda \sum_j (a_j)$$

where $I$ is an image, $\tilde{I}$ is the reconstructed version of an image, $i$ indexes across images, $a_j$ is the $j$th co-efficient in the array $\mathbf{a}$, and $\lambda$ controls the sparseness penalty. The key thing is that there are two terms in the loss function, one term corresponding to minimising the reconstruction error, and one term that encourages sparseness. [4 marks, 2 for each term.]
f) Something that looks like a 2D gabor function (an oriented gaussian-sine product, as shown in short q9). [4 marks]

 **END OF PAPER**