

**UNIVERSITY OF BRISTOL**

**May / June 2018 Examination Period**

**FACULTY OF ENGINEERING**

**Third Year / M Level Examination for the Degree of  
Bachelor of Science / Master of Engineering / Masters of Science**

**COMS30127 / COMSM2127  
Computational Neuroscience**

**TIME ALLOWED:  
2 hours**

**Answers to COMS30127 / COMSM2127: Computational  
Neuroscience**

**Intended Learning Outcomes:**

## **Section A: short questions - answer all questions**

**Q1.** Many synapses exhibit short-term plasticity: they temporarily change their efficacy when stimulated repeatedly, on a tens of milliseconds timescale. Some synapses tend to increase their strength while others tend to decrease their strength. What are the names given to these two forms of short-term plasticity, respectively?

**Solution:** Strengthening is facilitation, weakening is depression (1 mark each answer).

**Q2.** What glutamate receptor is crucial for Hebbian forms of long-term synaptic plasticity?

**Solution:** The NMDA receptor.

**Q3.** What is the difference between metabotropic and ionotropic neurotransmitter receptors?

**Solution:** Metabotropic receptors activate intracellular chemical signals (second messengers). Ionotropic receptors pass ionic currents. (1 mark each answer)

**Q4.** Sketch the nullclines for the Fitzhugh-Nagumo model.

**Solution:** This will show a cubic with positive cubic coefficient crossed by a line with positive slope.

**Q5.** What is the typical timescale of opening and closing of ion channels: nanoseconds, microsecond, milliseconds, seconds, minutes, hours or days?

**Solution:** Milliseconds.

**Q6.** What does the term 'rate coding' mean?

**Solution:** The assumption that the brain encodes information in the firing rates of neurons, and not in the specific timings of individual spikes.

**Q7.** Solve the equation

$$2 \frac{dv}{dt} = -v$$

with  $v(0) = 5$ .

**Solution:** Solved by ansatz or integrating factor this give  $v = 5 \exp(-t/2)$ .

**Q8.** The left and right hemispheres of the brain respond to different aspects of the visual field. What part of the visual field does the brain's right hemisphere respond to?

**Solution:** The left side of the visual field (not the left eye).

**Q9.** Classic spike-timing dependent plasticity learning rules such as that shown below, and taken from Dan, Yang, and Mu-ming Poo. "Spike timing-dependent plasticity of neural circuits." Neuron 44 (2004): 23-30, are typically unstable, meaning that they lead to synaptic strengths growing infinitely large. Explain how this can happen.

*this shows the usual spike timing dependent plasticity graph: the horizontal axis is 'pre/post spike interval', running from -200ms to 50ms, the vertical axis is called 'normalized EPSP slope' running from 0 to 250 percent. To the left of zero on the horizontal axis there is a curve that decays exponentially from about 50% to zero, on the right a curve decaying from about 20 % to zero so the right curve decays quicker than the left. Each curve has data points clustered around them.*

**Solution:** Positive feedback. If a presynaptic spike causes the post neuron to fire, then it will increase the synaptic strength. This makes it even more likely that the next presynaptic spike will cause a postsynaptic spike. And so on.

**Q10.** This image, taken from Newman, Jonathan P., et al. "Optogenetic feedback control of neural activity." eLife 4 (2015) shows the auto-correlogram for a cell in rat brain that responds to stimulation of a whisker. Speculate on the frequency the whisker is being stimulated at.

*The auto-correlogram runs from -0.2 seconds to 0.2 seconds, with two peaks each side of zero.*

**Solution:** The peaks show a 10Hz oscillation in the neuron so that probably corresponds to the whisker stimulation.

**Q11.** Name the region of the hippocampus marked as X.

*A drawing of the hippocampus, the X is roughly between CA3 and CA1.*

**Solution:** Dentate gyrus.

**Q12.** In the Hodgkin-Huxley squid axon model, which two membrane conductances from the following list are responsible for the rise and fall of the voltage during the action potential: chloride channels, potassium channels, calcium channels, or sodium channels? Be explicit about which channels drive the rising phase and which channels drive the falling phase.

**Solution:** Sodium for rise and potassium for fall. (1 mark each)

**Q13.** In a Hopfield network with three neurons there are two patterns to store:  $(-1, -1, -1)$  and  $(1, -1, 1)$ ; use the single-step approach to calculate the weights.

**Solution:**  $w_{ij} = \frac{1}{N} \sum_a x_i^a x_j^a$  so,  $w_{12} = 0$ ,  $w_{13} = 1$  and  $w_{23} = 0$ .

**Q14.** Describe briefly how varying the constant current input in the Fitzhugh-Nagumo model

$$\begin{aligned}\frac{dv}{dt} &= v - \frac{1}{3}v^3 - w + I \\ \frac{dw}{dt} &= 0.08(v + 0.7 - 0.8w)\end{aligned}$$

changes the behaviour from resting to spiking.

**Solution:** For low values of the current the  $w$  nullcline crosses the left, stable, branch of the  $v$  nullcline, so there is no activity, as  $I$  increases the cubic is lifted until the  $w$  nullcline crosses the  $v$  nullcline at the unstable middle branch. This causes the cycle.

**Q15.** What is meant by spike rate adaptation?

**Solution:** When a cell fires repeatedly its rate of firing falls.

## Section B: long questions - answer two questions

**Q1.** This question is about McCulloch-Pitts neurons.

- (a) Consider two McCulloch-Pitts neurons with zero thresholds; initially  $x_1 = -1$  and  $x_2 = 1$ .  $w_{12} = w_{21} = 2$  and  $w_{11} = w_{22} = 0$ . The neurons are updated synchronously, what are their values after the update. [5 marks]
- (b) Describe a perceptron; what is its update rule? [5 marks]
- (c) Explain carefully why a perceptron is only capable of learning the classification of groups of data which can be separated using a line? [5 marks]
- (d) It was proposed by David Marr that the cerebellum acts like a perceptron, explain this. [5 marks]

**Solution:** a) the formula for the MP neuron activation is

$$x_i = \Theta\left(\sum_j w_{ij}x_j + \theta_i\right)$$

[2 marks] so the activation afterwards is  $(+1, -1)$  [3 marks].

b) The perceptron is a two layer (input and output layers) feedforward network [2 marks] with

$$\delta w_{ij} = -\eta \text{error} x_i$$

where  $\eta$  is a small learning rate and the error is the difference in the outcome and desired outcome for the  $j$ th output [3 marks].

c) The argument of the Heaviside function specifies which side of a line the input is. [5 marks, 2 for some attempt]

d) There is an input layer, an output layer [1 mark] and an error signal in the form of signals from the climbing fibre. [4 marks, 2 if the climbing fibre isn't mentioned]

**Q2.** This question is about rate models. Consider a neuron that varies its mean firing rate  $\bar{r}(s)$  as a logarithmic function of the scalar intensity of stimulus  $s$ :  $\bar{r}(s) = \ln(s)$ . Imagine

(cont.)

we are trying to detect changes in the stimulus intensity from the neuron's firing rate alone. Due to the noisiness of the firing, assume our detector can only reliably detect firing rate changes of  $\sigma$  or larger,  $\Delta r_{min} = \sigma$ .

- (a) Using the identity  $\frac{d}{dx} \ln(x) = \frac{1}{x}$ , calculate the smallest detectable change in stimulus intensity  $\Delta s_{min}$ . [5 marks]
- (b) Sketch a plot with  $s$  on the x-axis and  $\frac{\Delta s_{min}}{s}$  on the y-axis. [5 marks]
- (c) This result is related to a psychophysical phenomenon known as Weber's law. Does Weber's law state that humans perceive relative or absolute changes in the intensity of sensory stimuli? Explain how Weber's law follows from the previous result in parts (a) and (b). [5 marks]
- (d) A more realistic scenario would be where the firing rate noise scales with the square root of  $r$ , so that smallest firing rate change our detector can resolve is  $\Delta r_{min} = \eta r^{1/2}$ . What would the smallest detectable change in stimulus intensity be now? [5 marks]

**Solution:** a)  $\Delta s_{min} = \sigma s$  [5 marks for correct answer, 2 marks if correctly begin by taking the derivative  $dr/ds$  but make subsequent errors when rearranging terms].  
b) Should be a plot with straight horizontal line [5 marks]. [2 marks for an attempt if they correctly plot  $\Delta s_{min}$  vs  $s$  instead, which would be an increasing straight line].  
c) Weber's law implies humans sense relative changes [3 marks], since  $\Delta s_{min}/s =$  constant [2 marks]. Also allow answers that invoke the logarithmic relationship between stimulus and response, students do not necessarily need to refer to part b to get full marks].  
d)  $\Delta s_{min} = \eta s \sqrt{\log s}$  [5 marks. 2 marks if correctly begin by taking the derivative  $dr/ds$  but make subsequent errors when rearranging terms].

**Q3.** This question is about integrate-and-fire neurons.

(a) In the leaky integrate-and-fire neuron the voltage,  $v$ , satisfies

$$\tau_m \frac{dv}{dt} = E_l - v + R_m I_e$$

with the rule that if  $v > V_t$  the voltage is reset to  $V_r$ . What is the term  $E_l$  and where does it come from? Describe two significant approximations that are made in this model. [7 marks]

(b) In an experiment a constant current input  $I_e$  is applied with successively larger values. What value of  $I_e$  will make the neuron spike? [5 marks]

(c) Derive a formula for the interspike interval for this neuron when there is a constant current large enough to cause spiking. [8 marks]

**Solution:** a)  $E_l$  is the reversal potential [1 mark] and is the result of chemical gradients, largely in potassium, across the cell membrane [2 marks]; the model is one compartment and there are no gated channels [2 marks each, also allow some discussion of sodium and spikes for the no gated channels bit].

b) The neuron will spike if  $I_e > (V_t - E_l)/R_m$  since then the equilibrium point is higher than threshold [5 marks, 2 for some attempt].

c)

In the model

$$\tau_m \frac{dV}{dt} = E_L - V + R_m I_e$$

which we can solve from our study of odes, it gives

$$V(t) = E_L + R_m I_e + [V(0) - E_L - R_m I_e]e^{-t/\tau_m}$$

[2 marks] so if the neuron has spiked and is reset at time  $t = 0$  and reaches threshold at time  $t = T$ , assume  $V_R = E_L$  we have [2 marks]

$$V_T = E_L + R_m I_e - R_m I_e e^{-T/\tau_m}$$

[1 mark] so

$$e^{-T/\tau_m} = \frac{E_L + R_m I_e - V_T}{R_m I_e}$$

[1 mark] Taking the log of both sides we get

$$T = \tau_m \log \left[ \frac{R_m I_e}{E_L + R_m I_e - V_T} \right]$$

[2 marks]