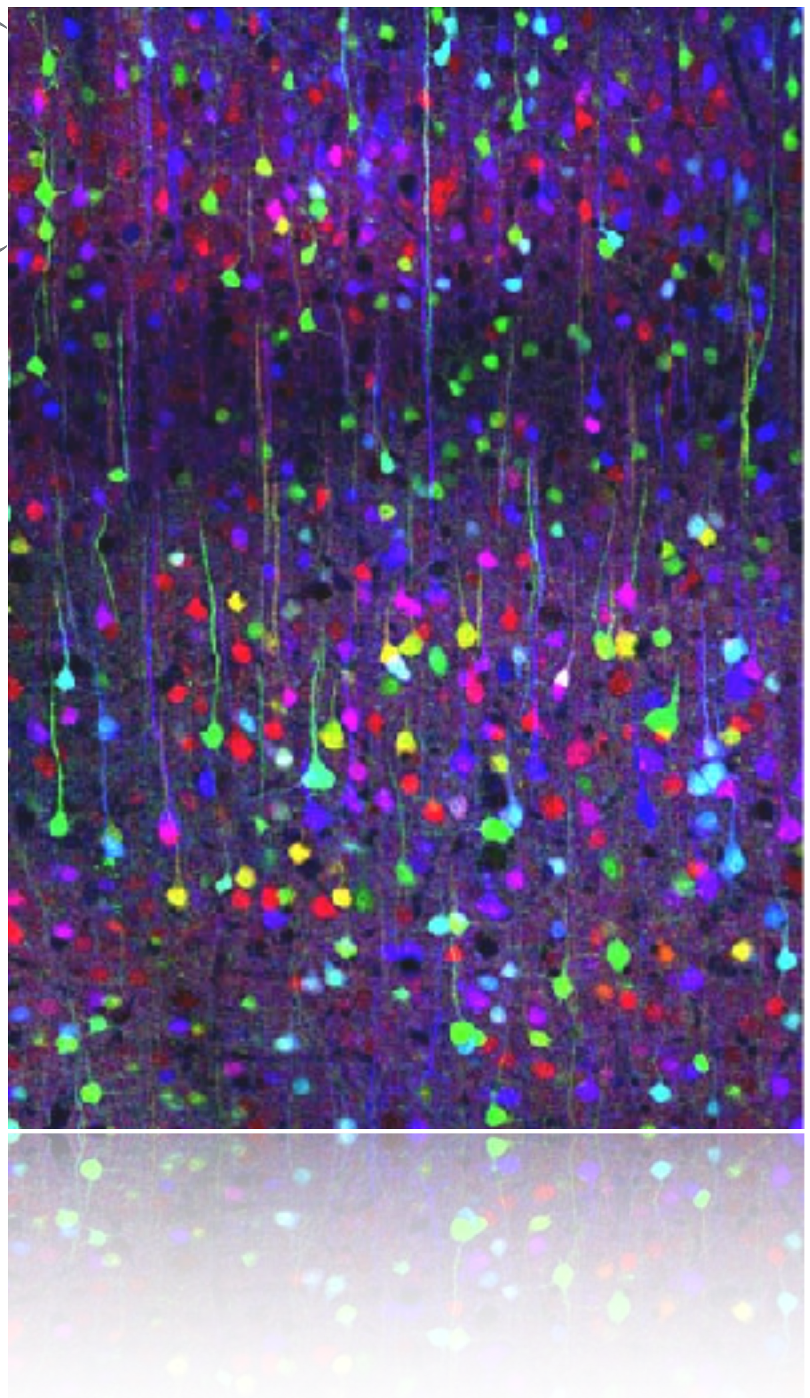
Neural Information Processing 2018/2019

University of BRISTOL

Brainbow (Litchman Lab)



Lecture 13
Neural circuits and learning:
Unsupervised learning:

Sparse coding and autoencoders

Extra lab session

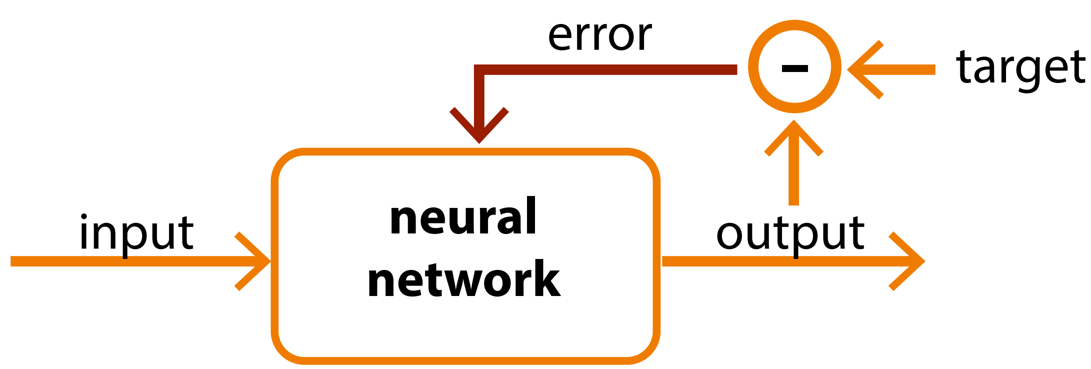
This Wednesday (12-2pm, 5th of December)!

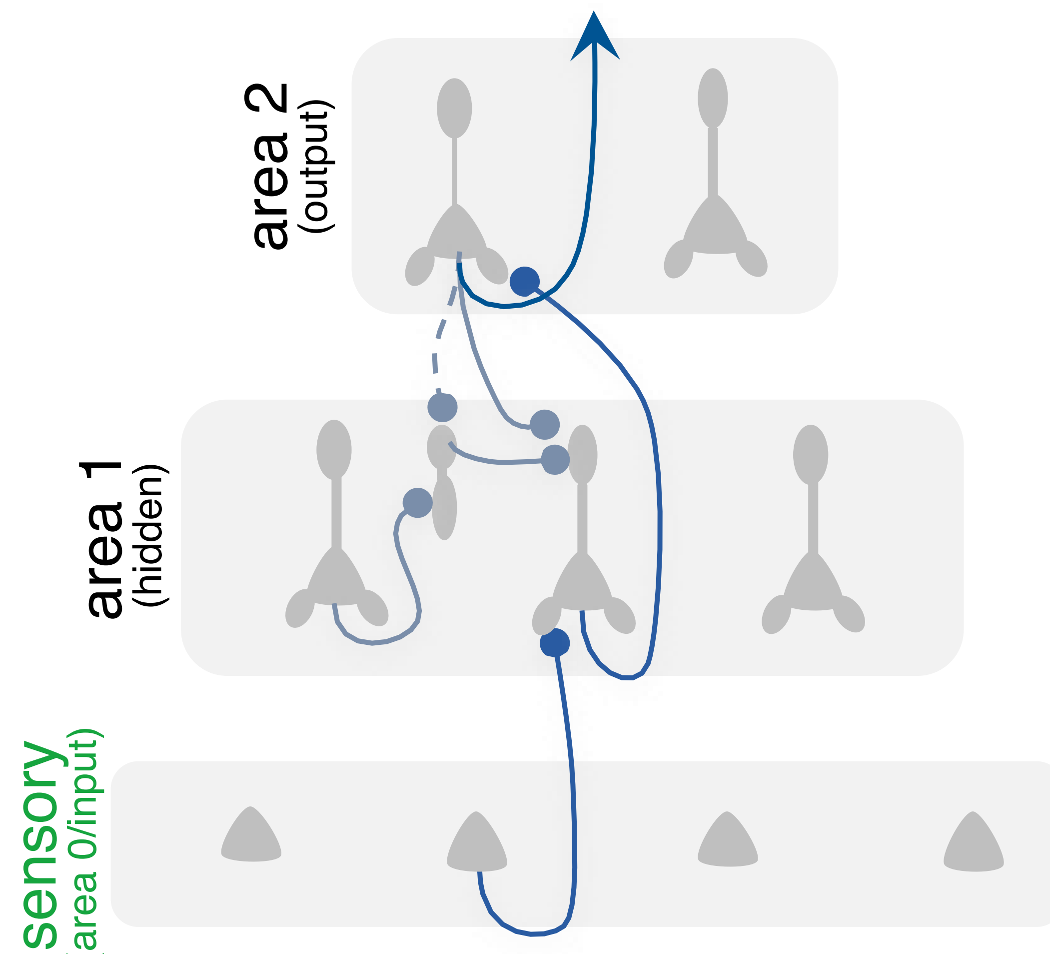
Sign up for help with course work:
https://doodle.com/poll/tbky2r8st4xxc3kb

Previously on *Neural Information Processing…*

Supervised learning and backprop:

Relies on a teaching signal





But, most sensory data is unsupervised

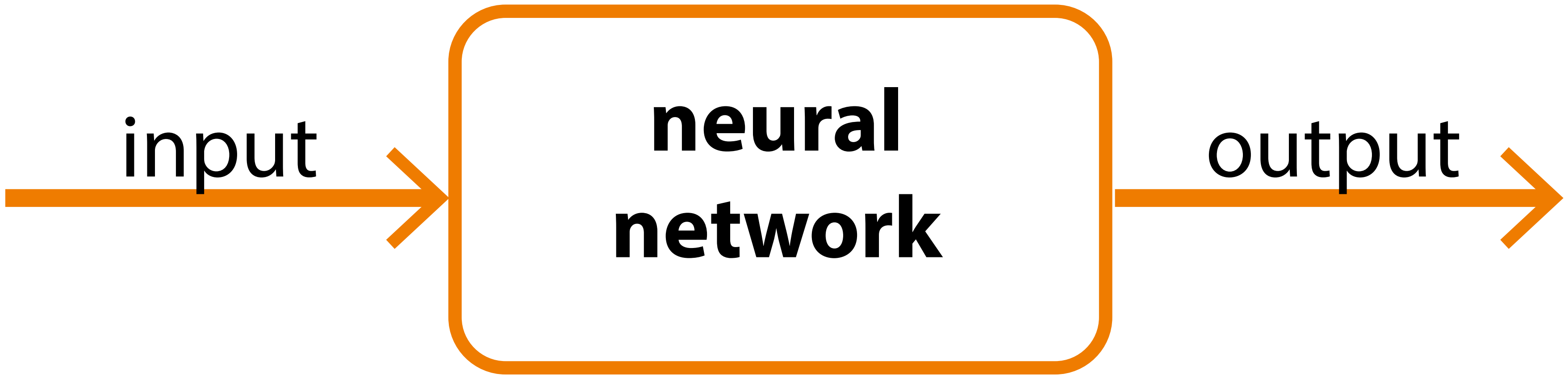
Every few milliseconds we experience a new input!

https://youtu.be/PIx9IJ1KOsg

But, most data animals experience is unsupervised

Unsupervised Learning:

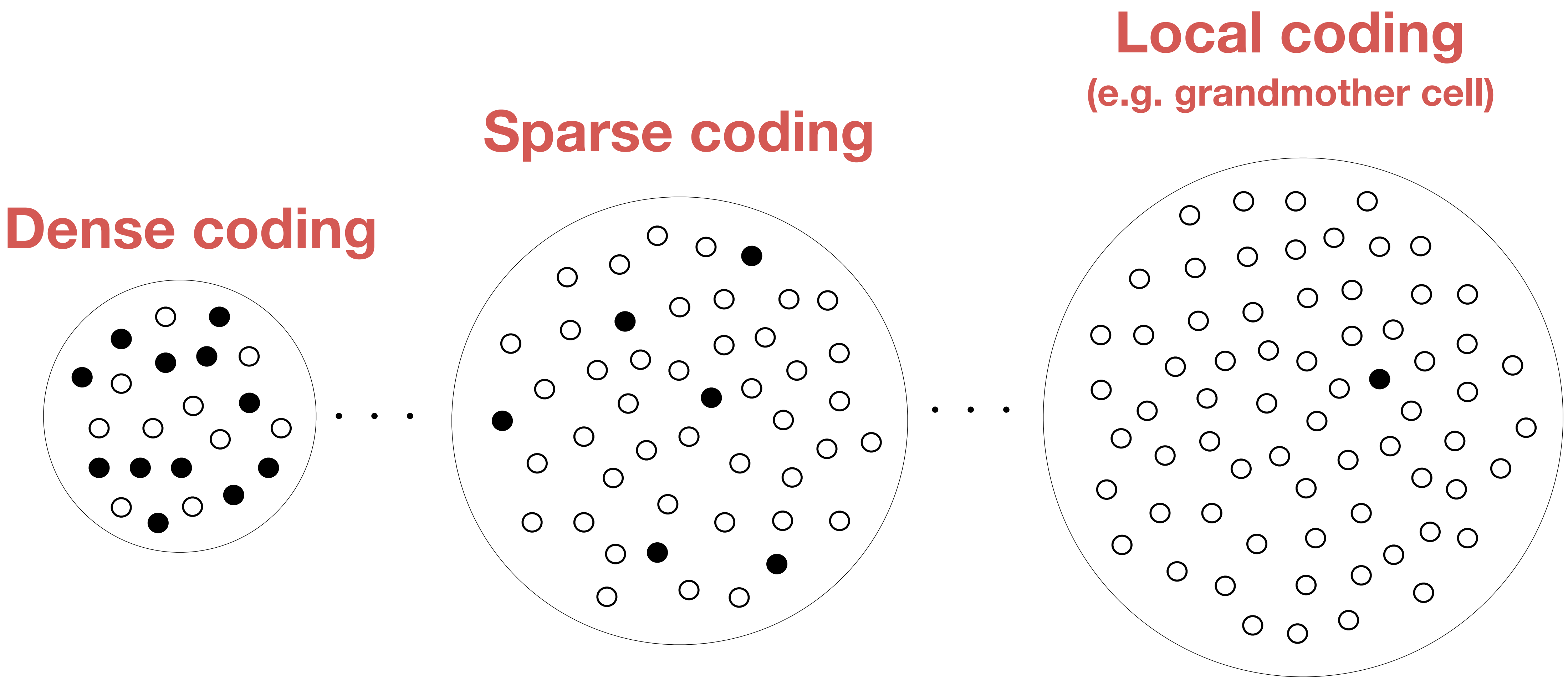
Extracts useful representations of input



Outline

1. **Sparse coding**
2. **Sparse coding in the brain**
3. **Autoencoders**

Sparse coding



Couzinie-Devy, (2010)

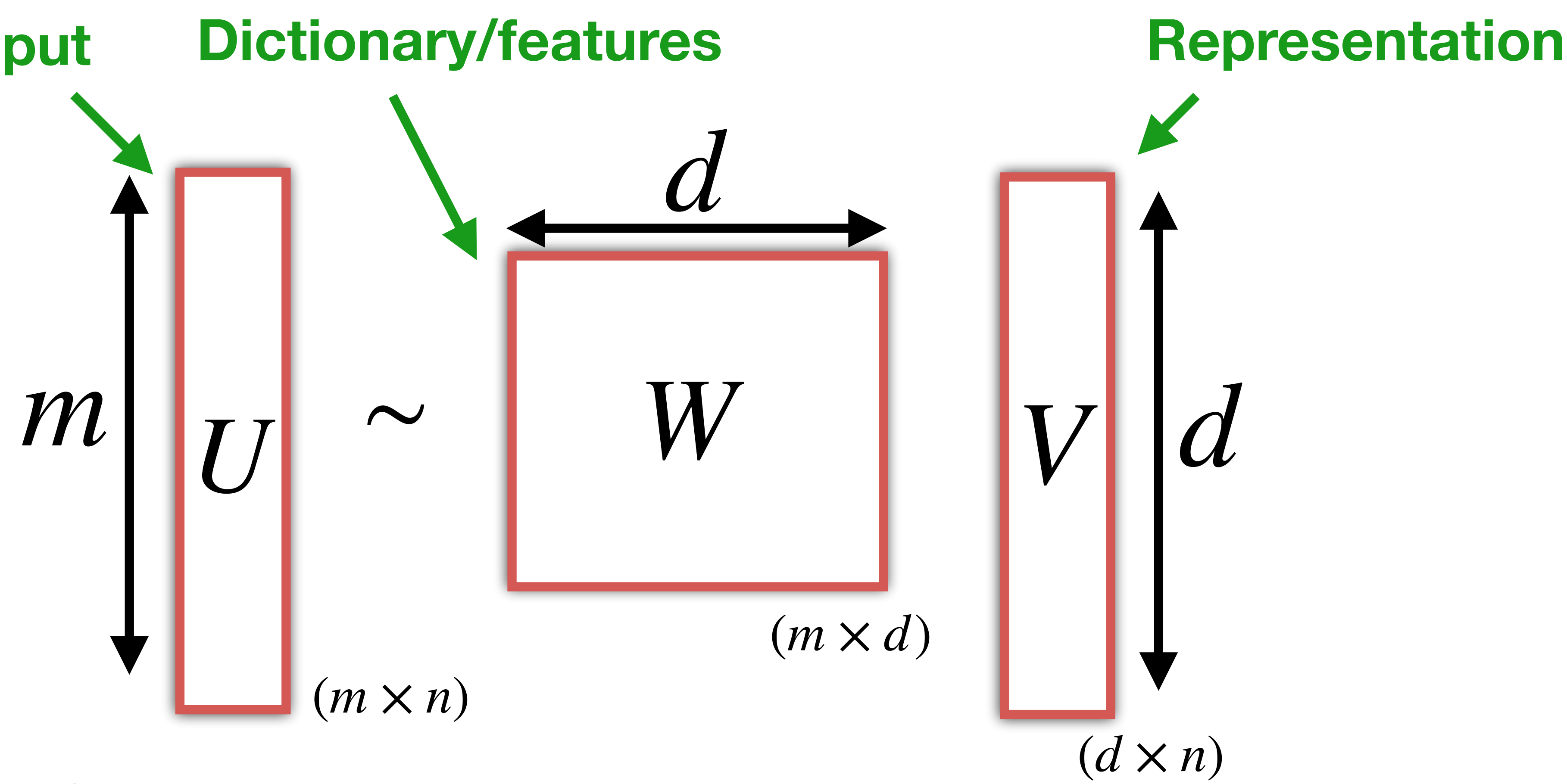
Sparse coding

- **Sparse coding** provides a framework for learning sparse representations of the input, typically through <u>unsupervised learning</u>.

- *Definition*: '**Sparse coding** is the representation of items by the strong activation of a relatively small set of neurons. For each stimulus, this is a different subset of all available neurons.'

Foldiak and Dominik, Scholarpedia (2008)

Sparse coding



Key properties of <u>sparse coding</u>:

- **Overcomplete dictionary *W*: d>m** (i.e. more dictionary elements than input elements)
- **Sparseness**: # of zero elements in ***v*** >> # non-zero elements in ***v***
- Overcomplete and sparse representation leads to <u>**richer features**</u>

Notes:

$$\|X\|_2^2 = \sum_{i=1}^{n} |x_i|^2$$

$$\|X\|_1 = \sum_{i=1}^{n} |x_i|$$

Sparse coding
L1-norm

$$\text{cost} = [\text{preserve information}] + \lambda[\text{sparseness}]$$

reconstruction cost

enforces sparseness

Sparse coding with L1-norm:

$$\text{argmin}_{W,V} = \|U - WV\|_2^2 + \lambda \|V\|_1$$

$$\text{subject to } \|W_i\|_2^2 \leq 1 \ \ \forall_i = 1,\ldots,m$$

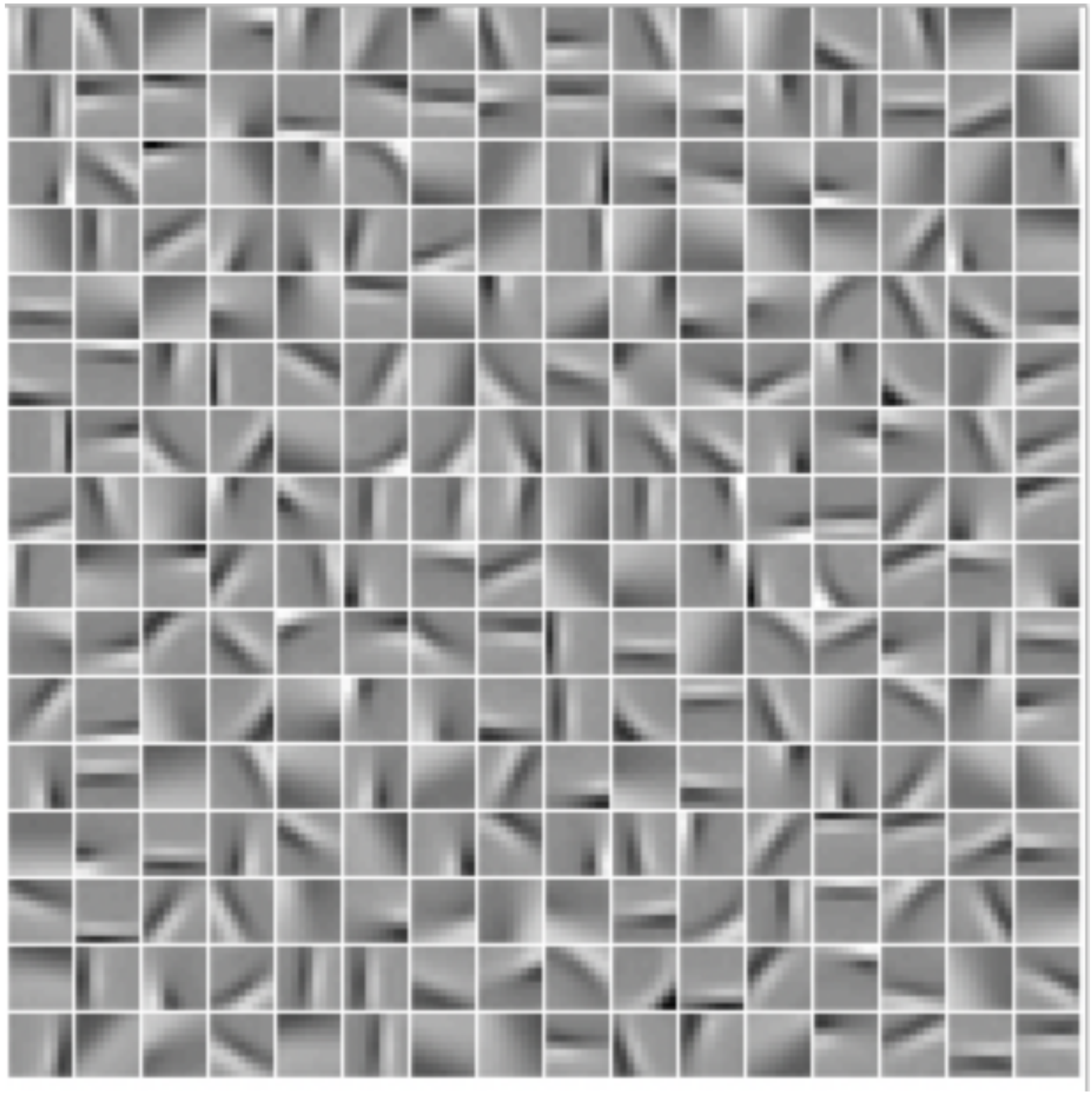
avoids arbitrarily high ***W***

- To find a good sparse code we need to optimise both the dictionary, ***W*** and the representation ***V***
- This optimisation problem is convex in ***V*** and ***W*** separately, but not both. Solution: interleaved optimisation of ***V*** and ***W***
- Several methods exist to optimise **V** and **W** (we will discuss one)
- λ controls the tradeoff between sparseness and reconstruction

Sparse coding

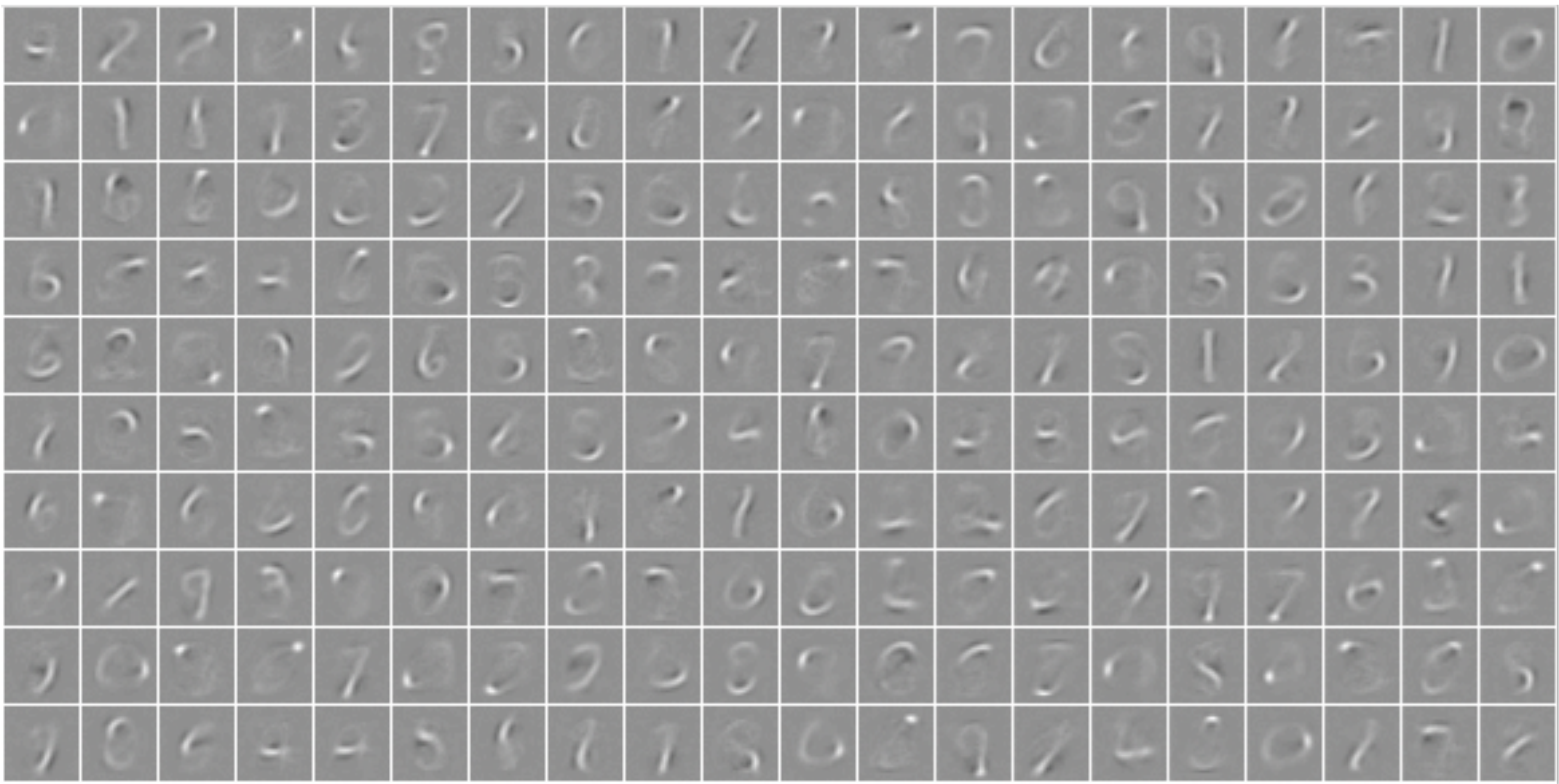
Input: Natural images

Simple cell like features:



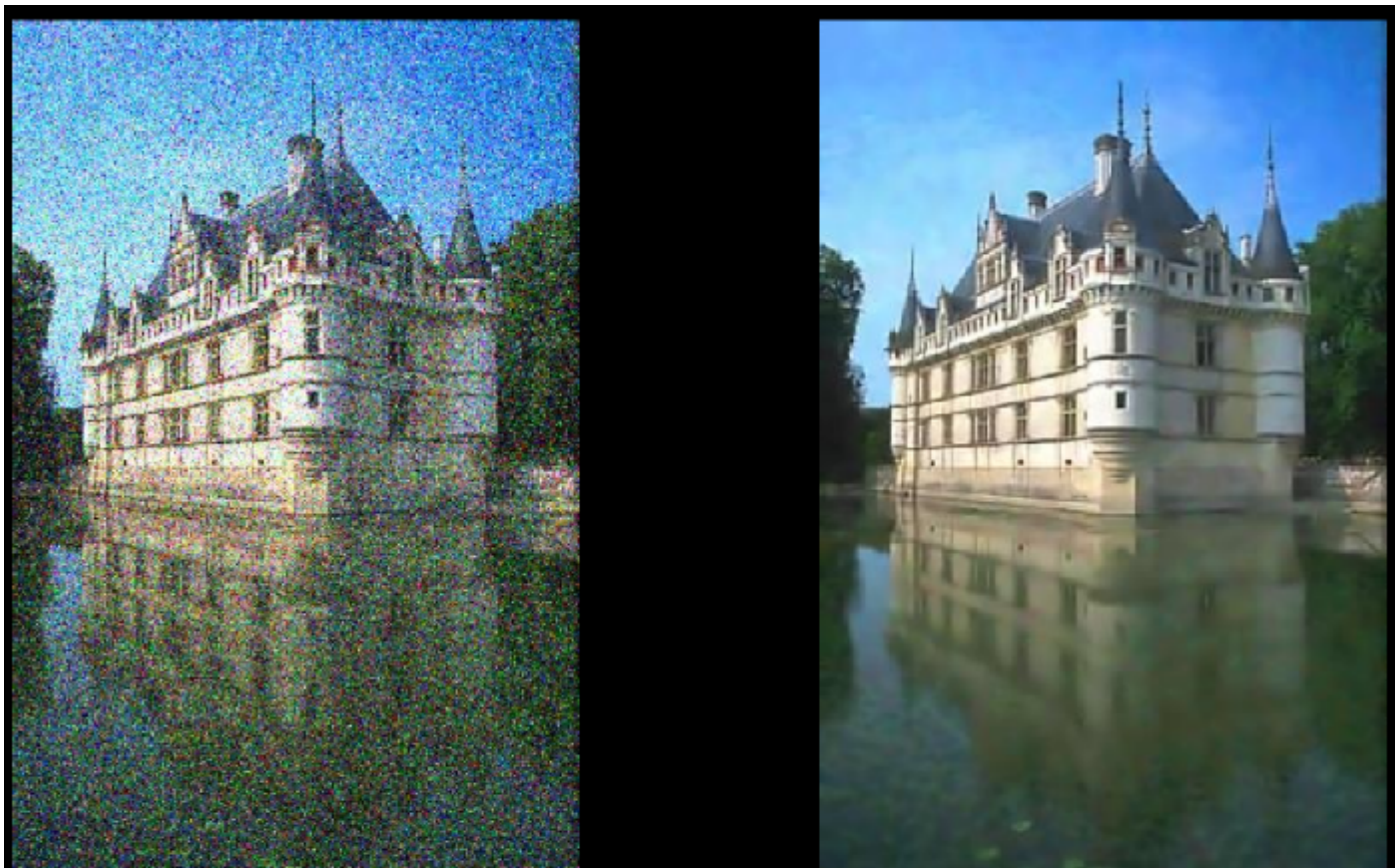
Sparse coding

Input: MNIST dataset (handwritten digits)



Sparse coding
Applications: denoising

Original | Sparse coding method



Sparse coding
Applications: image restoration

Original | Sparse coding method

Fig. 8. Underwater. Left: Image containing many high contrast edges, with text covering 16.19% of its area. Right: restored

Richard et al. VIIP 2001

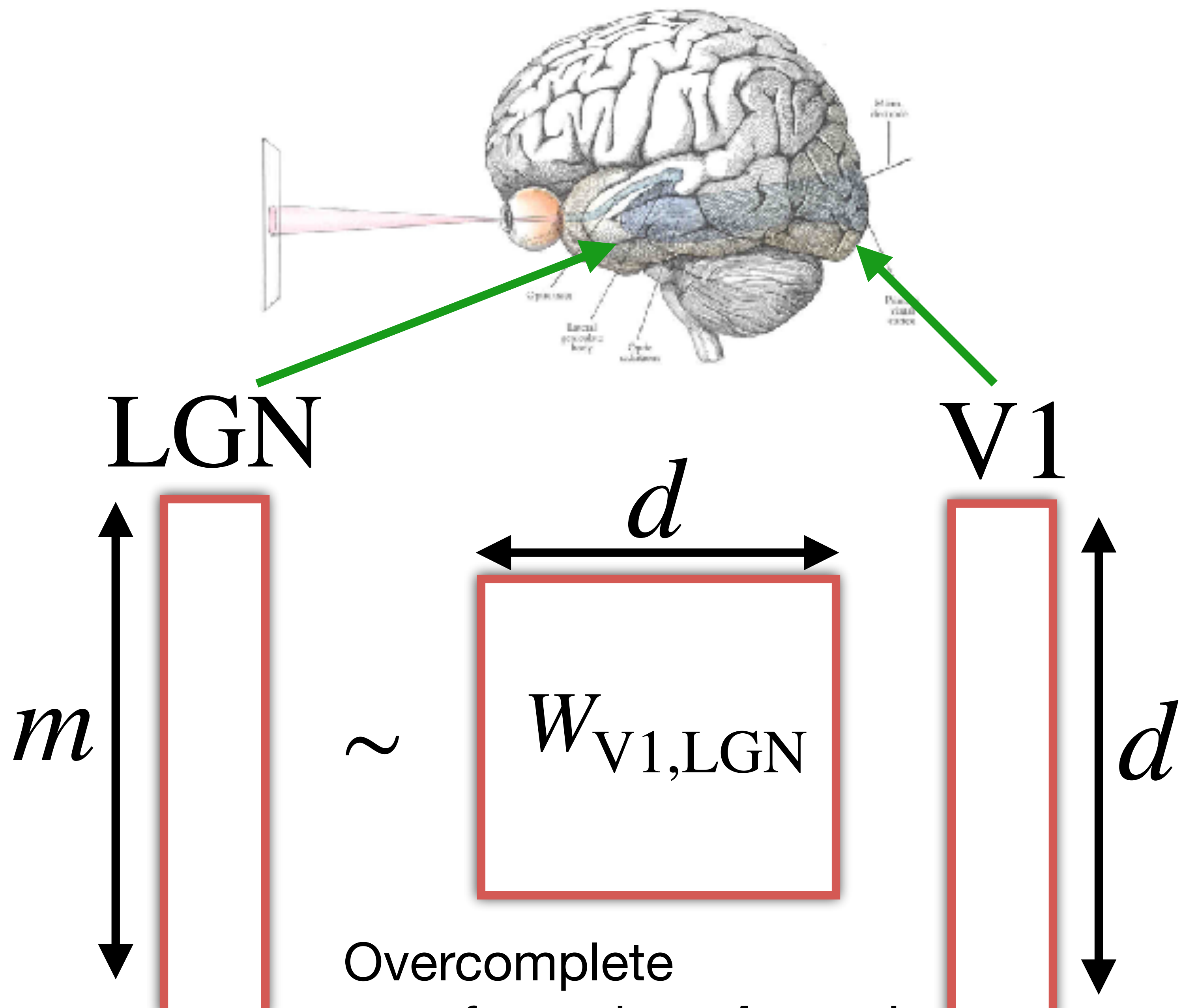
Sparse coding
Applications: digital zooming

Original

Sparse coding method

Couzinie-Devy, 2010

Sparse coding in the visual system



Overcomplete transformation: ***d*** >> ***m***!

Barlow (1981)

Sparse coding in the brain
Olshausen and Field 1996

New cost function:

new sparseness cost

$$\text{argmin}_{W,V} = \overbrace{\|U - WV\|_2^2 + \lambda \sum_i S(v_i)}^{\text{Cost}}$$

$$S(v_i) = \log(1 + v_i^2)$$

Notation in Olshausen and Field paper:

$$\text{argmin}_{\Phi,A} = \|I - \Phi A\|_2^2 + \lambda \sum_i C(a_i)$$

Olshausen and Field, Nature (1996)

Sparse coding in the brain

Olshausen and Field 1996

Compute *V* via gradient descent, and get dynamics for *v*:

$$\dot{v}_i = -\frac{\partial \mathrm{Cost}}{\partial v_i}$$

$$= w_i U - \sum_{j \neq i} G_{ij} v_j - f_\lambda(v_i)$$

where

$$G_{ij} = w_i w_j$$

$$f_\lambda(v_i) = v_i + \lambda S'(v_i)$$

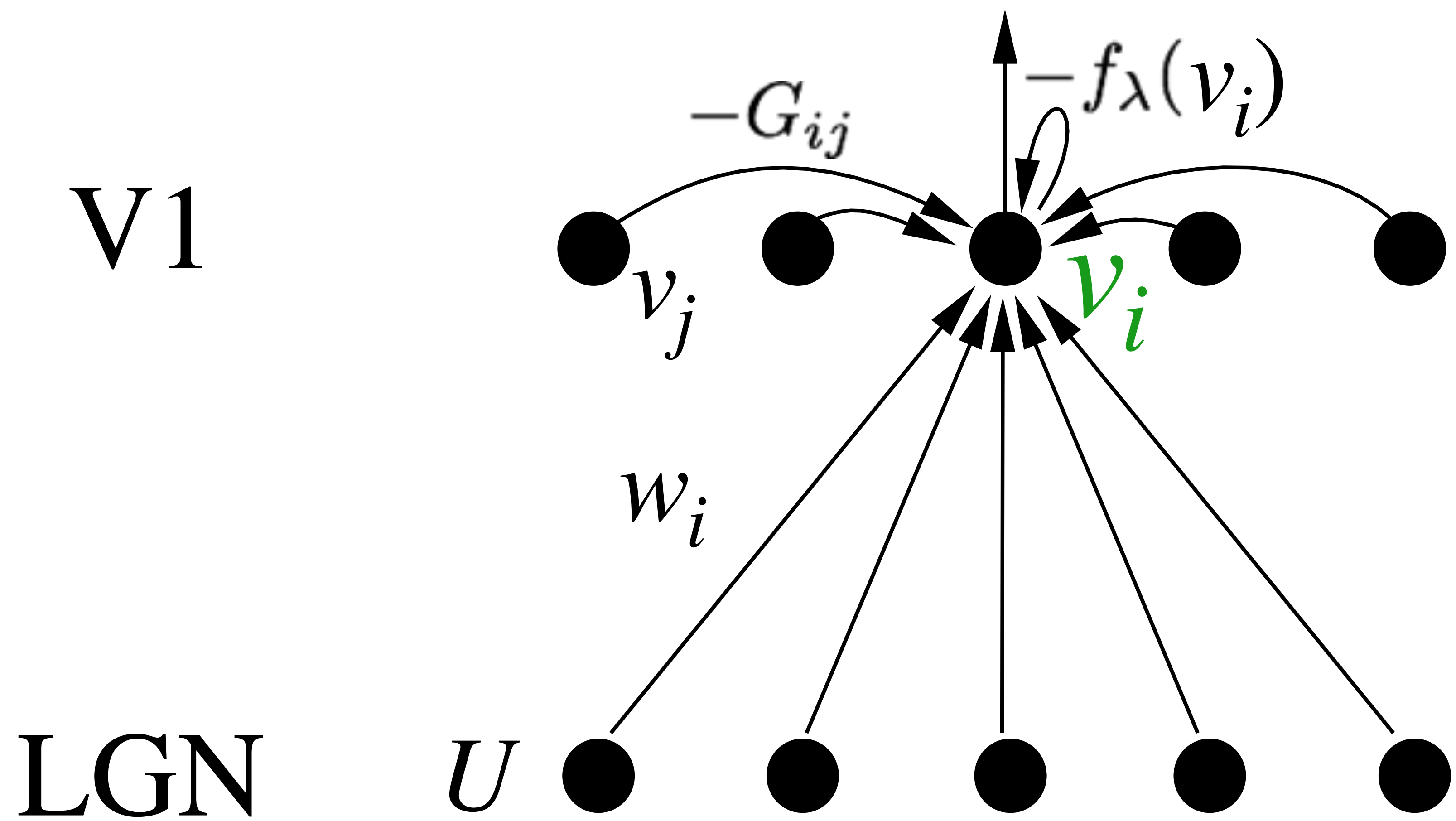
Olshausen and Field, Nature (1996)

Sparse coding in the brain

Olshausen and Field 1996

Potential network implementation of:

$$\dot{v}_i = w_i U - \sum_{j \neq i} G_{ij} v_j - f_\lambda(v_i)$$



Input image

Olshausen and Field, Nature (1996)

Sparse coding in the brain

Olshausen and Field 1996

With v we obtain the estimate, on a second phase we need to optimise the weights as:

$$\Delta w_i = -\eta \frac{\partial \mathrm{Cost}}{\partial w_i}$$

reconstructed image

$$= \eta < [U - \hat{U}]v_i >$$

$$= \eta < [U - \sum_{i}^{m} w_i v_i] v_i >$$

Learning rule is local, and hebbian:

$$\Delta w \propto \underbrace{UV}_{\text{Hebbian term}} - \underbrace{WV^2}_{\text{post. alone}}$$

Note: Hebbian learning rules depend on presynaptic and postsynaptic activity, e.g.:

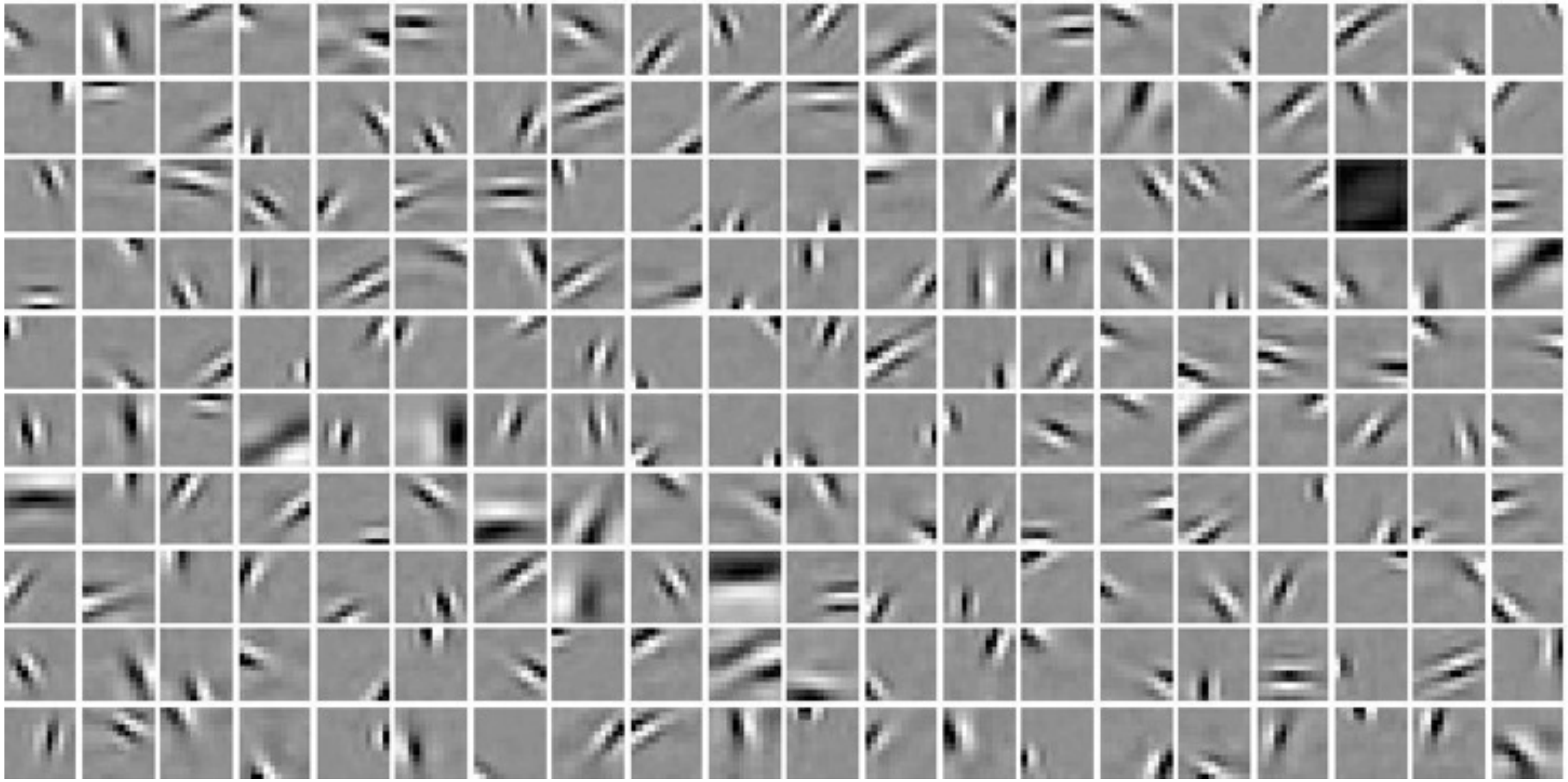
$$\Delta w = UV$$

Hebbian plasticity: Cells that fire together, wire together

Olshausen and Field, Nature (1996)

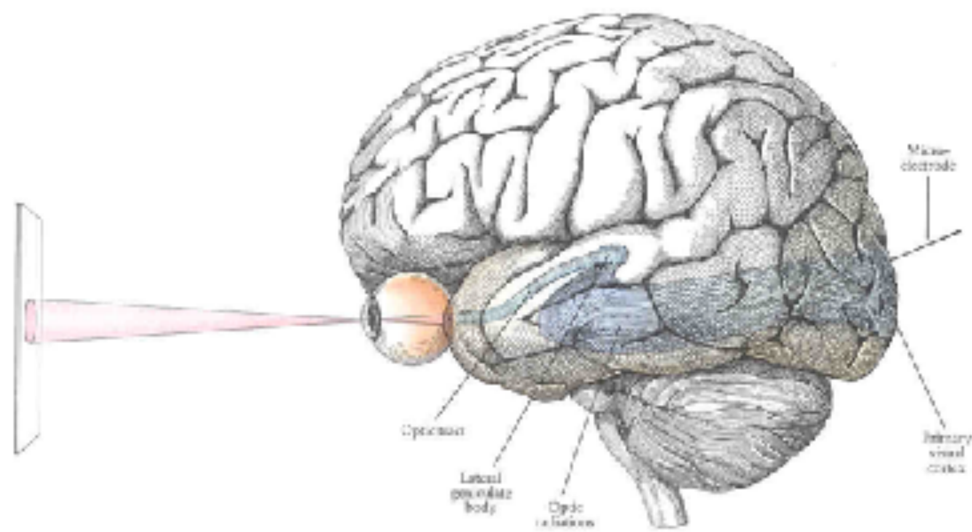
Sparse coding in the brain
Olshausen and Field 1996

Learned dictionaries/weights (simple cell-like)**:**



Olshausen and Field, Nature (1996)

Group discussion
groups of 2-3 (5min)



Can you think of advantages of using **sparse coding** in the brain?

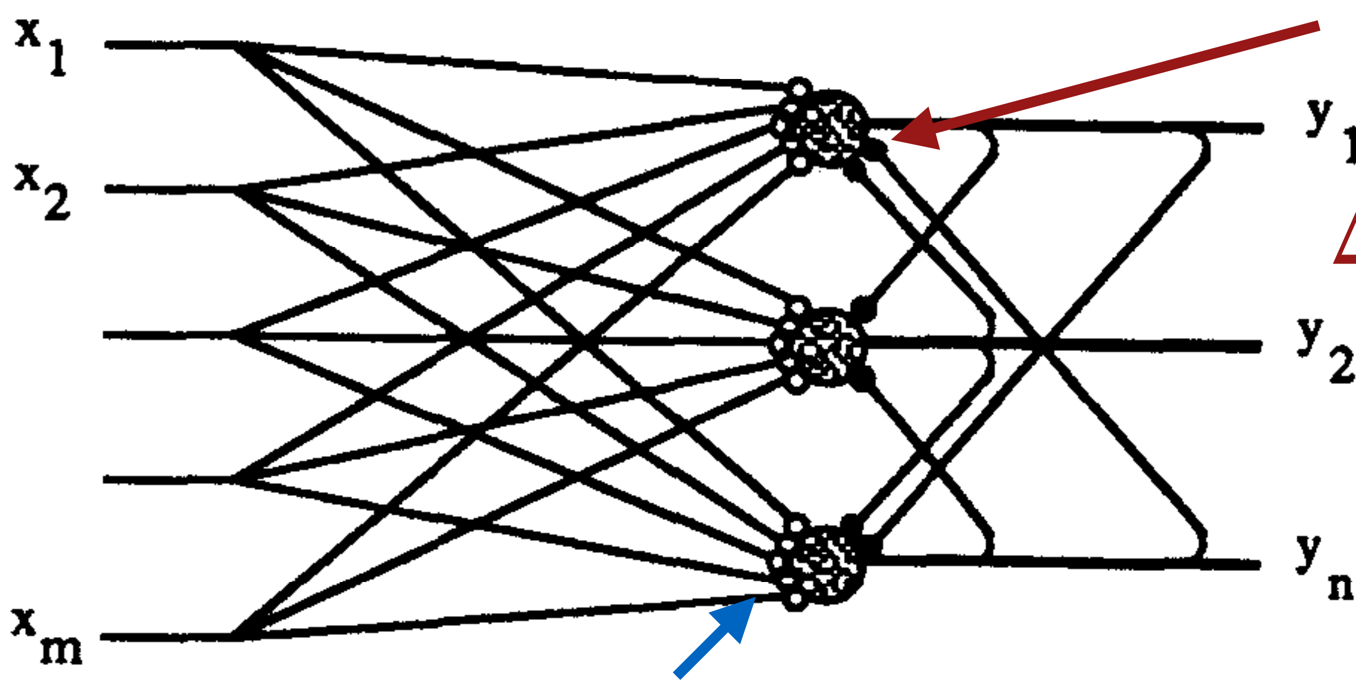
Group discussion
groups of 2-3 (5min)

Can you think of other advantages of using **sparse coding** in the brain?

- **Good representations** that can be used for decision making/ discrimination
- Sparse coding is **energy efficient** (only a few neurons active at a time)
- **More robust** to perturbations than purely localist codes

Sparse coding

Through Hebbian learning



$$\Delta w_{ij} = -\alpha(\underline{y_i y_j} - p^2)$$

Anti-Hebbian term
(because of the minus sign)

$$\Delta q_{ij} = \beta \underline{y_i (x_j} - q_{ij})$$

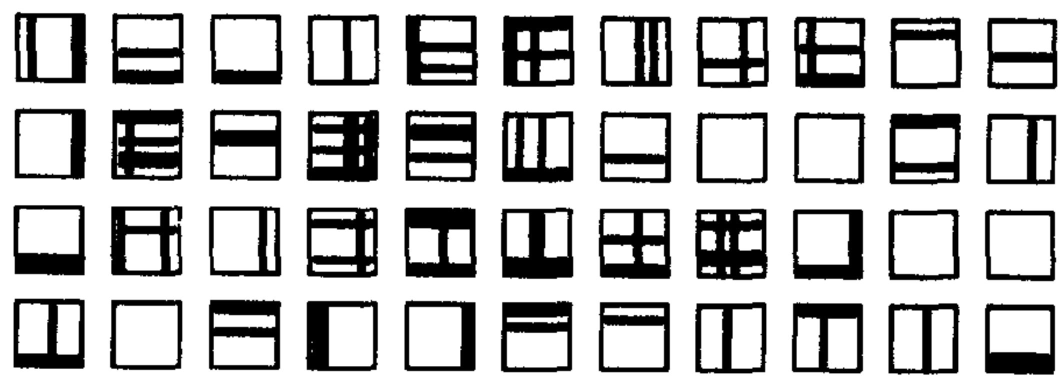
Hebbian term

Hebbian plasticity: Cells that fire together, wire together

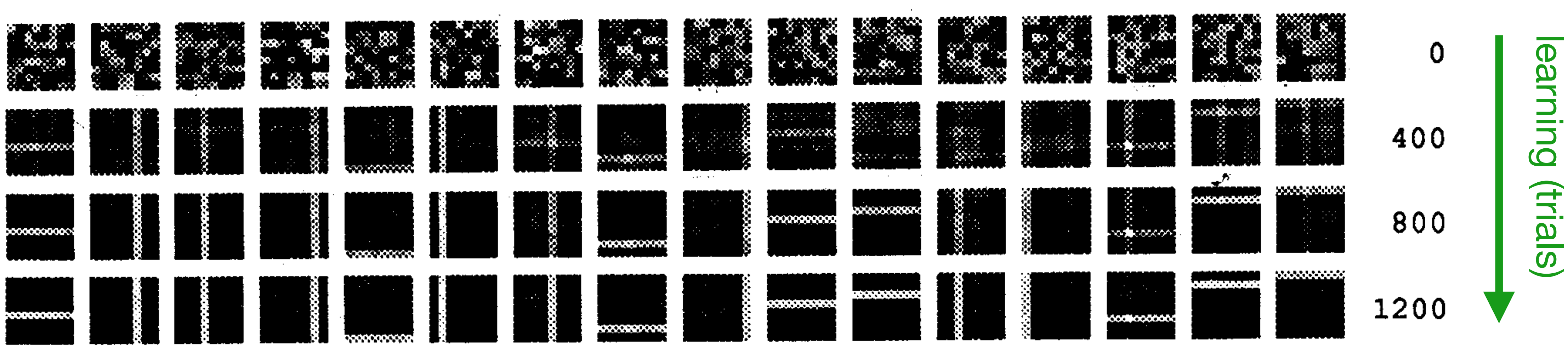
Foldiak, Biological Cybernetics (1990)

Sparse coding
Through Hebbian learning

Input defined as random patterns of vertical and horizontal lines:



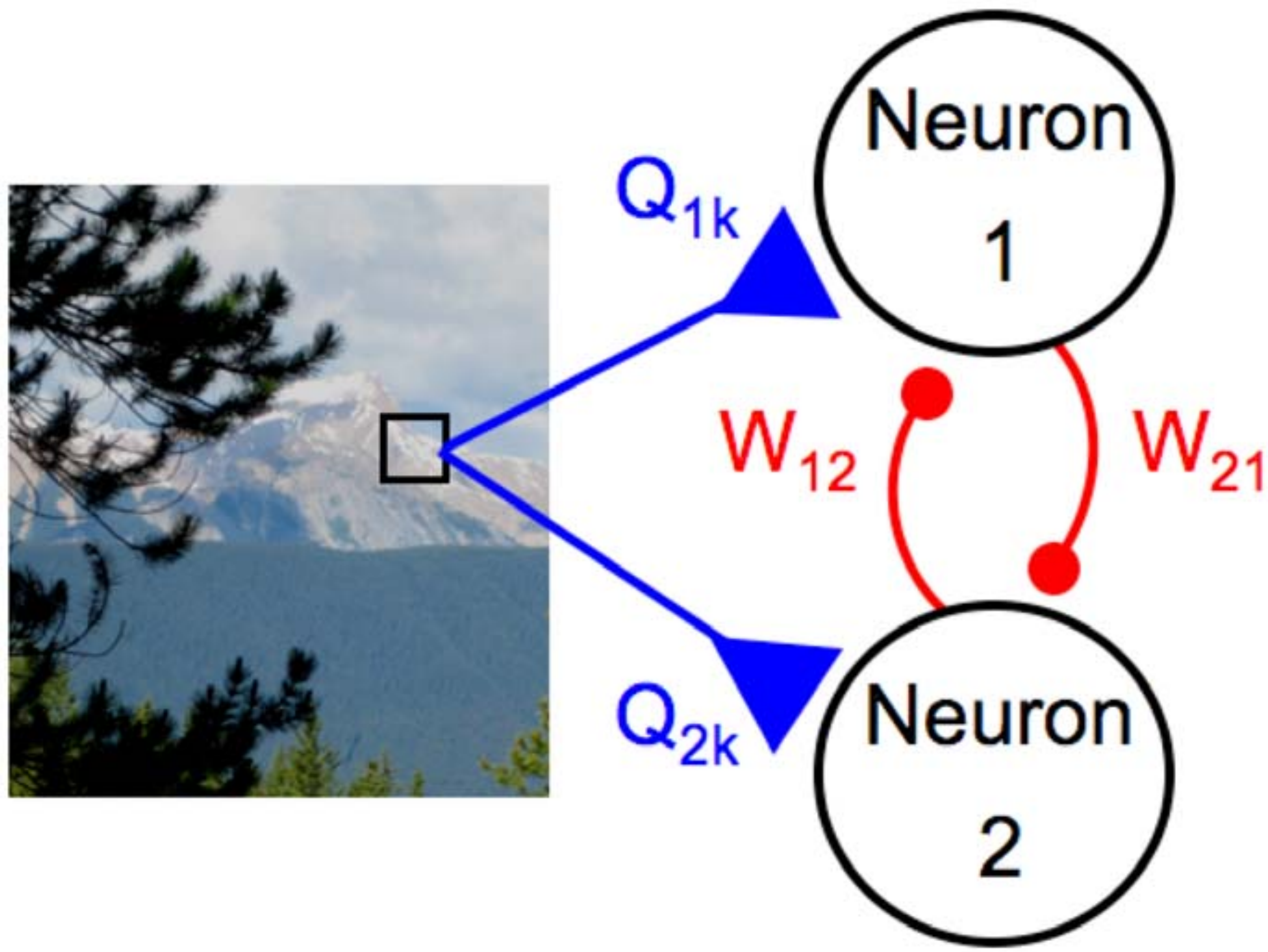
Learned features:



Foldiak, Biological Cybernetics (1990)

Sparse coding
Spiking neural network: SAILnet

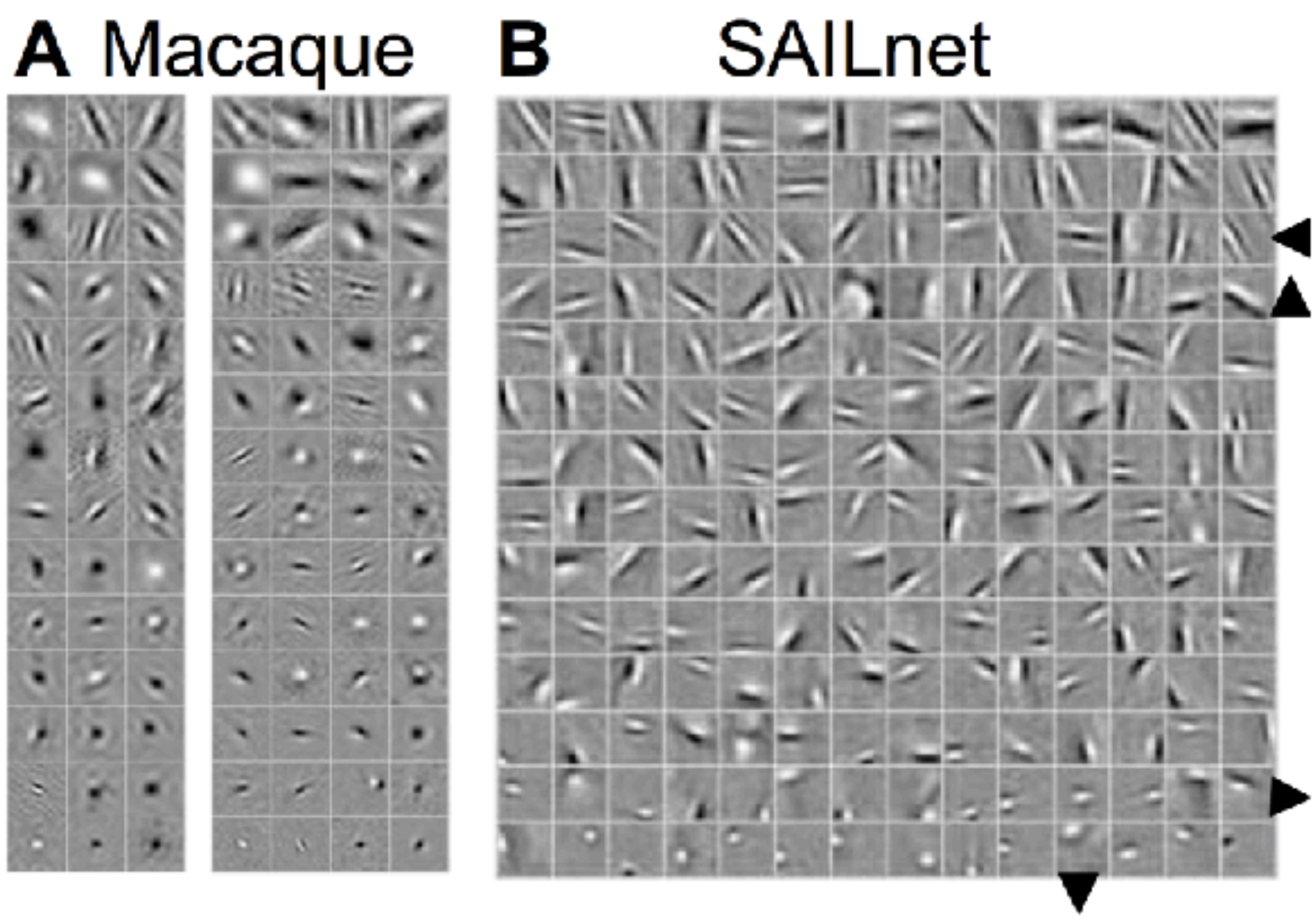
SAILnet:
A spiking version of Foldiak 1990 network with inhibitory and excitatory synapses and local learning rules:



Zylberberg et al. PLoS CompBIo (2011)

Sparse coding
SAILnet

Captures the diversity of receptive fields observed in animals:

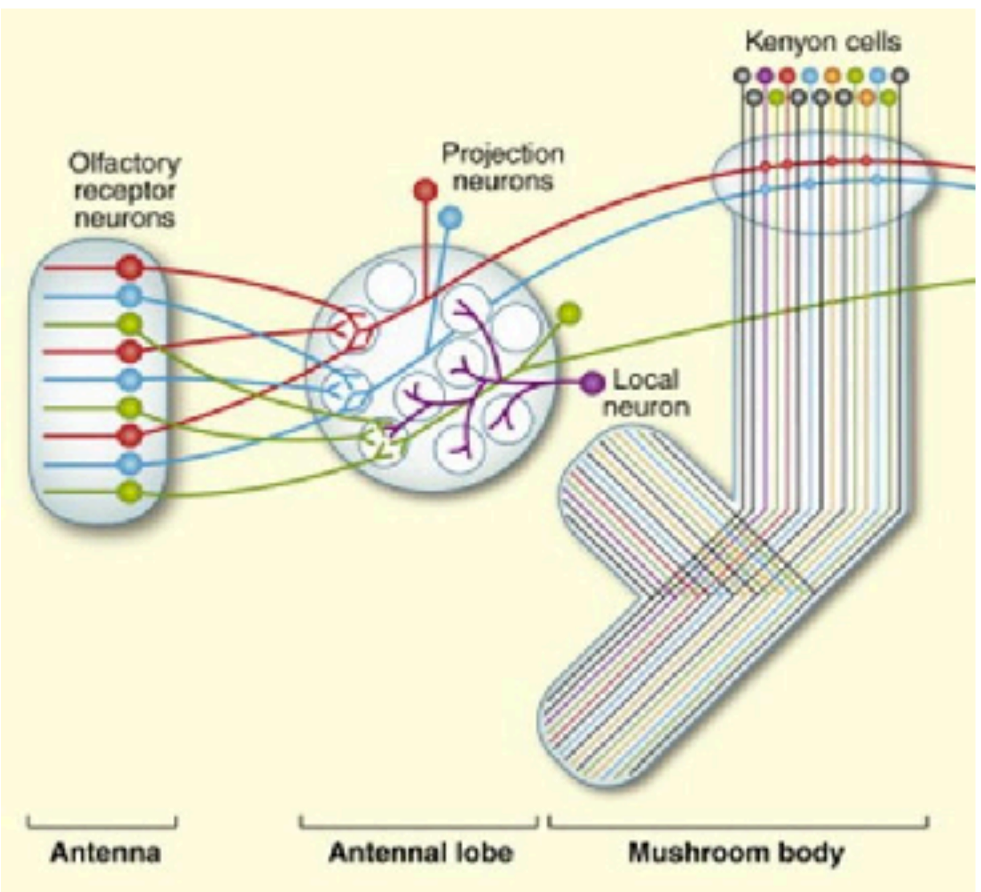


Zylberberg et al. PLoS CompBIo (2011)

Sparse coding across the animal kingdom:
from mammals to insets..

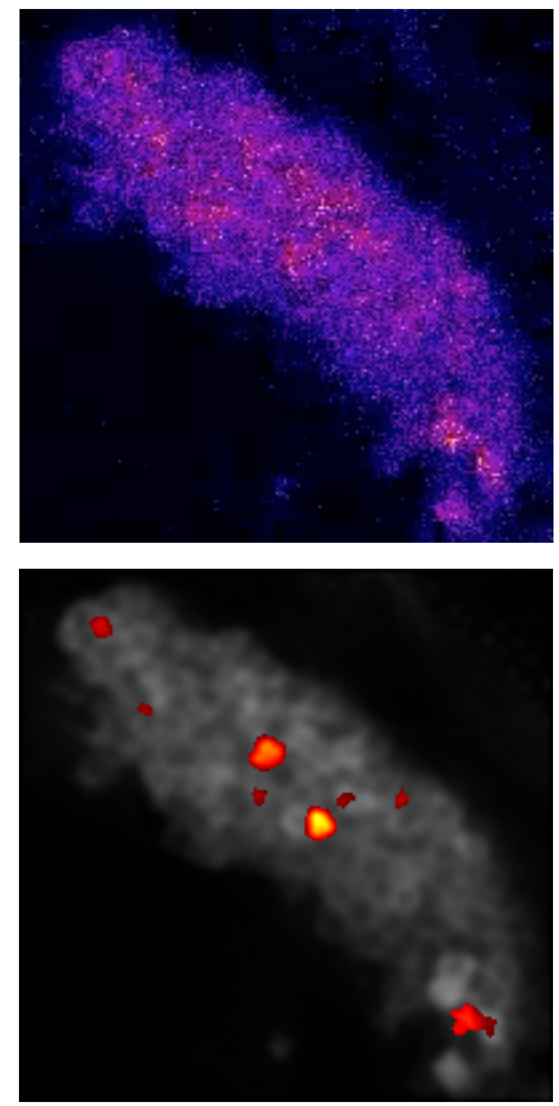
Sparse coding in fruit flies

Smell (odor) system in *Drosophila*



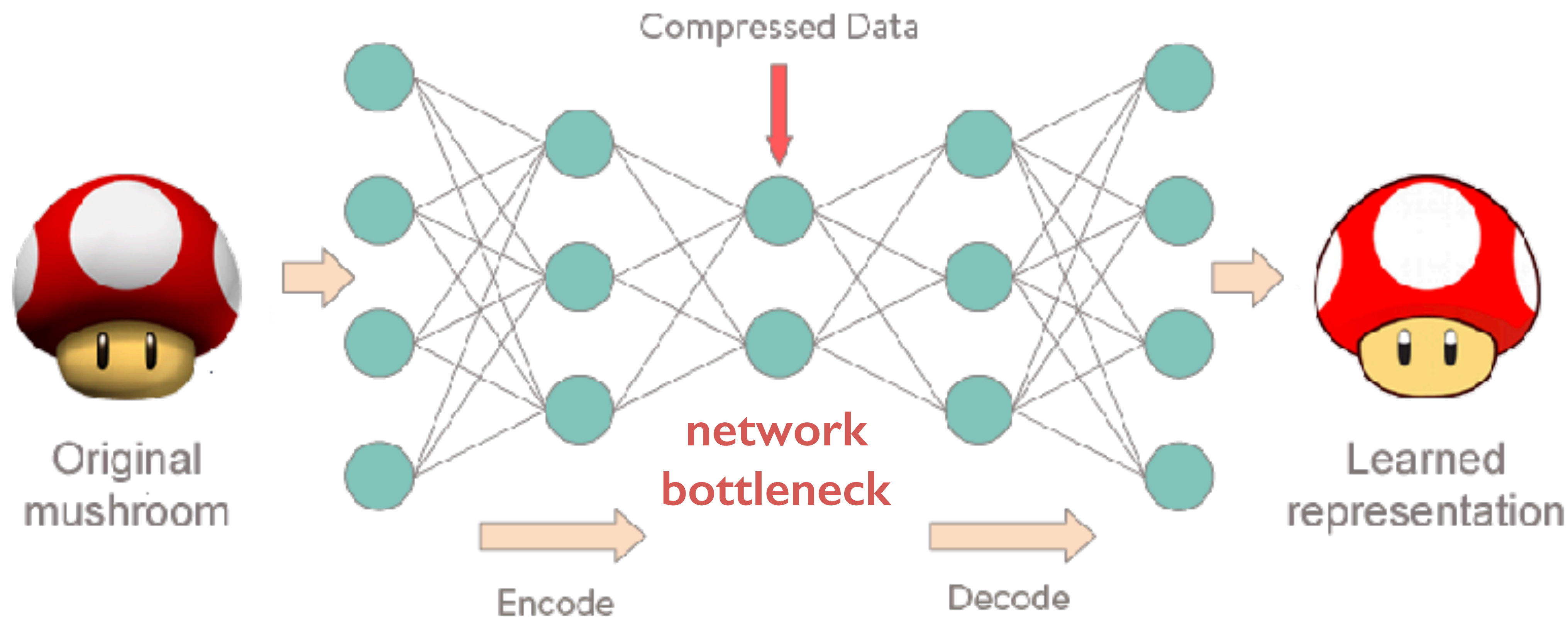
Masse et al. Current Biology (2009)

Sparse coding in Kenyon cells



Lin et al. Nature Neuroscience (2014)

Autoencoders

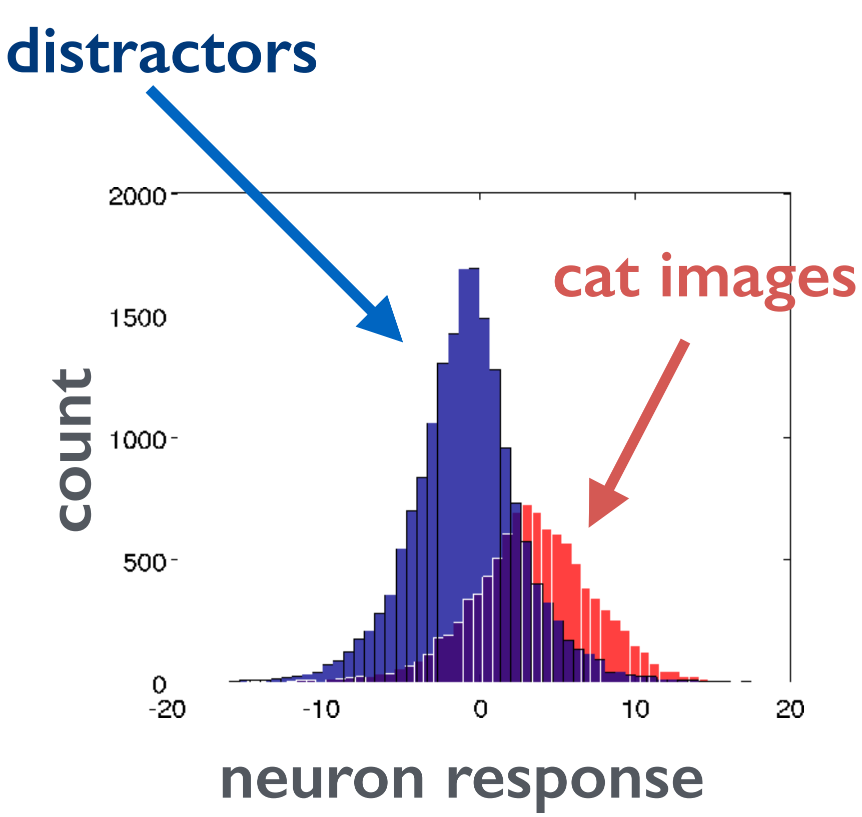


$$\text{argmin}_W = \|V - f(WU)\|_2^2$$

- **Autoencoders** also aim at reconstructing the input, but without an explicit constrain on sparsity (instead uses other forms of regularization)
- **Autoencoders** rely on a specific model ***f*** (i.e. neural network with bottleneck)
- If an **autoencoders** uses a sparsity constrain, then are equiv. to **sparse coding**

Autoencoders
Cat neuron

Autoencoder trained on **10 million images** from **YouTube** movies.
Showing that it is possible to learn high level feature detectors from unlabelled data (akin to grandmother cells).



Le et al. ICML (2012)

Summary

1. **Sparse coding** is an **optimisation** method that develops <u>useful representations</u> of the input

2. <u>Olshausen and Field</u> introduced a sparse coding framework that can be mapped onto the brain

3. **Sparse coding** develops **representations akin** to the brain

4. <u>Autoencoders</u> can also be used to extract useful representations, but do not (typically) rely on a sparsity cost and assume a specific neural network

References

Text books:

Natural Image Statistics, by Hyvarinen et al. 2009 [good reference on unsupervised learning from natural images]

Theoretical neuroscience: Dayan and Abbott 2001 [Unsupervised learning: chapter 3]

Deep Learning by Courville et al. 2015

Relevant papers:

- Foldiak, Biological Cybernetics (1990) [classical work on developing sparse representations with local Hebbian learning rules]
- Olshausen and Field, Nature (1996) [seminal work on sparse coding using natural images]
- Zylberberg et al., PLoS Computational Biology (2001) [more recent work on sparse coding in spiking networks with local learning rules]

Upcoming lectures

- **L10: Neural circuits and learning:** introduction
  - Visual processing
    - L11: Visual cortex
    - L11: Convolutional neural networks
  - Learning in the brain
    - L12: Supervised learning: The backpropagation algorithm/cerebellum
    - L13: Unsupervised learning: Sparse coding and autoencoders
    - **L14: Reinforcement learning: TD learning, Q learning, deep RL and dopamine**
  - Temporal processing in the brain
    - L14: Auditory cortex and recurrent neural networks
    - L15: Gated recurrent neural networks