# Worksheet

Many of these problems are taken from the excellent text book Cover and Thomas. Although the questions do vary a bit in difficulty each is worth two marks.

## Q1 - marginal and conditional distributions

Work out the marginal distributions and the $x = a$ conditional distribution for

|   | $a$ | $b$ |
|---|-----|-----|
| 1 | $\frac{1}{3}$ | $\frac{1}{6}$ |
| 2 | $0$ | $\frac{1}{4}$ |
| 3 | $\frac{1}{8}$ | $\frac{1}{8}$ |

## Q2 - working out entropy

A fair coin is flipped until the first head occurs. Let $X$ denote the number of flips required.

1. Find the entropy $H(X)$ in bits. The following expressions may be useful:

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$$
$$\sum_{n=0}^{\infty} nr^n = \frac{r}{(1-r)^2} \tag{1}$$

2. A random variable $X$ is drawn according to this distribution. Find an efficient sequence of yes-no questions of the form, 'Is $X$ contained in the set $S$?'. Compare $H(X)$ to the expected number of questions required to determine $X$.

## Q3 - A puzzle which lends itself to information type reasoning

Suppose that one has $n$ coins, among which there may or may not be one counterfeit coin. If there is a counterfeit coin it will weight either less or more than the other coins. The coins are weighed using a balance.

1. Find an upper bound on the number of coins $n$ so that $k$ weighings will find the counterfeit coin, if any, and correctly declare it to be heavier or lighter.

2. What is the coin-weighing strategy for $k = 3$ weighings and 12 coins,

## Q4 - Working out entropy and information

Let $p(x, y)$ be given by $p(0, 0) = p(0, 1) = p(1, 1) = 1/3$ and $p(1, 0) = 0$. Find $H(X)$, $H(Y)$, $H(X|Y)$, $H(Y|X)$, $H(X, Y)$, $H(Y) - H(Y|X)$ and $I(X; Y)$.

## Q5 - A question about information in the brain

On the way.