UNIVERSITY OF BRISTOL

September 2020 Examination Period

FACULTY OF ENGINEERING

**M Level Examination for the Degree of
Master of Engineering / Masters of Science**

**COMSM0034-J
Information Processing and the Brain**

**TIME ALLOWED:
2 hours**

This paper contains *two* parts.
The first section contains *15* short questions.
Each question is worth *two marks* and all should be attempted.
The second section contains *three* long questions.
Each long question is worth *20 marks*.
The best *two* long question answers will be used for assessment.
The maximum for this paper is *70 marks*.

Other Instructions:

Calculators must have the Faculty of Engineering Seal of Approval.

TURN OVER ONLY WHEN TOLD TO START WRITING

Section A: short questions - answer all questions

Q1. What is the maximum value Shannon's entropy could have for a random variable that takes five possible values. What is its probability distribution.

Q2. Define the mutual information for discrete random variables X and Y.

Q3. Show $I(X;Y) = H(X) + H(Y) - H(X,Y)$

Q4. For two dimensional mixed signals $\mathbf{r} = M\mathbf{s}$ the infomax algorithm attempts to find unmixing matrix $\mathbf{x} = W\mathbf{s}$. Explain why a saturating non-linearity is introduced to give $y_i = \sigma(x_i)$.

Q5. Given three random variables X, Y and Z with $p(z|x,y) \propto p(x|z)p(y|z)$, $p(x|z) \sim \mathcal{N}(z, \sigma_X^2)$ and $p(y|z) \sim \mathcal{N}(z, \sigma_Y^2)$ what are the mean and variance of Z?

Q6. List three assumptions that are made when applying a Kalman filter.

Q7. What is the key difference between reinforcement learning and other forms of learning?

Q8. Give two features specific to convolutional neural networks that are inspired by neuroscience.

Q9. Inhibition in the brain is key to information processing and balanced dynamics. Give the inhibitory learning rule discussed in the lectures that can be used to balance excitation-inhibition in the brain, and briefly explain the different terms.

Q10. Give an example of gated-recurrent neural networks, and briefly explain how the gates are believed to be implemented in biology.

Q11. What is the difference between sparse and dense coding?

Q12. Sparse coding usually relies on a neural network with $U = WV$. Explain what what are the different variables and what is referred to as a overcomplete dictionary.

Q13. Give two advantages of using sparse coding in the brain.

Q14. What is the main technical obstacle for fitting maximum-entropy-based models to data from large populations of neurons? What exactly is the difficulty?

Q15. The pairwise maximum entropy model accounts for pairwise correlations between neurons, but not higher-order correlations. What common extension of this model can capture higher-order correlations? And how does it work?

Section B: long questions - answer two questions

Q1. This question is about Shannon's entropy for both discrete and continuous distributions.

(a) Define Shannon's entropy $H(X)$. *[3 marks]*

(b) If X can take two values show Shannon's entropy is non-negative. *[5 marks]*

(c) Define the differential entropy $h(X)$. *[3 marks]*

(d) Show that the differential entropy is not non-negative in general. *[5 marks]*

(e) By calculating the entropy of a discretized distribution, explain why the differential entropy and discrete entropy are different. *[4 marks]*

Q2. This question is about the Kullback Leibler divergence and about reinforcement learning.

(a) Given a random variable X with two probability distributions $p(x)$ and $q(x)$ then roughly speaking the Kullback Leibler (KL) divergence measures how different q is from p. It is defined as

$$D(p\|q) = \sum_x p(x) \log \frac{p(x)}{q(x)}$$

Consider a random variable with the sample space $\{0, 1\}$ and two distribution: $p(0) = r$ so $p(1) = 1 - r$ and $q(0) = s$ so $q(1) = 1 - s$. In this case what is $D(p\|q)$ in terms of s and r? *[3 marks]*

(b) The random variable (X, Y) has two distributions on it, $p(x, y)$ and $p(x)p(y)$. Use this to relate the KL divergence to the mutual information. *[4 marks]*

(c) It can be proved that $D(p\|q) \geq 0$ for all distributions p and q with equality if and only if $p(x) = q(x)$ for all x; use this to show $H(X) \leq \log n$ where n is the size of the sample space, with equality if and only if X is uniformly distributed. *[3 marks]*

(d) Explain what problem the use of a replay memory addresses in deep reinforcement learning, and how does it relate to the brain. *[2 marks]*

(e) Give the value update function used in Q-learning and briefly describe the different terms. *[4 marks]*

(f) Using TD-learning and given the following reward sequence R and state space sequence S compute the value function for $V(S_{t=1})$, $V(S_{t=2})$ and $V(S_{t=3})$. Consider only the *first* and *second* updates. Assume that the value function starts at 0 before any update, and $\gamma = 0.9$. *[4 marks]*

Q3. This question is about supervised learning and backpropagation algorithm in the brain.

(a) Why is the credit assignment problem a central issue in neuroscience and machine learning? How does the backpropagation algorithm solve it? *[5 marks]*

(cont.)

(b) Give and explain three key features that have been suggested to make the back-propagation algorithm used in supervised learning biologically implausible. You should use a simple two layer neural network (with one hidden neuron *h*, one output neuron *v*, two input weights, and without biases, see Figure 1) to derive the weight updates and make a schematic of the network with the backprop algorithm to help illustrate your answer. Assume the cost function (or error) to be $E = (v - y)^2$, where *y* is the desired target. *[11 marks]*

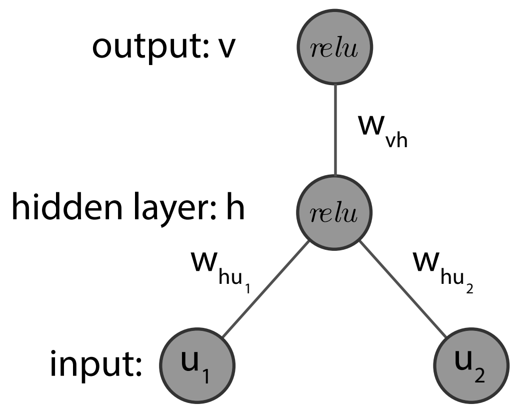

Figure 1: **Schematic of simple feedfoward neural network, where the activation function is given by the hyperbolic tangent** relu(x)**.**

(c) Which brain area has been proposed undergo supervised learning and why? In this system which component has been proposed to encode error signals? *[4 marks]*

 END OF PAPER