The data processing inequality: lecture 5

COMSM0075 Information Processing and Brain

`comsm0075.github.io`

September 2020

Snakes and Ladders or Moksha Patam

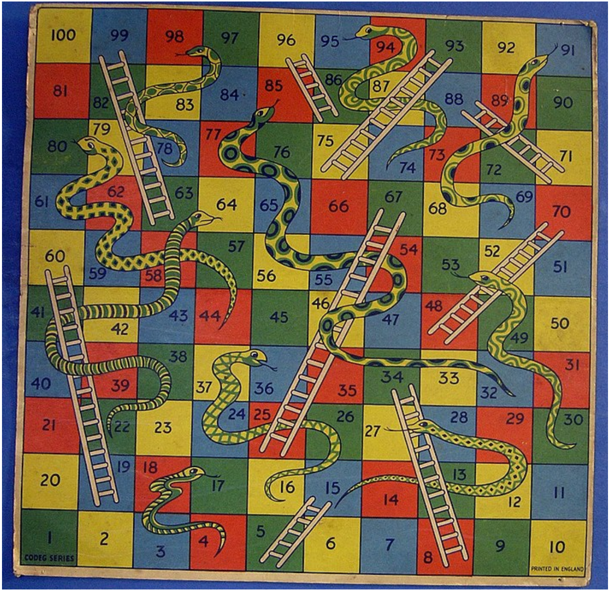

Image from wikipedia.

start at 1

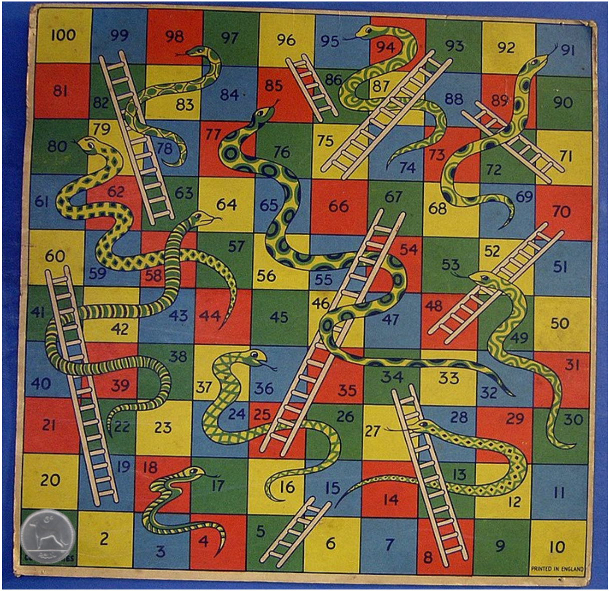

Image from wikipedia.

1→5 (roll 4)

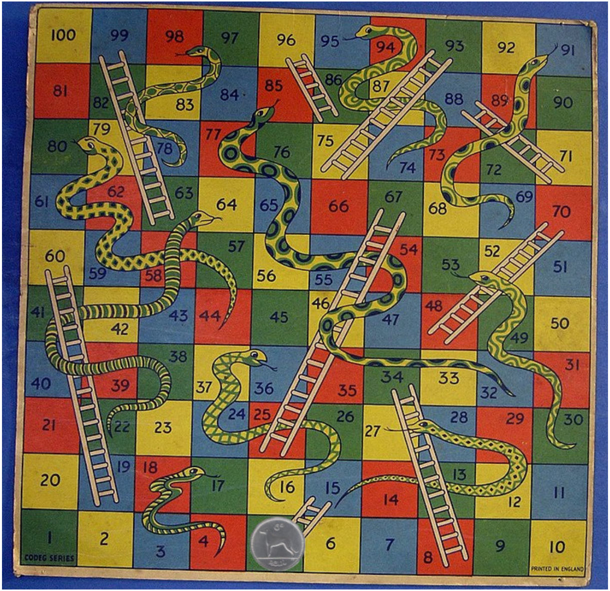

Image from wikipedia.

1→15 (up the ladder)

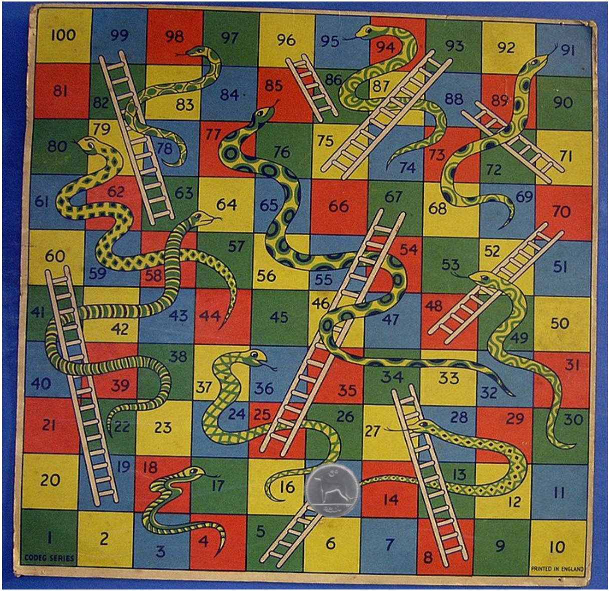

Image from wikipedia.

1→15→19 (roll another 4)

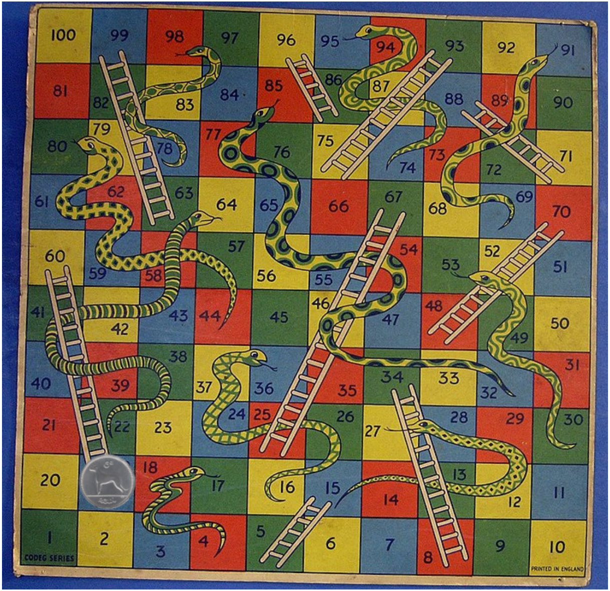

Image from wikipedia.

$1 \rightarrow 15 \rightarrow 19 \rightarrow 60$ (up the ladder)

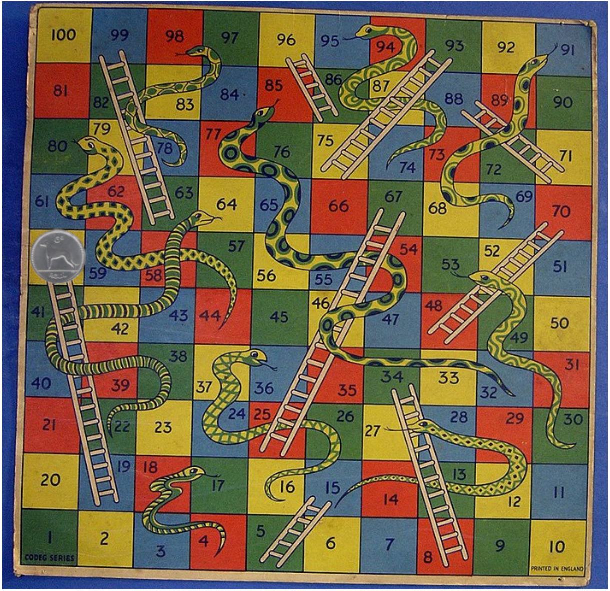

Image from wikipedia.

1→15→19→60→63 (roll a 3)

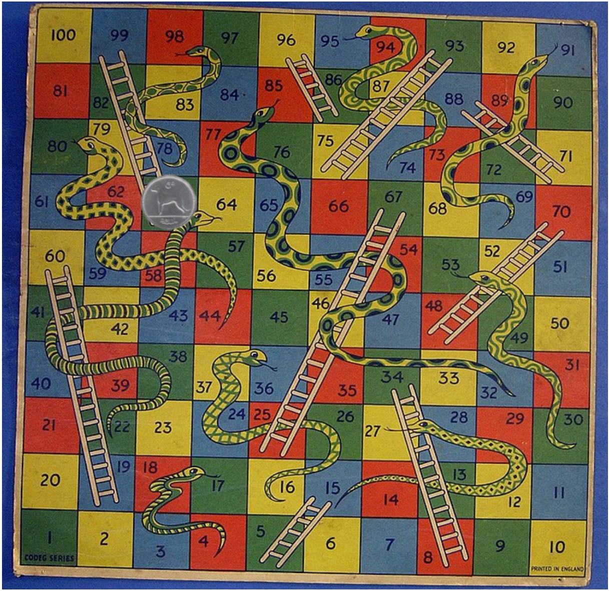

Image from wikipedia.

1→15→19→60→63→99 (up the ladder)

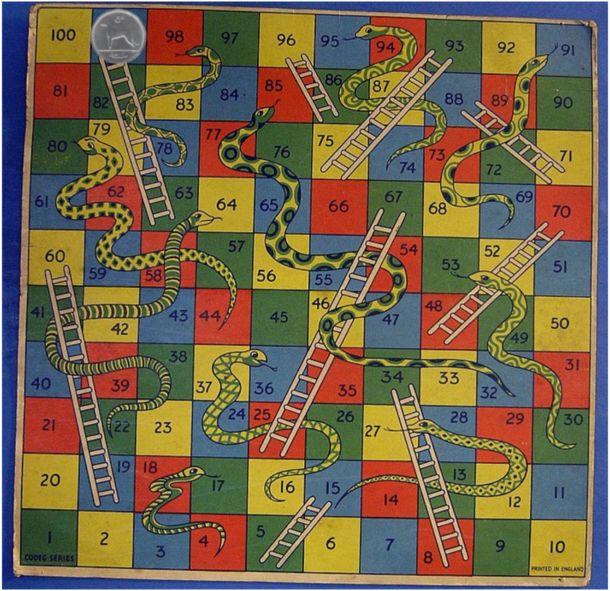

Image from wikipedia.

(4,4,3)

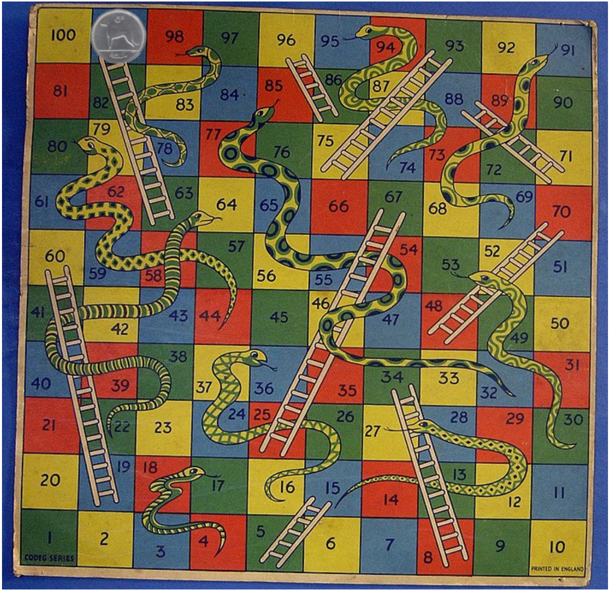

Image from wikipedia.

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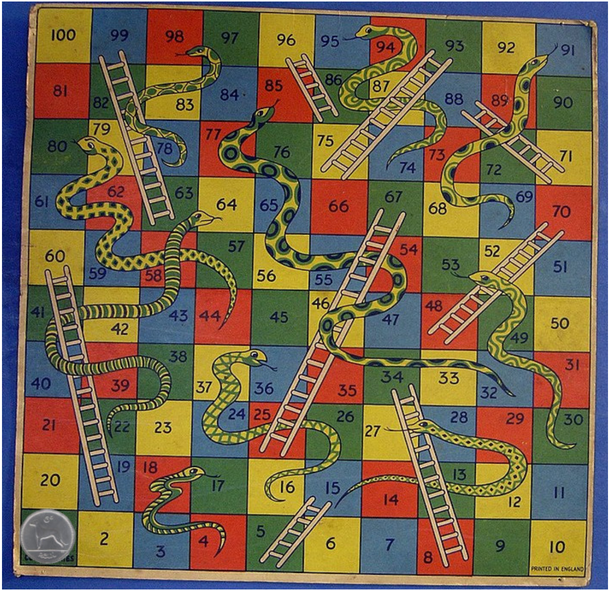


Image from wikipedia.

(8)

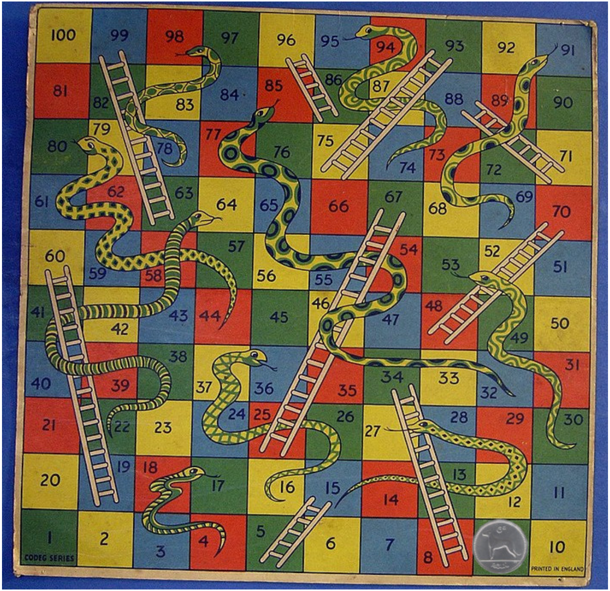


Image from wikipedia.

(8,10)

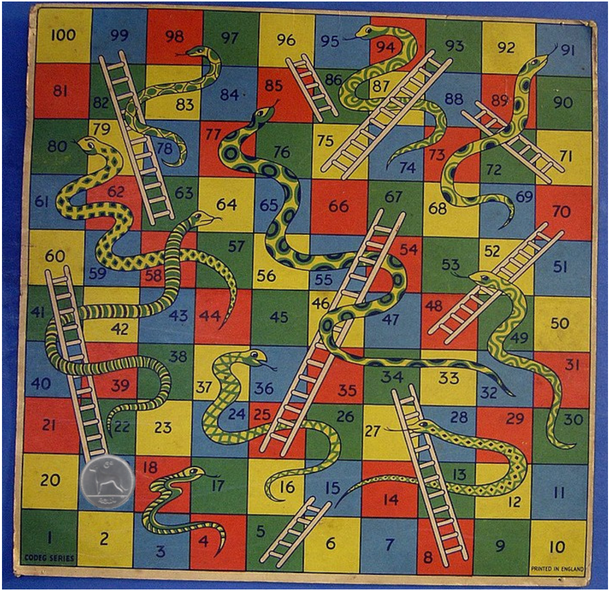


Image from wikipedia.

(8,10)

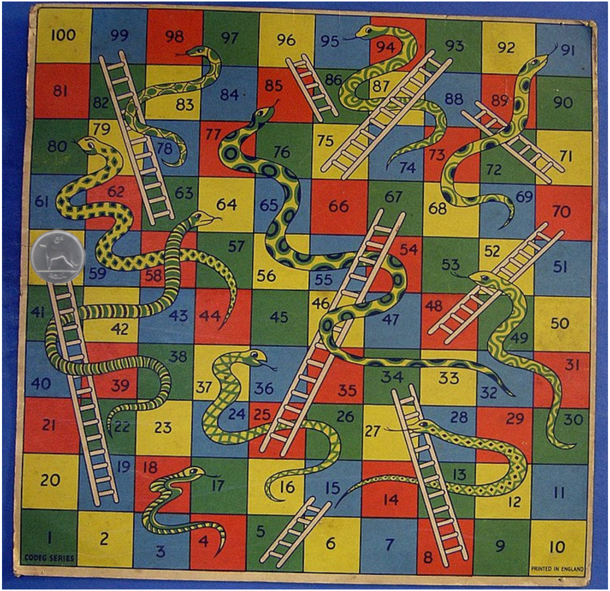


Image from wikipedia.

(8,10,3)

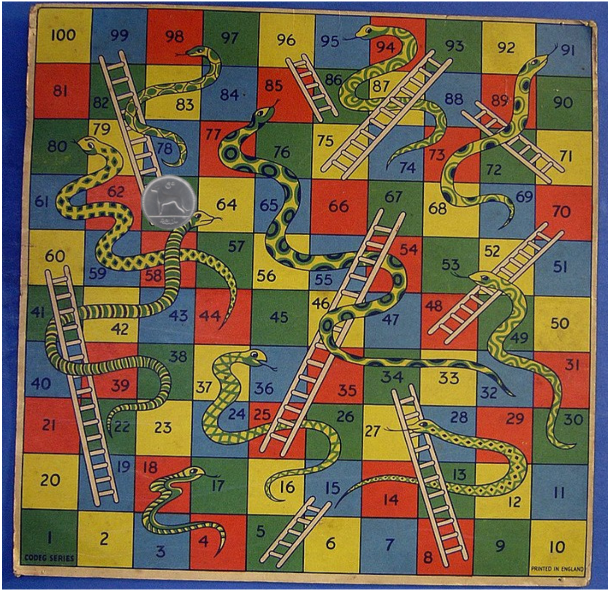


Image from wikipedia.

(8,10,3)

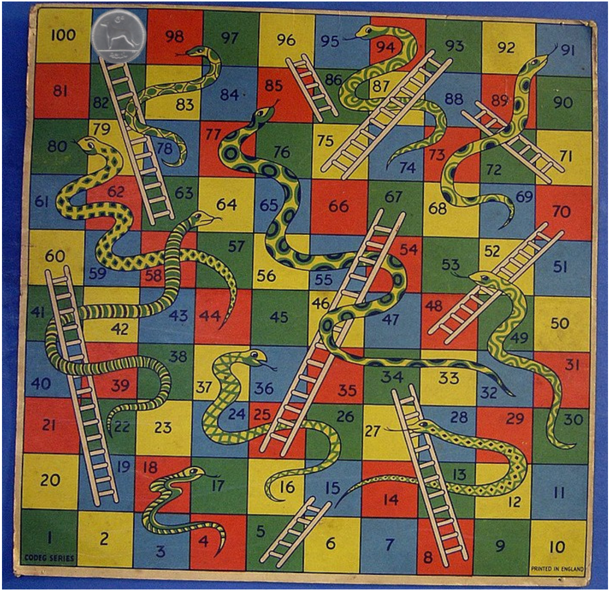
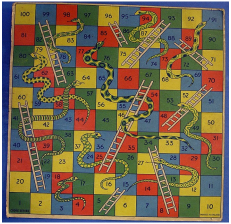


Image from wikipedia.

Probability

X, end of the first move, Y, end of the second move and Z, the end of the third after doing all the snakes and ladders stuff.

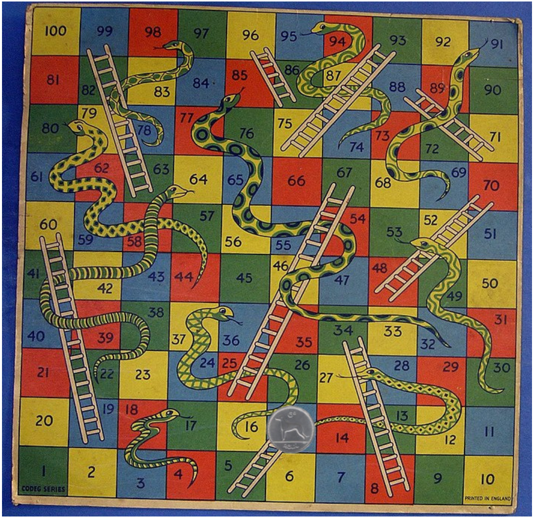


Probability

$$p(Z = 99) = 0.0031$$

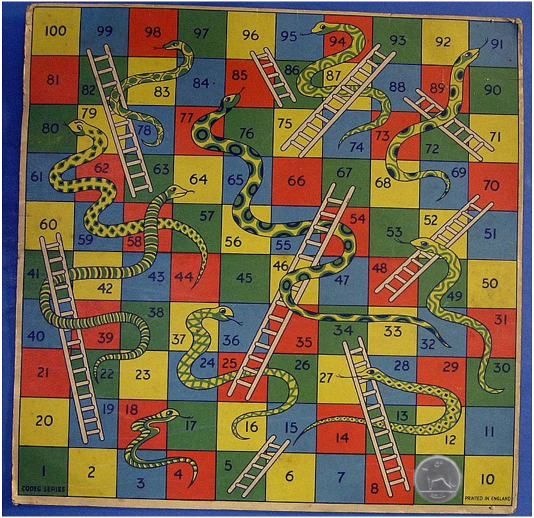
Conditional probability - rolling a four

$$p(Z = 99 | X = 15) = 0.0046$$



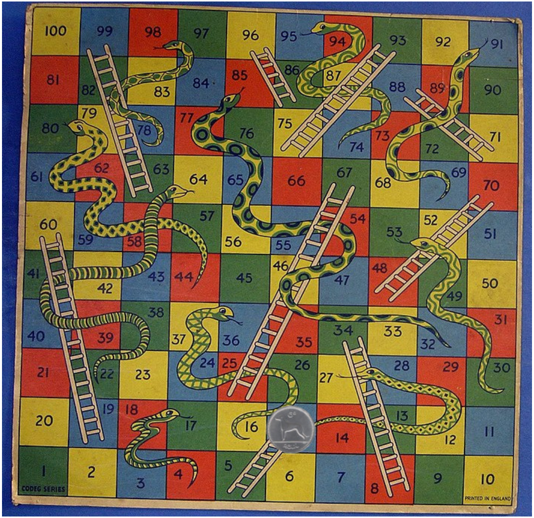
Conditional probability - rolling an eight

$$p(Z = 99 | X = 15) = 0.0046$$



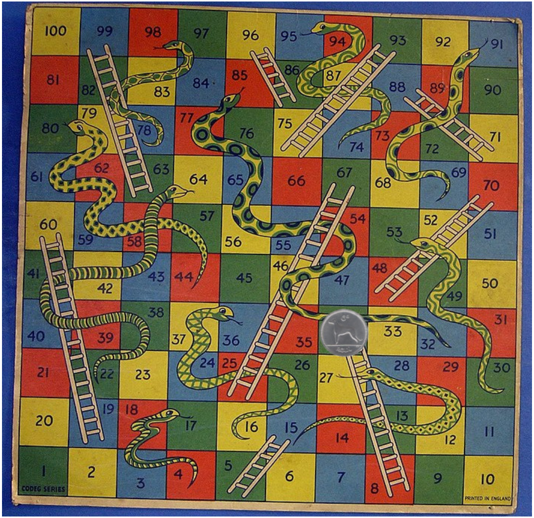
Conditional probability - rolling a 12

$$p(Z = 99|X = 15) = 0.0077$$



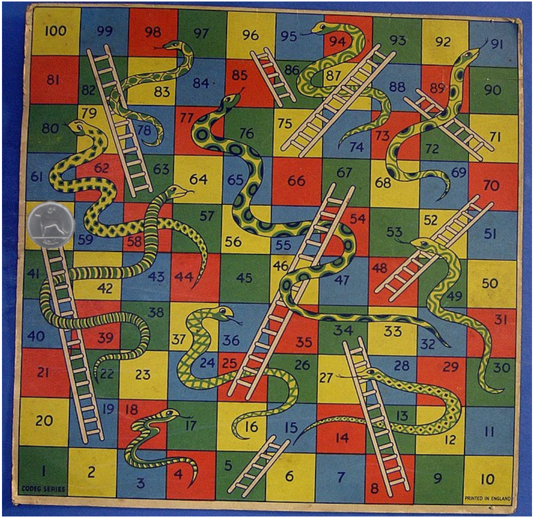
Conditional probability - rolling a seven

$$p(Z = 99|X = 34) = 0$$



Conditional probability - getting to 60

$$p(Z = 99 | Y = 60) = 0.0556$$



Conditional probability - X no longer matters

$$p(Z = 99|Y = 60) = p(Z = 99|Y = 60, X = 15)$$

Conditional probability - X no longer matters

$$p(Z = 99 | Y = 60) = p(Z = 99 | Y = 60, X = 10)$$

Conditional independence

X and Z are conditionally independent:

$$p_{X,Z|Y}(x,z|y) = p_{X|Y}(x|y)p_{X|Z}(z|y)$$

Markov chain

We write

$$X \to Y \to Z$$

if X and Z are conditional indepedent, conditioned on Y:

$$p_{X,Z|Y}(x,z|y) = p_{X|Y}(x|y)p_{X|Z}(z|y)$$

Data processing inequality

if

$$X \rightarrow Y \rightarrow Z$$

then

$$I(X,Y) \geq I(X,Z)$$

with equality if and only if $X \rightarrow Z \rightarrow Y$

Data processing inequality

Processing extracts information, it doesn't add to it.