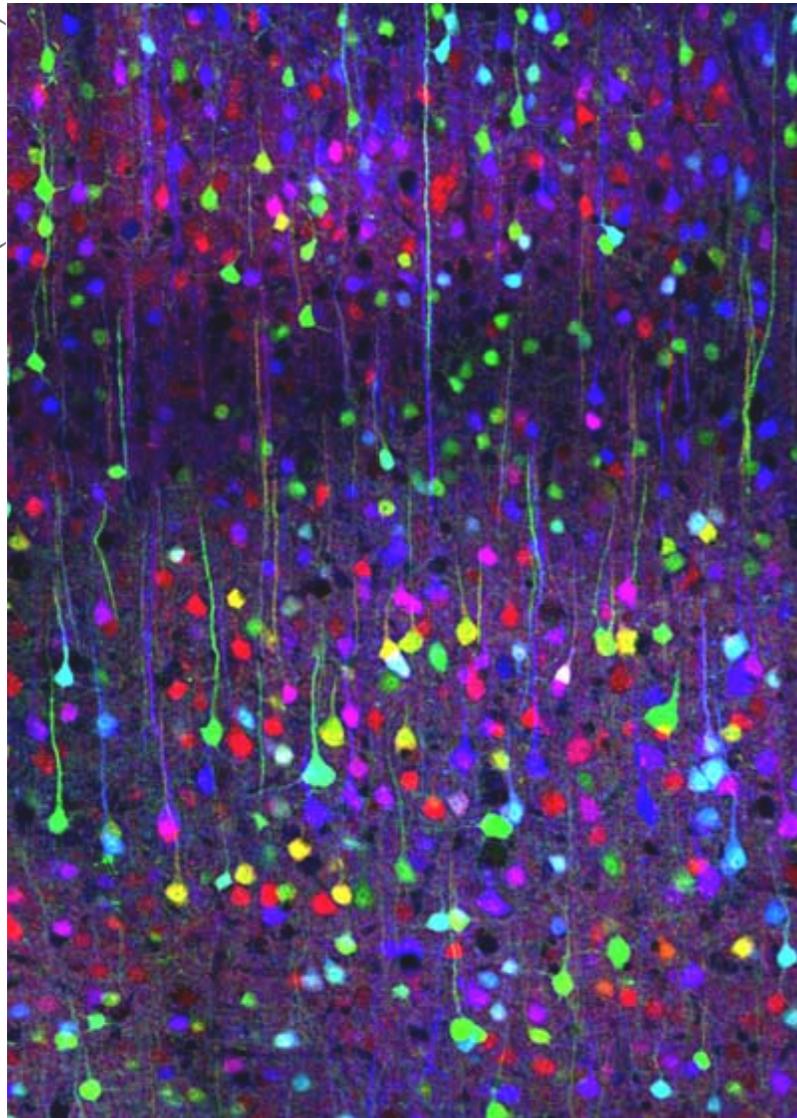


Information Processing & the Brain 2021/2022

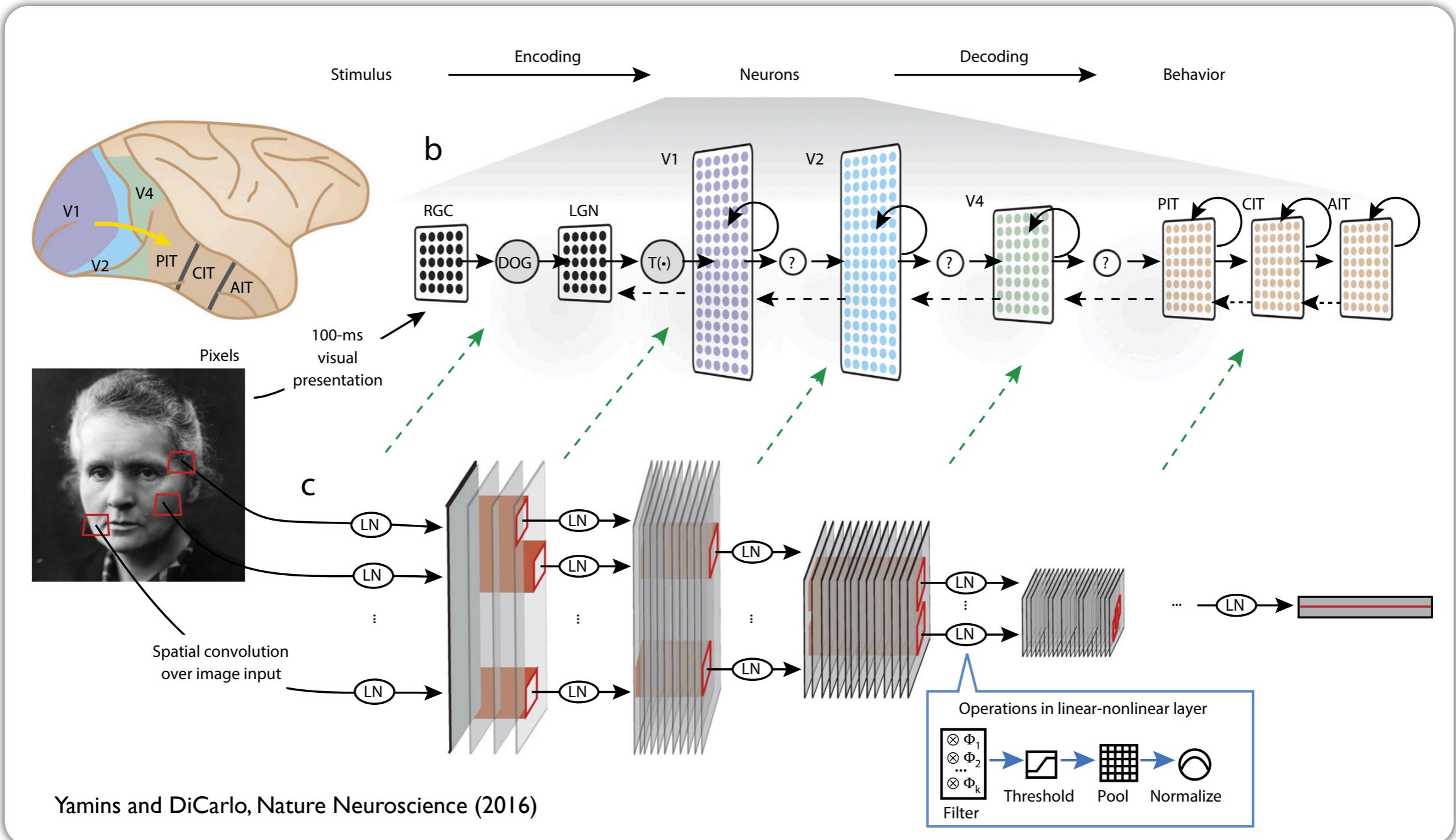


Brainbow (Litchman Lab)



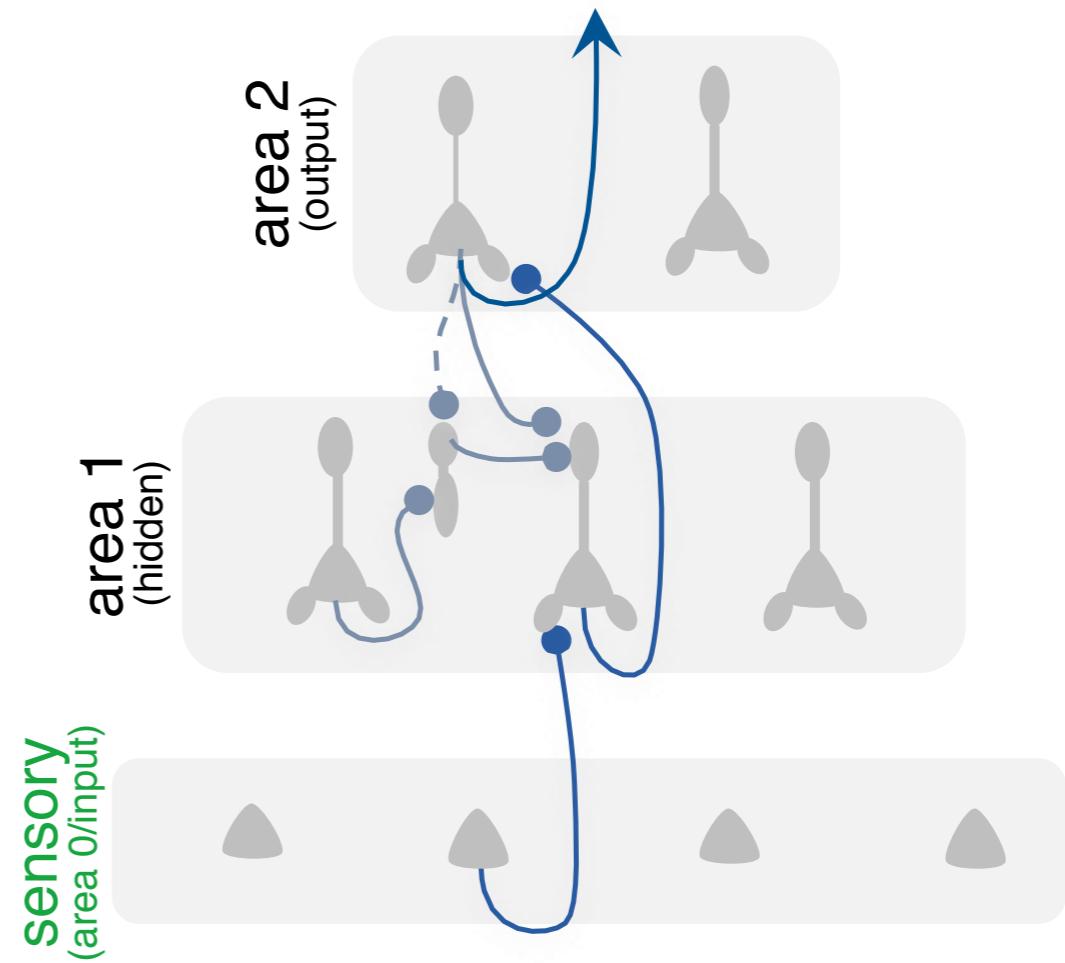
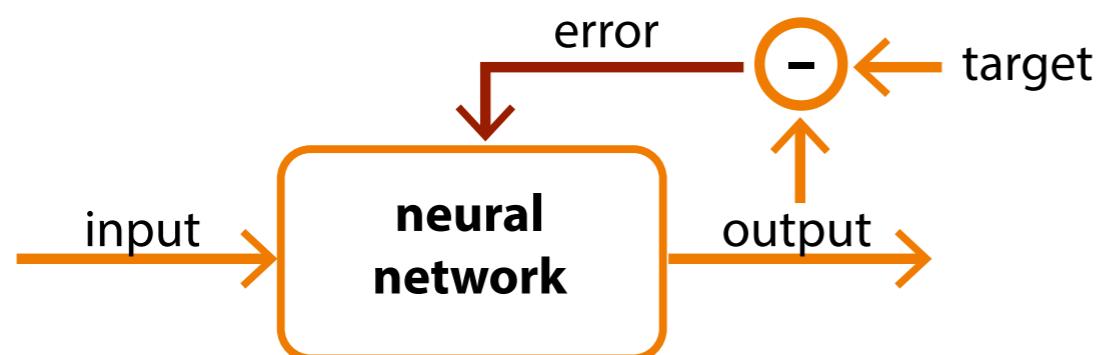
Lecture 4 Neural circuits and learning: Reinforcement Learning

Previously on IPB...



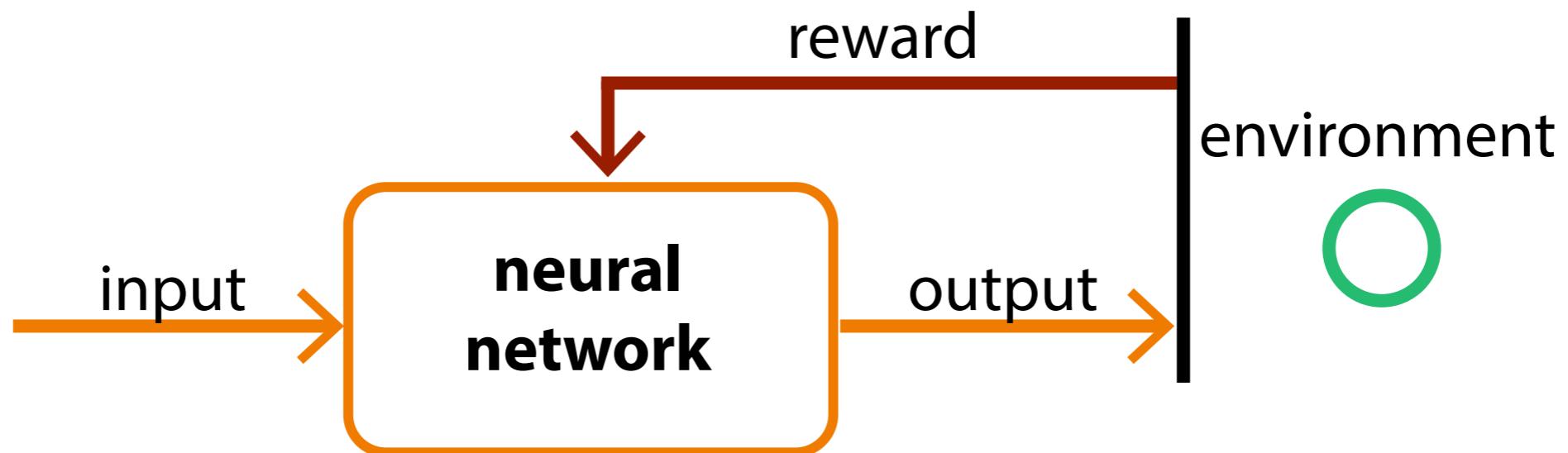
Previously on IPB...

Supervised learning:
Relies on a teaching signal



But, what if an agent/animal is exploring the unknown?

Reinforcement Learning: Learn to navigate in an environment



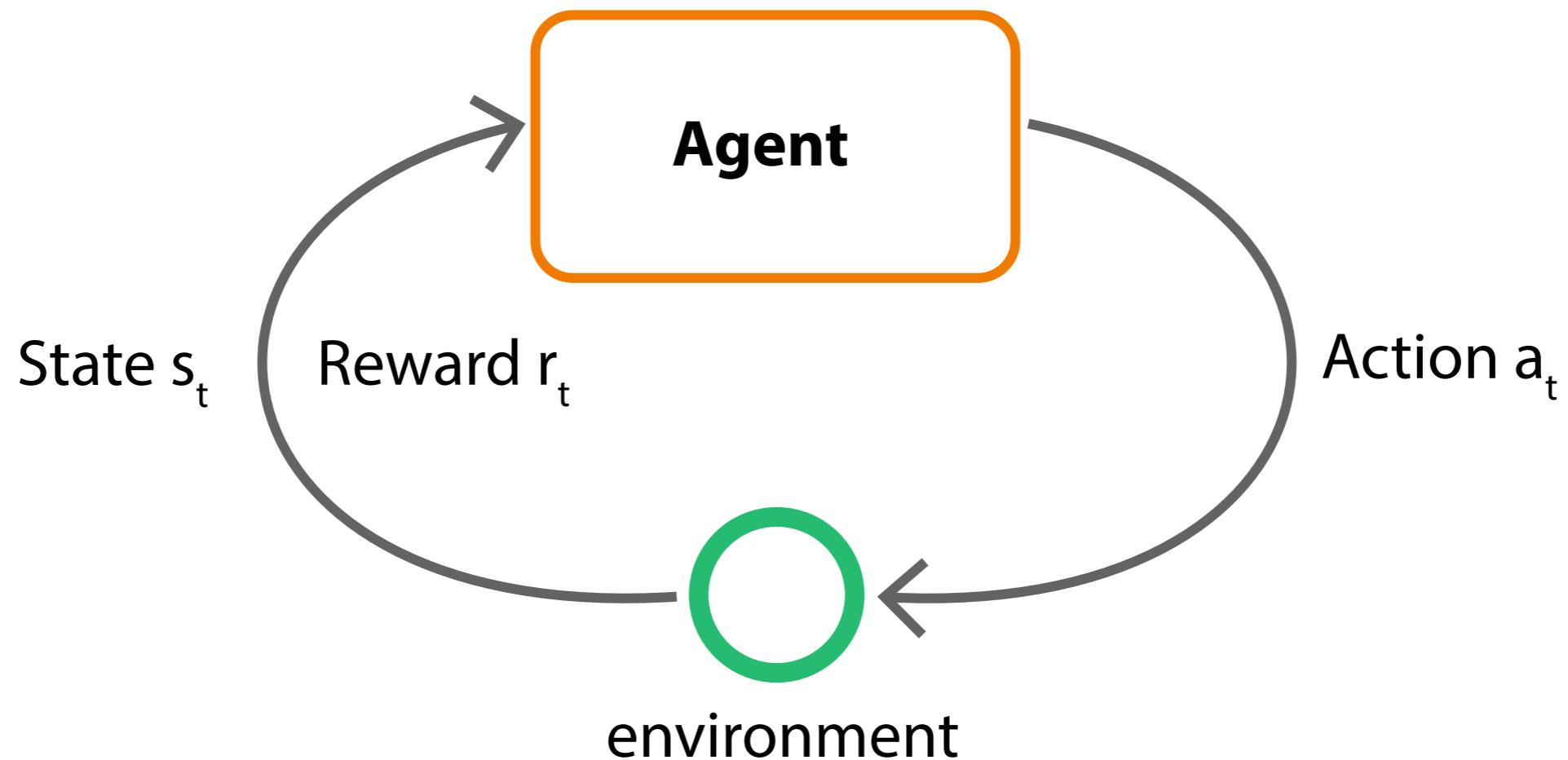
Outline

- I. Reinforcement learning: the basics**
- 2. Temporal difference and Q-learning**
- 3. RL in the brain**
- 4. Deep RL**

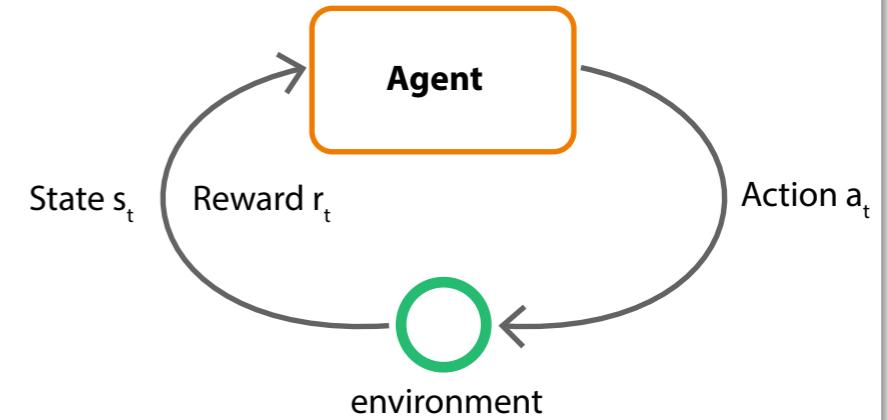
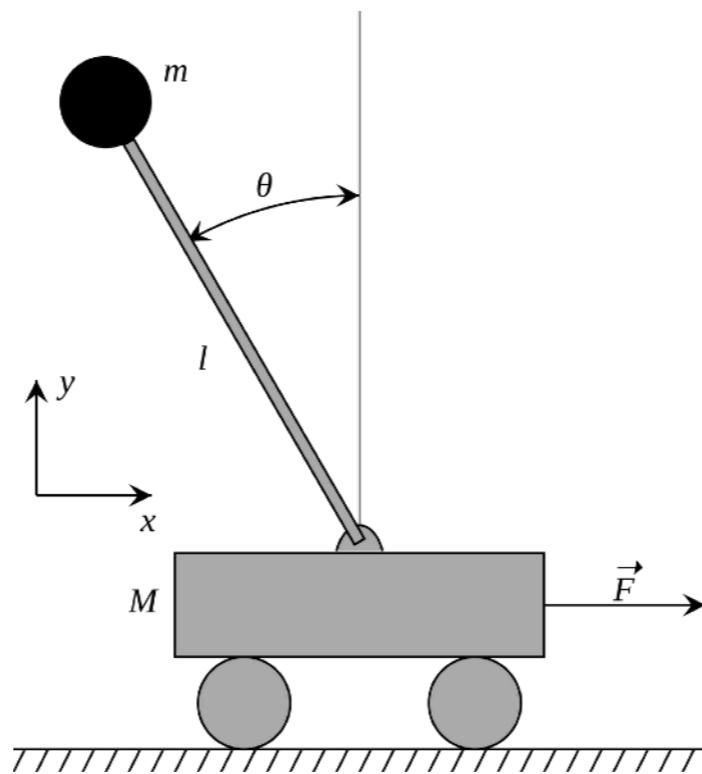
Reinforcement learning

the basics

Goal: Learn which actions to take in order to maximise rewards!



Reinforcement learning examples: cart-pole problem



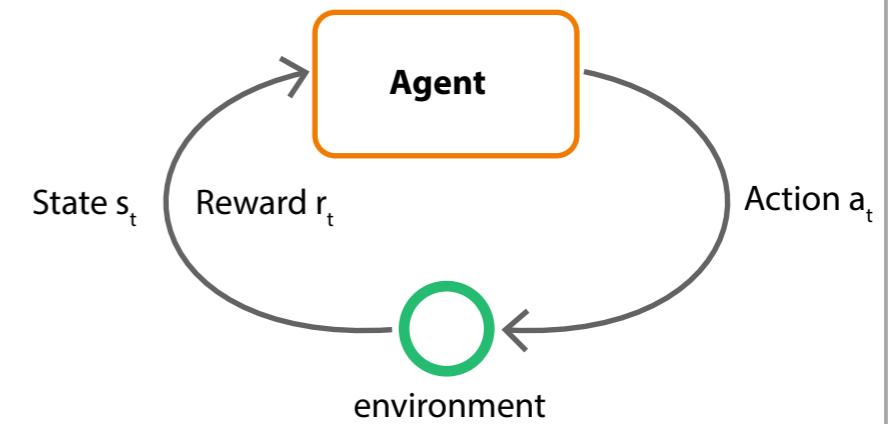
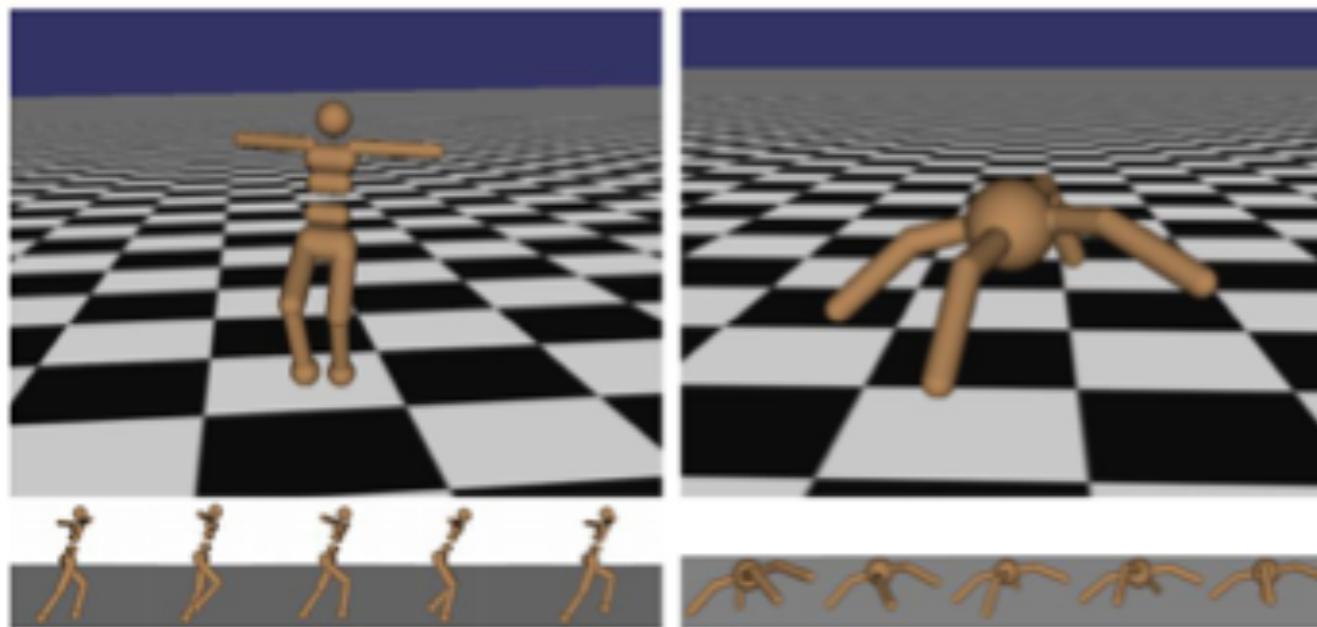
Goal: Balance a pole on top of a movable cart/car

State, s_t : angle, angular speed, position, horizontal velocity

Action(s), a_t : horizontal force applied on the cart

Reward, r_t : 1 at each time step if the pole is upright, 0 (or -1) otherwise

Reinforcement learning examples: learn to move (locomotion)



Goal: Make a simulated animal move forward

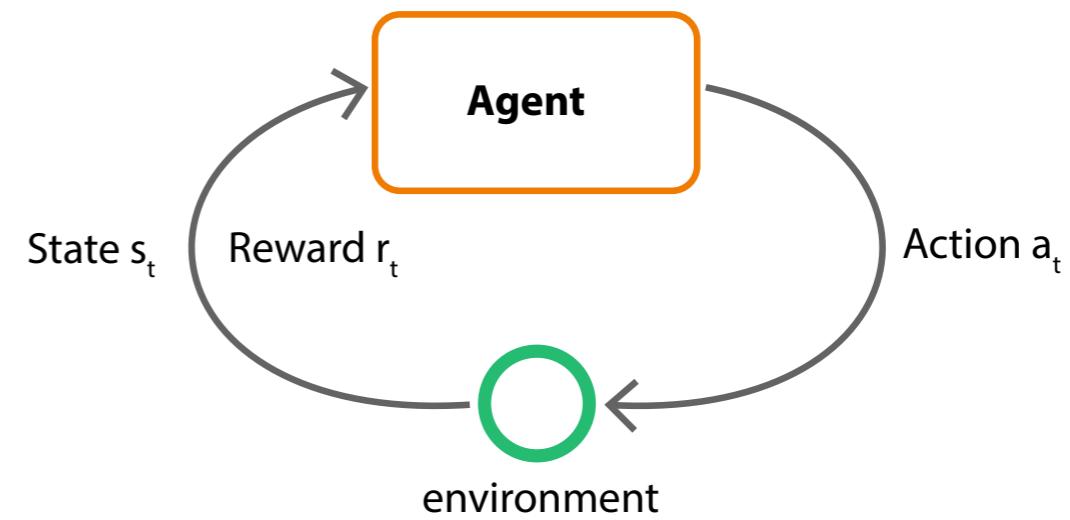
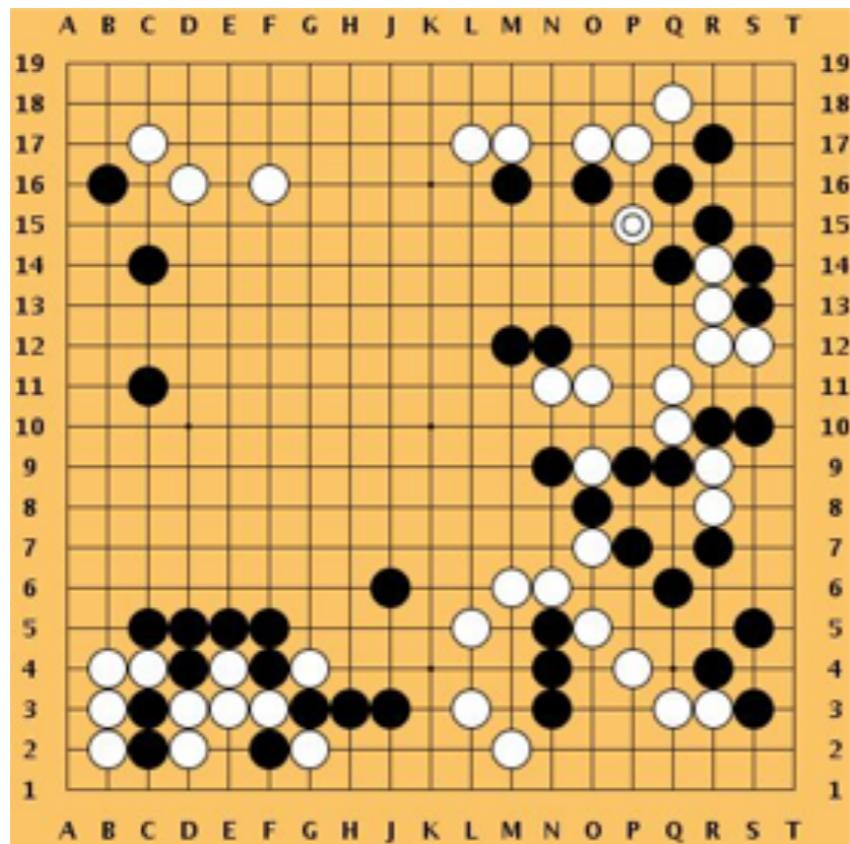
State, s_t : angle and position of joints

Action(s), a_t : Torque (or force) applied to joints

Reward, r_t : 1 for each time step the animal is upright, 0 (or -1) otherwise

Schulman et al. 2016 arXiv:1506.02438

Reinforcement learning examples: Board game Go



Goal: Win the game!

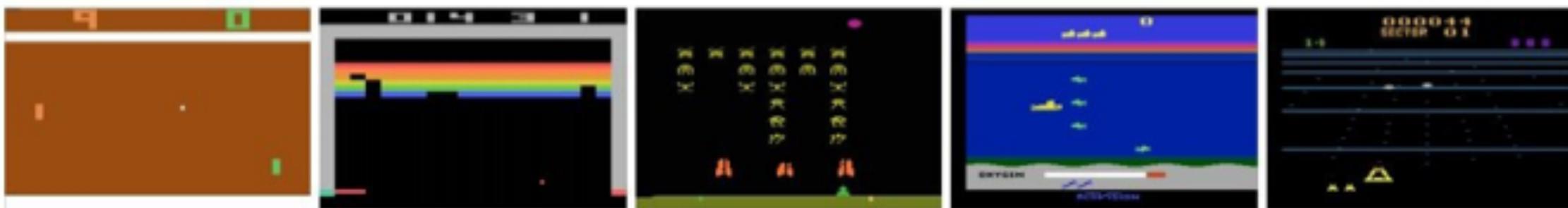
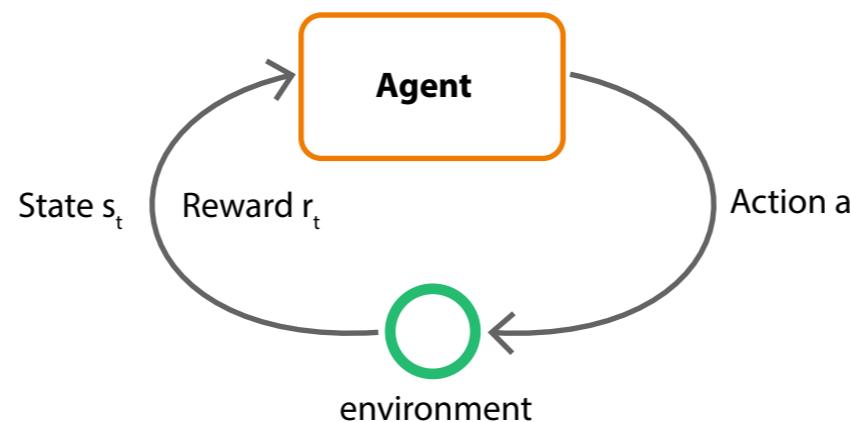
State, s_t : Position of the game pieces [agent has access to ‘clean’ state space!]

Action(s), a_t : Where to place next piece

Reward, r_t : 1 if won, -1 if lost (super sparse reward!)

Silver et al. Nature (2017)

Reinforcement learning examples: playing Atari games



Goal: Maximise game score

State, s_t : raw pixels of the current game state (high dimensional state!)

Action(s), a_t : Game controls (e.g. left, right, up, down, fire, etc.)

Reward, r_t : Score changes (increase, decrease) at each time step

[note: reward has more info. than just 1,0,-1]

Mnih et al, Nature (2013)

Reinforcement learning

General framework: Markov Decision Process

Markov property: “The future is independent of the past given the present.”

In mathematical terms, can be defined as the following transition probability of the future given the immediate past:

$$P(S_{t+1} | S_t) = P(S_{t+1} | S_1, \dots, S_t)$$

This assumes that the *current state* S_t *has all the information needed to make a decision about next state* S_{t+1} (note: information of the sequence of states that lead to a particular state is lost).

Sutton and Barto book (2018)

Reinforcement learning

General framework: Markov Decision Process

Markov decision process: Is a Markov Process defined by the following tuple:

$$(S, A, P, R, \gamma)$$

S : Set of states

A : Set of actions

P : Transition probability $P_{ss'}^a = \mathbb{P}(S_{t+1} = s' | S_t = s, A_t = a)$

R : Reward function (for state, action pairs) $R_s^a = \mathbb{E}(R_{t+1} | S_t = s, A_t = a)$

γ : discount factor [0,1]

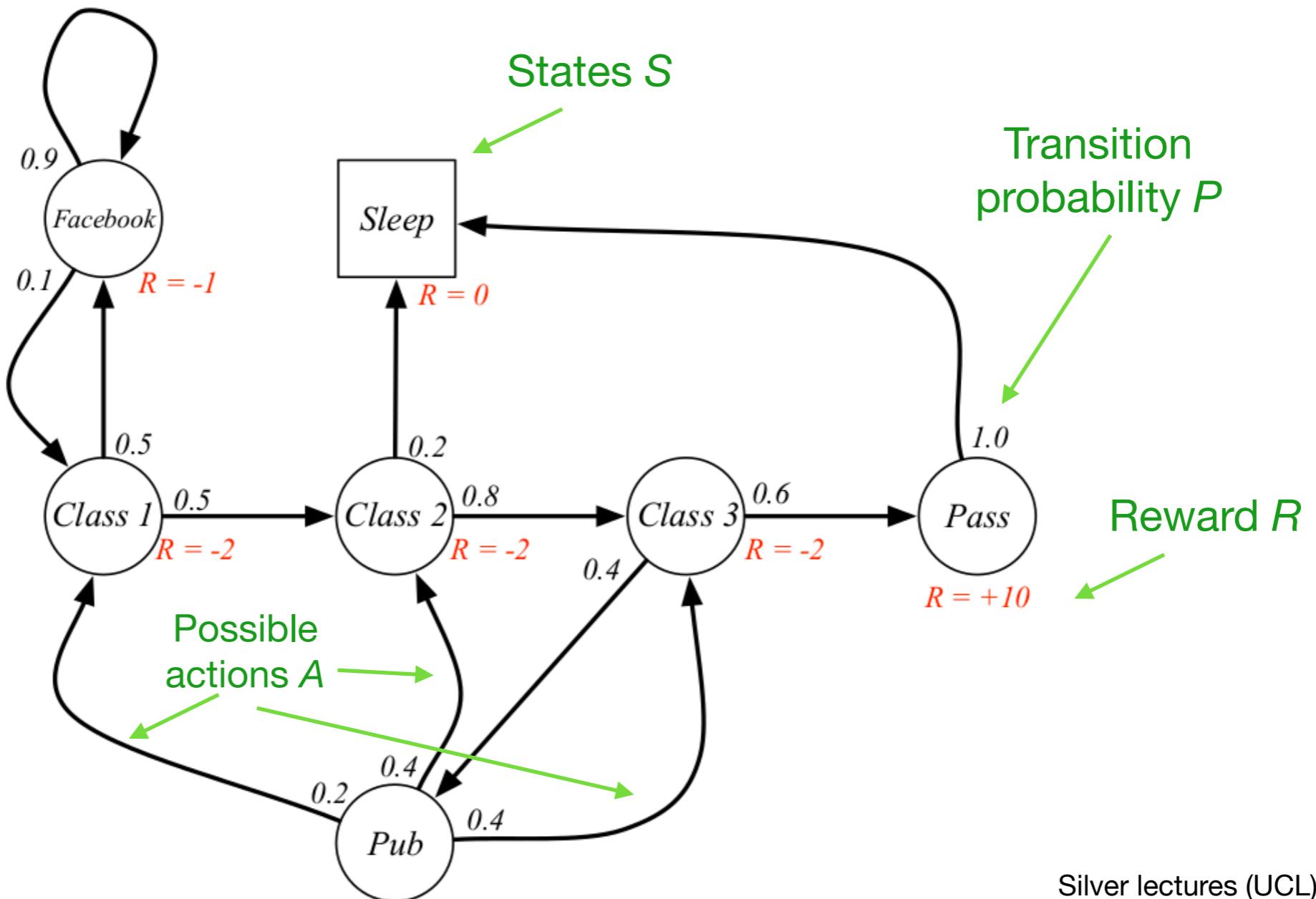
$\gamma \sim 0.9$ its usually set to 0.9 to gradually discount future rewards

Sutton and Barto book (2018)

Reinforcement learning

General framework: Markov Decision Process

Example of a **Markov decision process** for a student:



Reinforcement learning

Goal, policy and reward..

Goal: find **policy** that maximises **discounted reward**.

Policy: Distribution over actions given states, it defines the behaviour of an agent:

$$\pi(a | s) = P(A_t = a | S_t = s)$$

Discounted reward: Total (discounted) reward received by the agent:

$$R_t = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \dots = \sum_{t=0}^{\infty} \gamma^t r_t$$

Sutton and Barto book (2018)

Reinforcement learning

Value-state function

It is useful to define the **value** of a state as the expected cumulative reward:

$$V^\pi(s) = \mathbb{E} \left[\sum_{t=0}^{\infty} \gamma^t r_t \mid s_0 = s, \pi \right]$$

Sutton and Barto book (2018)

Reinforcement learning

Bellman equation

But how to update the Value function when faced with a new environment?

$$V^\pi(s) = \mathbb{E} \left[\sum_{t=0}^{\infty} \gamma^t r_t \mid s_0 = s, \pi \right]$$

For that we use the *Bellman equation* which gives the expectation over the policy for t and t+1:

$$V^\pi(s) = \mathbb{E}_\pi[r_t + \gamma V^\pi(s_{t+1}) \mid s_t = s]$$

Sutton and Barto book (2018)

Reinforcement learning

Temporal difference (TD) learning

TD learning: model-free reinforcement learning method which learns by bootstrapping based on current value function estimates.

From the *Bellman equation* we get an iterative method for updating the value function (TD learning):

$$\underbrace{V(S_t)}_{\text{value}} = V(S_t) + \overbrace{\left(\underbrace{R_{t+1}}_{\text{reward}} + \gamma \underbrace{V(S_{t+1})}_{\text{future value}} - V(S_t) \right)}^{\delta, \text{ prediction error}}$$

γ : discount factor

Note: this is tabular TD(0) because it only looks one state ahead.

Sutton and Barto book (2018)

Reinforcement learning

Temporal difference (TD) learning

$$\underbrace{V(S_t)}_{\text{value}} = V(S_t) + (\underbrace{R_{t+1}}_{\text{reward}} + \gamma \underbrace{V(S_{t+1})}_{\text{future value}} - V(S_t))$$

δ , prediction error
 γ : discount factor

Tabular TD(0) for estimating v_π

Input: the policy π to be evaluated

Algorithm parameter: step size $\alpha \in (0, 1]$

Initialize $V(s)$, for all $s \in \mathcal{S}^+$, arbitrarily except that $V(\text{terminal}) = 0$

Loop for each episode:

 Initialize S

 Loop for each step of episode:

$A \leftarrow$ action given by π for S

 Take action A , observe R, S'

$V(S) \leftarrow V(S) + \alpha[R + \gamma V(S') - V(S)]$

$S \leftarrow S'$

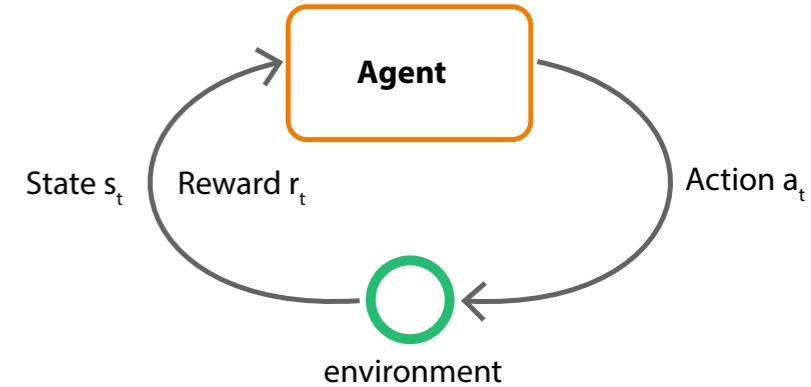
 until S is terminal

Note: In the algorithm above it is assumed that R is observed in $t+1$ (i.e. in the immediate future).

Sutton and Barto book (2018)

Reinforcement learning

Gridworld example

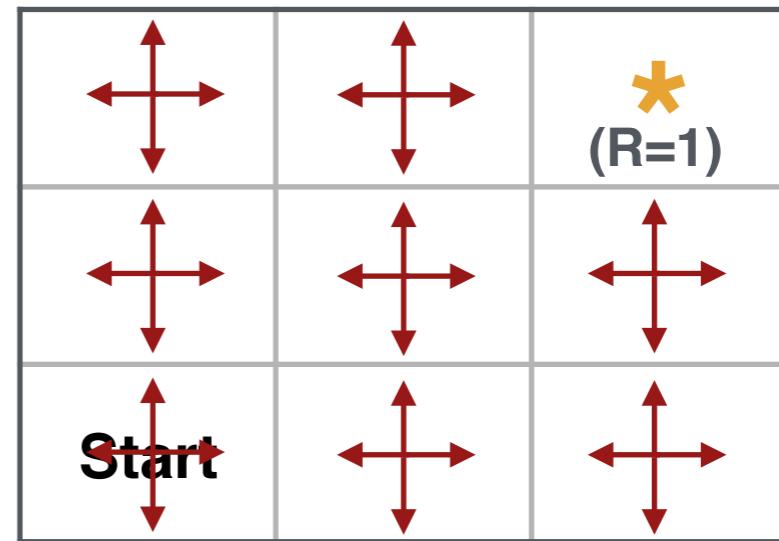


Actions = {left, right, up, down}

Values

0	0	(R=1)
0	0	0
Start	0	0

Policy

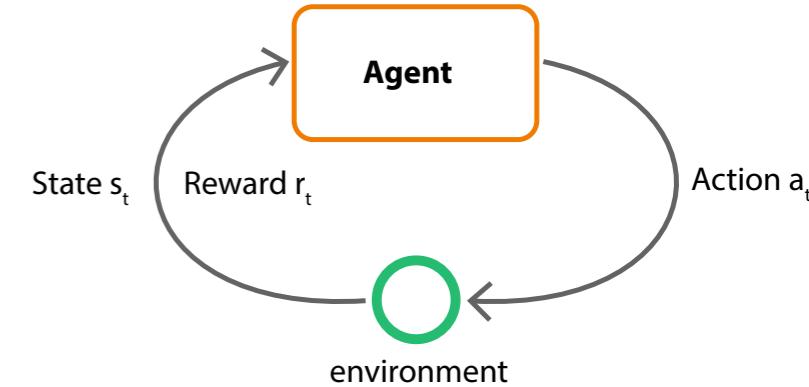


1. Define Value-state table (we assume that values are zero initially)
2. Define Policy (we use a random policy to start with)

Sutton and Barto book (2018)

Reinforcement learning

Gridworld example

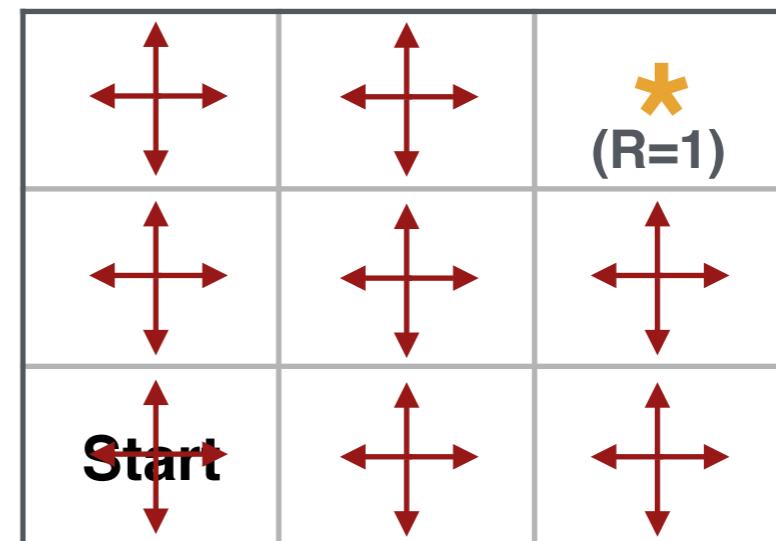


Values

(example after a few iterations; note that these values are just an illust.)

0.6	0.25	 (R=1)
0.3	0.5	0.8
Start	0.3	0.2

Policy

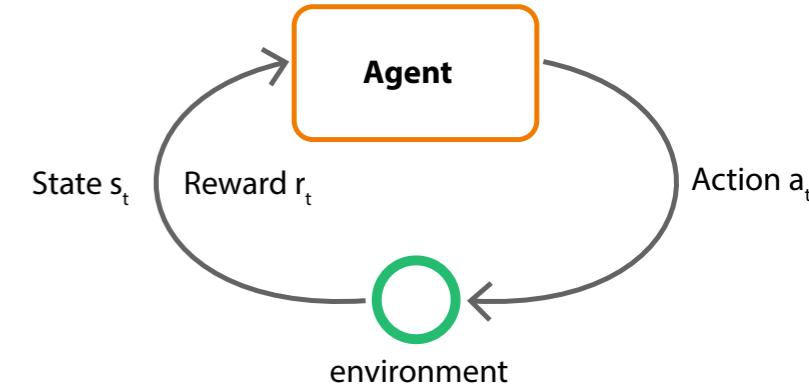


1. Define Value-state table (we assume that values are zero initially)
2. Define Policy (we use a random policy to start with)
3. Sample actions-states pairs (i.e. $s_0, a_0, s_1, a_1, s_2, a_2 \dots$)
4. Until a stop criteria is met (e.g. Terminal state is achieved or after n iterations)

Sutton and Barto book (2018)

Reinforcement learning

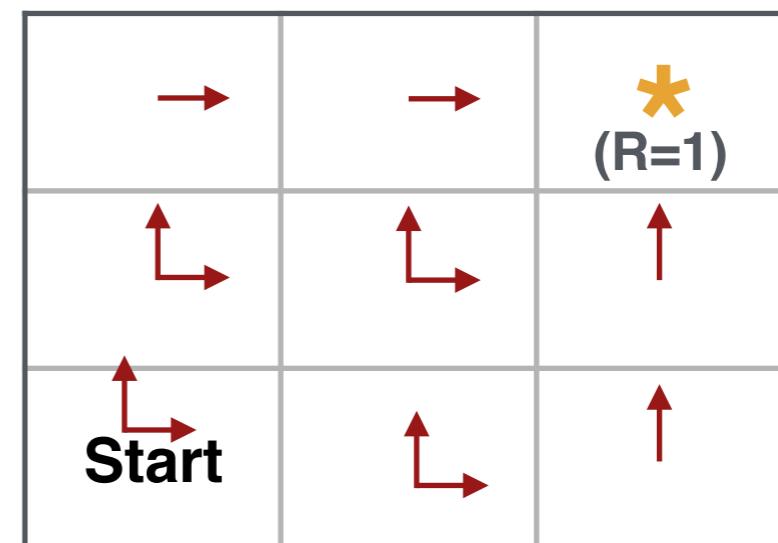
Gridworld example



Values
(example steady-state values)

0.6	0.8	(R=1)
0.3	0.6	0.8
Start	0.3	0.6

Policy *
(optimal policy)



1. Define Value-state table (we assume that values are zero initially)
2. Define Policy (we use a random policy to start with)
3. Sample actions-states pairs (i.e. $s_0, a_0, s_1, a_1, s_2, a_2 \dots$)
4. Until a stop criteria is met (e.g. Terminal state is achieved or after n iterations)

Sutton and Barto book (2018)

Quiz time!

**Please go to BB
and solve quiz 4.**

It should take you just a couple of minutes.

Reinforcement learning

Q-learning

In Q-learning we learn a action-value function.

$$Q(S_t, A_t) = \underbrace{Q(S_t, A_t)}_{\text{value}} + \overbrace{\left(\underbrace{R_{t+1}}_{\text{reward}} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t) \right)}^{\delta, \text{ prediction error}}$$
$$\qquad\qquad\qquad \underbrace{\qquad\qquad\qquad}_{\text{future value}}$$

Q-learning (off-policy TD control) for estimating $\pi \approx \pi_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$

Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(\text{terminal}, \cdot) = 0$

Loop for each episode:

 Initialize S

 Loop for each step of episode:

 Choose A from S using policy derived from Q (e.g., ε -greedy)

 Take action A , observe R, S'

$$Q(S, A) \leftarrow Q(S, A) + \alpha [R + \gamma \max_a Q(S', a) - Q(S, A)]$$

$$S \leftarrow S'$$

 until S is terminal

Sutton and Barto book (2018)

Reinforcement learning

TD vs Q-learning

TD-learning uses the value of the **state given by the policy (on policy method)**:

$$\underbrace{V(S_t)}_{\text{value}} = V(S_t) + \left(\underbrace{R_{t+1}}_{\text{reward}} + \gamma \underbrace{V(S_{t+1})}_{\text{future value}} - V_t \right)^{\delta, \text{ prediction error}}$$

Q-learning uses the **best possible action**, not necessarily the one given by the policy (off policy method):

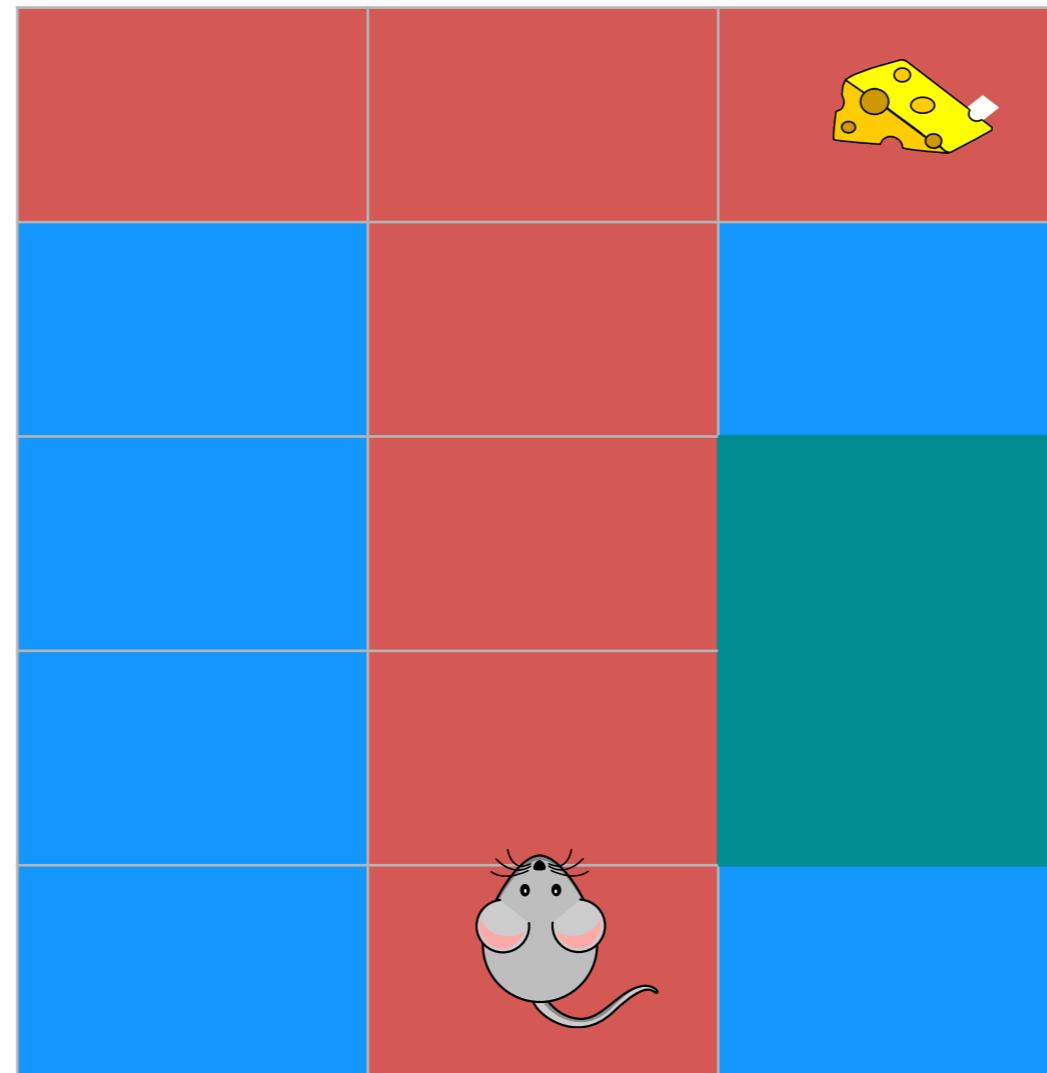
$$\underbrace{Q(S_t, A_t)}_{\text{value}} = Q(S_t, A_t) + \left(\underbrace{R_{t+1}}_{\text{reward}} + \gamma \max_a \underbrace{Q(S_{t+1}, a)}_{\text{future value}} - Q(S_t, A_t) \right)^{\delta, \text{ prediction error}}$$

Sutton and Barto book (2018)

Reinforcement learning in the brain

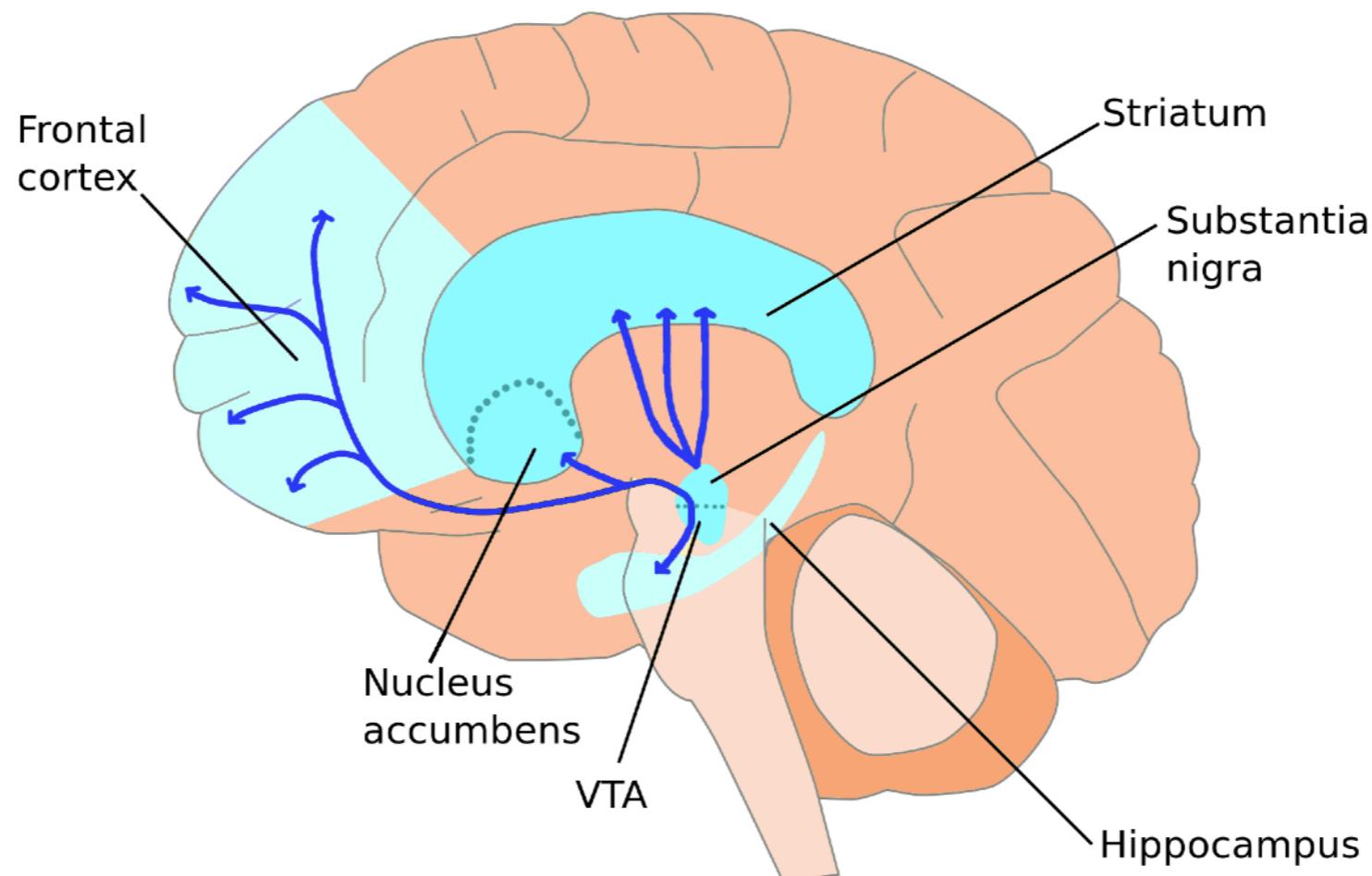
Typical experiment

Typical T-maze experiment:



Reward pathways in the brain

Specific brain regions encode reward signals. A classical example is **dopamine** which is the neuromodulator responsible for reward-driven behaviour, which is released from regions such as the **Ventral tegmental area** (VTA).



Reward prediction errors dopamine

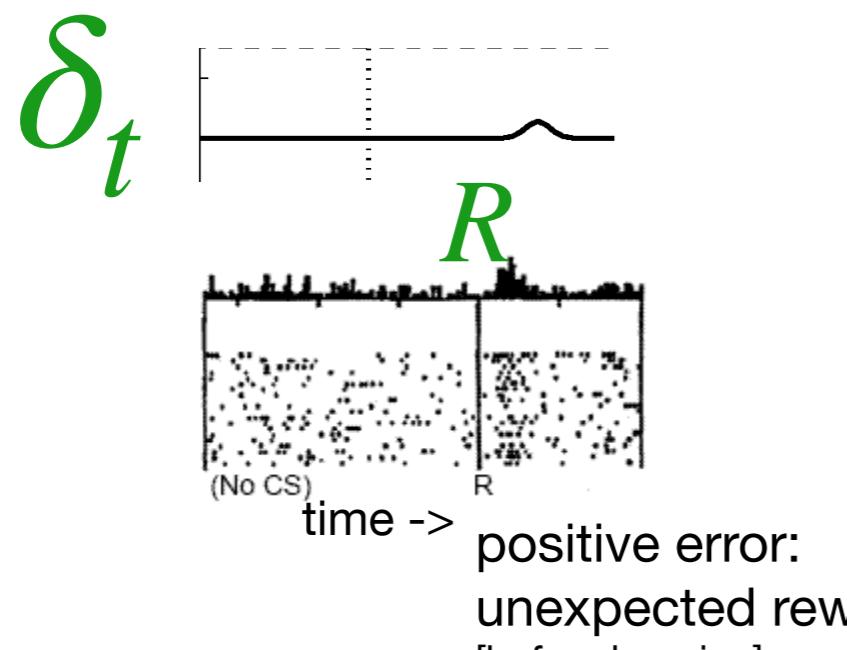
Recall that TD-learning suggests a specific
reward prediction error:

$$\underbrace{V(S_t)}_{\text{value}} = V(S_t) + \overbrace{\left(\underbrace{R_{t+1}}_{\text{reward}} + \gamma \underbrace{V(S_{t+1})}_{\text{future value}} - V(S_t) \right)}^{\delta, \text{ prediction error}}$$

Reward prediction errors dopamine

Reward prediction error:

$$\delta_t = R_{t+1} + \gamma V(S_{t+1}) - V(S_t)$$



positive error:
unexpected reward
[before learning]

CS: conditioned stimulus

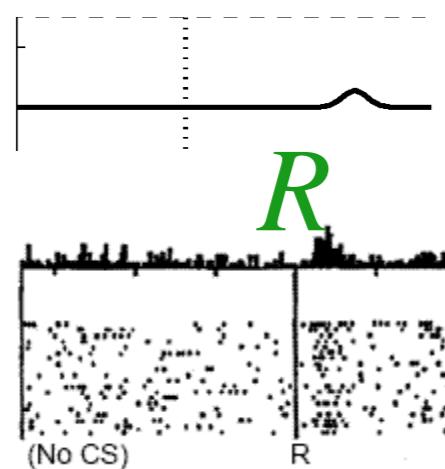
Schultz et al. (1997)

Reward prediction errors dopamine

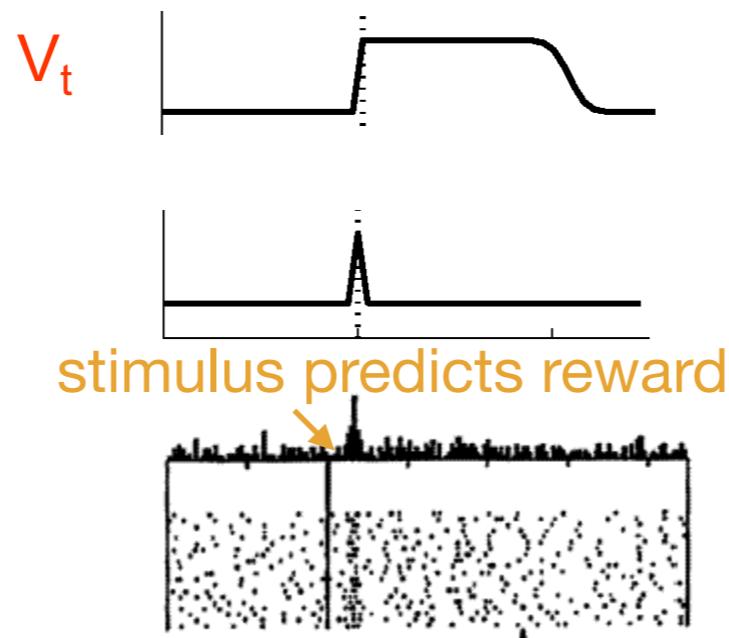
Reward prediction error:

$$\delta_t = R_{t+1} + \gamma V(S_{t+1}) - V(S_t)$$

$$\delta_t$$



positive error



no error: reward
expected and given

CS: conditioned stimulus

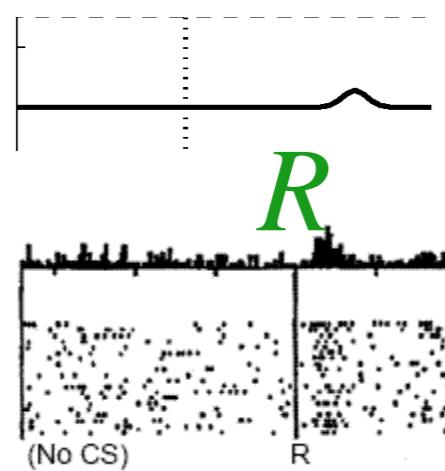
Schultz et al. (1997)

Reward prediction errors dopamine

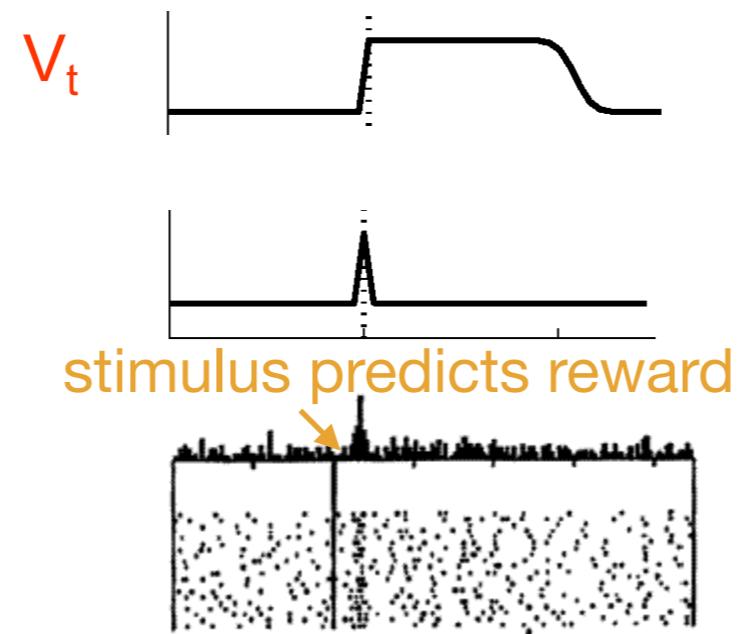
Reward prediction error:

$$\delta_t = R_{t+1} + \gamma V(S_{t+1}) - V(S_t)$$

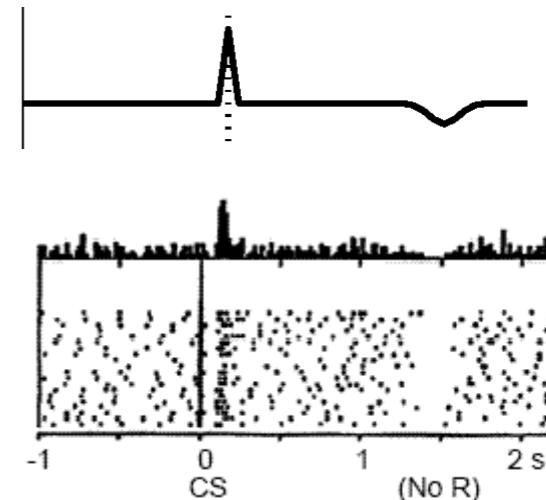
$$\delta_t$$



positive error



no error: reward
expected and given



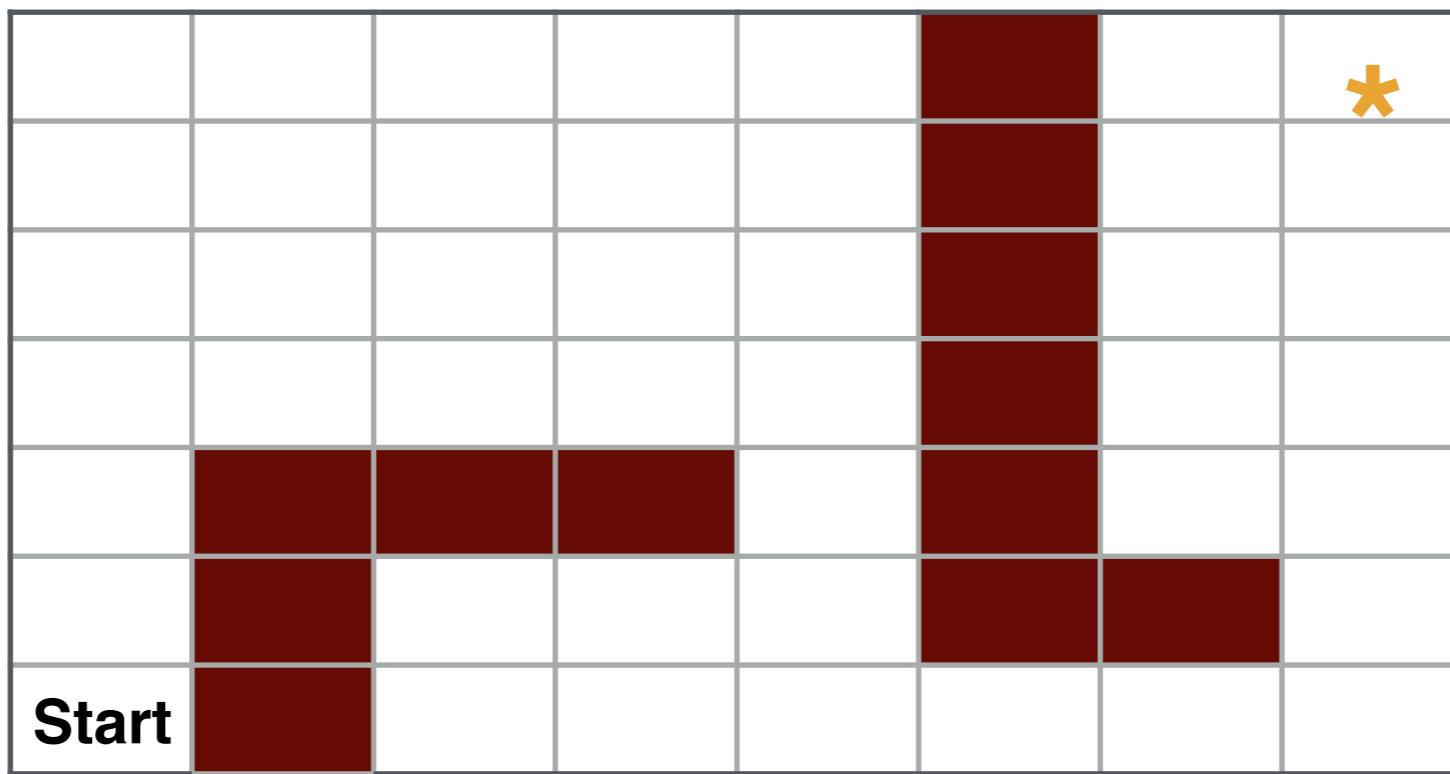
negative error: no reward

CS: conditioned stimulus

Schultz et al. (1997)

What if the state space is **much bigger**

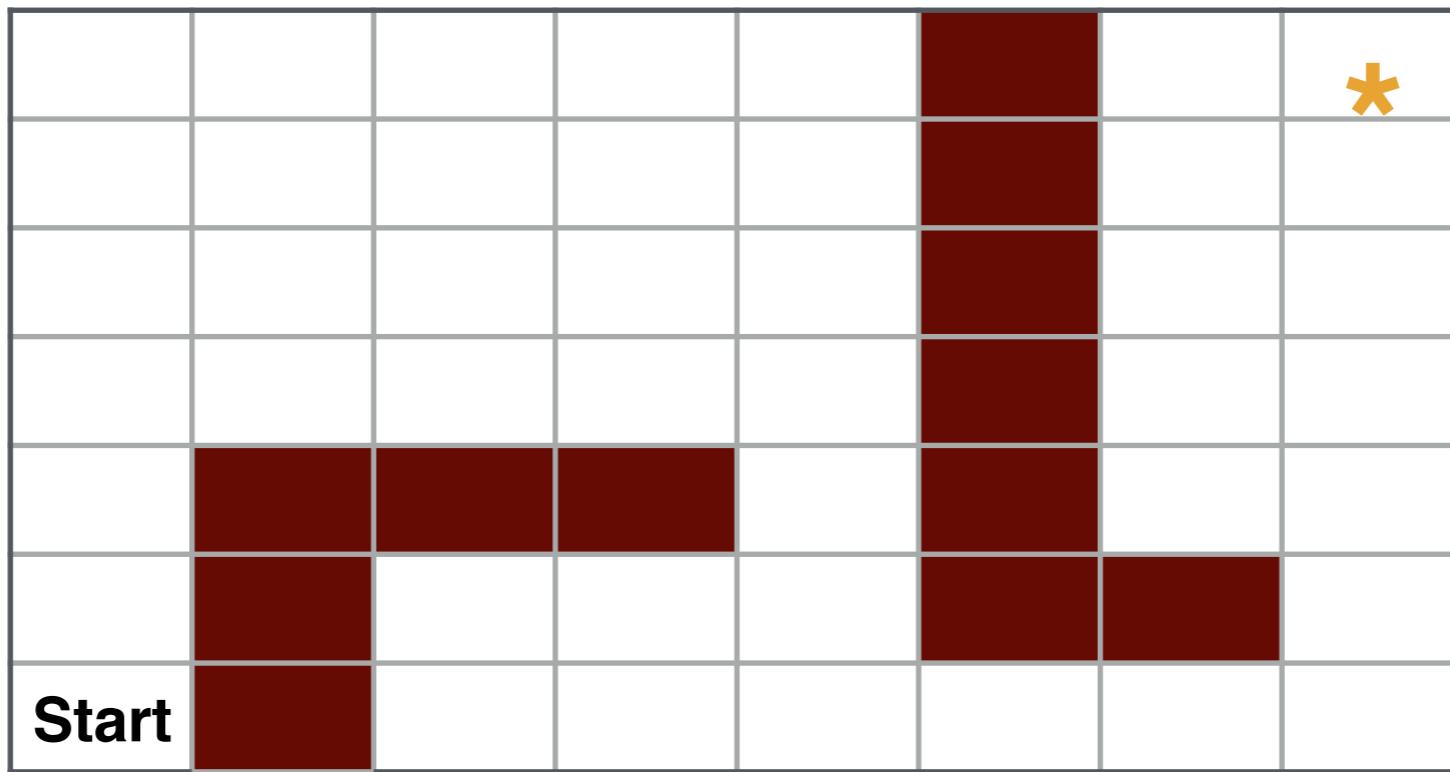
(for example hundreds or millions of states)?



What if the state space is **much bigger**

(for example hundreds or millions of states)?

Possible solutions: Use exploration-exploitation strategies, and/or function approximates (e.g. neural networks).



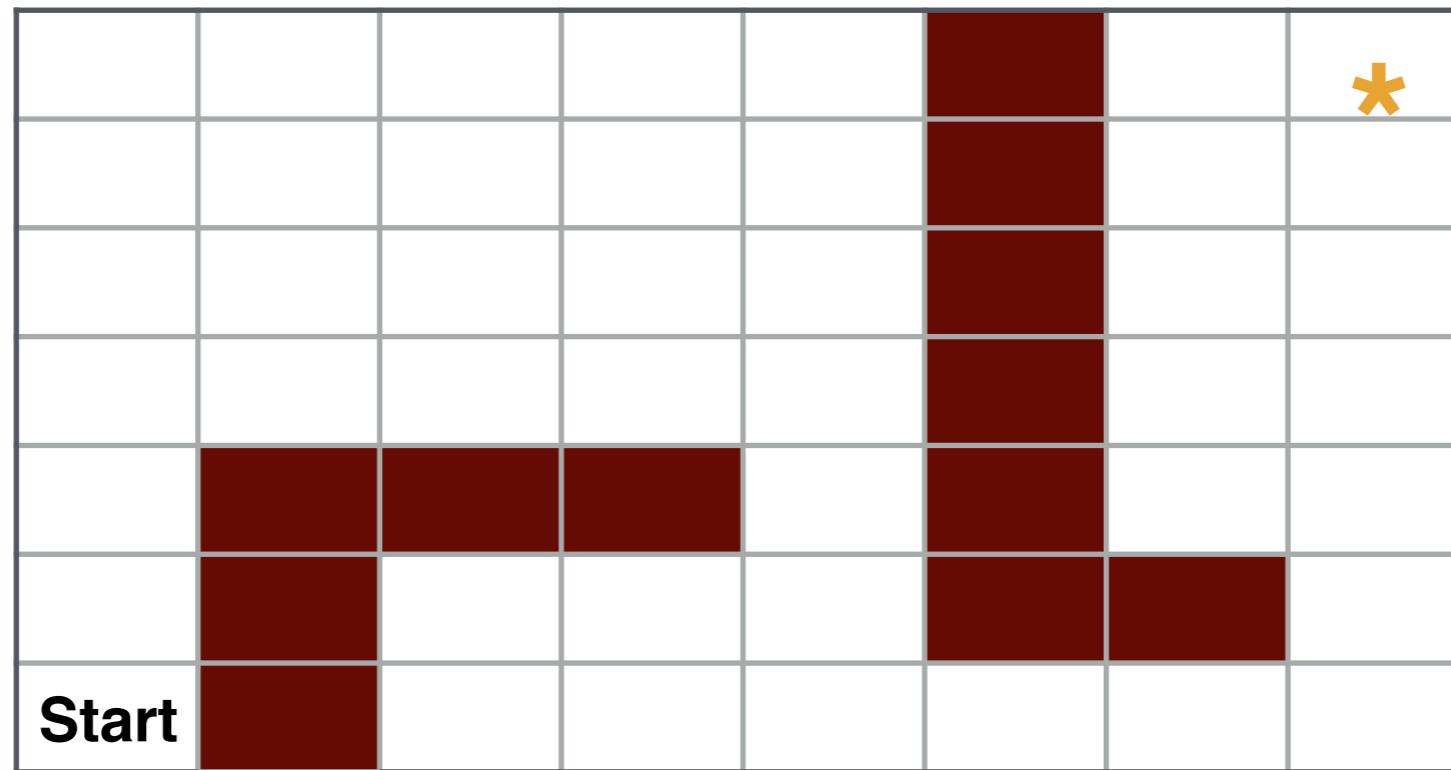
Reinforcement learning

Exploration-exploitation tradeoff

Use a good *exploration-exploitation tradeoff*.

Exploitation: make the best decision given current information

Exploration: gather more information



Reinforcement learning

Policy: ϵ -greedy

ϵ -greedy policy: a policy for when to choose the best available action a^* or a random action.

$$a_t = \begin{cases} a_t^* & \text{with probability } 1 - \epsilon \\ \text{random action} & \text{with probability } \epsilon \end{cases}$$

- $\epsilon=1$ gives a purely **exploratory** policy
- $\epsilon=0$ gives a purely **exploitative** policy
- $\epsilon \in [0,1]$ Somewhere in-between gives a tradeoff between exploration-exploitation.

It is often a good idea to explore more in the beginning and exploit more towards the end of optimization.

Sutton and Barto book (2018)

Deep Q-learning

Recent success in reinforcement learning relies on two tricks:

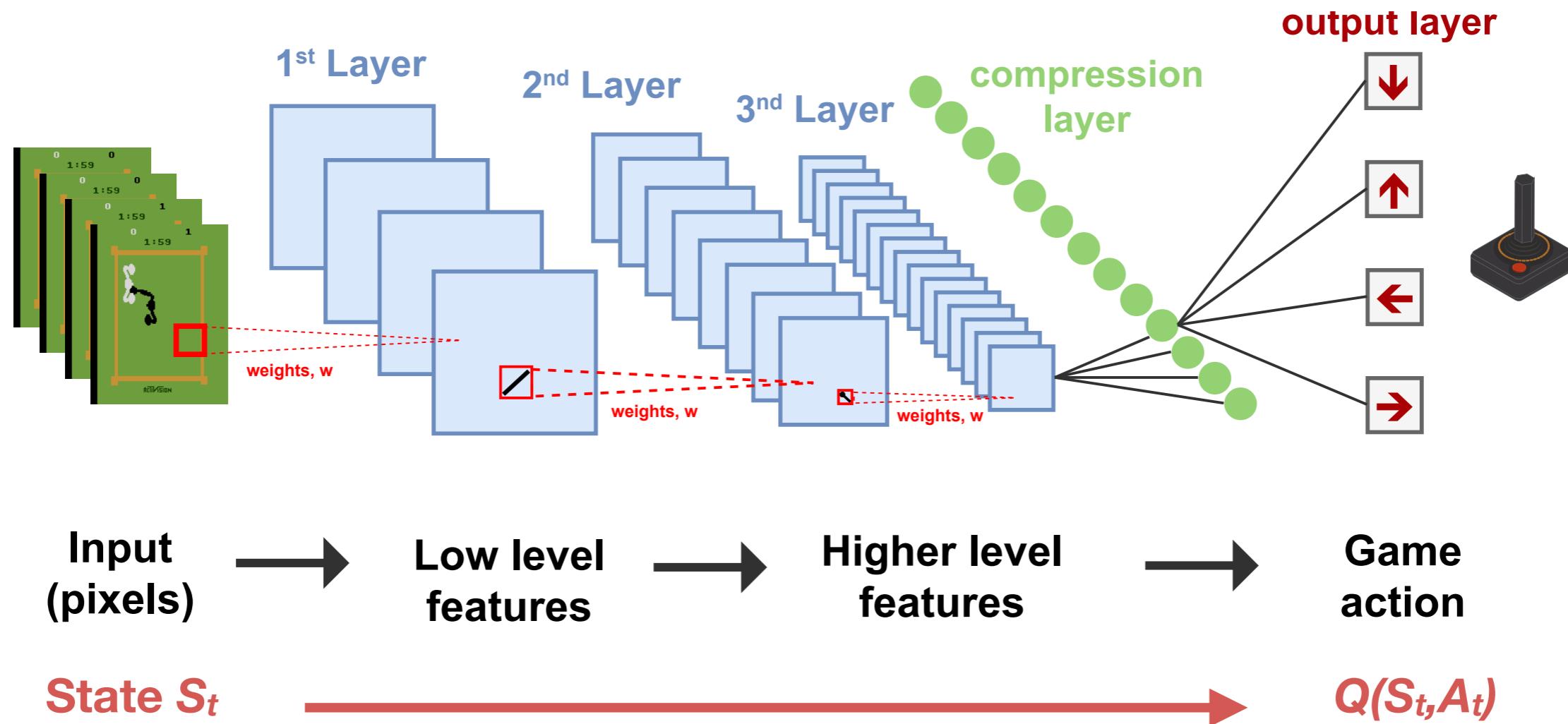
Trick 1: Use neural network as $Q(S,A;\theta)$ function approximator
with parameters θ

Trick 2: Use separate memory system to *avoid local optima*

Deep Q-learning

Trick I: deep net as function approximator

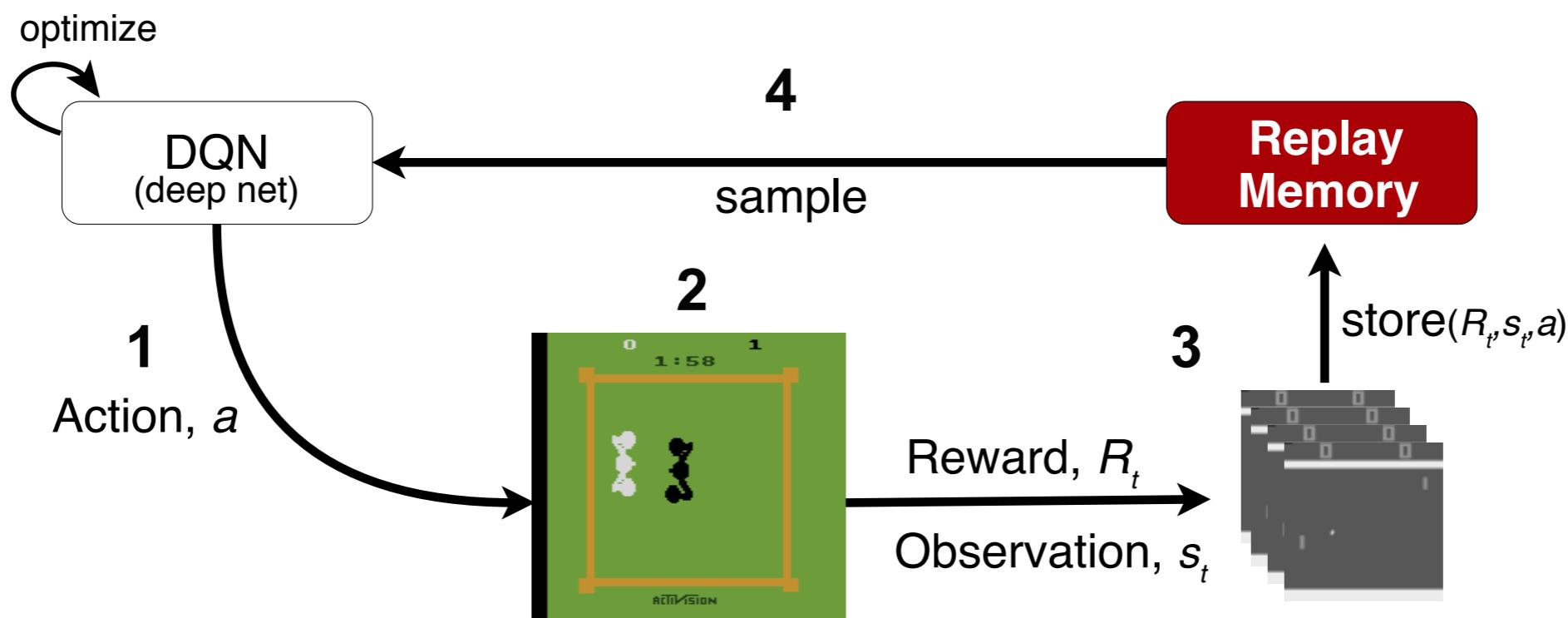
Use deep network as a $Q(S, A; \theta)$ function approximator
when in the presence of high-dimensional state spaces:



Mnih et al, Nature (2013)

Deep Q-learning

Trick 2: Replay previous episodes



1. Agent (player) performs an ***action a*** given by a **deep neural net**
2. This action triggers a new ***observation s_t*** in the game and a ***reward R_t***
3. The new observation and reward are stored in the **replay memory**
4. This memory is then used to train a **deep neural network (DQN)**

Mnih et al, Nature (2013)

Deep Q-learning

Algorithm 1: deep Q-learning with experience replay.

Initialize replay memory D to capacity N

Initialize action-value function Q with random weights θ

Initialize target action-value function \hat{Q} with weights $\theta^- = \theta$

For episode = 1, M **do**

 Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequence $\phi_1 = \phi(s_1)$

For $t = 1, T$ **do**

 With probability ϵ select a random action a_t

 otherwise select $a_t = \operatorname{argmax}_a Q(\phi(s_t), a; \theta)$

 Execute action a_t in emulator and observe reward r_t and image x_{t+1}

 Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

 Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in D

 Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from D

$$\text{Set } y_j = \begin{cases} r_j & \text{if episode terminates at step } j+1 \\ r_j + \gamma \max_{a'} \hat{Q}(\phi_{j+1}, a'; \theta^-) & \text{otherwise} \end{cases}$$

 Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ with respect to the network parameters θ

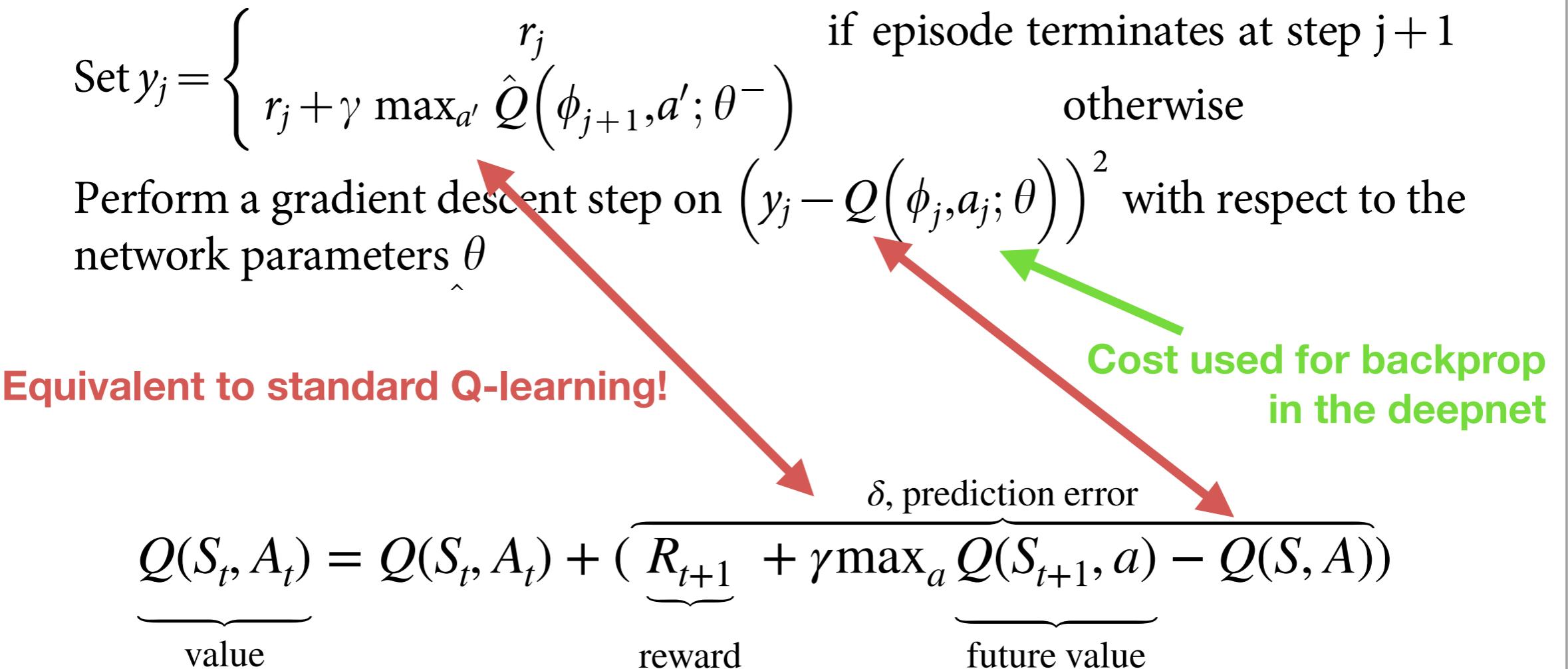
 Every C steps reset $\hat{Q} = Q$

End For

End For

Mnih et al, Nature (2013)

Deep Q-learning



Mnih et al, Nature (2013)

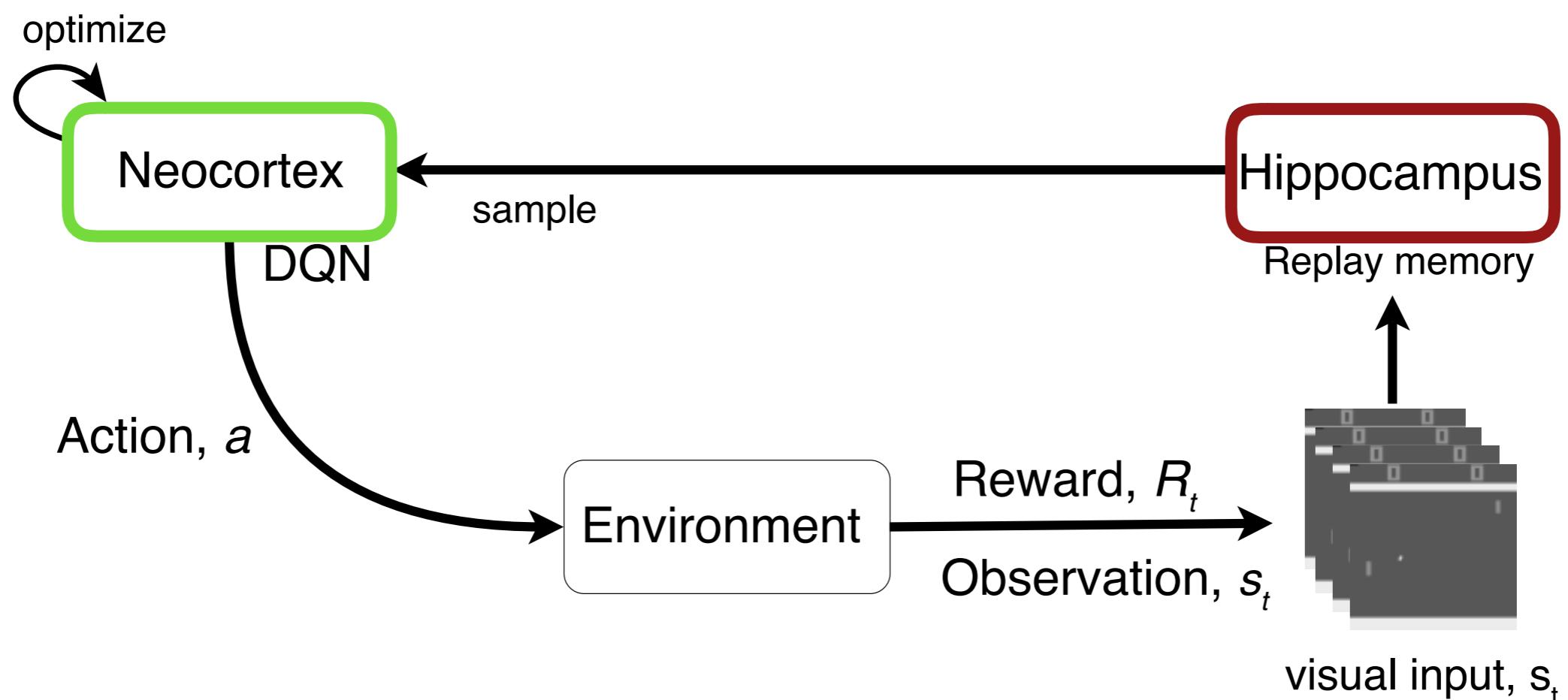
Deep Q-learning

Learning to play Atari games: Pong

Deep Q-learning

Learning to play Atari games: Seaquest

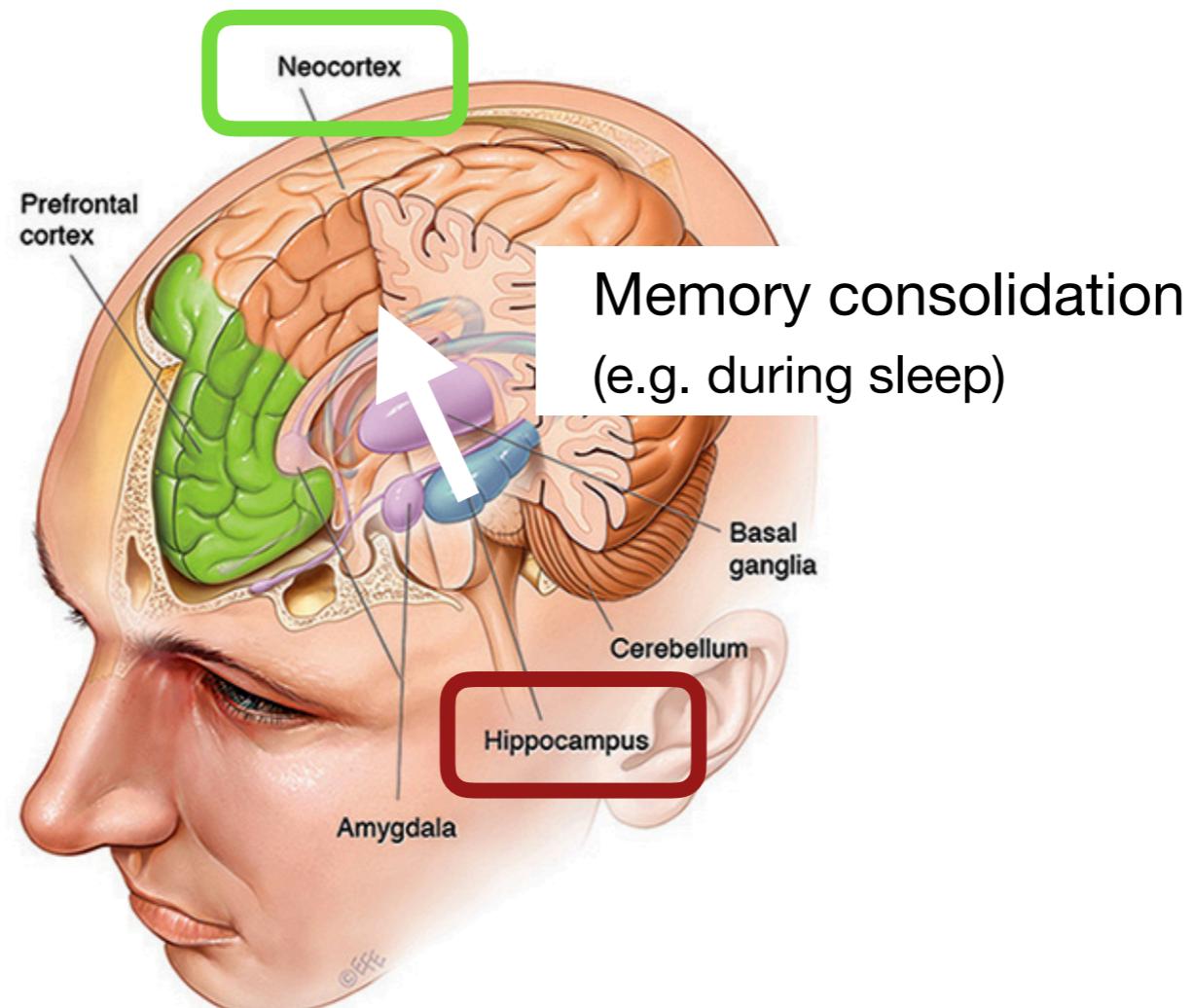
Deep Q-learning in the brain?



Mnih et al, Nature (2013)

Deep Q-learning in the brain?

Memory consolidation seems to rely on the *communication* between *hippocampus* (responsible for episodic memory) and the *neocortex*.



Summary

- I. Reinforcement learning relies on sparse rewards**
- 2. TD-learning and Q-learning are two of the most common methods**
- 3. The dopamine system encodes reward prediction errors**
- 4. Replay in deep RL is inspired in neuroscience to improve RL methods**

References

Text books:

Reinforcement Learning, Sutton and Barto 2018 (2nd edition available online)

Theoretical neuroscience: Dayan and Abbott 2001 [Unsupervised learning: chapter 3]

Note: Some of the slides are based on RL slides by Li, Johnson and Yeung, Stanford University

Relevant papers:

- Niv, Reinforcement learning in the brain, Journal of Mathematical Psychology (2009)
- Schultz et al. Science (1997) (seminal paper looking at reward prediction errors in the dopaminergic system)

Upcoming lectures

- L1^[4]: Neural circuits and learning: introduction
- L2^[4]: Supervised learning & backprop
- L3^[5]: Visual system: deep learning?
- L4^[5]: Reinforcement learning
- L5^[7]: **Unsupervised learning**
- L6^[7]: Temporal processing
- L7^[8]: Recurrent neural networks & brain vs. maquina