The Bayesian approach: the Bayesian Brain lecture 1

COMSM0094 Learning Computation and the Brain

`comsm0075.github.io`

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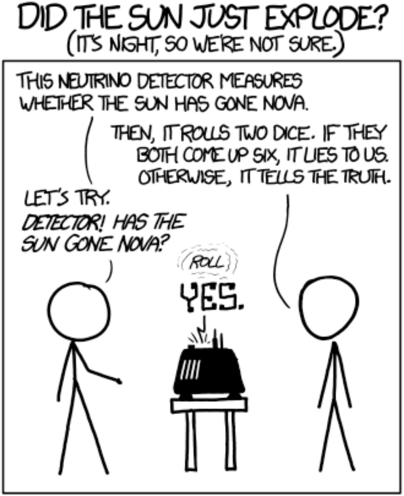
Bayes's rule

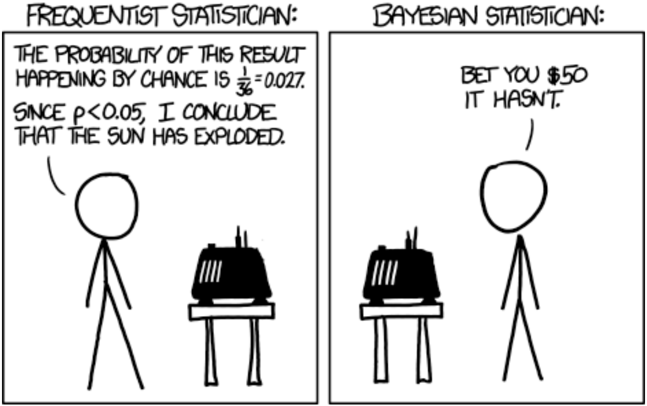
$$P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)}$$

The compulsory xkcd cartoon



xkcd.com/1132/

The compulsary xkcd cartoon



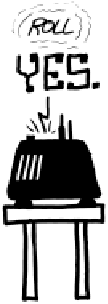
xkcd.com/1132/

Recall Bayes's rule

$$P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)}$$

xkcd maths

D is the state of the detector with $D = w$ corresponding to a warning.



xkcd maths

S is the state of the sun with $S = n$ corresponding to a nova and $S = f$ to it staying fine.

Picture of the Helix nebula from Wikipedia

xkcd math

The cartoon tells us that

$$P(D = w|S = f) = \frac{1}{36}$$

and since this is close to zero, the silly frequentist assumes the sun has exploded.

xkcd math

The cartoon tells us that

$$P(D = w|S = n) = \frac{35}{36}$$

Let's assume the probability the sun has exploded is one in a million; it is actually much less than that, so

$$P(S = n) = 10^{-6}$$

We want to know

$$P(S = f|D = w) = \frac{P(D = w|S = f)P(S = f)}{P(D = w)}$$

xkcd math

$$P(S = f|D = w) = \frac{P(D = w|S = f)P(S = f)}{P(D = w)}$$

Numerator is easy:

$$P(D = w|S = f)P(S = f) = \frac{1}{36}(1 - 10^{-6})$$

The denominator requires marginalizing:

$$P(D = w) = P(D = w, S = n) + P(D = w, S = f)$$

Doing what's required gives

$$P(S = f|D = w) \approx 0.999965$$

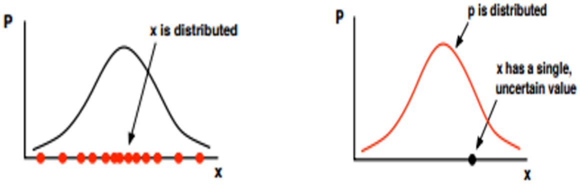
The Bayesian brain

The point of this section is that the brain appears to perform Bayesian calculation over multiple pieces of evidence incorporating recent observations and established models of the world.

What does probability model?

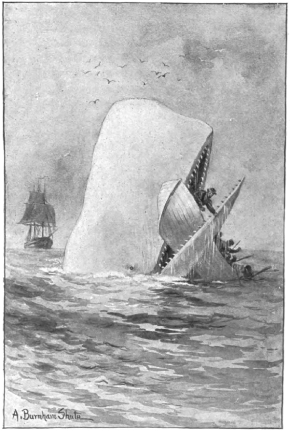
The laws of probability are mathematics - the mathematics don't determine what probabilities model.

Frequentists versus Bayesians



Picture from Rosalyn Moran

Guess the missing letter

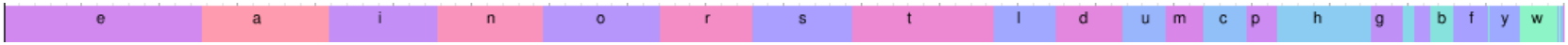


" Both jaws, like enormous shears, bit the craft completely in twain."

—*Page 510.*

CALL ME ISHMAE*

Guess the missing letter - letter frequencies



CALL ME ISHMAE*

with for example $p(E) = 0.13$, $p(L) = 0.04$ and $p(Q) = 0.001$.

Letter frequency table from Wikipedia

Guess the missing letter - 2-gram letter frequencies

Look at two letter pairs, *ER* and *EL* and so on, and work out conditional probabilities like

$$p(\text{second letter is L}|\text{first letter is E})$$

CALL ME ISHMAE*

with for example $p(R) = 0.14$, $p(L) = 0.04$ and $p(Q) = 0.003$.

Guess the missing letter - 3-gram letter frequencies

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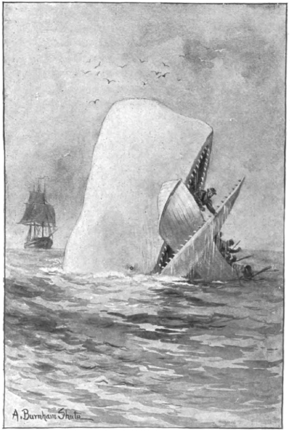
with for example $p(R) = 0.1$, $p(L) = 0.4$ and $p(P) = 0.001$.

Guess the missing letter



"Both jaws, like enormous shears, bit the craft completely in twain."

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CALL ME ISHMAEL

The Bayesian brain

The problem the brain faces is a Bayesian one, the brain gathers evidence about the world through the senses, these are unreliable and limited but the brain has to infer the state of the world from the information they provided.

Bayes's rule

In a Bayesian interpretation Bayes's rule

$$P(W|E) = \frac{P(E|W)P(W)}{P(E)}$$

allows us to update our view of the world W based on evidence E. The rule is described as an update rule:

$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{evidence}}$$

Bayes's rule

$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{evidence}}$$

The **prior**, is our belief about the world before our observation, the **likelihood** is the probability of the evidence given the state of the world $P(E|W)$, the **evidence** is the probability of the evidence $P(W)$ and the **posterior**, $P(W|E)$, is our new belief about the world given the evidence we have observed.