Note on the f-I curve

Introduction

The notes contain a figure, repeated here as Fig. 1 showing the f-I curve for an integrate and fire neuron. To save time the shape of the curve was never derived but for the curious I have writen it out here.

In the model

$$\tau_m \frac{dV}{dt} = E_L - V + R_m I_e \tag{1}$$

which we can solve from our study of odes, it gives

$$V(t) = E_L + R_m I_e + [V(0) - E_L - R_m I_e] e^{-t/\tau_m} \tag{2}$$

so if the neuron has spiked and is reset at time $t = 0$ and reaches threshold at time $t = T$, assume $V_R = E_L$ we have

$$V_T = E_L + R_m I_e - R_m I_e e^{-T/\tau_m} \tag{3}$$

so

$$e^{-T/\tau_m} = \frac{E_L + R_m I_e - V_T}{R_m I_e} \tag{4}$$

Taking the log of both sides we get

$$T = \tau_m \log \left[\frac{R_m I_e}{E_L + R_m I_e - V_T} \right] \tag{5}$$

Finally, this is the time between spikes, so the frequency is one over this. It is only defined for $R_m I_e > V_T - E_L$, below that there is no spiking and the frequency is zero. The actual gnuplot command used to make the figure was

```
plot [0:22] x>15 ?  1/(.01*log(x/(x-15))) :  0
```

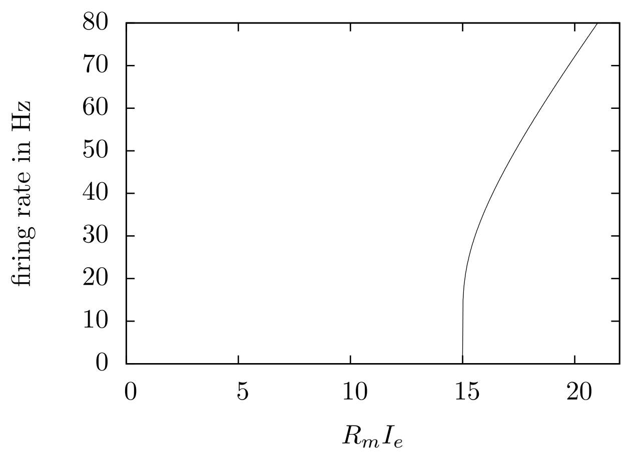

Figure 1: The firing rate, that is spikes per second, for the integrate and fire neuron with different constant inputs with $\tau_m = 10$ ms, $V_T = -55$ mV and both the leak and reset given by -70 mV. Notice how there is no firing until a threshold is reached and after that the firing increases very quickly.