Worksheet 8

Useful facts

- The **conditional probability** of event R given C:

$$P(R|C) = \frac{P(R \cap C)}{P(C)} \tag{1}$$

  This is the probability of getting an outcome in event R if we know the outcome is in event C.

- **Independence**: two events A and B are **independent** iff

$$P(A \cap B) = P(A)P(B) \tag{2}$$

  They are **conditionally independent**, given C, iff

$$P(A \cap B|C) = P(A|C)P(B|C) \tag{3}$$

- **Bayes's rule**

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \tag{4}$$

- **Set notation**:
  - The bar '|' in sets should be read as 'such that', so $A = \{x|\text{some stuff}\}$ should be read as A is the set of x **such that** 'some stuff' is true and $A = \{x \in \mathbf{Z}|x > 3 \text{ and } x < 10\}$ is the set $A = \{4, 5, 6, 7, 8, 9\}$. $\mathbf{Z}$ by the way is the set of integers.
  - $A \cup B$ is the union so $A \cup B = \{x|x \in A \text{ or } x \in B\}$. If $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$ then $A \cup B = \{1, 2, 3, 4, 5\}$
  - $A \cap B$ is the intersection so $A \cap B = \{x|x \in A \text{ and } x \in B\}$. If $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$ then $A \cup B = \{3\}$
  - $A \setminus B$ is the set minus so $A \setminus B = \{x|x \in A \text{ and } x \notin B\}$. If $A = \{1, 2, 3, 4\}$ and $B = \{1, 3, 5\}$ then $A \setminus B = \{2, 4\}$
  - If C is a subset, the complement of C, that is the set of all the elements not in C, is written $\bar{C}$. If $X = \{1, 2, 3, 4\}$ and $C = \{1, 2\}$ then $\bar{C} = \{3, 4\}$.

  For events, $A \cup B$ is the event of A or B happening, $A \cap B$ is the event of A and B happening, $A \setminus B$ is the event of A happening but B not happening and $\bar{C}$ is the event of C not happening.

Questions

This is an opportunity to revise the basics of using Bayes's rule!

1. In a library where all books have blue or yellow spines, four fifths of books with yellow spines are about mathematics but only a fifth of books with blue spines are about mathematics. There are the same number of yellow and blue spined books, you come upom a book open on a table; the book is about mathematics. What is the chance it has a yellow spine?

2. You want to go for a walk. However, when you wake up the day is cloudy and half of all raining days start off cloudy. On the other hand, two days in five start off cloudy and it's been rather dry recently with only rain only on one day in ten. What is the chance it will rain?

3. One night in a bar in Las Vegas you meet a dodgy character who tells you that there are two types of slot machine in the Topicana, one that pays out 10% of the time, the other 20%. One sort of machine is blue, the other red. Unfortunately the dodgy character is too drunk to remember which is which. The next day you randomly select red to try, you find a red machine and put in a coin. You lose. Assuming the dodgy character was telling the truth, what is the chance the red machine is the one that pays out more. If you had won instead of losing, what would the chance be?[1]

[1] I stole this problem from `courses.smp.uq.edu.au/MATH3104/`