

# The Bayesian approach: the Bayesian Brain

## lecture 2

COMSM0094 Learning Computation and the Brain

`comsm0094.github.io`

November 2021

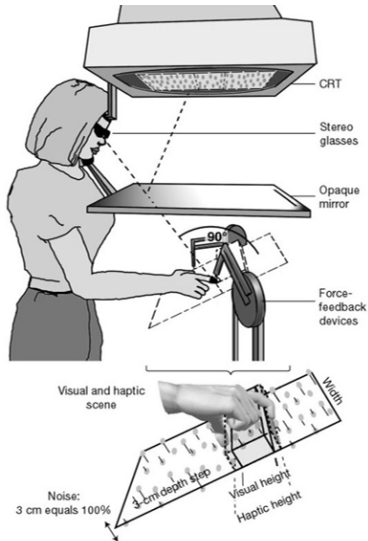
# Bayesian fusion

*The main topic here is **Bayesian fusion** but we will introduce it by talking about a nice experiment by Ernst and Banks which demonstrates Bayesian inference in human perception.*

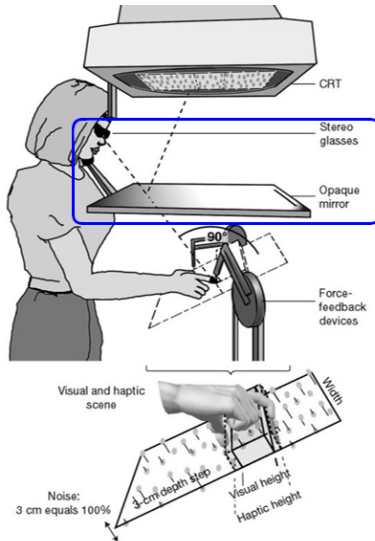
MO Ernst and MS Banks (2002) Humans integrate visual and haptic information in a statistically optimal fashion,

415:429 Nature

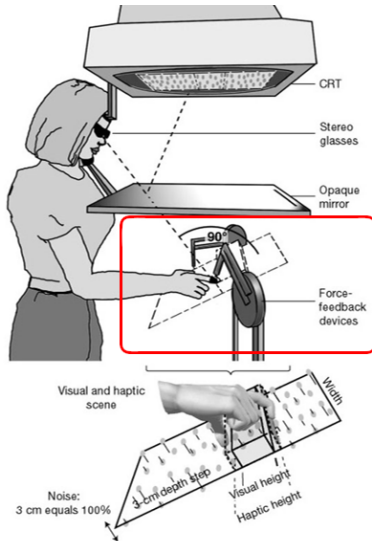
# Ernst and Banks



# Ernst and Banks - vision



# Ernst and Banks - touch



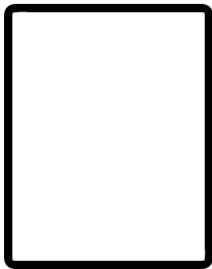
# Ernst and Banks

- ▶  $x$  is the **true** height of the box.
- ▶  $v$  is the **visual** estimate of the height of the box.
- ▶  $h$  is the **haptic** estimate of the height of the box.

haptic: of or relating to the sense of touch

Estimates are noisy

$$p(v|x)$$



Estimates are noisy

$$p(v|x)$$





# Markov chain

$$V \rightarrow X \rightarrow H$$

or

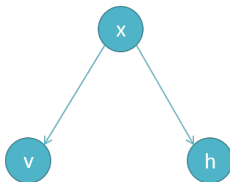
$$p(v, h|x) = p(v|x)p(h|x)$$

Markov chain - also called a directed acyclic graph

$$V \rightarrow X \rightarrow H$$

$$V \leftarrow X \leftarrow H$$

or



# Markov chain

$$p(v, h|x) = p(v|x)p(h|x)$$

so

$$p(v, h, x) = p(v, h|x)p(x) = p(v|x)p(h|x)p(x)$$

## Posterior judgment

$$p(x|v, h) = \frac{p(v, h|x)p(x)}{p(v, h)} = \frac{p(v|x)p(h|x)p(x)}{p(v, h)}$$

# Bayesian fusion

Let's assume the noise modelled by  $V$  and  $H$  are Gaussian so

$$\begin{aligned}(V|X=x) &\sim \mathcal{N}(x, \sigma_v^2) \\ p(v|x) &= \frac{1}{\sqrt{2\pi\sigma_v^2}} e^{-\frac{(v-x)^2}{2\sigma_v^2}}\end{aligned}$$

and

$$\begin{aligned}(H|X=x) &\sim \mathcal{N}(x, \sigma_h^2) \\ p(h|x) &= \frac{1}{\sqrt{2\pi\sigma_h^2}} e^{-\frac{(h-x)^2}{2\sigma_h^2}}\end{aligned}$$

It is also assumed that the participant has an estimate of the size of the noise, that is, the participant knows  $\sigma_v^2$  and  $\sigma_h^2$ .

# Bayesian fusion

So, ignoring any normalization factor

$$p(x|v, h) \propto p(v|x)p(h|x)$$

so we need to multiply the two Gaussians:

$$p(x|v, h) \propto e^{-\frac{(h-x)^2}{2\sigma_h^2}} e^{-\frac{(v-x)^2}{2\sigma_v^2}}$$

Looking at the exponent we get

$$-\frac{(h-x)^2}{2\sigma_h^2} - \frac{(v-x)^2}{2\sigma_v^2} = -\left(\frac{1}{2\sigma_h^2} + \frac{1}{2\sigma_v^2}\right)x^2 + \left(\frac{h}{\sigma_h^2} + \frac{v}{\sigma_v^2}\right)x + A$$

where  $A$  is other stuff with no  $x$ s.

# Bayesian fusion

$$-\frac{(h-x)^2}{2\sigma_h^2} - \frac{(v-x)^2}{2\sigma_v^2} = -\left(\frac{1}{2\sigma_h^2} + \frac{1}{2\sigma_v^2}\right)x^2 + \left(\frac{h}{\sigma_h^2} + \frac{v}{\sigma_v^2}\right)x + A$$

In short

$$p(x|v, h) \sim \mathcal{N}(\bar{x}, \sigma)$$

where

$$\frac{1}{\sigma^2} = \frac{1}{\sigma_v^2} + \frac{1}{\sigma_h^2}$$

and

$$\bar{x} = \frac{\sigma^2}{\sigma_v^2}v + \frac{\sigma^2}{\sigma_h^2}h$$

# Bayesian fusion

$$\frac{1}{\sigma^2} = \frac{1}{\sigma_v^2} + \frac{1}{\sigma_h^2}$$

Let

$$\lambda = \frac{\sigma^2}{\sigma_h^2}$$

then

$$1 - \lambda = \frac{\sigma^2}{\sigma_v^2}$$

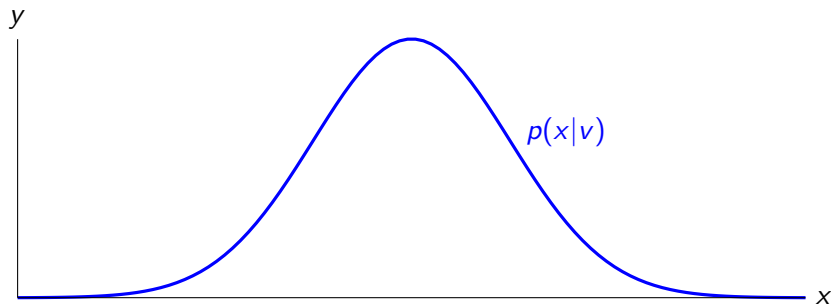
so

$$\bar{x} = (1 - \lambda)v + \lambda h$$



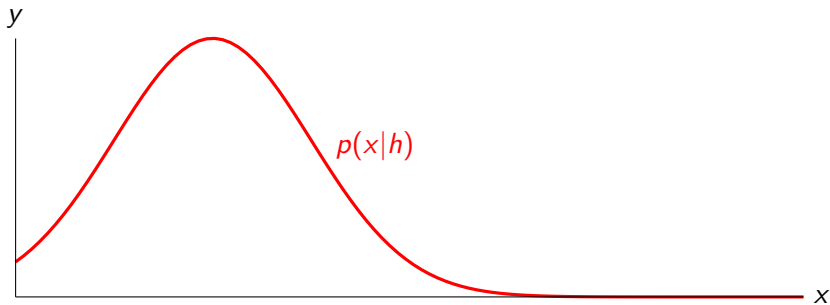
# Bayesian fusion

$$v = 4 \text{ and } \sigma_v = 1$$



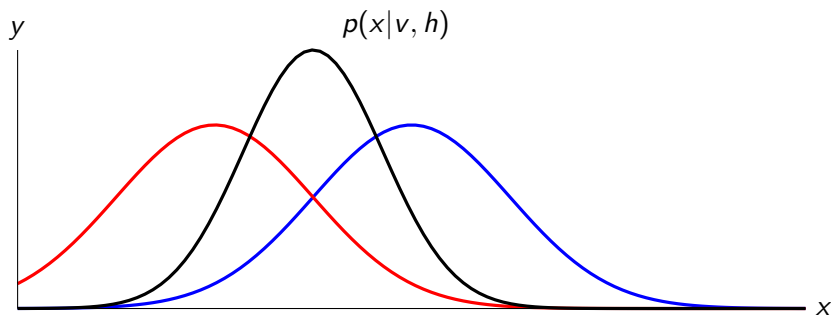
# Bayesian fusion

$$h = 2 \text{ and } \sigma_h = 1$$



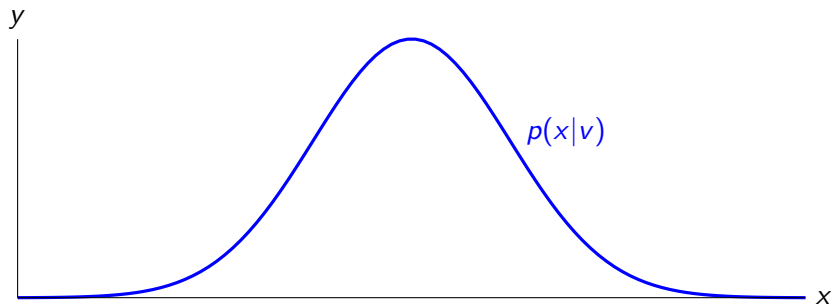
# Bayesian fusion

$$\bar{x} = 3 \text{ and } \sigma = 0.71$$



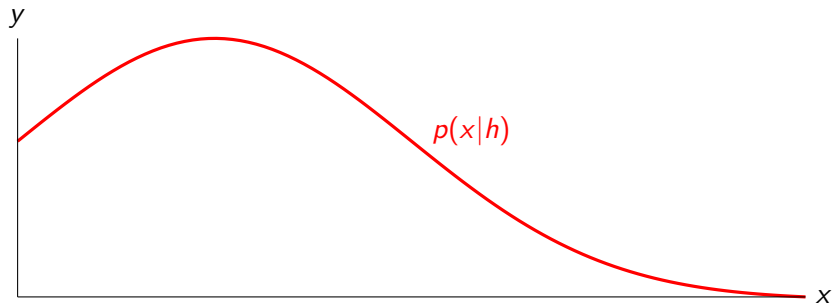
# Bayesian fusion

$$v = 4 \text{ and } \sigma_v = 1$$



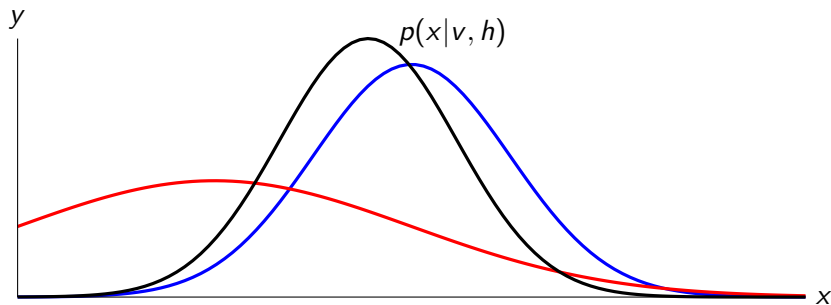
# Bayesian fusion

$$h = 2 \text{ and } \sigma_h = 2$$



# Bayesian fusion

$$\bar{x} = 3.56 \text{ and } \sigma = 0.9$$



# Ernst and Banks

Multiple trials in which the participants are asked to pick which of two blocks are larger. They use this to estimate the participant's estimate of the height  $\xi$  and fit this to

$$\xi = (1 - \mu)v + \mu h$$

and they then compare  $\mu$  to  $\lambda$ :

$$\bar{x} = (1 - \lambda)v + \lambda h$$

# Ernst and Banks

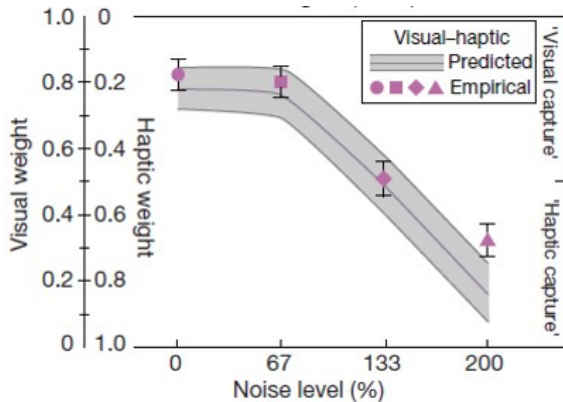


Figure from Ernst and Banks