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Strategic Deception in Poker: Mathematical Modeling and Dynamic Epistemic Logic in Game-Theoretic Analysis

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1 Introduction

This study aims to bridge the gap between theoretical game logic and practical applications by introducing a DEL-based framework tailored to the dynamics of Leduc Hold'em and its expansions. Our discussion captures the interaction between strategy, reasoning, and information flow, underpinned by fundamental assumptions such as rationality, common knowledge, and perfect recall. We start with a simplified model based on truthful behavior and then extend it to more complex strategies, including deceptive tactics such as slow-play and bluffing, and model complex games with DEL, illustrating the superiority of slow-play and bluff tactics in games and demonstrating that the use of deceptive tactics maximizes player benefits.

2 Background

2.1 Leduc Hold'em Game

There are many related variants of the game of poker, and the game presented in this article references a simplified and more accessible version known as Leduc Hold'em, a simplified poker game designed by game theory researcher David Leduc in the early 1990s[1]. The game's core objective is to solve the overly complex problems of mathematical analysis and strategic equilibrium modeling found in traditional poker games such as Texas Hold'em. Leduc Hold'Em provides an experimental platform for studying strategies such as bluffing, slow-playing, and information concealment, and helps researchers better understand the logic behind these behaviors.

2.2 Imperfect Information

All the nontrivial games played competitively by humans that has been solved to date is a perfect-information game[2–7]. In perfect-information games, all players are informed of everything that has occurred in the game before making a decision.In imperfect information games, players do not always have full knowledge of past events (e.g., cards dealt to other players in bridge and poker)[8]. These games are more challenging, with theory, computational algorithms, and instances of solved games lagging behind results in the perfect-information setting[7, 9]. In the Leduc Hold 'Em game, a poker game, imperfect information comes from the following sources: the opponent's hidden cards; the randomness of the game (e.g., the randomness of the cards dealt after shuffling); and the opponent's unknown strategy. Together, these uncertainties challenge modelling the Leduc Hold 'Em game.

2.3 Game Theory

Game theory is a science that studies multi-party decision-making interactions and strategy selection, and is widely used in fields such as economics, political science, and computer science. It analyzes the behavior and outcomes of participants through mathematical models. To accurately describe the behavior of decision-makers and their impact on outcomes, game theory models are usually based on several key assumptions. These assumptions ensure that the model is theoretical and analytically concise.

2.3.1 Rationality Assumption

All game participants in game theory are rational. Players will choose the optimal strategy based on their goals.[10] They can reason about all possible strategies, outcomes, and behaviors of other players in the game. They assume that their opponents will also try to maximize their utility and will base their strategies on the same logical reasoning.[11] In Leduc Hold 'Em, each player is assumed to be rational and will choose the action that maximizes their gains based on their information, strategy, and expected opponent behavior. This implies that agents avoid irrational folding strategies, such as folding at the start or folding after their opponent calls. Since agents have equal chip stacks, there is no reason to fold prematurely; they can always fold later if the opponent raises. Folding early would forfeit potentially better outcomes in future rounds, such as forming a pair.

2.3.2 Common Knowledge Assumption

Common knowledge is a fact that is known to each participant in an infinite recursive sense.[12] That is, everyone knows that event E, everyone knows that everyone knows that event E, everyone knows that everyone knows that everyone knows that event E, one to the infinite level.[13] This is a highly restrictive assumption in game theory. In Leduc Hold 'Em, the rules of the game, the rationality of the players, and the knowledge of each player are common to all players. This assumption ensures that all players have the same understanding of the basic rules of the game and assumes that opponents are also rational. The Common Knowledge Assumption allows players to speculate based on rules and opponent behavior without misinterpreting information.

2.3.3 Perfect Recall Assumption

Perfect recall in game theory is a theoretical assumption that builds on the fact that each game participant can accurately recall the history of the game[14]. In Leduc Hold 'Em, each player can completely remember all the information they have seen and all the actions they have taken during the game. This allows players to synthesize all the historical information when making decisions without making mistakes due to forgetfulness and helps simulate the strategic reasoning of players in actual poker games.

2.4 Dynamics Ways of Playing Game

The game tree is a static representation of all possible scenarios in the game. It shows all the possible paths a player can take, but does not address the dynamics of a particular run. Fig1, for example, can be used to explain the actions performed by the players in the first round of the Leduc Hold'em Game, which is the game tree of the first round of the Leduc Hold'em Game.

But in any specific run, it is the actual progression of a dynamic series of events. These events take the player into successive game states, each with different knowledge and ignorance. This requires us to scrutinize the logical "dynamics" of the game, especially the update mechanisms that generate or eliminate ignorance. Dynamic-epistemic logic (DEL) describes these dynamic processes, including the mechanisms by which the player's knowledge and ignorance change[15].

Fig2 shows how the different nodes reflect the knowledge or ignorance of the players during a dynamic game. Dynamic processes may increase or decrease player ignorance: the player takes action, such as raising, calling, etc.; information may be partially disclosed, affecting the basis of the player's decision. The dynamic process eventually brings the player to a defined state, possibly the end game of the game. At this point, the player's state of knowledge may be complete or partially restricted, depending on the rules of the game and the disclosure of information.

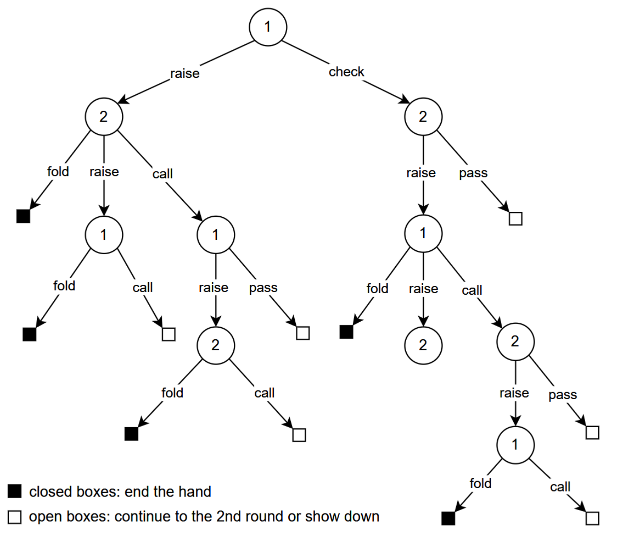


Fig1. Static representation of the Leduc Hold'em Game's decision tree. Circular nodes represent decision points in the game. The number inside the node (e.g., "1" or "2") indicates which player is making the decision. Closed boxes represent terminal states where the game ends. These nodes typically occur when a player chooses to "fold". Open boxes represent intermediate states where the game proceeds to the second round or enters the "showdown" phase.

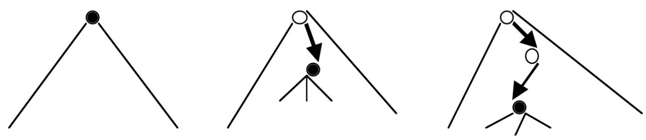


Fig2. Dynamic representation of the Game's tree[15]. The first panel (Left) represents the initial state of the game, where all possible paths exist; players may be in a state of ignorance about the state or information of other players (e.g., not seeing their opponent's hand). **The second panel (Middle)** represents the decision transition. The white circle indicates the updated state of the game after the first player has made a decision. The black node at the bottom indicates the decision point for the second player, who must now choose based on the updated state. **The third Panel (Right)** represents a further decision transition. The white circle and the black node show a progression in the game, where another decision point is reached.

3 Model

3.1 Epistemic Dynamic Logic

To begin with, we outline the epistemic model and its accessibility relationships. [16]Given a countable set of atomic propositions Φ and n agents, a Kripke model is a structure $M = (S, \pi, \mathcal{K}_1, \ldots, \mathcal{K}_n)$. S is a non-empty set of states; π is a valuation function that assigns truth values to atomic propositions in

each state; $\mathcal{K}_i$ is the accessibility relation for agent i on S.
In our model, we assume that all the relations $\mathcal{K}_i$ in M are equivalence relations, i.e., they are reflexive, symmetric, and transitive. Hence the model is a $\mathcal{S}5$ epistemic model, which is suitable for representing the knowledge of rational agents in this game.
Dynamic epistemic logic[17] studies logics of information change, such as inference, communication, and observation, to analyze changes in knowledge and beliefs over time in social interactions or rational inquiry.To express informational changes, DEL uses dynamic modalities $[!\alpha]\varphi$ (α are called epistemic actions) to represent if action α is performed, then φ will become true. An example of dynamic modalities is the truthful public announcement. $[!\varphi]\psi$ stands for "if a truth public announcement of φ is performed, then ψ will become true". The semantics of $!\varphi$ can be denoted by deleting the non-φ worlds, i,e., joint update with φ[18]. That is to say, the announcement restricts the worlds considered possible to those where φ is true. After the announcement:

$$M|\varphi = (S', \pi', \mathcal{K}'_1, \ldots, \mathcal{K}'_n)$$

where

$$S' := \{s \in S \mid (M, s) \vDash \varphi\}, \qquad \pi' := \pi \text{ restricted to } S', \qquad \mathcal{K}'_i := \mathcal{K}_i \cap (S' \times S').$$

And the model changes from (M, s) to $(M|\varphi, s)$.

3.2 Leduc Hold'em Game Rules

In this game, the deck consists of two identical sets of cards (e.g., J, Q, K), and two players predefine the amounts for raises. A two-raise maximum is enforced in each betting round. In the first round, each player is dealt a private card, which is visible only to themselves. In the second round, a community card is revealed. The result of the first round will dictate whether the community card is disclosed. Betting decisions alternate between the players. When a player checks, they relinquish their right to act as the first player in the current round. Folding ends the hand, awarding the pot to the opponent. Calling involves matching the total amount of chips contributed to the pot by the opponent. Raising adds a fixed number of chips beyond the opponent's total contribution. When the opponent chooses to call or check, ensuring the chips in the pot are consistent, the player can pass to proceed to the next round.
If neither player folds before the final betting round concludes, a showdown occurs. At this stage, both players reveal their private cards, and the player who can form the strongest poker hand using a combination of their private card and the community card wins. Specifically, a player wins if they can form a pair with the community card (e.g., JJ, QQ, or KK). If no player forms a pair, the winner is determined by the rank of the private card, following the hierarchy K > Q > J.
The round terminates under the following conditions: a player folds, a player calls (the call is made after two raises in the current round), continuing the hand to the next round.

3.3 Symbol Definition

We use the following symbol notations to represent the agents' knowledge states in the context of the Leduc Hold'em game. We assume there are two agents, Alice and Bob, denoted by Player A and Player B.

- $H_A :=$ The private card of player A. $H_B :=$ The private card of player B.
- $\alpha :=$ The strategies adopted by player A. $\beta :=$ The strategies adopted by player B.
- $C :=$ The community card. $G :=$ The historical sequence of events.

3.4 A Simplified Model

In this section, we use Public Announcement Logic (PAL) as an example of DEL to model the Leduc Hold'em poker game. By introducing an additional assumption, we construct a simplified epistemic logic model. We then provide an example to illustrate how agents utilize this model to iteratively eliminate possible worlds and infer the opponent's hand, aiding in their decision-making process.

3.4.1 Assumption and Definition

To simplify our model, we introduce an additional assumption:

- **Truthful Behavior** Agents are truthful in their actions, meaning their behavior directly reflects the strength of their actual hand. They do not adopt deceptive strategies, such as bluffing or slow play. For instance, agents tend to raise with strong hands and may fold or check with weaker hands.

In our poker scenario, agents' actions only influence their knowledge rather than the factual changes in the propositions representing the actual world. Thus, the actions are epistemic actions rather than ontic actions. This makes public announcement logic (PAL) particularly well-suited to model the epistemic states of agents regarding possible worlds in the game. Specifically, under the assumption of Truthful Behavior, we define the agent's actions as follows, using Alice as an example (the definitions for Bob can be obtained by swapping A and B):

1. $\phi_{A\text{Raise}} := K_A(H_A = K) \vee K_A(H_A > H_B)$: Alice raises iff she knows her hand is K, or she knows her hand is stronger than Bob's hand. Here, the action **raise** does not include cases where Alice is the initial aggressor at the start of the round.
2. $\phi_{A\text{Raise}^*} := K_A(H_A = K) \vee K_A(H_A = Q)$: Alice raises* iff she knows her hand is K or Q. Here, **raise*** specifically refers to raising as the initial aggressor at the start of the round.
3. $\phi_{A\text{Check}} := K_A(H_A = Q) \vee K_A(H_A = J)$: Alice checks iff she knows her hand is Q or J.
4. $\phi_{A\text{Fold}} := K_A(H_A < H_B)$: Alice folds iff she knows her hand is weaker than Bob's hand.
5. $\phi_{A\text{Call}} := K_A(H_A \geq H_B)$: Alice calls iff she knows her hand is at least as strong as Bob's hand.
6. $\phi_{A\text{Pass}} := K_A(H_A \leq H_B)$: Alice passes iff she knows her hand is weaker than or equal to Bob's hand.

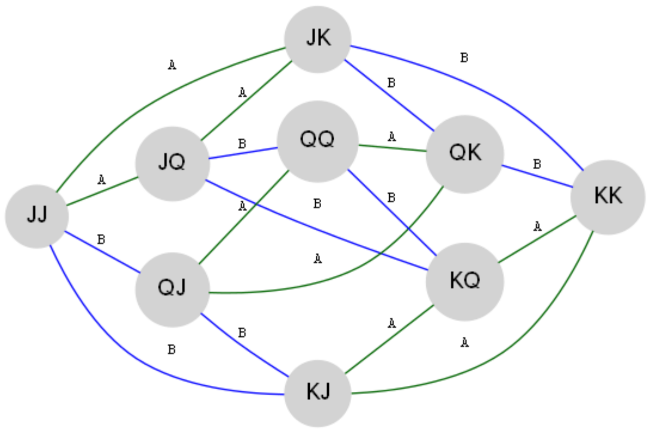
Here, the subscript A denotes Alice. For simplicity, we only define actions in the first round. In this model, S represents all possible combinations of hands, and the atomic propositions in Φ include information about the players' hands. Additionally, we assume that both agents have the aforementioned knowledge at the start of the game, and any action taken by one agent is observed by both agents, making it common knowledge. Note that in certain situations, multiple actions may satisfy the conditions for execution. For example, with a hand of Q, Alice could choose either raise* or check at the start of the round. Thus, the process is inherently uncertain. To extend the model, Probabilistic Dynamic Epistemic Logic (PDEL)[19] could be introduced to model the uncertainty in decision-making processes. However, our focus is not on why a particular decision is made but rather on the consequences of executing a specific action, i.e., the impact of a public announcement on the agents' knowledge. Thus, we simply assume agents randomly and uniformly choose an action when multiple options are available. Next, we provide a concrete example to demonstrate the role of PAL in this context.

3.4.2 Simulation

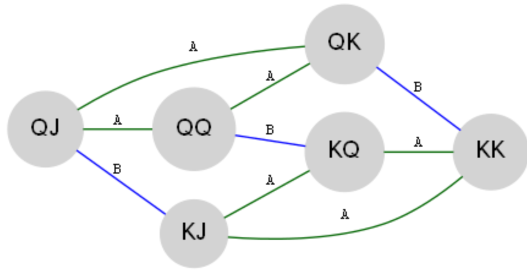
We assume that the first action is taken by Alice, and the next by Bob. The game flow corresponds to the decision tree shown in Fig1. In the actual world, Alice's hand is Q and Bob's hand is K, and both agents initially know only their own hand. Fig3a illustrates the complete set of possible game states before any player actions occur. Since the relations are equivalence relations, the reflexive edges and the bidirectional arrows are omitted for simplicity.

In this situation, Alice can choose to execute either raise* or check. Suppose Alice chooses to execute raise*, which is equivalent to issuing a public announcement $[!\phi_{A\text{Raise}^*}]$. This action removes all possible worlds where Alice's hand is J, along with their associated relations, as depicted in Fig3b. At this point, through the public announcement, Bob knows that Alice's hand must be either K or Q. Given that Bob's hand is K, he can choose either to raise or call. Suppose Bob chooses to raise, represented as $[!\phi_{B\text{Raise}}]$. This allows Alice to know that Bob's hand is K, because only $H_B = K$ satisfies $K_A(H_B = K) \vee K_B(H_B > H_A)$. The updated model is shown in Fig3c. Since Alice's hand Q is weaker than K, she can only choose to fold, represented as $[!\phi_{A\text{Fold}}]$. The game ends, and the final Kripke structure is illustrated in Fig3d. At this point, both Alice and Bob know each other's hands.

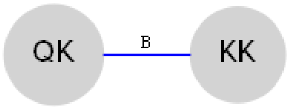
Now, let us simulate an alternative scenario. Suppose Bob chooses to call instead of raise, represented as $[!\phi_{B\text{Call}}]$. In this case, Alice can only deduce that Bob's hand is either K or Q. Suppose Alice then chooses to execute $[!\phi_{A\text{Pass}}]$, allowing the game to proceed to the second round. However, Bob cannot



(a) **Initial Kripke Structure**



(b) **After** $[!\phi_{A\mathbf{Raise*}}]$



(c) **After** $[!\phi_{B\mathbf{Raise}}]$

(d) **After** $[!\phi_{A\mathbf{Fold}}]$

Fig3. Overview of Kripke Structures. Each circle (node) represents a possible state in the game, corresponding to the private cards held by the players. Green edges(A) represent Alice, and blue edges(B) Bob. The four figures collectively illustrate the dynamic progression of Kripke structures in the Leduc Hold'em game.

determine whether Alice's hand is K or Q, because Alice, when deciding to pass, does not know Bob's hand. Consequently, whether Alice's hand is K or Q, both satisfy $K_A(H_A \leq H_B)$.

These examples illustrate how PAL can be applied to model the evolution of knowledge in a simplified poker game scenario. It also demonstrates that under the assumption of truthful behavior, agents' strategies become more straightforward. Additionally, based on the defined actions, certain branches of the decision tree are no longer reachable.

3.5 An Advanced Model

In this section, we remove the assumption of truthful behavior and consider more complex scenarios. We introduce a modal operator$\langle \mathcal{E}, e \rangle$to represent the updates to the event model[20]:

$$M, s \models \langle \mathcal{E}, e \rangle \varphi \iff M, s \models \text{pre}(e) \text{ and } M \otimes \mathcal{E}, (s, e) \models \varphi$$

The event model of this game is: $E := \{\text{Raise}, \text{Fold}, \text{Check}, \text{Call}, \text{Pass}\}$, the corresponding preconditions are:

$$\begin{aligned}
\text{pre(Raise)} &:= \text{No more than 2 Raises have occurred in the current round} \\
\text{pre(Fold)} &:= \text{The opponent player Raises} \\
\text{pre(Check)} &:= \text{The player is the lead player in the current round} \\
\text{pre(Call)} &:= \text{The opponent player Raises} \\
\text{pre(Pass)} &:= \text{The opponent chooses to call or check}
\end{aligned}$$

3.5.1 Case

To provide a clearer explanation of the application of DEL in games, we present a specific example. In this scenario:

$$S := (H_A, H_B, C, G, \alpha, \beta, \mathcal{E})$$

$$S_0 := \{S \mid H_A = \emptyset, H_B = \emptyset, C = \emptyset, G = \emptyset, \alpha = \emptyset, \beta = \emptyset, E\}$$

$$K_{A1} := \{(s, t) \in S_1 \times S_1 \mid \text{Alice cannot distinguish between } s \text{ and } t\}$$

$$K_{B1} := \{(s, t) \in S_1 \times S_1 \mid \text{Bob cannot distinguish between } s \text{ and } t\}$$

Reveal private cards:

$$M_0, s \models \langle E, e \rangle \phi_{reveal} \iff M_0, s \models \text{pre}(reveal) \text{ and } M_0 \otimes E, (s, reveal) \models \phi_{reveal}$$

M_0 update to M_1:

$$S_1 := \{s \in S_0 \mid (M_0, s) \vDash \phi_{reveal}\}$$

$$K_{A2} := \{(s,t) \in S_1 \times S_1 \mid H_{A_s} = H_{A_t} = J\}$$

$$K_{B2} := \{(s,t) \in S_1 \times S_1 \mid H_{B_s} = H_{B_t} = K\}$$

$$S_1 := \{S \mid H_A = J, H_B = K, C = \emptyset, G = \emptyset, \alpha = \emptyset, \beta = \emptyset, E\}$$

Alice chooses the bluff strategy; in the case where Alice raises:

$$M_1, s \models \langle E, e\rangle \phi_{ARaise} \iff M_1, s \models \text{pre}(Raise) \text{ and } M_1 \otimes E, (s, Raise) \models \phi_{ARaise}$$

M_1 update to M_2:
Bob will think that Alice either has a strong card or is bluffing.

$$[!\phi_{ARaise}]K_B\varphi_1, \quad \varphi_1 := (\text{Alice has a strong card in the first round}) \vee (\text{Alice is on a bluff})$$

$$S_2 := \{s \in S_1 \mid (M_1, s) \vDash \phi_{ARaise}\}, \quad K_{A3} := \{(s,t) \in S_2 \times S_2\}, \qquad K_{B3} := \{(s,t) \in S_2 \times S_2\}$$

Since Bob has K, the probability of Bob choosing to Raise is higher,

$$M_2, s \models \langle E, e\rangle \phi_{BRaise} \iff M_2, s \models \text{pre}(Raise) \text{ and } M_2 \otimes E, (s, Raise) \models \phi_{BRaise}$$

M_2 update to M_3

$$[!\phi_{BRaise}]K_A\varphi_2, \quad \varphi_2 := (\text{Bob has a strong card in the first round}) \vee (\text{Bob is on a bluff})$$

$$S_3 := \{s \in S_2 \mid (M_2, s) \vDash \phi_{BRaise}\}, \quad K_{A4} := \{(s,t) \in S_3 \times S_3\}, \qquad K_{B4} := \{(s,t) \in S_3 \times S_3\}$$

Based on higher-order reasoning $K_A\varphi_2(K_B\varphi_1)$, Alice will choose call,

$$M_3, s \models \langle E, e\rangle \phi_{ACall} \iff M_3, s \models \text{pre}(Call) \text{ and } M_3 \otimes E, (s, Call) \models \phi_{ACall}$$

M_3 update to M_4:

$$[!\phi_{ACall}]K_B\varphi_1$$

$$S_4 := \{s \in S_3 \mid (M_3, s) \vDash \phi_{ACall}\}, \quad K_{A5} := \{(s,t) \in S_4 \times S_4\}, \qquad K_{B5} := \{(s,t) \in S_4 \times S_4\}.$$

Reveal community cards:

$$M_4, s \models \langle E, e\rangle \phi_{reveal} \iff M_4, s \models \text{pre}(reveal) \text{ and } M_4 \otimes E, (s, reveal) \models \phi_{reveal}.$$

M_4 update to M_5:

$$[!\phi_{reveal}]K_A\varphi_3, \quad \varphi_3 := (\text{Alice must win.})$$

Alice will use slow-play, $S_5 := \{s \in S_4 \mid (M_4, s) \vDash \phi_{reveal}\}$

$$K_{A6} := \{(s,t) \in S_5 \times S_5 \mid C_s = C_t = J\}, \quad K_{B6} := \{(s,t) \in S_5 \times S_5 \mid C_s = C_t = J\}$$

Based on higher-order reasoning $K_B(K_A\varphi_2(K_B\varphi_1))$, Bob will choose to raise at this point.

$$M_5, s \models \langle E, e\rangle \phi_{BRaise} \iff M_5, s \models \text{pre}(Raise) \text{ and } M_5 \otimes E, (s, Raise) \models \phi_{BRaise}$$

M_5 update to M_6 :

$$S_6 := \{s \in S_5 \mid (M_5, s) \vDash \phi_{BRasie}\}, \quad K_{A7} := \{(s,t) \in S_6 \times S_6\}, \qquad K_{B7} := \{(s,t) \in S_6 \times S_6\}$$

Based on higher-order reasoning $K_A\varphi_3\big(K_B\big(K_A\varphi_2(K_B\varphi_1)\big)\big)$, Alice will choose Call at this point.

$$M_6, s \models \langle E, e\rangle \phi_{Acall} \iff M_6, s \models \text{pre(Call)} \text{ and } M_6 \otimes E, (s, \text{Call}) \models \phi_{Acall}$$

M_6 update to M_7:

$$[!\phi_{ACall}]K_A\varphi_4, \quad \varphi_4 := (\text{Alice has weak card in the second round}) \vee (\text{Alice on slow-play}),$$

$$S_7 := \{s \in S_6 \mid (M_6, s) \vDash \phi_{ACall}\}, \quad K_{A8} := \{(s,t) \in S_7 \times S_7\}, \qquad K_{B8} := \{(s,t) \in S_7 \times S_7\}$$

Based on higher-order reasoning $K_B\varphi_4\big(K_A\varphi_3\big(K_B\big(K_A\varphi_2\big(K_B\varphi_1\big)\big)\big)\big)$, Bob will choose Raise at this point.

$$M_7, s \models \langle E, e\rangle\phi_{BRaise} \iff M_7, s \models \text{pre}(Raise) \text{ and } M_7 \otimes E, (s, Raise) \models \phi_{BRaise}$$

M_7 update to M_8:

$$[!\phi_{BRasie}]K_A\varphi_5, \quad \varphi_5 := (\text{Alice will gets the maximum benefit}),$$

$$S_8 := \{s \in S_7 \mid (M_7, s) \vDash \phi_{Brasie}\}, \quad K_{A9} := \{(s,t) \in S_8 \times S_8\}, \qquad K_{B9} := \{(s,t) \in S_8 \times S_8\}$$

Based on higher-order reasoning $K_B\varphi_4\big(K_A\varphi_3\big(K_B\big(K_A\varphi_2\big(K_B\varphi_1\big)\big)\big)\big)$,Alice will choose Call at this point.

$$M_8, s \models \langle E, e\rangle\phi_{Acall} \iff M_8, s \models \text{pre}(Call) \text{ and } M_8 \otimes E, (s, Call) \models \phi_{Acall}$$

M_8 update to M_9 :

$$S_9 := \{s \in S_8 \mid (M_8, s) \vDash \phi_{BRasie}\}, \quad K_{A10} := \{(s,t) \in S_9 \times S_9\}, \qquad K_{B10} := \{(s,t) \in S_9 \times S_9\}$$

The game ends with Alice winning and gaining the most points!

3.5.2 Counter-Exploitation

Counter-exploitation refers to the strategic response that a player undertakes upon realizing that the opponent is attempting to exploit them.[1, 21, 22] This involves adjusting the strategy not only to avoid being exploited but also to take advantage of the opponent's exploitative tactics. Employing higher-level strategic thinking enables players to promptly mitigate losses and even transform potential weaknesses into advantages.

As outlined in the previous example, during the first round of the game, i.e., not advancing to the release of the community card, Bob can employ a counter-exploitation strategy when Alice attempts to win the pot through bluffing. By raising, Bob forces Alice to choose to fold, thereby securing the pot. When proceeding to the second round, where Alice holds a private card J, Bob holds a private card K, and the community card J is revealed. Bob recognizes that the community card does not form a pair with his own private card. Bob considers that Alice's behavior in the first round was a bluff, so conversely, Alice has the possibility of pairing with the community card. Bob will re-evaluate his strategy, initially choosing to check in the second round to control the pot size, cautiously calling, and folding if necessary to avoid becoming entangled in Alice's psychological game.

3.5.3 Code Stimulation

The code part we run each time is to carry out a simulation of 1000 games, the opponent cards according to the J, Q, K in order of sequence for 1, 2, 3, the pair of 4 to set the intensity, in each time the player to make a choice, the mechanism of its choice of strategy is based on the ε-greedy algorithm. Fig4 is a table of the results of running the code 5 times, from the results in the figure you can see that the use of bluffing or slow-playing is better than simply truthful.

	A(Truthful)	A(Slowplay)	A(Bluff)	B(Truthful)	B(Slowplay)	B(Bluff)
1	-815	-504	138	-458	-228	28
2	-914	-516	122	-444	-191	22
3	-938	-577	110	-334	-213	36
4	-812	-558	127	-472	-225	24
5	-982	-619	92	-176	-348	18

Fig4. Code stimulation results. Numerical results for two players (A and B) under three strategies (Truthful, Slowplay, Bluff), where each row represents the results of a different experiment. The contents of the table show the gains (or losses) of the two players under each combination of strategies.

4 Conclusion and Further Work

Our study demonstrates that DEL can effectively model dynamic decision-making processes in poker games, particularly in scenarios involving incomplete information. We not only explored players' behavior patterns under the principles of rational decision-making but also extended our research to encompass more complex deceptive strategies, such as slow-play and bluffing, as well as counter-exploitation tactics. Through detailed case analyses, we highlighted DEL's robust capacity to illustrate the impact of these strategies on game outcomes, showing that the strategic use of deception can significantly enhance players' gains. In the future, this research could be expanded to more complex poker games involving incomplete information, and the process of incomplete information games could be further probabilized by introducing mixed Nash equilibria to analyze players' strategies in greater depth.

Overall, DEL provides a powerful framework for simulating and analyzing decision-making processes in games with incomplete information. By modeling agents as rational entities with perfect recall and common knowledge, the framework effectively simulates realistic AI behaviors, enhancing the depth and interactivity of games.[1]

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[1] Please refer to the complete version uploaded on GitHub for all references and Contribution Table.

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Contributions

Task	s242519	s242598	s242610	s242649
Overall Framework		x		
Code Implementation				x
1		x		
2.1			x	x
2.2				x
2.3.1	x			x
2.3.2				x
2.3.3				x
2.4(Dynamic and writing)		x		x
2.4(Decision tree)	x			
3.1	x		x	x
3.2			x	
3.3		x	x	
3.4	x			
3.5.1		x	x	x
3.5.2		x	x	x
3.5.3				x
4		x		
Reformat				x