# Course 2BA1: Michaelmas Term 2002
# Section 1: The Principle of Mathematical Induction

David R. Wilkins



## Contentn


1 The Principle of Mathematical Induction 1
  1.1 Integers and Natural Numbers . . . . . . . . . . . . . . . . . . . 1
  1.2 Introduction to the Principle of Mathematical Induction . . . 2
  1.3 Some examples of proofs usinZ the Principle of Mathematical Induction . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4


## 1 The Principle of Mathematical Induction

## 1.3 Some examples of proofs using the Principle of Mathematical Induction

**Example** We claim that

$$\sum^{n}$$

To achieve this, we have to verify that the formula holds when $n = 1$, and that if the formula holds when $n = m$ for some natural number $m$, then the formula holds when $n = m$

The formula does indeed hold when $n = 1$, since $1 =$ $^{1}$

when $n = 1$. Suppose that the result is true when $n = m$

$k$ red balls from a collection of $n \quad 1$ red balls, and there are $\begin{matrix} n & 1 \\ & k \end{matrix}$ such choices. A *type II* choice requires us to choose $k \quad 1$ red balls from a collection of $n \quad 1$ red balls, and there are $\begin{matrix} n & 1 \\ k & 1 \end{matrix}$

1 in these cases. We conclude that if the proposition $P(n)$ is true for any natural number $n$ then the proposition $P(n+1)$ is also true. We can therefore conclude from the Principle of Mathematical Induction that the proposition $P(n)$ is true for all natural numbers $n$, which is what we are required to prove.

Example We can use the Principle of Mathematical Induction to prove that $(2n)! < 4^n(n!)^2$ for all natural numbers $n$. This inequality holds when $n = 1$, since in that case $(2n)! = 2! = 2$ and $4^n(n!)^2 = 4$. Suppose that the inequality holds when $n = m$

Then

$m+1$