Fibonacci
and the
The Perfect Microbes

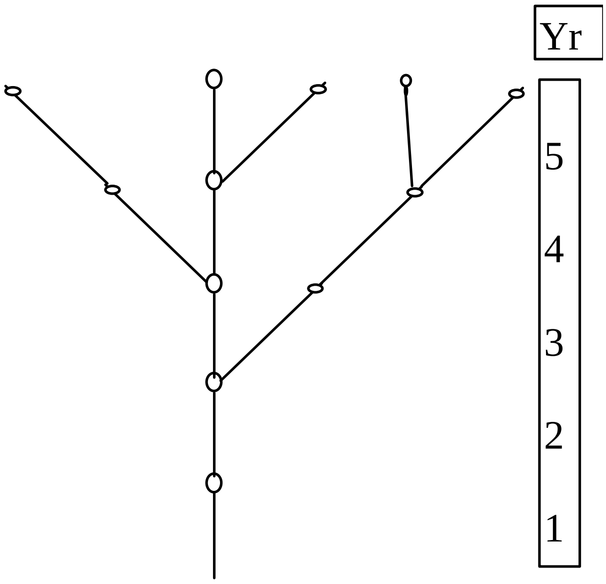
The perfect microbe is very-young for 1 second, young for the next, and old for the next and subsequent seconds. Each old microbe produces a new microbe, i.e. 2 seconds after the creation of a microbe, the microbe produces a new microbe and another new microbe for subsequent years.

Assume we start one just one microbe.

Second	#Microbes
1	1
2	1
3	2
4	3
5	5
6	8
...	...

Tree Growing Branches

Each branch grows during the 1st year and the end



of each subsequent year, it grows a new branch.

After Yr	1	tree has	1	branch
“	2	“	1	“
“	3	“	2	“
“	4	“	3	“
“	5	“	5	“
“	6	“	8	“
“	7	“	13	“

Fibonacci Sequence

f(0)	f(1)	f(2)	f(3)	f(4)	f(5)	f(6)	.....	f(12)
0	1	1	2	3	5	8	.....	144

Inductive/Recursive Definition

f(0) = 0
f(1) =1
f(n+2) = f(n+1) + f(n), if n ≥ 0

Golden Ratio, f

line L = |------------|------------------------|

1 ϕ

ϕ is Golden Ratio $\equiv \quad \frac{1}{\phi} = \frac{\phi}{1+\phi}$

$\equiv \quad \phi^2 - \phi - 1 = 0$

$\equiv \quad \phi = \frac{1+\sqrt{5}}{2}$ (= 1.618)

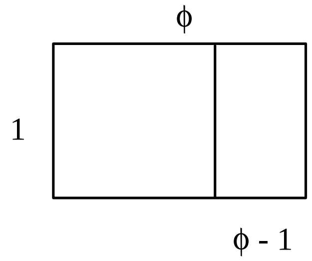
other root is

$\hat{\phi} = \frac{1-\sqrt{5}}{2}$ (= -0.618)

Golden Rectangle

A rectangle is said to be 'Golden' if it sides are in Golden Ratio

Consider a reactangle with width 1 and length ϕ. If the unit square is removed, the rectangle left is still a Golden Rectangle.



Lemma:

$\phi^n = \phi^{n-1} + \phi^{n-2}$(also $\hat{\phi}^n = \hat{\phi}^{n-1} + \hat{\phi}^{n-2}$)

Pf:

Given $\phi^2 = \phi + 1$

tf. $\phi^n = \phi^2 \phi^{n-2}$

$= (\phi+1)\phi^{n-2}$

$= \phi^{n-1} + \phi^{n-2}$

end.

Thm. $\quad f(n) = \dfrac{\phi^n - \hat{\phi}^n}{\sqrt{5}}$

Pf: (by induction)

Base Cases:

n= 0

$$f(0) = 0 = \frac{\phi^0 - \hat{\phi}^0}{\sqrt{5}}$$

n=1

f(1) = 1 and

$$\frac{\phi^1 - \hat{\phi}^1}{\sqrt{5}} = \frac{1+\sqrt{5}-1+\sqrt{5}}{2\sqrt{5}}$$

$$= 1$$

Induction Step:

$$
\begin{aligned}
f(n) &= f(n\text{-}1) + f(n\text{-}2) \\
&= \frac{\phi^{n-1} - \hat{\phi}^{n-1}}{\sqrt{5}} + \frac{\phi^{n-2} - \hat{\phi}^{n-2}}{\sqrt{5}} \\
&= \frac{\phi^{n-1} + \phi^{n-2} - (\hat{\phi}^{n-1} + \phi^{n-2})}{\sqrt{5}} \\
&= \{ \text{ by Lemma } \} \\
&= \frac{\phi^n - \hat{\phi}^n}{\sqrt{5}}
\end{aligned}
$$

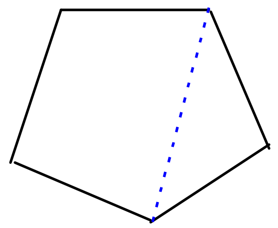
End.

Other Properties:

- $\lim_{n\to\infty} \dfrac{f(n+1)}{f(n)} = \phi$, the Golden Ratio.

- $f(2n+1) = f(n) * f(n) + f(n+1) * f(n+1)$
  $f(2n) = (2f(n+1) - f(n)) *f(n)$

- A regular pentagon with sides 1, has a 'diagonal' of length ϕ. $|\text{Diag}| = 2 \text{ Cos } \dfrac{\pi}{5} = \phi$



```
fib (k: INTEGER): INTEGER is
        --recursive version of fibonnaci
    require
        pre_fib: k >= 0
    do
        if  k = 0 then
            Result := 0
        elseif  k = 1 then
            Result := 1
        else
            Result := fib (k - 2) + fib (k - 1)
        end
    end ;
```

```
fib1 (k: INTEGER): INTEGER is
    require
        pre_fib: k > 0
    local
        i, p, c, n: INTEGER
    do
        from
            p := 0;
            c := 1;
            i := 1
        until
            i = k
        loop
            n := p + c;
            p := c;
            c := n;
            i := i + 1
        end ;
        Result := c
    end ;
```

```
fib2 (k: INTEGER): INTEGER is
    require
        pre_fib: k > 0
    local
        i, c, n: INTEGER
    do
        from
            c := 0;
            n := 1;
            i := 1
        until
            i = k
        loop
            n := n + c;
            c := n - c;
            i := i + 1
        end ;
        Result := n
    end ;
```

```
fib3 (k: INTEGER): INTEGER is
    local
        phi,r  : REAL;
        s: SINGLE_MATH
    do
        !! s;
        phi := (1 + s.sqrt (5.0)) / 2
        r := (phi ^ k) / s.sqrt (5.0);
        Result := r.rounded
    end ;
```