UNIVERSITY OF DUBLIN

MA1BA11

TRINITY COLLEGE

FACULTY OF SCIENCE

SCHOOL OF MATHEMATICS

JF CS/CSLL **Hilary Term 2000**

MATHEMATICS 135, CS/CSLL 1BA1: TEST 1

Friday, 28 January 2000 Exam Hall 9–11 a.m.

Dr. Ó Dúnlaing

Attempt all questions. Mathematical tables and graph paper are available. Calculators are permitted.

1. Let $P = (1, 3, 2)$, $Q = (2, 5, 3)$ in E^3. (i) Parametrise the plane containing O, P, and Q. (ii) Give an equation for the same plane. (iii) The point $X = (6, 5, -1)$ belongs to this plane: calculate the (unique) scalars s and t such that $X = sP + tQ$.

2. (i) Let $P = (3, 1)$ and $Q = (1, 3)$ in E^2. Calculate the result of projecting the point $(5, 4)$ onto the line OP in the direction parallel to OQ.

(ii) Let $S = (1, 1, -1)$, and $X = (2, 5, 1)$. Calculate the effect of rotating X through $\pi/3$ radians — $60°$ — anticlockwise around the axis OS.

3. Let

$$A = \begin{bmatrix} 1 & 2 & 3 & 3 & 4 \\ 2 & 4 & 6 & 10 & 12 \\ 2 & 5 & 8 & 9 & 9 \end{bmatrix}.$$

Calculate bases for (i) its row space and (ii) its column space. (iii) Express the non-basis columns of A in terms of the basis columns. (iv) Calculate a basis for the kernel of A.

4. Let $P = (1, 3, 2)$, $Q = (2, 5, 3)$, and $R = (1, 2, 2)$ in E^3. Let f be the linear map which takes

$$P \mapsto P + Q, \quad Q \mapsto -P + Q, \quad \text{and } R \mapsto P - Q + R.$$

(i) Give the matrix A' of the map f with respect to the basis P, Q, R, and (ii) calculate the standard matrix A of f.

5. Calculate the dterminant

$$\begin{vmatrix} 1 & 4 & 3 & 2 \\ 3 & 5 & 5 & 2 \\ 2 & 1 & 2 & 3 \\ 4 & 16 & 5 & 6 \end{vmatrix}$$

(i) by bringing to upper triangular form, and (ii) directly, by cofactor expansion along the second row.

6. (i) Using Cramer's Rule — no other method — solve

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \\ 8 \end{bmatrix}.$$

(ii) Using the Adjoint Matrix formula — no other method — invert the matrix given in part (i) of this question.