



Previously

- HDLC – High Level Data Link Control
- PPP – Point-to-Point Protocol



Error Detection & Correction

Introduction
Correction
Detection

- Error Correction
 - Hamming distance
- Error Detection
 - Parity bits
 - Block Sum Check
 - CRC

Errors

Introduction
Correction
Detection

■ Occurrence of errors

- Rare on _____
- More frequent on _____

■ Type of errors

- Errors generally occur as _____ or _____
- Burst errors mean that _____ but _____

■ Redundant Info is needed to

- _____
- _____

Basics

Introduction
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■ Frame consists of $n = m + r$ bits

- m _____
- r _____
- There are only _____

■ e.g. Parity bit: _____

- Even parity: _____
- Odd parity: _____
- 1001010 → _____ (even) or _____ (odd)
- There are _____
- We can detect _____
- No way to _____

Forward Error Correction

Introduction
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Detection

- Hamming Distance _____
 - Determine by _____ 10010100 00000111
 - For example.... 10011110 11001100
- Hamming distance of a code _____
 - E.g. 00111
- To detect an error of d bits... 01100
 - _____ 10010
- To correct an error of d bits... 11001
 - _____
 - where _____

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Single bit correction

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- To correct single bit errors...
 - Given 2^m messages, for each message there _____
(Corrupt each bit of the message)
 - $(n+1).2^m \leq 2^n$ _____
 - $(m+r+1) \leq 2^r$ _____
- Hamming codes achieves this lower limit

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Hamming codes

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- Hamming code achieves this lower limit
- Check bits: 1, 2, 4, 8, ... _____
- Data bit indices: 3, 5, 6, 7, 9, 10, 11, ... _____
- Check bits are parity bits computed from those data bits whose binary decomposition includes the check bit:

Data bit:	1	2	3	4	5	6	7	8
Data bit index:	3	5	6	7	9	10	11	12
Binary decomposition:	1	1		1	1		1	
	2		2	2		2	2	
		4	4	4				4
					8	8	8	8

- If one check bit is incorrect _____
- If multiple check bits are incorrect _____
- To correct burst errors we change _____

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Hamming code example

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ASCII

H	1001000	→
a	1100001	→
m	1101101	→
m	1101101	→
i	1101001	→
n	1101110	→
g	1100111	→

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Block Sum Check

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- Using a single parity bit the probability of an error not being detected is _____
- To increase this we can use a Block Sum Check
 - Compute parity for _____
 - Probability of an error not being detected is _____

P _R	B ₆	B ₅	B ₄	B ₃	B ₂	B ₁	B ₀	
0	0	0	0	0	0	1	0	= STX
1	0	1	0	1	0	0	0	Frame contents
0	1	1	0	0	0	1	0	
0	0	1	0	0	0	0	0	
1	0	1	0	1	1	0	1	
0	1	0	0	0	0	0	0	
1	1	0	0	0	1	1	1	
1	0	0	0	0	0	1	1	= ETX
1	1	0	0	0	0	0	1	= BCC

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Cyclic Redundancy Check

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- Treats the bits in the frame as _____
 - e.g. 110001 = _____
- Determine a Checksum which is
 - Data / Generator Polynomial
 - Checksum is usually _____
 - Generator Polynomial is _____
 - Checksum is referred to as the FCS (_____) or the CRC (_____)
 - Checksum appended _____

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Cyclic Redundancy Check

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Generator Polynomials

- High & low bits must _____
- $x^{32} + x^{26} + x^{23} + x^{22} + x^{16} + x^{12} + x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^2 + x^1 + 1$
- $x^{16} + x^{15} + x^2 + 1$

Computing the checksum

- Append zero bits $\rightarrow x^r M(x)$ where r _____
- $x^r M(x) / G(x)$ This division is done modulo 2
- Subtract the remainder from $x^r M(x)$

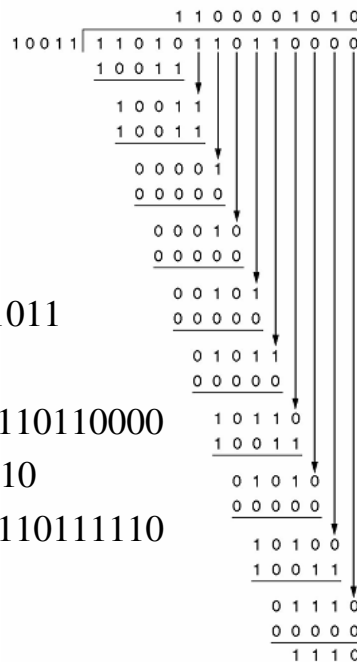
To detect errors

- Compute _____
- If zero then _____, otherwise _____

Example

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- $M(x) = 1101011011$
- $G(x) = 10011$
- $x^r M(x) = 11010110110000$
- Remainder = 1110
- Transmit 11010110111110



CRC Performance

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- $E(x)$: _____
- $E(x) = x^i$ _____. This is not divisible by $G(x)$ as long as _____
- $E(x) = x^i + x^j = x^j (x^{i-j} + 1)$ _____. Detectable as long $G(x)$ is not divisible by _____.
 – E.g. $x^{15} + x^{14} + 1$
- To catch all Odd errors we _____.
- Burst errors
 - Catch all errors _____ except _____
 - Probability (Long errors are unnoticed) = _____

Exercise

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- Given a message 100110111010 and a generator polynomial $(x^4 + x^3 + x^1 + 1)$ compute the CRC.

- Also, what errors can this CRC detect?

