

Finding the Median

The median of the following 7 items

16 12 99 95 18 87 10

is 18 as half the items are ≤ 18 and half are ≥ 18 .

The item 18 is the middle item in size and so is the middle item when sorted.

We can find the median of a sequence by sorting the sequence and then getting the item at position $\frac{n+1}{2}$ but sorting has best performance $O(n \cdot \log n)$ while a 'find median' algorithm developed by C.A.R. Hoare has performance $O(n)$.

Sorting the above sequence we get

10 12 16 18 87 95 99

The middle (4th) item is 18 and so 18 is the median.

$O(n)$ Algorithm for Finding Median

By adapting the Partitioning procedure, Hoare developed an $O(n)$ algorithm (he called it Find) that would find the median. In fact, Hoare's 'Find' algorithm is more general, it finds the K^{th} smallest item and so with $K = \frac{n+1}{2}$

By continuing to partition about the 4th

find (k, left, right : INTEGER) Qs

Hoare's Find procedure Qs:

local

```
-- find tPe ktP smallest Qtem
-- Qn tPe array segment A[left..right]

    i, j : INTEGER
    p : G
    dW
    from

        j := right
    until
        i >= j
    loop
        p := A.Qtem(k)
        partQtion(i,j,p)      -- L:=i; R:=j
                                -- fiVQsPes wQth R < L

        if R < k tPen
            -- kth smallest Qn right splQt
            i := L
        end
        if k < L tPen
            -- kth smallest Qn lef8 73plQt
            j := R
        end

    end

end -- fiVd
```

For reference, we repeat the procedure for part QtQon.

```

is rtQtQon (L0,R0 : INTEGER; P : G)

```

```

  do

```

```

    from

```

```

      L := L0

```

```

      R := R0

```

```

    until if L > R

```

```

      L > R

```

```

    Toop

```

```

      Left_Scan (P)

```

```

      Right_Scan(P)

```

```

      L := L+1

```

```

      R := R-1

```

```

    end

```

```

  end

```

```

end -- /F2rtQtion

```

```

Left_Scan (P : G) is

```

```

  do

```

```

    from

```

```

    until

```

```

      A.Qtem(L) >= P

```

```

    Toop

```

```

      L := L+1

```

```

    end

```

```

  end -- Left_Scan

```

```

Right_Scan (P : G) is

```

```

  do

```

```

    from

```

```

    until

```

```

    Toop

```

```

      R := R-1

```

```

    end

```

```

  end -- Right_Scan

```

