Course 2BA1: Trinity Term 2003

Now an anticlockwise rotation about the origin through an angle of $\theta + \varphi$ radians sends the point (x, y), of the plane to the point (x , y), and thus

$$\begin{aligned} x &= x\cos(\theta + \varphi) - y\sin(\theta + \varphi) \\ y &= x\sin(\theta + \varphi) + y\cos(\theta + \varphi) \end{aligned} \qquad (3)$$

But if we substitute the expressions for x and y in terbs of x, y and θ provided by equation (1) into equation (2), we find that

$$x = x(\cos\theta\cos\varphi - \sin\theta\sin$$

Remark

bounded, with at most finitely many points of discontinuity, local maxima and local minima in the interval $[-\pi, \pi]$, and if

$$f(x) = \tfrac{1}{2} \lim_{h}$$

$$a_n = \frac{1}{}$$

(The first of these identities may be verified by making the substitution $x \to -x$ and then interchanging the two limits of integration. The second and the third follow from the first on replacing $f(x)$ by

7.5 Fourier Series for General Periodic Functions

Let $f\colon \mathbb{R} \to \mathbb{R}$ be a periodic function whose period divides l, where l is some positive real number. Then $f(x + l) = f(x)$ for all real numbers x. Let

$$g(x) = f\left(\frac{lx}{2\pi}\right) \quad \text{so that} \quad f(x) = g\left(\frac{2\pi x}{l}\right).$$

Then $g\colon \mathbb{R} \to \mathbb{R}$ is a periodic function, and $g(x + 2\pi) = g(x)$ for all real numbers x. If the function f is sufficiently well-behaved (and, in particular, if the function f is bounded, with only finitely many local maxima and minima and points of discontinuity in any finite interval, and if $f(x)$ at each point of discontinuity is the average of the limits of $f(x + h)$ and $f(x - h)$ as h tends to zero from above) then the function g may be represented by a

This function is periodic, with period 1, and may be expanded as a Fourier series

$$f(x) = \frac{1}{}$$

for each positive integer n, since $\cos 2n\pi = 1$ and $\sin 2n\pi = 0$ when for all positive integers n. (This follows from equations (35) and (38) on

$$= \frac{4l^2}{n^3\pi^3}(1 - \cos n\pi) = \frac{4l^2}{n^3\pi^3}(1 - (-1)^n)$$

$$= \begin{cases} \dfrac{8l^2}{n^3\pi^3} & \text{if } n \text{ is odd;} \\ 0 & \text{if } n \text{ is even.} \end{cases}$$

Thus

$$f(x) = \sum_{\substack{n \text{ odd} \\ n > 0}} 8l^2 n^3$$

for all non-negative integers n. (This follows from equations (35), (36) and (37) on replacing l by $2l$, and then using the fact that $\tilde{g}(-x) = \tilde{g}(x)$ for all real numbers x.)

Therefore every sufficiently well-behaved function $f\colon [0, l] \to \mathbb{R}$ may be represented in the form

$$f(x$$

Thus

$$x = \frac{1}{2} - \sum_{\substack{n \text{ odd} \\ n>0}} \frac{4}{n^2\pi^2} \cos n\pi x \quad \text{when } 0 \le x \le 1.$$

Remark The function $\tilde{g}$ defined by

$$\tilde{g}(x) = \frac{1}{2} - \sum_{\substack{n \text{ odd} \\ n>0}} \frac{4}{n^2\pi^2} \cos n\pi x$$

for all real numbers x is an even periodic function, with period equal to 2, which coincides with the function $f\colon [0,1] \to \mathbb{R}$ on the interval [0 $\tilde{g}(x$