# Course 2BA1: Michaelmas Term 2002
# Section 2: Sets and Functions

David R. Wilkins



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$A = B$ if and only if every element of $A$ is an element of $B$ and every element of $B$ is an element of $A$.

If we have a list of entities, we denote the set consisting of these entities by enclosing the list within braces $f\colon:\colon:g$. For example the set consisting of

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**Definition** Let $A$ be a set. The *power set* $PA$ is the set whose elements are the subsets of $A$.

**Example** Let $A$ be a set consisting of exactly one element $a$, so that $A = \{a\}$. Then the subsets of $A$ are the empty set $\emptyset$ and $A$ itself. It follows that the power set $PA$ of $A$ is given by $PA = \{\emptyset, A\}$ in this case. Note that the set $A$ has 1 element and that its power set $PA$ has 2 elements.

**Example** Let $A = \{1, 2\}$. Then $PA = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$. Note that the set $A$ has 2 elements and that its power set $PA$ has 4 elements.

**Example** Let $A$ be the set consisting of the three colours red, green and blue. Let us for convenience denote these colours by R, G and B. Thus $A = \{\mathrm{R}, \mathrm{G}, \mathrm{B}\}$. Going systematically through the subsets of $A$ with 0, 1, 2, and 3 elements, we see that the power set of $A$ is given by the following:

$$PA = \{\emptyset, \{\mathrm{R}\}, \{\mathrm{G}\}, \{\mathrm{B}\}, \{\mathrm{G}, \mathrm{B}\}, \{\mathrm{B}, \mathrm{R}\}, \{\mathrm{R}, \mathrm{G}\}, \{\mathrm{R}, \mathrm{G}, \mathrm{B}\}\}.$$

Note that the set $A$ has 3 elements and its power set $PA$ has 8 elements.

**Example** Let $A$ be a set consisting of the four elements $a$, $b$, $c$ and $d$. Then the power set $PA$ of $A$ consists of the following subsets of $A$: the empty set $\emptyset$, $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$, $\{a, b\}$, $\{a, c\}$, $\{a, d\}$, $\{b, c\}$, $\{b, d\}$, $\{c, d\}$, $\{b, c, d\}$, $\{a, c, d\}$, $\{a, b, d\}$, $\{a, b, c\}$ and $\{a, b, c, d\}$. Thus the set $A$ 7(ts.246(one)-245(subsets.246(with)]TJ -

If $A$ has just one element then its power set $PA$ has two elements. Indeed if $A = \{a\}$ then $PA = \{\emptyset, A\}$.

An easy application of the Principle of Mathematical Induction proves

Example

**De nition** Let $R$ be a relation on a set $A$.

The relation $R$ is said to be *re exive* when it has the following property: $xRx$ for all elements $x$ of the set $A$.

The relation $R$ is said to be *symmetric* when it has the following property: if $x$ and $y$ are elements of the set $A$, and if $xRy$, then $yRx$.

The relation $R$ is said to be *transitive*

(i) $x$

(i) $x$ $x$ for all elements $x$ of $A$ (i.e., is *re exive*);

(ii) if $x$ and $y$ are elements of

Suppose then that there exists an element $z$ of $A$ that belongs to both [$x$] and [$y$]. Then $z$ $x$ and $z$ $y$. But then $x$ $z$ (since the relation is symmetric), and hence $x$ $y$ (since $x$ $z$

**Example** The relation (`less than or equal to') is a partial order on the set R of real numbers. (It clearly possesses all three properties listed above.) It is also a partial order when considered as a relation on the set Z of integers, or on the set N of natural numbers.

**Example** Let $A$ is a partial order on the power set $PA$ of $A$, where subsets $B$ and $C$ satisfy $B$ $C$ if and only if $B$

then $D$ $B$ and $D$ $C$

**Example** Let $A = \{1, 2,$

The collection of all such records contained in the database can be viewed

Whilst every function from $A$

!J/F33 33 11.955 Tf 15.62 0 Td[(W)]TJ/F15 11.955 Tf

## 2.14 Injective, Surjective and Bijective Functions

Many functions are not invertible. The following example illustrates some of the reasons why certain functions may not be invertible.

**Example** Let $W$ be the set of all English words occurring as headwords in some speci ed dictionary, l]TJ/F33 45 11.955 Tf 155.23 0 Td[(N)]TJ/F15 11.955 Tf 13.453 0 Td[

**Example** Let $\mathbb{R}^+$ denote the set of non-negative real numbers, an let $q\colon \mathbb{R}^+ \to \mathbb{R}^+$

An invertible function must also be surjective. For if $g\colon B \to A$ is an inverse of