UNIVERSITY OF DUBLIN

TRINITY COLLEGE

Faculty of Engineering and Systems Sciences

Department of Computer Science

B.A (Mod.) Computer Science
Senior Sophister Examination

Trinity Term 2004

4BA2 - Systems Modelling

Wednesday 2nd June | Sam. Beckett Rooms | 09:30 - 12:30

Dr. Tony Redmond, Dr. Donal O'Mahony

Attempt **FIVE** questions at least two from each section
Please use separate answer books for each section
Queuing Tables are attached to paper

Use separate answer books for each section.

Section A

Q1 Discuss the evolution of long-distance telecommunications links from simple phone circuits to the Gigabit links of today. As we move to providing links at many hundreds of megabits at the edge of the network, what sorts of technology do we need at the core to cope with this.

Q2 Describe the core idea behind Asynchronous Transfer Mode (ATM) technology. Explain the processes that must occur at the edge and the core of the network before the first cell can be switched end-to-end. Trace the evolution of this technology through to today's Multi-Protocol Label Switching (MPLS). Why might MPLS succeed where ATM failed?

Q3. Outline the protocol exchanges that take place when a page is fetched from a web server. What hardware and software mechanisms are used to scale up this simple system to one that can allow millions of simultaneous users to perform online transactions.

Q4. Describe the ways in which public key cryptosystem are operated and show how they can be used to produce a signed and/or encrypted message. What extra ingredients are needed before this technique can be used to secure communication between individuals who have never met.

Section B

Q5. **a.** Define and briefly discuss Hard Systems Theory (HST).
b. Define and discuss briefly Checkland's Soft System Theory (SST).
c. Define and briefly discuss Coverdale's Systematic Approach (SA). How does SA relate to HST and SST?
d. Show in a diagram the Coverdale Systematic Approach to a computer systems performance evaluation study.

Q6. **a.** A small business is considering either a configuration of 2 machines (with separate queues) of speed $\mu/2$ or a configuration of 1 machine of speed μ. Use the attached queueing theory formulae to estimate W2/W1 where W2 is the system response time in the 2 machine case and W1 is the response time in the 1 machine case. Which configuration is better and why?

b. The company is also considering a duplex (i.e. 2 machines with 1 queue) configuration of 2 machines of speed $\mu/2$ versus the 1 machine of speed μ configuration. Derive a formula for W2/W1 and hence calculate values of it for $\mu = 0.2, 0.4, 0.6$ and 0.8. Which configuration is better and why?

c. The company is also considering a duplex (i.e. 2 machines with 1 queue) configuration of 2 machines of speed μ versus the 1 machine of speed μ configuration. Derive a formula for W2/W1 and hence calculate values of it for $\mu = 0.2, 0.4, 0.6$ and 0.8. Which configuration is better and why?

d. What is meant by Streeter's scaling effect and why is it important?

Q7. **a.** The following sequence of random numbers was generated using the equation:
$r(n) = a * r(n-1)$ (modulo 10) where $r(0) = 123456789$ and $a = 100{,}003$.
Comment on the randomness of the numbers. How would you simply improve the randomness of such numbers?

n	un
1	0.60492 70367
2	0.51845 11101
3	0.66636 33303
4	0.33211 99909
5	0.99544 99727
6	0.98361 94181
7	0.94266 97543
8	0,80343 92629
9	0.33660 77887
10	0.78869 33661
11	0.70269 00983
12	0.11790 02949
13	0.38319 08847
14	0.23804 26541
15	0.97953 79623
16	0.73484 38869
17	0.59322 16607
18	0.94573 49821
19	0.33541 49463
20	0.50087 48389

b. Sketch the two basic time increment modes in simulation using diagrams.

c. Sketch how the inverse transform method is used for simulating random numbers drawn from an exponential distribution.

d. Give an example of a variance reduction technique and explain how it works. Apart from variance reduction what other main advantages does it have?

Q8. The characteristics of an interactive computer system, consisting essentially of a CPU, Drum, Disk and Terminals, have been estimated as follows:

Mean CPU service time per interaction = 7 ms

Mean Drum service time per interaction = 10 ms

Mean Disk service time per interaction = 70 ms.

After an interaction with the CPU, the probability of a job finishing is 0.1, the probability of the job going to the Drum is 0.7, and the probability of going to the Disk is 0.2.

The Central Server model formulae (using the usual notation) are as follows:

$$\rho_i = \left[\frac{G(K-1)}{G(K)}\right] \qquad i = 1$$

$$= \frac{\mu_1 \rho_1 p_i}{u_i} \qquad i = 2, 3, \ldots\ldots\ldots M.$$

$$\lambda_t = \mu_1 p_1 \rho_1$$

Buzen's algorithm:

$$x_1 = 1$$

$$x_i = \frac{\mu_1 p_i}{\mu_i} \qquad i = 2, 3, \ldots\ldots\ldots M.$$

$$G(K) = g(K, M)$$

$$g(k, m) = g(k, m-1) + x_m\, g(k-1, m) \qquad \text{if } k > 0 \text{ and } m > 1$$

$$g(k, 1) = 1 \qquad \text{for } k = 0, 1, \ldots, K$$

$$g(0, m) = 1 \qquad \text{for } m = 1, 2, \ldots, M$$

a. Use the Central Server model to model the CPU/ IO inner subsystem of the computer system and use Little's relation for other quantities. Draw a sketch of your model. Estimate the resource utilisations at a multiprogramming level of 4 for all resources. Calculate the throughput, and W, the response time, assuming there are (N =) 40 terminals in use with Think Time = 3 seconds. State which is the bottleneck resource.

b. Replace the original bottleneck processor with three of the same speed (assume the bottleneck traffic is evenly spread over the 3 processors). Compute resource utilisation, throughput and response time. State which is the bottleneck resource and give your comments.

c. Replace the bottleneck processor with one, which is twice as fast. Again compute the resource utilisations, throughput and response time. State the bottleneck resource and comment on your results.

d. Write a short note on extensions to the Central Server model.

TABLE 27

Steady State Equations of Central Server Model of Multiprogramming

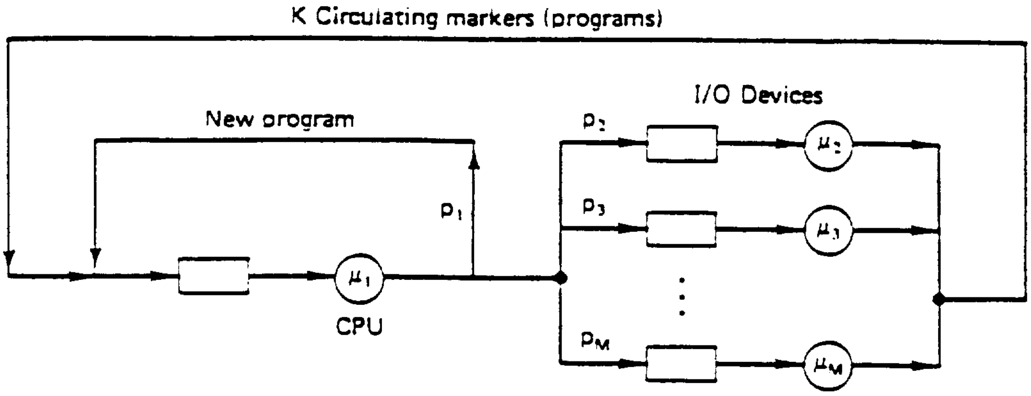

Fig. 6.3.4 Central server model of multiprogramming.

For the assumptions of the model see Section 6.3.4.
Calculate $G(0), G(1), \ldots, G(K)$ by Algorithm 6.3.1 (Buzen's Algorithm).
Then the server utilizations are given by

$$\rho_i = \begin{cases} G(K-1)/G(K) & i = 1 \\ \dfrac{\mu_1 \rho_1 p_i}{\mu_i} & i = 2, 3, \ldots, M. \end{cases} \tag{6.3.25}$$

The average throughput λ_T is given by

$$\lambda_T = \mu_1 \rho_1 p_1. \tag{6.3.26}$$

If the central server model is the central processor model for the interactive computing system of Fig. 6.3.1, then the average response time W is calculated by

$$W = \frac{N}{\lambda_T} - E[t] = \frac{N}{\mu_1 \rho_1 p_1} - E[t]. \tag{6.3.27}$$

TABLE 24
Steady State Formulas for the Finite Population Queueing Model of Interactive Computing With Processor-Sharing

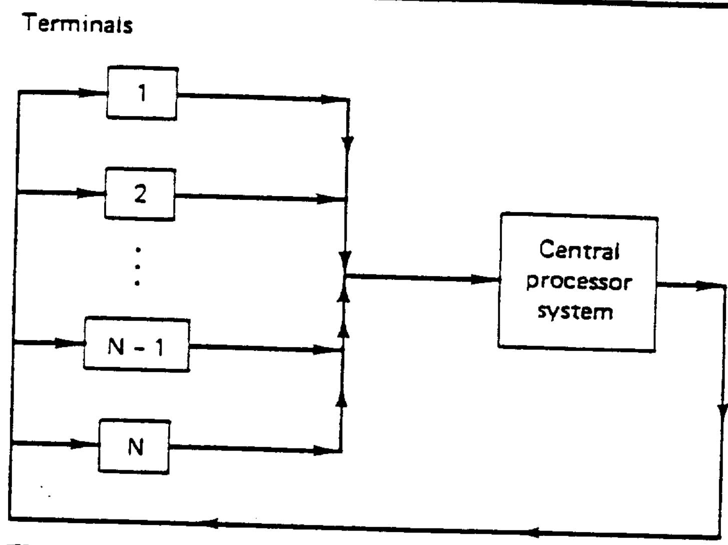

Fig. 6.3.1 Finite population queueing model of interactive computer system. Special case in which the central processor system consists of a single CPU with processor-sharing queue discipline.

The CPU operates with the processor-sharing queue discipline. CPU service time is general with the restriction that the Laplace–Stieltjes transform must be rational. The same restriction holds on think time. $E[t] = 1/\alpha$ is the average think time with $E[s] = 1/\mu$ the average CPU service time. Then

$$p_0 = \left[\sum_{n=0}^{N} \frac{N!}{(N-n)!}\left(\frac{E[s]}{E[t]}\right)^n\right]^{-1} = \left[\sum_{n=0}^{N} \frac{N!}{(N-n)!}\left(\frac{\alpha}{\mu}\right)^n\right]^{-1}.$$

The CPU utilization

$$\rho = 1 - p_0,$$

and the average throughput

$$\lambda_T = \frac{\rho}{E[s]} = \frac{1-p_0}{E[s]}.$$

The average response time

$$W = \frac{NE[s]}{1-p_0} - E[t].$$

TABLE 6

Steady State Formulas for M/M/2 Queueing System

$\rho = \lambda E[s]/2 = u/2.$

$p_0 = P[N = 0] = (1 - \rho)/(1 + \rho).$

$p_n = P[N = n] = 2p_0\rho^n = \dfrac{2(1-\rho)\rho^n}{(1+\rho)}, \qquad n = 1, 2, 3, \ldots.$

$\left.\begin{aligned}\pi_w(90) &\approx W + 1.3\sigma_w \\ \pi_w(95) &\approx W + 2\sigma_w\end{aligned}\right|$ Martin's estimate.

$L_q = E[N_q] = \dfrac{2\rho^3}{1-\rho^2}, \qquad \sigma^2_{N_q} = \dfrac{2\rho^3[(\rho+1)^2 - 2\rho^3]}{(1-\rho^2)^2}$

$C(2, u) = P[\text{both servers busy}] = 2\rho^2/(1+\rho).$

$L = E[N] = L_q + u = 2\rho/(1-\rho^2).$

$W_q(0) = P[q = 0] = (1 + \rho - 2\rho^2)/(1+\rho).$

$W_q(t) = P[q \le t] = 1 - [(2\rho^2)/(1+\rho)]e^{-2\mu t(1-\rho)}.$

$W_q = E[q] = \rho^2 E[s]/(1-\rho^2), \qquad E[q \mid q > 0] = E[s]/2(1-\rho).$

$\sigma_q{}^2 = \rho^2(1 + \rho - \rho^2)E[s]^2/(1-\rho^2)^2.$

$\pi_q(r) = \dfrac{E[s]}{2(1-\rho)} \ln\left(\dfrac{200\rho^2}{(100-r)(1+\rho)}\right).$

$\pi_q(90) = \dfrac{E[s]}{2(1-\rho)} \ln\left(\dfrac{20\rho^2}{1+\rho}\right), \qquad \pi_q(95) = \dfrac{E[s]}{2(1-\rho)} \ln\left(\dfrac{40\rho^2}{1+\rho}\right).$[a]

$$W(t) = P[w \le t] = \begin{cases} 1 - \dfrac{(1-\rho)}{1-\rho-2\rho^2}e^{-\mu t} + \dfrac{2\rho^2}{1-\rho-2\rho^2}e^{-2\mu t(1-\rho)} & \text{if } u \ne 1 \\ 1 - \left[1 + \dfrac{\mu t}{3}\right]e^{-\mu t} & \text{if } u = 1. \end{cases}$$

$W = E[s]/(1-\rho^2).$

$$E[w^2] = \begin{cases} \dfrac{\rho^2 E[s]^2[1 - 4(1-\rho)^2]}{(2\rho-1)(1-\rho)(1-\rho^2)} + 2E[s]^2, & u \ne 1. \\ \frac{10}{3}E[s]^2, & u = 1. \end{cases}$$

$\sigma_w{}^2 = E[w^2] - E[w]^2.$

[a] All percentile formulas for q yield negative values for low server utilization: all such should be replaced by zero.

TABLE 5
Steady State Formulas for M/M/c Queueing System

$u = \lambda/\mu = \lambda E[s], \qquad \rho = u/c.$

$$p_0 = P[N = 0] = \left[\sum_{n=0}^{c-1} \frac{u^n}{n!} + \frac{u^c}{c!\,(1-\rho)}\right]^{-1} = c!\,(1-\rho)C(c, u)/u^c.$$

$$p_n = \begin{cases} \dfrac{u^n}{n!} p_0 & \text{if } n = 0, 1, \ldots, c \\ \dfrac{u^n p_0}{c!\, c^{n-c}} & \text{if } n \geq c. \end{cases}$$

$$L_q = E[N_q] = \lambda W_q = \frac{uC(c, u)}{c(1-\rho)}, \qquad \sigma_{N_q}^2 = \frac{\rho C(c, u)[1 + \rho - \rho C(c, u)]}{(1-\rho)^2}.$$

where $C(c, u) = P[N \geq c]$ = probability all c servers are busy is called Erlang's C formula.

$$C(c, u) = \frac{u^c}{c!} \Big/ \left[\frac{u^c}{c!} + (1-\rho)\sum_{n=0}^{c-1} \frac{u^n}{n!}\right].$$

$L = E[N] = L_q + u = \lambda W.$

$$W_q(0) = P[q = 0] = 1 - \frac{p_c}{1-\rho} = 1 - C(c, u).$$

$$W_q(t) = P[q \leq t] = 1 - \frac{p_c}{1-\rho} e^{-ct(1-\rho)/E[s]} = 1 - C(c, u)e^{-t/E[q|q>0]}.$$

$$W_q = E[q] = \frac{C(c, u)E[s]}{c(1-\rho)}, \qquad E[q \mid q > 0] = \frac{E[s]}{c(1-\rho)}.$$

$$\sigma_q^2 = \frac{[2 - C(c, u)]C(c, u)E[s]^2}{c^2(1-\rho)^2}. \qquad \pi_q(r) = \frac{E[s]}{c(1-\rho)} \ln\left(\frac{100C(c, u)}{100 - r}\right).^*$$

$$\pi_q(90) = \frac{E[s]}{c(1-\rho)} \ln(10C(c, u)), \qquad \pi_q(95) = \frac{E[s]}{c(1-\rho)} \ln(20C(c, u)).^*$$

$$W(t) = P[w \leq t] = \begin{cases} 1 + C_1 e^{-\mu t} + C_2 e^{-c\mu t(1-\rho)} & \text{if } u \neq c - 1 \\ 1 - [1 + C(c, u)\mu t]e^{-\mu t} & \text{if } u = c - 1. \end{cases}$$

where $C_1 = \dfrac{u - c + W_q(0)}{c - 1 - u}$ and $C_2 = \dfrac{C(c, u)}{c - 1 - u}$.

$W = E[q] + E[s].$

$$E[w^2] = \begin{cases} \dfrac{2C(c, u)E[s]^2}{u + 1 - c}\left(\dfrac{1 - c^2(1-\rho)^2}{c^2(1-\rho)^2}\right) + 2E[s]^2, & u \neq c - 1. \\ 4C(c, u)E[s]^2 + 2E[s]^2, & u = c - 1. \end{cases}$$

$\sigma_w^2 = E[w^2] - E[w]^2$

$$\left.\begin{array}{l} \pi_w(90) \approx W + 1.3\sigma_w \\ \pi_w(95) \approx W + 2\sigma_w \end{array}\right\} \text{Martin's estimates}$$

* All percentile formulas for q yield negative values for low server utilization; all should be replaced by zero.

TABLE 4
Steady State Formulas for M/M/1/K Queueing System

($K \geq 1$ and $N \leq K$)

$$p_n = P[N = n] = \begin{cases} \dfrac{(1-u)u^n}{1-u^{K+1}} & \text{if } \lambda \neq \mu \text{ and } n = 0, 1, \ldots, K \\[2ex] \dfrac{1}{K+1} & \text{if } \lambda = \mu \text{ and } n = 0, 1, \ldots, K. \end{cases}$$

$p_K = P[N = K]$. Probability an arriving customer is lost.

$\lambda_a = (1 - p_K)\lambda$. λ_a is the actual arrival rate at which customers enter the system.

$$L = E[N] = \begin{cases} \dfrac{u[1-(K+1)u^K + Ku^{K+1}]}{(1-u)(1-u^{K+1})} & \text{if } \lambda \neq \mu \\[2ex] \dfrac{K}{2} & \text{if } \lambda = \mu. \end{cases}$$

$$L_q = E[N_q] = L - (1 - p_0)$$

$$q_n = \frac{p_n}{1-p_K}, \qquad n = 0, 1, 2, \ldots, K-1.$$

q_n is the probability that there are n customers in the system just before a customer enters.

$$W(t) = P[w \leq t] = 1 - \sum_{n=0}^{K-1} q_n \sum_{k=0}^{n} e^{-\mu t}\frac{(\mu t)^k}{k!}.$$

$$W = E[w] = L/\lambda_a.$$

$$W_q(t) = P[q \leq t] = 1 - \sum_{n=0}^{K-2} q_{n+1} \sum_{k=0}^{n} e^{-\mu t}\frac{(\mu t)^k}{k!}.$$

$$W_q = E[q] = L_q/\lambda_a.$$

$$E[q \mid q > 0] = W_q/(1 - p_0).$$

$$\rho = (1 - p_K)u.$$

ρ is the true server utilization (fraction of time the server is busy).

TABLE 3

Steady State Formulas for M/M/1 Queueing System

$p_n = P[N = n] = (1 - \rho)\rho^n, \qquad n = 0, 1, 2, \ldots$

$P[N \geq n] = \sum_{k=n}^{\infty} p_k = \rho^n, \qquad n = 0, 1, 2, \ldots$

$L = E[N] = \rho/(1 - \rho), \qquad \sigma_N^2 = \rho/(1 - \rho)^2.$

$L_q = E[N_q] = \rho^2/(1 - \rho), \qquad \sigma_{N_q}^2 = \rho^2(1 + \rho - \rho^2)/(1 - \rho)^2.$

$E[N_q \mid N_q > 0] = 1/(1 - \rho), \qquad \mathrm{Var}[N_q \mid N_q > 0] = \rho/(1 - \rho)^2.$

$W(t) = P[w \leq t] = 1 - e^{-t/W}, \qquad P[w > t] = e^{-t/W}.$

$W = E[w] = E[s]/(1 - \rho), \qquad \sigma_w = W.$

$\pi_w(90) = W \ln 10 \approx 2.3W, \qquad \pi_w(95) = W \ln 20 \approx 3W.$

$\pi_w(r) = W \ln [100/(100 - r)].$

$W_q(t) = P[q \leq t] = 1 - \rho e^{-t/W}, \qquad P[q > t] = \rho e^{-t/W}.$

$W_q = E[q] = \rho E[s]/(1 - \rho).$

$\sigma_q^2 = (2 - \rho)\rho E[s]^2/(1 - \rho)^2.$

$E[q \mid q > 0] = W, \qquad \mathrm{Var}[q \mid q > 0] = W^2.$

$\pi_q(90) = W \ln(10\rho), \qquad \pi_q(95) = W \ln(20\rho).$

$\pi_q(r) = W \ln \left(\frac{100\rho}{100 - r}\right).$

All percentile formulas for q will yield negative values when ρ is small: all negative values should be replaced by zero. For example, if ρ is 0.02, then 98 percent of all customers do not have to queue for service so the 95th percentile value of q is zero: so are the 90th and 95th percentile values.

TABLE 2
Relationships Between Random Variables of Queueing Theory Models

$u = E[s]/E[\tau] = \lambda E[s] = \lambda/\mu$	Traffic intensity in erlangs.
$\rho = u/c = \lambda E[s]/c = \lambda/c\mu$	Server utilization. The probability any particular server is busy.
$w = q + s$	Total waiting time in the system, including waiting in queue and service time.
$W = E[w] = E[q] + E[s] = W_q + W_s$	Average total waiting time in the steady state system.
$N = N_q + N_s$	Number of customers in the steady state system.
$L = E[N] = E[N_q] + E[N_s] = \lambda E[w] = \lambda W$	Average number of customers in the steady state system. $L = \lambda W$ is known as "Little's formula."
$L_q = E[N_q] = \lambda E[q] = \lambda W_q$	Average number in the queue for service for steady state system. $L_q = \lambda W_q$ is also called "Little's formula."

TABLE 1 *(Continued)*

λ_T	Average throughput of a computer system measured in jobs or interactions per unit time.
M	Symbol for exponential interarrival or service time distribution.
μ	Average (mean) service rate per server. Average service rate $\mu = 1/E[s]$, where $E[s]$ is the average (mean) service time.
N	Random variable describing number in queueing system when system is in the steady state.
N_q	Random variable describing number of customers in the steady state queue.
N_s	Random variable describing number of customers receiving service when the system is in the steady state.
O	Operating time of a machine in the machine repair queueing model (Sections 5.2.6 and 5.2.7). O is the time a machine remains in operation after repair before repair again is necessary.
$p_n(t)$	Probability that there are n customers in the queueing system at time t.
p_n	Steady state probability that there are n customers in the queueing system.
PRI	Symbol for priority queueing discipline.
PS	Abbreviation for "processor-sharing queue discipline." See Section 6.2.1.
$\pi_q(r)$	Symbol for rth percentile queueing time; that is, the queueing time that r percent of the customers do not exceed.
$\pi_w(r)$	Symbol for rth percentile waiting time in the system; that is, the time in the system (queueing time plus service time) that r percent of the customers do not exceed.
q	Random variable describing the time a customer spends in the queue (waiting line) before receiving service.
RSS	Symbol for queue discipline with "random selection for service."
ρ	Server utilization = traffic intensity/c = $\lambda E[s]/c = (\lambda/\mu)/c$. The probability that any particular server is busy.
s	Random variable describing service time for one customer.
SIRO	Symbol for queue discipline, "service in random order" which is identical with RSS. It means that each waiting customer has the same probability of being served next.
τ	Random variable describing interarrival time.
u	Traffic intensity = $E[s]/E[\tau] = \lambda E[s] = \lambda/\mu$. Unit of measure is the erlang.
w	Random variable describing the total time a customer spends in the queueing system, including both service time and time spent queueing for service.
$W(t)$	Distribution function for w. $W(t) = P[w \le t]$.
W	$E[w]$, expected (average or mean) time in the steady state system.
$W_q(t)$	Distribution function for time in the queue. $W_q(t) = P[q \le t]$.
W_q	$E[q]$, expected (average or mean) time in the queue (waiting line), excluding service time, for steady state system.
$W_{q\mid q>0}$	Expected (average or mean) queueing time for those who must queue. Same as $E[q \mid q > 0]$.
$W_s(t)$	Distribution function for service time. $W_s(t) = P[s \le t]$.
W_s	$E[s]$, expected (average or mean) service time. $1/\mu$.

Appendix C

QUEUEING THEORY DEFINITIONS AND FORMULAS

In Figs. 5.1.1 and 5.1.2, reproduced from Chapter 5, we indicate the elements and random variables used in queueing theory models. Table 1 is a compendium of the queueing theory definitions and notation used in this book. The remainder of Appendix C consists of tables of queueing theory formulas for the most useful models and figures to help with the calculations. APL functions are displayed in Appendix B to implement the formulas for most of the queueing models.

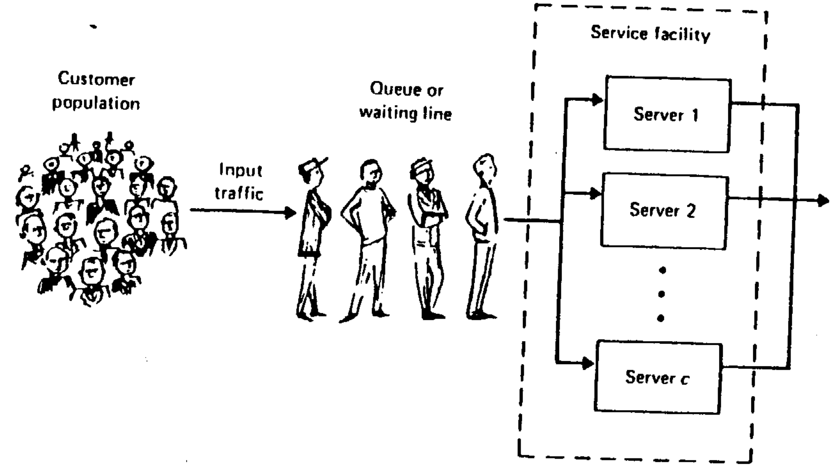

Fig. 5.1.1 Elements of a queueing system.

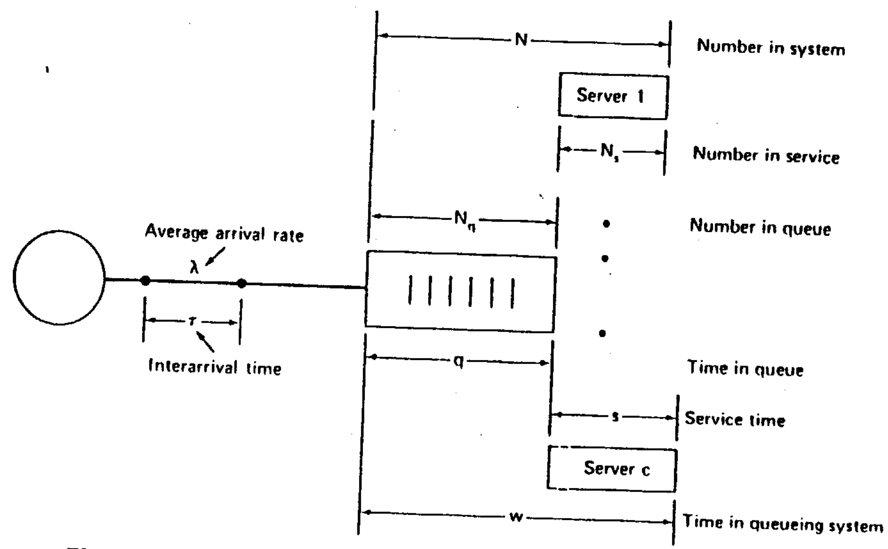

Fig. 5.1.2 Some random variables used in queueing theory models.

TABLE 1
Queueing Theory Notation and Definitions

$A(t)$	Distribution of interarrival time, $A(t) = P[\tau \leq t]$.
$B(c, u)$	Erlang's B formula or the probability all c servers are busy in an M/M/c/c queueing system.
$C(c, u)$	Erlang's C formula or the probability all c servers are busy in an M/M/c queueing system.
c	Symbol for the number of servers in the service facility of a queueing system.
D	Symbol for constant (deterministic) interarrival or service time distribution.
$E[N]$	Expected (average or mean) number of customers in the steady state queueing system. The letter L is also used for $E[N]$.
$E[N_q]$	Expected (average or mean) number of customers in the queue (waiting line) when the system is in the steady state. The symbol L_q is also used for $E[N_q]$.
$E[N_s]$	Expected (average or mean) number of customers receiving service when the system is in the steady state.
$E[q]$	Expected (average or mean) queueing time (does not include service time) when the system is in the steady state. The symbol W_q is also used for $E[q]$.
$E[s]$	Expected (average or mean) service time for one customer. The symbol W_s is also used for $E[s]$.
$E[\tau]$	Expected (average or mean) interarrival time. $E[\tau] = 1/\lambda$, where λ is average arrival rate.
$E[w]$	Expected (average or mean) waiting time in the system (this includes both queueing time and service time) when the system is in the steady state. The letter W is also used for $E[w]$.
E_k	Symbol for Erlang-k distribution of interarrival or service time.
$E[N_q \mid N_q > 0]$	Expected (average or mean) queue length of nonempty queues when the system is in the steady state.
$E[q \mid q > 0]$	Expected (average or mean) waiting time in queue for customers delayed when the system is in the steady state. Same as $W_{q\mid q>0}$.
FCFS	Symbol for "first come, first served," queue discipline.
FIFO	Symbol for "first in, first out," queue discipline which is identical with FCFS.
G	Symbol for general probability distribution of service time. Independence usually assumed.
GI	Symbol for general independent interarrival time distribution.
K	Maximum number allowed in queueing system, including both those waiting for service and those receiving service. Also size of population in finite population models.
L	$E[N]$, expected (average or mean) number in the queueing system when the system is in the steady state.
$\ln(\cdot)$	The natural logarithm function or the logarithm to the base e.
L_q	$E[N_q]$, expected (average or mean) number in the queue, not including those in service, for steady state system.
LCFS	Symbol for "last come, first served," queue discipline.
LIFO	Symbol for "last in, first out," queue discipline which is identical to LCFS.
λ	Average (mean) arrival rate to queueing system. $\lambda = 1/E[\tau]$, where $E[\tau]$ = average interarrival time.