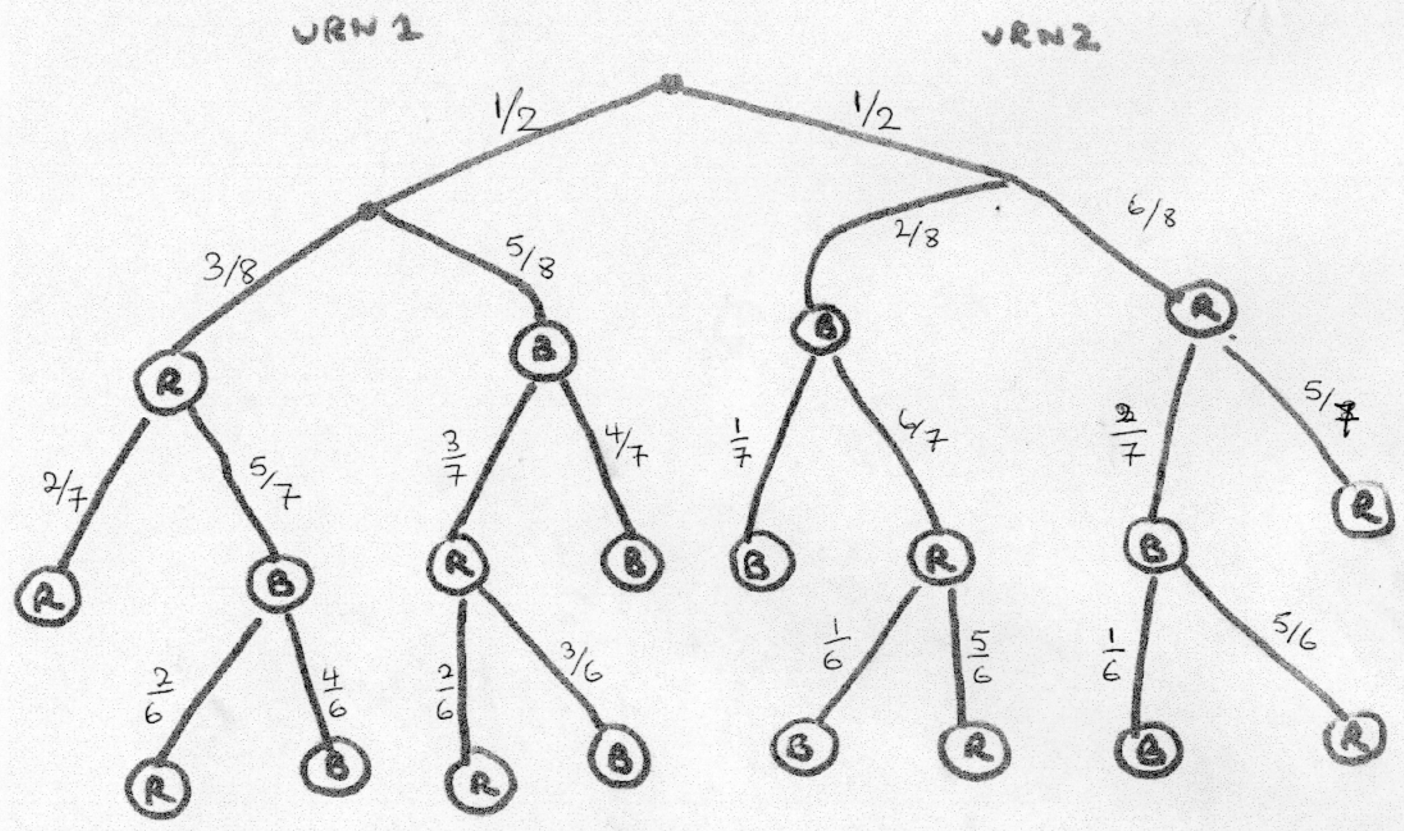
A game of chance

Urn 1 contains 3 red and 5 black balls
Urn 2 contains 6 red and 2 black balls.

As a contestant you do not know which Urn is which. You win a prize if succeed in drawing 2 red balls before two back balls.

Procedure:
Select an urn and draw from it. Compute the probability of wining. Given that you win what is the probability that you picked Urn 1?
If you initially pick Urn 1 what is the probability of Winning, what is it for urn 2?

The game becomes a little more interesting though if you are permitted to change urns in between draws. To keep the example to a reasonable size we shall only permit a swap after 2 draws (not after the first). You can now use the knowledge of the draws to improve your decision.


URN 1
URN 2
1/2
1/2
3/8
5/8
2/8
6/8
R
B
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2/7
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3/7
4/7
1/7
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R

The numbers on the arcs are the conditional probabilities of each colour given the state of the system – determined by the previous draws.
The probability of following any path down the tree is just the product of these probabilities.
Why?

$$P(Urn1 \cap R1 \cap B2 \cap R3) = P(R3 \mid Urn1 \cap R1 \cap B2) \times P(Urn1 \cap R1 \cap B2)$$
$$= P(R3 \mid Urn1 \cap R1 \cap B2) \times P(B2 \mid Urn1 \cap R1) \times P(Urn1 \cap R1) =$$
$$= P(R3 \mid Urn1 \cap R1 \cap B2) \times P(B2 \mid Urn1 \cap R1) \times P(R2 \mid Urn1) \times P(Urn1)$$

We use this method to compute the probability of every sequence in the tree and sum up the probability of winning.

So $P(Urn1 \cap R1 \cap R2) = \frac{1}{2} \times \frac{3}{8} \times \frac{2}{7} = \frac{3}{56}$

Complete the calculation as an exercise.

Making the decision.
We must have observed BR or RB. Win if get a Red , Lose if black.
If the two draws have been from Urn 1 then it now contains 2R and 4 B. so
P(Win | Urn1)= 2/6 if we stay with Urn 1.
If we swap
P(Win| next draw form 2) = 6/8

If Urn 2 it now has 5R and 1B

P(Win | Urn 2) =5/6
If we swap
P(win| next draw from 1) =3/8

If we don't swap

$$P(Win) = (1/3)P(inUrn1) + (5/6)P(inUrn2)$$

If we do Swap

$$P(Win) = (6/8)P(inUrn1) + (3/8)P(inUrn2)$$

What is probability inUrn1? Not ½ (the original) because we have drawn BR

$$P(inUrn1 \mid BR) = \frac{P(BR \mid Urn1)P(BR)}{P(BR)} = \frac{P(BR \mid Urn1)P(BR)}{P(BR \mid Urn1)P(Urn1) + P(BR \mid Urn2) * P(Urn2)}$$

$$P(BR \mid Urn1)P(Urn1) = \frac{1}{2} \times \frac{3}{8} \times \frac{5}{7} + \frac{1}{2} \times \frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$$

$$P(BR \mid Urn2)P(Urn2) = \frac{1}{2} \times \frac{2}{8} \times \frac{6}{7} + \frac{1}{2} \times \frac{6}{8} \times \frac{2}{7} = \frac{12}{56}$$

$$P(inUrn1 \mid BR) = \frac{\frac{15}{56}}{\frac{15}{56} + \frac{12}{56}} = \frac{15}{27} = \frac{5}{9}$$

Finally

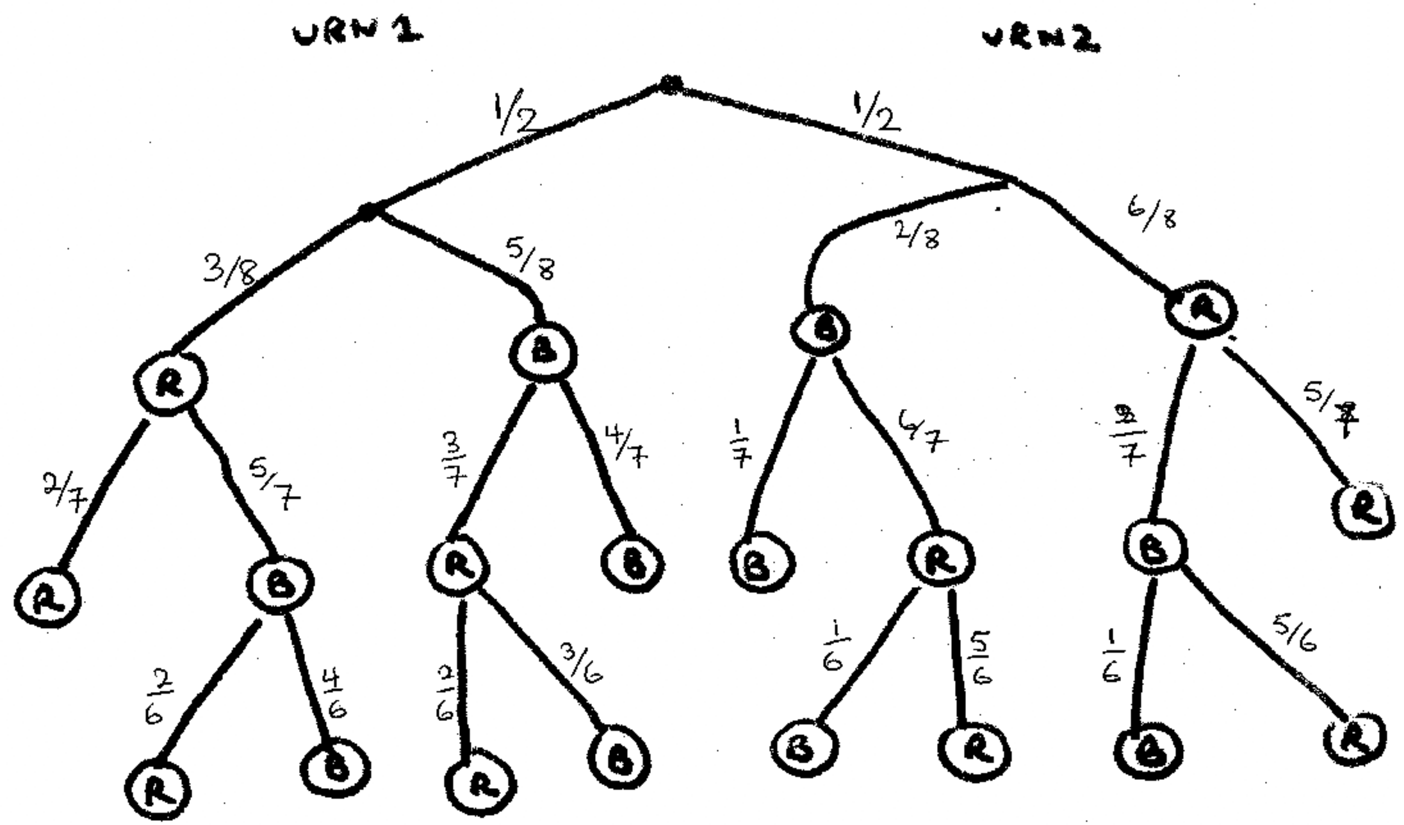
NoSwap

$$P(Win) = \frac{1}{3} \times \frac{5}{9} + \frac{5}{6} \times \frac{4}{9} = \frac{5}{27} + \frac{5}{27} = \frac{10}{27} = 0.37037$$

Swap

$$P(Win) = \frac{6}{8} \times \frac{5}{9} + \frac{3}{8} \times \frac{4}{9} = \frac{42}{72} = \frac{7}{12} = 0.58333$$

DO SWAP



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1/6
5/6
R
B