"Birthday Twins"

Given n people, what is the probability that two people have the same birthday.

Number of possible birthdays = 366 (leap year)

Consider probability of all n having different birthdays, call this p, then probability of (at least) 2 people having the same birthday = (1-p)

1st person has a birthday on any day

Prob. 2nd having b'day different to previous = $(1 - \frac{1}{366})$

$= \frac{365}{366}$

Prob. of 3rd having same b'day as 1st $= \frac{1}{366}$

also prob. of 3rd " " as 2nd $= \frac{1}{366}$

so prob. of 3rd having either same as 1st or 2nd

$= \frac{1}{366} + \frac{1}{366} = \frac{2}{366}$

tf.

prob. 3rd having b'day different to previous b'days

$= (1 - \frac{2}{366}) = \frac{364}{366}$

Prob 4th " " $= (1 - \frac{3}{366})$

$= \frac{363}{366}$

...

prob. Nth " " $= (1 - \frac{N-1}{366})$

$= \frac{366-(N-1)}{366}$

Prob N people having all different birthdays

$= \frac{365}{366} * \frac{364}{366} * \frac{363}{366} * \ldots * \frac{366-(N-1)}{366}$

$= \frac{365 * 364 * .. * 366-(N-1)}{366^{N-1}}$

$= \frac{(366-1) * (366-2) * .. * (366-(N-1))}{366^{N-1}}$

Example: N = 23

$$\frac{(366-1)*(366-2)*..*(366-(23-1))}{366^{23-1}}$$

$$\frac{365*364*...*344}{366^{22}}$$

$$= \frac{123,034,458,606,683,264,934,098,143,075,536,318,524,051,780,468,736,000,000}{249,220,566,387,204,098,009,877,496,558,393,544,293,430,769,946,781,024,256}$$

This simplifies to (lowest reducible fraction)

$$\frac{496,768,798,820,224,409,065,512,997,908,133,946,070,296,875}{1006,262,821,062,572,246,849,093,750,912,932,693,143,409,161}$$

$$\approx \frac{496}{1006}$$

$\approx$ 0.493 (Prob. of 23 not having same b'day)

therefore, in a group of 23 people, the probability of at least 2 people having the same birthday is

1 - 0.493

= 0.507

i.e. In a group of 23 people, there is a 51% chance that (at least) 2 people have the same birthday.

Birthday Probability Calculation

We can attempt to calculate

$$\frac{(366-1)*(366-2)*..*(366-(N-1))}{366^{N-1}}$$

$$= \frac{(366-(N-1))*(366-(N-2)*..*(366-1)}{366^{N-1}}$$

$$= \frac{\text{product}(366-(N-1),\ 366-1)}{366\text{^}(N-1)}$$

where *product*(m, n) = m*(m+1)* …* n

but for N=23, this will lead to 'overflow' and so we will need an alternate approach.

```
product(m,n : INTEGER) : INTEGER is
        -- returns product m * (m+1)  ... * n, if m ≤ n
        local
            k, r : INTEGER
        do
            from
                k := m;
                r := 1
            until
                k > n
            loop
                r := r*k;
                k := k+1
            end
            Result := r
        end -- product
```

```
bday_prob(n:INTEGER):REAL is
-- Probability that n people have different birthdays
        require
                n < 367
        local
                k       : INTEGER;
                p       : REAL
        do
                from
                        k := days - n + 1;
                        p := 1
                until
                        k = days
                loop
                        p := p*k/days;
                        k := k+1
                end
                Result := p
        ensure
                Result =   product(days - (n-1), days-1) / days^(n-1)
        end -- bday_prob
```