higher than n and if the nth derivative of y occurs non-trivially in the equation.

A di erential equation of order n may often be expressed in the form

$$\frac{d^n y}{dx^n} = G \quad x; y; \frac{dy}{dx}; \ldots; \; d^n$$

for all n-times di erentiable functions y_1 and y_2 and for 8d8dumen

6.3 Inhomogeneous Linear Differential Equations

A inhomogeneous linear differential equation of order n is a differential equation of the form

$$a_0(x)y + \sum_{j=1}^{n} a_j(x)\frac{d^j y}{dx^j} = f(x);$$

where $a_0; a_1; \dots; a_n$ and f are functions of t

where a

where

$$q'(x) = \frac{dq(x)}{dx}.$$

It follows that a function y of x is a solution of the di erential equation

$$y'(x) + p(x)y(x) = r(x).$$

if and only if

$$q(x)y'(x) + q'(x)y(x) = q(x)r(x).$$

But

$$q(x)y'(x) + q'(x)y(x) = \quad d$$

Example Consider the di erential equation

$$\frac{dy}{dx} + cy$$

We shall show the solutions of the differential equation $ay'' + by' + cy = 0$ are determined by the roots of the *auxiliary polynomial* $as^2 + bs + c$ determined by the differential equation.

We begin our investigation of the solutions of these differential equations

Finally suppose that $y = e^{px} \cos qx$. Then

$ay'' + by' + cy =$

Let a, b, c, p, q and r be real numbers with $a \neq 0$. We see that $y = e^{rx}$ is a solution of the differential equation $ay'' + by' + cy = 0$ if and only if r is a root of the quadratic polynomial $as^2 + bs + c$. Moreover $y = xe^{rx}$ is a solution of this differential equation if and only if r is a repeated root of this quadratic polynomial. Also $y = e^{px} \sin qx$ and $y = e^{px} \cos qx$ are solutions of this differential equation if and only if $p +$

6.6 Inhomogeneous Linear Di erential Equations of

Example Let us find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 7\frac{dy}{dx} + 10y = x^2$$

Remark Suppose that one is seeking a particular integral of an inhomogeneous di erential equation of the form

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x$$

polynomial $s^2 \quad 6s + 9$ has a repeated root, whose value is 3. The complementary function y_C is then given by $y_C = (Ax + B)e^{3x}$, where A and B are real constants. The general solution of the di erential equation

$$\frac{d^2y}{}$$

6.7 Initial Value Problems

In an *initial value problem* concerning a second order di erential equation

$$F\left(x; y; \frac{dy}{dx}; \frac{d^2y}{dx^2}\right) = 0;$$

the value of the solution $y(x_0$

6.8 Boundary Value Problems

In an *boundary value problem* concerning a second order di erential equation

$$F$$

with constant coe cients $a_0; a_1; : : : ; a_n$. The general solution of such a differential equation is of the form

$$A_1y_1 + A_2y_2 + \qquad + A_ny_n;$$

where A