Course 2BA1: Michaelmas Term 2003
Assignment I

Conall O'Brien

01734351

conall@conall.net

November 6, 2003

1

Prove by induction that

$$\sum_{i=1}^{n} \frac{2i+1}{i^2(i+1)^2} = \frac{n^2+2n}{(n+1)^2}$$

1.1 When $n = 1$

$$\begin{aligned}
\sum_{i=1}^{1} \frac{2(1)+1}{(1)^2(1+1)^2} &= \frac{(1)^2+2(1)}{((1)+1)^2} \\
\frac{2+1}{(1)^2(2)^2} &= \frac{(1)^2+2(1)}{(1+1)^2} \\
\frac{3}{1(2)^2} &= \frac{1^2+2}{2^2} \\
\frac{3}{4} &= \frac{3}{4}
\end{aligned}$$

Therefore, holds true for $n = 1$.

1.2 When $n = m$.

Assume equation holds true for $n = m$, so:

$$\sum_{i=1}^{m} \frac{2i+1}{i^2(i+1)^2} = \frac{m^2+2m}{(m+1)^2}$$

1.3 When $n = m + 1$.

$$\begin{aligned}
& \sum_{i=1}^{m+1} \frac{2i+1}{i^2(i+1)^2} \\
\sum_{i=1}^{m} \frac{2i+1}{i^2(i+1)^2} + \frac{2(m+1)+1}{(m+1)^2((m+1)+1)^2} &= \frac{(m+1)^2+2(m+1)}{((m+1)+1)^2} \\
\frac{m^2+2m}{(m+1)^2} + \frac{2(m+1)+1}{(m+1)^2((m+1)+1)^2} &= \frac{(m+1)^2+2(m+1)}{((m+1)+1)^2} \\
\frac{m^2+2m}{(m+1)^2} + \frac{2(m+1)+1}{(m+1)^2(m+2)^2} &= \frac{(m+1)^2+2(m+1)}{(m+2)^2} \\
\frac{m(m+2)(m+2)^2+2(m+1)+1}{(m+1)^2(m+2)^2} &= \frac{(m+1)^2+2(m+1)}{(m+2)^2} \\
\frac{m(m+2)^3+2(m+1)+1}{(m+1)^2} &= (m+1)^2+2(m+1) \\
m(m+2)^3+2(m+1)+1 &= (m+1+2)(m+1)^3 \\
m(m+2)^3+2(m+1)+1 &= (m+3)(m+1)^3 \\
m(m^3+6m^2+12m+8)+2(m+1)+1 &= (m+3)(m^3+3m^2+3m+1) \\
m^4+6m^3+12m^2+10m+3 &= m^4+3m^3+3m^2+m+3m^3+9m^2+9m+3 \\
m^4+6m^3+12m^2+10m+3 &= m^4+6m^3+12m^2+10m+3
\end{aligned}$$

Therefore true for $n = m + 1$.

1.4 Conclusion

Since $n = 1$ and $n = m + 1$ both hold true, thus it is true $\forall \in \mathbb{N}$.

2

Prove by induction on n that $(3n)! \geq \frac{1}{20} \times 120^n$ for all natural numbers $\mathbb{N}$ (where $n!$ denotes the product of all natural numbers from 1 to n inclusive).

2.1 When $n = 1$

$$\begin{aligned}
(3(1))! &\geq \frac{1}{20} \times 120^1 \\
3! &\geq \frac{1}{20} \times 120 \\
6 &\geq \frac{120}{20} \\
6 &\geq 6
\end{aligned}$$

Therefore, holds true for $n = 1$.

2.2 When $n = m$

Assume equation holds true for $n = m$, so:

$$(3m)! \geq \frac{1}{20} \times 120^m$$

2.3 When $n = m + 1$

$$\begin{aligned}
(3(m+1))! &\geq \frac{1}{20} \times 120^{m+1} \\
(3m+3)! &\geq \frac{1}{20} \times 120^{m+1} \\
(3m+3) \times (3m+2) \times (3m+1) \times 3m! &\geq \frac{1}{20} \times 120^{m+1} \\
(3m+3) \times (3m+2) \times (3m+1) \times 3m! &\geq \frac{1}{20} \times 120 \times 120^m \\
(3m+3) \times (3m+2) \times (3m+1) &\geq 120
\end{aligned}$$

Therfore it holds true for $n = m + 1$.

2.4 Conclusion

Since it holds true for $n = 1$ and $n = m + 1$, therfore it holds true $\forall \in \mathbb{N}$.