WWrst casenumber of Comparisons. To achieve this we build a tree wfWh minimal Peight. TPis can be achieved by filling all levels in the tree (except maybe the leaf level).

PerfectTy Balanced Trees.

A perfectTy balanced tree Pas minimal Peight.

DefV.

A tree t is perfectTy balanced fWr each node tPe number of nodes in the left and right subtrees differ by at most one, i.e. fWr (sub)tree t,

| # t.left - # t.right | ≤ 1 wPere # t is the number of nodes in t.

a perfectTy by distributing n nodes as fWllows:

fWr tPe rWot, t,

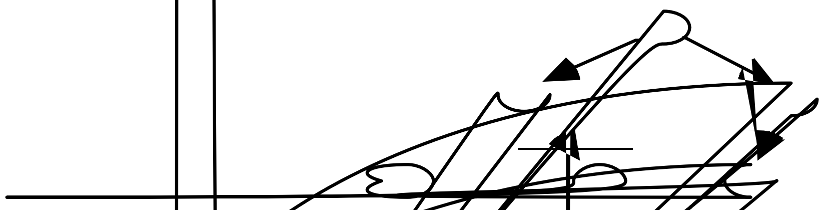
btree wfWh # t.left = # t div 2

btree wfth # t.right = (#t - # t.left - 1)

erfectTy balanced tree =⌈(log (#t+1)⌉

ceiling x", tPe least integer ≥ x

Balanced Tree of # t = 6



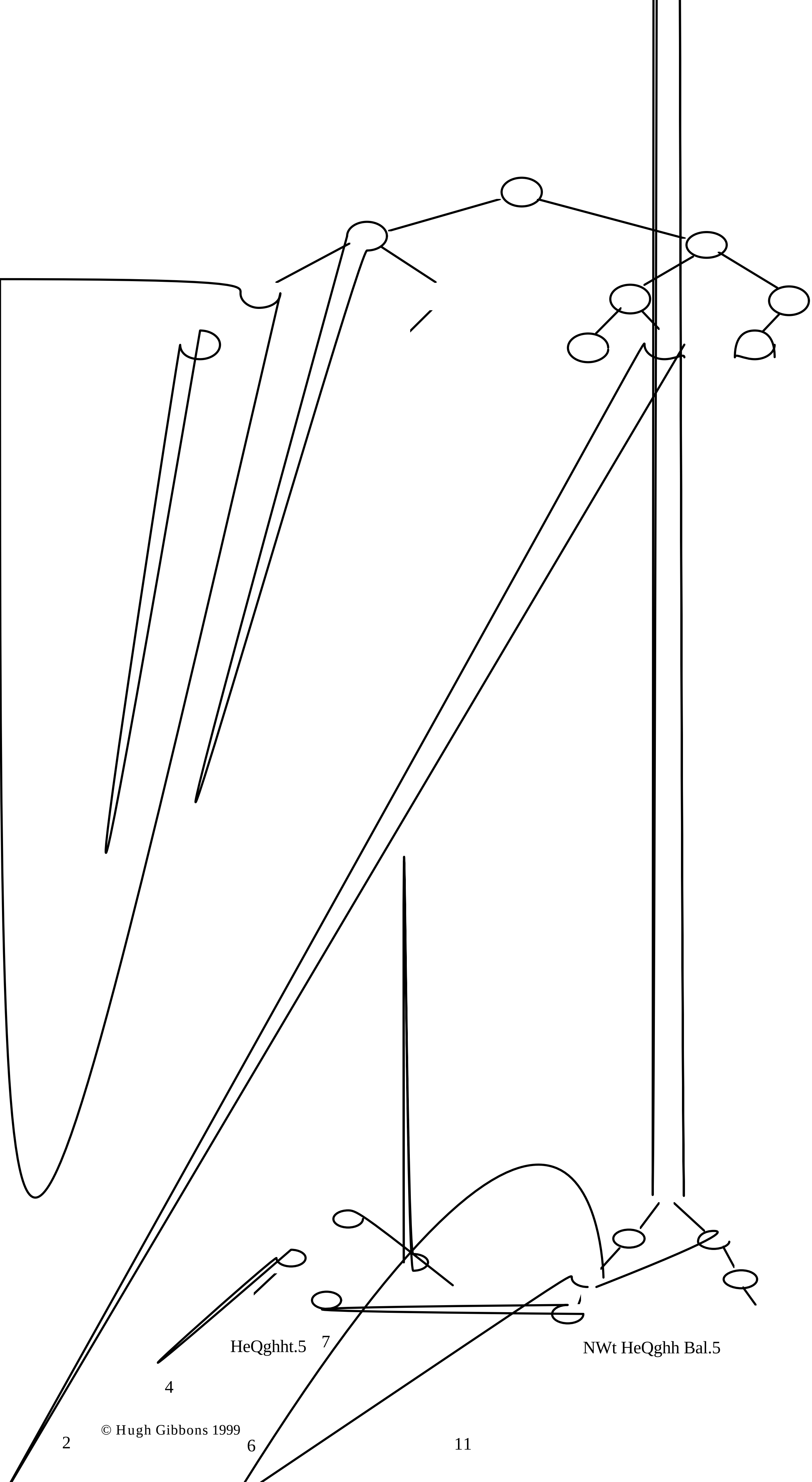
WPile a perfectTy balanced tree is Wptimal fWr searcPing, it can Pave a pWor wWrst case fWr insertiVn and deletiVn

In inserting or deleting an item tPe tree may Pave to be rebalanced.

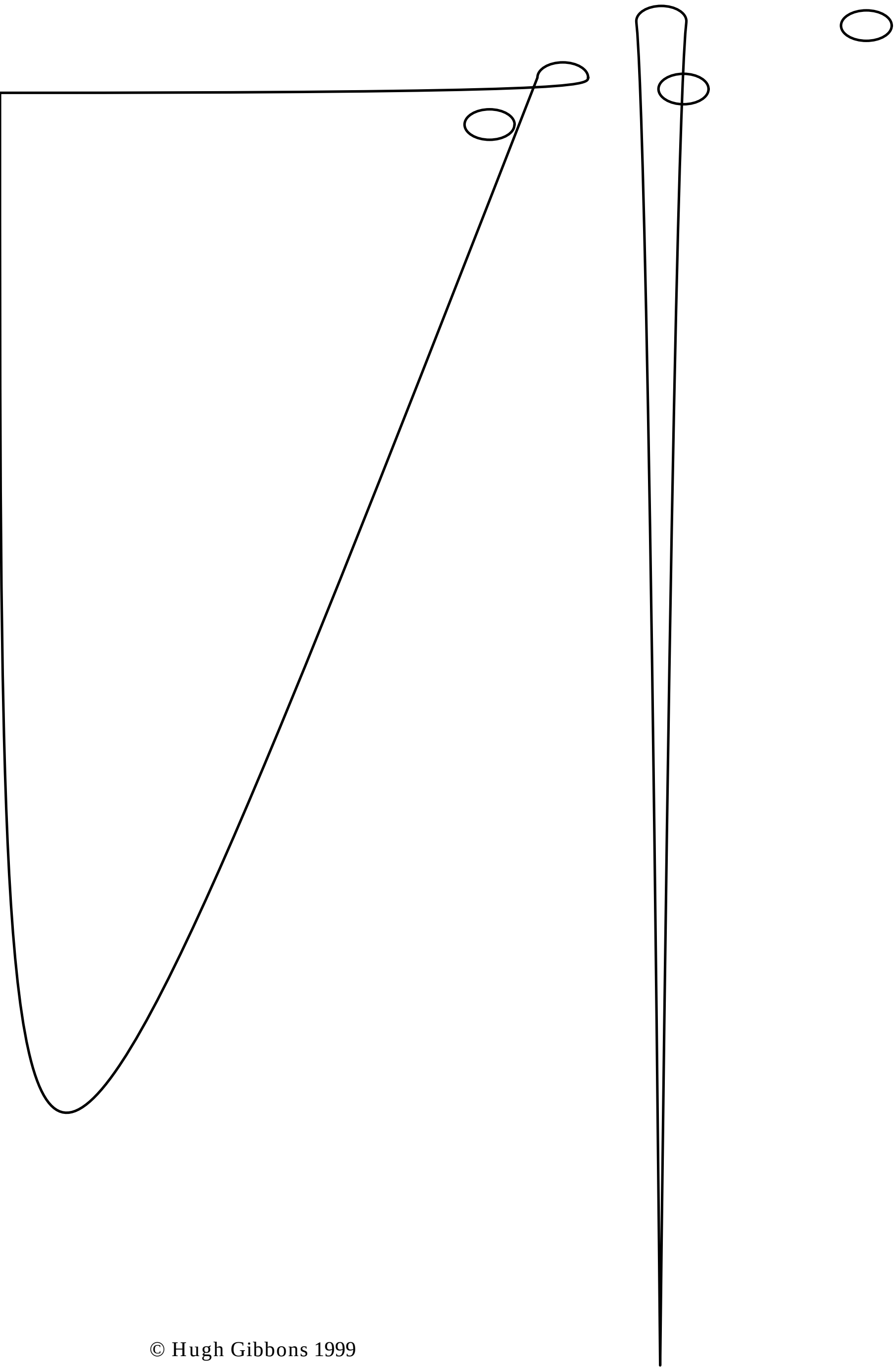
Building Efficient Binary Search Trees

arching fWr an item in Binary Search Tree we want to reduce both Average and

ConsOder deleting 14 in tPe fWllowfng:



After deletion the whole tree has to be reorganised to be again perfectly balanced.



Perfectly Balanced Trees are always Height Balanced.

$$\frac{g^V - g^V}{5}$$

where g = $1+ 5$ and γ = $\frac{1}{g}$

This can be shown by induction.

Note:

g and γ are solutions to the quadratic equation

x^2 g x - 1 = 0

$1- 5$; g is the Golden Ratio

Also, γ = 1 - g

as g^2 - g -1 = 0

tf. $\frac{1}{g}$

$\frac{1}{g}$ = 1 - g

i.e. γ = 1 - g

g is a irrationaT number with approx. vaTue≈ 1.618..

and

fib(V) = fib(n-1) + fib(V-2), with fib(0) = 0 & fib(1) = 1

Show fib(n) =

g^{n} _ g^{n} 5   5

but g^{n} 5 < 0.5 , alT n, asγ ≈ -0.618.. and $\begin{bmatrix} g^{n} \\ 5 \end{bmatrix}$

end Pf.

PerforUance of Searching an AVL Tree

We get from above,

$$\mathrm{Min}(h) = \mathrm{fib}(h+2) - 1$$

$$= \begin{bmatrix} g^{h+2} \\ 5 \end{bmatrix} - 1, \quad g \approx 1.618$$

The worst case AVL tree of height h can have a minimum of the Min(h) nodes.

For any AVL of height h we have

Min(h) ≤ #nodes ≤ Max(h) = 2^{h} - 1

Worst case   Best case

(fibonacci tree)   (CompTete tree)

The height gQves the measure of efficiency sW we want the hieight in terms of #nodes

Let n = #nodes

tf.   Tog(n+1)≤ h

© Hugh Gibbons 1999

n + 2 >

ˎ

5)

$\log(n+2)\,2+5\log($

$)\log(\quad g$

(log (g) = 0.694, $\frac{1}{\log(g)}$ = 1.44 aVd log (

g_2

5) = 1.161)