

Weights Problem

Assume we have N items such that item i weights $w(i)$ and that a carry weight, \max , is given. Select an optimal set of items such that the total weight of the items will be as close to (and less than) the carry weight, \max . This is sometimes referred to as the 'knapsack' problem.

The solution of this problem can be applied to many areas, for example, suppose I want to tape a selection from 20 songs to fit on a 30 minute side of a tape such that the total time of the selection will be a close to 30 minutes as possible.

A possible solution can be adapted from the program for generating sets. We generate all the subsets and keep track of the optimal set so far, and when all the subsets have been generated we output the optimal set. This solution calculates the weight of all subsets even though many will be deemed impossible to be included in a solution. For example, if the optimal weight so far is say 60 and we find that adding all the remaining items won't make the current set heavier than this, then we should ignore the current set and all super-sets of it.

Suppose we have the following 14 items with weights

1	2	3	4	5	6	7	8	9	10	11	12	13	14
180	227	126	163	144	186	152	191	150	180	182	166	191	208

Assume max carry weight is 1800 then an optimal selection (10 items) could be

{1, 2, 3, 4, 7, 8, 10, 11, 13, 14}

Total weight is: 1800

or if the max carry weight was 1790 then the selection (also 10 items) could be

{1, 2, 3, 4, 6, 7, 8, 12, 13, 14}

Total weight is: 1790

The total weight may not equal the carry weight. For example, if the carry weight was 13 and the weights were 6, 4 & 8 then the optimal weight would be 12.

The routine Add_El

The procedure call

`add_el (i, sum, achwgt)`

adds an item, i , to the current set whose current weight (before i is added) is sum . The weight achievable by the current set is achwgt . Suppose the weights were 6, 4 & 8. Assume we are adding item 2 under the condition that item 1 is to be excluded then the achievable weight is 12, as item 1 is not counted. The achievable weight is also governed by the carry weight, \max . If $\max=13$ and items 1 & 2 are in the current set, then adding item 3 (weight 8) is too much and so the achievable weight is current achievable weight minus

weight of item 3, achwgt - 8.

In the routine that follows, the attribute, s, indicates the current set. The attribute, opset, is the current optimal set with optimal weight, opt.

```
Add_El(i,sum,AchWgt : INTEGER) is
  local
    newsum, new_ach : INTEGER
  do
    newsum := sum + wgts.item(i)
    if newsum <= max then
      -- including item i
      s.put(true,i) -- add i to s
      if i < s.count then
        Add_El(i+1,newsum,AchWgt)
      elseif AchWgt > opt then
        opt := AchWgt
        opset.copy(s)
      end
      s.put(false,i) -- remove i
    end
    -- excluding item i
    new_ach := AchWgt - wgts.item(i)
    if opt < new_ach then
      if i < s.count then
        Add_El(i+1,sum,new_ach)
      else
        opt := new_ach
        opset.copy(s)
      end
    end
  end
end -- Ad_El
```

```

class
    WEIGHTS
creation
    make
feature
    s,opset : ARRAY[BOOLEAN] -- Integer Set, s@i iff i ∈ s
    wgts : ARRAY[INTEGER]
    opt, max : INTEGER

    Add_El(i,sum,AchWgt : INTEGER) is
        local
            newsum, new_ach : INTEGER
        do
            newsum := sum + wgts.item(i)
            if newsum <= max then
                -- include item i
                s.put(true,i) -- add i to s
                if i < s.count then
                    Add_El(i+1,newsum,AchWgt)
                elseif AchWgt > opt then
                    opt := AchWgt
                    opset.copy(s)
                end
                s.put(false,i) -- remove i
            end
            -- excluding i
            new_ach := AchWgt - wgts.item(i)
            if opt < new_ach then
                if i < s.count then
                    Add_El(i+1,sum,new_ach)
                else
                    opt := new_ach
                    opset.copy(s)
                end
            end
        end
    end -- Ad_El

```

```

make is
  local
    N,k, totw : INTEGER
  do
    io.put_string("%N Enter number of items: ")
    io.read_integer
    N := io.last_integer
    !!s.make(1,N)
    !!opset.make(1,N)
    !!wgts.make(1,N)
    opt := 0 -- by default it is 0
    read_file("wgts.txt",N)
    io.new_line
  from
    totw := 0
    k := 1
  until
    k > N
  loop
    totw := totw + wgts.item(k)
    io.put_integer(wgts.item(k))
    io.putchar(' ')
    k := k+1
  end
  io.put_string("%N Enter Max carry weight: ")
  io.read_integer
  max := io.last_integer
  Add_EI(1,0,totw)
  print_set(opset,opt)
end -- make

```

```

read_file(filename:STRING; size:INTEGER) is
  local
    in_file : PLAIN_TEXT_FILE
    w,i : INTEGER
  do
    from
      !!in_file.make_open_read(filename)
      i := 1
    until
      i > size
    loop
      in_file.read_integer
      w := in_file.last_integer
      wgts.put(w,i)
      i := i+1
    end
    in_file.close
  end -- read_file

Print_Set( s1:ARRAY[BOOLEAN]; wgt:INTEGER) is
  local
    i,k,tot : INTEGER
  do
    from
      tot := 0
      io.putchar('{')
      i := 1
      k := 0
    until
      i > s1.count
    loop
      if s1.item(i) then
        if k = 0 then
          io.put_integer(i)
        else
          io.putchar(',')
          io.put_integer(i)
        end
        k := k+1
      end
      i := i+1
    end
    io.putchar('}')
    io.put_string("%N Total weight is: ")
    io.put_integer(wgt)
  end -- Print_Set

end -- Class WEIGHTS

```