

1996 1998 2000 2002
 27.7 26.3 25.3 24.8
 regression model, with the input
 percentage. Take 1982 as the base
 has input value $x = 4$, and so on.

SLR: Why $\hat{\beta}_1$ (estimator of β_1) is a linear combination of the errors?

$$y_i = \beta_0 + \beta_1 x_i + e_i$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

$$= \frac{\sum_{i=1}^n [(x_i - \bar{x})y_i + (x_i - \bar{x})\bar{y}]}{S_{xx}}$$

$$= \frac{\sum_{i=1}^n (x_i - \bar{x})y_i + \sum_{i=1}^n (x_i - \bar{x})\bar{y}}{S_{xx}}$$

$$= \frac{\sum_{i=1}^n (x_i - \bar{x})y_i + \bar{y} \sum_{i=1}^n (x_i - \bar{x})}{S_{xx}}$$

$$= \frac{\sum_{i=1}^n (x_i - \bar{x})y_i}{S_{xx}}$$

$$= \frac{\sum_{i=1}^n (x_i - \bar{x})(\beta_0 + \beta_1 x_i + e_i)}{S_{xx}}$$

Work on the numerator. ✓

$$\sum_{i=1}^n (x_i - \bar{x})\beta_0 + \sum_{i=1}^n (x_i - \bar{x})\beta_1 x_i + \sum_{i=1}^n (x_i - \bar{x})e_i$$

First Term:

$$\sum_{i=1}^n (x_i - \bar{x})\beta_0 = \beta_0 \sum_{i=1}^n (x_i - \bar{x}) = \beta_0 \cdot 0 = 0$$

Second Term:

$$\sum_{i=1}^n (x_i - \bar{x})\beta_1 x_i = \beta_1 \sum_{i=1}^n (x_i^2 - \bar{x}x_i) =$$

$$= \beta_1 (\sum_{i=1}^n x_i^2 - \bar{x} \sum_{i=1}^n x_i)$$

$$= \beta_1 [(\sum_{i=1}^n x_i^2) - \bar{x} \cdot n \bar{x}]$$

$$= \beta_1 [(\sum_{i=1}^n x_i^2) - n \bar{x}^2]$$

$$= \beta_1 \sum_{i=1}^n (x_i - \bar{x})^2$$

$$= \beta_1 S_{xx}$$

$$(\sum_{i=1}^n (x_i - \bar{x})^2 = \sum_{i=1}^n (x_i^2 - 2x_i \bar{x} + \bar{x}^2)$$

$$= (\sum_{i=1}^n x_i^2) - (\sum_{i=1}^n 2x_i \bar{x}) + (\sum_{i=1}^n \bar{x}^2)$$

$$= (\sum_{i=1}^n x_i^2) - (2\bar{x} \cdot \sum_{i=1}^n x_i) + (n\bar{x}^2)$$

$$= (\sum_{i=1}^n x_i^2) - (2\bar{x} \cdot n \bar{x}) + (n\bar{x}^2)$$

$$= (\sum_{i=1}^n x_i^2) - n \bar{x}^2$$

$$= 0 + \beta_1 S_{xx} + \sum_{i=1}^n (x_i - \bar{x})e_i$$

$$\hat{\beta}_1 =$$

$$= \beta_1 + \sum \left[\frac{(x_i - \bar{x})}{S_{xx}} e_i \right] w_i = \frac{x_i - \bar{x}}{S_{xx}}$$

$w_i = \frac{x_i - \bar{x}}{S_{xx}}$ → linear combination of errors

so $\hat{\beta}_1$ is a normal distribution by CLT