Let μ denote the mean value of some population. Suppose that in order to test

$H_0 : \mu \le 1.5$

against the alternative hypothesis
$H_1 : \mu > 1.5$

a sample is chosen from the population.

(a) Suppose for this sample that H_0 was not rejected. Does this imply that the sample data would have resulted in rejection of the null hypothesis if we had been testing the following?

$H_0 : \mu > 1.5 \quad \text{against} \quad H_1 : \mu \le 1.5$

(b) Suppose this sample resulted in the rejection of H_0. Does this imply that the same sample data would have resulted in not rejecting the null hypothesis if we had been testing the following?

$H_0 : \mu > 1.5 \quad \text{against} \quad H_1 : \mu \le 1.5$

Given tests (one-sided, α = 0.05)

- Test 1: $H_0 : \mu \le 1.5$ vs $H_1 : \mu > 1.5$
  Reject H_0 when $T = \frac{\bar{X} - 1.5}{SE} > c_{0.95}$.
- "Reversed" test: $H_0 : \mu > 1.5$ vs $H_1 : \mu \le 1.5$
  Reject H_0 when $T < c_{0.05} = -c_{0.95}$.

(a)

No. Failing to reject $H_0 : \mu \le 1.5$ only tells us that
$T \le c_{0.95}$. That includes both the **central region** and the **lower tail**.

To reject the reversed H_0 we would need $T < -c_{0.95}$ (the lower 5%).

It's entirely possible that T is between $-c_{0.95}$ and $c_{0.95}$, in which case **neither** null is rejected.

So non-rejection in the first test **does not imply** rejection in the reversed test.

(b)

Yes. If the first test **rejects** $H_0 : \mu \le 1.5$, then
$T > c_{0.95}$ (upper 5%). But the reversed test rejects only when
$T < -c_{0.95}$ (lower 5%). These rejection regions are **disjoint**.

Hence, if you rejected in the first test, you **cannot** rejec the reversed test; you would **not reject** $H_0 : \mu > 1.5$.