Universidade do Vale do Itajaí - UNIVALI
Curso de Ciência da Computação
Disciplina de Cálculo Numérico
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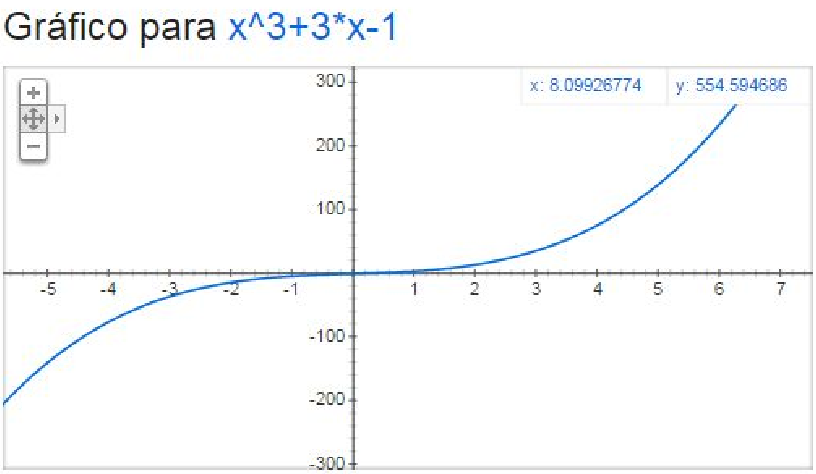
Linguagem de programação: C++
Ambiente de desenvolvimento: Qt Creator

Resolução de equações não-lineares através de métodos iterativos

1. Encontrar as raízes das funções através do método da **Bissecção**, com erro inferior a 10^{-10}.
   a) $f(x) = x^3 + 3x - 1$

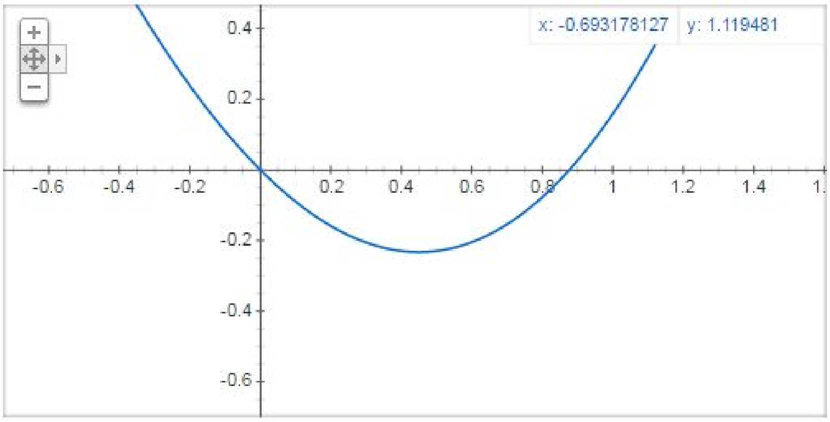


Solução: raiz no intervalo [-1, 1]

k	a	b	x_k	$f(x_k)$	epsilon
0	-1	1	0	-1	-
1	0	1	0.5	0.625	1
2	0	0.5	0.25	-0.234375	1
3	0.25	0.5	0.375	0.177734	0.333333
33	0.32218535454	0.32218535477	0.32218535466	1.01E-10	3.61E-10
34	0.32218535454	0.32218535466	0.32218535460	-9.19E-11	1.81E-10
35	0.32218535460	0.32218535466	0.32218535463	4.47E-12	9.03E-11

b) $f(x) = x^2 - \text{sen } x$

Gráfico para x^2-sin(x)



Solução: raiz no intervalo [0.8, 1]

k	a	b	x_k	$f(x_k)$	epsilon
0	0.8	1	0.9	0.0266731	-
1	0.8	0.9	0.85	-0.0287804	0.0588235
2	0.85	0.9	0.875	-0.0019185	0.0285714
3	0.875	0.9	0.8875	0.0121605	0.0140845
29	0.87672621533	0.87672621571	0.87672621552	1.38E-10	2.12E-10
30	0.87672621533	0.87672621552	0.87672621543	3.43E-11	1.06E-10
31	0.87672621533	0.87672621543	0.87672621538	-1.75E-11	5.31E-11

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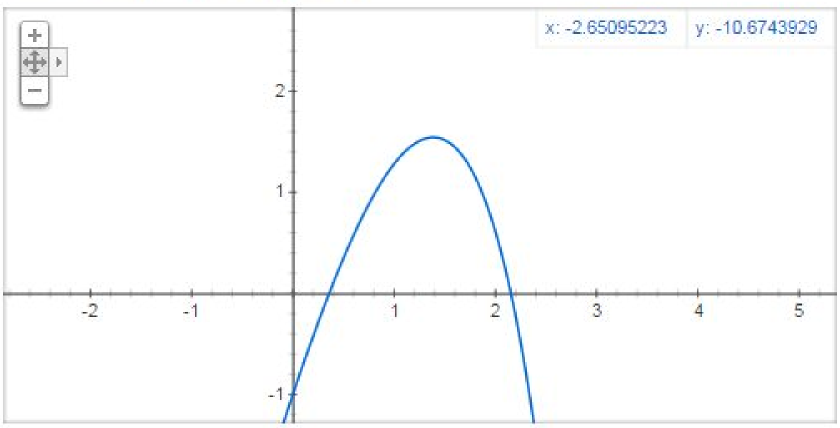
c) $f(x) = 4x - e^x$

Gráfico para 4*x-e^x



Solução 1: raiz no intervalo [0, 1]

k	a	b	x_k	$f(x_k)$	epsilon
0	0	1	0.5	0.351279	-
1	0	0.5	0.25	-0.284025	1
2	0.25	0.5	0.375	0.0450086	0.333333
3	0.25	0.375	0.3125	-0.116838	0.2
32	0.35740295611	0.35740295635	0.35740295623	1.24E-10	3.26E-10
33	0.35740295611	0.35740295623	0.35740295617	-2.56E-11	1.63E-10
34	0.35740295617	0.35740295623	0.35740295620	4.92E-11	8.14E-11

Solução 2: raiz no intervalo [2, 3]

k	a	b	x_k	$f(x_k)$	epsilon
0	2	3	2.5	-2.18249	-
1	2	2.5	2.25	-0.487736	0.111111
2	2	2.25	2.125	0.127103	0.0588235
3	2.125	2.25	2.1875	-0.162903	0.0285714
30	2.15329236351	2.15329236444	2.15329236398	6.23E-10	2.16E-10
31	2.15329236398	2.15329236444	2.15329236421	-4.51E-10	1.08E-10
32	2.15329236398	2.15329236421	2.15329236409	8.64E-11	5.41E-11

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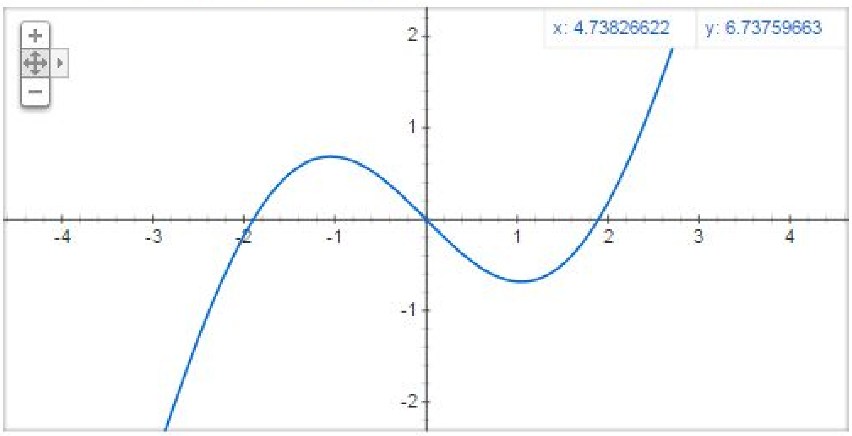
2. Encontrar as raízes das funções através da **Posição Falsa**, com erro inferior a 10^{-10}.

a) $f(x) = x - 2\,\text{sen}\,x$

Gráfico para x-2*sin(x)



Solução 1: raiz no intervalo [-3, -1]

k	a	b	x_k	$f(x_k)$	epsilon
0	-3	-1	-1.40165	0.569809	-
1	-3	-1.40165	-1.67868	0.309695	0.165029
2	-3	-1.67868	-1.81384	0.127375	0.0745187
3	-3	-1.81384	-1.86695	0.0459872	0.0284441
19	-3	-1.8954942649	-1.8954942663	1.16E-09	7.47E-10
20	-3	-1.8954942663	-1.8954942668	3.89E-10	2.50E-10
21	-3	-1.8954942668	-1.8954942670	1.30E-10	8.35E-11

Solução 2: raiz no intervalo [1, 3]

k	a	b	x_k	$f(x_k)$	epsilon
0	1	3	1.40165	-0.569809	-
1	1.40165	3	1.67868	-0.309695	0.165029
2	1.67868	3	1.81384	-0.127375	0.0745187
3	1.81384	3	1.86695	-0.0459872	0.0284441
19	1.8954942649	3	1.8954942663	-1.16E-09	7.47E-10
20	1.8954942663	3	1.8954942668	-3.89E-10	2.50E-10
21	1.8954942668	3	1.8954942670	-1.30E-10	8.35E-11

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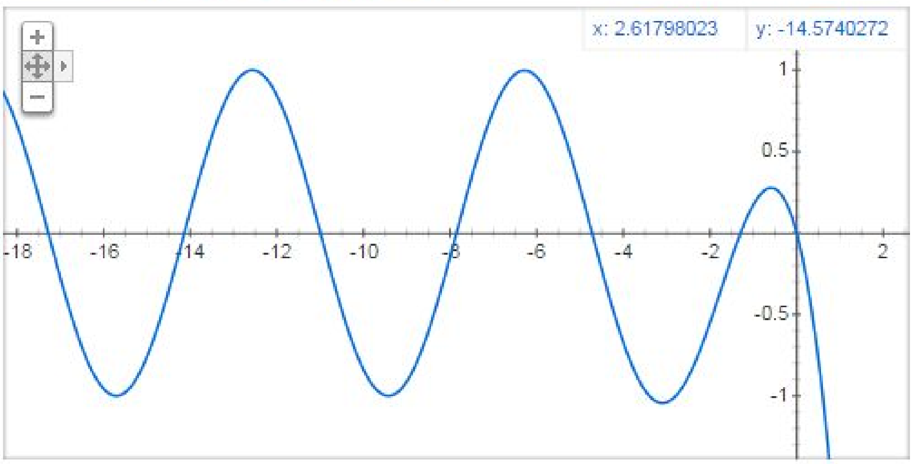
b) $f(x) = \cos x - e^x$ (duas primeiras raízes negativas)

Gráfico para cos(x)-e^x



Solução 1: raiz no intervalo [-2, -0.5]

k	a	b	x_k	$f(x_k)$	epsilon
0	-2	-0.5	-0.994299	0.175108	-
1	-2	-0.994299	-1.23667	0.0375927	0.195988
2	-2	-1.23667	-1.28539	0.00500767	0.0378975
3	-2	-1.28539	-1.29182	0.000604084	0.00497794
10	-2	-1.2926957169	-1.2926957191	2.02E-10	1.69E-09
11	-2	-1.2926957191	-1.2926957193	2.41E-11	2.01E-10
12	-2	-1.2926957193	-1.2926957194	2.86E-12	2.39E-11

Solução 2: raiz no intervalo [-6, -4]

k	a	b	x_k	$f(x_k)$	epsilon
0	-6	-4	-4.82467	0.104013	-
1	-4.82467	-4	-4.71413	-0.00722983	0.0234486
2	-4.82467	-4.71413	-4.72131	1.84E-05	0.00152165
3	-4.7213110037	-4.7141268408	-4.7212927599	1.01E-09	3.86E-06
4	-4.7212927599	-4.7141268408	-4.7212927589	5.45E-14	2.12E-10
5	-4.7212927589	-4.7141268408	-4.7212927589	-1.51E-16	1.15E-14

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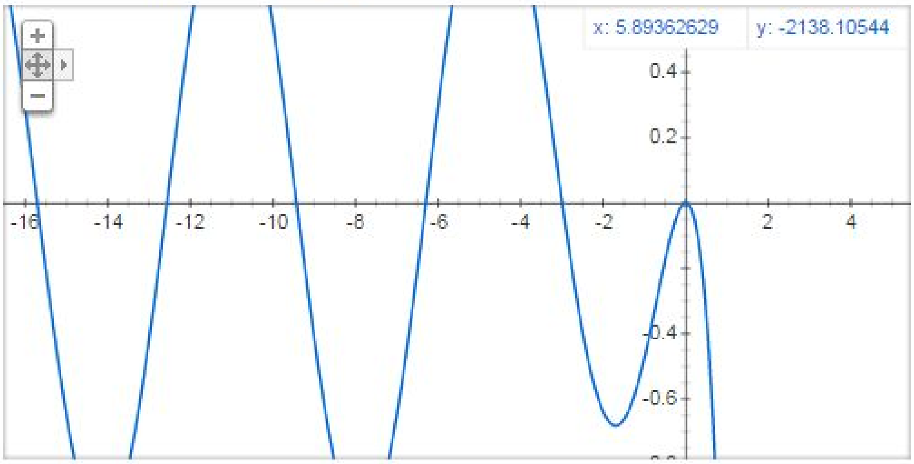
c) $f(x) = \text{sen}\, x - xe^x$ (duas primeiras raízes negativas)

Gráfico para sin(x)-x*e^x



Solução 1: raiz no intervalo [-4, -2]

k	a	b	x_k	$f(x_k)$	epsilon
0	-4	-2	-2.86965	-0.105837	-
1	-4	-2.86965	-2.99748	0.00599753	0.0426444
2	-2.99748	-2.86965	-2.99062	-9.85E-05	0.0022922
3	-2.9974795293	-2.9906244057	-2.9907351466	-7.39E-08	3.70E-05
4	-2.9974795293	-2.9907351466	-2.9907352297	-5.54E-11	2.78E-08
5	-2.9974795293	-2.9907352297	-2.9907352297	-4.16E-14	2.08E-11

Solução 2: raiz no intervalo [-7, -6]

k	a	b	x_k	$f(x_k)$	epsilon
0	-7	-6	-6.31145	-0.0168048	-
1	-6.31145	-6	-6.29463	0.000179208	0.00267279
2	-6.3114516652	-6.2946274708	-6.2948049924	-3.69E-08	2.82E-05
3	-6.2948049924	-6.2946274708	-6.2948049558	-6.33E-14	5.81E-09
4	-6.2948049558	-6.2946274708	-6.2948049558	-1.40E-15	9.74E-15

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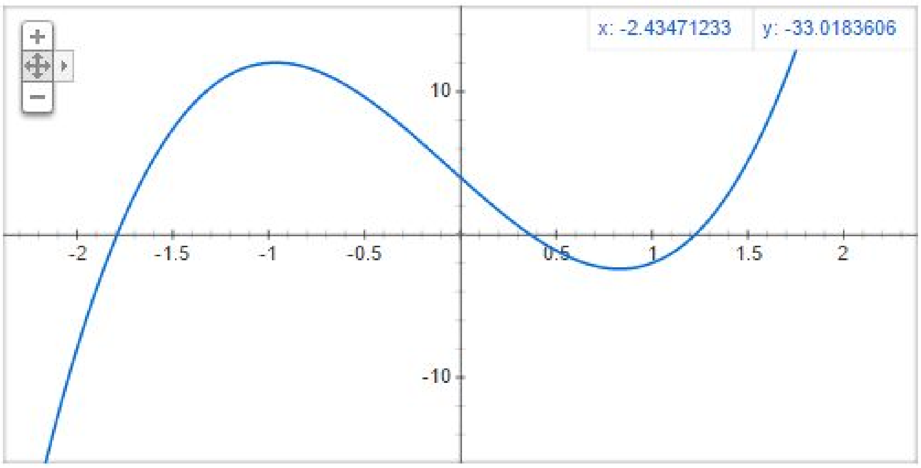
3. Encontrar as raízes das funções através do **Método de Newton**, com erro inferior a 10^{-10}.

a) $f(x) = 5x^3 + x^2 - 12x + 4$

$f'(x) = 15x^2 + 2x - 12$

Gráfico para 5*x^3+x^2-12*x+4



Solução 1: raiz próxima de -2

k	x_k	$f(x_k)$	$f'(x_k)$	epsilon
0	-2	-8	44	-
1	-1.81818	-0.928625	33.9504	0.1
2	-1.79083	-0.0195537	32.5244	0.0152736
3	-1.79023	-9.35E-06	32.4933	0.000335824
4	-1.790227935	-2.14E-12	32.493285	1.61E-07
5	-1.790227935	-8.88E-16	32.493285	3.67E-14

Solução 2: raiz próxima de 0.5

k	x_k	$f(x_k)$	$f'(x_k)$	epsilon
0	0.5	-1.125	-7.25	-
1	0.344828	0.185985	-9.52675	0.45
2	0.36435	0.00238967	-9.28004	0.0535816
3	0.364608	4.29E-07	-9.27671	0.000706256
4	0.3646075792	1.39E-14	-9.276704539	1.27E-07
5	0.3646075792	-2.78E-17	-9.276704539	4.11E-15

Solução 3: raiz próxima de 1.5

k	x_k	$f(x_k)$	$f'(x_k)$	epsilon
0	1.5	5.125	24.75	-
1	1.29293	0.963245	15.6609	0.160156
2	1.23142	0.0759881	13.2089	0.0499476
3	1.22567	0.000643447	12.9853	0.0046936
4	1.225620359	4.76E-08	12.98341968	4.04E-05
5	1.225620355	4.44E-16	12.98341954	2.99E-09
6	1.225620355	4.44E-16	12.98341954	0

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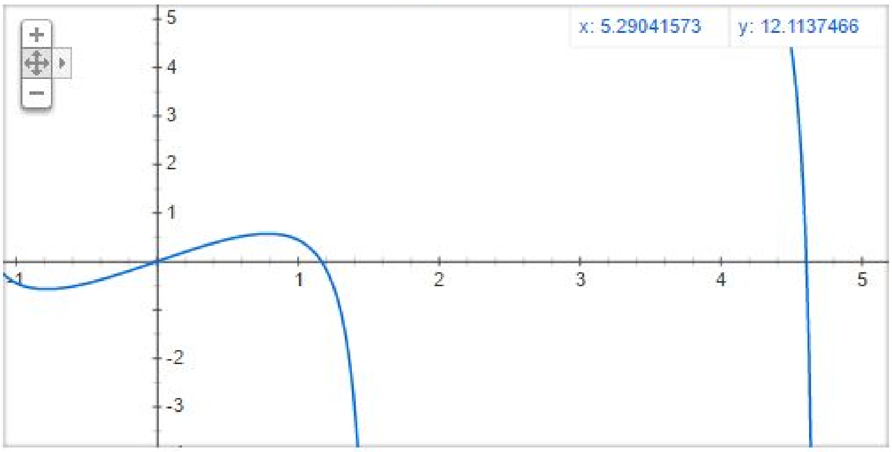
b) $f(x) = 2x - \tan x$ (duas primeiras raízes positivas)

$f'(x) = 2 - \sec^2 x$

Gráfico para 2*x-tan(x)



Solução 1: raiz próxima de 1

k	x_k	$f(x_k)$	$f'(x_k)$	epsilon
0	1	0.442592	-1.42552	-
1	1.31048	-1.13333	-13.0946	0.23692
2	1.22393	-0.318529	-6.6529	0.070714
3	1.17605	-0.0482071	-4.76148	0.040711
5	1.16556	-2.00E-06	-4.43415	0.000313044
6	1.165561185	-3.05E-12	-4.434131506	3.87E-07
7	1.165561185	-4.44E-16	-4.434131506	5.91E-13

Solução 2: raiz próxima de 4.7

k	x_k	$f(x_k)$	$f'(x_k)$	epsilon
0	4.7	-71.3128	-6513.55	-
1	4.68905	-33.4639	-1834.44	0.00233488
2	4.67081	-14.6949	-576.754	0.00390555
3	4.64533	-5.59944	-220.715	0.00548478
7	4.604216802	-2.05E-06	-83.79528714	1.11E-05
8	4.604216777	-4.76E-13	-83.79524853	5.31E-09
9	4.604216777	-3.02E-14	-83.79524853	1.16E-15

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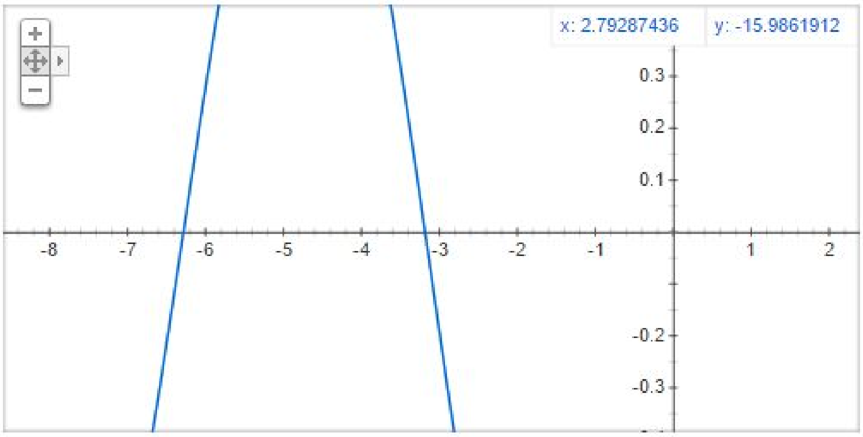
c) $f(x) = \text{sen}\, x - e^x$ (duas primeiras raízes negativas)

$f'(x) = \cos x - e^x$

Gráfico para sin(x)-e^x



Solução 1: raiz próxima de -6

k	x_k	$f(x_k)$	$f'(x_k)$	epsilon
0	-6	0.276937	0.957692	-
1	-6.28917	-0.00784209	0.998126	0.0459792
2	-6.28131	4.65E-08	0.998127	0.00125082
3	-6.281314366	4.00E-16	0.9981273099	7.41E-09
4	-6.281314366	4.00E-16	0.9981273099	0

Solução 2: raiz próxima de -3

k	x_k	$f(x_k)$	$f'(x_k)$	epsilon
0	-3	-0.190907	-1.03978	-
1	-3.1836	0.000562328	-1.04055	0.0576716
2	-3.18306	-1.22E-08	-1.0406	0.000169777
3	-3.183063012	-2.15E-16	-1.040598701	3.67E-09
4	-3.183063012	-2.15E-16	-1.040598701	0

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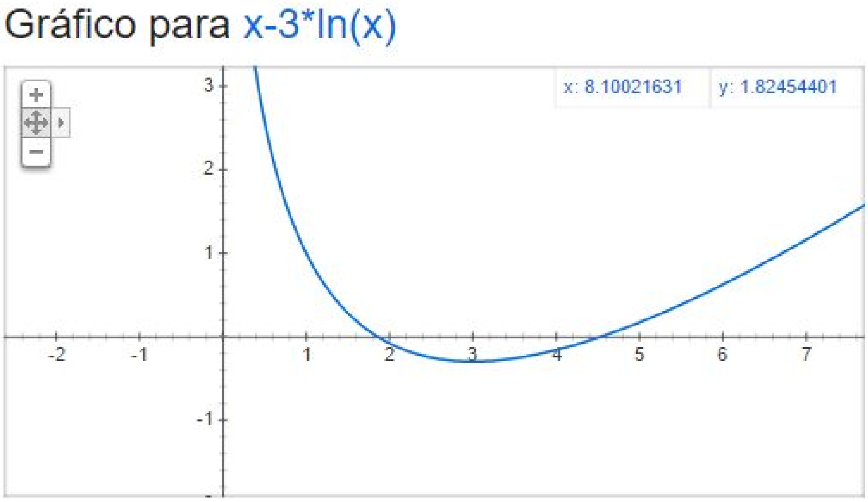
3. Encontrar as raízes das funções pelo **Método das Secantes**, com erro inferior a 10^{-10}.

   a) $f(x) = x - 3 \ln x$



Solução 1: $x_0 = 1$ e $x_1 = 1.5$

k	x_k	$f(x_k)$	$f'(x_k)$	epsilon
0	1	1	-	-
1	1.5	0.283605	-1.43279	-
2	1.69794	0.109694	-0.87861	0.116576
3	1.82279	0.0216865	-0.704909	0.0684935
6	1.857183637	1.37E-07	-0.6153869359	4.68E-05
7	1.85718386	8.46E-12	-0.6153490479	1.20E-07
8	1.85718386	-6.07E-17	-0.6153489446	7.40E-12

Solução 2: $x_0 = 3.5$ e $x_1 = 4$

k	x_k	$f(x_k)$	$f'(x_k)$	epsilon
0	3.5	-0.258289	-	-
1	4	-0.158883	0.198812	-
2	4.79916	0.0938387	0.316233	0.166521
3	4.50242	-0.0114236	0.35473	0.0659066
6	4.53640365	-1.70E-09	0.3386841471	2.90E-06
7	4.536403655	-4.68E-15	0.3386831883	1.11E-09
8	4.536403655	1.35E-16	0.338684082	3.13E-15

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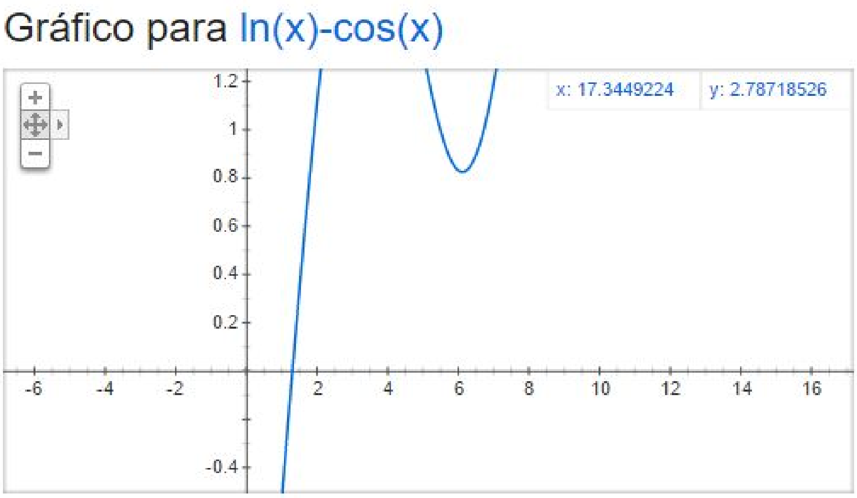
b) $f(x) = \ln x - \cos x$



Solução: $x_0 = 0.5$ e $x_1 = 1$

k	x_k	$f(x_k)$	$f'(x_k)$	epsilon
0	0.5	-1.57073	-	-
1	1	-0.540302	2.06085	-
2	1.26217	-0.0709109	1.79038	0.207716
3	1.30178	-0.0020499	1.73863	0.0304249
5	1.302964001	-8.63E-10	1.73182846	3.45E-06
6	1.302964001	-2.22E-16	1.731827772	3.82E-10
7	1.302964001	1.67E-16	1.75	1.70E-16

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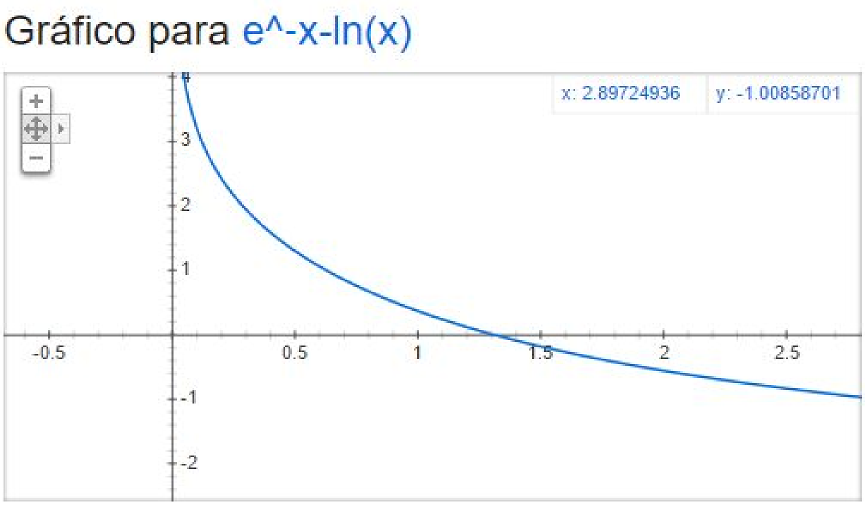
c) $f(x) = e^{-x} - \ln x$



Solução: $x_0 = 0.5$ e $x_1 = 1$

k	x_k	$f(x_k)$	$f'(x_k)$	epsilon
0	0.5	1.29968	-	-
1	1	0.367879	-1.8636	-
2	1.1974	0.121822	-1.24647	0.164859
3	1.29514	0.015244	-1.0905	0.0754622
6	1.309799585	1.21E-09	-1.033351484	3.16E-06
7	1.309799586	2.06E-15	-1.033349736	8.92E-10
8	1.309799586	3.85E-17	-1.013481988	1.53E-15