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Assignment 1 Companion Notebook

In this notebook you'll be applying Bayesian analysis to a couple of problems. The presentation will follow Allen's workshop from PyCon Bayes Made Simple.

First, install some packages and import the necessary libraries.

In [1]:

```
!pip install empyrical-dist

%matplotlib inline

import numpy as np
import pandas as pd

import seaborn as sns
sns.set_style('white')
sns.set_context('talk')

import matplotlib.pyplot as plt

from empyrical_dist import Pmf
```

```
Requirement already satisfied: empyrical-dist in /anaconda3/lib/python3.6/site-packages (0.2.0)
```

Working with Pmfs

Recall from the main assignment document that a PMF (probability mass function) specifies the probability that a random variable takes on a particular value. In this problem we can think of the random variable X as representing the outcome of rolling a single, six-sided die.

Create a Pmf object to represent a six-sided die.

In [2]:

```
d6 = Pmf()
```

A Pmf is a map from possible outcomes to their probabilities.

In [3]:

```
for x in [1,2,3,4,5,6]:
    d6[x] = 1
```

Initially the probabilities don't add up to 1. *Just as a probability measure must add to 1 when summed over a set of mutually exclusive, exhaustive events, a PMF must also add to 1*

In [4]:

```
d6
```

Out[4]:

	probs
1	1
2	1
3	1

	probs
4	1
5	1
6	1

`normalize` adds up the probabilities and divides through. The return value is the total probability before normalizing.

In [5]:

```
d6.normalize()
```

Out[5]:

6

Now the Pmf is normalized.

In [6]:

```
d6
```

Out[6]:

	probs
1	0.166667
2	0.166667
3	0.166667
4	0.166667
5	0.166667
6	0.166667

And we can compute its mean (which only works if it's normalized).

In [7]:

```
d6.mean()
```

Out[7]:

```
3.5
```

The cookie problem

The setup of this problem is in the chapter 1 reading from Think Bayes. The relevant part is that you have two bowls of cookies. Bowl 1 contains 3/4 vanilla cookies and 1/4 chocolate and bowl 2 contains 1/2 vanilla cookies and 1/2 chocolate.

`Pmf.from_seq` makes a `Pmf` object from a sequence of values.

Here's how we can use it to create a `Pmf` with two equally likely hypotheses.

In [8]:

```
cookie = Pmf.from_seq(['Bowl 1', 'Bowl 2'])
cookie
```

Out[8]:

	probs
Bowl 1	0.5
Bowl 2	0.5

Now we can update each hypothesis with the likelihood of the data (a vanilla cookie). *This is Bayes' rule in action!*

In [9]:

```
cookie['Bowl 1'] *= 0.75
cookie['Bowl 2'] *= 0.5
cookie.normalize()
```

Out[9]:

```
0.625
```

And display the posterior probabilities.

In [10]:

```
cookie
```

Out[10]:

	probs
Bowl 1	0.6
Bowl 2	0.4

You might be wondering what happened to the denominator in Bayes' rule. You would think that to update the probability of bowl 1 you should be doing the following calculation:

$$p(\text{Bowl 1} \,|\, \text{vanilla}) = \frac{p(\text{vanilla} \,|\, \text{Bowl 1}) p(\text{Bowl 1})}{p(\text{vanilla})}$$

.

in Allen's code, all he does is multiply the prior by the likelihood (the numerator of the previous equation). It turns out that applying the *normalize* function will have the same effect as dividing by $p(\text{vanilla})$. You can take this on faith, or show why this is true yourself (it should be pretty straightforward, post on NB if you need help).

Notebook Exercise 1

Suppose we put the first cookie back, stir, choose again from the same bowl, and get a chocolate cookie.

What are the posterior probabilities after the second cookie?

Hint: The posterior (after the first cookie) becomes the prior (before the second cookie).

Solution

In [11]:

```
# ***Solution***
cookie['Bowl 1'] *= 0.25
cookie['Bowl 2'] *= 0.5
cookie.normalize()
cookie
```

Out[11]:

	probs
Bowl 1	0.428571
Bowl 2	0.571429

The dice problem

Create a Suite to represent dice with different numbers of sides (in this case 4, 6, 8, and 12).

In [12]:

```
dice = Pmf.from_seq([4, 6, 8, 12])
dice
```

Out[12]:

	probs
4	0.25
6	0.25
8	0.25
12	0.25

Notebook Exercise 2

We'll solve this problem two ways. First we'll do it "by hand", as we did with the cookie problem; that is, we'll multiply each hypothesis by the likelihood of the data, and then renormalize.

Update `dice` based on the likelihood of the data (rolling a 6), then normalize and display the results.

Solution

In [13]:

```
# ***Solution***
dice[4] *= 0
dice[6] *= 1/6
dice[8] *= 1/8
dice[12] *= 1/12
dice.normalize()
dice
```

Out[13]:

	probs
4	0.000000
6	0.444444
8	0.333333
12	0.222222

Notebook Exercise 3

Now let's do the same calculation using `Pmf.update`, which encodes the structure of a Bayesian update.

Define a function called `likelihood_dice` that takes `data` and `hypo` and returns the probability of the data (the outcome of rolling the die) for a given hypothesis (number of sides on the die).

Hint: What should you do if the outcome exceeds the hypothetical number of sides on the die?

Here's an outline to get you started.

In [14]:

```
def likelihood_dice(data, hypo):
    """Likelihood function for the dice problem.

    data: outcome of the die roll
    hypo: number of sides

    returns: float probability
    """
```

```
    # TODO: fill this in!
    return 1
```

Solution

In [15]:

```
# ***Solution***
def likelihood_dice(data, hypo):
    """Likelihood function for the dice problem.

    data: outcome of the die roll
    hypo: number of sides

    returns: float probability
    """
    if data > hypo:
        return 0
    return 1/hypo
```

In [16]:

```
# ***Solution***
# make sure to start with a fresh Pmf (what would happen if you don't?)
dice = Pmf.from_seq([4, 6, 8, 12])
dice.update(likelihood_dice, 6)
dice
```

Out[16]:

	probs
4	0.000000
6	0.444444
8	0.333333
12	0.222222

More Fun with Dice

If we get more data, we can perform more updates.

In [17]:

```
for roll in [8, 7, 7, 5, 4]:
    dice.update(likelihood_dice, roll)
```

Here are the results.

In [18]:

```
dice
```

Out[18]:

	probs
4	0.000000
6	0.000000
8	0.919294
12	0.080706

The German tank problem

The German tank problem is actually identical to the dice problem. This one is a cool example, because it was actually used to estimate German monthly tank production during WWII. Here are the results of the analysis.

	Month	Statistical estimate	Intelligence estimate	German records
June 1940	169	1,000	122	
June 1941	244	1,550	271	
August 1942	327	1,550	342	

In [19]:

```python
def likelihood_tank(data, hypo):
    """Likelihood function for the tank problem.

    data: observed serial number
    hypo: number of tanks

    returns: float probability
    """
    if data > hypo:
        return 0
    else:
        return 1 / hypo
```

Here is the update after seeing Tank #42.

In [20]:

```python
tank = Pmf.from_seq(range(100))
tank.update(likelihood_tank, 42)
tank.mean()
```

Out[20]:

```
66.32784309363326
```

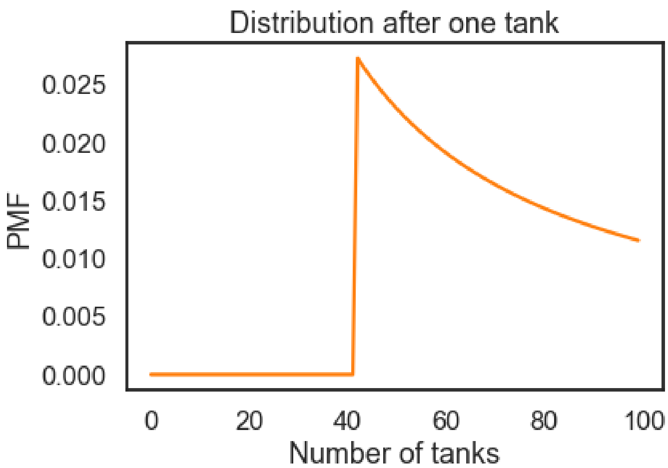
And here's what the posterior distribution looks like.

In [21]:

```python
def decorate_tank(title):
    """Labels the axes.

    title: string
    """
    plt.xlabel('Number of tanks')
    plt.ylabel('PMF')
    plt.title(title)

tank.plot()
decorate_tank('Distribution after one tank')
```



Notebook Exercise 4

Suppose we see another tank with serial number 17. What effect does this have on the posterior probabilities?

Update the `Pmf` with the new data and plot the results.

Solution

In [22]:

```
# ***Solution***
tank.update(likelihood_tank, 17)
tank.mean()
```

Out[22]:

```
62.25994473449364
```

In [23]:

```
# ***Solution***
tank.plot()
decorate_tank('Distribution after two tanks')
```