Problem 1 AIME 2025

Sunday, July 27, 2025 5:04 PM

Find the sum of all integer bases $b > 9$ for which 17_b is a divisor of 97_b.

$17_b = b+7$

$97_b = 9b+7$

$b+7 \mid 9b+7 \Rightarrow 9b+7 \equiv 0 \pmod{b+7}$

$\Rightarrow 9b \equiv 0 \pmod{b+7}$ (substituting b for -7 as this is mod $b+7$)

$\Rightarrow 8(-7) \equiv 0$

$\Rightarrow -56 \equiv 0$

$\Rightarrow 56 \equiv 0 \pmod{b+7}$

$\Rightarrow b+7 \mid 56$

56 can be factored into $2 \cdot 2 \cdot 2 \cdot 7$

which means that $b+7$ can be 56, 28, 14, ...

$b = 49, 21, \cancel{7}$ (discarding $b \leq 9$)

$49 + 21 = \boxed{70}$