2.) $a = \log 36 = 2\log 2 + 2\log 3$

$\frac{1}{3}b = \frac{1}{3}\log 125 = \log 5$

$c = \log \frac{1}{12} = -(\log 3 + 2\log 2)$

$1 + \frac{1}{3}b = \log 10 + \log 5 = \log 2$

$\frac{1}{2}a + \frac{1}{3}b = \log 6 + \log 3$

$= 1 + \log 3$

$+2(1 - \frac{1}{3}b) \qquad +2(1 - \frac{1}{3}b)$

$\frac{1}{2}a + \frac{2}{3}b + 2 = 1 + 2\log 2 + \log 3$

$\frac{1}{2}a - \frac{2}{3}b + 1 = 2\log 2 + \log 3$

$\boxed{\frac{2}{3}b - \frac{1}{2}a - 1} = -(2\log 2 + \log 3)$

$= -\log 12$

$= \log \frac{1}{12}$

3.) $y = 2\log x, \quad y = \log 2x$

$10^{2\log x} = 10^{\log 2x}$

$x^2 = 2x$

$x^2 - 2x = 0$

$x(x-2)=0$

$x=0, \boxed{2}$

4) $\log_{10}(x^{\log x}) = \log_{10}\left(\frac{x^3}{100}\right)$

$\log x \log x = 3\log x - \log 100$

$(\log x)^2 - 3\log x + 2 = 0$

$(\log x - 1)(\log x - 2) = 0$

$\log x = 1, 2$

$\boxed{x = 10, 100}$

5) $p = \frac{\log_b(\log_b a)}{\log_b a}$

$= \log_a(\log_b a)$

$= \log_a\left(\frac{\log_a a}{\log_a b}\right)$

$= \log_a(1) - \log_a(\log_a b)$

$= -\log_a(\log_a b)$

$a^p = \frac{1}{\log_a b} = \boxed{\log_b a}$

6) $2^x = 10^{(\log_{10} 2)x}$

$2^{10} > 10^3$

$x = 10 \Rightarrow \log_{10} 2 > \frac{3}{10}$

$2^{13} < 10^4 \Rightarrow \log_{10} 2 < \frac{4}{13}$

7. $\frac{1}{\log_3 x} + \frac{1}{\log_4 x} + \frac{1}{\log_5 x}$

$= \log_x 3 + \log_x 4 + \log_x 5$

$= \log_x 60 = \frac{\log 60}{\log x} = \frac{1}{\log_{60} x}$

8.) $5^{44} = 10^{44} 2^{-44}$ $\sim 10^{44 - \log_{10}(2)\,44}$

$= 10^{44 - 13.2\,44}$

$= 10^{30.75\,4}$

31 digits

9.) $\log_6 3 = P$

$\log_3 5 = Q$

$\log_{10} 5 = N = 1 - \log_{10} 2$

$\frac{1}{3P} = \frac{1}{3}\log_3 6$

$= \log_3 2$

$3PQ = \frac{1}{\log_5 2}$ $\frac{1}{3PQ} = \frac{\log_5 6}{3\log_3 5}$

$= \frac{1}{3}\log_5 6$

$3PQ + 1 = \frac{1}{\log_5 2} + \frac{1}{\log_2 2}$

$= \frac{1}{\log_{10} 2}$

$= \log_3 2$

$+Q \qquad +Q$

$\frac{1}{3P} + Q = \log_3 10$

$\frac{1}{3PQ+1} = \log_{10} 2$

$1 - \frac{1}{3PQ+1} = 1 - \log_{10} 2 = \log_{10} 5$

$$1 - \frac{1}{3\log_8 3 \log_3 5 + 1}$$

$$= 1 - \frac{1}{\frac{1}{\log_3 2} \log_3 5 + 1}$$

$$= 1 - \frac{1}{\log_2 5 + \log_2 2}$$

$$= 1 - \frac{1}{\log_2 10}$$

$$= 1 - \log_{10} 2$$

$$= \log_{10} 10 - \log_{10} 2 = \log_{10} 5$$

10.) $\log_q p = x \qquad p = q^x$

$\log_{12} q = x \qquad q = 12^x$

$\log_{16}(p+q) = x \qquad p+q = 16^x$

$\frac{q}{p} = \frac{12^x}{q^x} \qquad p+q = 16^x$

$\frac{q}{p} = \left(\frac{4}{3}\right)^x \qquad \frac{p+q}{p} = \frac{16^x}{q}$

$\frac{p+q}{p} = \frac{16}{9}^x$

$1 + \frac{q}{p} = \left(\frac{16}{9}\right)^x$

$\frac{q}{p} = \left(\frac{16}{9}\right)^x - 1$

$\left(\frac{4}{3}\right)^x = \left(\frac{16}{9}\right)^x - 1$

$4^x = \frac{16}{3}^x - 3^x$

$12^x = 16^x - 9^x$

$3^x 4^x = 4^{2x} - 3^{2x}$

$4^{2x} - 3^x 4^x - 3^{2x} = 0$

$4^x = \frac{3^x \pm \sqrt{3^{2x} + 4(3^{2x})}}{2}$

$= \frac{3^x \pm \sqrt{5}\, 3^x}{2}$

$4^x = 3^x \left(\frac{1 \pm \sqrt{5}}{2}\right)$

$\frac{q}{p} = \frac{1 \pm \sqrt{5}}{2}$

$q, p > 0 \Rightarrow \boxed{\frac{q}{p} = \frac{1+\sqrt{5}}{2}}$

(c) $\log_4 40\sqrt{3} = \log_4 45 = x$

(c) $\log_{4n} 40\sqrt{3} = \log_{3n} 45 = x$

$40\sqrt{3} = 4^x n^x$

$45 = 3^x n^x$

$\left(\frac{4}{3}\right)^x = \frac{40\sqrt{3}}{9}$

$x = \log_{\frac{4}{3}} \frac{40\sqrt{3}}{9}$

$= \log_{\frac{4}{3}} \frac{16}{9} \cdot \frac{\sqrt{3}}{2}$

$= \log_{\frac{4}{3}} \frac{16}{9} - \frac{1}{2}$

$\therefore 2 - \frac{1}{2} = \frac{3}{2}$

$40\sqrt{3} = 4^{3/2} n^{3/2}$

$n^{3/2} = \frac{40\sqrt{3}}{8}$

$= 5\sqrt{3}$

$n^3 = 75$

$45 = 3^{3/2} n^{3/2}$

$45 = \sqrt{27}\, n^{3/2}$

$n^{3/2} = \frac{45}{}$

$45 = \sqrt{27}\ n^{3/2}$

$n^{3/2} = \frac{45}{3\sqrt{3}}$

$= \frac{15\sqrt{3}}{3}$

$= 5\sqrt{3}$

$n^3 = 75$

12) $\log_a a = \log_{15} b = \log_{25}(a+2b) = x$

$a = 9^x$

$b = 15^x$

$a + 2b = 25^x$

$\frac{b}{a} = \frac{15^x}{9^x} = \left(\frac{5}{3}\right)^x$

$\frac{a+2b}{2a} = \frac{25^x}{2(9)^x}$

$\frac{1}{2} + 2\frac{b}{a} = \frac{1}{2}\left(\frac{25}{9}\right)^x$

$\frac{b}{a} = \frac{1}{4}\left(\frac{25}{9}\right)^x - \frac{1}{4}$

$\frac{1}{4}\left(\frac{25}{9}\right)^x - \frac{1}{4} = \left(\frac{5}{3}\right)^x$

$\left(\frac{25}{9}\right)^x - 1 = 4\left(\frac{5}{3}\right)^x$

$25^x - 9^x = 4(15)^x$

$5^{2x} - 3^{2x} = 4(3)^x 5^x$

$5^{2x} - 4(3)^x 5^x - 3^{2x} = 0$

$$5^x = \frac{4(3)^x \pm \sqrt{4(3)^{2x} + 4(3)^{2x}}}{2}$$

$$= \frac{4(3)^x \pm 3^x\sqrt{20}}{2}$$

$$= \frac{3^x(4 \pm \sqrt{20})}{2}$$

$$\left(\frac{5}{3}\right)^x = \frac{(4 \pm \sqrt{20})}{2}$$

$$\boxed{\frac{b}{a} = \frac{4 + \sqrt{20}}{2}}$$

13) $60^a = 3$

$60^b = 5$

$12^{\frac{1-a-b}{2-2b}} = ?$

$a = \log_{60} 3$

$b = \log_{60} 5$

$$12^{\log_{60} \frac{60 - \log_{60} 3 - \log_{60} 5}{2\log_{60} 60 - 2\log_{60} 5}}$$

$$= 12^{\frac{\log_{60} 4}{2\log_{60} 12}} = 12^{\frac{1}{2}\log_{12} 4}$$

$$= \sqrt{2}$$

$= 4^{\frac{1}{2}} = \boxed{2}$