COMP 302 / Assign. 3 / Question 2 / Connor Sullivan / 260 421 531

We want to show that:

$$acc*pow\ (n, k) == pow\_tl\ (n, k, acc) == acc*n\hat{}k$$

for all n and k (as long as they are the same in both functions).

Base Case

$$acc*pow\ (n, 0) == n\hat{}0*acc = acc$$
$$pow\_tl\ (n, 0, acc) == acc$$

Inductive Step

Assume acc*pow (n, k) == pow_tl (n, k, acc) == acc*n^k

We now show that

$$acc*pow\ (n\ , k+1) == pow\_tl\ (n, k+1, acc) == acc*n*\ n\hat{}k == acc*n\hat{}(k+1)$$

i) acc*pow (n, k+1) == acc*n*pow(n, k) == acc*n*n^k == acc*n^(k+1)

ii) pow_tl (n , k+1, acc) == pow_tl (n, k, acc*n) == (acc*n)*n^k == acc*n^(k+1)

Hence these functions return the same result when applied to the same arguments.