Assignment #4 / Q. 4 / Connor Sullivan / 260 421 531 / COMP 302/ Win. 2015

We want to prove that :

> **(i)** reduce(lst, base, op) == reduce_tr(lst, base, op)

To do this we first prove:

> **(ii)** op(h, reduce(l, n, op)) == reduce_tr(l, op (h, n), op)
>
> using the lemmas **(a), (b),** and **(c)** provided in the assignment.

<u>Proof of (ii):</u>

Base Case__

op (h, reduce(nil, n, op)) == op(h, n)
reduce_tr(nil, op(h, n), op) == op(h, n)

Inductive Step__

*Assume **(ii)** holds.*

op(h, reduce(x::l, n, op)) == op(h, op(x, reduce(l, n, op)))
    == op(op(x, h), reduce(l, n, op))

reduce_tr(x::l, op(h, n), op) == reduce_tr(l, op(x, op(h, n)), op)
    == reduce_tl(l, op(op(x, h), n), op)
    == op(op(x, h), reduce(l, n, op))   **by assuming (ii) and the lemmas*

Therefore op(h, reduce(l, n, op)) == reduce_tr(l, op(h, n), op) is true.

Now that we have proved (ii) we now prove (i) using (ii).

<u>Proof of (i):</u>

Base Case__

reduce(lst, base, op) == base
reduce_tr(lst, base, op) == base

Inductive Step__

*Assume **(i)** holds.*

reduce(x::lst, base, op) == op(x, reduce(lst, base, op))

== reduce_tr(lst, op(x, base), op) *by (ii)*

reduce_tr(x::lst, base, op) == reduce_tr(lst, op(x, base), op)

Therefore reduce(lst, base, op) == reduce_tr(lst, base, op) is true and we conclude our proof.