**Assignment #4 / Q. 4 / Connor Sullivan / 260 421 531 / COMP 302/ Win. 2015**

We want to prove that :

**(i)** reduce(lst, base, op) == reduce\_tr(lst, base, op)

To do this we first prove:

**(ii)** op(h, reduce(l, n, op)) == reduce\_tr(l, op (h, n), op)

using the lemmas **(a), (b),** and **(c)** provided in the assignment.

**Proof of (ii):**

**Base Case\_\_**

op (h, reduce(nil, n, op)) == op(h, n)

reduce\_tr(nil, op(h, n), op) == op(h, n)

**Inductive Step\_\_**

*Assume* ***(ii)*** *holds.*

op(h, reduce(x::l, n, op)) == op(h, op(x, reduce(l, n, op)))

== op(op(x, h), reduce(l, n, op))

reduce\_tr(x::l, op(h, n), op) == reduce\_tr(l, op(x, op(h, n)), op)

== reduce\_tl(l, op(op(x, h), n), op)

== op(op(x, h), reduce(l, n, op)) *\*by assuming (ii) and the lemmas*

Therefore op(h, reduce(l, n, op)) == reduce\_tr(l, op(h, n), op) is true.

Now that we have proved (ii) we now prove (i) using (ii).

**Proof of (i):**

**Base Case\_\_**

reduce(lst, base, op) == base

reduce\_tr(lst, base, op) == base

**Inductive Step\_\_**

*Assume* ***(i)*** *holds.*

reduce(x::lst, base, op) == op(x, reduce(lst, base, op))

== reduce\_tr(lst, op(x, base), op) *\*by (ii)*

reduce\_tr(x::lst, base, op) == reduce\_tr(lst, op(x, base), op)

**Therefore reduce(lst, base, op) == reduce\_tr(lst, base, op) is true and we conclude our proof.**