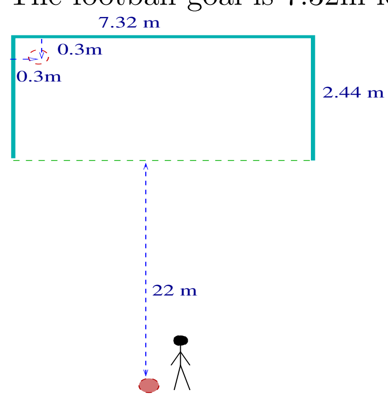
Computational Physics

Assignment 2

Due date: 29th April

1. You are practicing free-kicks for a football match. You are taking shots standing 22 m from the goal in a central position, aiming for a point (0.3 m, 0.3 m) away from the top corner of the goal (see fig). The football goal is 7.32m long and 2.44 m high.



   If your kick gives the ball an initial speed $v_0 = 70$ km/hr, at which angle should you shoot? Take air drag to be Dv^2, with D =3 g/m and mass of the football 450 g.

   How will the angle need to be changed if you kick the ball harder? Plot the angle as a function of v_0, varying v_0 from 70 km/hr to 85 km/hr. Here I am asking only about straight shots (but feel free to explore swerving shots also).

2. In the class we calculated the low-lying energy eigenvalues and eigenstates of a particle in the one-dimensional potential

$$V(x) \;=\; \frac{1}{2}\,k\,x^2 \;+\; \frac{\lambda}{4!}\,x^4\,.$$

   We took λ=1 and looked at the two cases $k = \pm 1$, using the Dirichlet boundary condition, $\Psi(-L/2) = \Psi(L/2) = 0$ where the system was put in a finite box, $x \in [-L/2, L/2)$.

   Solve the problem instead with the periodic boundary condition, $\Psi(x) = \Psi(x + L)$. Compare the size of the finite L correction in the two boundary conditions.

3. The attached data file *corr.dat* gives $N = 50$ estimates of a correlation function $C_m(t)$, where t ranges from 0 to 95. The different estimates deviate from the true correlation function by an error (you can think of them as N "measurements" of the correlation function). The data file has t in the first column and $C_m(t)$ in the second, and each measurement is placed as a set (such that the first 96 rows correspond to the first measurement, etc).

(a) Find the average, $\bar{C}(t)$, and the rms error,

$$\epsilon(t) = \sqrt{\frac{\sigma^2(t)}{N-1}}$$

of the measurements, and plot $\bar{C}(t)$ (with error) as function of t.

Note that the variance can be written in two equivalent ways:

$$\sigma^2(t) = \frac{1}{N}\sum_{m=1}^{N}\left(C_m(t) - \bar{C}(t)\right)^2 = \frac{1}{N}\sum_{m=1}^{N} C_m(t)^2 - \bar{C}(t)^2 \cdot$$

The second definition is more convenient to code, as you need to run the loop over m just once (how?). Check, however, that this has a larger roundoff error than the first definition.

Can you write a code that does the calculation in one loop, without increasing the round-off error?

(b) A mesure of the correlation between the different measurements can be obtained by looking at the covariance matrix

$$\begin{aligned}\mathrm{cov}(t_1, t_2) &= \sigma(t_1)\,\sigma(t_2)\,\mathrm{cor}(t_1, t_2) \\ &= \frac{1}{N}\sum_{m=1}^{N}\left(C_m(t_1) - \bar{C}(t_1)\right)\left(C_m(t_2) - \bar{C}(t_2)\right) \cdot\end{aligned}$$

Find the eigenvalues of the 30x30 correlation matrix $\mathrm{cor}(t_1, t_2 \in [33, 62])$.

Do not use any in-built statistical package for calculating mean, variance, covariance. Arrange your calculation so as to minimize round-off error. You can use linear algebra packages for calculating the eigenvalues.