Computational Physics: Final Exam

Date: 15th June, 2022
Time: 9:30 a.m. – 12:30 p.m.

Please prepare a proper answersheet, which can be handwritten (hand it over to me) or a pdf file (upload in moodle). The figures and all the codes used should be uploaded to moodle, and the answersheet should clearly mention the names of the figure files and the codes relevant for each answer. All the codes (written in python/c/c++/fortran) should have proper instructions on how to run them to get the required output.

You can use codes you have written before. Use of standard packages like scipy, lapack etc are allowed (please clearly mention which all routines you have used). If you ar eusing any other code written by someone else, that should be properly attributed. Unattributed use of others' code will be considered as unfair practice. Sharing of codes, or any discussion, *during the exam* is not allowed.

1. Solve the differential equation (20)

$$y'' \;=\; -\frac{2}{x}\,y' \;+\; \frac{2}{x^2}\,y \;+\; \frac{\sin\left(\log x\right)}{x^2}$$

for $1 < x < 2$, with the boundary conditions $y(1) = 1$ and $y(2) = 2$, by converting it into a linear algebra problem (through discretization of the derivatives).

How does the discretization error in your solution depend on the discretization scale dx?

2. In an experiment measuring the dimuons from the $Z \to \mu^+\mu^-$ process, the average dimuon counts in different energy bins of width 1 GeV are shown in the attached file ***dimuon-yield***. (20)

Assume that the detector smearing in this energy regime is Gaussian with width 2.3 GeV, i.e., the signal observed is

$$f(E) \;=\; \int dE'\; g(E')\; h(E-E'), \qquad h(E-E') \;=\; \frac{1}{\sqrt{2\pi}\Delta}\; e^{-\frac{(E-E')^2}{2\Delta^2}}$$

where $g(E)$ is the *true* signal and $h(E)$ is the detector smearing function, with $\Delta = 2.3$ GeV.

Extract the *true* signal from the data given.

(This is a toy problem. The detector smearing in experiments is done differently.)

3. The area of a circle with unit radius is given by the integral (20)

$$I = \int_{-1}^{1}\int_{-1}^{1} f(x,y)\,\mathrm{d}x\,\mathrm{d}y,$$

where

$$f(x) = \begin{cases} 1 & \text{if } x^2+y^2 \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Write a Python code to calculate this area using Monte Carlo integration. You are allowed to use Numpy or Scipy.

(b) Calculate the volume of a ten-dimensional unit sphere.

4. Use the Metropolis algorithm to get a sample from a density that is uniform for $3 < x < 7$ and zero elsewhere. (20)

   (a) Make a plot to show the histogram of your sample.

   (b) Add the expected histogram to the plot.

   (c) Use the KS test to label your sample as "not sufficiently random", "suspect", "almost suspect", or "sufficiently random".
   (The Scipy function `scipy.stats.ks_1samp` implements the KS test.)