

Задача 1032(j).

Найти собственные значения и собственные векторы матрицы $\begin{pmatrix} 2 & 5 & -6 \\ 4 & 6 & -9 \\ 3 & 6 & -8 \end{pmatrix}$

Решение:

$$\det(A - \lambda E) = \begin{vmatrix} 2-\lambda & 5 & -6 \\ 4 & 6-\lambda & -9 \\ 3 & 6 & -8-\lambda \end{vmatrix} = \begin{vmatrix} 2-\lambda & 5 & -6 \\ 1 & -\lambda & \lambda-1 \\ 3 & 6 & -8-\lambda \end{vmatrix} = \begin{vmatrix} 2-\lambda & 5 & -1 \\ 1 & -\lambda & -1 \\ 3 & 6 & -2-\lambda \end{vmatrix} =$$

$$\begin{vmatrix} 1-\lambda & 5+\lambda & 0 \\ 1 & -\lambda & -1 \\ 3 & 6 & -2-\lambda \end{vmatrix} = (1-\lambda) \begin{vmatrix} -\lambda & -1 \\ 6 & -2-\lambda \end{vmatrix} - (5+\lambda) \begin{vmatrix} 1 & -1 \\ 3 & -2-\lambda \end{vmatrix} = (1-\lambda)(\lambda^2 + 2\lambda + 6) - (5+\lambda)(-2-\lambda+3) = 1 - \lambda^3$$

$$f(\lambda) = 0 \Rightarrow \lambda = 1$$

$$\lambda = 1 : \begin{pmatrix} 1 & 5 & -6 \\ 4 & 5 & -9 \\ 3 & 6 & -9 \end{pmatrix} X = 0$$

$$\left(\begin{array}{ccc|c} 1 & 5 & -6 & 0 \\ 4 & 5 & -9 & 0 \\ 3 & 6 & -9 & 0 \end{array} \right) \xrightarrow{S_2-4S_1; S_3-3S_1} \left(\begin{array}{ccc|c} 1 & 5 & -6 & 0 \\ 0 & -15 & 15 & 0 \\ 0 & -9 & 9 & 0 \end{array} \right) \xrightarrow{5S_3-3S_2} \left(\begin{array}{ccc|c} 1 & 5 & -6 & 0 \\ 0 & -15 & 15 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\begin{cases} x_1 + 5x_2 - 6x_3 = 0 \\ -15x_2 + 15x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = x_3 \\ x_2 = x_3 \end{cases}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} x_3 \Rightarrow \vartheta = (1 \ 1 \ 1)^T$$

$$\text{Ответ: } \lambda = 1; \vartheta = (1 \ 1 \ 1)^T$$

Задача 1032(h).

Найти собственные значения и собственные векторы матрицы $\begin{pmatrix} 0 & 2 & 1 \\ -2 & 0 & 3 \\ -1 & -3 & 0 \end{pmatrix}$

Решение:

$$\begin{vmatrix} -\lambda & 2 & 1 \\ -2 & -\lambda & 3 \\ -1 & -3 & -\lambda \end{vmatrix} = (-\lambda^3 - 6 + 6) - (\lambda + 4\lambda + 9\lambda) = -\lambda^3 - 15\lambda = 0$$

$$-\lambda(\lambda^2 + 15) = 0$$

$\lambda \in \{0\}$ Найдем собственный вектор:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}^T * \begin{pmatrix} 0 & 2 & 1 \\ -2 & 0 & 3 \\ -1 & -3 & 0 \end{pmatrix} = \begin{pmatrix} -2y - z \\ 2x - 3z \\ x + 3y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \text{ Решим систему}$$

$$\begin{pmatrix} 0 & -2 & -1 & 0 \\ 2 & 0 & -3 & 0 \\ 1 & 3 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & -2 & -1 & 0 \\ 0 & -6 & -3 & 0 \\ 1 & 3 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -2 & -1 & 0 \\ 1 & 3 & 0 & 0 \end{pmatrix}$$

Итого:

$$\lambda = 0$$

$$v = \begin{pmatrix} 3 \\ 2 \\ -1 \\ 1 \end{pmatrix}$$

Задача .

Найти собственные значения и собственные векторы матрицы $\begin{pmatrix} 0 & 1 & 1 & -1 \\ 1 & 0 & -1 & 1 \\ 1 & -1 & 0 & 1 \\ -1 & 1 & 1 & 0 \end{pmatrix}$

Решение:

$$\det(A - \lambda E) = \begin{vmatrix} -\lambda & 1 & 1 & -1 \\ 1 & -\lambda & -1 & 1 \\ 1 & -1 & -\lambda & 1 \\ -1 & 1 & 1 & -\lambda \end{vmatrix} \xrightarrow{S_4+S_3} \begin{vmatrix} -\lambda & 1 & 1 & -1 \\ 1 & -\lambda & -1 & 1 \\ 1 & -1 & -\lambda & 1 \\ 0 & 0 & 1-\lambda & 1-\lambda \end{vmatrix} =$$

$$(1-\lambda)(-1)^{4+3} \begin{vmatrix} -\lambda & 1 & -1 \\ 1 & -\lambda & 1 \\ 1 & -1 & 1 \end{vmatrix} + (1-\lambda)(-1)^{4+4} \begin{vmatrix} -\lambda & 1 & 1 \\ 1 & -\lambda & -1 \\ 1 & -1 & -\lambda \end{vmatrix} =$$

$$(\lambda-1)(\lambda^2-2\lambda+1) + (1-\lambda)(-\lambda^3+3\lambda-2) = \lambda^4-6\lambda^2+8\lambda-3 = (\lambda-1)^3(\lambda+3)$$

$$f(\lambda) = 0 \Rightarrow \begin{cases} \lambda = 1, \\ \lambda = -3 \end{cases}$$

$$\lambda = 1: \begin{pmatrix} -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \end{pmatrix} X = 0$$

$$\left(\begin{array}{cccc|c} -1 & 1 & 1 & -1 & 0 \\ 1 & -1 & -1 & 1 & 0 \\ 1 & -1 & -1 & 1 & 0 \\ -1 & 1 & 1 & -1 & 0 \end{array} \right) \Rightarrow \left(\begin{array}{cccc|c} -1 & 1 & 1 & -1 & 0 \\ 1 & -1 & -1 & 1 & 0 \\ 1 & -1 & -1 & 1 & 0 \\ -1 & 1 & 1 & -1 & 0 \end{array} \right) \xrightarrow{S_2+S_1; S_3+S_1; S_4-S_1} \left(\begin{array}{cccc|c} -1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$-x_1 + x_2 + x_3 - x_4 = 0 \Rightarrow x_1 = -x_2 - x_3 + x_4$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} x_2 + \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} x_3 + \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} x_4$$

$$\vartheta_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}; \vartheta_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}; \vartheta_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\lambda = -3: \begin{pmatrix} 3 & 1 & 1 & -1 \\ 1 & 3 & -1 & 1 \\ 1 & -1 & 3 & 1 \\ -1 & 1 & 1 & 3 \end{pmatrix} X = 0$$

$$\left(\begin{array}{cccc|c} 3 & 1 & 1 & -1 & 0 \\ 1 & 3 & -1 & 1 & 0 \\ 1 & -1 & 3 & 1 & 0 \\ -1 & 1 & 1 & 3 & 0 \end{array} \right) \xrightarrow{3S_2-S_1; 3S_3-S_1; 3S_4+S_1} \left(\begin{array}{cccc|c} 3 & 1 & 1 & -1 & 0 \\ 0 & 8 & -6 & 6 & 0 \\ 0 & -6 & 8 & 6 & 0 \\ 0 & 4 & 4 & 8 & 0 \end{array} \right) \Rightarrow$$

$$\begin{pmatrix} 3 & 1 & 1 & -1 & | & 0 \\ 0 & 8 & -6 & 6 & | & 0 \\ 0 & -6 & 8 & 6 & | & 0 \\ 0 & 4 & 4 & 8 & | & 0 \end{pmatrix} \xrightarrow{4S_3+3S_2; 2S_4-S_2} \begin{pmatrix} 3 & 1 & 1 & -1 & | & 0 \\ 0 & 8 & -6 & 6 & | & 0 \\ 0 & 0 & 14 & 42 & | & 0 \\ 0 & 0 & 14 & 10 & | & 0 \end{pmatrix} \xrightarrow{S_4-S_3} \begin{pmatrix} 3 & 1 & 1 & -1 & | & 0 \\ 0 & 8 & -6 & 6 & | & 0 \\ 0 & 0 & 14 & 42 & | & 0 \\ 0 & 0 & 0 & -32 & | & 0 \end{pmatrix}$$

$$\begin{cases} 3x_1 + x_2 + x_3 - x_4 = 0 \\ 8x_2 - 6x_3 + 6x_4 = 0 \\ 14x_3 + 42x_4 = 0 \\ -32x_4 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \\ x_4 = 0 \end{cases}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\vartheta_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Ответ: $\lambda_1 = 1; \vartheta_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}; \vartheta_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}; \vartheta_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}; \lambda_2 = -3; \vartheta_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

Задача 1033.б.

Найти собственные значения матрицы

$$\begin{pmatrix} 0 & 1 & 0 & \dots & 0 & 0 & 0 \\ -1 & 0 & 1 & \dots & 0 & 0 & 0 \\ 0 & -1 & 0 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 1 & 0 \\ 0 & 0 & 0 & \dots & -1 & 0 & 1 \\ 0 & 0 & 0 & \dots & 0 & -1 & 0 \end{pmatrix}$$

Решение:

$$\det(A - \lambda E) = \begin{vmatrix} -\lambda & 1 & 0 & \dots & 0 & -\lambda & 0 \\ -1 & 0 & 1 & \dots & 0 & 0 & 0 \\ 0 & -1 & -\lambda & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & -\lambda & 1 & 0 \\ 0 & 0 & 0 & \dots & -1 & -\lambda & 1 \\ 0 & 0 & 0 & \dots & 0 & -1 & -\lambda \end{vmatrix} = -\lambda \begin{vmatrix} 0 & 1 & 0 & \dots & 0 & 0 & 0 \\ -1 & -\lambda & 1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & -\lambda & 1 & 0 \\ 0 & 0 & 0 & \dots & -1 & -\lambda & 1 \\ 0 & 0 & 0 & \dots & 0 & -1 & -\lambda \end{vmatrix} +$$

$$1(-1)^{1+1} \begin{vmatrix} -1 & 1 & 0 & \dots & 0 & 0 & 0 \\ 0 & -\lambda & 1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & -\lambda & 1 & 0 \\ 0 & 0 & 0 & \dots & -1 & -\lambda & 1 \\ 0 & 0 & 0 & \dots & 0 & -1 & -\lambda \end{vmatrix} = -\lambda \Delta_{n-1} + \Delta_{n-2}$$

Получаем рекуррентную формулу: $\Delta_n = -\lambda \Delta_{n-1} + \Delta_{n-2}$ $\Delta_2 = -\lambda \Delta_1 + \Delta_0$

$$\Delta_2 = \lambda^2 + 1; \Delta_1 = -\lambda$$

$$\Delta_0 = 1$$

$$G(z) = \frac{c_0 + c_1 z - \alpha c_0 z}{1 - \alpha z - \beta z^2}; \alpha = -\lambda; \beta = 1$$

$$G(z) = \frac{1 - \lambda z + \lambda z}{1 + \lambda z - z^2}$$

$$z_{1|2} = \frac{-\lambda \pm \sqrt{\lambda^2 + 4}}{-2} = \frac{\lambda \mp \sqrt{\lambda^2 + 4}}{2}$$

$$\text{Пусть } z_{1|2} = \cos \theta + i \sin \theta$$

$$\frac{\lambda \mp \sqrt{\lambda^2 + 4}}{2} = \cos \theta + i \sin \theta \Rightarrow \lambda^2 + 4 = 4(\cos 2\theta + i \sin 2\theta) - 4\lambda(\cos \theta + i \sin \theta) + \lambda^2 \Rightarrow$$

$$1 = \cos 2\theta - \lambda \cos \theta \Rightarrow \lambda = \frac{\cos 2\theta - 1}{\cos \theta}$$

$$\frac{1}{1 + \lambda z - z^2} = \frac{A}{z_1 - z} + \frac{B}{z_2 - z} = \frac{A(z_2 - z) + B(z_1 - z)}{(z_1 - z)(z_2 - z)} \Rightarrow$$

$$\begin{cases} A(z_2 - z_1) = 1 \\ B(z_1 - z_2) = 1 \end{cases} \Rightarrow \begin{cases} A = \frac{i}{2 \sin \theta} \\ B = -\frac{i}{2 \sin \theta} \end{cases}$$

$$\Delta_n = \frac{A}{z_1^{n+1}} + \frac{B}{z_2^{n+1}} = \frac{i}{2 \sin \theta} \frac{1}{(\cos \theta + i \sin \theta)^{n+1}} - \frac{i}{2 \sin \theta} \frac{1}{(\cos \theta - i \sin \theta)^{n+1}} = \frac{i}{2 \sin \theta} \left(\frac{1}{(\cos \theta + i \sin \theta)^{n+1}} - \frac{1}{(\cos \theta - i \sin \theta)^{n+1}} \right) = \frac{i}{2 \sin \theta} \frac{-2i \sin \theta (n+1)}{\cos^2 \theta (n+1) + \sin^2 \theta (n+1)} = \frac{\sin \theta (n+1)}{\sin \theta (\cos^2 \theta (n+1) + \sin^2 \theta)} = \frac{\sin \theta (n+1)}{\sin \theta} \Rightarrow$$

$$\Delta_n = \frac{\sin \theta (n+1)}{\sin \theta}$$

$$\Delta_n X = 0$$

$$\frac{\sin \theta (n+1)}{\sin \theta} \sin \theta = 0$$

$$\sin \theta (n+1) = 0$$

$$\theta(n+1) = \frac{\pi}{2} + 2\pi k, k \in \mathbb{Z}$$

$$\theta = \frac{\pi + 4\pi k}{2(n+1)}, k \in \mathbb{Z}$$

$$\text{В итоге получаем, что } \theta = \frac{\cos \frac{\pi + 4\pi k}{2(n+1)} - 1}{\cos \frac{\pi + 4\pi k}{2(n+1)}}$$

$$\text{Ответ: } \theta = \frac{\cos \frac{\pi + 4\pi k}{2(n+1)} - 1}{\cos \frac{\pi + 4\pi k}{2(n+1)}}$$

Задача 1034.

Найти собственные значения матрицы

$$\begin{pmatrix} -1 & 1 & 0 & \dots & 0 & 0 \\ 1 & 0 & 1 & \dots & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & 0 & \dots & 1 & 0 \end{pmatrix}$$

Решение:

$$\begin{pmatrix} -1-\lambda & 1 & 0 & \dots & 0 & 0 \\ 1 & -\lambda & 1 & \dots & 0 & 0 \\ 0 & 1 & -\lambda & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & -\lambda & 1 \\ 0 & 0 & 0 & \dots & 1 & -\lambda \end{pmatrix} = (-1-\lambda) \begin{pmatrix} -\lambda & 1 & \dots & 0 & 0 \\ 1 & -\lambda & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & -\lambda & 1 \\ 0 & 0 & \dots & 1 & -\lambda \end{pmatrix} - \\
\begin{pmatrix} -\lambda & 1 & \dots & 0 & 0 \\ 0 & -\lambda & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & -\lambda & 1 \\ 0 & 0 & \dots & 1 & -\lambda \end{pmatrix} = - \begin{pmatrix} -\lambda & 1 & \dots & 0 & 0 \\ 1 & -\lambda & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & -\lambda & 1 \\ 0 & 0 & \dots & 1 & -\lambda \end{pmatrix} - \begin{pmatrix} -\lambda & 1 & \dots & 0 & 0 \\ 1 & -\lambda & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & -\lambda & 1 \\ 0 & 0 & \dots & 1 & -\lambda \end{pmatrix} \\
\begin{pmatrix} -\lambda & 1 & \dots & 0 & 0 \\ 1 & -\lambda & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & -\lambda & 1 \\ 0 & 0 & \dots & 1 & -\lambda \end{pmatrix} = \Delta_n$$

$$\Delta_n = -\lambda \Delta_{n-1} - \Delta_{n-2}$$

$$G(z) = \frac{c_0 + c_1 z - \alpha c_0 z}{1 - \alpha z - \beta z^2}$$

$$\alpha = -\lambda; \beta = -1$$

$$c_0 = 1; c_1 = -\lambda; c_2 = \lambda^2 - 1$$

$$G(z) = \frac{1}{1 + \lambda z + z^2}$$

$$z_{1|2} = \frac{\lambda \pm \sqrt{\lambda^2 - 4}}{2}$$

$$\text{Пусть } z_{1|2} = \cos \theta \pm i \sin \theta$$

$$\frac{\lambda \pm \sqrt{\lambda^2 - 4}}{2} = \cos \theta \pm i \sin \theta$$

$$\lambda^2 - 4 = 4(\cos 2\theta + i \sin 2\theta) + 4\lambda(\cos \theta + i \sin \theta) + \lambda^2$$

$$-1 = \cos 2\theta + \lambda \cos \theta$$

$$\lambda = \frac{-1 - \cos 2\theta}{\cos \theta}$$

$$\frac{1}{1 + \lambda z + z^2} = \frac{A(z_2 - z) + B(z_1 - z)}{(z_1 - z)(z_2 - z)}$$

$$\begin{cases} A(z_2 - z_1) = 1 \\ B(z_1 - z_2) = 1 \end{cases}; \begin{cases} A = \frac{i}{2 \sin \theta} \\ B = -\frac{i}{2 \sin \theta} \end{cases}$$

$$\Delta_{n-1} = \frac{A}{z_1^n} + \frac{B}{z_2^n} = \frac{i}{2 \sin \theta} \left(\frac{1}{(\cos \theta + i \sin \theta)^n} - \frac{1}{(\cos \theta - i \sin \theta)^n} \right) = \frac{\sin \theta n}{\sin \theta (\cos^2 \theta n + \sin^2 \theta n)} = \frac{\sin \theta n}{\sin \theta}$$

$$\Delta_{n-2} = \frac{\sin \theta}{\sin \theta (n-1)}$$

$$-\frac{\sin \theta n}{\sin \theta} - \frac{\sin \theta}{\sin \theta (n-1)} = 0 \Rightarrow -\sin \theta n = \sin(\theta n - \theta) \Rightarrow \theta = -\pi$$

$$\lambda = \frac{-1 - \cos 2\theta}{\cos \theta} = 2$$

$$\text{Ответ: } 2$$

Задача 1035.

Найти собственные значения матрицы

$$\begin{pmatrix} 0 & x & x & \dots & x \\ y & 0 & x & \dots & x \\ y & y & 0 & \dots & x \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ y & y & y & \dots & 0 \end{pmatrix}$$

Решение:

$$\begin{pmatrix} -\lambda & x & x & \dots & x \\ y & -\lambda & x & \dots & x \\ y & y & -\lambda & \dots & x \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ y & y & y & \dots & -\lambda \end{pmatrix} \xrightarrow{S_{i+1}-S_i, i=\overline{1,n}} \begin{pmatrix} -\lambda & x & x & \dots & x \\ y+\lambda & -\lambda-x & 0 & \dots & 0 \\ 0 & y+\lambda & -\lambda-x & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & -\lambda-x \end{pmatrix}$$

$$-\lambda \begin{pmatrix} -\lambda-x & 0 & \dots & 0 \\ y+\lambda & -\lambda-x & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & -\lambda-x \end{pmatrix} - x \begin{pmatrix} -\lambda-x & 0 & \dots & 0 \\ 0 & -\lambda-x & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & -\lambda-x \end{pmatrix} \dots$$

$$-\lambda(-\lambda-x)^{n-1} - x(y+\lambda)(-\lambda-x)^{n-2} + \dots$$

При четном n

$$\det(A - \lambda E) = -\lambda(-\lambda-x)^{n-1} - x(y+\lambda)(-\lambda-x)^{n-2}$$

$$-\lambda(-\lambda-x)^{n-1} - x(y+\lambda)(-\lambda-x)^{n-2} = 0$$

$$\lambda_1 = \sqrt{xy}$$

$$\lambda_2 = -\sqrt{xy}$$

При нечетном n

$$\det(A - \lambda E) = -\lambda(-\lambda-x)^{n-1}$$

$$-\lambda(-\lambda-x)^{n-1} = 0$$

$$\lambda_1 = 0$$

$$\lambda_2 = -x$$

Ответ:

$$1) \lambda_1 = \sqrt{xy}; \lambda_2 = -\sqrt{xy}$$

$$2) \lambda_1 = 0; \lambda_2 = -x$$