6.1 Линии пересекаются в точке: $P(u=1, v=2)$
$\vec{r_u} = \{\cos v, \sin v, 2u\}$
$\vec{r_v} = \{-u\sin v, u\cos v, 0\}$
$E = \vec{r_u}^2 = \cos v^2 + \sin v^2 + 4u^2 = 1 + 4u^2$
$F = \vec{r_u}\vec{r_v} = -u\sin v\cos v + u\sin v\cos v = 0$
$G = \vec{r_v}^2 = u^2\sin v^2 + u^2\cos v^2 = u^2$
$dv = du$
$\delta v = -\delta u$
$\cos\theta = \frac{(1+4u^2)du\delta u + u^2 dv\delta v}{\sqrt{(1+4u^2)du^2+u^2dv^2}\sqrt{(1+4u^2)\delta u^2+u^2\delta v^2}} = \frac{du\delta u+4u^2du\delta u-u^2du\delta u}{\sqrt{du^2+4u^2du^2+u^2du^2}\sqrt{\delta u^2+4u^2\delta u^2+u^2\delta u^2}} = \frac{du\delta u(3u^2+1)}{\sqrt{(1+5u^2)du^2}\sqrt{(1+5u^2)\delta u^2}} =$
$\frac{1+3u^2}{1+5u^2} = (u=1) = \frac{2}{3}$
Ответ:
$\arccos\frac{2}{3}$
6.2
$I = du^2 + dv^2$
$E = 1$
$F = 0$
$G = 1$
$$\begin{cases} dv = -\dfrac{\alpha}{\beta}du \\ \\ \delta v = -\dfrac{\gamma}{\tau}dv \end{cases}$$
$\cos\theta = \frac{du\delta u+dv\delta v}{\sqrt{du^2+dv^2}\sqrt{\delta u^2+\delta v^2}} = \frac{du\delta u+\frac{\alpha}{\beta}du\frac{\gamma}{\tau}\delta u}{\sqrt{du^2+\frac{\alpha^2}{\beta^2}du^2}\sqrt{\delta u^2+\frac{\gamma^2}{\tau^2}\delta u^2}} = \frac{(1+\frac{\alpha\gamma}{\beta\tau})du\delta u}{\sqrt{\frac{\beta^2+\alpha^2}{\beta^2}}\sqrt{\frac{\tau^2+\gamma^2}{\tau^2}}du\delta u} = \frac{\beta\tau+\alpha\gamma}{\beta\tau\sqrt{\frac{\beta^2+\alpha^2}{\beta^2}}\sqrt{\frac{\tau^2+\gamma^2}{\tau^2}}} = \frac{\beta\tau+\alpha\gamma}{\sqrt{\beta^2+\alpha^2}\sqrt{\tau^2+\gamma^2}} =$
$\frac{(\beta\tau+\alpha\gamma)\sqrt{(\beta^2+\alpha^2)(\tau^2+\gamma^2)}}{(\beta^2+\alpha^2)(\tau^2+\gamma^2)}$
Ответ:
$\arccos\frac{(\beta\tau+\alpha\gamma)\sqrt{(\beta^2+\alpha^2)(\tau^2+\gamma^2)}}{(\beta^2+\alpha^2)(\tau^2+\gamma^2)}$
6.3
$I = 2du^2 - dudv + 4dv^2$
$E = 2$
$F = -\frac{1}{2}$
$G = 4$
$$\begin{cases} dv = 2du \\ \\ \delta v = -2\delta u \end{cases}$$
$\cos\theta = \frac{2du\delta u-\frac{1}{2}(du\delta v+dv\delta u)+4dv\delta v}{\sqrt{2du^2-dudv+4dv^2}\sqrt{2\delta u^2-\delta u\delta v+4\delta v^2}} = \frac{2du\delta u-\frac{1}{2}(-2du\delta u+2du\delta u)-16du\delta u}{\sqrt{2du^2-2du^2+16du^2}\sqrt{2\delta u^2+2\delta u^2+16\delta u^2}} = \frac{-14du\delta u}{4du2\sqrt{5}\delta u} = -\frac{7}{4\sqrt{5}} = -\frac{7\sqrt{5}}{20}$
Ответ:
$\arccos -\frac{7\sqrt{5}}{20}$