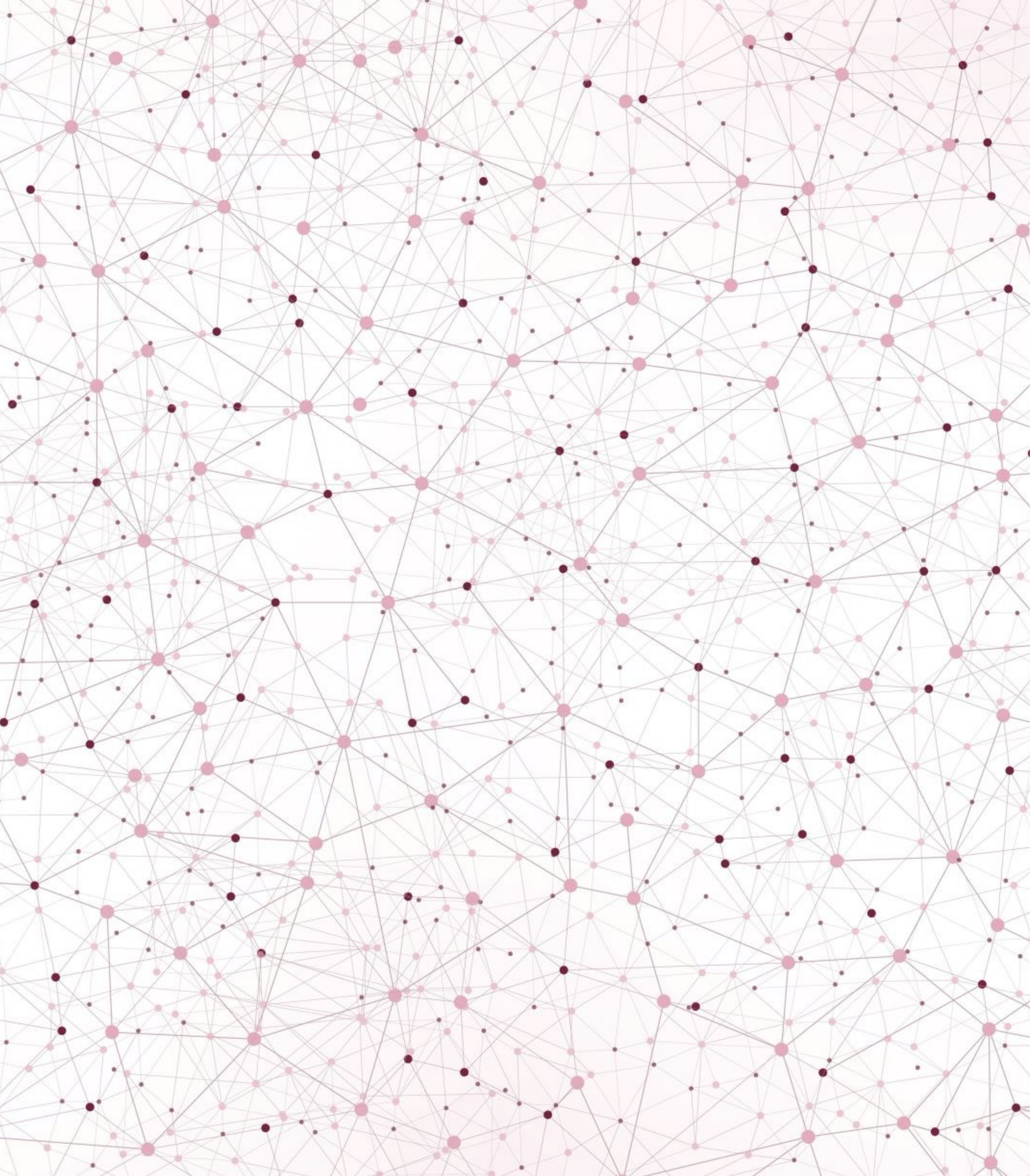

The Elusive Phase Operator: Unravelling Quantum Mysteries

Exploring breakthroughs from Dirac’s dilemma to modern formulations

Presented by: Pranav Srikanth

Introduction to Phase Operators

- Overview of the quantum harmonic oscillator (QHO)
- Importance of phase operators in quantum mechanics
- Brief history and challenges

Dirac's Dilemma

Dirac’s Attempt and Its Issues

- Dirac's definition: $\hat{a} = e^{i\hat{\phi}}\hat{N}^{1/2}$
- Problems: non-unitarity of: $e^{i\hat{\phi}}$
- expectation value issues

Dirac, 1927

$$[cos\hat{\phi}, \hat{N}] = i\ sin\hat{\phi}$$

$$[sin\hat{\phi}, \hat{N}] = -i\ cos\hat{\phi}$$

Louisell's Contribution

Louisell’s Alternative Relations

- Commutation relations as given above.
- Hermitian phase operator challenge

Louisell, 1963

Susskind-Glogower Operators

Susskind and Glogower's Approach

- Cosine and sine operators

$$\hat{C} = \frac{1}{2}[(\hat{E} + \hat{E}^\dagger)]$$

$$\hat{S} = \frac{1}{2i}[(\hat{E} - \hat{E}^\dagger)]$$

- Definitions and commutation relations:

$$\hat{a} = (\hat{N} + 1)^{1/2}\hat{E}$$

$$[\hat{C}, \hat{N}] = i\hat{S}, [\hat{S}, \hat{N}] = -i\hat{C}$$

Susskind and Glogower, 1964

$$\hat{C}|cos\ \varphi_c\rangle = cos\ \varphi_c|cos\ \varphi_c\rangle,$$
$$\varphi_c \in (0, \pi)$$

$$\hat{S}|sin\ \varphi_s\rangle = sin\ \varphi_s|sin\ \varphi_s\rangle,$$
$$\varphi_s \in (-\pi/2, \pi/2)$$

$$[\hat{C}, \hat{S}] = \frac{1}{2\hat{i}}|0\rangle\langle 0|$$

Carruthers and Nieto's Findings

Eigenvalue Spectra and Non-commutation

- Continuous spectra for the cosine and sin operators on [−1,1]
- Non-commuting nature

CARRUTHERS, 1968

Pegg-Barnett Phase Operator

Pegg-Barnett Formalism

- Phase operator in a finite-dimensional subspace
- Definition:

Barnett, 1986

$$\hat{a}_s = e^{i\hat{\phi}_s} \hat{N}_s^{1/2},$$

where $e^{i\hat{\phi}_s}$ is a cycling operator unitary in Ψ_s,

$$e^{i\hat{\phi}_s} = |0\rangle\langle 1| + |1\rangle\langle 2| + \cdots + |s-1\rangle\langle s| + |s\rangle\langle 0|.$$

Desired Properties of Phase Operators

Properties of an Ideal Phase Operator

$$-Hermiticity : \hat{\phi}^{\dagger} = \hat{\phi}$$

$$-TrigonometricIdentity : \cos^2 \hat{\phi} + \sin^2 \hat{\phi} = \hat{I}$$

$$-ProperTimeDependence : \hat{\phi}(t) = \hat{\phi}(0) - \omega t$$

Rhodes, 2015

Recent Advances and Applications

Modern Formulations and Applications

- Recent theoretical developments
- Practical applications in quantum optics and information

Conclusion and Future Directions

SUMMARY OF THE KEY POINTS

OPEN QUESTIONS AND FUTURE RESEARCH DIRECTIONS

THANK YOU!