Search Algorithms and Markov Decision Processes
CS3IS7 Artificial Intelligence - Assignment 1

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1 Introduction

2 Maze Generation

In this project, we generate mazes using the Recursive Backtracking algorithm. In the first instance, we generate what is called a "perfect maze", and then we create shortcuts through it to allow for multiple paths to be drawn between entrance and exit (and therefore the existence of "optimal" and "sub-optimal" paths). See Figure 1 for an illustration of these two types of mazes.

2.1 Perfect Mazes

The definition of a perfect maze is one where there is exactly *one* path between any two points, where there are no loops, and there are no standalone walls (i.e. all walls must be connected to the borders of the maze). Here, we represent the maze as a 2d array, where `1`'s are walls and `0`'s are paths. We use the Recursive Backtracking algorithm to generate these mazes, as follows:

1. The entire maze is initialised with walls (i.e. `1`'s)
2. The algorithm begins at the starting position `(1, 1)`, marking it as a path
3. It checks each direction (up, down, left, right) and, when it observes a wall **2 steps away**, it draws a path between the current point and that new point (i.e. joining them with a path). It then adds it to the stack, to be visited later.
4. When the algorithm visits a cell with no wall neighbours, it "backtracks" and pops the cell off the stack.
5. Once the stack is empty, all accessible cells have been visited.

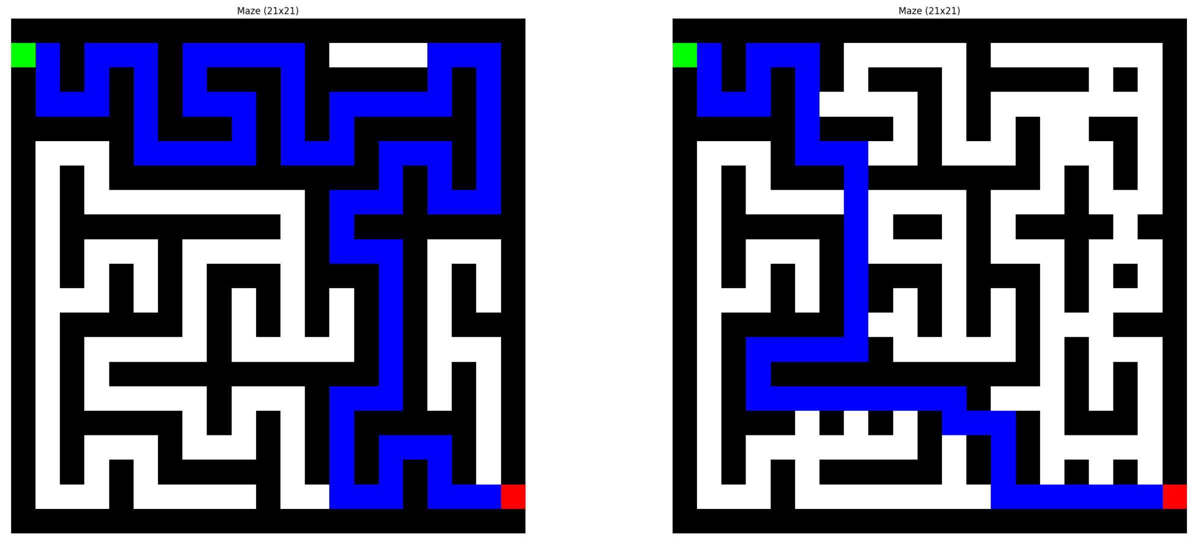
Since the algorithm only connects cells that are *two blocks away*, where one of them is currently a wall, it never creates two paths between any two points. This makes it a perfect maze. An example if this can be seen in Figure 1a.

2.2 Imperfect Mazes

Once we have a perfect maze, we can turn it into an imperfect maze by removing some of the internal walls. Our algorithm identifies these walls, and turns a particular fraction of them into paths according to a provided parameter - `removal_percentage`. This allows us to demonstrate how optimal the various algorithms are. In Figure 1b, you can see how an imperfect maze has multiple ways to get from entrance to exit.

3 Search Algorithms

In this section, we compare the performance of three path finding algorithms, namely, depth-first search (DFS), breadth-first search (BFS), and A*. For the A* algorithm, we use the Manhattan distance from the current cell and the goal as a heuristic to guide the search. This choice is motivated in the second subsection, where we compare the performance of three different heuristics for the A* algorithm. But first, we'll examine the performance of the algorithms.



(a) Perfect Maze

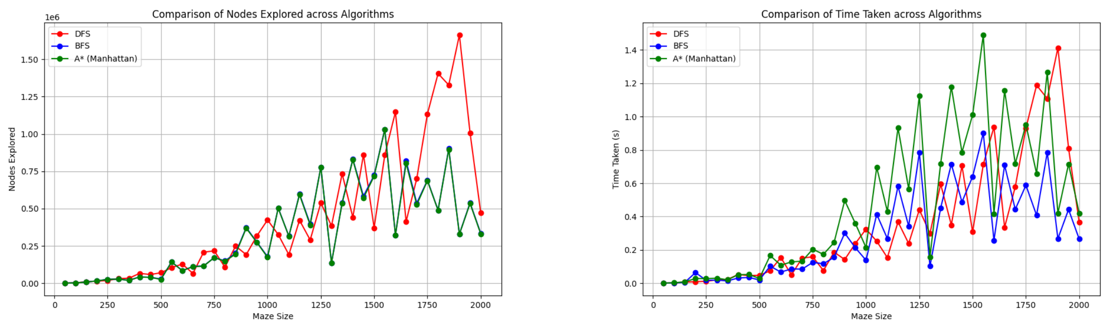
(b) Imperfect Maze

Figure 1: Perfect and Imperfect mazes with size (21x21)

3.1 Comparison Between Search Algorithms

3.1.1 Perfect Maze

We start by comparing the performance of the three algorithms on a perfect maze. We vary the maze sizes between (50x50) and (1000x1000), plotting the time taken for each of the algorithms to find the path as well as the number of nodes explored. The results can be seen in Figure 2.



(a) Nodes Explored

(b) Time Taken

Figure 2: Performance Comparison of DFS, BFS, and A* for Different Perfect Maze Sizes

In Figure 2a, we see that all algorithms explore a similar number of nodes. While we might expect DFS to explore fewer nodes, in these types of perfect mazes, the algorithms don't have much choice. We can see that for the last few very large mazes, DFS is more consistently "unlucky" in how it explores the maze. DFS and A* will not explore any paths longer than the path to the goal, where as DFS may get stuck exploring longer paths with dead ends before it finds the correct path to the goal.

It's clear from Figure 2b that the time taken by each algorithm is a different story. The overall time taken by BFS and DFS almost perfectly match the number of nodes explored. A*, on the other hand, suffers from the additional overhead of calculating the heuristic. For a perfect maze, this heurstic is not particularly helpful and does not help A* to find the path any faster. There is not a strong reason to believe that, at any junction in the maze, the correct path is the one that heads in the direction of the goal. Additionally, since there is only one path to the goal, all algorithms are optimal.

The behaviour of the algorithms is characterised by Figure 3. We can see that DFS manages to avoid some of the dead ends explored by the other algorihms at first, but then takes a catestrophic wrong turn near the exit, causing it to explore many more nodes than the others. A* explores marginally fewer

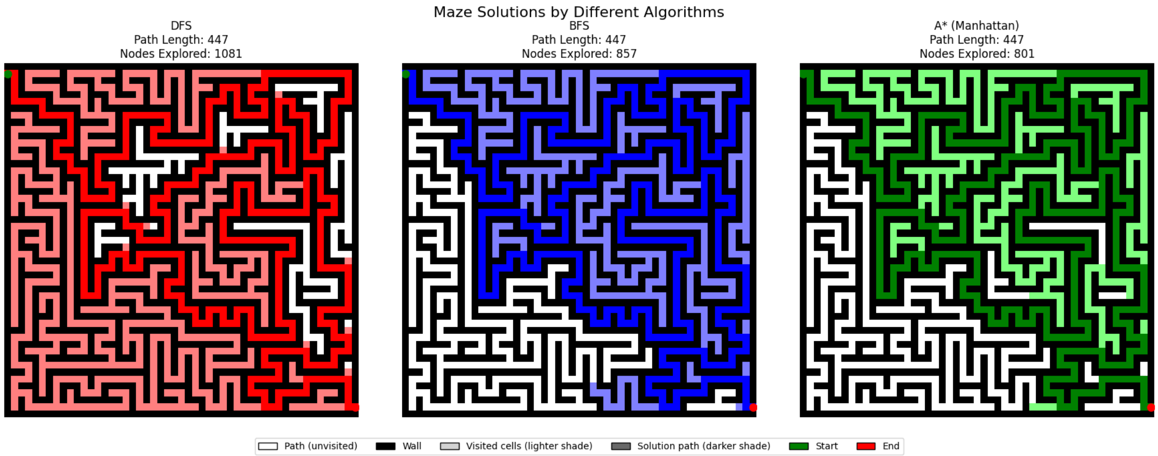


Figure 3: Solutions from DFS, BFS, and A* for a Perfect Maze

nodes than BFS, as it avoids going down paths that head away from the goal towards the end.

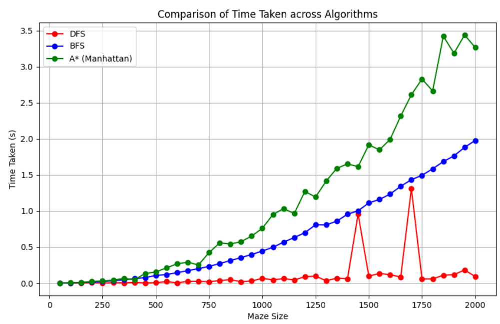
In Table 1, we observe that A* takes roughly 1.75 times longer than DFS and 1.62 times longer than BFS to find the solution. Additionally, it seems to explore marginally more nodes than BFS, suggesting that the heuristic is holding the algorithm back on this type of maze.

Metric	**A* vs DFS**	**A* vs BFS**
Nodes Explored	1.0240	0.9816
Time Taken	1.7557	1.6262

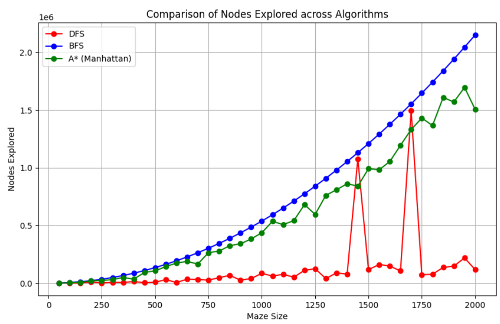
Table 1: Average relative performance of A* compared to other algorithms (perfect maze)

3.1.2 Imperfect Maze

In order to observe the differences between the algorithms in terms of optimality (finding the shortest path) and efficiency (in terms of nodes explored), we test them on an *imperfect* maze. We remove 10% of the walls from a perfect maze, and then test the algorithms on the resulting maze. The results can seen in Figure 4.



(a) Time Taken



(b) Nodes Explored

Figure 4: Performance Comparison of DFS, BFS, and A* for Different Imperfect Maze Sizes

Looking at Figure 4a, we see that A* takes the longest out of all of the algorithms, nudging ahead of BFS. This is now more in line with the expected complexity of these algorithms - BFS shows a broadly linear time complexity ($O(n)$) while A* is more like $O(n \log n)$. The time taken for DFS to find a solution is now much quicker, as it benefits from finding *any* path to the goal.

If we now consider the number of nodes explored, as well as the optimality of the path found, the results are somewhat flipped on their head. First, considering path length, BFS and A* are both *optimal*, and will always find the shortest path to the goal. This is not true for DFS. On average, across all maze sizes tested, the path length found by DFS was on average **12.05 times longer** than the optimal paths found by BFS and A*.

If we turn to the number of nodes expored, DFS naturally explores the least. However, among the optimal algorithms, A* explores fewer nodes than BFS, guided by the heuristic which is now more helpful given the altered structure of the maze. These results are summarised in Table 2.

Metric	**A* vs DFS**	**A* vs BFS**
Nodes Explored	9.2184	0.7768
Time Taken	21.0405	1.6368

Table 2: Average relative performance of A* compared to other algorithms (imperfect maze)

3.2 Comparison of A* Heuristics

4 Markov Decision Processes

4.1 Implementation Decisions

4.2 Policy Iteration vs. Value Iteration