MAXFLOW & MINCUT

DormLads Presents

WHAT IS NETWORK FLOW ?

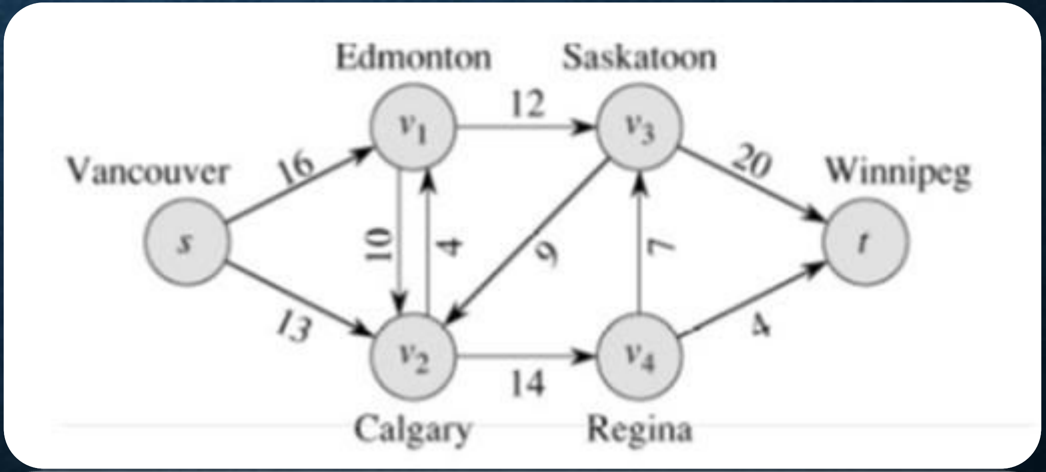
- Flow network is a directed graph G=(V,E) such that eachedge has a non-negative capacity $c(u,v) \geq 0$.
- Two distinguished vertices exist in G namely :

- Source (denoted by s) : In-degree of this vertex is 0.
- Sink (denoted by t) : Out-degree of this vertex is 0.

- Flow in a network is an integer-valued function f defined On the edges of G satisfying $0 \leq f(u,v) \leq c(u,v)$, for every Edge (u,v) in E.

WHAT IS NETWORK FLOW ?

- Each edge (u,v) has a non-negative capacity c(u,v).
- If (u,v) is not in E assume c(u,v)=0.
- We have source s and sink t.
- Assume that every vertex v in V is on some path from s to t.
- Following is an illustration of a network flow:
- c(s,v1) = 16
- c(v1,s) = 0
- c(v2,s) = 0



THE VALUE OF A FLOW.

- **The value of a flow is given by :**

$$|f| = \sum_{v \in V} f(s,v) = \sum_{v \in V} f(v,t)$$

- The flow into the node is same as flow going out from the node and thus the flow is conserved. Also the total amount of flow from sources = total amount of flow into the sink t.

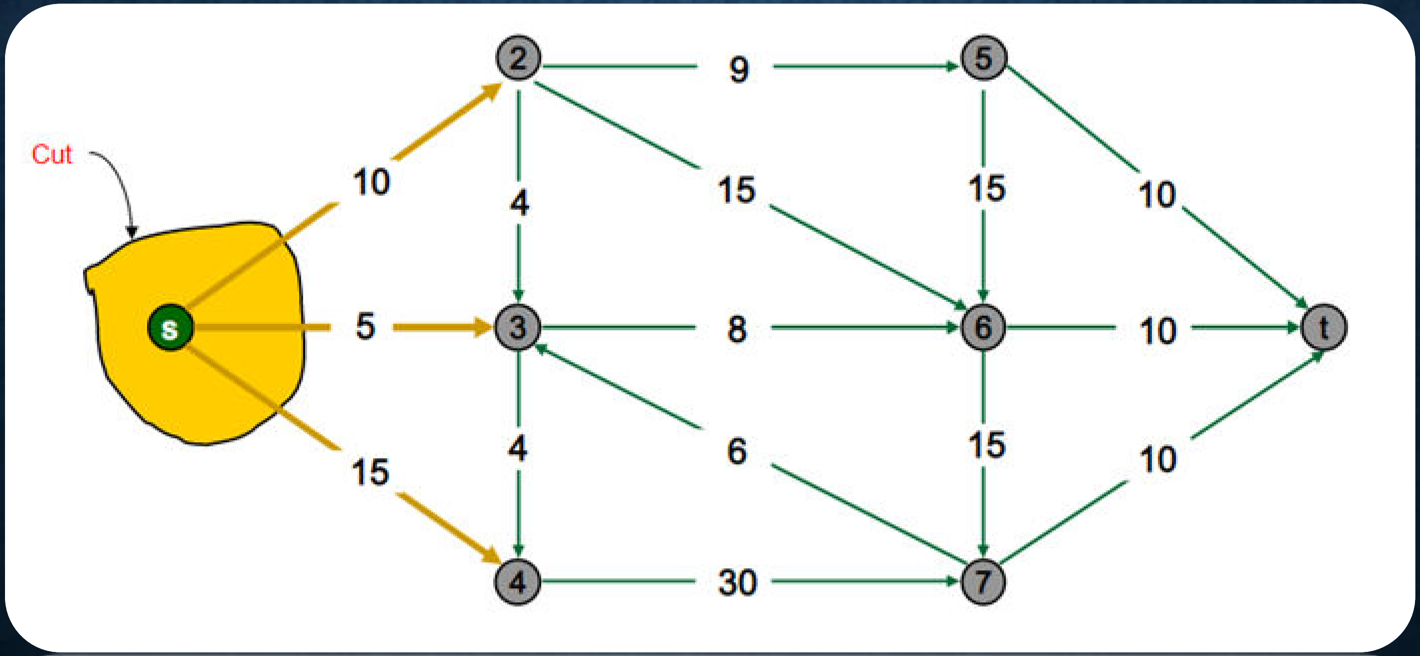
THE MAXIMUM FLOW PROBLEM

- In simple terms maximizethe s to t flow, while ensuringthat the flow is feasible.
- Given a Graph G (V,E) such that:
  - $x_{i,j}$ = flow on edge (i,j)
  - $u_{i,j}$= capacity of edge (i,j)
  - s = source node
  - t = sink node

- Maximize v
- Subject To $\Sigma_j x_{ij} - \Sigma_j x_{ji} = 0$ for each $i \neq s,t$
- $\Sigma_j x_{sj} = v$
- $0 \leq x_{ij} \leq u_{ij}$ for all $(i,j) \in E$.

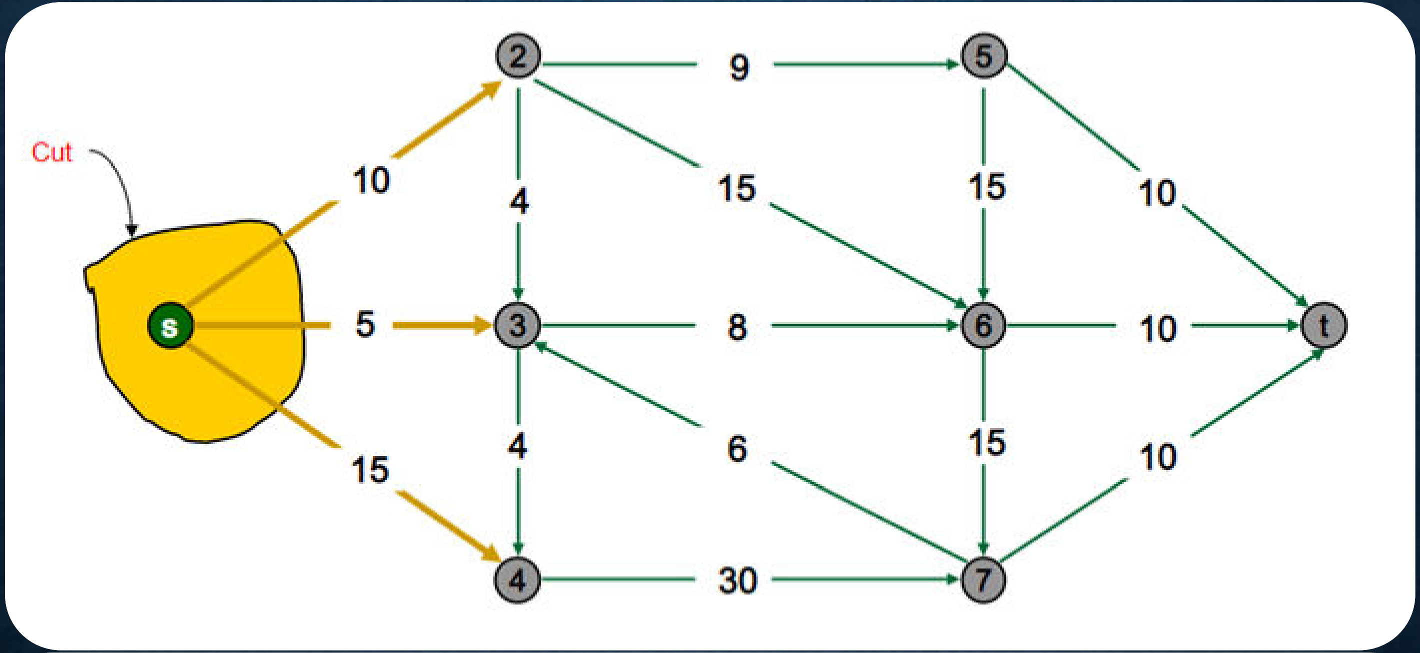
CUTS OF FLOW NETWORKS

- A Cut in a network is a partition of V into S and T (T=V-S) such thats (source) is in S and t (target) is in T.



CAPACITY OF CUT (S,T)

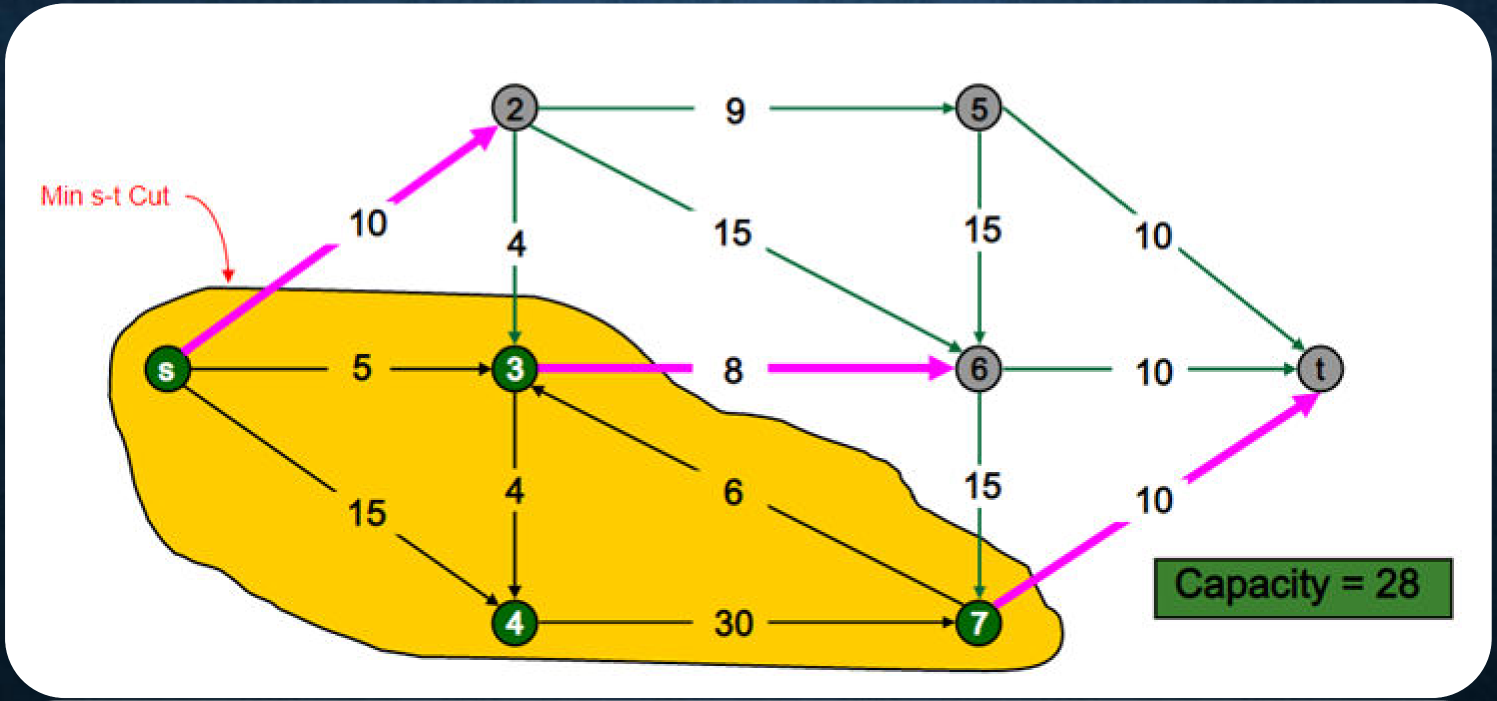
$$c(S,T) = \sum_{u \in S, v \in T} c(u,v)$$



Capacity is 30

MIN CUT

- Min cut (Also called as a Min s-t cut) is a cut of minimum capacity.



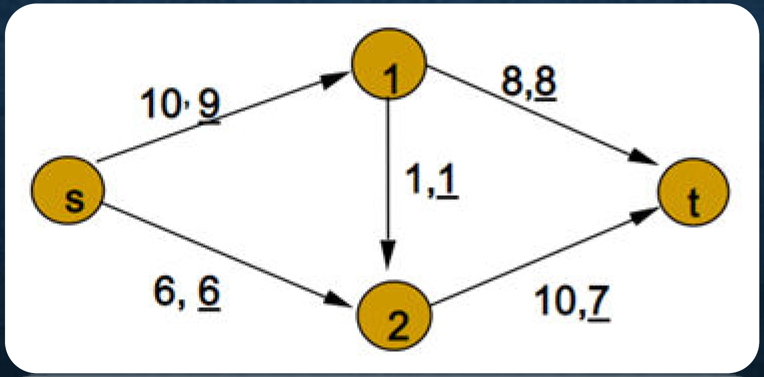
MAX FLOW & MIN CUT THEOREM

- The main theorem links the maximum flow through a network with the minimum cut of the network.
- The maximum value of an s-t flow is equal to the minimum capacity over all s-t cuts.
- The theorem states that the maximum flow through any network from a given source to a given sink is exactly the sum of the edge weights that, if removed, would totally disconnect the source from the sink.

THE FORD-FULKERSON METHOD

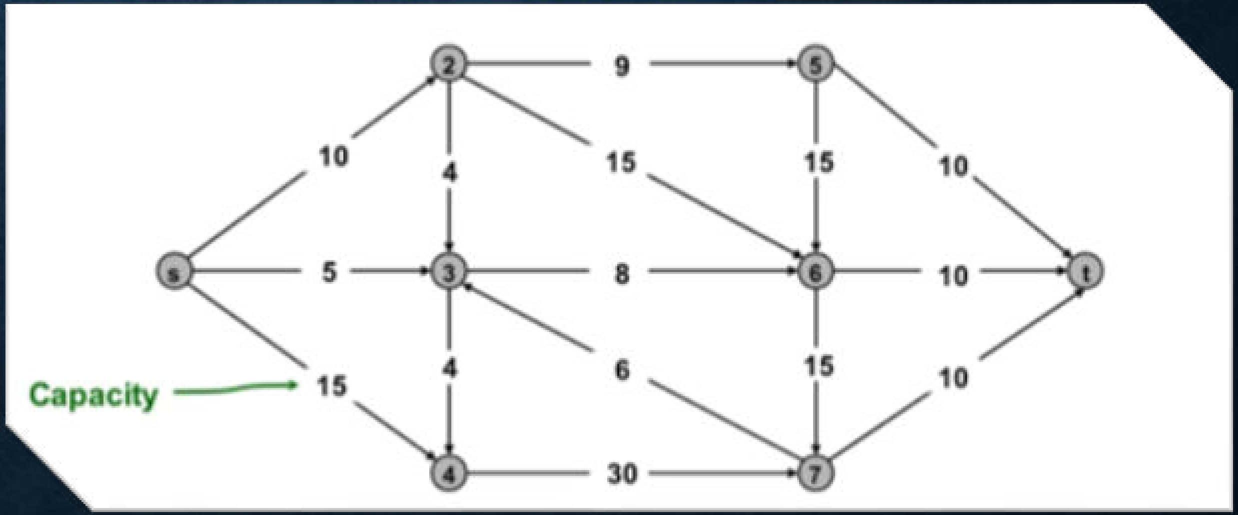
- Try to improve the flow, until we reach the maximum value of the flow
- The residual capacity of the network with a flow f is given by:
- The residual capacity (rc) of an edge (i,j) equals c(i,j) – f(i,j) when (i,j)is a forward edge, and equals f(i,j) when (i,j) is a backward edge.Moreover the residual capacity of an edge is always non-negative.

$$c_f(u,v) = c(u,v) - f(u,v)$$



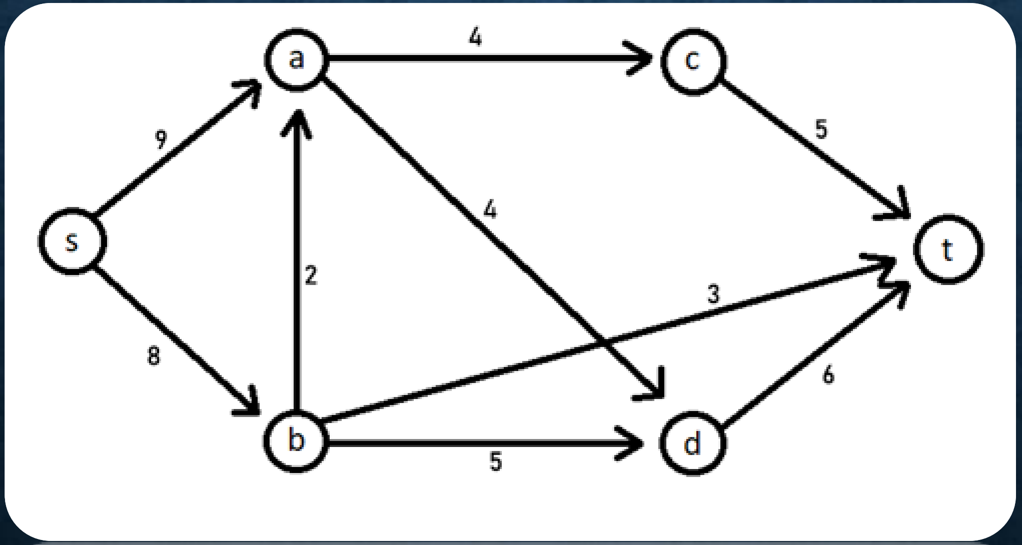
MAX FLOW NETWORK

- Max Flow Network: G = (V, E, s, t, u).
- (V, E) = Directed graph.
- Two distinguished nodes: s is called source, t called as sink.
- u(e) = Capacity of arc e.

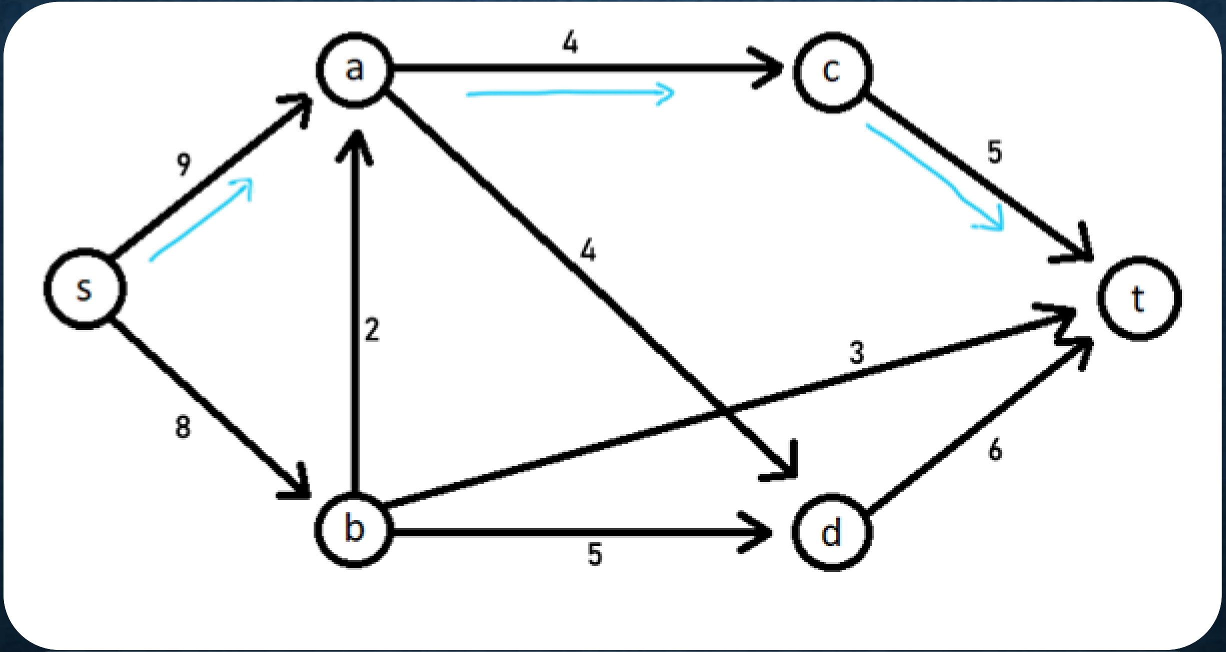


PROBLEM

- Find Max Flow in the transport network and determine the corresponding Min Cut.

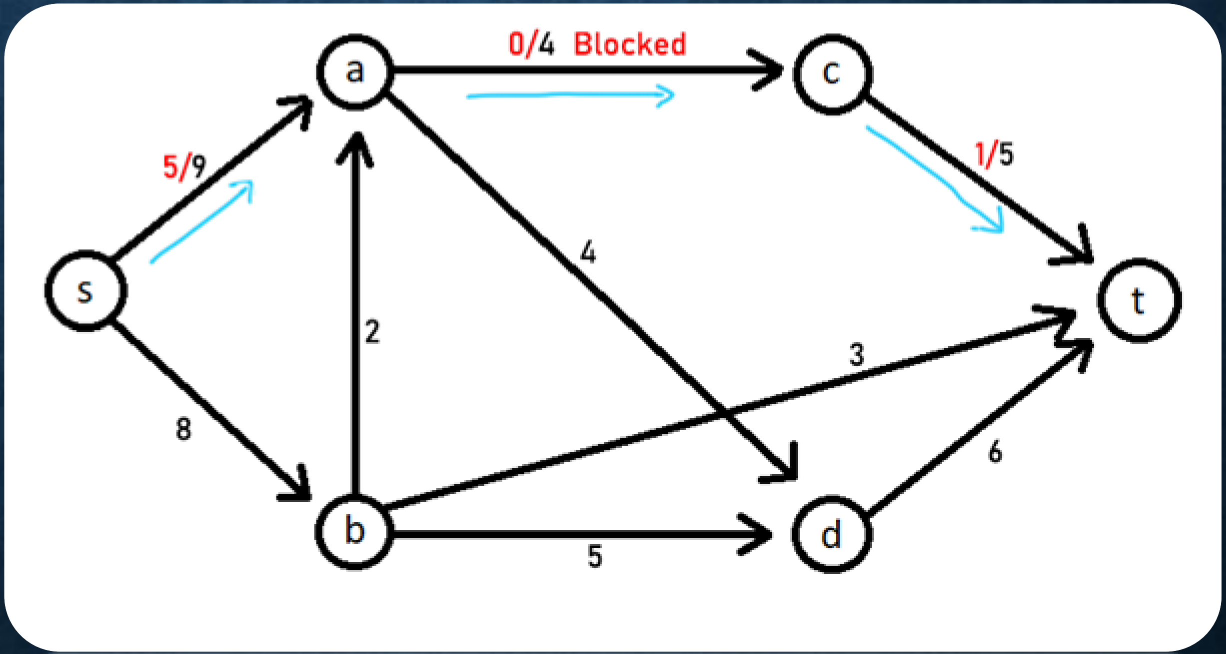


STEP ONE: CHOOSE YOUR PATH



STEP TWO: SUBTRACT THE MAX FLOW

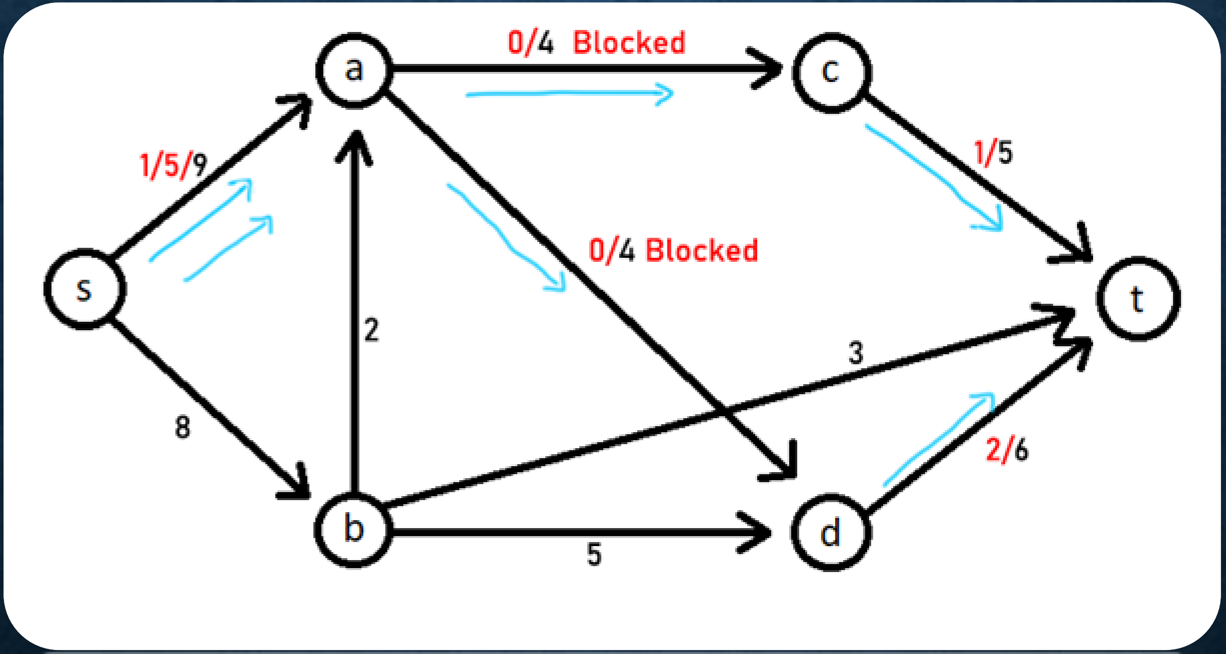
- Red colored numbers are new values of the edges.
- If an edge equals to 0, that edge cannot be used again.



STEP THREE: WRITE DOWN YOUR PATH

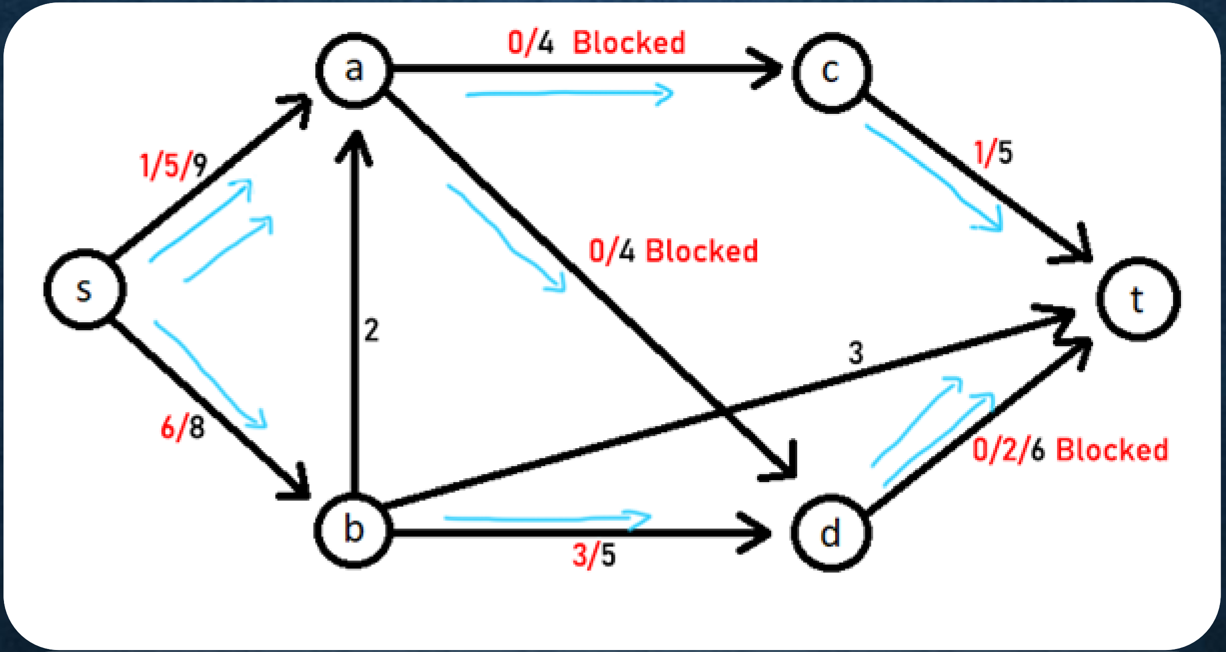
- $s \rightarrow a \rightarrow c \rightarrow t = 4$

FOLLOW SAME STEPS WITH ANOTHER PATH



WRITE DOWN YOUR PATH

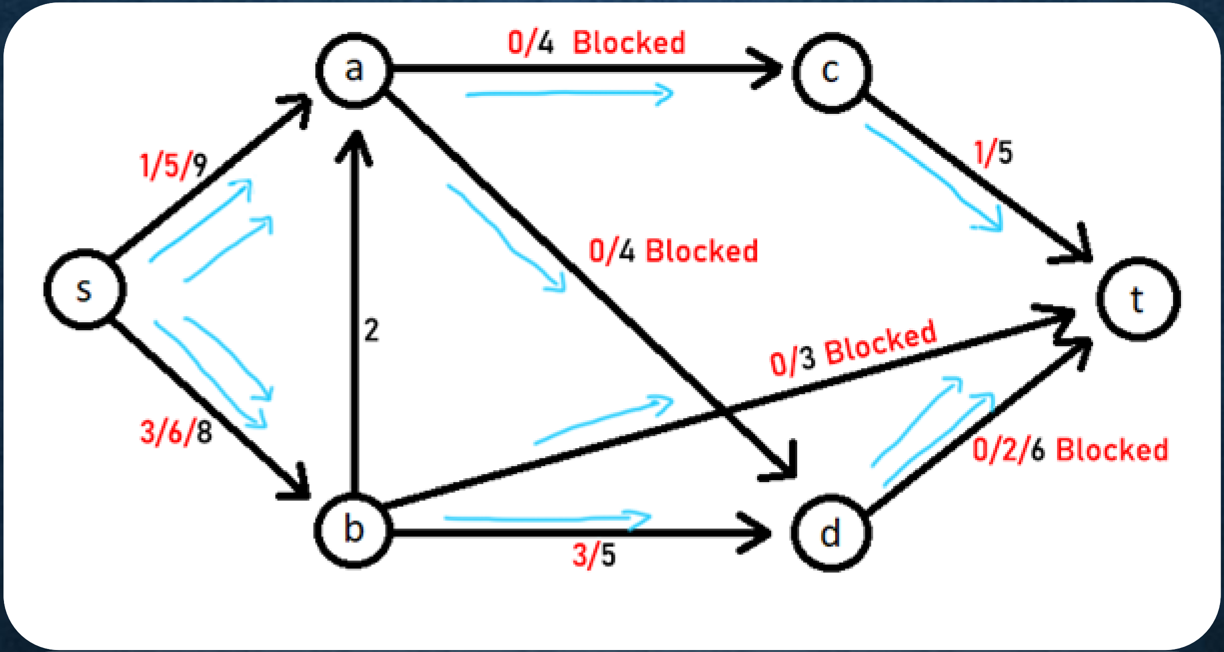
- $s \rightarrow a \rightarrow c \rightarrow t = 4$
- $s \rightarrow a \rightarrow d \rightarrow t = 4$



a
c
s
t
b
d
0/4 Blocked
1/5/9
1/5
0/4 Blocked
2
3
6/8
0/2/6 Blocked
3/5

WRITE DOWN YOUR PATH

- $s \rightarrow a \rightarrow c \rightarrow t = 4$
- $s \rightarrow a \rightarrow d \rightarrow t = 4$
- $s \rightarrow b \rightarrow d \rightarrow t = 2$

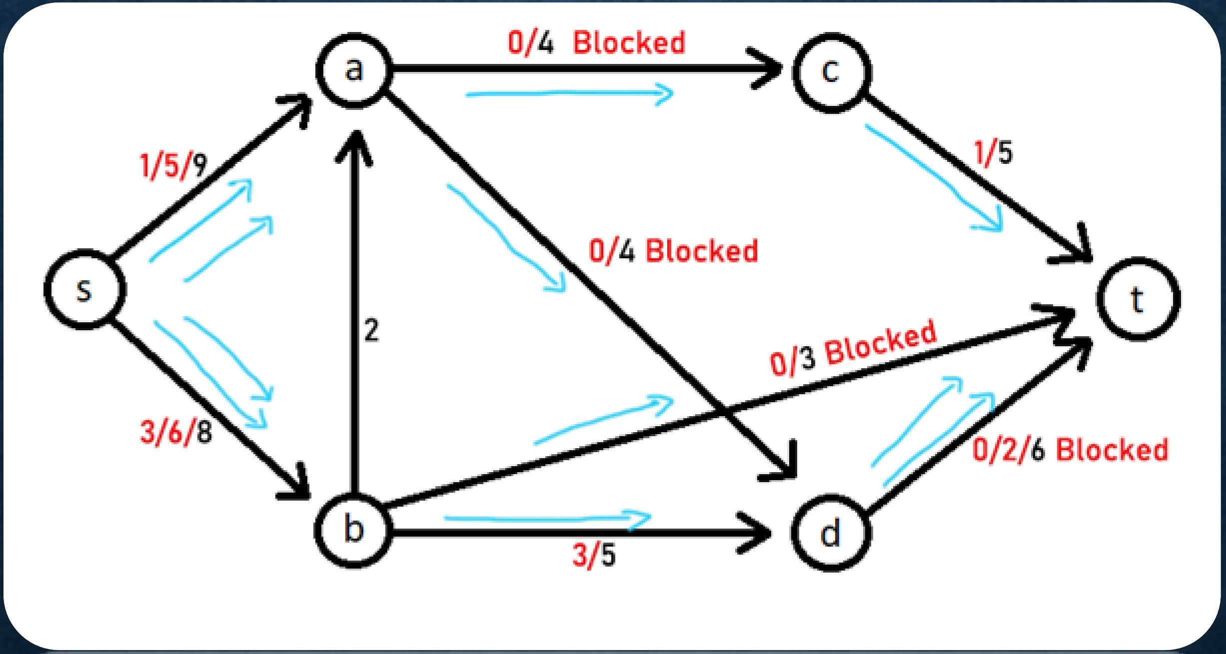


a
c
s
t
b
d
0/4 Blocked
1/5/9
1/5
0/4 Blocked
2
0/3 Blocked
3/6/8
0/2/6 Blocked
3/5

WRITE DOWN YOUR PATH

- $s \rightarrow a \rightarrow c \rightarrow t = 4$
- $s \rightarrow a \rightarrow d \rightarrow t = 4$
- $s \rightarrow b \rightarrow d \rightarrow t = 2$
- $s \rightarrow b \rightarrow t = 3$

- As you can see, all the paths are closed.



SUM THEM ALL TO FIND MAX FLOW

- $s \rightarrow a \rightarrow c \rightarrow t = 4$
- $s \rightarrow a \rightarrow d \rightarrow t = 4$
- $s \rightarrow b \rightarrow d \rightarrow t = 2$
- $s \rightarrow b \rightarrow t = 3$

Maximum Flow =13

FIND MIN CUT

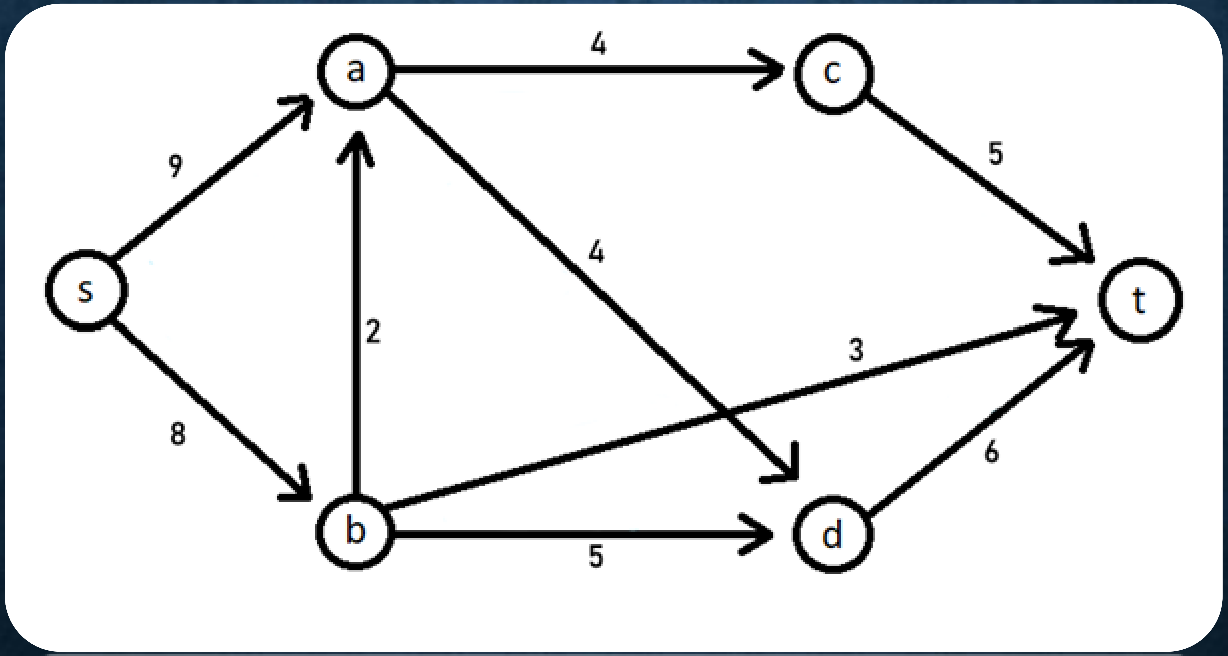
- Cut:

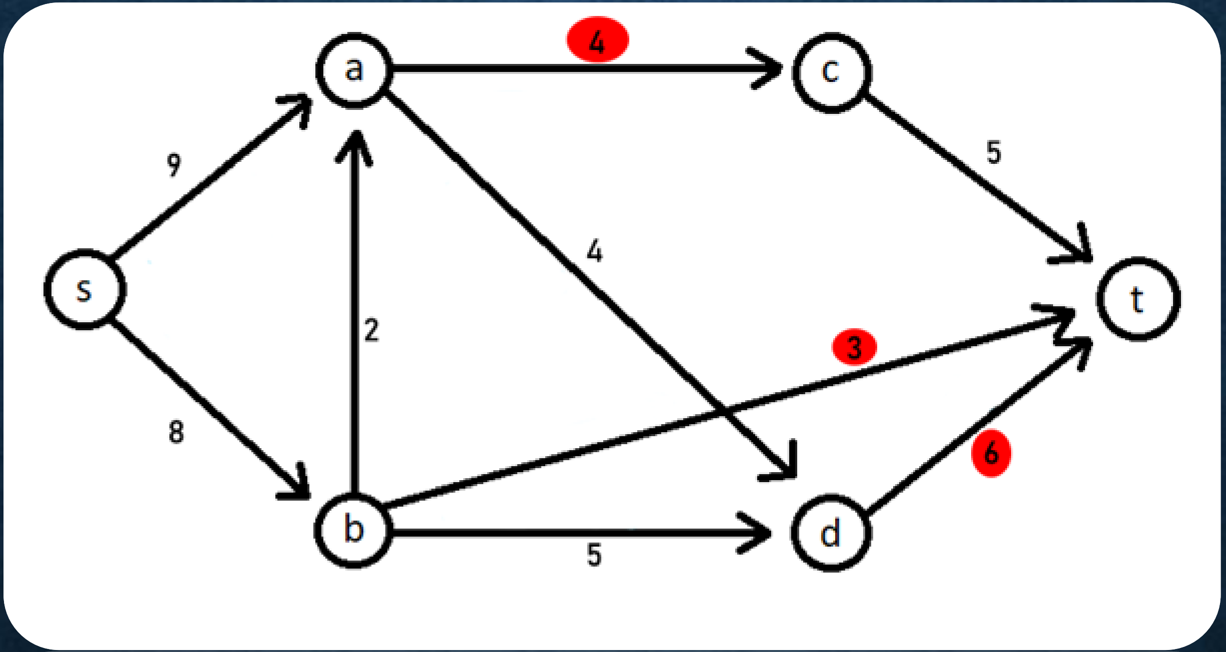
Set of edges whose removal divides network into two part; X and Y where

$s \in X$ and $t \in Y$

The maximum value of an s-t flow is equal to the minimum capacity over all s-t cuts.

Here we try to find edges that weigh 13. Because our Max Flow is 13.




a
c
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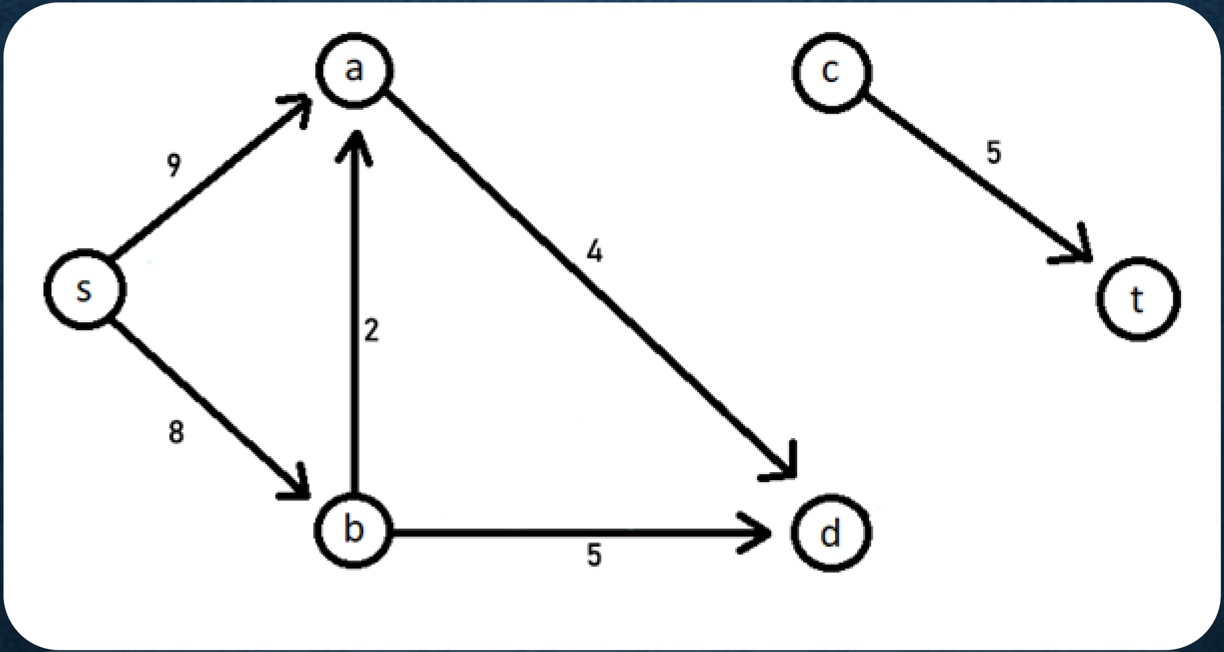
REMOVE THE SELECTED EDGES



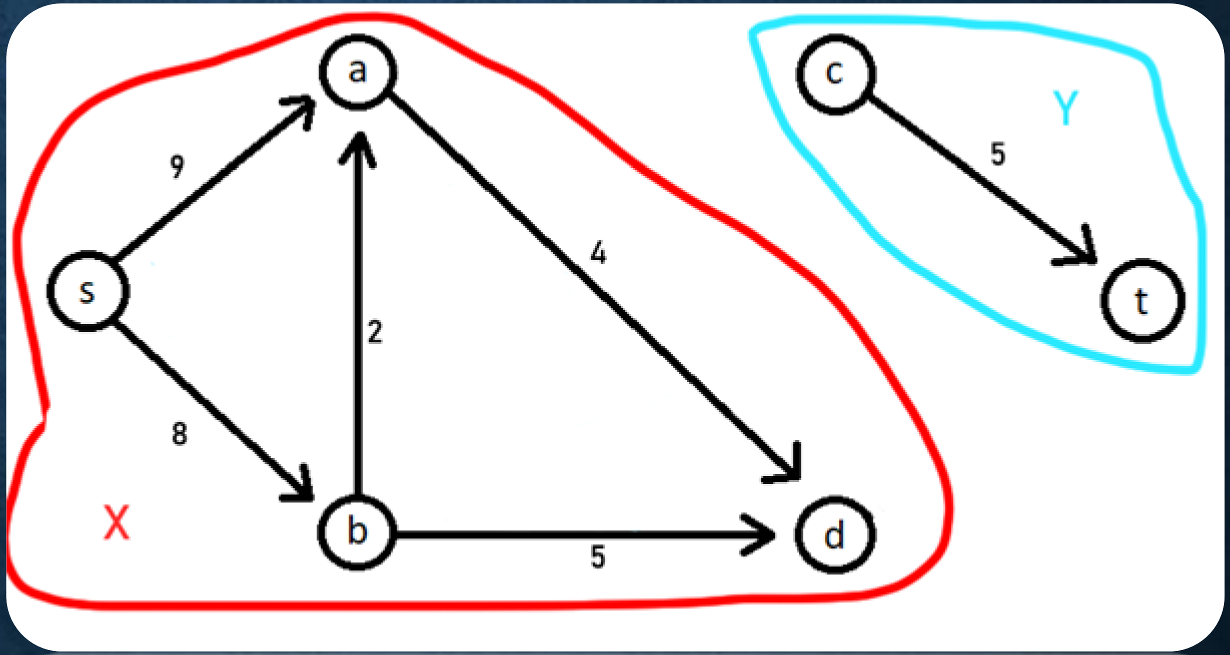
- Removed Edges

$a \rightarrow c = 4$

$b \rightarrow t = 3$

$d \rightarrow t = 6$

Total = 13



Min Cut - Capacity of Cut = Max Flow