MAXFLOW & MINCUT

DormLads Presents

MAXFLOW & MINCUT

- Two very rich algorithmic problems

- The max-flow min-cut theorem is a special case of the duality theorem for linear programs

MAIN THEOREM

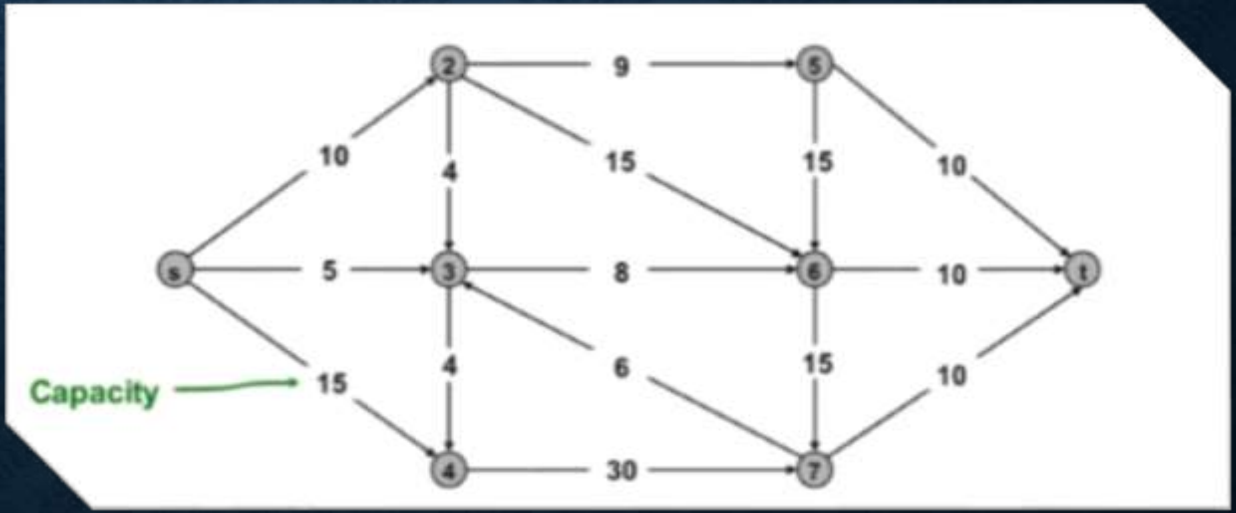
- The main theorem links the maximum flow through a network with the minimum cut of the network.

- The maximum value of an s-t flow is equal to the minimum capacity over all s-t cuts.

- The theorem states that the maximum flow through any network from a given source to a given sink is exactly the sum of the edge weights that, if removed, would totally disconnect the source from the sink.

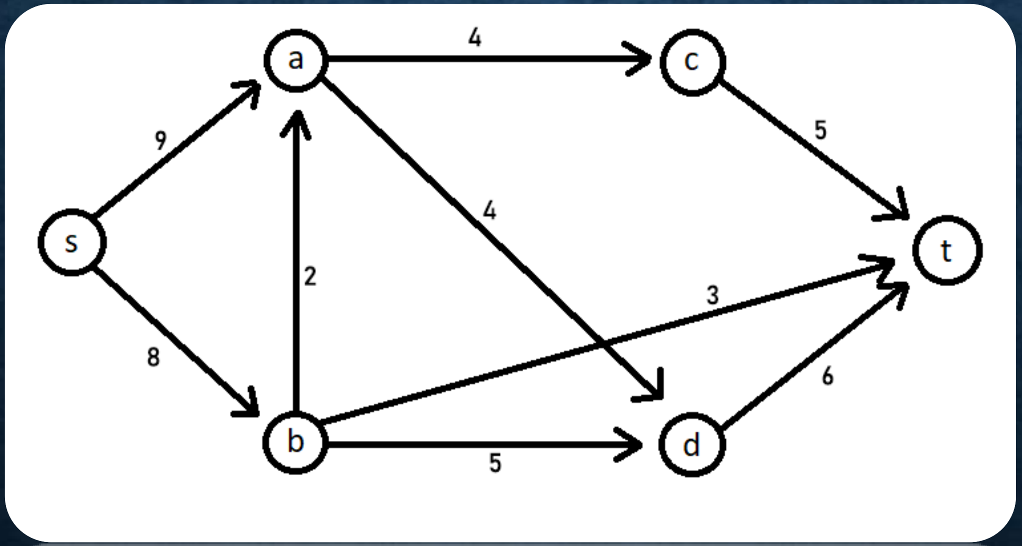
MAX FLOW NETWORK

- Max Flow Network: G = (V, E, s, t, u).
- (V, E) = Directed graph.
- Two distinguished nodes: s is called source, t called as sink.
- u(e) = Capacity of arc e.

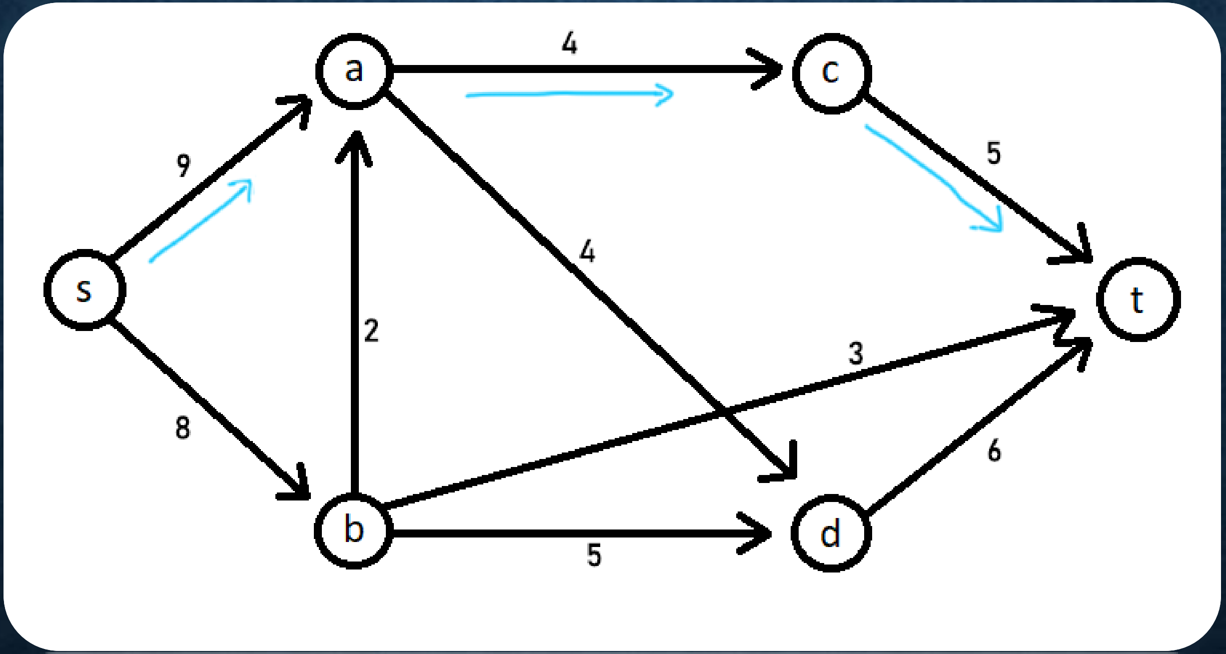


PROBLEM

- Find Max Flow in the transport network and determine the corresponding Min Cut.

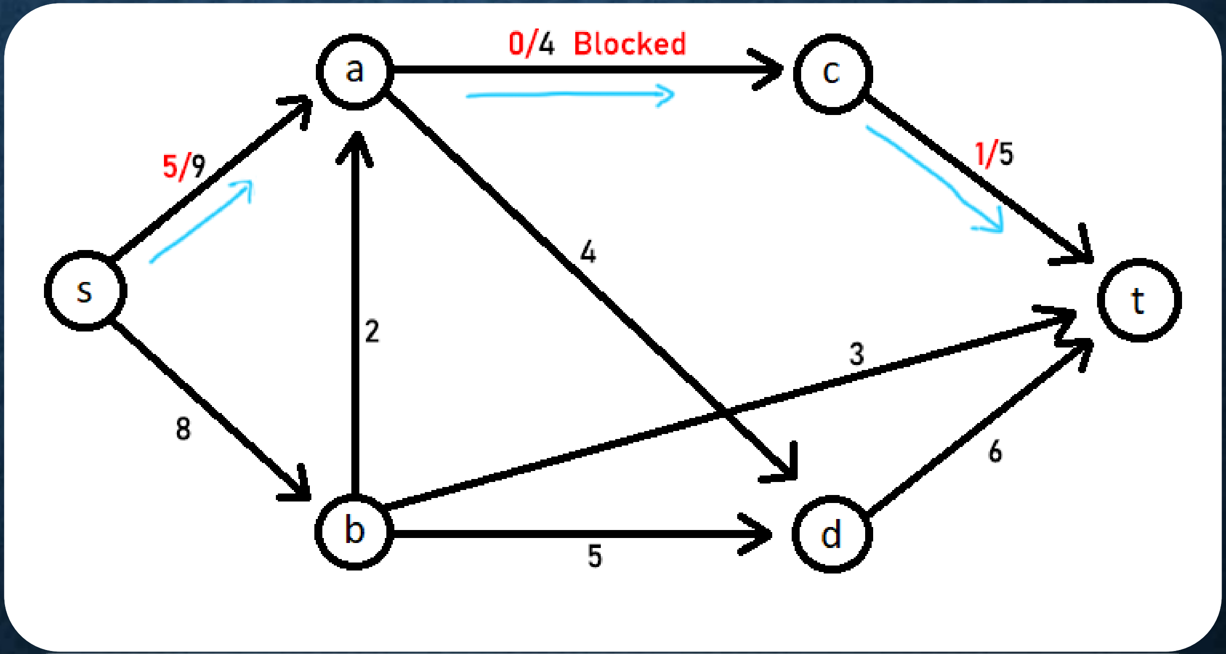


STEP ONE: CHOOSE YOUR PATH



STEP TWO: SUBTRACT THE MAX FLOW

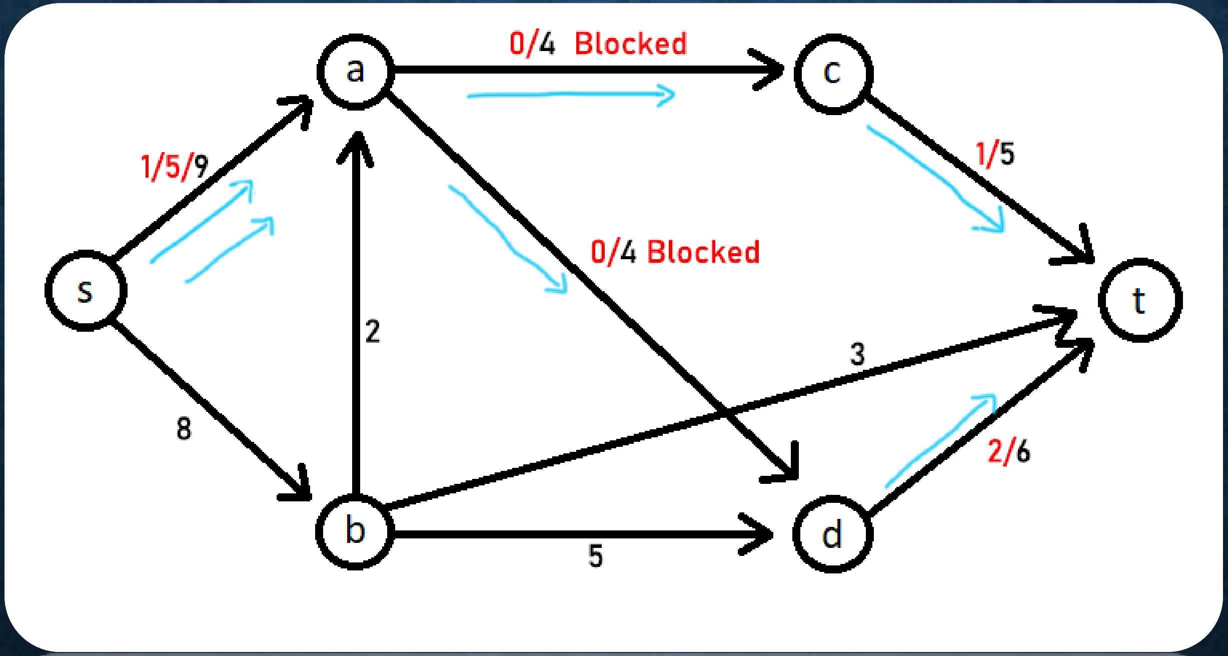
- Red colored numbers are new values of the edges.
- If an edge equals to 0, that edge cannot be used again.



STEP THREE: WRITE YOUR PATH DOWN

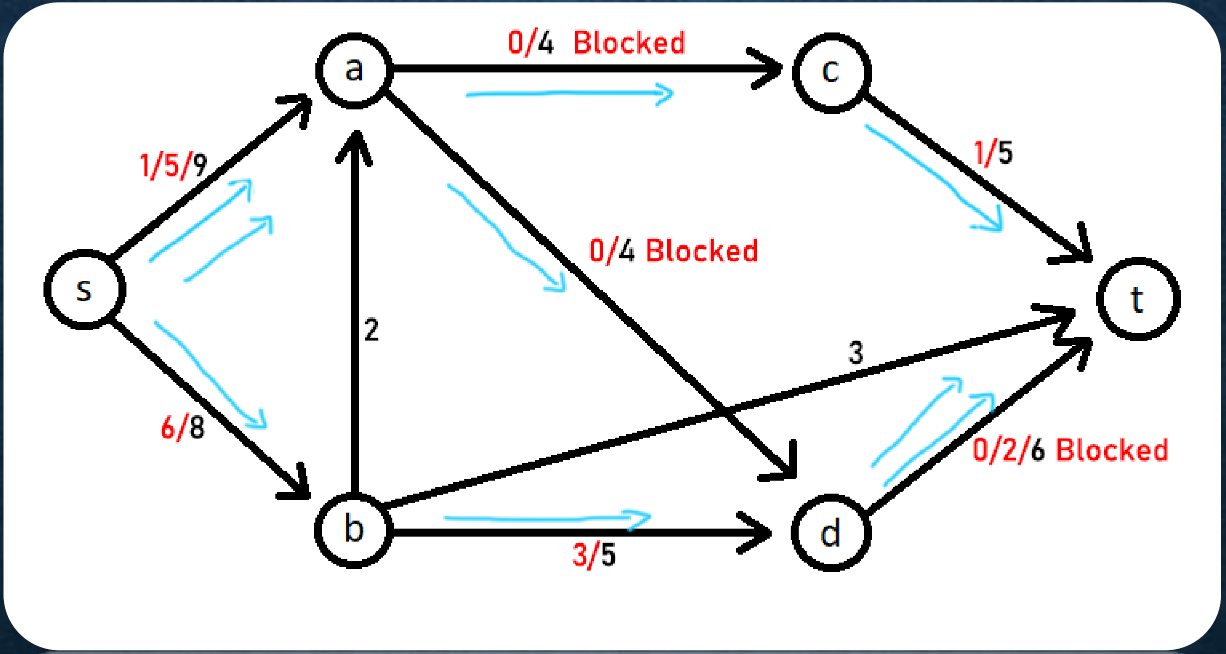
- $s \rightarrow a \rightarrow c \rightarrow t = 4$

FOLLOW SAME STEPS WITH ANOTHER PATH



WRITE YOUR PATH DOWN

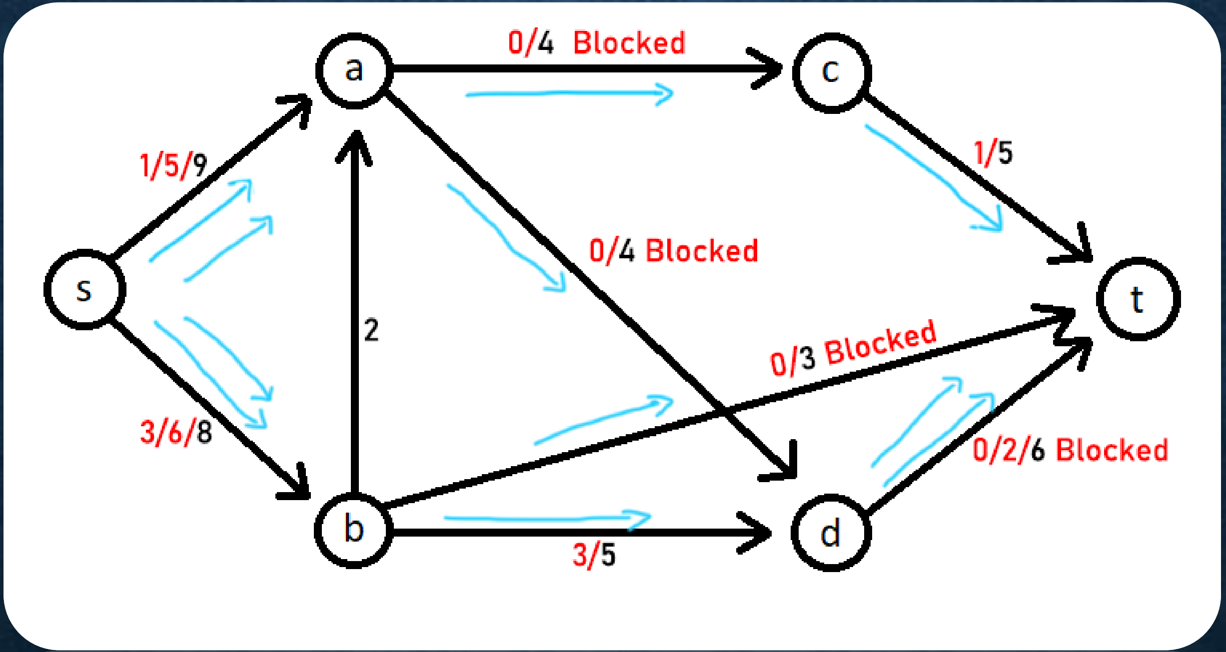
- $s \rightarrow a \rightarrow c \rightarrow t = 4$
- $s \rightarrow a \rightarrow d \rightarrow t = 4$



a
c
s
t
b
d
0/4 Blocked
1/5/9
0/4 Blocked
1/5
2
3
6/8
0/2/6 Blocked
3/5

WRITE YOUR PATH DOWN

- $s \rightarrow a \rightarrow c \rightarrow t = 4$
- $s \rightarrow a \rightarrow d \rightarrow t = 4$
- $s \rightarrow b \rightarrow d \rightarrow t = 2$

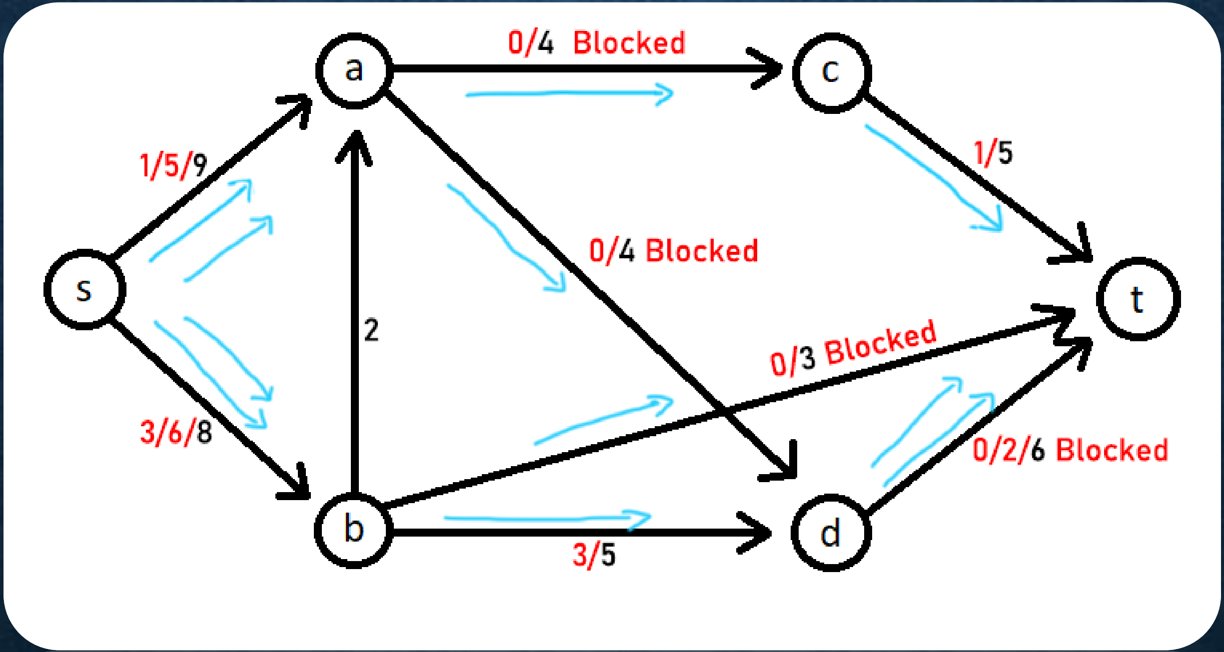


s
a
b
c
d
t
1/5/9
3/6/8
2
0/4 Blocked
0/4 Blocked
1/5
0/3 Blocked
3/5
0/2/6 Blocked

WRITE YOUR PATH DOWN

- $s \rightarrow a \rightarrow c \rightarrow t = 4$
- $s \rightarrow a \rightarrow d \rightarrow t = 4$
- $s \rightarrow b \rightarrow d \rightarrow t = 2$
- $s \rightarrow b \rightarrow t = 3$

- As you can see, all the paths are closed.



SUM THEM ALL TO FIND MAX FLOW

- $s \rightarrow a \rightarrow c \rightarrow t = 4$
- $s \rightarrow a \rightarrow d \rightarrow t = 4$
- $s \rightarrow b \rightarrow d \rightarrow t = 2$
- $s \rightarrow b \rightarrow t = 3$

Maximum Flow =13

FIND MIN CUT

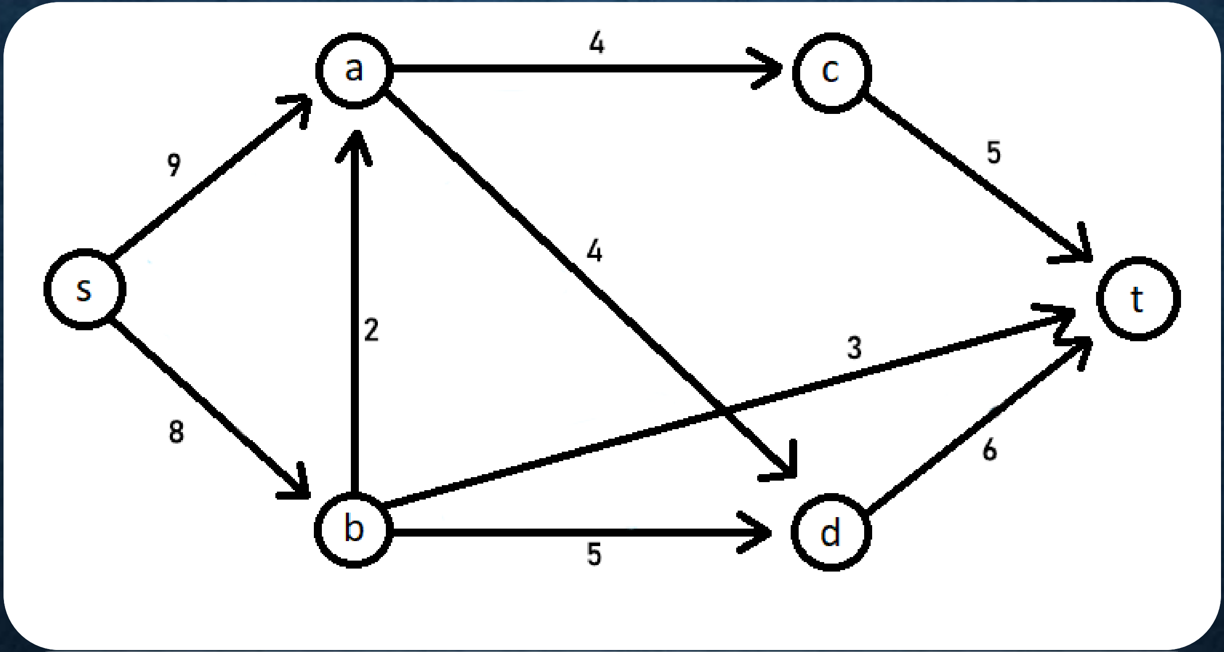
- Cut:

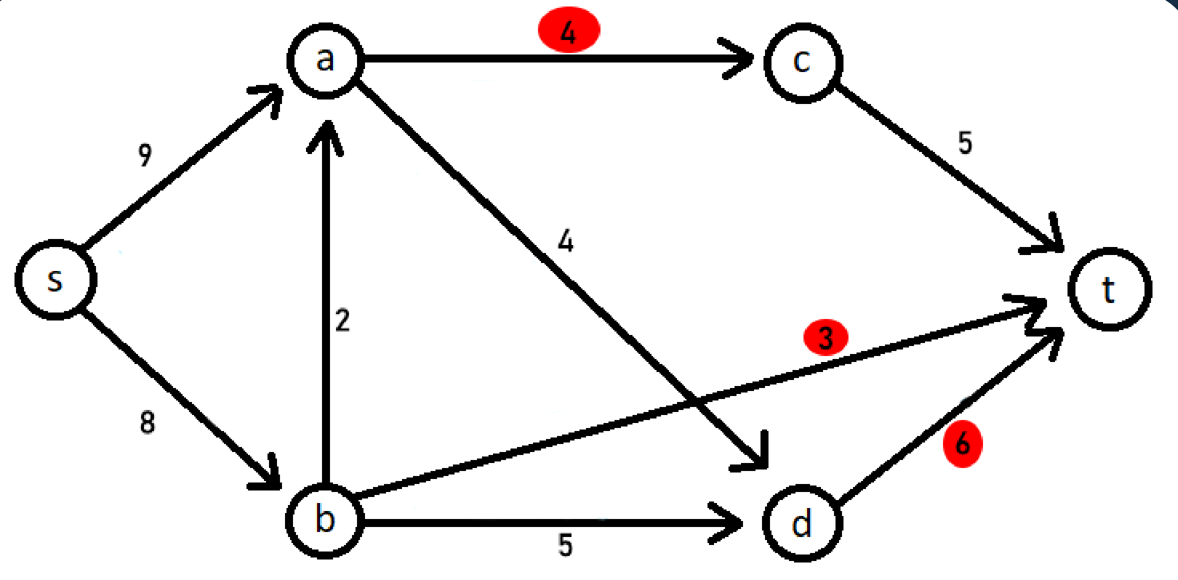
Set of edges whose removal divides network into two part; X and Y where

$s \in X$ and $t \in Y$

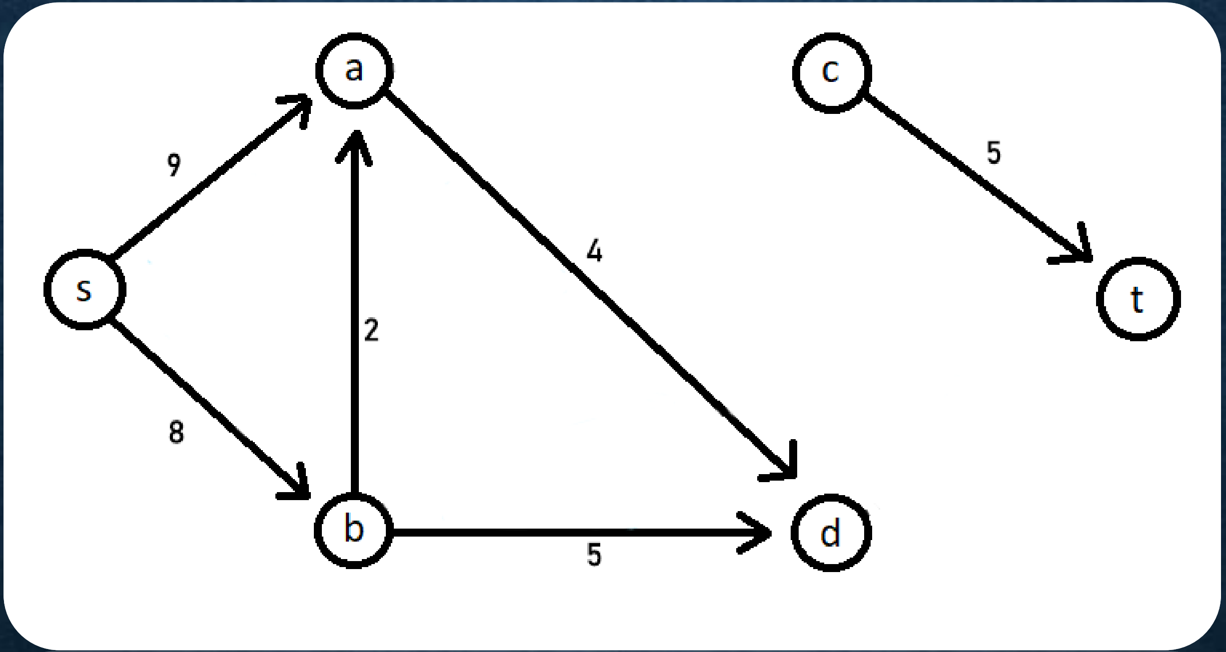
The maximum value of an s-t flow is equal to the minimum capacity over all s-t cuts.

Here we try to find edges that weigh 13. Because our Max Flow is 13.




a
4
c
9
5
4
s
2
t
3
8
6
b
d
5

REMOVE THE SELECTED EDGES



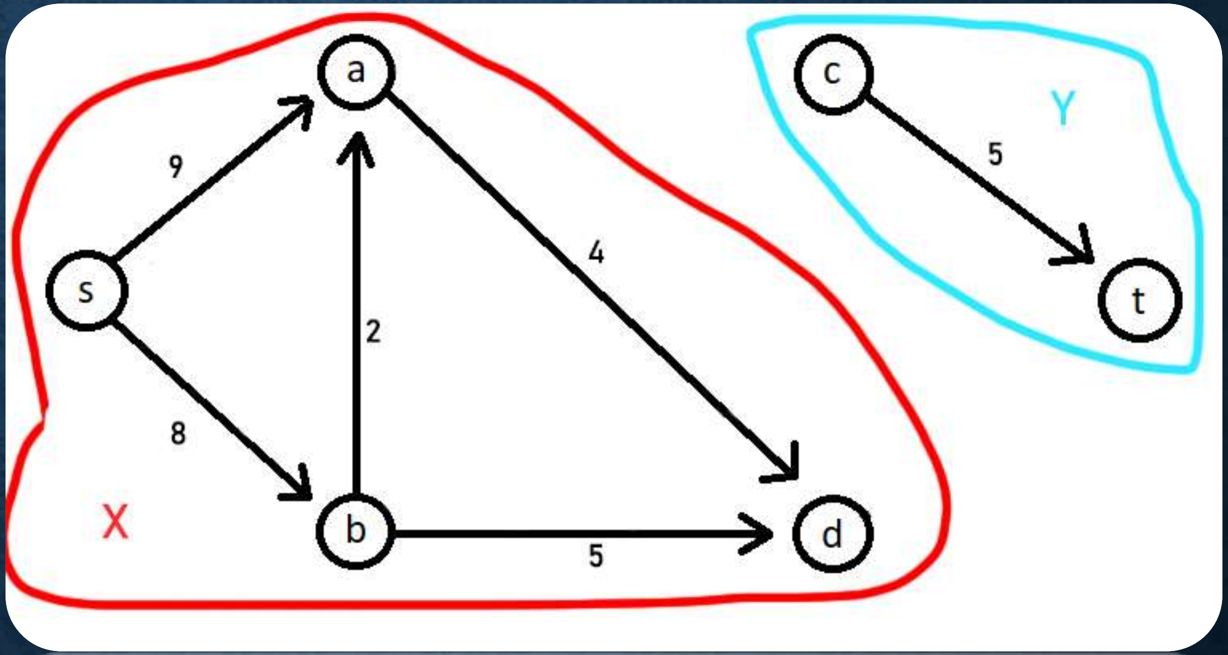
- Removed Edges

$a \rightarrow c = 4$

$b \rightarrow t = 3$

$d \rightarrow t = 6$

Total = 13



Min Cut - Capacity of Cut = Max Flow