

The Quantum-to-Classical Transition:
Decoherence and Beyond

Maximilian Schlosshauer-Selbach

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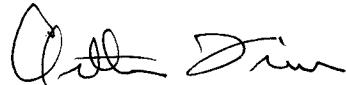
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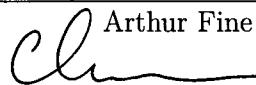


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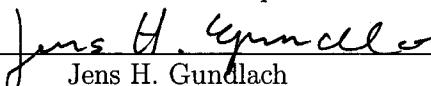
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Abstract

The Quantum-to-Classical Transition:
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Maximilian Schlosshauer-Selbach

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The main theme of this dissertation is the emergence of the “classical” world of our experience from quantum mechanics. The underlying key ingredient to be discussed is decoherence, i.e., the dislocalization of local phase relations through entanglement with environmental degrees of freedom. The analysis of a decoherence-based quantum-to-classical transition is approached from four different points of view.

First, key features of the decoherence program are clarified, and implications of decoherence for the quantum measurement problem and for the existing interpretations of quantum mechanics are extensively discussed.

Second, a critical analysis of the “self-induced decoherence” approach is presented. This approach has claimed to provide an alternative viewpoint on decoherence and to bypass some of the conceptual difficulties often associated with environmental decoherence. It is shown that this approach not only falls short of describing a physically relevant decoherence mechanism, but also fails in a large class of model systems.

Third, a derivation of the quantum-probability concept and the Born rule from symmetries in quantum entanglement is analyzed. In particular, important key assumptions not mentioned in the original exposition of the derivation are identified.

Fourth, recent experiments are discussed that have demonstrated the existence of superpositions of macroscopically distinct states. The crucial role of decoherence in these experiments is analyzed, and the implications for minimal non-collapse quantum mechanics and physical collapse models are investigated.

Based on these studies, it is concluded that pure quantum features, most notably, quantum entanglement and the resulting decoherence effects, are well capable of explaining the emergence of classicality in unitary quantum mechanics.

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DEDICATION

To Kari, with all my love.

Chapter 1

INTRODUCTION

The birth of quantum theory is closely tied to difficulties in explaining microscopic phenomena by means of the laws of classical physics. Starting with Planck's hypothesis of quantized harmonic-oscillator energies to account for the spectrum of black-body radiation [258], Einstein's explanation of the photoelectric effect [121], and Bohr's quantum model of the atomic structure and spectra [55], quantum theory was initially exclusively applied to the microscopic domain, for two main reasons. First, the known macroscopic phenomena could be well accounted for by classical physics, so no need seemed to exist for employing quantum theory in the description of the macroworld. Second, it became increasingly clear during the further development of quantum theory that the extrapolation of the predictions of the quantum formalism to macroscopic scales led to flagrantly nonclassical results that seemed irreconcilable with our observations in the “everyday world.” Moreover, even if without directly applying quantum theory to the latter domain, quantum entanglement appeared to inevitably imprint the quantum features associated with the microscopic world realm on the macroscopic world. Most poignantly, this consequence was pointed out by Einstein in a “crude macroscopic example” that featured a mixture of exploded and not exploded gunpowder [133, p. 78], and by Schrödinger in his famous cat paradox [283, p. 812].

In more abstract terms, such microscopic–macroscopic entanglement was recognized as the result of a general measurement process formulated in quantum-mechanical terms, as first described by von Neumann [312], in which a macroscopic apparatus

measures the state of a microscopic system. The linearity of the Schrödinger equation then implies that, if the system is described by a superposition $\sum_i \alpha_i |s_i\rangle$ of states $|s_i\rangle$, the quantum state of the system-apparatus combination after the entangling measurement interaction will also be a superposition of the form $\sum_i \alpha_i |s_i\rangle |a_i\rangle$, where the states $|a_i\rangle$ represent the “pointer positions” of the measuring device corresponding to the microscopic states $|s_i\rangle$. Much of the controversy about the interpretation of quantum theory has been centered around the resulting “measurement problem”: How to reconcile the observation of the pointer’s always being in a definite position with the fact that the system-apparatus combination is described by a superposition in which no particular “pointer state” $|a_i\rangle$ is singled out. This issue can be viewed as part of the more general problem of the *quantum-to-classical transition*.

Two main strands of interpretation have been pursued in order to make sense of this apparent contradiction. The first one suggests a breakdown of the laws of quantum mechanics (laws that have been well-confirmed on microscopic scales) in the macroscopic domain. This is done either by postulating an intrinsic inapplicability of quantum theory to the macroworld (as stated by the Copenhagen interpretation), or by introducing a physical mechanism that collapses superpositions of macroscopically distinct states, while leaving microscopic superpositions sufficiently unaffected as to ensure consistency with observed quantum effects. The second route has consisted of maintaining the universal validity of Schrödinger dynamics on all length scales, while reinterpreting superpositions in such a way as to, in spite of the presence of a quantum-superposition state, either allow for the assignment of definite values of physical quantities, or to explain through observer-based accounts why definiteness is subjectively perceived on the macroscopic level.

These interpretive developments have been immensely influenced by the recognition of the importance of taking into account the openness of essentially all quantum systems in order to arrive at a realistic description of these systems. Interaction of the system with its surroundings leads to rapid entanglement with (typically macro-

scopic) environmental degrees of freedom and thus to *decoherence*, i.e., the “diffusion” of phase relations defining the superposition of the state of the local system into the combined system–environment state. In other words, the phase relations become a shared property of the global state and cannot be recovered through measurements performed on the system only. Simultaneously, interaction with the environment also selects the set of preferred states for the system between which local phase coherence is damped. Since decoherence is simply a consequence of quantum entanglement, it is interpretation-neutral and is regarded as an experimentally confirmed fact. However, as the consequences of decoherence are intimately tied to the question of the quantum-to-classical transition spelled out in the measurement problem, they have also been widely discussed in the context of interpretations of quantum mechanics and with respect to their ability to motivate resolutions to the measurement problem.

This has led to a gradual shift in the status of different interpretive strategies. Earlier, the main route had consisted of resolving the measurement problem by adding formal rules for establishing correspondences between the quantum formalism and the properties of the observed macroworld. Usually, this was done by assigning different interpretations to the wave function. For example, in the Bohm–de Broglie theory the wave function only defines possible particle trajectories, but not the particles itself. Modal interpretations use the density matrix, derived from the wave function, only as a formal catalog of possible and mutually exclusive definite values of physical quantities. By contrast, proposals that identify the wave function as directly describing physical reality, such as Schrödinger’s attempted description of particles as narrow wave packets [282] or Everett’s relative-state interpretation, immediately faced the seemingly insurmountable measurement problem.

Then, with the recognition of the importance of decoherence effects, the latter category of interpretations has gained new-found viability, while it has become possible to impose stringent, physically motivated constraints on the admissible range of formal rules in the former proposals, in order to ensure conformity of the predictions

derived from these rules with those of decoherence theory (and, therefore, of quantum mechanics *per se*). Different ways to resolve the measurement problem that previously had seemed a matter of formalities and taste only had suddenly become much more precisely defined due to decoherence.

These, and other, topics related to decoherence will form the main subject of this dissertation. In each chapter, we shall approach the question of the role of decoherence in formalizing, understanding, and interpreting, the quantum-to-classical transition from a slightly different direction. Collectively, these chapters are intended to form a unifying picture.

In Chapter 2 (p. 7), we review the formalism and key features of the decoherence program and explore the implications of decoherence for various interpretations of quantum mechanics. We discuss, for example, the following questions: What, precisely, is environment-induced decoherence and superselection, and how is it formally represented? How does it relate to the problem of the quantum-to-classical transition and that of the preferred basis? What are the conceptual difficulties and formal caveats associated with the theory of decoherence? How can decoherence support, disprove, unify, and change existing and future interpretations of quantum mechanics?

In Chapter 3 (p. 105), we analyze a novel approach (termed *self-induced decoherence*) to the “classical limit” of quantum mechanics that has claimed to represent a new way of describing decoherence. We show why this approach does not describe a physically relevant decoherence effect and also demonstrate, through analytical and numerical studies, that it fails in a large category of model systems. This chapter emphasizes the importance of a clear definition of decoherence in order to avoid the misinterpretation of certain formal features.

In Chapter 4 (p. 131), we analyze a recently proposed derivation of the quantum-probability concept and the Born rule from properties of entangled bipartite states under certain transformations. This derivation is based on principles similar to those of decoherence theory, mainly, quantum entanglement with a partner system. We

identify a set of crucial assumptions that the derivation relies on and investigate to what extent the approach is relevant to an understanding of the meaning of probabilities in relative-state interpretations of quantum mechanics that are often viewed as quite naturally arising from a purely unitary, decoherence-based framework.

Chapter 5 (p. 152) explores the role of decoherence in quantum mechanics along a somewhat different route than in the previous chapters. As we have mentioned earlier, the measurement problem deals with the difficulty of reconciling superpositions of macroscopically distinct states with what we actually observe. On the other hand, as we discuss in this chapter, there exist experiments that have recently allowed for the creation and observation of such quantum states. We analyze three of these experiments, namely, setups employing superconducting quantum interference devices, matter-wave diffraction experiments using massive fullerene molecules, and Bose-Einstein condensation. These experiments are so remarkable because they allow for the experimental verification of the existence of quantum states that were previously only associated with the *Gedanken* category, such as the Schrödinger-cat setup. Moreover, we demonstrate how decoherence can quantify precisely the lack of robustness of such superpositions. The main goal of this chapter is to argue that a purely unitary quantum theory, based on the smallest possible set of axioms and on a quantum-mechanical description of observations and observers, is both supported by the experimental evidence and is likely to be empirically adequate. If such a decoherence-based “minimal” theory could suffice to construct a complete account of quantum mechanics that is in agreement with our observations, why introduce additional formal rules to tame the measurement problem by fiat?

Finally, in Chapter 6 (p. 225), we summarize our main results and discuss the emerging broader picture with respect to the quantum-to-classical transition and some interpretive stances in quantum mechanics. An extensive bibliography can be found starting on p. 261.

We note that the main chapters 2–5 of this dissertation are represented by a

collection of articles that have been written and published during the course of this PhD research. In many places, the original papers have been slightly modified for the purpose of inclusion in this dissertation (for instance, by adding crossreferences between chapters). They also have been updated in some places to reflect the most recent progress in the field. Inevitably, the author's thinking about certain aspects discussed in the papers has evolved over time, so different articles (and therefore chapters) may evaluate similar questions from distinct angles and therefore arrive at somewhat different conclusions. For example, in Chap. 2 (corresponding to the earliest article), we give a rather critical account of the ability of decoherence to account for the perception of single measurement outcomes, because this question is discussed from the perspective of the standard ("orthodox") interpretation. By contrast, in Chap. 5 (representing the most recent paper), the question is evaluated in the context of a relative-state ("many-branches") framework, leading to a much more positive assessment. Similarly, the discussion of envariance and Zurek's derivation of the Born rule in Chap. 4 takes a rather critical stance with respect to the success of the derivation. Several of the criticisms, however, disappear when the derivation is considered, again, within a relative-state view, as is done in Chapters 2 and 5.

As we shall demonstrate and argue in the course of this dissertation, a consistent and universal application of pure quantum principles, such as entanglement and the resulting decoherence effects, across all length scales and levels of description may, in fact, yield the most fundamental explanation for the emergence of classicality.

Chapter 2

DECOHERENCE, THE MEASUREMENT PROBLEM, AND INTERPRETATIONS OF QUANTUM MECHANICS

Summary

Environment-induced decoherence and superselection have been a subject of intensive research over the past two decades, yet their implications for the foundational problems of quantum mechanics, most notably the quantum measurement problem, have remained a matter of great controversy. This paper is intended to clarify key features of the decoherence program, including its more recent results, and to investigate their application and consequences in the context of the main interpretive approaches of quantum mechanics.

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2.1 *Introduction*

The implications of the decoherence program for the foundations of quantum mechanics have been the subject of an ongoing debate since the first precise formulation of the program in the early 1980s. The key idea promoted by decoherence is the insight that realistic quantum systems are never isolated, but are immersed in the surrounding environment and interact continuously with it. The decoherence program then studies, entirely within the standard quantum formalism (i.e., without adding any new elements in the mathematical theory or its interpretation), the resulting formation of quantum correlations between the states of the system and its environment and the often surprising effects of these system-environment interactions. In short,

decoherence brings about a local suppression of interference between preferred states selected by the interaction with the environment.

Bub [65] termed decoherence part of the “new orthodoxy” of understanding quantum mechanics—as the working physicist’s way of motivating the postulates of quantum mechanics from physical principles. Proponents of decoherence called it an “historical accident” [196, p. 13] that the implications for quantum mechanics and for the associated foundational problems were overlooked for so long. Zurek [344, p. 717] suggests

The idea that the “openness” of quantum systems might have anything to do with the transition from quantum to classical was ignored for a very long time, probably because in classical physics problems of fundamental importance were always settled in isolated systems.

When the concept of decoherence was first introduced to the broader scientific community by Zurek’s article [339] in *Physics Today*, it elicited a series of contentious comments from the readership (see the April 1993 issue of *Physics Today*). In response to his critics, Zurek [344, p. 718] states

In a field where controversy has reigned for so long this resistance to a new paradigm [namely, to decoherence] is no surprise.

Omnès [242, p. 2] had this assessment:

The discovery of decoherence has already much improved our understanding of quantum mechanics. (...) [B]ut its foundation, the range of its validity and its full meaning are still rather obscure. This is due most probably to the fact that it deals with deep aspects of physics, not yet fully investigated.

In particular, the question whether decoherence provides, or at least suggests, a solution to the measurement problem of quantum mechanics has been discussed for several years. For example, Anderson [16, p. 492] writes in an essay review

The last chapter (...) deals with the quantum measurement problem (...). My main test, allowing me to bypass the extensive discussion, was a quick, unsuccessful search in the index for the word “decoherence” which describes the process that used to be called “collapse of the wave function.”

Zurek speaks in various places of the “apparent” or “effective” collapse of the wave function induced by the interaction with environment (when embedded into a minimal additional interpretive framework) and concludes [341, p. 1793]

A “collapse” in the traditional sense is no longer necessary. (...) [The] emergence of “objective existence” [from decoherence] (...) significantly reduces and perhaps even eliminates the role of the “collapse” of the state vector.

D’Espagnat, who considers the explanation of our experiences (i.e., of “appearances”) as the only “sure” requirement of a physical theory, states [96, p. 136]

For macroscopic systems, the appearances are those of a classical world (no interferences etc.), even in circumstances, such as those occurring in quantum measurements, where quantum effects take place and quantum probabilities intervene (...). Decoherence explains the just mentioned appearances and this is a most important result. (...) As long as we remain within the realm of mere predictions concerning what we shall observe (i.e., what will appear to us)—and refrain from stating anything concerning “things as they must be before we observe them”—no break in the linearity of quantum dynamics is necessary.

In his monumental book on the foundations of quantum mechanics (QM), Auletta [23, p. 791] concludes that

the Measurement theory could be part of the interpretation of QM only to the extent that it would still be an open problem, and we think that this is largely no longer the case.

This is mainly so because, according to Auletta [23, p. 289],

decoherence is able to solve practically all the problems of Measurement which have been discussed in the previous chapters.

On the other hand, even leading adherents of decoherence have expressed caution or even doubt that decoherence has solved the measurement problem. Joos [196, p. 14] writes

Does decoherence solve the measurement problem? Clearly not. What decoherence tells us, is that certain objects appear classical when they are observed. But what is an observation? At some stage, we still have to apply the usual probability rules of quantum theory.

Along these lines, Kiefer and Joos [203, p. 5] warn that

One often finds explicit or implicit statements to the effect that the above processes are equivalent to the collapse of the wave function (or even solve the measurement problem). Such statements are certainly unfounded.

In a response to Anderson's comment [16, p. 492], Adler [2, p. 136] states

I do not believe that either detailed theoretical calculations or recent experimental results show that decoherence has resolved the difficulties associated with quantum measurement theory.

Similarly, Bacciagaluppi [26, p. 3] writes

Claims that *simultaneously* the measurement problem is real [and] decoherence solves it are confused at best.

Zeh asserts [198, Ch. 2]

Decoherence by itself does not yet solve the measurement problem (...). This argument is nonetheless found wide-spread in the literature. (...) It does seem that the measurement problem can only be resolved if the Schrödinger dynamics (...) is supplemented by a nonunitary collapse (...).

The key achievements of the decoherence program, apart from their implications for conceptual problems, do not seem to be universally understood either. Zurek [341, p. 1800] remarks

[The] eventual diagonality of the density matrix (...) is a byproduct (...) but not the essence of decoherence. I emphasize this because diagonality of [the density matrix] in some basis has been occasionally (mis-)interpreted as a key accomplishment of decoherence. This is misleading. Any density matrix is diagonal in some basis. This has little bearing on the interpretation.

These remarks show that a balanced discussion of the key features of decoherence and their implications for the foundations of quantum mechanics is overdue. The decoherence program has made great progress over the past decade, and it would be inappropriate to ignore its relevance in tackling conceptual problems. However, it is equally important to realize the limitations of decoherence in providing consistent and noncircular answers to foundational questions.

An excellent review of the decoherence program has recently been given by Zurek [344]. It deals primarily with the technicalities of decoherence, although it contains some discussion on how decoherence can be employed in the context of a relative-state interpretation to motivate basic postulates of quantum mechanics. A helpful first orientation and overview, the entry by Bacciagaluppi [25] in the *Stanford Encyclopedia of Philosophy* features a relatively short (in comparison to the present paper) introduction to the role of decoherence in the foundations of quantum mechanics, including comments on the relationship between decoherence and several popular interpretations of quantum theory. In spite of these valuable recent contributions to

the literature, a detailed and self-contained discussion of the role of decoherence in the foundations of quantum mechanics seems still to be lacking. This review article is intended to fill the gap.

To set the stage, we shall first, in Sec. 2.2, review the measurement problem, which illustrates the key difficulties that are associated with describing quantum measurement within the quantum formalism and that are all in some form addressed by the decoherence program. In Sec. 2.3, we then introduce and discuss the main features of the theory of decoherence, with a particular emphasis on their foundational implications. Finally, in Sec. 2.4, we investigate the role of decoherence in various interpretive approaches of quantum mechanics, in particular with respect to the ability to motivate and support (or disprove) possible solutions to the measurement problem.

2.2 The measurement problem

One of the most revolutionary elements introduced into physical theory by quantum mechanics is the superposition principle, mathematically founded in the linearity of the Hilbert state space. If $|1\rangle$ and $|2\rangle$ are two states, then quantum mechanics tells us that any linear combination $\alpha|1\rangle + \beta|2\rangle$ also corresponds to a possible state. Whereas such superpositions of states have been experimentally extensively verified for microscopic systems (for instance, through the observation of interference effects), the application of the formalism to macroscopic systems appears to lead immediately to severe clashes with our experience of the everyday world. A book has never been ever observed to be in a state of being both “here” and “there” (i.e., to be in a superposition of macroscopically distinguishable positions), nor does a Schrödinger cat that is a superposition of being alive and dead bear much resemblance to reality as we perceive it. The problem is, then, how to reconcile the vastness of the Hilbert space of possible states with the observation of a comparatively few “classical” macroscopic states, defined by having a small number of determinate and robust properties such as position and momentum. Why does the world appear classical to us, in spite of its

supposed underlying quantum nature, which would, in principle, allow for arbitrary superpositions?

2.2.1 Quantum measurement scheme

This question is usually illustrated in the context of quantum measurement where microscopic superpositions are, via quantum entanglement, amplified into the macroscopic realm and thus lead to very “nonclassical” states that do not seem to correspond to what is actually perceived at the end of the measurement. In the ideal measurement scheme devised by von Neumann [312], a (typically microscopic) system \mathcal{S} , represented by basis vectors $\{|s_n\rangle\}$ in a Hilbert space \mathcal{H}_S , interacts with a measurement apparatus \mathcal{A} , described by basis vectors $\{|a_n\rangle\}$ spanning a Hilbert space \mathcal{H}_A , where the $|a_n\rangle$ are assumed to correspond to macroscopically distinguishable “pointer” positions that correspond to the outcome of a measurement if \mathcal{S} is in the state $|s_n\rangle$.¹

Now, if \mathcal{S} is in a (microscopically “unproblematic”) superposition $\sum_n c_n |s_n\rangle$, and \mathcal{A} is in the initial “ready” state $|a_r\rangle$, the linearity of the Schrödinger equation entails that the total system $\mathcal{S}\mathcal{A}$, assumed to be represented by the Hilbert product space $\mathcal{H}_S \otimes \mathcal{H}_A$, evolves according to

$$\left(\sum_n c_n |s_n\rangle \right) |a_r\rangle \xrightarrow{t} \sum_n c_n |s_n\rangle |a_n\rangle. \quad (2.1)$$

This dynamical evolution is often referred to as a *premeasurement* in order to emphasize that the process described by Eq. (2.1) does not suffice to directly conclude that a measurement has actually been completed. This is so for two reasons. First, the right-hand side is a *superposition* of system-apparatus states. Thus, without supplying an additional physical process (say, some collapse mechanism) or giving a suitable

¹Note that von Neumann’s scheme is in sharp contrast to the Copenhagen interpretation, where measurement is not treated as a system-apparatus interaction described by the usual quantum-mechanical formalism, but instead as an independent component of the theory, to be represented entirely in fundamentally classical terms.

interpretation of such a superposition, it is not clear how to account, given the final composite state, for the definite pointer positions that are perceived as the result of an actual measurement—i.e., why do we seem to perceive the pointer to be in one position $|a_n\rangle$ but not in a superposition of positions? This is the *problem of definite outcomes*. Second, the expansion of the final composite state is in general not unique, and therefore the measured observable is not uniquely defined either. This is the *problem of the preferred basis*. In the literature, the first difficulty is typically referred to as the measurement problem, but the preferred-basis problem is at least equally important, since it does not make sense even to inquire about specific outcomes if the set of possible outcomes is not clearly defined. We shall therefore regard the measurement problem as composed of both the problem of definite outcomes and the problem of the preferred basis, and discuss these components in more detail in the following.

2.2.2 *The problem of definite outcomes*

Superpositions and ensembles

The right-hand side of Eq. (2.1) implies that after the premeasurement the combined system \mathcal{SA} is left in a pure state that represents a linear superposition of system-pointer states. It is a well-known and important property of quantum mechanics that a superposition of states is fundamentally different from a classical ensemble of states, where the system actually is in only one of the states but we simply do not know in which (this is often referred to as an “ignorance-interpretable,” or “proper” ensemble).

This can be shown explicitly, especially on microscopic scales, by performing experiments that lead to the direct observation of interference patterns instead of the realization of one of the terms in the superposed pure state, for example, in a setup where electrons pass individually (one at a time) through a double slit. As

is well known, this experiment clearly shows that, within the standard quantum-mechanical formalism, the electron must not be described by either one of the wave functions describing the passage through a particular slit (ψ_1 or ψ_2), but only by the superposition of these wave functions ($\psi_1 + \psi_2$), since the correct density distribution ϱ of the pattern on the screen is not given by the sum of the squared wave functions describing the addition of individual passages through a single slit ($\varrho = |\psi_1|^2 + |\psi_2|^2$), but only by the square of the sum of the individual wave functions ($\varrho = |\psi_1 + \psi_2|^2$).

Put differently, if an ensemble interpretation could be attached to a superposition, the latter would simply represent an ensemble of more fundamentally determined states, and based on the additional knowledge brought about by the results of measurements, we could simply choose a subensemble consisting of the definite pointer state obtained in the measurement. But then, since the time evolution has been strictly deterministic according to the Schrödinger equation, we could backtrack this subensemble in time and thus also specify the initial state more completely (“postselection”), and therefore this state necessarily could not be physically identical to the initially prepared state on the left-hand side of Eq. (2.1).

Superpositions and outcome attribution

In the standard (“orthodox”) interpretation of quantum mechanics, an observable corresponding to a physical quantity has a definite value if and only if the system is in an eigenstate of the observable; if the system is, however, in a superposition of such eigenstates, as in Eq. (2.1), it is, according to the orthodox interpretation, meaningless to speak of the state of the system as having any definite value of the observable at all. (This is frequently referred to as the so-called eigenvalue-eigenstate link, or “e-e link” for short.) The e-e link, however, is by no means forced upon us by the structure of quantum mechanics or by empirical constraints [65]. The concept of (classical) “values” that can be ascribed through the e-e link based on observables and the existence of exact eigenstates of these observables has therefore

frequently been either weakened or altogether abandoned. For instance, outcomes of measurements are typically registered in position space (pointer positions, etc.), but there exist no exact eigenstates of the position operator, and the pointer states are never exactly mutually orthogonal. One might then (explicitely or implicitly) promote a “fuzzy” e-e link, or give up the concept of observables and values entirely and directly interpret the time-evolved wave functions (working in the Schrödinger picture) and the corresponding density matrices. Also, if it is regarded as sufficient to explain our perceptions rather than describe the “absolute” state of the entire universe (see the argument below), one might only require that the (exact or fuzzy) e-e link hold in a “relative” sense, i.e., for the state of the rest of the universe relative to the state of the observer.

Then, to solve the problem of definite outcomes, some interpretations (for example, modal interpretations and relative-state interpretations) interpret the final-state superposition in such a way as to explain the existence, or at least the subjective perception, of “outcomes” even if the final composite state has the form of a superposition. Other interpretations attempt to solve the measurement problem by modifying the strictly unitary Schrödinger dynamics. Most prominently, the orthodox interpretation postulates a collapse mechanism that transforms a pure-state density matrix into an ignorance-interpretable ensemble of individual states (a “proper mixture”). Wave-function collapse theories add stochastic terms to the Schrödinger equation that induce an effective (albeit only approximate) collapse for states of macroscopic systems [251, 152, 149, 252], while other authors suggested that collapse occurs at the level of the mind of a conscious observer [323, 290]. Bohmian mechanics, on the other hand, upholds a unitary time evolution of the wavefunction, but introduces an additional dynamical law that explicitely governs the always-determinate positions of all particles in the system.

Objective vs subjective definiteness

In general, (macroscopic) definiteness—and thus a solution to the problem of outcomes in the theory of quantum measurement—can be achieved either on an *ontological* (objective) or an *observational* (subjective) level. Objective definiteness aims at ensuring “actual” definiteness in the macroscopic realm, whereas subjective definiteness only attempts to explain why the macroscopic world appears to be definite—and thus does not make any claims about definiteness of the underlying physical reality (whatever this reality might be). This raises the question of the significance of this distinction with respect to the formation of a satisfactory theory of the physical world. It might appear that a solution to the measurement problem based on ensuring subjective, but not objective, definiteness is merely good “for all practical purposes”—abbreviated, rather disparagingly, as “FAPP” by Bell [45]—and thus not capable of solving the “fundamental” problem that would seem relevant to the construction of the “precise theory” that Bell demanded so vehemently.

It seems to the author, however, that this criticism is not justified, and that subjective definiteness should be viewed on a par with objective definiteness with respect to a satisfactory solution to the measurement problem. We demand objective definiteness because we experience definiteness on the subjective level of observation, and it should not be viewed as an *a priori* requirement for a physical theory. If we knew independently of our experience that definiteness existed in nature, subjective definiteness would presumably follow as soon as we had employed a simple model that connected the “external” physical phenomena with our “internal” perceptual and cognitive apparatus, where the expected simplicity of such a model can be justified by referring to the presumed identity of the physical laws governing external and internal processes. But since knowledge is based on experience, that is, on observation, the existence of objective definiteness could only be derived from the observation of definiteness. And, moreover, observation tells us that definiteness is in fact not a universal property of

nature, but rather a property of macroscopic objects, where the borderline to the macroscopic realm is difficult to draw precisely; mesoscopic interference experiments have demonstrated clearly the blurriness of the boundary (see Chap. 5). Given the lack of a precise definition of the boundary, any demand for fundamental definiteness on the objective level should be based on a much deeper and more general commitment to a definiteness that applies to every physical entity (or system) across the board, regardless of spatial size, physical property, and the like.

Therefore, if we realize that the often deeply felt commitment to a general objective definiteness is only based on our experience of macroscopic systems, and that this definiteness in fact fails in an observable manner for microscopic and even certain mesoscopic systems, the author sees no compelling grounds on which objective definiteness must be demanded as part of a satisfactory physical theory, provided that the theory can account for subjective, observational definiteness in agreement with our experience. Thus the author suggests that the same legitimacy be attributed to proposals for a solution of the measurement problem that achieve “only” subjective but not objective definiteness—after all, the measurement problem arises solely from a clash of our experience with certain implications of the quantum formalism. D’Espagnat [96, pp. 134–135] has advocated a similar viewpoint:

The fact that we perceive such “things” as macroscopic objects lying at distinct places is due, partly at least, to the structure of our sensory and intellectual equipment. We should not, therefore, take it as being part of the body of sure knowledge that we have to take into account for defining a quantum state. (...) In fact, scientists most rightly claim that the purpose of science is to describe human experience, not to describe “what really is”; and as long as we only want to describe human experience, that is, as long as we are content with being able to predict what will be observed in all possible circumstances (...) we need not postulate the existence—in some absolute sense—of unobserved (i.e., not

yet observed) objects lying at definite places in ordinary 3-dimensional space.

2.2.3 The preferred-basis problem

The second difficulty associated with quantum measurement is known as the preferred-basis problem, which demonstrates that the measured observable is in general not uniquely defined by Eq. (2.1). For any choice of system states $\{|s_n\rangle\}$, we can find corresponding apparatus states $\{|a_n\rangle\}$, and vice versa, to equivalently rewrite the final state emerging from the premeasurement interaction, i.e., the right-hand side of Eq. (2.1). In general, however, for some choice of apparatus states the corresponding new system states will not be mutually orthogonal, so that the observable associated with these states will not be Hermitian, which is usually not desired (however, not forbidden—see the discussion in [344]). Conversely, to ensure distinguishable outcomes, we must, in general, require the (at least approximate) orthogonality of the apparatus (pointer) states, and it then follows from the biorthogonal decomposition theorem that the expansion of the final premeasurement system-apparatus state of Eq. (2.1),

$$|\psi\rangle = \sum_n c_n |s_n\rangle |a_n\rangle, \quad (2.2)$$

is unique, but only if all coefficients c_n are distinct. Otherwise, we can in general rewrite the state in terms of different state vectors,

$$|\psi\rangle = \sum_n c'_n |s'_n\rangle |a'_n\rangle, \quad (2.3)$$

such that the same postmeasurement state seems to correspond to two different measurements, that is, of the observables $\hat{A} = \sum_n \lambda_n |s_n\rangle \langle s_n|$ and $\hat{B} = \sum_n \lambda'_n |s'_n\rangle \langle s'_n|$ of the system, respectively, although in general \hat{A} and \hat{B} do not commute.

As an example, consider a Hilbert space $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ where \mathcal{H}_1 and \mathcal{H}_2 are two-dimensional spin spaces with states corresponding to spin up or spin down along a given axis. Suppose we are given an entangled spin state of the Einstein-Podolsky-

Rosen form [126]

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|z+\rangle_1|z-\rangle_2 - |z-\rangle_1|z+\rangle_2), \quad (2.4)$$

where $|z\pm\rangle_{1,2}$ represents the eigenstates of the observable σ_z corresponding to spin up or spin down along the z axis of the two systems 1 and 2. The state $|\psi\rangle$ can however equivalently be expressed in the spin basis corresponding to any other orientation in space. For example, when using the eigenstates $|x\pm\rangle_{1,2}$ of the observable σ_x (which represents a measurement of the spin orientation along the x axis) as basis vectors, we get

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|x+\rangle_1|x-\rangle_2 - |x-\rangle_1|x+\rangle_2). \quad (2.5)$$

Now suppose that system 2 acts as a measuring device for the spin of system 1. Then Eqs. (2.4) and (2.5) imply that the measuring device has established a correlation with both the z and the x spin of system 1. This means that, if we interpret the formation of such a correlation as a measurement in the spirit of the von Neumann scheme (without assuming a collapse), our apparatus (system 2) could be considered as having measured also the x spin once it has measured the z spin, and vice versa—in spite of the noncommutativity of the corresponding spin observables σ_z and σ_x . Moreover, since we can rewrite Eq. (2.4) in infinitely many ways, it appears that once the apparatus has measured the spin of system 1 along one direction, it can also be regarded as having measured the spin along any other direction, again in apparent contradiction with quantum mechanics due to the noncommutativity of the spin observables corresponding to different spatial orientations.

It thus seems that quantum mechanics has nothing to say about which observable(s) of the system is (are) being recorded, via the formation of quantum correlations, by the apparatus. This can be stated in a general theorem [337, 23]: When quantum mechanics is applied to an isolated composite object consisting of a system S and an apparatus A , it cannot determine which observable of the system has been measured—in obvious contrast to our experience of the workings of measuring devices

that seem to be “designed” to measure certain quantities.

2.2.4 The quantum-to-classical transition and decoherence

In essence, as we have seen above, the measurement problem deals with the transition from a quantum world, described by essentially arbitrary linear superpositions of state vectors, to our perception of “classical” states in the macroscopic world, that is, a comparatively small subset of the states allowed by the quantum-mechanical superposition principle, having only a few, but determinate and robust, properties, such as position, momentum, etc. The question of why and how our experience of a “classical” world emerges from quantum mechanics thus lies at the heart of the foundational problems of quantum theory.

Decoherence has been claimed to provide an explanation for this *quantum-to-classical transition* by appealing to the ubiquitous immersion of virtually all physical systems in their environment (“environmental monitoring”). This trend can also be read off nicely from the titles of some papers and books on decoherence, for example, “The emergence of classical properties through interaction with the environment” [197], “Decoherence and the transition from quantum to classical” [339], and “Decoherence and the appearance of a classical world in quantum theory” [198]. We shall critically investigate in this paper to what extent the appeal to decoherence for an explanation of the quantum-to-classical transition is justified.

2.3 The decoherence program

As remarked earlier, the theory of decoherence is based on a study of the effects brought about by the interaction of physical systems with their environment. In classical physics, the environment is usually viewed as a kind of disturbance, or noise, that perturbs the system under consideration in such a way as to negatively influence the study of its “objective” properties. Therefore science has established the idealization of isolated systems, with experimental physics aiming at eliminating any outer

sources of disturbance as much as possible in order to discover the “true” underlying nature of the system under study.

The distinctly nonclassical phenomenon of quantum entanglement, however, has demonstrated that the correlations between two systems can be of fundamental importance and can lead to properties that are not present in the individual systems.² The earlier view of phenomena arising from quantum entanglement as “paradoxa” has generally been replaced by the recognition of entanglement as a fundamental property of nature.

The decoherence program³ is based on the idea that such quantum correlations are ubiquitous; that nearly every physical system must interact in some way with its environment (for example, with the surrounding photons that then create the visual experience within the observer), which typically consists of a large number of degrees of freedom that are hardly ever fully controlled. Only in very special cases of typically microscopic (atomic) phenomena, so goes the claim of the decoherence program, is the idealization of isolated systems applicable so that the predictions of linear quantum mechanics (i.e., a large class of superpositions of states) can actually be observationally confirmed. In the majority of the cases accessible to our experience, however, interaction with the environment is so dominant as to preclude the observation of the “pure” quantum world, imposing effective superselection rules [318, 319, 140, 320, 84, 153] onto the space of observable states that lead to states corresponding to the “classical” properties of our experience. Interference between such states gets locally suppressed and is thus claimed to become inaccessible to the observer.

Probably the most surprising aspect of decoherence is the effectiveness of the system-environment interactions. Decoherence typically takes place on extremely

²Broadly speaking, this means that the (quantum-mechanical) whole is different from the sum of its parts.

³For key ideas and concepts, see Refs. [326, 208, 327, 329, 330, 332, 197, 337, 338, 339, 340, 344, 198].

short time scales and requires the presence of only a minimal environment [197]. Due to the large number of degrees of freedom of the environment, it is usually very difficult to undo system-environment entanglement, which has been claimed as a source of our impression of irreversibility in nature (see, for example, Refs. [338, 344, 350, 203, 334]). In general, the effect of decoherence increases with the size of the system (from microscopic to macroscopic scales), but it is important to note that there exist, admittedly somewhat exotic, examples for which the decohering influence of the environment can be sufficiently shielded to lead to mesoscopic and even macroscopic superpositions. One such example would be the case of superconducting quantum interference devices (SQUIDs), in which superpositions of macroscopic currents become observable [136, 303] (see also Chap. 5). Conversely, some microscopic systems (for instance, certain chiral molecules that exist in different distinct spatial configurations) can be subject to remarkably strong decoherence.

The decoherence program has dealt with the following two main consequences of environmental interaction:

- (1) *Environment-induced decoherence.* The fast local suppression of interference between different states of the system. However, since only unitary time evolution is employed, global phase coherence is not actually destroyed—it becomes absent from the local density matrix that describes the system alone, but remains fully present in the total system-environment composition.⁴ We shall discuss environment-induced local decoherence in more detail in Sec. 2.3.4.
- (2) *Environment-induced superselection.* The selection of preferred sets of states, often referred to as “pointer states,” that are robust (in the sense of retaining correlations over time) in spite of their immersion in the environment. These

⁴Note that the persistence of coherence in the total state is important to ensure the possibility of describing special cases in which mesoscopic or macroscopic superpositions have been experimentally realized (see Chap. 5).

states are determined by the form of the interaction between the system and its environment and are suggested to correspond to the “classical” states of our experience. We shall consider this mechanism in Sec. 2.3.5.

Another, more recent aspect of the decoherence program, termed *environment-assisted invariance* or “envariance,” was introduced by Zurek [344, 345, 347] and further developed in Ref. [346]. In particular, Zurek used envariance to explain the emergence of probabilities in quantum mechanics and to derive Born’s rule based on certain assumptions. We shall review envariance and Zurek’s derivation of the Born rule in Sec. 2.3.6.

Finally, let us emphasize that decoherence arises from a direct application of the quantum mechanical formalism to a description of the interaction of a physical system with its environment. By itself, decoherence is therefore neither an interpretation nor a modification of quantum mechanics. Yet the implications of decoherence need to be interpreted in the context of the different interpretations of quantum mechanics. Also, since decoherence effects have been studied extensively in both theoretical models and experiments (for a survey, see, for example, Refs. [198, 344] and Chap. 5), their existence can be taken as a well-confirmed fact.

2.3.1 *Resolution into subsystems*

Note that decoherence derives from the presupposition of the existence and the possibility of a division of the world into “system(s)” and “environment.” In the decoherence program, the term “environment” is usually understood as the “remainder” of the system, in the sense that its degrees of freedom are typically not (cannot be, do not need to be) controlled and are not directly relevant to the observation under consideration (for example, the many microscopic degrees of freedom of the system), but that nonetheless the environment includes “all those degrees of freedom which contribute significantly to the evolution of the state of the apparatus” [337, p. 1520].

This system–environment dualism is generally associated with quantum entanglement, which always describes a correlation between parts of the universe. As long as the universe is not resolved into individual subsystems, there is no measurement problem: the state vector $|\Psi\rangle$ of the entire universe⁵ evolves deterministically according to the Schrödinger equation $i\hbar\frac{\partial}{\partial t}|\Psi\rangle = \hat{H}|\Psi\rangle$, which poses no interpretive difficulty. Only when we decompose the total Hilbert-state space \mathcal{H} of the universe into a product of two spaces $\mathcal{H}_1 \otimes \mathcal{H}_2$, and accordingly form the joint-state vector $|\Psi\rangle = |\Psi_1\rangle|\Psi_2\rangle$, and want to ascribe an individual state (besides the joint state that describes a correlation) to one of the two systems (say, the apparatus), does the measurement problem arise. Zurek [344, p. 718] puts it like this:

In the absence of systems, the problem of interpretation seems to disappear.
There is simply no need for “collapse” in a universe with no systems. Our experience of the classical reality does not apply to the universe as a whole, seen from the outside, but to the systems within it.

Moreover, terms like “observation,” “correlation,” and “interaction” will naturally make little sense without a division into systems. Zeh has suggested that the locality of the observer defines an observation in the sense that any observation arises from the ignorance of a part of the universe; and that this also defines the “facts” that can occur in a quantum system. Landsman [210, pp. 45–46] argues similarly:

The essence of a “measurement,” “fact” or “event” in quantum mechanics lies in the non-observation, or irrelevance, of a certain part of the system in question.
(...) A world without parts declared or forced to be irrelevant is a world without facts.

However, the assumption of a decomposition of the universe into subsystems—as necessary as it appears to be for the emergence of the measurement problem and

⁵If we dare to postulate this total state—see counterarguments by Auletta [23].

for the definition of the decoherence program—is definitely nontrivial. By definition, the universe as a whole is a closed system, and therefore there are no “unobserved degrees of freedom” of an external environment which would allow for the application of the theory of decoherence to determine the space of quasiclassical observables of the universe in its entirety. Also, there exists no general criterion for how the total Hilbert space is to be divided into subsystems, while at the same time much of what is called a property of the system will depend on its correlation with other systems. This problem becomes particularly acute if one would like decoherence not only to motivate explanations for the subjective perception of classicality (as in Zurek’s “existential interpretation,” see Refs. [340, 341, 344] and Sec. 2.4.3 below), but moreover to allow for the definition of quasiclassical “macrofacts.” Zurek [341, p. 1820] admits this severe conceptual difficulty:

In particular, one issue which has been often taken for granted is looming big, as a foundation of the whole decoherence program. It is the question of what are the “systems” which play such a crucial role in all the discussions of the emergent classicality. (...) [A] compelling explanation of what are the systems—how to define them given, say, the overall Hamiltonian in some suitably large Hilbert space—would be undoubtedly most useful.

A frequently proposed idea is to abandon the notion of an “absolute” resolution and instead postulate the intrinsic relativity of the distinct state spaces and properties that emerge through the correlation between these relatively defined spaces (see, for example, the proposals, unrelated to decoherence, in Refs. [129, 226, 227, 269]). This relative view of systems and correlations has counterintuitive, in the sense of nonclassical, implications. However, as in the case of quantum entanglement, these implications need not be taken as paradox that demand further resolution. Accepting some properties of nature as counterintuitive is indeed a satisfactory path to take in order to arrive at a description of nature that is as complete and objective as is allowed

by the range of our experience (which is based on inherently local observations).

2.3.2 The concept of reduced density matrices

Since reduced density matrices are a key tool of decoherence, it will be worthwhile to briefly review their basic properties and interpretation in the following. The concept of reduced density matrices emerged in the earliest days of quantum mechanics [209, 312, 139]; for some historical remarks, see Ref. [257]. In the context of a system of two entangled systems in a pure state of the Einstein-Podolsky-Rosen-type,

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|+\rangle_1|-\rangle_2 - |-\rangle_1|+\rangle_2), \quad (2.6)$$

it had been realized early that for an observable \widehat{O} that pertains only to system 1, $\widehat{O} = \widehat{O}_1 \otimes \widehat{I}_2$, the pure-state density matrix $\rho = |\psi\rangle\langle\psi|$ yields, according to the trace rule $\langle\widehat{O}\rangle = \text{Tr}(\rho\widehat{O})$ and given the usual Born rule for calculating probabilities, exactly the same statistics as the reduced density matrix ρ_1 obtained by tracing over the degrees of freedom of system 2 (i.e., the states $|+\rangle_2$ and $|-\rangle_2$),

$$\rho_1 = \text{Tr}_2|\psi\rangle\langle\psi| = {}_2\langle+|\psi\rangle\langle\psi|+\rangle_2 + {}_2\langle-|\psi\rangle\langle\psi|-\rangle_2, \quad (2.7)$$

since it is easy to show that, for this observable \widehat{O} ,

$$\langle\widehat{O}\rangle_\psi = \text{Tr}(\rho\widehat{O}) = \text{Tr}_1(\rho_1\widehat{O}_1). \quad (2.8)$$

This result holds in general for any pure state $|\psi\rangle = \sum_i \alpha_i |\phi_i\rangle_1 |\phi_i\rangle_2 \cdots |\phi_i\rangle_N$ of a resolution of a system into N subsystems, where the $\{|\phi_i\rangle_j\}$ are assumed to form orthonormal basis sets in their respective Hilbert spaces \mathcal{H}_j , $j = 1 \cdots N$. For any observable \widehat{O} that pertains only to system j , $\widehat{O} = \widehat{I}_1 \otimes \widehat{I}_2 \otimes \cdots \otimes \widehat{I}_{j-1} \otimes \widehat{O}_j \otimes \widehat{I}_{j+1} \otimes \cdots \otimes \widehat{I}_N$, the statistics of \widehat{O} generated by applying the trace rule will be identical regardless of whether we use the pure-state density matrix $\rho = |\psi\rangle\langle\psi|$ or the reduced density matrix $\rho_j = \text{Tr}_{1,\dots,j-1,j+1,\dots,N}|\psi\rangle\langle\psi|$, since again $\langle\widehat{O}\rangle = \text{Tr}(\rho\widehat{O}) = \text{Tr}_j(\rho_j\widehat{O}_j)$.

The typical situation in which the reduced density matrix arises is this: Before a premeasurement-type interaction, the observer knows that each individual system is in some (unknown) pure state. After the interaction, i.e., after the correlation between the systems is established, the observer has access to only one of the systems, say, system 1; everything that can be known about the state of the composite system must therefore be derived from measurements on system 1, which will yield the possible outcomes of system 1 and their probability distribution. All information that can be extracted by the observer is then, exhaustively and correctly, contained in the reduced density matrix of system 1, assuming that the Born rule for quantum probabilities holds.

Let us return to the Einstein-Podolsky-Rosen-type example, Eqs. (2.6) and (2.7). If we assume that the states of system 2 are orthogonal, $\langle +| - \rangle_2 = 0$, ρ_1 becomes diagonal,

$$\rho_1 = \text{Tr}_2 |\psi\rangle\langle\psi| = \frac{1}{2}(|+\rangle\langle+|)_1 + \frac{1}{2}(|-\rangle\langle-|)_1. \quad (2.9)$$

But this density matrix is formally identical to the density matrix that would be obtained if system 1 were in a mixed state, i.e., in either one of the two states $|+\rangle_1$ and $|-\rangle_1$ with equal probabilities—as opposed to the superposition $|\psi\rangle$, in which both terms are considered present, which could in principle be confirmed by suitable interference experiments. This implies that a measurement of an observable that only pertains to system 1 cannot discriminate between the two cases, pure vs mixed state.⁶

However, note that the formal identification of the reduced density matrix with a mixed-state density matrix is easily misinterpreted as implying that the state of the system can be viewed as mixed too (see also the discussion by d'Espagnat [94]). Density matrices are only a calculational tool for computing the probability distribution of a set of possible outcomes of measurements; they do not specify the state of the

⁶As discussed by Bub [65, pp. 208–210], this result also holds for any observable of the composite system that factorizes into the form $\hat{O} = \hat{O}_1 \otimes \hat{O}_2$, where \hat{O}_1 and \hat{O}_2 do not commute with the projection operators $(|\pm\rangle\langle\pm|)_1$ and $(|\pm\rangle\langle\pm|)_2$, respectively.

system.⁷ Since the two systems are entangled and the total composite system is still described by a superposition, it follows from the standard rules of quantum mechanics that no individual definite state can be attributed to one of the systems. The reduced density matrix looks like a mixed-state density matrix because, if one actually measured an observable of the system, one would expect to get a definite outcome with a certain probability; in terms of measurement statistics, this is equivalent to the situation in which the system is in one of the states from the set of possible outcomes from the beginning, that is, before the measurement. As Pessoa [257, p. 432] puts it, “taking a partial trace amounts to the statistical version of the projection postulate.”

2.3.3 A modified von Neumann measurement scheme

Let us now reconsider the von Neumann model for ideal quantum-mechanical measurement, Eq. (2.10), but now with the environment included. We shall denote the environment by \mathcal{E} and represent its state before the measurement interaction by the initial state vector $|e_0\rangle$ in a Hilbert space $\mathcal{H}_{\mathcal{E}}$. As usual, let us assume that the state space of the composite object system-apparatus-environment is given by the tensor product of the individual Hilbert spaces, $\mathcal{H}_{\mathcal{S}} \otimes \mathcal{H}_{\mathcal{A}} \otimes \mathcal{H}_{\mathcal{E}}$. The linearity of the Schrödinger equation then yields the following time evolution of the entire system $\mathcal{S}\mathcal{A}\mathcal{E}$,

$$\left(\sum_n c_n |s_n\rangle \right) |a_r\rangle |e_0\rangle \xrightarrow{(1)} \left(\sum_n c_n |s_n\rangle |a_n\rangle \right) |e_0\rangle \xrightarrow{(2)} \sum_n c_n |s_n\rangle |a_n\rangle |e_n\rangle, \quad (2.10)$$

where the $|e_n\rangle$ are the states of the environment associated with the different pointer states $|a_n\rangle$ of the measuring apparatus. Note that while for two subsystems, say, \mathcal{S} and \mathcal{A} , there always exists a diagonal (“Schmidt”) decomposition of the final state of the form $\sum_n c_n |s_n\rangle |a_n\rangle$, for three subsystems (for example, \mathcal{S} , \mathcal{A} , and \mathcal{E}), a decomposition of the form $\sum_n c_n |s_n\rangle |a_n\rangle |e_n\rangle$ is not always possible. This implies that the total

⁷In this context we note that any nonpure density matrix can be written in many different ways, demonstrating that any partition in a particular ensemble of quantum states is arbitrary.

Hamiltonian that induces a time evolution of the above kind, Eq. (2.10), must be of a special form.⁸

Typically, the $|e_n\rangle$ will be product states of many microscopic subsystem states $|\varepsilon_n\rangle_i$ corresponding to the individual parts that form the environment, i.e., $|e_n\rangle = |\varepsilon_n\rangle_1|\varepsilon_n\rangle_2|\varepsilon_n\rangle_3\dots$. We see that a nonseparable and in most cases, for all practical purposes, irreversible (due to the enormous number of degrees of freedom of the environment) correlation has been established between the states of the system-apparatus combination \mathcal{SA} and the different states of the environment \mathcal{E} . Note that Eq. (2.10) also implies that the environment has recorded the state of the system—and, equivalently, the state of the system-apparatus composition. The environment, composed of many subsystems, thus acts as an amplifying, higher-order measuring device.

2.3.4 Decoherence and local suppression of interference

Interaction with the environment typically leads to a rapid vanishing of the diagonal terms in the local density matrix describing the probability distribution for the outcomes of measurements on the system. This effect has become known as environment-induced decoherence, and it has also frequently been claimed to imply at least a partial solution to the measurement problem.

General formalism

In Sec. 2.3.2, we have already introduced the concept of local (or reduced) density matrices and pointed out some caveats on their interpretation. In the context of the decoherence program, reduced density matrices arise as follows. Any observation will typically be restricted to the system-apparatus component, \mathcal{SA} , while the many degrees of freedom of the environment \mathcal{E} remain unobserved. Of course, typically some

⁸For an example of such a Hamiltonian, see the model of Zurek [337, 338] and its outline in Sec. 2.3.4 below. For a critical comment regarding limitations on the form of the evolution operator and the possibility of a resulting disagreement with experimental evidence, see Ref. [257].

degrees of freedom of the environment will always be included in our observation (e.g., some of the photons scattered off the apparatus) and we shall accordingly include them in the “observed part \mathcal{SA} of the universe.” The crucial point is that there still remains a comparatively large number of environmental degrees of freedom that will not be observed directly.

Suppose then that the operator $\hat{O}_{\mathcal{SA}}$ represents an observable of \mathcal{SA} only. Its expectation value $\langle \hat{O}_{\mathcal{SA}} \rangle$ is given by

$$\langle \hat{O}_{\mathcal{SA}} \rangle = \text{Tr}(\hat{\rho}_{\mathcal{S}\mathcal{A}\mathcal{E}}[\hat{O}_{\mathcal{SA}} \otimes \hat{I}_{\mathcal{E}}]) = \text{Tr}_{\mathcal{SA}}(\hat{\rho}_{\mathcal{S}\mathcal{A}}\hat{O}_{\mathcal{SA}}), \quad (2.11)$$

where the density matrix $\hat{\rho}_{\mathcal{S}\mathcal{A}\mathcal{E}}$ of the total $\mathcal{S}\mathcal{A}\mathcal{E}$ combination,

$$\hat{\rho}_{\mathcal{S}\mathcal{A}\mathcal{E}} = \sum_{mn} c_m c_n^* |s_m\rangle |a_m\rangle |e_m\rangle \langle s_n| \langle a_n| \langle e_n|, \quad (2.12)$$

has, for all practical purposes of statistical prediction, been replaced by the local (or reduced) density matrix $\hat{\rho}_{\mathcal{S}\mathcal{A}}$, obtained by “tracing out the unobserved degrees of the environment,” that is,

$$\hat{\rho}_{\mathcal{S}\mathcal{A}} = \text{Tr}_{\mathcal{E}}(\hat{\rho}_{\mathcal{S}\mathcal{A}\mathcal{E}}) = \sum_{mn} c_m c_n^* |s_m\rangle |a_m\rangle \langle s_n| \langle a_n| \langle e_n| e_m\rangle. \quad (2.13)$$

So far, $\hat{\rho}_{\mathcal{S}\mathcal{A}}$ contains characteristic interference terms $|s_m\rangle |a_m\rangle \langle s_n| \langle a_n|$, $m \neq n$, since we cannot assume from the outset that the basis vectors $|e_m\rangle$ of the environment are necessarily mutually orthogonal, i.e., that $\langle e_n| e_m\rangle = 0$ if $m \neq n$. Many explicit physical models for the interaction of a system with the environment (see Sec. 2.3.4 below for a simple example), however, have shown that due to the large number of subsystems that compose the environment, the pointer states $|e_n\rangle$ of the environment rapidly approach orthogonality, $\langle e_n| e_m\rangle(t) \rightarrow \delta_{n,m}$, such that the reduced density matrix $\hat{\rho}_{\mathcal{S}\mathcal{A}}$ becomes approximately orthogonal in the “pointer basis” $\{|a_n\rangle\}$; that is,

$$\hat{\rho}_{\mathcal{S}\mathcal{A}} \xrightarrow{t} \hat{\rho}_{\mathcal{S}\mathcal{A}}^d \approx \sum_n |c_n|^2 |s_n\rangle |a_n\rangle \langle s_n| \langle a_n| = \sum_n |c_n|^2 \hat{P}_n^{(\mathcal{S})} \otimes \hat{P}_n^{(\mathcal{A})}. \quad (2.14)$$

Here, $\widehat{P}_n^{(\mathcal{S})}$ and $\widehat{P}_n^{(\mathcal{A})}$ are the projection operators onto the eigenstates of \mathcal{S} and \mathcal{A} , respectively. Therefore the interference terms have vanished in this local representation, i.e., phase coherence has been locally lost. This is precisely the effect referred to as environment-induced decoherence. The decohered local density matrices describing the probability distribution of the outcomes of a measurement on the system-apparatus combination is formally (approximately) identical to the corresponding mixed-state density matrix. But as we pointed out in Sec. 2.3.2, we must be careful in interpreting this state of affairs, since full coherence is retained in the total density matrix $\rho_{\mathcal{S}\mathcal{A}\mathcal{E}}$.

An exactly solvable two-state model for decoherence

To see how the approximate mutual orthogonality of the environmental state vectors arises, let us discuss a simple model first introduced by Zurek [338]. Consider a system \mathcal{S} with two spin states $\{| \uparrow \rangle, | \downarrow \rangle\}$ that interacts with an environment \mathcal{E} described by a collection of N other two-state spins represented by $\{| \uparrow_k \rangle, | \downarrow_k \rangle\}$, $k = 1 \dots N$. The self-Hamiltonians $\widehat{H}_{\mathcal{S}}$ and $\widehat{H}_{\mathcal{E}}$ and the self-interaction Hamiltonian $\widehat{H}_{\mathcal{E}\mathcal{E}}$ of the environment are taken to be equal to zero. Only the interaction Hamiltonian $\widehat{H}_{\mathcal{S}\mathcal{E}}$ that describes the coupling of the spin of the system to the spins of the environment is assumed to be nonzero and of the form

$$\widehat{H}_{\mathcal{S}\mathcal{E}} = (| \uparrow \rangle \langle \uparrow | - | \downarrow \rangle \langle \downarrow |) \otimes \sum_k g_k (| \uparrow_k \rangle \langle \uparrow_k | - | \downarrow_k \rangle \langle \downarrow_k |) \bigotimes_{k' \neq k} \widehat{I}_{k'}, \quad (2.15)$$

where the g_k are coupling constants and $\widehat{I}_k = (| \uparrow_k \rangle \langle \uparrow_k | + | \downarrow_k \rangle \langle \downarrow_k |)$ is the identity operator for the k th environmental spin. Applied to the initial state before the interaction is turned on,

$$|\psi(0)\rangle = (a| \uparrow \rangle + b| \downarrow \rangle) \bigotimes_{k=1}^N (\alpha_k| \uparrow_k \rangle + \beta_k| \downarrow_k \rangle), \quad (2.16)$$

this Hamiltonian yields a time evolution of the state given by

$$|\psi(t)\rangle = a| \uparrow \rangle |E_{\uparrow}(t)\rangle + b| \downarrow \rangle |E_{\downarrow}(t)\rangle, \quad (2.17)$$

where the two environmental states $|E_{\uparrow}(t)\rangle$ and $|E_{\downarrow}(t)\rangle$ are

$$|E_{\uparrow}(t)\rangle = |E_{\downarrow}(-t)\rangle = \bigotimes_{k=1}^N (\alpha_k e^{ig_k t} |\uparrow_k\rangle + \beta_k e^{-ig_k t} |\downarrow_k\rangle). \quad (2.18)$$

The reduced density matrix $\rho_S(t) = \text{Tr}_{\mathcal{E}}(|\psi(t)\rangle\langle\psi(t)|)$ is then

$$\rho_S(t) = |a|^2 |\uparrow\rangle\langle\uparrow| + |b|^2 |\downarrow\rangle\langle\downarrow| + z(t) ab^* |\uparrow\rangle\langle\downarrow| + z^*(t) a^* b |\downarrow\rangle\langle\uparrow|, \quad (2.19)$$

where the interference coefficient $z(t)$ which determines the weight of the off-diagonal elements in the reduced density matrix is given by

$$z(t) = \langle E_{\uparrow}(t) | E_{\downarrow}(t) \rangle = \prod_{k=1}^N (|\alpha_k|^2 e^{ig_k t} + |\beta_k|^2 e^{-ig_k t}), \quad (2.20)$$

and thus

$$|z(t)|^2 = \prod_{k=1}^N \{1 + [(|\alpha_k|^2 - |\beta_k|^2)^2 - 1] \sin^2 2g_k t\}. \quad (2.21)$$

At $t = 0$, $z(t) = 1$, i.e., the interference terms are fully present, as expected. If $|\alpha_k|^2 = 0$ or 1 for each k , i.e., if the environment is in an eigenstate of the interaction Hamiltonian $\hat{H}_{S\mathcal{E}}$ of the type $|\uparrow_1\rangle|\uparrow_2\rangle|\downarrow_3\rangle\cdots|\uparrow_N\rangle$, and/or if $2g_k t = m\pi$ ($m = 0, 1, \dots$), then $z(t)^2 \equiv 1$ so coherence is retained over time. However, under realistic circumstances, we can typically assume a random distribution of the initial states of the environment (i.e., of coefficients α_k, β_k) and of the coupling coefficients g_k . Then, in the long-time average,

$$\langle |z(t)|^2 \rangle_{t \rightarrow \infty} \simeq 2^{-N} \prod_{k=1}^N [1 + (|\alpha_k|^2 - |\beta_k|^2)^2] \xrightarrow{N \rightarrow \infty} 0, \quad (2.22)$$

so the off-diagonal terms in the reduced density matrix become strongly damped for large N .

It can also be shown directly that, given very general assumptions about the distribution of the couplings g_k (namely, requiring their initial distribution to have finite variance), $z(t)$ exhibits a Gaussian time dependence of the form $z(t) \propto e^{iAt} e^{-B^2 t^2/2}$, where A and B are real constants [348]. For the special case in which $\alpha_k = \alpha$ and

$g_k = g$ for all k , this behavior of $z(t)$ can be immediately seen by first rewriting $z(t)$ as the binomial expansion

$$z(t) = (|\alpha|^2 e^{igt} + |\beta|^2 e^{-igt})^N = \sum_{l=0}^N \binom{N}{l} |\alpha|^{2l} |\beta|^{2(N-l)} e^{ig(2l-N)t}. \quad (2.23)$$

For large N , the binomial distribution can then be approximated by a Gaussian,

$$\binom{N}{l} |\alpha|^{2l} |\beta|^{2(N-l)} \approx \frac{e^{-(l-N|\alpha|^2)^2/(2N|\alpha|^2|\beta|^2)}}{\sqrt{2\pi N|\alpha|^2|\beta|^2}}, \quad (2.24)$$

in which case $z(t)$ becomes

$$z(t) = \sum_{l=0}^N \frac{e^{-(l-N|\alpha|^2)^2/(2N|\alpha|^2|\beta|^2)}}{\sqrt{2\pi N|\alpha|^2|\beta|^2}} e^{ig(2l-N)t}, \quad (2.25)$$

that is, $z(t)$ is the Fourier transform of an (approximately) Gaussian distribution and is therefore itself (approximately) Gaussian.

Detailed model calculations, in which the environment is typically represented by a more sophisticated model consisting of a collection of harmonic oscillators [349, 344, 198, 68, 301, 189], have shown that the damping occurs on extremely short decoherence time scales τ_D , which are typically many orders of magnitude shorter than the thermal relaxation. Even microscopic systems such as large molecules are rapidly decohered by the interaction with thermal radiation on a time scale that is much shorter than any practical observation could resolve; for mesoscopic systems such as dust particles, the 3K cosmic microwave background radiation is sufficient to yield strong and immediate decoherence [197, 339].

Within τ_D , $|z(t)|$ approaches zero and remains close to zero, fluctuating with an average standard deviation of the random-walk type $\sigma \sim \sqrt{N}$ [338]. However, the multiple periodicity of $z(t)$ implies that coherence, and thus the purity of the reduced density matrix, will reappear after a certain time τ_r , which can be shown to be very long and of the Poincaré type with $\tau_r \sim N!$. For macroscopic environments of realistic but finite sizes, τ_r can exceed the lifetime of the universe [338], but nevertheless always remains finite.

From a conceptual point of view, recurrence of coherence is of little relevance. The recurrence time could only be infinitely long in the hypothetical case of an infinitely large environment. In this situation, off-diagonal terms in the reduced density matrix would be irreversibly damped and lost in the limit $t \rightarrow \infty$, which has sometimes been regarded as describing a physical collapse of the state vector [182]. But the assumption of infinite sizes and times is never realized in nature [43], nor can information ever be truly lost (as achieved by a “true” state vector collapse) through unitary time evolution—full coherence is always retained at all times in the total density matrix $\rho_{S,AE}(t) = |\psi(t)\rangle\langle\psi(t)|$.

We can therefore state the general conclusion that, except for exceptionally well-isolated and carefully prepared microscopic and mesoscopic systems, the interaction of the system with the environment causes the off-diagonal terms of the local density matrix, expressed in the pointer basis and describing the probability distribution of the possible outcomes of a measurement on the system, to become extremely small in a very short period of time, and that this process is irreversible for all practical purposes.

2.3.5 Environment-induced superselection

Let us now turn to the second main consequence of the interaction with the environment, namely, the environment-induced selection of stable preferred-basis states. We discussed in Sec. 2.2.3 the fact that the quantum-mechanical measurement scheme as represented by Eq. (2.1) does not uniquely define the expansion of the postmeasurement state and thereby leaves open the question of which observable can be considered as having been measured by the apparatus. This situation is changed by the inclusion of the environment states in Eq. (2.10), for the following two reasons:

- (1) *Environment-induced superselection of a preferred basis.* The interaction between the apparatus and the environment singles out a set of mutually com-

muting observables.

- (2) *The existence of a tridecompositional uniqueness theorem* [128, 86, 65]. If a state $|\psi\rangle$ in a Hilbert space $\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3$ can be decomposed into the diagonal (“Schmidt”) form $|\psi\rangle = \sum_i \alpha_i |\phi_i\rangle_1 |\phi_i\rangle_2 |\phi_i\rangle_3$, the expansion is unique provided that the $\{|\phi_i\rangle_1\}$ and $\{|\phi_i\rangle_2\}$ are sets of linearly independent, normalized vectors in \mathcal{H}_1 and \mathcal{H}_2 , respectively, and that $\{|\phi_i\rangle_3\}$ is a set of mutually noncollinear normalized vectors in \mathcal{H}_3 . This can be generalized to an N -decompositional uniqueness theorem, in which $N \geq 3$. Note that it is not always possible to decompose an arbitrary pure state of more than two systems ($N \geq 3$) into the Schmidt form $|\psi\rangle = \sum_i \alpha_i |\phi_i\rangle_1 |\phi_i\rangle_2 \cdots |\phi_i\rangle_N$, but if the decomposition exists, its uniqueness is guaranteed.

The tridecompositional uniqueness theorem ensures that the expansion of the final state in Eq. (2.10) is unique, which fixes the ambiguity in the choice of the set of possible outcomes. It demonstrates that the inclusion of (at least) a third “system” (here referred to as the environment) is necessary to remove the basis ambiguity.

Of course, given any pure state in the composite Hilbert space $\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3$, the tridecompositional uniqueness theorem neither tells us whether a Schmidt decomposition exists nor specifies the unique expansion itself (provided the decomposition is possible), and since the precise states of the environment are generally not known, an additional criterion is needed that determines what the preferred states will be.

Stability criterion and pointer basis

The decoherence program has attempted to define such a criterion based on the interaction with the environment and the idea of robustness and preservation of correlations. The environment thus plays a double role in suggesting a solution to the preferred-basis problem: it selects a preferred pointer basis, and it guarantees its uniqueness via the tridecompositional uniqueness theorem.

In order to motivate the basis superselection approach proposed by the decoherence program, we note that in step (2) of Eq. (2.10) we tacitly assumed that interaction with the environment does not disturb the established correlation between the state of the system, $|s_n\rangle$, and the corresponding pointer state $|a_n\rangle$. This assumption can be viewed as a generalization of the concept of “faithful measurement” to the realistic case in which the environment is included. Faithful measurement in the usual sense concerns step (1), namely, the requirement that the measuring apparatus \mathcal{A} act as a reliable “mirror” of the states of the system \mathcal{S} by forming only correlations of the form $|s_n\rangle|a_n\rangle$ but not $|s_m\rangle|a_n\rangle$ with $m \neq n$. But since any realistic measurement process must include the inevitable coupling of the apparatus to its environment, the measurement could hardly be considered faithful as a whole if the interaction with the environment disturbed the correlations between the system and the apparatus.⁹

It was therefore first suggested by Zurek [337] that the preferred pointer basis be taken as the basis which “contains a reliable record of the state of the system \mathcal{S} ” [337, p. 1519], i.e., the basis in which the system-apparatus correlations $|s_n\rangle|a_n\rangle$ are left undisturbed by the subsequent formation of correlations with the environment (the *stability criterion*). One can then find a sufficient criterion for dynamically stable pointer states that preserve the system-apparatus correlations in spite of the interaction of the apparatus with the environment by requiring all pointer state projection operators $\hat{P}_n^{(\mathcal{A})} = |a_n\rangle\langle a_n|$ to commute with the apparatus-environment interaction Hamiltonian $\hat{H}_{\mathcal{AE}}$,¹⁰

$$[\hat{P}_n^{(\mathcal{A})}, \hat{H}_{\mathcal{AE}}] = 0 \quad \text{for all } n. \quad (2.26)$$

This implies that any correlation of the measured system (or any other system, for

⁹For fundamental limitations on the precision of von Neumann measurements of operators that do not commute with a globally conserved quantity, see the Wigner-Araki-Yanase theorem [321, 20].

¹⁰For simplicity, we assume here that the environment \mathcal{E} interacts directly only with the apparatus \mathcal{A} , but not with the system \mathcal{S} .

instance an observer) with the eigenstates of a *preferred apparatus observable*,

$$\hat{O}_{\mathcal{A}} = \sum_n \lambda_n \hat{P}_n^{(\mathcal{A})}, \quad (2.27)$$

is preserved, and that the states of the environment reliably mirror the pointer states $\hat{P}_n^{(\mathcal{A})}$. In this case, the environment can be regarded as carrying out a nondemolition measurement on the apparatus. The commutativity requirement, Eq. (2.26), is obviously fulfilled if $\hat{H}_{\mathcal{AE}}$ is a function of $\hat{O}_{\mathcal{A}}$, $\hat{H}_{\mathcal{AE}} = \hat{H}_{\mathcal{AE}}(\hat{O}_{\mathcal{A}})$. Conversely, system-apparatus correlations in which the states of the apparatus are not eigenstates of an observable that commutes with $\hat{H}_{\mathcal{AE}}$ will in general be rapidly destroyed by the interaction.

Put the other way around, this implies that the environment determines through the form of the interaction Hamiltonian $\hat{H}_{\mathcal{AE}}$, a preferred apparatus observable $\hat{O}_{\mathcal{A}}$, Eq. (2.27), and thereby also the states of the system that are measured by the apparatus, that is, reliably recorded through the formation of dynamically stable quantum correlations. The tridecompositional uniqueness theorem then guarantees the uniqueness of the expansion of the final state $|\psi\rangle = \sum_n c_n |s_n\rangle |a_n\rangle |e_n\rangle$ (where no constraints on the c_n have to be imposed) and thereby the uniqueness of the preferred pointer basis.

Other criteria similar to the commutativity requirement, Eq. (2.26), have been suggested for the selection of the preferred pointer basis because it turns out that in realistic cases the simple relation of Eq. (2.26) can usually only be fulfilled approximately [349, 340]. More general criteria, for example, have been based on the von Neumann entropy $-\text{Tr} \rho_{\Psi}^2(t) \ln \rho_{\Psi}^2(t)$, or the purity $\text{Tr} \rho_{\Psi}^2(t)$, with the goal of finding the most robust states or the states which become least entangled with the environment in the course of the evolution [349, 340, 341, 344]. Pointer states are obtained by extremizing the measure (i.e., minimizing entropy, or maximizing purity, etc.) over the initial state $|\Psi\rangle$ and requiring the resulting states to be robust when varying the time t . Application of this method leads to a ranking of the possible pointer states

with respect to their “classicality,” i.e., their robustness with respect to interaction with the environment, and thus allows for the selection of preferred pointer basis in terms of the “most classical” pointer states (the “predictability sieve” [340, 349]). Although the proposed criteria differ somewhat and other meaningful criteria are likely to be suggested in the future, it is hoped that in the macroscopic limit the resulting stable pointer states obtained from different criteria will turn out to be very similar [344]. For some toy models (in particular, for harmonic-oscillator models that lead to coherent states as pointer states), this has already been verified explicitly (see, for example, Refs. [208, 340, 110, 198, 127]).

Selection of quasiclassical properties

System-environment interaction Hamiltonians frequently describe a scattering process of surrounding particles (photons, air molecules, etc.) interacting with the system under study. Since the force laws describing such processes typically depend on some power of distance (such as $\propto r^{-2}$ in Newton’s or Coulomb’s force law), the interaction Hamiltonian will usually commute with the position basis, such that, according the commutativity requirement of Eq. (2.26), the preferred basis will be in position space. The fact that position is frequently the determinate property of our experience can then be explained by referring to the dependence of most interactions on distance [337, 338, 339].

This holds, in particular, for mesoscopic and macroscopic systems, as demonstrated, for instance, by the pioneering study of Joos and Zeh [197], in which surrounding photons and air molecules are shown to continuously “measure” the spatial structure of dust particles, leading to rapid decoherence into an apparent (improper) mixture of wave packets that are sharply peaked in position space. Similar results sometimes even hold for microscopic systems (usually found in energy eigenstates; see below) when they occur in distinct spatial structures that couple strongly to the surrounding medium. For instance, chiral molecules such as sugar are always observed

to be in chirality eigenstates (left-handed and right-handed) which are superpositions of different energy eigenstates [178, 332]. This is explained by the fact that the spatial structure of these molecules is continuously “monitored” by the environment, for example, through the scattering of air molecules, which gives rise to a much stronger coupling than could typically be achieved by a measuring device that was intended to measure, say, parity or energy; furthermore, any attempt to prepare such molecules in energy eigenstates would lead to immediate decoherence into environmentally stable (“dynamically robust”) chirality eigenstates, thus selecting position as the preferred basis.

On the other hand, it is well known that many systems, especially in the microscopic domain, are typically found in energy eigenstates, even if the interaction Hamiltonian depends on a different observable than energy, e.g., position. Paz and Zurek [245] have shown that this situation arises when the predominant frequencies present in the environment are significantly lower than the intrinsic frequencies of the system, that is, when the separation between the energy states of the system is greater than the largest energies available in the environment. Then, the environment will only be able to monitor quantities that are constants of motion. In the case of nondegeneracy, this will be energy, thus leading to the environment-induced superselection of energy eigenstates for the system.

Another example of environment-induced superselection that has been studied is related to the fact that only eigenstates of the charge operator are observed, but never superpositions of different charges. The existence of the corresponding superselection rules was first only postulated [318, 319], but could subsequently be explained in the framework of decoherence by referring to the interaction of the charge with its own Coulomb (far) field, which takes the role of an “environment,” leading to immediate decoherence of charge superpositions into an apparent mixture of charge eigenstates [154, 153].

In general, three different cases have typically been distinguished (for example, in

Ref. [245]) for the kind of pointer observable emerging from an interaction with the environment, depending on the relative strengths of the system's self-Hamiltonian \hat{H}_S and of the system-environment interaction Hamiltonian \hat{H}_{SE} :

- (1) When the dynamics of the system are dominated by \hat{H}_{SE} , i.e., the interaction with the environment, the pointer states will be eigenstates of \hat{H}_{SE} (and thus typically eigenstates of position). This case corresponds to the typical quantum measurement setting; see, for example, the model of Zurek [337, 338], which is outlined in Sec. 2.3.4 above.
- (2) When the interaction with the environment is weak and \hat{H}_S dominates the evolution of the system (that is, when the environment is “slow” in the above sense), a case that frequently occurs in the microscopic domain, pointer states will arise that are energy eigenstates of \hat{H}_S [245].
- (3) In the intermediate case, when the evolution of the system is governed by \hat{H}_{SE} and \hat{H}_S in roughly equal strength, the resulting preferred states will represent a “compromise” between the first two cases; for instance, the frequently studied model of quantum Brownian motion has shown the emergence of pointer states localized in phase space, i.e., in both position and momentum [349, 344, 198, 301, 127].

Implications for the preferred-basis problem

The decoherence program proposes that the preferred basis be selected by the requirement that correlations be preserved in spite of the interaction with the environment, and thus be chosen through the form of the system-environment interaction Hamiltonian. This seems certainly reasonable, since only such “robust” states will in general be observable—and, after all, we solely seek an explanation for our experience (see the discussion in Sec. 2.2.2). Although only particular examples have been

studied (for a survey and references, see, for example, Refs. [198, 48, 344]), the results thus far suggest that the selected properties are in agreement with our observation: for mesoscopic and macroscopic objects the distance-dependent scattering interaction with surrounding air molecules, photons, etc., will in general give rise to immediate decoherence into spatially localized wave packets and thus select position as the preferred basis. On the other hand, when the environment is comparably “slow,” as is frequently the case for microscopic systems, environment-induced superselection will typically yield energy eigenstates as the preferred states.

The clear merit of the approach of environment-induced superselection lies in the fact that the preferred basis is not chosen in an *ad hoc* manner simply to make our measurement records determinate or to match our experience of which physical quantities are usually perceived as determinate (for example, position). Instead the selection is motivated on physical, observer-free grounds, that is, through the system-environment interaction Hamiltonian. The vast space of possible quantum-mechanical superpositions is reduced so much because the laws governing physical interactions depend only on a few physical quantities (position, momentum, charge, and the like), and the fact that precisely these are the properties that appear determinate to us is explained by the dependence of the preferred basis on the form of the interaction. The appearance of “classicality” is therefore grounded in the structure of the physical laws—certainly a highly satisfying and reasonable approach.

The above argument in favor of the approach of environment-induced superselection could, of course, be considered as inadequate on a fundamental level: All physical laws are discovered and formulated by us, so they can contain only the determinate quantities of our experience. These are the only quantities we can perceive and thus include in a physical law. Thus the derivation of determinacy from the structure of our physical laws might seem circular. However, we argue again that it suffices to demand a subjective solution to the preferred-basis problem—that is, to provide an answer to the question of why we perceive only such a small subset of properties as

determinate, not whether there really are determinate properties (on an ontological level) and what they are (cf. the remarks in Sec. 2.2.2).

We might also worry about the generality of this approach. One would need to show that any such environment-induced superselection leads, in fact, to precisely those properties that appear determinate to us. But this would require precise knowledge of the system and the interaction Hamiltonian. For simple toy models, the relevant Hamiltonians can be written down explicitly. In more complicated and realistic cases, this will in general be very difficult, if not impossible, since the form of the Hamiltonian will depend on the particular system or apparatus and the monitoring environment under consideration, where, in addition, the environment is not only difficult to define precisely, but also constantly changing, uncontrollable, and, in essence, infinitely large.

But the situation is not as hopeless as it might sound, since we know that the interaction Hamiltonian will, in general, be based on the set of known physical laws which, in turn, employ only a relatively small number of physical quantities. So as long as we assume the stability criterion and consider the set of known physical quantities as complete, we can automatically anticipate that the preferred basis will be a member of this set. The remaining, yet very relevant, question is then which subset of these properties will be chosen in a specific physical situation (for example, will the system preferably be found in an eigenstate of energy or position?), and to what extent this will match the experimental evidence. To give an answer, one will usually need a more detailed knowledge of the interaction Hamiltonian and of its relative strength with respect to the self-Hamiltonian of the system in order to verify the approach. Besides, as mentioned in Sec. 2.3.5, there exist other criteria than the commutativity requirement, and whether they all lead to the same determinate properties is a question that has not yet been fully explored.

Finally, a fundamental conceptual difficulty of the decoherence-based approach to the preferred-basis problem is the lack of a general criterion for what defines the sys-

tems and the “unobserved” degrees of freedom of the environment (see the discussion in Sec. 2.3.1). While in many laboratory-type situations, the division into system and environment might seem straightforward, it is not clear *a priori* how quasiclassical observables can be defined through environment-induced superselection on a larger and more general scale, when larger parts of the universe are considered where the split into subsystems is not suggested by some specific system-apparatus-surroundings setup.

To summarize, environment-induced superselection of a preferred basis (i) proposes an explanation for why a particular pointer basis gets chosen at all—by arguing that it is only the pointer basis that leads to stable, and thus perceivable, records when the interaction of the apparatus with the environment is taken into account; and (ii) argues that the preferred basis will correspond to a subset of the set of the determinate properties of our experience, since the governing interaction Hamiltonian will depend solely on these quantities. But it does not tell us precisely what the pointer basis will be in any given physical situation, since it will usually be hardly possible to write down explicitly the relevant interaction Hamiltonian in realistic cases. This also means that it will be difficult to argue that any proposed criterion based on the interaction with the environment will always and in all generality lead to exactly those properties that we perceive as determinate.

More work remains to be done, therefore, to fully explore the general validity and applicability of the approach of environment-induced superselection. But since the results obtained thus far from toy models have been in promising agreement with empirical data, there is little reason to doubt that the decoherence program has proposed a very valuable criterion for explaining the emergence of preferred states and their robustness. The fact that the approach is derived from physical principles should be counted additionally in its favor.

Pointer basis vs instantaneous Schmidt states

The so-called Schmidt basis, obtained by diagonalizing the (reduced) density matrix of the system at each instant of time, has been frequently studied with respect to its ability to yield a preferred basis (see, for example, Refs. [327, 9, 10]), having led some to consider the Schmidt basis states as describing “instantaneous pointer states” [9]. However, as it has been emphasized (for example, by Zurek [340]), any density matrix is diagonal in some basis, and this basis will in general not play any special interpretive role. Pointer states that are supposed to correspond to quasi-classical stable observables must be derived from an explicit criterion for classicality (typically, the stability criterion); the simple mathematical diagonalization procedure of the instantaneous density matrix will generally not suffice to determine a quasi-classical pointer basis (see the studies by Barvinsky and Kamenshchik [35] and Kent and McElwaine [202]).

In a more refined method, one refrains from computing instantaneous Schmidt states and instead allows for a characteristic decoherence time τ_D to pass during which the reduced density matrix decoheres (a process that can be described by an appropriate master equation) and becomes approximately diagonal in the stable pointer basis, the basis that is selected by the stability criterion. Schmidt states are then calculated by diagonalizing the decohered density matrix. Since decoherence usually leads to rapid diagonality of the reduced density matrix in the stability-selected pointer basis to a very good approximation, the resulting Schmidt states are typically very similar to the pointer basis except when the pointer states are very nearly degenerate. The latter situation is readily illustrated by considering the approximately diagonalized decohered density matrix

$$\rho = \begin{pmatrix} 1/2 + \delta & \omega^* \\ \omega & 1/2 - \delta \end{pmatrix}, \quad (2.28)$$

where $|\omega| \ll 1$ (strong decoherence) and $\delta \ll 1$ (near-degeneracy; see Ref. [10]).

If decoherence led to exact diagonality, $\omega = 0$, the eigenstates would be, for any fixed value of δ , proportional to $(0, 1)$ and $(1, 0)$ (corresponding to the “ideal” pointer states). However, for fixed $\omega > 0$ (approximate diagonality) and $\delta \rightarrow 0$ (degeneracy), the eigenstates become proportional to $(\pm|\omega|/\omega, 1)$, which implies that, in the case of degeneracy, the Schmidt decomposition of the reduced density matrix can yield preferred states that are very different from the stable pointer states, even if the decohered, rather than the instantaneous, reduced density matrix is diagonalized.

In summary, it is important to emphasize that stability (or a similar criterion) is the relevant requirement for the emergence of a preferred quasiclassical basis, which cannot, in general, be achieved by simply diagonalizing the instantaneous reduced density matrix. However, the eigenstates of the decohered reduced density matrix will, in many situations, approximate the quasiclassical stable pointer states well, especially when these pointer states are sufficiently nondegenerate.

2.3.6 Envariance, quantum probabilities, and the Born rule

In the following, we shall review an interesting and promising approach introduced recently by Zurek [344, 345, 347, 346] that aims to explain the emergence of quantum probabilities and to deduce the Born rule based on a mechanism termed “environment-assisted invariance,” or “envariance” for short, a particular symmetry property of entangled quantum states. The original exposition [344] was followed up by several articles by other authors, who assessed the approach, pointed out more clearly the assumptions entering into the derivation, and presented variants of the proof [281, 33, 230]. An expanded treatment of envariance and quantum probabilities that addresses some of the issues discussed in these papers and that offers an interesting outlook on further implications of envariance can be found in Ref. [346]. In our outline of the theory of envariance, we shall follow this most recent treatment, as it spells out the derivation and the required assumptions more explicitly and in greater detail and clarity than in Zurek’s earlier papers [344, 345, 347] (cf. also the remarks of

Schlosshauer and Fine [281]).

We include a discussion of Zurek's proposal here for two reasons. First, the derivation is based on the inclusion of an environment \mathcal{E} , entangled with the system \mathcal{S} of interest to which probabilities of measurement outcomes are to be assigned, and thus it matches well the spirit of the decoherence program. Second, and more importantly, despite the contributions of decoherence to explaining the emergence of subjective classicality from quantum mechanics, a consistent derivation of classicality (including a motivation for some of the axioms of quantum mechanics, as suggested by Zurek [344]) requires the separate derivation of the Born rule. The decoherence program relies heavily on the concept of reduced density matrices and the related formalism and interpretation of the trace operation, see Eq. (2.11), which *presuppose* Born's rule. Therefore decoherence itself cannot be used to derive the Born rule (as was tried, for example, by Deutsch [99] and Zurek [341]) since otherwise the argument would be rendered circular [330, 344].

There have been various attempts in the past to replace the postulate of the Born rule by a derivation. Gleason's theorem [155] has shown that if one imposes the condition that for any orthonormal basis of a given Hilbert space the sum of the probabilities associated with each basis vector must add up to one, the Born rule is the only possibility for the calculation of probabilities. However, Gleason's proof provides little insight into the physical meaning of the Born probabilities, and therefore various other attempts, typically based on a relative frequencies approach (i.e., on a counting argument), have been made towards a derivation of the Born rule in a no-collapse (and usually relative-state) setting (see, for example, Refs. [129, 103, 179, 102, 159, 147, 131, 99]). However, it was pointed out that these approaches fail due to the use of circular arguments [293, 200, 288, 34]; cf. also Refs. [315, 279].

Zurek's recently developed theory of envariance provides a promising new strategy for deriving, given certain assumptions, the Born rule in a manner that avoids the circularities of the earlier approaches. We shall outline the concept of envariance in

the following and show how it can lead to Born's rule.

Environment-assisted invariance

Zurek introduces his definition of envariance as follows. Consider a composite state $|\psi_{S\mathcal{E}}\rangle$ (where, as usual, S refers to the “system” and \mathcal{E} to some “environment”) in a Hilbert space given by the tensor product $\mathcal{H}_S \otimes \mathcal{H}_{\mathcal{E}}$, and a pair of unitary transformations $\widehat{U}_S = \widehat{u}_S \otimes \widehat{I}_{\mathcal{E}}$ and $\widehat{U}_{\mathcal{E}} = \widehat{I}_S \otimes \widehat{u}_{\mathcal{E}}$ acting on S and \mathcal{E} , respectively. If $|\psi_{S\mathcal{E}}\rangle$ is invariant under the combined application of \widehat{U}_S and $\widehat{U}_{\mathcal{E}}$,

$$\widehat{U}_{\mathcal{E}}(\widehat{U}_S|\psi_{S\mathcal{E}}\rangle) = |\psi_{S\mathcal{E}}\rangle, \quad (2.29)$$

$|\psi_{S\mathcal{E}}\rangle$ is called *envariant under \widehat{u}_S* . In other words, the change in $|\psi_{S\mathcal{E}}\rangle$ induced by acting on S via \widehat{U}_S can be undone by acting on \mathcal{E} via $\widehat{U}_{\mathcal{E}}$. Note that envariance is a distinctly quantum feature, absent from pure classical states, and a consequence of quantum entanglement.

The main argument of Zurek's derivation is based on a study of a composite pure state in the diagonal Schmidt decomposition

$$|\psi_{S\mathcal{E}}\rangle = \frac{1}{\sqrt{2}}(e^{i\varphi_1}|s_1\rangle|e_1\rangle + e^{i\varphi_2}|s_1\rangle|e_1\rangle), \quad (2.30)$$

where the $\{|s_k\rangle\}$ and $\{|e_k\rangle\}$ are sets of orthonormal basis vectors that span the Hilbert spaces \mathcal{H}_S and $\mathcal{H}_{\mathcal{E}}$, respectively. The case of higher-dimensional state spaces can be treated similarly, and a generalization to expansion coefficients of different magnitudes can be made by application of a standard counting argument [345, 346]. The Schmidt states $|s_k\rangle$ are identified with the outcomes, or “events” [347, p. 12], to which probabilities are to be assigned.

Zurek now states three simple assumptions, called “facts” [346, p. 4] (see also the discussion in Ref. [281]):

- (A1) A unitary transformation of the form $\cdots \otimes \widehat{I}_S \otimes \cdots$ does not alter the state of S .

- (A2) All measurable properties of \mathcal{S} , including probabilities of outcomes of measurements on \mathcal{S} , are fully determined by the state of \mathcal{S} .
- (A3) The state of \mathcal{S} is completely specified by the global composite state vector $|\psi_{\mathcal{SE}}\rangle$.

Given these assumptions, one can show that the state of \mathcal{S} and any measurable properties of \mathcal{S} cannot be affected by envariant transformations. The proof goes as follows. The effect of an envariant transformation $\hat{u}_{\mathcal{S}} \otimes \hat{I}_{\mathcal{E}}$ acting on $|\psi_{\mathcal{SE}}\rangle$ can be undone by a corresponding “countertransformation” $\hat{I}_{\mathcal{S}} \otimes \hat{u}_{\mathcal{E}}$ that restores the original state vector $|\psi_{\mathcal{SE}}\rangle$. Since it follows from (A1) that the latter transformation has left the state of \mathcal{S} unchanged, but (A3) implies that the final state of \mathcal{S} (after the transformation and countertransformation) is identical to the initial state of \mathcal{S} , the first transformation $\hat{u}_{\mathcal{S}} \otimes \hat{I}_{\mathcal{E}}$ cannot have altered the state of \mathcal{S} either. Thus, using assumption (A2), it follows that an envariant transformation $\hat{u}_{\mathcal{S}} \otimes \hat{I}_{\mathcal{E}}$ acting on $|\psi_{\mathcal{SE}}\rangle$ leaves any measurable properties of \mathcal{S} unchanged, in particular the probabilities associated with outcomes of measurements performed on \mathcal{S} .

Let us now consider two different envariant transformations: A *phase transformation* of the form

$$\hat{u}_{\mathcal{S}}(\xi_1, \xi_2) = e^{i\xi_1} |s_1\rangle\langle s_1| + e^{i\xi_2} |s_2\rangle\langle s_2| \quad (2.31)$$

that changes the phases associated with the Schmidt product states $|s_k\rangle|e_k\rangle$ in Eq. (2.30), and a *swap transformation*

$$\hat{u}_{\mathcal{S}}(1 \leftrightarrow 2) = e^{i\xi_{12}} |s_1\rangle\langle s_2| + e^{i\xi_{21}} |s_2\rangle\langle s_1| \quad (2.32)$$

that exchanges the pairing of the $|s_k\rangle$ with the $|e_l\rangle$. Based on the assumptions (A1)–(A3) mentioned above, envariance of $|\psi_{\mathcal{SE}}\rangle$ under these transformations means that measurable properties of \mathcal{S} cannot depend on the phases φ_k in the Schmidt expansion of $|\psi_{\mathcal{SE}}\rangle$, Eq. (2.30). Similarly, it follows that a swap $\hat{u}_{\mathcal{S}}(1 \leftrightarrow 2)$ leaves the state of \mathcal{S} unchanged, and that the consequences of the swap cannot be detected by any measurement that pertains to \mathcal{S} alone.

Deducing the Born rule

Together with an additional assumption, this result can then be used to show that the probabilities of the “outcomes” $|s_k\rangle$ appearing in the Schmidt decomposition of $|\psi_{S\mathcal{E}}\rangle$ must be equal, thus arriving at Born’s rule for the special case of a state-vector expansion with coefficients of equal magnitude. Zurek [346] offers three possibilities for such an assumption. Here we shall limit our discussion to one of these possible assumptions (see also the comments in Ref. [281]):

- (A4) The Schmidt product states $|s_k\rangle|e_k\rangle$ appearing in the state-vector expansion of $|\psi_{S\mathcal{E}}\rangle$ imply a direct and perfect correlation of the measurement outcomes associated with the $|s_k\rangle$ and $|e_k\rangle$. That is, if an observable $\hat{O}_S = \sum s_{kl}|s_k\rangle\langle s_l|$ is measured on S and $|s_k\rangle$ is obtained, a subsequent measurement of $\hat{O}_{\mathcal{E}} = \sum e_{kl}|e_k\rangle\langle e_l|$ on \mathcal{E} will yield $|e_k\rangle$ with certainty (i.e., with probability equal to one).

This assumption explicitly introduces a probability concept into the derivation. (Similarly, the two other possible assumptions suggested by Zurek establish a connection between the state of S and probabilities of outcomes of measurements on S .)

Then, denoting the probability for the outcome $|s_k\rangle$ by $p(|s_k\rangle; |\psi_{S\mathcal{E}}\rangle)$ when the composite system $S\mathcal{E}$ is described by the state vector $|\psi_{S\mathcal{E}}\rangle$, this assumption implies that

$$p(|s_k\rangle; |\psi_{S\mathcal{E}}\rangle) = p(|e_k\rangle; |\psi_{S\mathcal{E}}\rangle). \quad (2.33)$$

After acting on $|\psi_{S\mathcal{E}}\rangle$ with the envariant swap transformation $\hat{U}_S = \hat{u}_S(1 \leftrightarrow 2) \otimes \hat{I}_{\mathcal{E}}$ [see Eq. (2.32)] and using assumption (A4) again, we get

$$\begin{aligned} p(|s_1\rangle; \hat{U}_S|\psi_{S\mathcal{E}}\rangle) &= p(|e_2\rangle; \hat{U}_S|\psi_{S\mathcal{E}}\rangle), \\ p(|s_2\rangle; \hat{U}_S|\psi_{S\mathcal{E}}\rangle) &= p(|e_1\rangle; \hat{U}_S|\psi_{S\mathcal{E}}\rangle). \end{aligned} \quad (2.34)$$

Now, when a “counterswap” $\hat{U}_{\mathcal{E}} = \hat{I}_{\mathcal{S}} \otimes u_{\mathcal{E}}(1 \leftrightarrow 2)$ is applied to $|\psi_{\mathcal{SE}}\rangle$, the original state vector $|\psi_{\mathcal{SE}}\rangle$ is restored, i.e., $\hat{U}_{\mathcal{E}}(\hat{U}_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle) = |\psi_{\mathcal{SE}}\rangle$. It then follows from assumptions (A2) and (A3) listed above that

$$p(|s_k\rangle; \hat{U}_{\mathcal{E}}\hat{U}_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle) = p(|s_k\rangle; |\psi_{\mathcal{SE}}\rangle). \quad (2.35)$$

Furthermore, assumptions (A1) and (A2) imply that the first and second swap cannot have affected the measurable properties of \mathcal{E} and \mathcal{S} , respectively, particularly not the probabilities for outcomes of measurements on \mathcal{E} (\mathcal{S}),

$$\begin{aligned} p(|s_k\rangle; \hat{U}_{\mathcal{E}}\hat{U}_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle) &= p(|s_k\rangle; \hat{U}_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle), \\ p(|e_k\rangle; \hat{U}_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle) &= p(|e_k\rangle; |\psi_{\mathcal{SE}}\rangle). \end{aligned} \quad (2.36)$$

Combining Eqs. (2.33)–(2.36) yields

$$\begin{aligned} p(|s_1\rangle; |\psi_{\mathcal{SE}}\rangle) &\stackrel{(2.35)}{=} p(|s_1\rangle; \hat{U}_{\mathcal{E}}\hat{U}_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle) \\ &\stackrel{(2.36)}{=} p(|s_1\rangle; \hat{U}_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle) \\ &\stackrel{(2.34)}{=} p(|e_2\rangle; \hat{U}_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle) \\ &\stackrel{(2.36)}{=} p(|e_2\rangle; |\psi_{\mathcal{SE}}\rangle) \\ &\stackrel{(2.33)}{=} p(|s_2\rangle; |\psi_{\mathcal{SE}}\rangle), \end{aligned} \quad (2.37)$$

which establishes the desired result $p(|s_1\rangle; |\psi_{\mathcal{SE}}\rangle) = p(|s_2\rangle; |\psi_{\mathcal{SE}}\rangle)$. The general case of unequal coefficients in the Schmidt decomposition of $|\psi_{\mathcal{SE}}\rangle$ can then be treated by means of a simple counting method [345, 346], leading to Born’s rule for probabilities that are rational numbers. Using a continuity argument, this result can be further generalized to include probabilities that cannot be expressed as rational numbers [346].

Summary and outlook

If one grants the stated assumptions, Zurek’s development of the theory of envariance offers a novel and promising way of deducing Born’s rule in a noncircular

manner. Compared to the relatively well-studied field of decoherence, envariance and its consequences have only begun to be explored. In this review, we have focused on envariance in the context of a derivation of the Born rule, but other far-reaching implications of envariance have recently been suggested by Zurek [346]. For example, envariance could also account for the emergence of an environment-selected preferred basis (that is, for environment-induced superselection) without an appeal to the trace operation or to reduced density matrices. This could open up the possibility of a re-development of the decoherence program based on fundamental quantum-mechanical principles that do not require one to presuppose the Born rule; this also might shed new light, for example, on the interpretation of reduced density matrices that has led to much controversy in discussions of decoherence (see Sec. 2.3.2). As of now, the development of such ideas is at a very early stage, but we can expect further interesting results derived from envariance in the near future.

2.4 The role of decoherence in interpretations of quantum mechanics

It was not until the early 1970s that the importance of the interaction of physical systems with their environments for a realistic quantum-mechanical description of these systems was realized and a proper viewpoint on such interactions was established [326, 327]. It took another decade for the first concise formulation of the theory of decoherence [337, 338] to be worked out and for numerical studies to be made that showed the ubiquity and effectiveness of decoherence effects [197]. Of course, by that time, several interpretive approaches to quantum mechanics had already been established, for example, Everett-style relative-state interpretations [129], the concept of modal interpretations introduced by van Fraassen [304, 305], and the pilot-wave theory of de Broglie and Bohm [52].

When the relevance of decoherence effects was recognized by (parts of) the scientific community, decoherence provided a motivation for a fresh look at the existing interpretations and for the introduction of changes and extensions to these interpreta-

tions, as well as for new interpretations. Some of the central questions in this context were, and still are, the following:

1. Can decoherence by itself solve certain foundational issues at least for all practical purposes, such as to make certain interpretive additives superfluous? What, then, are the crucial remaining foundational problems?
2. Can decoherence protect an interpretation from empirical disproof?
3. Conversely, can decoherence provide a mechanism to exclude an interpretive strategy as incompatible with quantum mechanics and/or as empirically inadequate?
4. Can decoherence physically motivate some of the assumptions on which an interpretation is based and give them a more precise meaning?
5. Can decoherence serve as an amalgam that would unify and simplify a spectrum of different interpretations?

These and other questions have been widely discussed, both in the context of particular interpretations and with respect to the general implications of decoherence for any interpretation of quantum mechanics. In particular, interpretations that uphold the universal validity of the unitary Schrödinger time evolution, most notably relative-state and modal interpretations, have frequently incorporated environment-induced superselection of a preferred basis and decoherence into their framework. It is the purpose of this section to critically investigate the implications of decoherence for the existing interpretations of quantum mechanics, with particular attention to the questions outlined above.

2.4.1 General implications of decoherence for interpretations

When measurements are understood as ubiquitous interactions that lead to the formation of quantum correlations, the selection of a preferred basis becomes in most cases a fundamental requirement. This also corresponds, in general, to the question of what properties are being ascribed to systems (or worlds, minds, etc.). Thus the preferred-basis problem is at the heart of any interpretation of quantum mechanics. Some of the difficulties that must be faced in solving the preferred-basis problem are

- (i) to decide whether the selection of any preferred basis (or quantity or property) is justified at all or only an artefact of our subjective experience;
- (ii) if we decide on (i) in the positive, to select those determinate quantity or quantities (what appears determinate to us does not need to appear determinate to other kinds of observers, nor does it need to be the “true” determinate property);
- (iii) to avoid any *ad hoc* character of the choice and any possible empirical inadequacy or inconsistency with the confirmed predictions of quantum mechanics;
- (iv) if a multitude of quantities is selected that apply differently among different systems, to be able to formulate specific rules that specify the determinate quantity or quantities under every circumstance;
- (v) to ensure that the basis is chosen such that if the system is embedded in a larger (composite) system, the principle of property composition holds, i.e., the property selected by the basis of the original system should also persist when the system is considered as part of a larger composite system.¹¹

¹¹This is a problem encountered in some modal interpretations [87].

The hope is then that environment-induced superselection of a preferred basis can provide a universal mechanism that fulfills the above criteria and solves the preferred-basis problem on strictly physical grounds.

A popular reading of the decoherence program typically goes as follows. First, the interaction of the system with the environment selects a preferred basis, i.e., a particular set of quasiclassical robust states that commute, at least approximately, with the Hamiltonian governing the system–environment interaction. Since the form of the interaction Hamiltonians usually depends on familiar “classical” quantities, the preferred states will typically also correspond to the small set of “classical” properties. Decoherence then quickly damps superpositions between the localized preferred states when only the system is considered. This is taken as an explanation of the appearance to a local observer of a “classical” world of determinate, “objective” (in the sense of being robust) properties. The tempting interpretation of these achievements is then to conclude that this accounts for the observation of unique (via environment-induced superselection) and definite (via decoherence) pointer states at the end of the measurement, and the measurement problem appears to be solved, at least for all practical purposes.

However, the crucial difficulty in the above reasoning is justifying the second step: How is one to interpret the local suppression of interference in spite of the fact that full coherence is retained in the total state that describes the system-environment combination? While the local destruction of interference allows one to infer the emergence of an (improper) ensemble of individually localized components of the wave function, one still needs to impose an interpretive framework that explains why only one of the localized states is realized and/or perceived. This has been done in various interpretations of quantum mechanics, typically on the basis of the decohered reduced density matrix to ensure consistency with the predictions of the Schrödinger dynamics and thus empirical adequacy.

In this context, one might raise the question whether retention of full coherence in the composite state of the system-environment combination could ever lead to empirical conflicts with the ascription of definite values to (mesoscopic and macroscopic) systems in some decoherence-based interpretive approach. After all, one could think of enlarging the system so as to include the environment in such a way that measurements could now actually reveal the persisting quantum coherence even on a macroscopic level. However, Zurek [338] asserted that such measurements would be impossible to carry out in practice, a statement that was supported by a simple model calculation by Omnes [240, p. 356] for a body with a macroscopic number (10^{24}) of degrees of freedom.

2.4.2 The standard and the Copenhagen interpretation

As is well known, the standard interpretation (“orthodox” quantum mechanics) postulates that every measurement induces a discontinuous break in the unitary time evolution of the state through the collapse of the total wave function onto one of its terms in the state-vector expansion (uniquely determined by the eigenbasis of the measured observable), which selects a single term in the superposition as representing the outcome. The nature of the collapse is not at all explained, and thus the definition of measurement remains unclear. Macroscopic superpositions are not *a priori* forbidden, but are never observed since any observation would entail a measurement-like interaction. In the following, we shall also consider a “Copenhagen” variant of the standard interpretation, which adds an additional key element, postulating the necessity of classical concepts in order to describe quantum phenomena, including measurements.

The problem of definite outcomes

The interpretive rule of orthodox quantum mechanics that tells us when we can speak of outcomes is given by the e-e link.¹² This is an “objective” criterion since it allows us to infer the existence of a definite state of the system to which a value of a physical quantity can be ascribed. Within this interpretive framework (and without presuming the collapse postulate) decoherence cannot solve the problem of outcomes: Phase coherence between macroscopically different pointer states is preserved in the state that includes the environment, and we can always enlarge the system so as to include (at least parts of) the environment. In other words, the superposition of different pointer positions still exists, coherence is only “*delocalized* into the larger system” [203, p. 5], that is, into the environment—or, as Joos and Zeh [197, p. 224] put it, “the interference terms still exist, but they are not *there*”—and the process of decoherence could in principle always be reversed. Therefore, if we assume the orthodox e-e link to establish the existence of determinate values of physical quantities, decoherence cannot ensure that the measuring device actually ever is in a definite pointer state (unless, of course, the system is initially in an eigenstate of the observable), or that measurements have outcomes at all. Much of the general criticism directed against decoherence with respect to its ability to solve the measurement problem (at least in the context of the standard interpretation) has been centered on this argument.

Note that, with respect to the global postmeasurement state vector, given by the final step in Eq. (2.10), the interaction with the environment has only led to additional entanglement. It has not transformed the state vector in any way, since the rapidly increasing orthogonality of the states of the environment associated with the different pointer positions has not influenced the state description at all. In brief,

¹²It is not particularly relevant for the subsequent discussion whether the e-e link is assumed in its “exact” form, i.e., requiring the exact eigenstates of an observable, or a “fuzzy” form that allows the ascription of definiteness based on only approximate eigenstates or on wave functions with (tiny) “tails.”

the entanglement brought about by interaction with the environment could even be considered as making the measurement problem worse. Bacciagaluppi [25, Sec. 3.2] puts it like this:

Intuitively, if the environment is carrying out, without our intervention, lots of approximate position measurements, then the measurement problem ought to apply more widely, also to these spontaneously occurring measurements. (...) The state of the object and the environment could be a superposition of zillions of very well localised terms, each with slightly different positions, and which are collectively spread over a macroscopic distance, even in the case of everyday objects. (...) If everything is in interaction with everything else, everything is entangled with everything else, and that is a worse problem than the entanglement of measuring apparatuses with the measured probes.

Only once we have formed the reduced pure-state density matrix $\hat{\rho}_{S,A}$, Eq. (2.13), can the orthogonality of the environmental states have an effect; then, $\hat{\rho}_{S,A}$ dynamically evolves into the improper ensemble $\hat{\rho}_{S,A}^d$ [Eq. (2.14)]. However, as pointed out in our general discussion of reduced density matrices in Sec. 2.3.2, the orthodox rule of interpreting superpositions prohibits regarding the components in the sum of Eq. (2.14) as corresponding to individual well-defined quantum states.

Rather than considering the postdecoherence state of the system (or, more precisely, of the system-apparatus combination S,A), we can instead analyze the influence of decoherence on the *expectation values* of observables pertaining to S,A ; after all, such expectation values are what local observers would measure in order to arrive at conclusions about S,A . The diagonalized reduced density matrix, Eq. (2.14), together with the trace relation, Eq. (2.11), implies that, for all practical purposes, the statistics of the system S,A will be indistinguishable from that of a proper mixture (ensemble) by any local observation on S,A . That is, given (i) the trace rule $\langle \hat{O} \rangle = \text{Tr}(\hat{\rho}\hat{O})$ and (ii) the interpretation of $\langle \hat{O} \rangle$ as the expectation value of an observable \hat{O} , the

expectation value of any observable $\hat{O}_{S,A}$ restricted to the local system S,A will be for all practical purposes identical to the expectation value of this observable if S,A had been in one of the states $|s_n\rangle|a_n\rangle$ (as if S,A were described by an ensemble of states). In other words, decoherence has effectively removed any interference terms (such as $|s_m\rangle|a_m\rangle\langle a_n|\langle s_n|$ where $m \neq n$) from the calculation of the trace $\text{Tr}(\hat{\rho}_{S,A}\hat{O}_{S,A})$ and thereby from the calculation of the expectation value $\langle\hat{O}_{S,A}\rangle$. It has therefore been claimed that *formal equivalence*—i.e., the fact that decoherence transforms the reduced density matrix into a form identical to that of a density matrix representing an ensemble of pure states—yields *observational equivalence* in the sense above, namely, the (local) indistinguishability of the expectation values derived from these two types of density matrices via the trace rule.

But we must be careful in interpreting the correspondence between the mathematical formalism (such as the trace rule) and the common terms employed in describing “the world.” In quantum mechanics, the identification of the expression “ $\text{Tr}(\rho A)$ ” as the expectation value of a quantity relies on the mathematical fact that, when writing out this trace, it is found to be equal to a sum over the possible outcomes of the measurement, weighted by the Born probabilities for the system to be “thrown” into a particular state corresponding to each of these outcomes in the course of a measurement. This certainly represents our common-sense intuition about the meaning of expectation values as the sum over possible values that can appear in a given measurement, multiplied by the relative frequency of actual occurrence of these values in a series of such measurements. This interpretation, however, presumes (i) that measurements have outcomes, (ii) that measurements lead to definite “values,” (iii) that measurable physical quantities are identified as operators (observables) in a Hilbert space, and (iv) that the modulus square of the expansion coefficients of the state in terms of the eigenbasis of the observable can be interpreted as representing probabilities of actual measurement outcomes (Born rule).

Thus decoherence brings about an apparent (and approximate) mixture of states

that seem, based on the models studied, to correspond well to those states that we perceive as determinate. Moreover, our observation tells us that this apparent mixture indeed looks like a proper ensemble in a measurement situation, as we observe that measurements lead to the “realization” of precisely one state in the “ensemble.” But within the framework of the orthodox interpretation, decoherence cannot explain this crucial step from an apparent mixture to the existence and/or perception of single outcomes.

Observables, measurements, and environment-induced superselection

In the standard and Copenhagen interpretationS, property ascription is determined by an observable that represents the measurement of a physical quantity and that in turn defines the preferred basis. However, any Hermitian operator can play the role of an observable, and thus any given state has the potential for an infinite number of different properties whose attribution is usually mutually exclusive unless the corresponding observables commute (in which case they share a common eigenbasis which preserves the uniqueness of the preferred basis). What then determines the observable that is being measured? As our discussion in Sec. 2.2.3 has demonstrated, the derivation of the measured observable from the particular form of a given state-vector expansion can lead to paradoxical results since this expansion is in general nonunique, so the observable must be chosen by other means. In the standard and Copenhagen interpretations, it is essentially the “user” who simply “chooses” the particular observable to be measured and thus determines which properties the system possesses.

This positivist point of view has, of course, led to a lot of controversy, since it runs counter to the attempt to establish an observer-independent reality that has been the central pursuit of natural science since its beginning. Moreover, in practice, one certainly does not have the freedom to choose any arbitrary observable and measure it; instead, one has “instruments” (including one’s senses) that are designed to

measure a particular observable. For most (and maybe all) practical purposes, this will ultimately boil down to a single relevant observable, namely, position. But what, then, makes the instruments designed for such a particular observable?

Answering this crucial question essentially means abandoning the orthodox view of treating measurements as a “black box” process that has little, if any, relation to the workings of actual physical measurements (where measurements can here be understood in the broadest sense of a “monitoring” of the state of the system). The first key point, the formalization of measurements as a formation of quantum correlations between system and apparatus, goes back to the early years of quantum mechanics and is reflected in the measurement scheme of von Neumann [312], but it does not resolve the issue of how the choice of observables is made. The second key point, the explicit inclusion of the environment in a description of the measurement process, was brought into quantum theory by the studies of decoherence. Zurek’s stability criterion [337] discussed in Sec. 2.3.5 has shown that measurements must be of such a nature as to establish stable records, where stability is to be understood as preserving the system-apparatus correlations in spite of the inevitable interaction with the surrounding environment. The “user” cannot choose the observables arbitrarily, but must design a measuring device whose interaction with the environment is such as to ensure stable records in the sense above (which, in turn, defines a measuring device for this observable). In the reading of orthodox quantum mechanics, this can be interpreted as the environment determining the properties of the system.

In this sense, the decoherence program has embedded the rather formal concept of measurement as proposed by the standard and Copenhagen interpretations—with its vague notion of observables that are seemingly freely chosen by the observer—in a more realistic and physical framework. This is accomplished via the specification of observer-free criteria for the selection of the measured observable through the physical structure of the measuring device and its interaction with the environment, which is, in most cases, needed to amplify the measurement record and thereby to make it

accessible to the external observer.

The concept of classicality in the Copenhagen interpretation

The Copenhagen interpretation additionally postulates that classicality is not to be derived from quantum mechanics, for example, as the macroscopic limit of an underlying quantum structure (as is in some sense assumed, but not explicitly derived, in the standard interpretation), but instead that it be viewed as an indispensable and irreducible element of a complete quantum theory—and, in fact, be considered as a concept prior to quantum theory. In particular, the Copenhagen interpretation assumes the existence of macroscopic measurement apparatuses that obey classical physics and that are not supposed to be described in quantum mechanical terms (in sharp contrast to the von Neumann measurement scheme, which rather belongs to the standard interpretation); such a classical apparatus is considered necessary in order to make quantum-mechanical phenomena accessible to us in terms of the “classical” world of our experience. This strict dualism between the system \mathcal{S} , to be described by quantum mechanics, and the apparatus \mathcal{A} , obeying classical physics, also entails the existence of an essentially fixed boundary between \mathcal{S} and \mathcal{A} , which separates the microworld from the macroworld (the “Heisenberg cut”). This boundary cannot be moved significantly without destroying the observed phenomenon (i.e., the full interacting compound \mathcal{SA}).

Especially in the light of the insights gained from decoherence it seems impossible to uphold the notion of a fixed quantum–classical boundary on a fundamental level of the theory. Environment-induced superselection and suppression of interference have demonstrated how quasiclassical robust states can emerge, or remain absent, using the quantum formalism alone and over a broad range of microscopic to macroscopic scales, and have established the notion that the boundary between \mathcal{S} and \mathcal{A} is to a large extent movable towards \mathcal{A} . Similar results have been obtained from the general study of quantum nondemolition measurements (see, for example, Chap. 19

of Ref. [23]) which include the monitoring of a system by its environment. Also note that since the apparatus is described in classical terms, it is macroscopic by definition; but not every apparatus must be macroscopic: the actual “instrument” could well be microscopic. Only the “amplifier” must be macroscopic. As an example, consider Zurek’s toy model of decoherence [337], outlined in Sec. 2.3.4, in which the instrument can be represented by a bistable atom while the environment plays the role of the amplifier; a more realistic example is a macroscopic detector of gravitational waves that is treated as a quantum-mechanical harmonic oscillator.

Based on the progress already achieved by the decoherence program, it is reasonable to anticipate that decoherence embedded in some additional interpretive structure could lead to a complete and consistent derivation of the classical world from quantum mechanical principles. This would make the assumption of an intrinsically classical apparatus (which has to be treated outside of the realm of quantum mechanics) appear as neither a necessary nor a viable postulate. Bacciagaluppi [26, p. 22] refers to this strategy as “having Bohr’s cake and eating it”: acknowledging the correctness of Bohr’s notion of the necessity of a classical world (“having Bohr’s cake”), but being able to view the classical world as part of and as emerging from a purely quantum-mechanical world.

2.4.3 Relative-state interpretations

Everett’s original proposal [129] of a relative-state interpretation of quantum mechanics has motivated several strands of interpretation, presumably owing to the fact that Everett himself never clearly spelled out how his theory was supposed to work. The system-observer duality of orthodox quantum mechanics introduces into the theory external “observers” who are not described by the deterministic laws of quantum systems but instead follow a stochastic indeterminism. This approach obviously runs into problems when the universe as a whole is considered: by definition, there cannot be any external observers. The central idea of Everett’s proposal is then to abandon

duality and instead (i) to assume the existence of a total state $|\Psi\rangle$ representing the state of the entire universe and (ii) to uphold the universal validity of the Schrödinger evolution, while (iii) postulating that all terms in the superposition of the total state at the completion of the measurement actually correspond to physical states. Each such physical state can be understood as relative (a) to the state of the other part in the composite system (as in Everett's original proposal; also see Refs. [269, 226]), (b) to a particular "branch" of a constantly "splitting" universe (the *many-worlds interpretations*, popularized by DeWitt [101] and Deutsch [97]), or (c) to a particular "mind" in the set of minds of the conscious observer (the *many-minds interpretation*; see, for example, Ref. [217]). In other words, every term in the final-state superposition can be viewed as representing an equally "real" physical state of affairs that is realized in a different "branch of reality."

Decoherence adherents have typically been inclined towards relative-state interpretations (see, for instance, Refs. [326, 327, 328, 341]), presumably because the Everett approach takes unitary quantum mechanics essentially "as is," with a minimum of added interpretive elements. This matches well the spirit of the decoherence program, which attempts to explain the emergence of classicality purely from the formalism of basic quantum mechanics. It may also seem natural to identify the decohering components of the wave function with different Everett branches. Conversely, proponents of relative-state interpretations have frequently employed the mechanism of decoherence in solving the difficulties associated with this class of interpretations (see, for example, Refs. [276, 277, 278, 97, 98, 100, 302, 313, 314]).

There are many different readings and versions of relative-state interpretations, especially with respect to what defines the "branches," "worlds," and "minds"; whether we deal with one, a multitude, or an infinity of such worlds and minds; and whether there is an actual (physical) or only perspectival splitting of the worlds and minds into different branches corresponding to the terms in the superposition. Does the world or mind split into two separate copies (thus somehow doubling all the mat-

ter contained in the original system), or is there just a “reassignment” of states to a multitude of worlds or minds of constant (typically infinite) number, or is there only one physically existing world or mind in which each branch corresponds to different “aspects” (whatever they are). Regardless, in the following discussion of the key implications of decoherence, the precise details and differences of these various strands of interpretation will, for the most part, be largely irrelevant.

Relative-state interpretations face two core difficulties. First, the preferred-basis problem: If states are only relative, the question arises, relative to what? What determines the particular basis terms that are used to define the branches, which in turn define the worlds or minds in the next instant of time? When precisely does the “splitting” occur? Which properties are made determinate in each branch, and how are they connected to the determinate properties of our experience? Second, what is the meaning of probabilities, since *every* outcome actually occurs in some world or mind, and how can Born’s rule be motivated in such an interpretive framework?

Everett branches and the preferred-basis problem

Stapp [292, p. 1043] stated the requirement that “a many-worlds interpretation of quantum theory exists only to the extent that the associated basis problem is solved.” In the context of relative-state interpretations, the preferred-basis problem is not only much more severe than in the orthodox interpretation, but also more fundamental for several reasons: (i) The branching occurs continuously and essentially everywhere; in the general sense of measurements understood as the formation of quantum correlations, every newly formed correlation, whether it pertains to microscopic or macroscopic systems, corresponds to a branching. (ii) The ontological implications are much more drastic, at least in those relative-state interpretations, which assume an actual “splitting” of worlds or minds, since the choice of the basis determines the resulting “world” or “mind” as a whole.

The environment-based basis superselection criteria of the decoherence program

have frequently been employed to solve the preferred-basis problem of relative-state interpretations (see, for example, Refs. [341, 67, 313, 314]). There are several advantages in a decoherence-related approach to selecting the preferred Everett bases: First, no *a priori* existence of a preferred basis needs to be postulated, but instead the preferred basis arises naturally from the physical criterion of robustness. Second, the selection will be likely to yield empirical adequacy, since the decoherence program is derived solely from the well-confirmed Schrödinger dynamics (modulo the possibility that robustness may not be the universally valid criterion). Lastly, the decohered components of the wave function evolve in such a way that they can be reidentified over time (forming “trajectories” in the preferred state spaces) and thus can be used to define stable, temporally extended Everett branches. Similarly, such trajectories can be used to ensure robust observer record states and/or environmental states that make information about the state of the system of interest widely accessible to observers (see, for example, Zurek’s “existential interpretation,” outlined in Sec. 2.4.3 below).

While the idea of directly associating the environment-selected basis states with Everett worlds seems natural and straightforward, it has also been subject to criticism. Stapp [292] has argued that an Everett-type interpretation must aim at determining a denumerable set of distinct branches that correspond to the apparently discrete events of our experience. Among these branches one must be able to assign determinate values and finite probabilities according to the usual rules and therefore one would need to be able to specify a denumerable set of mutually orthogonal projection operators. It is well known, however [341], that the preferred states chosen through the interaction with the environment via the stability criterion frequently form an overcomplete set of states—often a continuum of narrow Gaussian-type wave packets, for example, the coherent states of harmonic-oscillator models that are not necessarily orthogonal (i.e., the Gaussians may overlap; see Refs. [208, 349]). Stapp therefore considers this approach to the preferred-basis problem in relative-state interpretations

to be unsatisfactory. Zurek [343] has rebutted this criticism by pointing out that a collection of harmonic oscillators that would lead to such overcomplete sets of Gaussians cannot serve as an adequate model of the human brain, and it is ultimately only in the brain where the perception of denumerability and mutual exclusiveness of events must be accounted for (see Sec. 2.2.2); when neurons are more appropriately modeled as two-state systems, the issue raised by Stapp disappears (for a discussion of decoherence in a simple two-state model, see Sec. 2.3.4).¹³

The approach of using environment-induced superselection and decoherence to define the Everett branches has also been criticized on grounds of being “conceptually approximate,” since the stability criterion generally leads only to an approximate specification of a preferred basis and therefore cannot give an “exact” definition of the Everett branches (see, for example, the comments of Zeh [327], Kent [200], and also the well-known “anti-FAPP” position of Bell [44]). Wallace [314, pp. 90–91] has argued against such an objection as

(...) arising from a view implicit in much discussion of Everett-style interpretations: that certain concepts and objects in quantum mechanics must either enter the theory formally in its axiomatic structure, or be regarded as illusion. (...) [Instead] the emergence of a classical world from quantum mechanics is to be understood in terms of the emergence from the theory of certain sorts of structures and patterns, and ... this means that we have no need (as well as no hope!) of the precision which Kent [200] and others (...) demand.

Accordingly, in view of our argument in Sec. 2.2.2 for considering subjective solutions to the measurement problem as sufficient, there is no *a priori* reason to doubt that an “approximate” criterion for the selection of the preferred basis can give a meaningful definition of the Everett branches—one that is empirically adequate and

¹³For interesting quantitative results on the role of decoherence in neuronal processes, see Ref. [296].

that accounts for our experiences. The environment-superselected basis emerges naturally from the physically very reasonable criterion of robustness together with the purely quantum mechanical effect of decoherence. It would be rather difficult to imagine what an axiomatically introduced “exact” rule could be able to select preferred bases in a manner that is similarly physically motivated and capable of ensuring empirical adequacy.

Besides using the environment-superselected pointer states to describe the Everett branches, various authors have directly used the instantaneous Schmidt decomposition of the composite state (or, equivalently, the set of orthogonal eigenstates of the reduced density matrix) to define the preferred basis (see also Sec. 2.3.5). This approach is easier to implement than the explicit search for dynamically stable pointer states since the preferred basis follows directly from a simple mathematical diagonalization procedure at each instant of time. Furthermore, it has been favored by some (see, e.g., Ref. [327]) since it gives an “exact” rule for basis selection in relative-state interpretations; the consistently quantum origin of the Schmidt decomposition, which matches well the “pure quantum-mechanics” spirit of Everett’s proposal (where the formalism of quantum mechanics supplies its own interpretation), has also been counted as an advantage [35]. In an earlier work, Deutsch [97] attributed a fundamental role to the Schmidt decomposition in relative-state interpretations as defining an “interpretation basis” that imposes the precise structure that is needed to give meaning to Everett’s basic concept.

However, as pointed out in Sec. 2.3.5, emerging basis states based on the instantaneous Schmidt states will frequently have properties that are very different from those selected by the stability criterion and that are undesirably nonclassical. For example, they may lack the spatial localization of the robustness-selected Gaussians [292]. The question to what extent the Schmidt basis states correspond to classical properties in Everett-style interpretations was investigated in detail by Barvinsky and Kamenshchik [35]. The authors study the similarity of the states selected by

the Schmidt decomposition to coherent states (i.e., minimum-uncertainty Gaussians) that are chosen as the “yardstick states” representing classicality (see also Ref. [127]). For the investigated toy models it is found that only subsets of the Everett worlds corresponding to the Schmidt decomposition exhibit classicality in this sense; furthermore, the degree of robustness of classicality in these branches is very sensitive to the choice of the initial state and the interaction Hamiltonian, such that classicality emerges typically only temporarily, and the Schmidt basis generally lacks robustness under time evolution. Similar difficulties with the Schmidt basis approach have been described by Kent and McElwaine [202].

These findings indicate that a selection criterion based on robustness provides a much more meaningful, physically transparent, and general rule for the selection of quasiclassical branches in relative-state interpretations, especially with respect to its ability to lead to wave-function components representing quasiclassical properties that can be reidentified over time (which a simple diagonalization of the reduced density matrix at each instant of time does not, in general, allow for).

Probabilities in Everett interpretations

Various attempts unrelated to decoherence have been made to find a consistent derivation of the Born probabilities (see, for instance, Refs. [129, 179, 102, 159, 147, 99]) in the explicit or implicit context of a relative-state interpretation, but several arguments have been presented that show that these approaches fail.¹⁴ When the effects of decoherence and environment-induced superselection are included, it seems natural to identify the diagonal elements of the decohered reduced density matrix (in the environment-superselected basis) with the set of possible elementary events and to interpret the corresponding coefficients as relative frequencies of worlds (or minds, etc.) in the Everett theory, assuming a typically infinite multitude of worlds, minds,

¹⁴See, for example, Refs. [293, 200, 288, 34]; however, also note the arguments of Wallace [315] and Gill [151], defending the approach of Deutsch [99]; see also Ref. [279].

etc. Since decoherence enables one to reidentify the individual localized components of the wave function over time (describing, for example, observers and their measurement outcomes attached to individual well-defined “worlds”), this leads to a natural interpretation of the Born probabilities as empirical frequencies.

However, decoherence cannot yield an actual derivation of the Born rule (for attempts in this direction, see Refs. [99, 341]). As mentioned before, this is so because the key elements of the decoherence program, the formalism and the interpretation of reduced density matrices and the trace rule, *presume* the Born rule. Attempts to consistently derive probabilities from reduced density matrices and from the trace rule are therefore subject to the charge of circularity [330, 344]. In Sec. 2.3.6, we outlined a recent proposal by Zurek [345] that evades this circularity and deduces the Born rule from envariance, a symmetry property of entangled systems, and from certain assumptions about the connection between the state of the system \mathcal{S} of interest, the state vector of the composite system \mathcal{SE} that includes an environment \mathcal{E} entangled with \mathcal{S} , and probabilities of outcomes of measurements performed on \mathcal{S} . Decoherence combined with this approach provides a framework in which quantum probabilities and the Born rule can be given a rather natural motivation, definition, and interpretation in the context of relative-state interpretations.

The “existential interpretation”

A well-known Everett-type interpretation that relies heavily on decoherence has been proposed by Zurek [340, 341] (see also the recent reevaluation in Ref. [346]). This approach, termed the “existential interpretation,” defines the reality, or objective existence, of a state as the possibility of finding out what the state is and simultaneously leaving it unperturbed, similar to a classical state.¹⁵ Zurek assigns a “rel-

¹⁵This intrinsically requires the notion of open systems, since in isolated systems, the observer would need to know in advance what observables commute with the state of the system, in order to perform a nondemolition measurement that avoids repreparing the state of the system.

ative objective existence” to the robust states (identified with elementary “events”) selected by the environmental stability criterion. By measuring properties of the system-environment interaction Hamiltonian and employing the robustness criterion, the observer can, at least in principle, determine the set of observables that can be measured on the system without perturbing it and thus find out its “objective” state. Alternatively, the observer can take advantage of the redundant records of the state of the system as monitored by the environment. By intercepting parts of this environment, for example, a fraction of the surrounding photons, he can determine the state of the system essentially without perturbing it (cf. also the related recent ideas of “quantum Darwinism” and the role of the environment as a “witness,” see Refs. [342, 344, 347, 234]).¹⁶

Zurek emphasizes the importance of stable records for observers, i.e., robust correlations between the environment-selected states and the memory states of the observer. Information must be represented physically, and thus the “objective” state of the observer who has detected one of the potential outcomes of a measurement must be physically distinct and objectively different from the state of an observer who has recorded an alternative outcome (since the record states can be determined from the outside without perturbing them—see the previous paragraph). The different objective states of the observer are, via quantum correlations, attached to different branches defined by the environment-selected robust states; they thus ultimately label the different branches of the universal state vector. This is claimed to lead to the perception of classicality; the impossibility of perceiving arbitrary superpositions is explained via the quick suppression of interference between different memory states induced by decoherence, where each (physically distinct) memory state represents an individual observer identity.

A similar argument has been given by Zeh [328] who employs decoherence together

¹⁶The partial ignorance is necessary to avoid redefinition of the state of the system.

with an (implicit) branching process to explain the perception of definite outcomes:

[A]fter an observation one need not necessarily conclude that only one component now *exists* but only that only one component *is observed*. (...) Superposed world components describing the registration of different macroscopic properties by the “same” observer are dynamically entirely independent of one another: they describe different observers. (...) He who considers this conclusion of an indeterminism or splitting of the observer’s identity, derived from the Schrödinger equation in the form of dynamically decoupling (“branching”) wave packets on a fundamental global configuration space, as unacceptable or “extravagant” may instead dynamically formalize the superfluous hypothesis of a disappearance of the “other” components by whatever method he prefers, but he should be aware that he may thereby also create his own problems: Any deviation from the global Schrödinger equation must in principle lead to observable effects, and it should be recalled that none have ever been discovered.

The existential interpretation has recently been connected to the theory of envariance (see Ref. [346] and Sec. 2.3.6). In particular, the derivation of Born’s rule based on envariance as outlined in Sec. 2.3.6 can be recast in the framework of the existential interpretation such that probabilities refer explicitly to the future record state of an observer. Such a concept of probability bears similarities with classical probability theory (for more details on these ideas, see Ref. [346]).

The existential interpretation continues Everett’s goal of interpreting quantum mechanics using the quantum-mechanical formalism itself. Zurek takes the standard no-collapse quantum theory “as is” and explores to what extent the incorporation of environment-induced superselection and decoherence (and recently also envariance) could form a viable interpretation that would, with a minimal additional interpretive framework, be capable of accounting for the perception of definite outcomes and of explaining the origin and nature of probabilities.

2.4.4 *Modal interpretations*

The first type of modal interpretation was suggested by van Fraassen [304, 305], based on his program of “constructive empiricism,” which proposes to take only empirical adequacy, but not necessarily “truth,” as the goal of science. Since then, a large number of interpretations of quantum mechanics have been suggested that can be considered as modal (for a review and discussion of some of the basic properties and problems of such interpretations, see Ref. [87]).

In general, the approach of modal interpretations consists in weakening the orthodox e-e link by allowing for the assignment of definite measurement outcomes even if the system is not in an eigenstate of the observable representing the measurement. In this way, one can preserve a purely unitary time evolution without the need for an additional collapse postulate to account for definite measurement results. Of course, this immediately raises the question of how physical properties that are perceived through measurements and measurement results are connected to the state, since the bidirectional link is broken between the eigenstate of the observable (which corresponds to the physical property) and the eigenvalue (which represents the manifestation of the value of this physical property in a measurement). The general goal of modal interpretations is then to specify rules that determine a catalog of possible properties of a system described by the density matrix ρ at time t . Two different views are typically distinguished: a semantic approach that only changes the way of talking about the connection between properties and state; and a realistic view that provides a different specification of what the possible properties of a system really are, given the state vector (or the density matrix).

Such an attribution of possible properties must fulfill certain requirements. For instance, probabilities for outcomes of measurements should be consistent with the usual Born probabilities of standard quantum mechanics; it should be possible to recover our experience of classicality in the perception of macroscopic objects; and

an explicit time evolution of properties and their probabilities should be definable that is consistent with the results of the Schrödinger equation. As we shall see in the following, decoherence has frequently been employed in modal interpretations to motivate and define rules for property ascription. Dieks [105, 106] has argued that one of the central goals of modal approaches is to provide an interpretation for decoherence.

Property assignment based on environment-induced superselection

The intrinsic difficulty of modal interpretations is to avoid any *ad hoc* character of the property assignment, yet to find generally applicable rules that lead to a selection of possible properties that include the determinate properties of our experience. To solve this problem, various modal interpretations have embraced the results of the decoherence program. A natural approach would be to employ the environment-induced superselection of a preferred basis—since it is based on an entirely physical and very general criterion (namely, the stability requirement) and has, for the cases studied, been shown to give results that agree well with our experience, thus matching van Fraassen’s goal of empirical adequacy—to yield sets of possible quasiclassical properties associated with the correct probabilities.

Furthermore, since the decoherence program is based solely on Schrödinger dynamics, the task of defining a time evolution of the “property states” and their associated probabilities that is in agreement with the results of unitary quantum mechanics would presumably be easier than in a model of property assignment in which the set of possibilities does not arise dynamically via the Schrödinger equation alone (for a detailed proposal for modal dynamics of the latter type, see Ref. [27]). The need for explicit dynamics of property states in modal interpretations is controversial. One can argue that it suffices to show that at each instant of time, the set of possibly possessed properties that can be ascribed to the system is empirically adequate, in the sense of containing the properties of our experience, especially with respect to

the properties of macroscopic objects (this is essentially the view of, for example, van Fraassen [304, 305]). On the other hand, this cannot ensure that these properties behave over time in agreement with our experience (for instance, that macroscopic objects that are left undisturbed do not change their position in space spontaneously in an observable manner). In other words, the emergence of classicality is to be tied not only to determinate properties at each instant of time, but also to the existence of quasiclassical “trajectories” in property space. Since decoherence allows one to reidentify components of the decohered density matrix over time, this could be used to derive property states with continuous, quasiclassical trajectorylike time evolution based on Schrödinger dynamics alone. For some discussions of this approach, see Refs. [181, 27].

The fact that the states emerging from decoherence and the stability criterion are sometimes nonorthogonal or form a continuum will presumably be of even less relevance in modal interpretations than in Everett-style interpretations (see Sec. 2.4.3) since the goal here is solely to specify sets of possible properties, of which only one set actually gets assigned to the system. Hence an “overlap” of the sets is not necessarily a problem (modulo the potential difficulty of a straightforward assignment of probabilities in such a situation).

Property assignment based on instantaneous Schmidt decompositions

Since it is usually rather difficult to determine explicitly the robust “pointer states” through the stability (or a similar) criterion, it would not be easy to specify a general rule for property assignment based on environment-induced superselection. To simplify this situation, several modal interpretations have restricted themselves to the orthogonal decomposition of the density matrix to define the set of properties that can be assigned (see, for instance, Refs. [204, 180, 104, 310, 65]). For example, the approach of Dieks [104] recognizes, by referring to the decoherence program, the relevance of the environment by considering a composite system-environment state

vector and its diagonal Schmidt decomposition, $|\psi\rangle = \sum_k \sqrt{p_k} |\phi_k^S\rangle |\phi_k^E\rangle$, which always exists. Possible properties that can be assigned to the system are then represented by the Schmidt projectors $\hat{P}_k = \lambda_k |\phi_k^S\rangle\langle\phi_k^S|$. Although all terms are present in the Schmidt expansion (that Dieks calls the “mathematical state”), the “physical state” is postulated to be given by only one of the terms, with probability p_k . A generalization of this approach to a decomposition into any number of subsystems has been described by Vermaas and Dieks [310]. In this sense, the Schmidt decomposition itself is taken to define an interpretation of quantum mechanics. Dieks [107] suggested a physical motivation for the Schmidt decomposition in modal interpretations based on the assumed requirement of a one-to-one correspondence between the properties of the system and its environment. For a comment on the violation of the property composition principle in such interpretations, see the analysis of Clifton [87].

A central problem associated with the approach of orthogonal decomposition is that it is not at all clear that the properties determined by the Schmidt diagonalization represent the determinate properties of our experience. As outlined in Sec. 2.3.5, the states selected by the (instantaneous) orthogonal decomposition of the reduced density matrix will in general differ from the robust “pointer states” chosen by the stability criterion of the decoherence program and may have distinctly nonclassical properties. That this will be the case especially when the states selected by the orthogonal decomposition are close to degeneracy has already been indicated in Sec. 2.3.5. It has also been explored in more detail in the context of modal interpretations by Bacciagaluppi [28] and Donald [111], who showed that in the case of near degeneracy (as it typically occurs for macroscopic systems with many degrees of freedom), the resulting projectors will be extremely sensitive to the precise form of the state [28]. Clearly such sensitivity is undesired since the projectors, and thus the properties of the system, will not be well behaved under the inevitable approximations employed in physics [111].

Property assignment based on decompositions of the decohered density matrix

Other authors therefore have appealed to the orthogonal decomposition of the decohered reduced density matrix (instead of the decomposition of the instantaneous density matrix) which has led to noteworthy results. When the system is represented by only a finite-dimensional Hilbert space, a discrete model of decoherence, the resulting states were indeed found to be typically close to the robust states selected by the stability criterion (for macroscopic systems, this typically meant localization in position space), unless again the final composite state was very nearly degenerate [29, 47] (see also Sec. 2.3.5). Thus, in sufficiently nondegenerate cases, decoherence can ensure that the definite properties selected by modal interpretations of the Dieks type will be appropriately close to the properties corresponding to the ideal pointer states if the modal properties are based on the orthogonal decomposition of the reduced decohered density matrix.

On the other hand, Bacciagaluppi [24] showed that in the more general and realistic case of an infinite-dimensional state space of the system, when one employs a continuous model of decoherence (namely, that of Joos and Zeh [197]), the predictions of the modal approach [104, 310] and those of decoherence can differ significantly. It was demonstrated that the definite properties obtained from the orthogonal decomposition of the decohered density matrix were highly delocalized (that is, smeared out over the entire spread of the state), although the coherence length of the density matrix itself was shown to be very small, so that decoherence indicated very localized properties. Thus, based on these results (and similar ones of Donald [111]), decoherence can be used to argue for the physical inadequacy of the rule for the assignment of definite properties as proposed by Dieks [104] and Vermaas and Dieks [310].

More generally, if the definite properties selected by the modal interpretation fail to mesh with the results of decoherence (in particular, when they also lack the desired classicality and correspondence to the determinate properties of our experience), we

are given reason to doubt whether the proposed rules for property assignment have sufficient physical motivation, legitimacy, or generality.

Concluding remarks

There are many different proposals that can be grouped under the heading of modal interpretations. They all share the problem of motivating and verifying a consistent system of property assignment. Using the robust pointer states selected by interaction with the environment and by the stability criterion is a step in the right direction, but the difficulty remains to derive a general rule for property assignment from this method that would yield explicitly the sets of possibilities in every situation. In certain cases, for example, close to degeneracy and in Hilbert-state spaces of infinite dimension, the simpler approach of deriving the possible properties from the orthogonal decomposition of the decohered reduced density matrix fails to yield the sharply localized, quasiclassical pointer states as selected by environmental robustness criteria. These are the cases in which decoherence can play a vital role in helping to identify inadequate rules for property assignment in modal interpretations.

2.4.5 Physical collapse theories

The basic idea of physical collapse theories is to introduce an explicit modification of the Schrödinger time evolution to achieve a physical mechanism for state-vector reduction (for an extensive recent review, see Ref. [37]). This is in general motivated by a “realist” interpretation of the state vector, that is, the state vector is directly identified with a physical state, which then requires reduction to one of the terms in the superposition to establish equivalence to the observed determinate properties of physical states, at least as far as the macroscopic realm is concerned.

The first proposals for theories of this type were made by Pearle [246, 251, 247] and Gisin [152], who developed dynamical reduction models that modify unitary dynamics such that a superposition of quantum states evolves continuously into one of its terms

(see also the review by Pearle [252]). Typically, terms representing external white noise are added to the Schrödinger equation, causing the squared amplitudes $|c_n(t)|^2$ in the state-vector expansion $|\Psi(t)\rangle = \sum_n c_n(t)|\psi_n\rangle$ to fluctuate randomly in time, while maintaining the normalization condition $\sum_n |c_n(t)|^2 = 1$ for all t . This process is known as *stochastic dynamical reduction*. Eventually one amplitude $|c_n(t)|^2 \rightarrow 1$, while all other squared coefficients $\rightarrow 0$ (the “gambler’s ruin game” mechanism), where $|c_n(t)|^2 \rightarrow 1$ with probability $|c_n(t=0)|^2$ (the squared coefficients in the initial precollapse state-vector expansion) in agreement with the Born probability interpretation of the expansion coefficients.

These early models exhibit two main difficulties. First, the preferred-basis problem: What determines the terms in the state-vector expansion into which the state vector gets reduced? Why does reduction lead to precisely the distinct macroscopic states of our experience and not superpositions thereof? Second, how can one account for the fact that the effectiveness of collapsing superpositions increases when going from microscopic to macroscopic scales?

These problems motivated *spontaneous localization* models, initially proposed by Ghirardi, Rimini, and Weber (GRW) [149]. Here state-vector reduction is not implemented as a dynamical process (i.e., as a continuous evolution over time), but instead occurs instantaneously and spontaneously, leading to a spatial localization of the wave function. To be precise, the N -particle wave function $\psi(\mathbf{x}_1, \dots, \mathbf{x}_N)$ is at random intervals multiplied by a Gaussian of the form $\exp[-(\mathbf{X} - \mathbf{x}_k)^2/2\Delta^2]$ (this process is often called a “hit” or a “jump”), and the resulting product is subsequently normalized. The occurrence of these hits is not explained, but rather postulated as a new fundamental physical mechanism. Both the coordinate \mathbf{x}_k and the “center of the hit” \mathbf{X} are chosen at random, but the probability for a specific \mathbf{X} is postulated to be given by the squared inner product of $\psi(\mathbf{x}_1, \dots, \mathbf{x}_N)$ with the Gaussian (and therefore hits are more likely to occur where $|\psi|^2$, viewed as a function of \mathbf{x}_k only, is large). The mean frequency ν of hits for a single microscopic particle is chosen so as to ef-

fectively preserve unitary time evolution for microscopic systems, while ensuring that for macroscopic objects containing a very large number N of particles the localization occurs rapidly (on the order of $N\nu$), in such a way as to preclude the persistence of spatially separated macroscopic superpositons (such as the pointer's being in a superpositon of "up" and "down") on time scales shorter than realistic observations could resolve. Ghirardi *et al.* [149] chose $\nu \approx 10^{-16} \text{ s}^{-1}$, so a macroscopic system with $N \approx 10^{23}$ particles undergoes localization on average every 10^{-7} s . Inevitable coupling to the environment can in general be expected to lead to a further drastic increase of N and therefore to an even higher localization rate. Note, however, that the localization process itself is independent of any interaction with environment, in sharp contrast to the decoherence approach.

Subsequently, the ideas of the stochastic dynamical reduction and GRW theory were combined into *continuous spontaneous localization* models [250, 148] in which localization of the GRW type can be shown to emerge from a nonunitary, nonlinear Itô stochastic differential equation, namely, the Schrödinger equation augmented by spatially correlated Brownian motion terms (see also Refs. [108, 109]). The particular choice of stochastic term determines the preferred basis. Frequently, the stochastic term has been based on the mass density which yields a GRW-type spatial localization [109, 250, 148], but stochastic terms driven by the Hamiltonian, leading to a reduction on an energy basis, have also been studied [39, 40, 229, 254, 255, 190, 135, 1, 4, 5]. If we focus on the first type of term, the Ghirardi-Rimini-Weber theory and continuous spontaneous localization become phenomenologically similar, and we shall refer to them jointly as "spontaneous localization" models in the following discussion whenever it is unnecessary to distinguish them explicitly.

The preferred-basis problem

Physical reduction theories typically remove wave-function collapse from the restrictive context of the orthodox interpretation (where the external observer arbi-

trarily selects the measured observable and thus determines the preferred basis), and rather understand reduction as a universal mechanism that acts constantly on every state vector regardless of an explicit measurement situation. In view of this it is particularly important to provide a definition for the states into which the wave function collapses.

As mentioned before, the original stochastic dynamical reduction models suffer from this preferred-basis problem. Taking into account environment-induced superselection of a preferred basis could help resolve this issue. Decoherence has been shown to occur, especially for mesoscopic and macroscopic objects, on extremely short time scales, and thus would presumably be able to bring about basis selection much faster than the time required for dynamical fluctuations to establish a “winning” expansion coefficient.

In contrast, the GRW theory solves the preferred-basis problem by postulating a mechanism that leads to reduction to a particular state vector in an expansion on a position basis, i.e., position is assumed to be the universal preferred basis. State-vector reduction then amounts to simply modifying the functional shape of the projection of the state vector $|\psi\rangle$ onto the position basis $\langle \mathbf{x}_1, \dots, \mathbf{x}_N \rangle$. This choice can be motivated by the insight that essentially all (human) observations must be grounded in a position measurement.¹⁷

On the one hand, the selection of position as the preferred basis is supported by the decoherence program, since physical interactions frequently are governed by distance-dependent laws. Given the stability criterion or a similar requirement, this leads to position as the preferred observable. In this sense, decoherence provides a physical motivation for the assumption of the GRW model. On the other hand, it makes this assumption appear as too restrictive as it cannot account for cases in

¹⁷This measurement may ultimately occur only in the brain of the observer; see the objection to the GRW model by Albert and Vaidman [7]. With respect to the general preference for position as the basis of measurements, see also the comment by Bell [44].

which position is not the preferred basis—for instance, in microscopic systems where typically energy is the robust observable, or in the superposition of (macroscopic) currents in SQUIDs (see Chap. 5). The GRW model simply excludes such cases by choosing the parameters of the spontaneous localization process such that microscopic systems remain generally unaffected by any state vector reduction. The basis selection approach proposed by the decoherence program is therefore much more general and also avoids the *ad hoc* character of the GRW theory by allowing for a range of preferred observables and motivating their choice on physical grounds.

A similar argument can be made with respect to the continuous spontaneous localization approach. Here, one essentially preselects a preferred basis through the particular choice of the stochastic terms added to the Schrödinger equation. This allows for a greater range of possible preferred bases, for instance by combining terms driven by the Hamiltonian and by the mass density, leading to a competition between localization in energy and position space (corresponding to the two most frequently observed eigenstates). Nonetheless, any particular choice of terms will again be subject to the charge of possessing an *ad hoc* flavor, in contrast to the physical definition of the preferred basis derived from the structure of the unmodified Hamiltonian as suggested by environment-induced selection.

Simultaneous presence of decoherence and spontaneous localization

Since decoherence can be considered as an omnipresent phenomenon that has been extensively verified both theoretically and experimentally, the assumption that a physical collapse theory holds means that the evolution of a system must be guided by both decoherence effects *and* the reduction mechanism.

Let us first consider the situation in which decoherence and the localization mechanism act constructively in the same direction, i.e., towards a common preferred basis. This raises the question in which order these two effects influence the evolution of the system [25]. If localization occurs on a shorter time scale than environment-induced

superselection of a preferred basis and suppression of local interference, decoherence will in most cases have very little influence on the evolution of the system, since typically the system will already have evolved into a reduced state. Conversely, if decoherence effects act more quickly on the system than the localization mechanism, the interaction with the environment will presumably lead to the preparation of quasiclassical robust states that are subsequently chosen by the localization mechanism. As pointed out in Sec. 2.3.4, decoherence usually occurs on extremely short time scales, which can be shown to be significantly smaller than the action of the spontaneous localization process for most cases (for studies related to the GRW model, see Refs. [294, 46]). This indicates that decoherence will typically play an important role even in the presence of physical wave-function reduction.

The second case occurs when decoherence leads to the selection of a different preferred basis than the reduction basis specified by the localization mechanism. As remarked by Bacciagaluppi [25, 26] in the context of the GRW theory, one might then imagine the collapse either to occur only at the level of the environment (which would then serve as an amplifying and recording device with different localization properties than the system under study), or to lead to an explicit competition between decoherence and localization effects.

The tails problem

The clear advantage of physical collapse models over the consideration of decoherence-induced effects alone for a solution to the measurement problem lies in the fact that an actual state reduction is achieved such that one may be tempted to conclude that at the conclusion of the reduction process the system actually is in a determinate state. However, all collapse models achieve only an approximate (“for all practical purposes”) reduction of the wave function. In the case of dynamical reduction models, the state will always retain small interference terms for finite times. Similarly, in the GRW theory the width Δ of the multiplying Gaussian cannot be made arbitrary

ily small, and therefore the reduced wave packet cannot be made infinitely sharply localized in position space, since this would entail an infinitely large energy gain by the system via the time-energy uncertainty relation, which would certainly show up experimentally (Ghirardi *et al.* [149], chose $\Delta \approx 10^{-5}$ cm). This need for only an approximate reduction leads to wave function “tails” [6], that is, in any region in space and at any time $t > 0$, the wave function will remain nonzero if it has been nonzero at $t = 0$ (before the collapse), and thus there will be always a part of the system that is not “here.”

Physical collapse models that achieve reduction only “for all practical purposes” require a modification, namely, a weakening, of the orthodox e-e link to allow one to speak of the system’s actually being in a definite state, and thereby to ensure the objective attribution of determinate properties to the system.¹⁸ In this sense, collapse models are as much “fine for all practical purposes” (to paraphrase Bell [45]) as decoherence is, where perfect orthogonality of the environment states is only attained as $t \rightarrow \infty$. The severity of the consequences, however, is not equivalent for the two strategies. Since collapse models directly change the state vector, a single outcome is at least approximately selected, and it only requires a “slight” weakening of the e-e link to make this state of affairs correspond to the (objective) existence of a determinate physical property. In the case of decoherence, the lack of a precise destruction of interference terms is not the main problem; even if exact orthogonality of the environment states were ensured at all times, the resulting reduced density matrix would represent an improper mixture, with no outcome having been singled out according to the e-e link. This would be the case regardless of whether the e-e link is expressed in the strong or weakened form, and we would still have to supply some additional interpretative framework to explain our perception of outcomes (see also the comment by Ghirardi *et al.* [150]).

¹⁸It should be noted, however, that such “fuzzy” e-e links may in turn lead to difficulties, as the discussion of Lewis’s “counting anomaly” has shown [216].

Connecting decoherence and collapse models

It was realized early that there exists a striking formal similarity of the equations that govern the time evolution of density matrices in the GRW approach and in models of decoherence. For example, the GRW equation for a single free mass point reads [149, Eq. (3.5)],

$$i\frac{\partial\rho(x,x',t)}{\partial t}=\frac{1}{2m}\left[\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial x'^2}\right]\rho-i\Lambda(x-x')^2\rho, \quad (2.38)$$

where the second term on the right-hand side accounts for the destruction of spatially separated interference terms. A simple model for environment-induced decoherence yields a very similar equation [197, Eq. (3.75)] (see also the comment by Joos [195]). Thus the physical justification for an *ad hoc* postulate of an explicit reduction-inducing mechanism could be questioned (of course modulo the important interpretive difference between the approximately proper ensembles arising from collapse models and the improper ensembles resulting from decoherence; see also Ref. [150]). More constructively, the similarity of the governing equations might enable one to choose the free parameters in collapse models on physical grounds rather than on the basis of empirical adequacy. Conversely, this similarity can also be viewed as leading to a “protection” of physical collapse theories from empirical disproof. This is so because the inevitable and ubiquitous interaction with the environment will always, for all practical purposes of observation (that is, of statistical prediction), result in (local) density matrices that are formally very similar to those of collapse models. What is measured is not the state vector itself, but the probability distribution of outcomes, i.e., values of a physical quantity and their frequency, and this information is equivalently contained in the state vector and the density matrix. Measurements with their intrinsically local character will presumably be unable to distinguish between the probability distribution given by the decohered reduced density matrix and the probability distribution defined by an (approximately) proper mixture obtained from a physical collapse. In other words, as long as the free parameters of collapse

theories are chosen in agreement with those determined from decoherence, models for state-vector reduction can be expected to be empirically adequate since decoherence is an effect that will be present with near certainty in every realistic (especially macroscopic) physical system.

One might of course speculate that the simultaneous presence of both decoherence and reduction effects might actually allow for an experimental disproof of collapse theories by preparing states that differ in an observable manner from the predictions of the reduction models.¹⁹ If we acknowledge the existence of interpretations of quantum mechanics that employ only decoherence-induced suppression of interference to explain the perception of apparent collapses (as is, for example, claimed by the “existential interpretation” of Zurek [340, 341]; see Sec. 2.4.3), we will not be able to distinguish experimentally between a “true” collapse and a mere suppression of interference as explained by decoherence. Instead, an experimental situation is required in which the collapse model predicts a collapse, but in which no suppression of interference through decoherence arises. Again, the problem in the realization of such an experiment is that it is very difficult to shield a system from decoherence effects, especially since we will typically require a mesoscopic or macroscopic system in which the reduction is efficient enough to be observed. For example, based on explicit numerical estimates, Tegmark [294] has shown that decoherence due to scattering of environmental particles such as air molecules or photons will have a much stronger influence than the proposed GRW effect of spontaneous localization (see also Refs. [37, 46]; for different results for energy-driven reduction models, cf. Ref. [1]).

¹⁹For proposed experiments to detect the GRW collapse, see, for example, Refs. [289, 262]. For experiments that could potentially demonstrate deviations from the predictions of quantum theory when dynamical state-vector reduction is present, see Refs. [248, 249]. See also Chap. 5.

Summary and outlook

Decoherence has the distinct advantage of being derived directly from the laws of standard quantum mechanics, whereas current collapse models are required to postulate their reduction mechanism as a new fundamental law of nature. On the other hand, collapse models yield, at least for all practical purposes, proper mixtures, so they are capable of providing an “objective” solution to the measurement problem. The formal similarity between the time evolution equations of the collapse and decoherence models nourishes hopes that the postulated reduction mechanisms of collapse models could possibly be derived from the ubiquitous and inevitable interaction of every physical system with its environment and the resulting decoherence effects. We may therefore regard collapse models and decoherence not as mutually exclusive alternatives for a solution to the measurement problem, but rather as potential candidates for a fruitful unification. For a vague proposal along these lines, see Ref. [257]; cf. also Refs. [109, 252] for speculations that quantum gravity might act as a collapse-inducing universal “environment.”

2.4.6 Bohmian mechanics

Bohm’s approach [52, 53, 54] is a modification of de Broglie’s original “pilot-wave” proposal [93]. In Bohmian mechanics, a system containing N (nonrelativistic) particles is described by a wave function $\psi(t)$ and the configuration $\mathcal{Q}(t) = (\mathbf{q}_1(t), \dots, \mathbf{q}_N(t)) \in \mathbb{R}^{3N}$ of particle positions $\mathbf{q}_i(t)$, i.e., the state of the system is represented by (ψ, \mathcal{Q}) for each instant t . The evolution of the system is guided by two equations. The wave function $\psi(t)$ is transformed as usual via the standard Schrödinger equation, $i\hbar(\partial/\partial t)\psi = \hat{H}\psi$, while the particle positions $\mathbf{q}_i(t)$ of the configuration $\mathcal{Q}(t)$ evolve according to the “guiding equation”

$$\frac{d\mathbf{q}_i}{dt} = \mathbf{v}_i^\psi(\mathbf{q}_1, \dots, \mathbf{q}_N) \equiv \frac{\hbar}{m_i} \text{Im} \frac{\psi^* \nabla_{\mathbf{q}_i} \psi}{\psi^* \psi}(\mathbf{q}_1, \dots, \mathbf{q}_N), \quad (2.39)$$

where m_i is the mass of the i th particle. Thus the particles follow determinate trajectories described by $\mathcal{Q}(t)$, with the distribution of $\mathcal{Q}(t)$ being given by the quantum equilibrium distribution $\rho = |\psi|^2$.

Particles as fundamental entities

Bohm's theory has been criticized for ascribing fundamental ontological status to particles. General arguments against particles on a fundamental level of any relativistic quantum theory have been frequently given (see, for instance, Refs. [221, 175]).²⁰ Moreover, and this is the point we would like to discuss in this section, it has been argued that the appearance of particles ("discontinuities in space") could be derived from the continuous process of decoherence, leading to claims that no fundamental role need be attributed to particles [328, 331, 335]. Based on decohered density matrices of mesoscopic and macroscopic systems that essentially always represent quasi-ensembles of narrow wave packets in position space, Zeh [328, p. 190] holds that such wave packets can be viewed as representing individual "particle" positions.²¹

All particle aspects observed in measurements of quantum fields (like spots on a plate, tracks in a bubble chamber, or clicks of a counter) can be understood by taking into account this decoherence of the relevant local (*i.e.*, subsystem) density matrix.

The first question is then whether a narrow wave packet in position space can be identified with the subjective experience of a "particle." The answer appears to be yes: our notion of "particles" hinges on the property of localizability, *i.e.*, the

²⁰On the other hand, there are proposals for a "Bohmian mechanics of quantum fields," *i.e.*, a theory that embeds quantum field theory into a Bohmian-style framework [120, 119].

²¹Schrödinger [282] had made an attempt into a similar direction but had failed since the Schrödinger equation tends to continuously spread out any localized wavepacket when it is considered as describing an isolated system. The inclusion of an interacting environment and thus decoherence counteracts the spread and opens up the possibility of maintaining narrow wave packets over time [197].

definition of a region of space $\Omega \in \mathbb{R}^3$ in which the system (that is, the support of the wave function) is entirely contained. Although the nature of the Schrödinger dynamics implies that any wave function will have nonvanishing support (“tails”) outside of any finite spatial region Ω and therefore exact localizability will never be achieved, we only need to demand approximate localizability to account for our experience of particle aspects.

However, to interpret the ensembles of narrow wave packets resulting from decoherence as leading to the perception of individual particles, we must embed standard quantum mechanics (with decoherence) into an additional interpretive framework that explains why only one of the wavepackets is perceived;²² that is, we do need to add some interpretive rule to get from the improper ensemble emerging from decoherence to the perception of individual terms, so decoherence alone does not necessarily make Bohm’s particle concept superfluous. But it suggests that the postulate of particles as fundamental entities could be unnecessary, and taken together with the difficulties in reconciling such a particle theory with a relativistic quantum field theory, Bohm’s *a priori* assumption of particles at a fundamental level of the theory appears seriously challenged.

Bohmian trajectories and decoherence

A well-known property of Bohmian mechanics is the fact that its trajectories are often highly nonclassical (see, for example, Refs. [54, 184, 19]). This poses the serious problem of how Bohm’s theory can explain the existence of quasiclassical trajectories on a macroscopic level.

Bohm and Hiley [54] considered the scattering of a beam of environmental particles on a macroscopic system, a process that is known to give rise to decoherence [197, 198]. The authors demonstrate that this scattering yields quasiclassical trajectories for the

²²Zeh himself, like Zurek [341], adheres to an Everett-style branching to which distinct observers are attached [328]; see also the quote in Sec. 2.4.3.

system. It has further been shown that for isolated systems, the Bohm theory will typically not give the correct classical limit [19]. It was thus suggested that the inclusion of the environment and of the resulting decoherence effects might be helpful in recovering quasiclassical trajectories in Bohmian mechanics [18, 331, 11, 13, 12, 275].

We mentioned before that the interaction between a macroscopic system and its environment will typically lead to a rapid approximate diagonalization of the reduced density matrix in position space, and thus to spatially localized wave packets that follow (approximately) Hamiltonian trajectories. [This observation also provides a physical motivation for the choice of position as the fundamental preferred basis in Bohm's theory, in agreement with Bell's well-known comment [44] that "in physics the only observations we must consider are position observations, if only the positions of instrument pointers."] The intuitive step is then to associate these trajectories with the particle trajectories $Q(t)$ of the Bohm theory. As pointed out by Bacciagaluppi [26], a great advantage of this strategy lies in the fact that the same approach would allow for a recovery of both quantum and classical phenomena.

However, a careful analysis by Appleby [18] showed that this decoherence-induced diagonalization in the position basis alone will in general not suffice to yield quasiclassical trajectories in Bohm's theory; only under certain additional assumptions will processes that lead to decoherence also give correct quasiclassical Bohmian trajectories for macroscopic systems (Appleby described the example of the long-time limit of a system that has initially been prepared in an energy eigenstate). Interesting results were also reported by Allori and co-workers [11, 13, 12]. They demonstrated that decoherence effects can play the role of preserving classical properties of Bohmian trajectories. Furthermore, they showed that while in standard quantum mechanics it is important to maintain narrow wave packets to account for the emergence of classicality, the Bohmian description of a system by both its wave function and its configuration allows for the derivation of quasiclassical behavior from highly delocal-

ized wave functions. Sanz and Borondo [275] studied the double-slit experiment in the framework of Bohmian mechanics and in the presence of decoherence and showed that even when coherence is fully lost, and thus interference is absent, nonlocal quantum correlations remain that influence the dynamics of the particles in the Bohm theory, demonstrating that in this example decoherence does not suffice to achieve the classical limit in Bohmian mechanics.

In conclusion, while the basic idea of employing decoherence-related processes to yield the correct classical limit of Bohmian trajectories seems reasonable, many details of this approach still need to be worked out.

2.4.7 Consistent histories interpretations

The consistent- (or decoherent-) histories approach was introduced by Griffiths [162, 163, 164] and further developed by Omnès [236, 237, 238, 239, 240, 241, 242], Gell-Mann and Hartle [142, 143, 145, 144], Dowker and Halliwell [115], and others. Reviews of the program can be found in the papers by Omnès [240] and Halliwell [170, 171], as well as in the recent book by Griffiths [165]. Thoughtful critiques investigating key features and assumptions of the approach have been given, for example, by d’Espagnat [95], Dowker and Kent [116, 117], [201], and Bassi and Ghirardi [36]. The basic idea of the consistent-histories approach is to eliminate the fundamental role of measurements in quantum mechanics, and instead study quantum histories, defined as sequences of events represented by sets of time-ordered projection operators, and to assign probabilities to such histories. The approach was originally motivated by quantum cosmology, i.e., the study of the evolution of the entire universe, which, by definition, represents a closed system. Therefore no external observer (which is, for example, an indispensable element of the Copenhagen interpretation) can be invoked.

Definition of histories

We assume that a physical system \mathcal{S} is described by a density matrix ρ_0 at some initial time t_0 and define a sequence of arbitrary times $t_1 < t_2 < \dots < t_n$ with $t_1 > t_0$. For each time point t_i in this sequence, we consider an exhaustive set $\mathcal{P}^{(i)} = \{\widehat{P}_{\alpha_i}^{(i)}(t_i) \mid \alpha_i = 1 \dots m_i\}$, $1 \leq i \leq n$, of mutually orthogonal Hermitian projection operators $\widehat{P}_{\alpha_i}^{(i)}(t_i)$, obeying

$$\sum_{\alpha_i} \widehat{P}_{\alpha_i}^{(i)}(t_i) = 1, \quad \widehat{P}_{\alpha_i}^{(i)}(t_i) \widehat{P}_{\beta_i}^{(i)}(t_i) = \delta_{\alpha_i, \beta_i} \widehat{P}_{\alpha_i}^{(i)}(t_i), \quad (2.40)$$

and evolving, using the Heisenberg picture, according to

$$\widehat{P}_{\alpha_i}^{(i)}(t) = U^\dagger(t_0, t) \widehat{P}_{\alpha_i}^{(i)}(t_0) U(t_0, t), \quad (2.41)$$

where $U(t_0, t)$ is the operator that dynamically propagates the state vector from t_0 to t .

A possible, “maximally fine-grained” history is defined by the sequence of times $t_1 < t_2 < \dots < t_n$ and by the choice of one projection operator in the set $\mathcal{P}^{(i)}$ for each time point t_i in the sequence, i.e., by the set

$$\mathcal{H}_{\{\alpha\}} = \{\widehat{P}_{\alpha_1}^{(1)}(t_1), \widehat{P}_{\alpha_2}^{(2)}(t_2), \dots, \widehat{P}_{\alpha_n}^{(n)}(t_n)\}. \quad (2.42)$$

We also define the set $\mathfrak{H} = \{\mathcal{H}_{\{\alpha\}}\}$ of all possible histories for a given time sequence $t_1 < t_2 < \dots < t_n$. The natural interpretation of a history $\mathcal{H}_{\{\alpha\}}$ is then to take it as a series of propositions of the form “the system \mathcal{S} was, at time t_i , in a state of the subspace spanned by $\widehat{P}_{\alpha_i}^{(i)}(t_i)$.”

Maximally fine-grained histories can be combined to form “coarse-grained” sets which assign to each time point t_i a linear combination

$$\widehat{Q}_{\beta_i}^{(i)}(t_i) = \sum_{\alpha_i} \pi_{\alpha_i}^{(i)} \widehat{P}_{\alpha_i}^{(i)}(t_i), \quad \pi_{\alpha_i}^{(i)} \in \{0, 1\} \quad (2.43)$$

of the original projection operators $\widehat{P}_{\alpha_i}^{(i)}(t_i)$.

So far, the projection operators $\hat{P}_{\alpha_i}^{(i)}(t_i)$ chosen at a certain instant t_i in time in order to form a history $\mathcal{H}_{\{\alpha\}}$ were independent of the choice of the projection operators at earlier times $t_0 < t < t_i$ in $\mathcal{H}_{\{\alpha\}}$. This situation was generalized by Omnès [236, 237, 238, 239, 240] to include “branch-dependent” histories of the form (see also Ref. [144])

$$\mathcal{H}_{\{\alpha\}} = \{\hat{P}_{\alpha_1}^{(1)}(t_1), \hat{P}_{\alpha_2}^{(2,\alpha_1)}(t_2), \dots, \hat{P}_{\alpha_n}^{(n,\alpha_1,\dots,\alpha_{n-1})}(t_n)\}. \quad (2.44)$$

Probabilities and consistency

In standard quantum mechanics, we can always assign probabilities to single events, represented by the eigenstates of some projection operator $\hat{P}^{(i)}(t)$, via the rule

$$p(i, t) = \text{Tr}[\hat{P}^{(i)\dagger}(t)\rho(t_0)\hat{P}^{(i)}(t)]. \quad (2.45)$$

The natural extension of this formula to the calculation of the probability $p(\mathcal{H}_{\{\alpha\}})$ of a history $\mathcal{H}_{\{\alpha\}}$ is given by

$$p(\mathcal{H}_{\{\alpha\}}) = \mathcal{D}(\alpha, \alpha), \quad (2.46)$$

where the so-called *decoherence functional* $\mathcal{D}(\alpha, \beta)$ is defined by Gell-Mann and Hartle [142]

$$\mathcal{D}(\alpha, \beta) = \text{Tr}[\hat{P}_{\alpha_n}^{(n)}(t_n) \cdots \hat{P}_{\alpha_1}^{(1)}(t_1)\rho_0\hat{P}_{\beta_1}^{(1)}(t_1) \cdots \hat{P}_{\beta_n}^{(n)}(t_n)]. \quad (2.47)$$

If we instead work in the Schrödinger picture, the decoherence functional is

$$\mathcal{D}(\alpha, \beta) = \text{Tr}[\hat{P}_{\alpha_n}^{(n)}U(t_{n-1}, t_n) \cdots \hat{P}_{\alpha_1}^{(1)}\rho(t_1)\hat{P}_{\beta_1}^{(1)} \cdots U^\dagger(t_{n-1}, t_n)\hat{P}_{\beta_n}^{(n)}(t_n)]. \quad (2.48)$$

Consider now the coarse-grained history that arises from a combination of the two maximally fine-grained histories $\mathcal{H}_{\{\alpha\}}$ and $\mathcal{H}_{\{\beta\}}$,

$$\mathcal{H}_{\{\alpha \vee \beta\}} = \{\hat{P}_{\alpha_1}^{(1)}(t_1) + \hat{P}_{\beta_1}^{(1)}(t_1), \hat{P}_{\alpha_2}^{(2)}(t_2) + \hat{P}_{\beta_2}^{(2)}(t_2), \dots, \hat{P}_{\alpha_n}^{(n)}(t_n) + \hat{P}_{\beta_n}^{(n)}(t_n)\}. \quad (2.49)$$

We interpret each combined projection operator $\hat{P}_{\alpha_i}^{(i)}(t_i) + \hat{P}_{\beta_i}^{(i)}(t_i)$ as stating that, at time t_i , the system was in the range described by the union of $\hat{P}_{\alpha_i}^{(i)}(t_i)$ and $\hat{P}_{\beta_i}^{(i)}(t_i)$.

Accordingly, we would like to require that the probability for a history containing such a combined projection operator be equivalently calculable from the sum of the probabilities of the two histories containing the individual projectors $\widehat{P}_{\alpha_i}^{(i)}(t_i)$ and $\widehat{P}_{\beta_i}^{(i)}(t_i)$, that is,

$$\begin{aligned} \text{Tr} & [\widehat{P}_{\alpha_n}^{(n)}(t_n) \cdots (\widehat{P}_{\alpha_i}^{(i)}(t_i) + \widehat{P}_{\beta_i}^{(i)}(t_i)) \cdots \widehat{P}_{\alpha_1}^{(1)}(t_1) \rho_0 \widehat{P}_{\alpha_1}^{(1)}(t_1) \cdots \\ & \cdots (\widehat{P}_{\alpha_i}^{(i)}(t_i) + \widehat{P}_{\beta_i}^{(i)}(t_i)) \cdots \widehat{P}_{\alpha_n}^{(n)}(t_n)] \\ & \stackrel{!}{=} \text{Tr} [\widehat{P}_{\alpha_n}^{(n)}(t_n) \cdots \widehat{P}_{\alpha_i}^{(i)}(t_i) \cdots \widehat{P}_{\alpha_1}^{(1)}(t_1) \rho_0 \widehat{P}_{\alpha_1}^{(1)}(t_1) \cdots \widehat{P}_{\alpha_i}^{(i)}(t_i) \cdots \widehat{P}_{\alpha_n}^{(n)}(t_n)] \\ & + \text{Tr} [\widehat{P}_{\alpha_n}^{(n)}(t_n) \cdots \widehat{P}_{\beta_i}^{(i)}(t_i) \cdots \widehat{P}_{\alpha_1}^{(1)}(t_1) \rho_0 \widehat{P}_{\alpha_1}^{(1)}(t_1) \cdots \widehat{P}_{\beta_i}^{(i)}(t_i) \cdots \widehat{P}_{\alpha_n}^{(n)}(t_n)]. \end{aligned}$$

It can be easily shown that this relation holds if and only if

$$\text{Re}\{\text{Tr}[\widehat{P}_{\alpha_n}^{(n)}(t_n) \cdots \widehat{P}_{\alpha_i}^{(i)}(t_i) \cdots \widehat{P}_{\alpha_1}^{(1)}(t_1) \rho_0 \widehat{P}_{\alpha_1}^{(1)}(t_1) \cdots \widehat{P}_{\beta_i}^{(i)}(t_i) \cdots \widehat{P}_{\alpha_n}^{(n)}(t_n)]\} = 0 \quad (2.50)$$

for $\alpha_i \neq \beta_i$. Generalizing this two-projector case to the coarse-grained history $\mathcal{H}_{\{\alpha \vee \beta\}}$ of Eq. (2.49), we arrive at the (sufficient and necessary) *consistency condition* for two histories $\mathcal{H}_{\{\alpha\}}$ and $\mathcal{H}_{\{\beta\}}$ [162, 239, 240],

$$\text{Re}[\mathcal{D}(\alpha, \beta)] = \delta_{\alpha, \beta} \mathcal{D}(\alpha, \alpha). \quad (2.51)$$

If this relation is violated, the usual sum rule for calculating probabilities does not apply. This situation arises when quantum interference between the two combined histories $\mathcal{H}_{\{\alpha\}}$ and $\mathcal{H}_{\{\beta\}}$ is present. Therefore, to ensure that the standard laws of probability theory also hold for coarse-grained histories, the set \mathfrak{H} of possible histories must be consistent in the above sense.

However, Gell-Mann and Hartle [142] have pointed out that when decoherence effects are included to model the emergence of classicality, it is more natural to require

$$\mathcal{D}(\alpha, \beta) = \delta_{\alpha, \beta} \mathcal{D}(\alpha, \alpha). \quad (2.52)$$

Condition (2.51) has often been referred to as *weak decoherence*, and Eq. (2.52) as *medium decoherence* (for a proposal of a criterion for *strong decoherence*, see

Ref. [146]). The set \mathfrak{H} of histories is called consistent (or decoherent) when all its members $\mathcal{H}_{\{\alpha\}}$ fulfill the consistency condition, Eqs. (2.51) or (2.52), i.e., when they can be regarded as independent (noninterfering).

Selection of histories and classicality

Even when the stronger consistency criterion (2.52) is imposed on the set \mathfrak{H} of possible histories, the number of mutually incompatible consistent histories remains relatively large [95, 117]. It is not at all clear *a priori* that at least some of these histories should represent any meaningful set of propositions about the world of our observation. Even if a collection of such “meaningful” histories is found, it leaves open the question how to select such histories and which additional criteria would need to be invoked.

Griffith’s original aim [162] in formulating the consistency criterion was only to allow for a consistent description of sequences of events in closed quantum systems without running into logical contradictions.²³ Commonly, however, consistency has also been tied to the emergence of classicality. For example, the consistency criterion corresponds to the demand for the absence of quantum interference—a property of classicality—between two combined histories. It has become clear that most consistent histories are in fact flagrantly nonclassical [142, 145, 340, 244, 10, 116, 117]. For instance, when the projection operators $\widehat{P}_{\alpha_i}^{(i)}(t_i)$ are chosen to be the time-evolved eigenstates of the initial density matrix $\rho(t_0)$, the consistency condition will automatically be fulfilled, yet the histories composed of these projection operators have been shown to result in highly nonclassical macroscopic superpositions when applied to standard examples such as quantum measurement or Schrödinger’s cat. This demonstrates that the consistency condition cannot serve as a sufficient criterion for classi-

²³However, Goldstein [157] used a simple example to argue that the consistent-histories approach can lead to contradictions with respect to a combination of joint probabilities, even if the consistency criterion is imposed; see also the subsequent exchange of letters in the February 1999 issue of *Physics Today*.

cality.

Consistent histories of open systems

Various authors have appealed to interaction with the environment and the resulting decoherence effects in defining additional criteria that would select quasiclassical histories and would also lead to a physical motivation for the consistency criterion (see, for example, Refs. [142, 115, 9, 10, 340, 244, 300, 134, 14, 146, 174]). This approach intrinsically requires the notion of local, open systems and the split of the universe into subsystems, in contrast to the original aim of the consistent-histories approach to describe the evolution of a single closed, undivided system (typically the entire universe). The decoherence-based studies then assume the usual decomposition of the total Hilbert space \mathcal{H} into a space \mathcal{H}_S , corresponding to the system S , and \mathcal{H}_E of an environment E . One then describes the histories of the system S by employing projection operators that act only on the system, i.e., that are of the form $\widehat{P}_{\alpha_i}^{(i)}(t_i) \otimes \widehat{I}_E$, where $\widehat{P}_{\alpha_i}^{(i)}(t_i)$ acts only on \mathcal{H}_S and \widehat{I}_E is the identity operator in \mathcal{H}_E .

This raises the question of when, i.e., under which circumstances, the reduced density matrix $\rho_S = \text{Tr}_E \rho_{SE}$ of the system S suffices to calculate the decoherence functional. The reduced density matrix arises from a nonunitary trace over E at every time point t_i , whereas the decoherence functional of Eq. (2.48) employs the full, unitarily evolving density matrix ρ_{SE} for all times $t_i < t_f$ and only applies a nonunitary trace operation (over both S and E) at the final time t_f . Paz and Zurek [244] have answered this (rather technical) question by showing that the decoherence functional can be expressed entirely in terms of the reduced density matrix if the time evolution of the reduced density matrix is independent of the correlations dynamically formed between the system and the environment. A necessary (but not always sufficient) condition for this requirement to be satisfied is given by demanding that the reduced dynamics be governed by a master equation that is local in time.

When a “reduced” decoherence functional exists, at least to a good approximation,

i.e., when the reduced dynamics are sufficiently insensitive to the formation of system-environment correlations, the consistency of whole-universe histories, described by a unitarily evolving density matrix ρ_{SE} and sequences of projection operators of the form $\widehat{P}_{\alpha_i}^{(i)}(t_i) \otimes \widehat{I}_E$, will be directly related to that of open-system histories, represented by a nonunitarily evolving reduced density matrix $\rho_S(t_i)$ and “reduced” projection operators $\widehat{P}_{\alpha_i}^{(i)}(t_i)$ [340].

Schmidt states vs pointer basis as projectors

The ability of the instantaneous eigenstates of the reduced density matrix (Schmidt states; see also Sec. 2.3.5) to serve as projectors for consistent histories and possibly to lead to the emergence of quasiclassical histories has been studied in much detail [9, 10, 340, 244, 202]. Paz and Zurek [244] have shown that Schmidt projectors $\widehat{P}_{\alpha_i}^{(i)}$, defined by their commutativity with the evolved, path-projected reduced density matrix,

$$[\widehat{P}_{\alpha_i}^{(i)}, U(t_{i-1}, t_i)\{\cdots U(t_1, t_2)\widehat{P}_{\alpha_1}^{(1)}\rho_S(t_1)\widehat{P}_{\alpha_1}^{(1)}U^\dagger(t_1, t_2)\cdots\}U^\dagger(t_{i-1}, t_i)] = 0, \quad (2.53)$$

will always give rise to an infinite number of sets of consistent histories (“Schmidt histories”). However, these histories are branchdependent [see Eq. (2.44)] and usually extremely unstable, since small modifications of the time sequence used for the projections (for instance by deleting a time point) will typically lead to drastic changes in the resulting history, indicating that Schmidt histories are usually very nonclassical [340, 244].

This situation is changed when the time points t_i are chosen such that the intervals $(t_{i+1} - t_i)$ are larger than the typical decoherence time τ_D of the system over which the reduced density matrix becomes approximately diagonal in the preferred pointer basis chosen through environment-induced superselection (see also the discussion in Sec. 2.3.5). When the resulting pointer states, rather than the instantaneous Schmidt states, are used to define the projection operators, stable quasiclassical histories will

typically emerge [340, 244]. In this sense, it has been suggested that interaction with the environment can provide the missing selection criterion that ensures the quasiclassicality of histories, i.e., their stability (predictability), and the correspondence of the projection operators (the pointer basis) to the preferred determinate quantities of our experience.

The approximate noninterference, and thus consistency, of histories based on local density operators (energy, number, momentum, charge etc.) as quasiclassical projectors (the so-called *hydrodynamic observables*, see Refs. [145, 115, 172]) has been attributed to the dynamical stability exhibited by the eigenstates of the local density operators. This stability leads to decoherence in the corresponding basis [172, 173]. It has been argued by Zurek [344] that this behavior and thus the special quasiclassical properties of hydrodynamic observables can be explained by the fact that these observables obey the commutativity criterion, Eq. (2.26), of the environment-induced superselection approach.

Exact vs approximate consistency

In the idealized case where the pointer states lead to an exact diagonalization of the reduced density matrix, histories composed of the corresponding *pointer projectors* will automatically be consistent. However, under realistic circumstances decoherence will typically lead only to approximate diagonality in the pointer basis. This implies that the consistency criterion will not be fulfilled exactly and that hence the probability sum rules will only hold approximately—although usually, due to the efficiency of decoherence, to a very good approximation [162, 145, 240, 241, 9, 10, 340, 244, 300]. Hence, the consistency criterion has been viewed both as overly restrictive, since the quasiclassical pointer projectors rarely obey the consistency equations exactly, and as insufficient, because it does not give rise to constraints that would single out quasiclassical histories.

A relaxation of the consistency criterion has therefore been suggested, leading to

“approximately consistent histories” whose decoherence functional would be allowed to contain nonvanishing off-diagonal terms (corresponding to a violation of the probability sum rules) as long as the net effect of all the off-diagonal terms was “small” in the sense of remaining below the experimentally detectable level (see, for example, Refs. [145, 115]. Gell-Mann and Hartle [145]) have even ascribed a fundamental role to such approximately consistent histories, a move that has sparked much controversy and has been considered as unnecessary and irrelevant by some [116, 117]. Indeed, if only approximate consistency is demanded, it is difficult to regard this condition as a fundamental concept of a physical theory, and the question of how much consistency is required will inevitably arise.

Consistency and environment-induced superselection

The relationship between consistency and environment-induced superselection, and therefore the connection between the decoherence functional and the diagonalization of the reduced density matrix through environmental decoherence, has been investigated by various authors. The basic idea, promoted, for example, in Refs. [340, 244], is to suggest that if the interaction with the environment leads to rapid superselection of a preferred basis, which approximately diagonalizes the local density matrix, coarse-grained histories defined in this basis will automatically be (approximately) consistent.

This approach has been explored by Twamley [300], who carried out detailed calculations in the context of a quantum optical model of phase-space decoherence and compared the results with two-time projected phase-space histories of the same model system. It was found that when the parameters of the interacting environment were changed such that the degree of diagonality of the reduced density matrix in the emerging preferred pointer basis was increased, histories in that basis also became more consistent. For a similar model, however, Twamley [299] also showed that consistency and diagonality in a pointer basis as possible criteria for the emergence

of quasiclassicality may exhibit a very different dependence on the initial conditions.

Extensive studies on the connection between Schmidt states, pointer states and consistent quasiclassical histories have also been made by Albrecht [9, 10], based on analytical calculations and numerical simulations of toy models for decoherence, including detailed numerical results on the violation of the sum rule for histories composed of different (Schmidt and pointer) projector bases. It was demonstrated that the presence of stable system-environment correlations (“records”), as demanded by the criterion for the selection of the pointer basis, was of great importance in making certain histories consistent. The relevance of “records” for the consistent-histories approach in ensuring the “permanence of the past” has also been emphasized by other authors, for example, in Refs. [244, 340, 344], and in the “strong decoherence” criterion by Gell-Mann and Hartle [146]. The redundancy with which information about the system is recorded in the environment and can thus be found out by different observers without measurably disturbing the system itself has been suggested to allow for the notion of “objectively existing histories,” based on environment-selected projectors that represent sequences of “objectively existing” quasiclassical events [340, 244, 344, 347].

In general, damping of quantum coherence caused by decoherence will necessarily lead to a loss of quantum interference between individual histories (but not vice versa—see the discussion by Twamley [300]), since the final trace operation over the environment in the decoherence functional will make the off-diagonal elements very small due to the decoherence-induced approximate mutual orthogonality of the environmental states. Finkelstein [134] has used this observation to propose a new decoherence condition that coincides with the original definition, Eqs. (2.47) and (2.48), except for restricting the trace to \mathcal{E} , rather than tracing over both \mathcal{S} and \mathcal{E} . It was shown that this condition not only implies the consistency condition of Eq. (2.52), but also characterizes those histories that decohere due to interaction with the environment and that lead to the formation of “records” of the state of the

system in the environment.

Summary and discussion

The core difficulty associated with the consistent-histories approach has been to explain the emergence of the classical world of our experience from the underlying quantum nature. Initially, it was hoped that classicality could be derived from the consistency criterion alone. Soon, however, the status and the role of this criterion in the formalism and its proper interpretation became rather unclear and diffuse, since exact consistency was shown to provide neither a necessary nor a sufficient criterion for the selection of quasiclassical histories.

The inclusion of decoherence effects into the consistent histories approach, leading to the emergence of stable quasiclassical pointer states, has been found to yield a highly efficient mechanism and a sensitive criterion for singling out quasiclassical observables that simultaneously fulfill the consistency criterion to a very good approximation due to the suppression of quantum coherence in the state of the system. The central question is then: What is the meaning and the remaining role of an explicit consistency criterion in the light of such “natural” mechanisms for the decoherence of histories? Can one dispose of this criterion as a key element of the fundamental theory by noting that for all “realistic” histories consistency will be likely to arise naturally from environment-induced decoherence alone?

The answer to this question may actually depend on the viewpoint one takes with respect to the aim of the consistent-histories approach. As we have noted before, the original goal was simply to provide a formalism in which one could, in a measurement-free context, assign probabilities to certain sequences of quantum events without logical inconsistencies. The more recent and rather opposite aim would be to provide a formalism that selects only a very small subset of “meaningful” quasiclassical histories, all of which are consistent with our world of experience, and whose projectors can be directly interpreted as objective physical events.

The consideration of decoherence effects that can give rise to effective superselection of possible quasiclassical (and consistent) histories certainly falls into the latter category. It is interesting to note that this approach has also led to a departure from the original “closed systems only” view to the study of local open quantum systems and thus to the decomposition of the total Hilbert space into subsystems, within the consistent-histories formalism. Besides the fact that decoherence intrinsically requires the openness of systems, this move might also reflect the insight that the notion of classicality itself can be viewed as only arising from a conceptual division of the universe into parts (see the discussion in Sec. 2.3.1).

Therefore environment-induced decoherence and superselection have played a remarkable role in consistent-histories interpretations: a practical one by suggesting a physical selection mechanism for quasiclassical histories; and a conceptual one by contributing to a shift in our view of originally rather fundamental concepts, such as consistency, and of the aims of the consistent-histories approach, like the focus on description of closed systems.

2.5 Concluding remarks

We have presented an extensive discussion of the role of decoherence in the foundations of quantum mechanics, with a particular focus on the implications of decoherence for the measurement problem in the context of various interpretations of quantum mechanics.

A key achievement of the decoherence program is the recognition that openness in quantum systems is important for their realistic description. The well-known phenomenon of quantum entanglement had already, early in the history of quantum mechanics, demonstrated that correlations between systems can lead to “paradoxical” properties of the composite system that cannot be composed from the properties of the individual systems. Nonetheless, the viewpoint of classical physics that the idealization of isolated systems is necessary to arrive at an “exact description” of

physical systems has influenced quantum theory for a long time. It is the great merit of the decoherence program to have emphasized the ubiquity and essential inescapability of system-environment correlations and to have established the important role of such correlations as factors in the emergence of “classicality” from quantum systems. Decoherence also provides a realistic physical modeling and a generalization of the quantum measurement process, thus enhancing the “black-box” view of measurements in the standard (“orthodox”) interpretation and challenging the postulate of fundamentally classical measuring devices in the Copenhagen interpretation.

With respect to the preferred-basis problem of quantum measurement, decoherence provides a very promising definition of preferred pointer states via a physically meaningful requirement, namely, the robustness criterion, and it describes methods for selecting operationally such states, for example, via the commutativity criterion or by extremizing an appropriate measure such as purity or von Neumann entropy. In particular, the fact that macroscopic systems virtually always decohere into position eigenstates gives a physical explanation for why position is the ubiquitous determinate property of the world of our experience.

We have argued that, within the standard interpretation of quantum mechanics, decoherence cannot solve the problem of definite outcomes in quantum measurement: We are still left with a multitude of (albeit individually well-localized quasiclassical) components of the wave function, and we need to supplement or otherwise to interpret this situation in order to explain why and how single outcomes are perceived. Accordingly, we have discussed how environment-induced superselection of quasiclassical pointer states together with the local suppression of interference terms can be put to great use in physically motivating, or potentially disproving, rules and assumptions of alternative interpretive approaches that change (or altogether abandon) the strict orthodox eigenvalue-eigenstate link and/or modify the unitary dynamics to account for the perception of definite outcomes and classicality in general. For example, to name just a few applications, decoherence can provide a universal criterion for the selection

of the branches in relative-state interpretations and a physical argument for the non-interference between these branches from the point of view of an observer; in modal interpretations, it can be used to specify empirically adequate sets of properties that can be ascribed to systems; in collapse models, the free parameters (and possibly even the nature of the reduction mechanism itself) might be derivable from environmental interactions; decoherence can also assist in the selection of quasiclassical particle trajectories in Bohmian mechanics, and it can serve as an efficient mechanism for singling out quasiclassical histories in the consistent-histories approach. Moreover, it has become clear that decoherence can ensure the empirical adequacy and thus empirical equivalence of different interpretive approaches, which has led some to the claim that the choice, for example, between the orthodox and the Everett interpretation becomes “purely a matter of taste, roughly equivalent to whether one believes mathematical language or human language to be more fundamental” [295, p. 855].

It is fair to say that the decoherence program sheds new light on many foundational aspects of quantum mechanics. It paves a physics-based path towards motivating solutions to the measurement problem; it imposes constraints on the strands of interpretations that seek such a solution and thus makes them also more and more similar to each other. Decoherence remains an ongoing field of intense research, in both the theoretical and experimental domain, and we can expect further implications for the foundations of quantum mechanics from such studies in the near future.

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Chapter 3

THE “SELF-INDUCED DECOHERENCE” APPROACH: STRONG LIMITATIONS ON ITS VALIDITY IN A SIMPLE SPIN BATH MODEL AND ON ITS GENERAL PHYSICAL RELEVANCE

Summary

The “self-induced decoherence” (SID) approach suggests that (1) the expectation value of any observable becomes diagonal in the eigenstates of the total Hamiltonian for systems endowed with a continuous energy spectrum, and that (2) this process can be interpreted as decoherence. We evaluate the first claim in the context of a simple spin bath model. We find that even for large environments, corresponding to an approximately continuous energy spectrum, diagonalization of the expectation value of random observables does in general not occur. We explain this result and conjecture that SID is likely to fail also in other systems composed of discrete subsystems. Regarding the second claim, we emphasize that SID does not describe a physically meaningful decoherence process for individual measurements, but only involves destructive interference that occurs collectively within an ensemble of pre-supposed “values” of measurements. This leads us to question the relevance of SID for treating observed decoherence effects.

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3.1 Introduction

In a series of papers [79, 78, 71, 77, 80, 76, 73, 75, 74, 213, 70, 212, 211, 72], the authors claim to present a “new approach to decoherence” [78], termed “self-induced decoherence” (SID). Their main assertion is that, for systems endowed with a continuous energy spectrum, the expectation value of an observable will become diagonal in the eigenbasis of the Hamiltonian of the system, and that this effect can be viewed as decoherence.

The basic idea underlying SID goes back to well-known arguments in the context of quantum measurement and the theory of irreversible processes [243, 309, 90, 182, 256, 308, 306, 307]. It rests on the observation that a superposition of a large number of terms with random phases in the expression for the expectation value of a typical observable, or for the matrix elements of the density operator, leads to destructive interference. The phase differences are either due to a random-phase assumption [243], or, as in SID, are created dynamically through the time evolution factor $e^{iEt/\hbar}$ associated with each energy eigenstate in the superposition. These destructive interference effects are then responsible for the diagonalization of the expectation value in the energy eigenbasis as described by SID.

However, this process differs strongly from the mechanism of environment-induced decoherence (EID) [198, 337, 338, 340, 341, 344, 280, 326, 327]. EID understands decoherence as the practically irreversible dislocalization of local phase relations between environment-selected preferred basis states due to entanglement with an environment. The approximate diagonality of the expectation value of local observables expressed in the preferred basis is only a formal phenomenological consequence of the relative states of the environment becoming rapidly orthogonal during the decoherence process. The fact that SID does not require an explicit environment interacting with the system motivated the term “self-induced” and was suggested [78] to circumvent the question of a proper interpretation of the concept of “observational ignorance of the

environment” in EID [94, 198, 341, 280].

This paper pursues two main goals. First, after formalizing the basic idea of SID (Sec. 3.2), we shall discuss the question to what extent SID can claim to describe a physically relevant decoherence process (Sec. 3.3). In particular, we will argue that, contrary to the claim of its proponents [78], SID does not constitute a “new viewpoint” on decoherence in the usual definition of EID. Second, we shall study whether diagonalization of the expectation value of random observables in the energy eigenbasis is obtained in the context of an explicit spin bath model (Sec. 3.4). Deliberately, we have chosen a discrete model to investigate the required degree of “quasicontinuity” for SID to work as claimed. To anticipate, we find that even for bath sizes large compared to what is typically considered in EID, no general decay of off-diagonal terms is found, unless both the observable and the initial state of the bath are appropriately restricted. We explain and discuss this result in Sec. 3.5, and present our conclusions in Sec. 3.6.

3.2 Self-induced decoherence

The basic formalism of SID as developed in Refs. [78, 71, 77, 80, 76, 73, 75, 74, 213, 70, 212, 211, 72] considers an arbitrary observable

$$\widehat{O} = \int dE \int dE' \tilde{O}(E, E') |E\rangle\langle E'|, \quad (3.1)$$

expanded in the eigenstates $|E\rangle$ of the Hamiltonian $\widehat{H} = \int dE E |E\rangle\langle E|$ with continuous spectrum. In the general treatment, only observables with

$$\tilde{O}(E, E') = O(E)\delta(E - E') + O(E, E') \quad (3.2)$$

are considered, where $O(E)$ and the $O(E, E')$ are assumed to be regular functions. The time evolution of the expectation value $\langle \widehat{O} \rangle_{\Psi(t)}$ of \widehat{O} in the pure state $|\Psi(t)\rangle = e^{-iHt}|\Psi_0\rangle = \int dE e^{-iEt}\langle E|\Psi_0\rangle|E\rangle$ (setting $\hbar = 1$) is then given by

$$\langle \widehat{O} \rangle_{\Psi(t)} = \int dE O(E) |\langle E|\Psi_0\rangle|^2 + \int dE \int dE' e^{-i\Delta Et} O(E, E') \langle E|\Psi_0\rangle \langle \Psi_0|E'\rangle, \quad (3.3)$$

where $\Delta E = E - E'$. For large t , the phase factor $e^{-i\Delta Et}$ fluctuates rapidly with ΔE , which leads to destructive interference in the double integral if the multiplying function $f(E, E') \equiv O(E, E')\langle E|\Psi_0\rangle\langle\Psi_0|E'\rangle$ varies comparably slowly. To formalize this argument, SID employs the Riemann-Lebesgue theorem [264], which prescribes that

$$\lim_{t \rightarrow \infty} \int dz g(z)e^{izt} = 0, \quad (3.4)$$

if $g(z)$ is a regular function and L^1 integrable (i.e., $\int dz |g(z)| < \infty$). Provided these conditions are satisfied by $f(E, E')$, it is concluded that

$$\langle \hat{O} \rangle_{\Psi(t)} \longrightarrow \int dE O(E) |\langle E|\Psi_0\rangle|^2, \quad (3.5)$$

for large t . Thus, the off-diagonal terms $E \neq E'$ have collectively disappeared, which in SID is interpreted as “decoherence in the expectation value.” Formally, the SID program introduces a “diagonal-equivalent” density matrix ρ_d ,

$$\rho_d = \int dE |\langle E|\Psi_0\rangle|^2 |E\rangle\langle E|, \quad (3.6)$$

which satisfies $\langle \hat{O} \rangle_{\rho_d} \equiv \lim_{t \rightarrow \infty} \langle \hat{O} \rangle_{\rho(t)}$. Note that ρ_d is only a formal equivalent and is not obtained through any dynamical process. Also, expectation values of a nonexhaustive set of observables [see Eq. (3.2)] do not uniquely determine the density matrix. Therefore, one must not derive any conclusions about the possibility for certain states of the system from ρ_d .

To summarize, the main result Eq. (3.5) has been obtained from two key assumptions: (1) The energy spectrum of the system is continuous; and (2) the coefficients used in expanding the initial state and the observable in the energy eigenbasis form regular (and integrable) functions of the energy variable.

The first requirement of a continuous energy spectrum can be viewed as an implicit inclusion of an internal “environment” with an infinite number of degrees of freedom. However, any realistic physical system is of finite size, and therefore the energy spacing will be discrete. An approximate suppression of off-diagonal terms as

given by Eq. (3.5) should therefore occur also for quasicontinuous energy spectra, i.e., for small but discrete energy spacings.

The regularity assumption (2) is crucial, since it ensures that the phase factors $e^{i\Delta E t}$ are able to lead to the required destructive interference of the expansion coefficients for large times. However, especially in the realistic case of systems of finite size where the expansion coefficients will be a finite set of discrete values, this condition will not hold. It is therefore important to understand the physical meaning and the consequences of a violation of this assumption.

Note also that the strict mathematical limit $t \rightarrow \infty$ employed in the Riemann-Lebesgue theorem, Eq. (3.4), is not physically meaningful, and approximate suppression must therefore occur already over finite time scales, as indicated in Eq. (3.5). Also, for the realistic case of only quasicontinuous (i.e., essentially discrete) energy spectra, no conclusions about an “irreversibility” of the decay should be derived from the limit $t \rightarrow \infty$ (as it is done, for example, in Ref. [78, p. 88]), since the off-diagonal terms will return to their initial values within a finite recurrence time scale.

The issues outlined above will be illustrated and investigated in the context of a particular model system in Sec. 3.4.

3.3 Does SID describe decoherence?

Despite the fact that SID and EID share the term “decoherence” in their name, we shall demonstrate in this section that their foundations, scope, and physical implications are fundamentally different.¹ Keeping these differences in mind is very important for a proper interpretation of the study of the bath model described in the following Sec. 3.4.

As already briefly outlined in the Introduction, the standard approach of environmental decoherence [198, 337, 338, 340, 341, 344, 280, 326, 327] describes the conse-

¹The author is indebted to H.-D. Zeh and E. Joos for drawing strong attention to this point.

quences of the ubiquitous interaction of any system with its environment. This leads to entanglement between the system and the environment and singles out a preferred basis of the system that is dynamically determined by the Hamiltonian governing the interaction. The relative environmental states associated with these preferred states rapidly approach orthogonality (i.e., macroscopic distinguishability). Phase relations between the preferred states that were initially associated with the system alone are now “dislocalized” into the system-environment combination due to the entanglement, which constitutes the decoherence process. In this sense, interference between the preferred states becomes locally suppressed, i.e., decoherence leads *locally* to a transition from a superposition to an *apparent* (“improper” [94]) ensemble. This can be used to define dynamically independent relative local wave-function components that can be related to local quasiclassical properties, thereby mimicking an *apparent* “collapse” of the wave function [326, 327, 328, 333, 341, 340, 346, 344, 280, 198].

The interaction between the system and its environment, often referred to as a “continuous measurement by the environment,” is observer independent and can be formulated entirely in terms of wave functions, without reference to presumed (classical) concepts such as “values of observables” and expectation values (see, for example, Chap. 2 of Ref. [198]). As it has been emphasized frequently [94, 280, 198], the formalism of local (“reduced”) density matrices and expectation values presupposes the probabilistic interpretation of the wave function and ultimately relies on the occurrence of a “collapse” of the wave function at some stage (or on the description of an observationally equivalent “branching” process in a relative-state framework [326, 327, 328, 333, 341, 340, 346, 344, 280]). The approximate diagonalization of the reduced density matrix $\rho_S = \text{Tr}_{\mathcal{E}}\rho_{S\mathcal{E}}$ (describing the probability distribution of outcomes of measurements on the “system S of interest” immersed into an environment \mathcal{E}) in the environment-selected basis should therefore be considered only as a phenomenological consequence of EID, but not as its essence (see also Ref. [341, p. 1800]). Given an ensemble of results of measurements of a local observable \hat{O}_S , the suppres-

sion of off-diagonal terms in ρ_S can then be related to the approximate diagonality of the expectation value of \hat{O}_S in the preferred basis, since $\langle \hat{O}_S \rangle_{\rho_{SE}} = \text{Tr}_S(\rho_S \hat{O}_S)$.

In contrast with EID, SID focuses solely on the derivation of a suppression of off-diagonal terms (in the energy eigenbasis only) in the expectation value of observables pertaining to a single undivided closed system; entanglement through interactions between subsystems plays no role in SID. As indicated earlier, the damping effect is due to destructive interference between a large number of terms with dynamically induced phase differences. Thus it is only the averaging process contained in the concept of expectation values that leads to a disappearance of interference terms. Individually, each term remains present at all times and is not suppressed independently of the other terms. The fact that collectively the off-diagonal terms may lead to a mutual canceling-out must not be misinterpreted as implying that the measurement “outcomes” corresponding to these terms do not occur. Thus SID cannot pertain to the relevant problem of a loss of interference in individual measurements. In view of this argument, the concept of the “diagonal-equivalent density matrix” ρ_d , as introduced by the SID program [see Eq. (3.6)], is rather misleading, since it gives the incorrect impression of an absence of interference terms $|E\rangle\langle E'|$, while the corresponding terms in the expression for the expectation value are individually present at all times. Derivations of a “classical limit” based on ρ_d [71] appear to have overlooked this issue.

While SID rests on the concept of expectation values, i.e., of weighted averages over an ensemble of measurement outcomes, it does not explain the physical origin of these outcomes and their ensembles. In contrast with EID, SID does not contain a dynamical account of the measurement process itself that could motivate explanations for how measurement outcomes arise (if only, as in EID, in an “apparent,” relative-state sense). Consequently, the assumption of an *a priori* existence of an ensemble of measurement outcomes, as it is inherent in SID, could be viewed as a particular application of the Copenhagen interpretation. One might then argue that in this case decoherence would not even be necessary in explaining the observed absence of

(macroscopic) interference effects.

Note that EID makes crucial use of the concept of locality in deriving a loss of interference, since globally the quantum-mechanical superposition remains unchanged, as required by the unitarity of the time evolution of the total wave function. As frequently emphasized by Zeh (e.g., in Refs. [326, 333]) and others (see, for example, Ref. [210]), this locality can be grounded in the (nontrivial) empirical insight that all observers and interactions are intrinsically local. On the other hand, the decomposition into a “system of interest” and an environment that is ignored from an observational point of view, as required in EID, and the resulting implication that the relevance of environmental decoherence is restricted to local subsystems of the total (nonlocal) quantum Universe, has been a subject of ongoing critical discussions (see, for example, Refs. [94, 198, 341, 280]). Furthermore, no general rule is available that would indicate where the split between system and environment is to be placed, a conceptual difficulty admitted also by proponents of EID [341, p. 1820]. These issues seem to have motivated the attempt of the SID program to derive decoherence for closed, undivided systems.

However, it is important to note that EID has clearly demonstrated that the assumption of the existence of closed system is unrealistic in essentially all cases [197, 294]. Enlarging the system by including parts of its environment, as it is implicitly done in SID in order to arrive at a quasicontinuous spectrum, will render the closed-system assumption even less physically viable: The combined system will in turn interact with its surroundings, and the degree of environmental interaction will increase with the number of degrees of freedom in the system. Also, since some interaction with the external measuring device will be required, the assumption of a closed system simply bypasses the question of how the information contained in the ensemble is acquired in the first place. Ultimately, the only truly closed “system” is the Universe in its entirety, and one can therefore question the physical relevance and motivation for a derivation of decoherence for subsystems that are presumed to be

closed.

Furthermore, a general measurement in SID would pertain also to the environment implicitly contained in the “closed system,” posing the question of how this could translate into an experimentally realizable situation. And even if such a measurement can be carried out, its result would usually be of rather little physical interest in the typical situation of observing decoherence for a particular object due to its largely unobserved environment.

Finally, in SID, suppression of off-diagonal terms always occurs in the energy eigenbasis, which can therefore be viewed as the universal “preferred basis” in this approach. However, this basis will generally not be useful in accounting for our observation of different preferred bases for the relevant local systems of interest (e.g., spatial localization of macroscopic bodies [198, 294, 186, 188, 141], chirality eigenstates for molecules such as sugar [197, 178, 48], and energy eigenstates in atoms [245]). Furthermore, the energy eigenbasis cannot be used to describe the emergence of time-dependent, quasiclassical properties.

In conclusion, not only is the scope of SID more limited than that of EID, but the two approaches also rest on different foundations. The interpretation of the processes described by these theories is fundamentally different, even though phenomenological effects of EID can manifest themselves in a manner formally similar to that of SID, i.e., as a disappearance of off-diagonal terms in expectation values. Any proposed derivations of an “equivalence” between SID and EID [71, 76, 79] can therefore at most claim to describe coincidental formal similarities in the context of very particular models, and only if the scope of EID is reduced to the influence on expectation values. On the basis of our arguments, we question the justification for labeling the process referred to by SID as “decoherence.”

3.4 Analysis of the spin bath model

By studying an explicit model, we shall now directly investigate the claim of SID, that terms not diagonal in energy in the expectation value of arbitrary observables of the system decay if the system is endowed with a continuous energy spectrum. We shall also illustrate formal and numerical differences in the time evolution of the expectation value of local observables that take into account only the degrees of freedom of the system \mathcal{S} while ignoring the environment \mathcal{E} (the situation encountered in EID), and global observables that pertain to both \mathcal{S} and \mathcal{E} (the case treated by SID). However, in view of our arguments in the preceding Sec. 3.3, this should not be misunderstood as a side-by-side comparison of SID and EID. While expectation values may share formal similarities in both approaches, they also obliterate fundamental differences between SID and EID that lead to very different implications of these expectation values for the question of decoherence.

3.4.1 The model and its time evolution

The probably most simple exactly solvable model for decoherence was introduced some years ago by Zurek [338]. Here, the system \mathcal{S} consists of a spin-1/2 particle (a single qubit) with two possible states $|0\rangle$ (representing spin up) and $|1\rangle$ (corresponding to spin down), interacting with a collection of N environmental qubits (described by the states $|\uparrow_i\rangle$ and $|\downarrow_i\rangle$) via the total Hamiltonian

$$\hat{H}_{\mathcal{SE}} = \frac{1}{2}(|0\rangle\langle 0| - |1\rangle\langle 1|) \sum_{i=0}^N g_i (|\uparrow_i\rangle\langle \uparrow_i| - |\downarrow_i\rangle\langle \downarrow_i|) \bigotimes_{i' \neq i} \hat{I}_{i'}. \quad (3.7)$$

Here, the g_i are coupling constants, and $\hat{I}_i = (|\uparrow_i\rangle\langle \uparrow_i| + |\downarrow_i\rangle\langle \downarrow_i|)$ is the identity operator for the i th environmental qubit. The self-Hamiltonians of \mathcal{S} and \mathcal{E} are taken to be equal to zero. Note that $\hat{H}_{\mathcal{SE}}$ has a particularly simple form, since it contains only terms diagonal in the $\{|0\rangle, |1\rangle\}$ and $\{|\uparrow_i\rangle, |\downarrow_i\rangle\}$ bases.

It follows that the eigenstates of $\hat{H}_{S\mathcal{E}}$ are product states of the form $|\phi_\lambda\rangle = |0\rangle|\uparrow_1\rangle|\downarrow_2\rangle\cdots|\uparrow_N\rangle$, etc. A general state $|\Psi_0\rangle$ can then be written as a linear combination of product eigenstates,

$$|\Psi_0\rangle = (a|0\rangle + b|1\rangle) \bigotimes_{i=1}^N (\alpha_i|\uparrow_i\rangle + \beta_i|\downarrow_i\rangle). \quad (3.8)$$

This state evolves under the action of $\hat{H}_{S\mathcal{E}}$ into

$$|\Psi(t)\rangle = a|0\rangle|\mathcal{E}_0(t)\rangle + b|1\rangle|\mathcal{E}_1(t)\rangle, \quad (3.9)$$

where

$$|\mathcal{E}_0(t)\rangle = |\mathcal{E}_1(-t)\rangle = \bigotimes_{i=1}^N (\alpha_i e^{ig_i t/2}|\uparrow_i\rangle + \beta_i e^{-ig_i t/2}|\downarrow_i\rangle). \quad (3.10)$$

The density matrix is

$$\begin{aligned} \rho(t) = & |a|^2|0\rangle|\mathcal{E}_0(t)\rangle\langle\mathcal{E}_0(t)|\langle 0| + |b|^2|1\rangle|\mathcal{E}_1(t)\rangle\langle\mathcal{E}_1(t)|\langle 1| \\ & + ab^*|0\rangle|\mathcal{E}_0(t)\rangle\langle\mathcal{E}_1(t)|\langle 1| + a^*b|1\rangle|\mathcal{E}_1(t)\rangle\langle\mathcal{E}_0(t)|\langle 0|, \end{aligned} \quad (3.11)$$

and its part diagonal in energy (i.e., diagonal in the eigenstates $|\phi_\lambda\rangle$ of $\hat{H}_{S\mathcal{E}}$) is

$$\begin{aligned} \rho_d(t) = & (|a|^2|0\rangle\langle 0| + |b|^2|1\rangle\langle 1|) \\ & \times \underbrace{[\dots + |\alpha_1|^2|\beta_2|^2\cdots|\alpha_N|^2|\uparrow_1\rangle|\downarrow_2\rangle\cdots|\uparrow_N\rangle\langle\uparrow_N|\cdots\langle\downarrow_2|\langle\uparrow_1| + \dots]}_{\text{same-direction pairing}} \\ & + (ab^*|0\rangle\langle 1| + a^*b|1\rangle\langle 0|) \\ & \times \underbrace{[\dots + \beta_1\alpha_1^*\beta_2\alpha_2^*\cdots\alpha_N\beta_N^*|\downarrow_1\rangle|\downarrow_2\rangle\cdots|\downarrow_N\rangle\langle\downarrow_N|\cdots\langle\uparrow_2|\langle\uparrow_1| + \dots]}_{\text{opposite-direction pairing}}. \end{aligned} \quad (3.12)$$

3.4.2 Expectation values of local observables

Focusing, in the spirit of EID, on the system \mathcal{S} alone, we trace out the degrees of freedom of the spin bath in the density operator $\rho_{S\mathcal{E}} = |\Psi(t)\rangle\langle\Psi(t)|$. This yields the reduced density operator

$$\rho_{\mathcal{S}} = \text{Tr}_{\mathcal{E}}\rho_{S\mathcal{E}} = |a|^2|0\rangle\langle 0| + |b|^2|1\rangle\langle 1| + ab^*r(t)|0\rangle\langle 1| + a^*br^*(t)|1\rangle\langle 0|, \quad (3.13)$$

where the time dependence of the off-diagonal terms $|0\rangle\langle 1|$ and $|1\rangle\langle 0|$ is given by the decoherence factor

$$r(t) = \langle \mathcal{E}_1(t) | \mathcal{E}_0(t) \rangle = \prod_{i=1}^N (|\alpha_i|^2 e^{ig_it} + |\beta_i|^2 e^{-ig_it}). \quad (3.14)$$

The expectation value of any local \mathcal{S} observable

$$\hat{O}_{\mathcal{S}} = \left[\sum_{s,s'=0,1} s_{ss'} |s\rangle\langle s'| \right] \bigotimes_{i=1}^N \hat{I}_i, \quad (3.15)$$

is then given by

$$\langle \hat{O}_{\mathcal{S}} \rangle_{\Psi(t)} = \text{Tr}_{\mathcal{S}\mathcal{E}}(\rho \hat{O}_{\mathcal{S}}) = \text{Tr}_{\mathcal{S}}(\rho_{\mathcal{S}} \hat{O}_{\mathcal{S}}) = |a|^2 s_{00} + |b|^2 s_{11} + 2 \text{Re}[ab^* s_{10} r(t)]. \quad (3.16)$$

We can formally rewrite $r(t)$ as a sum,

$$r(t) = \sum_{\lambda} |\langle \Psi_0 | \phi_{\lambda} \rangle|^2 e^{iE_{\lambda}t}, \quad (3.17)$$

where the sum runs over all eigenstates $|\phi_{\lambda}\rangle$ of the total Hamiltonian $\hat{H}_{\mathcal{S}\mathcal{E}}$, with eigenvalues E_{λ} .

A concrete illustration for the time dependence of $r(t)$, Eq. (3.14), for two different bath sizes is shown in Fig. 3.1. We see that $|r(t)|$ decays quickly by several orders of magnitude and then continues to oscillate about a very small mean value. Thus, for local observables, terms corresponding to interference between the two \mathcal{S} states $|0\rangle$ and $|1\rangle$ become quickly and strongly suppressed.

3.4.3 Expectation values of global observables

An arbitrary global observable $\hat{O} \equiv \hat{O}_{\mathcal{S}\mathcal{E}}$ can be written as a linear combination of the form $\hat{O} = \sum_{\lambda\lambda'} O_{\lambda\lambda'} |\phi_{\lambda}\rangle\langle\phi_{\lambda'}|$, where the $|\phi_{\lambda}\rangle$ are product eigenstates of the total Hamiltonian $\hat{H}_{\mathcal{S}\mathcal{E}}$, Eq. (3.7). Explicitly,

$$\begin{aligned} \hat{O} = & \sum_j (s_{00}^{(j)} |0\rangle\langle 0| + s_{01}^{(j)} |0\rangle\langle 1| + s_{10}^{(j)} |1\rangle\langle 0| + s_{11}^{(j)} |1\rangle\langle 1|) \\ & \bigotimes_{i=1}^N (\epsilon_{\uparrow\uparrow}^{(ij)} |\uparrow_i\rangle\langle \uparrow_i| + \epsilon_{\uparrow\downarrow}^{(ij)} |\uparrow_i\rangle\langle \downarrow_i| + \epsilon_{\downarrow\uparrow}^{(ij)} |\downarrow_i\rangle\langle \uparrow_i| + \epsilon_{\downarrow\downarrow}^{(ij)} |\downarrow_i\rangle\langle \downarrow_i|). \end{aligned} \quad (3.18)$$

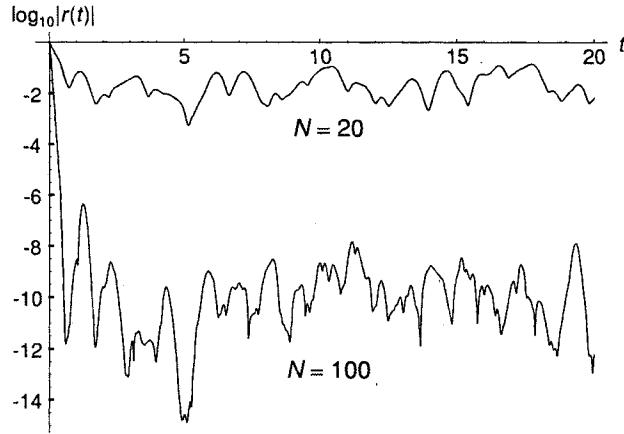


Figure 3.1: Plot of $\log_{10}|r(t)|$, with the decoherence factor $r(t)$ given by Eq. (3.14), for two different bath sizes $N = 20$ and 100 . Fast decay of $r(t)$, corresponding to local decoherence, is observed, and the degree of decoherence is seen to increase with N . The squared coefficients $|\alpha_i|^2$ and the couplings g_i were drawn from a uniform random distribution over the intervals $[0, 1]$ and $[-\pi, \pi]$, respectively.

Since \hat{O} must be Hermitian, s_{00} , s_{11} , $\epsilon_{\uparrow\uparrow}^{(i)}$, and $\epsilon_{\downarrow\downarrow}^{(i)}$ are real numbers, and $s_{01} = (s_{10})^*$, $\epsilon_{\downarrow\uparrow}^{(i)} = (\epsilon_{\uparrow\downarrow}^{(i)})^*$. To keep the notation simple, we shall omit the sum over j (and thus the index j) in the following.

The expectation value of \hat{O} in the state $|\Psi(t)\rangle$, Eq. (3.9), is

$$\begin{aligned} \langle \hat{O} \rangle_{\Psi(t)} &= (|a|^2 s_{00} + |b|^2 s_{11}) \prod_{i=1}^N [|\alpha_i|^2 \epsilon_{\uparrow\uparrow}^{(i)} + \alpha_i^* \beta_i \epsilon_{\uparrow\uparrow}^{(i)} e^{-ig_i t} + (\alpha_i^* \beta_i \epsilon_{\uparrow\uparrow}^{(i)})^* e^{ig_i t} + |\beta_i|^2 \epsilon_{\downarrow\downarrow}^{(i)}] \\ &\quad + 2 \operatorname{Re} \left(ab^* s_{10} \prod_{i=1}^N [|\alpha_i|^2 \epsilon_{\uparrow\uparrow}^{(i)} e^{ig_i t} + \alpha_i^* \beta_i \epsilon_{\uparrow\uparrow}^{(i)} + (\alpha_i^* \beta_i \epsilon_{\uparrow\uparrow}^{(i)})^* + |\beta_i|^2 \epsilon_{\downarrow\downarrow}^{(i)} e^{-ig_i t}] \right) \\ &\equiv (|a|^2 s_{00} + |b|^2 s_{11}) \Gamma_0(t) + 2 \operatorname{Re} [ab^* s_{10} \Gamma_1(t)]. \end{aligned} \quad (3.19)$$

The special case of the expectation value of local observables, as considered in the preceding Sec. 3.4.2, can easily be recovered by remembering that tracing out the degrees of freedom of \mathcal{E} is equivalent to choosing all coefficients $\epsilon_{\uparrow\uparrow}^{(i)} = \epsilon_{\downarrow\downarrow}^{(i)} = 1$ and $\epsilon_{\uparrow\downarrow}^{(i)} = (\epsilon_{\downarrow\uparrow}^{(i)})^* = 0$, which yields $\Gamma_0(t) = 1$ and $\Gamma_1(t) = r(t)$ [see Eq. (3.14)], in agreement

with Eq. (3.16).

Suppression of terms in $\langle \hat{O} \rangle_{\Psi(t)}$ that are not diagonal in the energy eigenbasis would be represented by the vanishing of all time-dependent terms in the above expression, i.e.,

$$\begin{aligned}\langle \hat{O} \rangle_d &= (|a|^2 s_{00} + |b|^2 s_{11}) \prod_{i=1}^N (|\alpha_i|^2 \epsilon_{\uparrow\uparrow}^{(i)} + |\beta_i|^2 \epsilon_{\downarrow\downarrow}^{(i)}) + 2 \operatorname{Re} \left(ab^* s_{10} \prod_{i=1}^N 2 \operatorname{Re}(\alpha_i^* \beta_i \epsilon_{\uparrow\downarrow}^{(i)}) \right) \\ &\equiv (|a|^2 s_{00} + |b|^2 s_{11}) \Gamma_0^d + 2 \operatorname{Re}(ab^* s_{10} \Gamma_1^d),\end{aligned}\quad (3.20)$$

because we can easily show that $\langle \hat{O} \rangle_d = \operatorname{Tr}(\rho_d \hat{O})$, where ρ_d , Eq. (3.12), is the part of the density matrix that is diagonal in the eigenstates of the total Hamiltonian. We also see that $\langle \hat{O} \rangle_d = \operatorname{Tr}(\rho \hat{O}_d)$, where

$$\begin{aligned}\hat{O}_d &= (s_{00}|0\rangle\langle 0| + s_{11}|1\rangle\langle 1|) \bigotimes_{i=1}^N (\epsilon_{\uparrow\uparrow}^{(i)}|\uparrow_i\rangle\langle \uparrow_i| + \epsilon_{\downarrow\downarrow}^{(i)}|\downarrow_i\rangle\langle \downarrow_i|) \\ &\quad + (s_{01}|0\rangle\langle 1| + s_{10}|1\rangle\langle 0|) \bigotimes_{i=1}^N (\epsilon_{\uparrow\downarrow}^{(i)}|\uparrow_i\rangle\langle \downarrow_i| + \epsilon_{\downarrow\uparrow}^{(i)}|\downarrow_i\rangle\langle \uparrow_i|).\end{aligned}\quad (3.21)$$

is the part of \hat{O} diagonal in energy. Thus, as expected, diagonality of $\langle \hat{O} \rangle_{\Psi(t)}$ in energy can also be characterized by the presence of only those product expansion coefficients that are contained in \hat{O}_d .

The form of the two product terms $\Gamma_0(t)$ and $\Gamma_1(t)$ is similar: They only differ in the order of the pairing of the product expansion coefficients with the exponential factors. Also, since the coefficients $s_{jj'}$ are independent, diagonalization in energy will in general require that individually $\Gamma_j(t) \rightarrow \Gamma_0^d$ and $\Gamma_1(t) \rightarrow \Gamma_1^d$ for large t . We can therefore restrict our following analysis to $\Gamma_0(t)$ alone. (We shall also omit the subscript “0” in the following.)

First of all, let us rewrite $\Gamma(t)$ as a sum of 4^N terms,

$$\Gamma(t) = \sum_{\lambda} c_{\lambda} e^{iE_{\lambda}t}. \quad (3.22)$$

where the c_{λ} represent products of expansion coefficients,

$$c_{\lambda} = \left(\prod_{i \in I_1(\lambda)} |\alpha_i|^2 \epsilon_{\uparrow\uparrow}^{(i)} \right) \left(\prod_{i \in I_2(\lambda)} |\beta_i|^2 \epsilon_{\downarrow\downarrow}^{(i)} \right) \left(\prod_{i \in I_3(\lambda)} \alpha_i^* \beta_i \epsilon_{\uparrow\downarrow}^{(i)} \right) \left(\prod_{i \in I_4(\lambda)} (\alpha_i^* \beta_i \epsilon_{\uparrow\downarrow}^{(i)})^* \right). \quad (3.23)$$

Here the sets $I_k(\lambda)$ specify over which indices i each product runs, namely, they are subsets of the set $I = \{1, \dots, N\}$ of all integers between 1 and N such that $\cup_k I_k(\lambda) = I$ and $\cap_k I_k(\lambda) = \emptyset$. The total energy E_λ associated with each term in the sum, Eq. (3.22), is

$$E_\lambda = \sum_{i \in I_4(\lambda)} g_i - \sum_{i \in I_3(\lambda)} g_i. \quad (3.24)$$

We choose the index λ such that $E_{\lambda-1} \leq E_\lambda \leq E_{\lambda+1}$ for all λ . Clearly, $E_\lambda = 0$ whenever $I_3(\lambda) = I_4(\lambda) = \emptyset$ [i.e., if $I_1(\lambda) \cup I_2(\lambda) = I$], canceling out the time dependence of the associated product term in the expression for $\Gamma(t)$. Thus, we can split $\Gamma(t)$ into a time-independent and a time-dependent part,

$$\Gamma(t) = \sum_{\lambda} c_{\lambda} + \sum_{\lambda} c_{\lambda} e^{iE_{\lambda}t} \equiv \Gamma^d + \Lambda(t), \quad (3.25)$$

where now the first sum runs over all λ for which $I_1(\lambda) \cup I_2(\lambda) = I$, while the second sum runs over all λ for which $I_3(\lambda) \cup I_4(\lambda) \neq \emptyset$.

Diagonality in energy would require $\Lambda(t) \rightarrow 0$ as $t \rightarrow \infty$. Written this way, we see that $\Lambda(t)$ is formally similar to the function $r(t)$ derived for local observables, Eq. (3.17). This might not come as a surprise, since also the expression for $r(t)$ can be derived from the calculation of an expectation value of an observable, namely, that of the local observable $\hat{O}_r = (|0\rangle\langle 1| + |1\rangle\langle 0|) \otimes_{i=1}^N \hat{I}_k$ that measures the degree of local interference between the S states $|0\rangle$ and $|1\rangle$. However, in the case of $r(t)$, $c_{\lambda} = |\langle \phi_{\lambda} | \Psi_0 \rangle|^2$ is a product of N real and non-negative coefficients $|\alpha_i|^2$ and $|\beta_i|^2 = 1 - |\alpha_i|^2$, while the c_{λ} of Eq. (3.23) contain cross terms of the form $\alpha_i \beta_i^*$ and $\alpha_i^* \beta_i$, arbitrary real coefficients $\epsilon_{\uparrow\uparrow}^{(i)}$ and $\epsilon_{\downarrow\downarrow}^{(i)}$, and arbitrary complex coefficients $\epsilon_{\uparrow\downarrow}^{(i)}$.

We expect this difference to have strong influence on the time evolution of $\Lambda(t)$ vs that of $r(t)$. The destructive interference needed to obtain suppression of the off-diagonal part of the expectation value relies on the idea that, when a function $f(z)$ is multiplied by a phase factor e^{izt} whose variation with z is much faster than that of $f(z)$, neighboring values $f(z)$ and $f(z + \delta z)$ will have similar magnitude and

phases, but will be weighted with two strongly different phase factors, which leads to an averaging-out effect in the sum $\sum_z f(z)e^{izt}$.

In our case, writing

$$\Lambda(t) = \sum_{\lambda} r_{\lambda} e^{i\varphi_{\lambda}} e^{iE_{\lambda}t}, \quad (3.26)$$

with $r_{\lambda} = |c_{\lambda}|$, the phases φ_{λ} will in general vary very rapidly with λ and, thus, with E_{λ} . This is a consequence of the fact that the $c_{\lambda} = r_{\lambda} e^{i\varphi_{\lambda}}$ are composed of products of coefficients, such that changing a single term in the product will in general result in a drastic change in the overall phase associated with the c_{λ} . (The variation in magnitude among the c_{λ} can be expected to be comparably insignificant for larger N .) Such discontinuous phase fluctuations are absent in the formally similar function $r(t)$, Eq. (3.14), since there only the absolute value of the coefficients α_i and β_i enters. Note that the impact of the phase fluctuations cannot be diminished by going to larger t , since the 2π periodicity of phases implies that the effect of a phase difference between terms λ and $\lambda + 1$ induced by $e^{i(E_{\lambda+1}-E_{\lambda})t}$ will in average be similar to that induced by $e^{i(\varphi_{\lambda+1}-\varphi_{\lambda})}$ for all (larger) values of t .

We anticipate the described phase-variation effect to counteract the averaging-out influence of the multiplying phase factor e^{-iEt} , and to thus make it more difficult, if not entirely impossible, for $\Lambda(t)$, Eq. (3.25), to converge to zero. On the other hand, if the average difference between the phases associated with the individual coefficients is decreased, we would expect that the rate and degree of decay of $\Lambda(t)$ will be improved.

3.4.4 Numerical results for the expectation value of random global observables

To check this prediction and to generally gain more insight into the behavior of $\langle \hat{O} \rangle_{\Psi(t)}$, Eq. (3.19), we studied numerically the time evolution of $\Lambda(t)$, Eq. (3.26), normalized by its initial value at $t = 0$, for sets of random observables \hat{O} . Diagonalization of $\langle \hat{O} \rangle_{\Psi(t)}$ in energy would then be represented by a decay of $\Lambda(t)$ from its initial value of one.

Figure 3.2 shows three typical examples for the time evolution of $\log_{10} \Lambda(t)$ for a fixed bath size of $N = 100$. All couplings g_i were taken to be random real numbers between $-\pi$ and π . To investigate the influence of phase fluctuations of the c_λ , Eq. (3.23), we considered three different cases for selecting the coefficients α_i , β_i , $\epsilon_{\uparrow\downarrow}^{(i)}$, $\epsilon_{\downarrow\uparrow}^{(i)}$, and $\epsilon_{\uparrow\uparrow}^{(i)}$, i.e., for choosing the initial state of the environment and the observable. In the completely random case (A), the coefficients α_i , β_i , and $\epsilon_{\uparrow\downarrow}^{(i)}$ were taken to be random complex numbers, with magnitudes and phases drawn from a uniform distribution over the intervals $[0, 1]$ and $[0, 2\pi]$, respectively (and such that $|\beta_i|^2 = 1 - |\alpha_i|^2$). Similarly, the coefficients $\epsilon_{\uparrow\uparrow}^{(i)}$ and $\epsilon_{\downarrow\downarrow}^{(i)}$ were random real numbers drawn from a uniform distribution over the interval $[-1, 1]$. In the second case (B), the initial state of the environment was prepared such that the phases of the α_i and β_i were restricted to the interval $[0, \pi/2]$. Also, only observables with non-negative values of $\epsilon_{\uparrow\downarrow}^{(i)}$ and $\epsilon_{\downarrow\uparrow}^{(i)}$ were considered, such that sign reversals of c_λ due to a change of product terms containing these coefficients were prevented. Finally, in the third case (C), only the absolute values of the α_i , β_i , $\epsilon_{\uparrow\uparrow}^{(i)}$, $\epsilon_{\downarrow\downarrow}^{(i)}$, and $\epsilon_{\uparrow\downarrow}^{(i)}$ were used, which implies that the c_λ fluctuated only in magnitude.

We observed a drastic influence of the range of phases and signs associated with the individual coefficients α_i , β_i , $\epsilon_{\uparrow\downarrow}^{(i)}$, $\epsilon_{\uparrow\uparrow}^{(i)}$, and $\epsilon_{\downarrow\downarrow}^{(i)}$, on the evolution of $\Lambda(t)$. In the special case (C) of all coefficients being real non-negative numbers, $\Lambda(t)$ exhibited a consistently strong and fast decay behavior, similar to the decay of the function $r(t)$, Eq. (3.14), describing suppression of off-diagonal terms for local observables (see Fig. 3.1). In the intermediate case (B), with restricted phases and signs, the degree of decay of $\Lambda(t)$ was decreased, while the decay rate stayed roughly the same. In the general random case (A), in which no restriction on the spread of phases and on the signs of the coefficients was imposed, the time evolution of $\Lambda(t)$ was observed to be sensitive to the particular set of random numbers used for the coefficients α_i , β_i , $\epsilon_{\uparrow\downarrow}^{(i)}$, $\epsilon_{\uparrow\uparrow}^{(i)}$, and $\epsilon_{\downarrow\downarrow}^{(i)}$ in each run. For some of the sets, $\Lambda(t)$ was seen to lack any decay behavior at all. In other cases, the baseline of oscillation was located below zero,

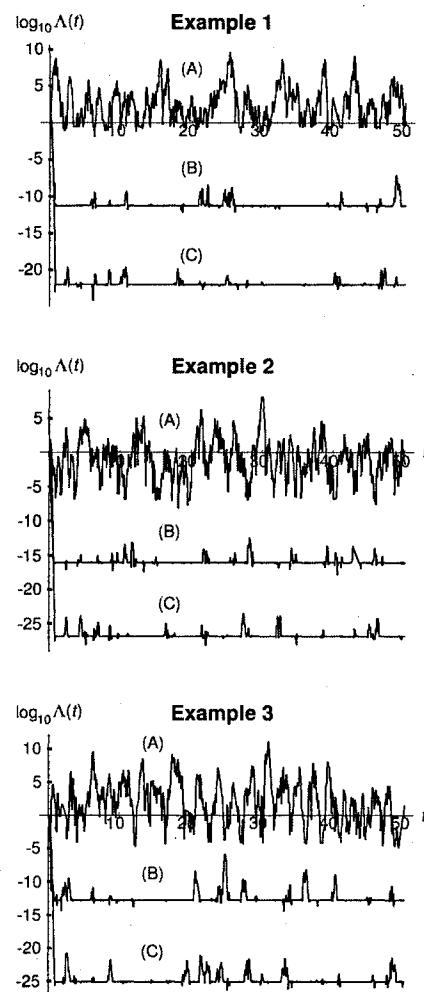


Figure 3.2: Time evolution of $\log_{10} \Lambda(t)$ [see Eq. (3.26)] for $N = 100$ bath spins and three different random observables and initial states of the environment. The function $\Lambda(t)$ quantifies the time dependence of the terms in the expectation value $\langle \hat{O} \rangle_{\Psi(t)}$ [Eq. (3.19)] that are not diagonal in energy. Suppression of these terms is represented by a decay of $\Lambda(t)$ from its initial value of one [i.e., $\log_{10} \Lambda(t) \rightarrow -\infty$]. It is observed that in the general case (A) of completely random observables and initial states of the environment, collective decay of off-diagonal terms does, in general, not occur. However, if the phases of the coefficients describing the observable and the environment are moderately restricted (B) or all set equal to zero (C), decay of increasing strength is found.

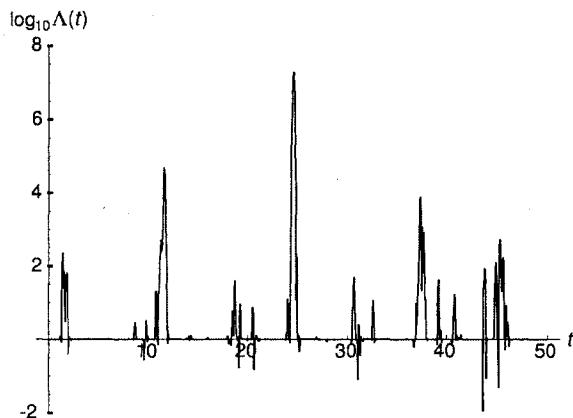


Figure 3.3: Time evolution of $\log_{10} \Lambda(t)$ for $N = 100$ bath spins [see Eq. (3.26)] when no restrictions on the initial state of the environment are imposed. Such restrictions are physically unrealistic and require a preparing measurement on the unrestricted environment, which would in turn be in conflict with the desired generality of the derivation of decay effects. No collective decay of off-diagonal terms is observed, regardless of any restrictions imposed on the observable.

indicating a very weak damping effect, albeit with the peaks of the large-amplitude oscillation frequently reaching values greater than zero.

These results show that, for the bath size studied here, a consistent occurrence of a decay of $\Lambda(t)$ hinges on the phase restrictions imposed on the coefficients describing the observable and the initial state of the environment. If these restrictions are given up, the time evolution of $\Lambda(t)$ and any occurrence of a (comparably weak) decay will exhibit strong dependence on the particular set of values chosen for the coefficients.

However, it is important to realize that the assumption of a restricted initial state of \mathcal{E} is not only unrealistic, since the environment is typically uncontrollable, but it will also lead to a circular argument when aiming at a derivation of a universal decay effect. This is so because any restriction would require an appropriate preparation of the initial state through a measurement on the entire \mathcal{E} , which implies that suppression

of off-diagonal terms would then in general be absent for the observable corresponding to this measurement, if the restriction of the initial state of \mathcal{E} is relevant to the occurrence of the suppression. Consequently, the α_i and β_i must be allowed to possess arbitrary phases. Then, since the $\epsilon_{\uparrow\uparrow}^{(i)}$, $\epsilon_{\downarrow\downarrow}^{(i)}$, and $\epsilon_{\uparrow\downarrow}^{(i)}$ are always paired with the α_i and β_i in the expression for the c_λ that make up $\Lambda(t)$ [see Eq. (3.23)], we anticipate that giving up phase restrictions on the α_i and β_i will render the restrictions imposed on the \hat{O} -coefficients less effective, if not entirely irrelevant, in bringing about a decay of $\Lambda(t)$.

To study this prediction, in Fig. 3.3 we show a representative plot of $\Lambda(t)$ using only the absolute values of the \hat{O} coefficients $\epsilon_{\uparrow\uparrow}^{(i)}$, $\epsilon_{\downarrow\downarrow}^{(i)}$, and $\epsilon_{\uparrow\downarrow}^{(i)}$, but with the \mathcal{E} coefficients α_i and β_i possessing random phases between 0 and 2π . We found that decay is either entirely absent or strongly diminished in strength, despite the fact that the strongest possible restriction on the phases and signs of the \hat{O} coefficients is imposed. Similar to the case of completely random coefficients, the behavior of $\Lambda(t)$ was observed to depend crucially on the particular set of random numbers chosen for the coefficients. These results lead us to conclude that a universal decay of off-diagonal terms does not occur for the studied bath size and time scale.

To be sure, SID is based on the assumption of a quasicontinuous energy spectrum and very long time scales, corresponding to “sufficiently large” N and t (the existing derivations of SID [78, 71, 77, 80, 76, 73, 75, 74, 213, 70, 212, 211, 72] even assume the strict limits $N \rightarrow \infty$ and $t \rightarrow \infty$, in order to allow for a direct application of the Riemann-Lebesgue theorem), while so far we have only considered relatively modest values for these parameters. However, since we know from Fig. 3.1 that for expectation values of local observables, strong and fast decay of off-diagonal terms is obtained for the value of N and over the time scale used in the plots shown in Fig. 3.2, it is clear that, if a general global disappearance of interference terms is to occur in our model, it will require a much larger number of environmental qubits and/or longer time scales than typically considered for local observables.

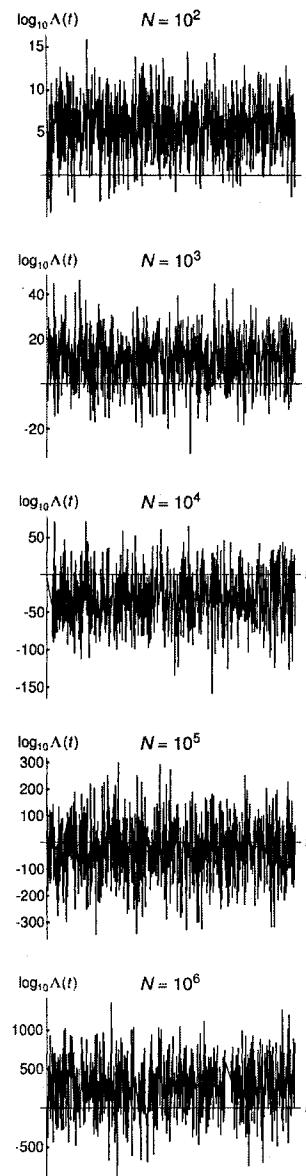


Figure 3.4: Example for the time evolution of $\log_{10} \Lambda(t)$ [see Eq. (3.26)] using a random observable and random initial bath state, for bath sizes between $N = 10^2$ and 10^6 and a long time scale $t = 0\text{--}10^6$. No connection between the size of the spin bath and the occurrence and the degree of damping is observed. Therefore, no consistent collective decay of interference terms occurs. The increased time scale is seen to be irrelevant.

Accordingly, in Fig. 3.4 we show a typical example for the time evolution of $\Lambda(t)$ over the time scale $t = 0\text{--}10^6$ for the case of a completely random observable and initial state of \mathcal{E} , using comparably large bath sizes N between $N = 10^2$ and 10^6 . We observed that even for these values of N , no consistent occurrence of a decay became apparent. In particular, no generally valid direct correlation between the value of N and the time evolution of $\Lambda(t)$ was visible. Instead, it was again the particular set of random numbers included in the computation of $\Lambda(t)$ for a given value of N (but not to the size N of the set itself) that determined whether the baseline of oscillation of $\Lambda(t)$ was located above or below the zero line. In agreement with analytical predictions in the preceding section, we also found that the choice of a longer timescale is irrelevant, since neither the baseline nor the amplitude of oscillation changed significantly over the investigated time interval after a comparably short initial period. Furthermore, we observed that even if $\Lambda(t)$ “decayed” for a particular set of random numbers, the function sustained a large-amplitude oscillation whose peaks often attained values much larger than the initial value of $\Lambda(t)$.

Our results show that, in general, for the bath sizes and time scales studied, destructive interference of off-diagonal terms in the expectation value expressed in the energy eigenbasis [as quantified by $\Lambda(t)$, see Eq. (3.26)] does not occur in our model. Instead, the time evolution of $\Lambda(t)$ is simply determined by the particular random numbers used to describe the observable and the initial state of the environment. Therefore, no general suppression of interference terms can be inferred.

3.5 Discussion

The process described by SID appears to be neither formally nor conceptually nor physically related to the decoherence mechanism in the standard sense of environmental decoherence. EID accounts for the absence of interference from the perspective of the local (open) system by describing interactions with an environment in quantum-mechanical terms of wave-function entanglement. In contrast, SID de-

scribes dynamically induced destructive interference between time-dependent terms in the expression for expectation values. SID does not, however, explain the physical origin of the measurement outcomes and their probability-weighted ensembles needed to define the expectation values. Even if this purely phenomenological basis of SID is accepted, the described process has no bearing on a loss of coherence in individual measurements, since it is only a consequence of averaging over a large number of measurement results. This is in fundamental contrast to EID, where each measurementlike interaction leads to a dislocalization of interference and thus, locally, to a disappearance of interference.

The main result of our study of the spin bath model is the finding that the destructive interference predicted by SID will in general fail to occur in our model even for bath sizes and over time scales much larger than typically considered in treatments of the same model in environmental decoherence. The source of this failure lies in the random relative phases associated with the individual initial bath spin states and the expansion coefficients of the observable. The resulting discontinuous phase fluctuations in the coefficient function c_λ , as defined in Eq. (3.23), counteract the supposed averaging-out effect of the dynamical phase factors e^{iEt} in a way that is, due to the 2π periodicity of the phase, effectively independent of the value of t .

Even when the bath size is increased, the function c_λ remains a set of discrete values with discontinuously varying phases. This can be explained by noting that, while the total energy is a sum of the energies of each subsystem, such that enlarging the number of contributing subsystems will in general lead to an improved quasicontinuity of the energy spectrum, the 2π periodicity of the phases implies that the degree of phase discontinuity of the c_λ will not be diminished by increasing the number of subsystems. It is therefore unlikely that a consistent decay behavior could become apparent for spin baths much larger than those considered here.

This indicates that it is not the degree of continuity of the energy spectrum that represents the determining factor for obtaining destructive interference. Rather, it

is the discrete nature of the model itself that seems to lead to difficulties. Only if restrictions are imposed on both the measured observable and the initial state of the environment, a consistent and general suppression of off-diagonal terms can occur. But, as we have argued, the corresponding preparation of the initial state of the environment is physically unrealistic and renders the derivation of a universal decay effect circular.

We conjecture that the diagonalization of the expectation value, as described by SID, is likely to fail also in other systems composed of discrete individual subentities. For, in such models, the relevant function will typically be represented by a large product of discrete expansion coefficients, similar to the c_λ of our model, whose discontinuous phase fluctuations will again be likely to counteract the averaging-out influence of the dynamical phases. It is therefore clear that the seemingly innocuous mathematical requirement of regularity and integrability of the coefficient functions (see Sec. 3.2) is far from “valid in all relevant cases” where the condition of a sufficiently continuous energy spectrum holds. The suggestion to approximate such discrete functions by a continuous function through interpolation [78] does not appear to be viable, since the interpolated function would describe a physically different situation.

On a general note, it is also important to realize that dynamical phases are correlated. Thus one could always construct an observable for which the initial phases of the coefficients seem completely random, but are in fact chosen such that recurrence of coherence will show up within a finite time interval, thus disproving the claimed universality of SID without any further argument.

3.6 Summary and conclusions

We have investigated the two main claims of the “self-induced decoherence” approach, namely, (1) that expectation values of observables pertaining to a closed system become diagonal in the eigenbasis of the Hamiltonian, provided the system is

endowed with a continuous energy spectrum; and (2) that this process represents a new way of describing quantum decoherence, and that it leads to results equivalent to the standard approach of environment-induced decoherence.

We have evaluated the first claim in the context of a simple spin bath model of finite size by studying, analytically and numerically, the time evolution of expectation values of random global observables. We have found that, in general, collective decay of terms off-diagonal in the energy eigenbasis does not occur over the large range of bath sizes and time scales considered. This result is not due to an insufficient quasicontinuity of the energy spectrum, but is rather rooted in the randomness of the phases associated with the observable and the initial state of the environment. Even in the limit of large bath sizes, the discrete functions for which destructive interference is to be derived do not approach their sufficiently smoothly varying interpolated approximations required for the dynamical phase averaging to have an effect.

These results represent an example for a simple model system that, although endowed with a quasicontinuous energy spectrum, fails to exhibit the decay of off-diagonal terms that would be expected from an extrapolation of SID to discrete models in the limit of comparably large sizes of the system. Such an extrapolation should be possible if the approach is to have general physical relevance. We have also anticipated that the decay effect described by SID will likely be absent also in other similar models that are composed of discrete subsystems.

With respect to the second claim of the SID program, we have questioned the suggestion that SID represents a “new viewpoint” [78] on the theory of environment-induced decoherence, since the two approaches are based on conceptually, formally, and physically unrelated mechanisms. In particular, we have pointed out the following key differences and objections.

- (i) SID does not describe the suppression of interference for individual measurements, since interference terms in the expectation value are not damped individually.
- (ii) SID simply presupposes the existence of an ensemble of measurement out-

comes, without giving an account of its origin in terms of a physical description of measurement.

(iii) The assumption of closed systems is unrealistic, especially for systems containing the many degrees of freedom needed to obtain the required quasicontinuous energy spectrum.

(iv) The physical feasibility and relevance of measurements pertaining to the total system-environment combination is doubtful.

(v) Energy as the universal preferred basis of the global closed system can usually not account for the different observed preferred bases for the local system of interest.

Our study leads us to two main conclusions. First, it points to the need for more precise, physically motivated criteria for the occurrence of the destructive interference effect described by SID. Most importantly, however, the physical interpretation and relevance of this effect need to be explained. We suspect that the SID approach may have mistakenly interpreted and labeled an unrelated process as “decoherence.”

Acknowledgments

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Chapter 4

ON ZUREK'S DERIVATION OF THE BORN RULE¹

Summary

Recently, W. H. Zurek presented a novel derivation of the Born rule based on a mechanism termed environment-assisted invariance, or “envariance” [W. H. Zurek, *Phys. Rev. Lett.* **90**(2), 120404 (2003)]. We review this approach and identify fundamental assumptions that have implicitly entered into it, emphasizing issues that any such derivation is likely to face.

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4.1 Introduction

In standard quantum mechanics, Born’s rule [59] is simply postulated. A typical formulation of this rule reads:

If an observable \hat{O} , with eigenstates $\{|o_i\rangle\}$ and spectrum $\{o_i\}$, is measured on a system described by the state vector $|\psi\rangle$, the probability for the measurement to yield the value o_i is given by $p(o_i) = |\langle o_i | \psi \rangle|^2$.

Born’s rule is of paramount importance to quantum mechanics as it introduces a probability concept into the otherwise deterministic theory and relates it mathematically to the Hilbert space formalism. No violation of Born’s rule has ever been discovered experimentally—which has certainly supported the rôle of the Born rule as the favorite ingredient of what has been nicknamed the “shut up and calculate”

¹Co-authored with Arthur Fine.

interpretation of quantum mechanics. (Although often attributed to Feynman, it appears that the nickname was actually coined by David Mermin [224]. For an example of such a stance, see [138]).

Replacing the postulate of Born's rule by a derivation would be a highly desirable goal within quantum theory in general. The famous theorem of Gleason [155] presented a mathematical motivation for the form of the Born probabilities by showing that if one would like to assign a non-negative real valued function $p(v)$ to every vector v of a vector space \mathcal{V} of dimension greater than two such that for every orthonormal basis $\{v_1, \dots, v_n\}$ of \mathcal{V} the sum of the $p(v_i)$ is equal to one,

$$\sum_i p(v_i) = 1, \quad (4.1)$$

then the only possible choice is $p(v) = |\langle v|w \rangle|^2$ for all vectors v and an arbitrary but fixed vector w , provided that probabilities are assumed to be non-contextual. The normalization requirement of Eq. (4.1) for $p(v)$ with respect to any orthonormal basis can be physically motivated by remembering that any orthonormal basis $\{v_1, \dots, v_n\}$ can be viewed as the eigenbasis of observables $\hat{O} = \sum_i \lambda_i |v_i\rangle\langle v_i|$, and by referring to the fact that in every measurement of such an observable \hat{O} on a system with state vector w one outcome (represented by the eigenvalue λ_j corresponding to one of the eigenvectors v_j) will occur, such that $p(v_j) = 1$ and $p(v_i) = 0$ for $i \neq j$, and Eq. (4.1) follows. In spite of its mathematical elegance, Gleason's theorem is usually considered as giving rather little physical insight into the emergence of quantum probabilities and the Born rule.

Other attempts towards a consistent derivation of the Born probabilities have previously been made in particular in the context of relative-state interpretations where both the meaning of probabilities and their relation to Born's rule requires explicit elucidation (see, for example, [129, 179, 102, 159, 147, 131]), but the success of these approaches is controversial [293, 200, 288]. A widely disputed derivation of the Born rule that is solely based on the non-probabilistic axioms of quantum

mechanics and on classical decision theory (and that is more physically motivated than Gleason’s argument) has been proposed by Deutsch [99]. It was criticized by Barnum *et al.* [34] but was subsequently defended by Wallace [315] and put into an operational framework by Saunders [279]; no decisive conclusion seems to have been reached on the success of these derivations thus far.

A novel and interesting proposal towards a derivation of Born’s rule has recently been put forward by Zurek [345] (see also the follow-ups in [347, 344]). Zurek is a key figure in the development of the decoherence program (for a recent survey of the program and further references, see [344, 281]) that is based on a study of open quantum systems and their interaction with the many degrees of freedom of their environment, leading to explanations for the emergence of the “classical” world of our observation. However, one of the remaining loopholes in a consistent derivation of classicality from decoherence and standard non-collapse quantum mechanics alone has been tied to the fact that the formalism of decoherence and its interpretation rely implicitly on Born’s rule, but that decoherence does not yield an independent motivation for the connection between the quantum mechanical state space formalism and probabilities. Any derivation of the Born rule from decoherence [341] is therefore subject to the charge of circularity [330].

To address this criticism, Zurek has suggested a derivation of Born’s rule that is based on the inclusion of the environment—thus matching well the spirit of the decoherence program—but without relying on the key elements of decoherence that presume Born’s rule and would thus render the argument circular. Zurek’s derivation is of course not only relevant in the context of the decoherence program.

Because we consider Zurek’s approach promising, we would like to bring out the assumptions that enter into the derivation but have not been explicitly mentioned in Refs. [345, 347, 344]. Hopefully such an analysis will help in a careful evaluation of the question to what extent Zurek’s derivation can be regarded as fundamental. In fact, after this paper had been posted online as a preprint, two other discussions

of Zurek's argument have appeared that also describe variants of the proof [33, 230]. Moreover, Zurek himself [346] has recently revised his original derivation in a way that addresses several of the issues raised in this article and that is more explicit about the assumptions (some designated now as "facts") used in his proof. These correspond to what we identify in the following discussion.

To anticipate, we find that Zurek's derivation is based on at least the following assumptions:

- (1) The probability for a particular outcome, i.e., for the occurrence of a specific value of a measured physical quantity, is identified with the probability for the eigenstate of the measured observable with eigenvalue corresponding to the measured value—an assumption that would follow from the *eigenvalue-eigenstate link*.
- (2) Probabilities of a system \mathcal{S} entangled with another system \mathcal{E} are a function of the *local* properties of \mathcal{S} only, which are exclusively determined by the state vector of the *composite* system \mathcal{SE} .
- (3) For a composite state in the Schmidt form $|\psi_{\mathcal{SE}}\rangle = \sum_k \lambda_k |s_k\rangle |e_k\rangle$, the probability for $|s_k\rangle$ is *equal* to the probability for $|e_k\rangle$.
- (4) Probabilities associated with a system \mathcal{S} entangled with another system \mathcal{E} remain *unchanged* when certain transformations (namely, Zurek's "envariant transformations") are applied that only act on \mathcal{E} (and similarly for \mathcal{S} and \mathcal{E} interchanged).

Our paper is organized as follows. First, we review Zurek's derivation of the Born rule as given in his original papers [345, 347, 344], and also include a line of reasoning presented in his recent follow-up [346] (that in turn takes issues raised in the following discussion into account). We then elucidate and discuss step by step the assumptions

that we believe have entered into Zurek's approach. In the final section, we summarize our main points.

4.2 Review of Zurek's derivation

(I) Zurek suggests a derivation of Born's rule for the following pure state that describes an entanglement between a system \mathcal{S} , described by a Hilbert space $\mathcal{H}_{\mathcal{S}}$, and its environment \mathcal{E} , represented by a Hilbert space $\mathcal{H}_{\mathcal{E}}$:

$$|\psi_{\mathcal{SE}}\rangle = \sum_k \lambda_k |s_k\rangle |e_k\rangle, \quad (4.2)$$

where $\{|s_k\rangle\}$ and $\{|e_k\rangle\}$ are orthonormal bases of $\mathcal{H}_{\mathcal{S}}$ and $\mathcal{H}_{\mathcal{E}}$, respectively. Zurek holds that after the \mathcal{SE} correlation has been established, the system no longer interacts with the environment [347, p. 10], i.e., that \mathcal{E} is “dynamically decoupled” [345, p. 120404-1] and thus “causally disconnected” [344, p. 754] from \mathcal{S} .

For the sake of clarity and simplicity, we shall in the following restrict ourselves to the case of coefficients of equal magnitude and to two-dimensional state spaces $\mathcal{H}_{\mathcal{S}}$ and $\mathcal{H}_{\mathcal{E}}$, i.e., we consider the state

$$|\psi_{\mathcal{SE}}\rangle = \frac{1}{\sqrt{2}} \left(e^{i\alpha_1} |s_1\rangle |e_1\rangle + e^{i\alpha_2} |s_2\rangle |e_2\rangle \right). \quad (4.3)$$

Once a valid derivation of Born's rule is accomplished for this situation, the case of non-equal probabilities and of state spaces of more than two dimensions can be treated by means of a relatively straightforward counting argument [345] (at least for probabilities that are rational numbers). What Zurek's derivation now aims to establish is the result that for an observer of \mathcal{S} , the probabilities for $|s_1\rangle$ and $|s_2\rangle$ will be equal. That claim is the focus of our analysis.

(II) Zurek considers pairs of unitary transformations $\widehat{U}_{\mathcal{S}} = \widehat{u}_{\mathcal{S}} \otimes \widehat{I}_{\mathcal{E}}$ and $\widehat{U}_{\mathcal{E}} = \widehat{I}_{\mathcal{S}} \otimes \widehat{u}_{\mathcal{E}}$. Here $\widehat{u}_{\mathcal{S}}$ acts only on the Hilbert state space $\mathcal{H}_{\mathcal{S}}$ of \mathcal{S} , and $\widehat{I}_{\mathcal{E}}$ is the identity operator in $\mathcal{H}_{\mathcal{E}}$. Similarly $\widehat{u}_{\mathcal{E}}$ acts only on the Hilbert state space $\mathcal{H}_{\mathcal{E}}$ of \mathcal{E} , and $\widehat{I}_{\mathcal{S}}$ is the identity operator in $\mathcal{H}_{\mathcal{S}}$.

If the composite state $|\psi_{S\mathcal{E}}\rangle$ is invariant under the combined application of \hat{U}_S and $\hat{U}_{\mathcal{E}}$,

$$\hat{U}_{\mathcal{E}}(\hat{U}_S|\psi_{S\mathcal{E}}\rangle) = |\psi_{S\mathcal{E}}\rangle, \quad (4.4)$$

the composite state is called *envariant under \hat{u}_S* . (The word “envariant” stems from the abbreviation “envariance” of the term “environment-assisted invariance,” an expression coined by Zurek). Zurek gives the following interpretation of envariance [345, p. 120404-1]:

When the transformed property of the system can be so “untransformed” by acting only on the environment, it is not the property of S . Hence, when $S\mathcal{E}$ is in the state $|\psi_{S\mathcal{E}}\rangle$ with this characteristic, it follows that the envariant properties of S must be completely unknown.

It is difficult to understand just what the term “property” refers to here, since it is the composite state that is transformed and untransformed, and so the “properties” involved would seem to be features of the state, not of the system. It seems that envariance under \hat{u}_S is taken to imply that an observer who “in the spirit of decoherence” [347, p. 10] only has access to S will not be able to determine features of the combined state that are affected by \hat{u}_S (or, more properly, by \hat{U}_S). For such an observer a local description of S will be independent of these features, which may depend on a particular decomposition. While this general description is far from precise, the uses to which Zurek puts envariance are clear enough.

(IIa) The first type of an envariant transformation that Zurek considers is the pair

$$\hat{u}_S^{(\beta_1, \beta_2)} = e^{i\beta_1}|s_1\rangle\langle s_1| + e^{i\beta_2}|s_2\rangle\langle s_2|, \quad (4.5a)$$

$$\hat{u}_{\mathcal{E}}^{(\beta_1, \beta_2)} = e^{-i\beta_1}|e_1\rangle\langle e_1| + e^{-i\beta_2}|e_2\rangle\langle e_2|. \quad (4.5b)$$

The effect of the first transformation $\widehat{U}_S = \widehat{u}_S^{(\beta_1, \beta_2)} \otimes \widehat{I}_{\mathcal{E}}$ is to change the phases associated with the terms in the Schmidt state, Eq. (4.3), that is,

$$\widehat{U}_S |\psi_{S\mathcal{E}}\rangle = \frac{1}{\sqrt{2}} \left(e^{i(\alpha_1 + \beta_1)} |s_1\rangle |e_1\rangle + e^{i(\alpha_2 + \beta_2)} |s_2\rangle |e_2\rangle \right). \quad (4.6)$$

It is easy to see that if one subsequently acts on this state with $\widehat{U}_{\mathcal{E}} = \widehat{I}_S \otimes \widehat{u}_{\mathcal{E}}^{(\beta_1, \beta_2)}$, the original $|\psi_{S\mathcal{E}}\rangle$ will be restored. Thus, $|\psi_{S\mathcal{E}}\rangle$, and in particular the phases associated with the states in the Schmidt decomposition of $|\psi_{S\mathcal{E}}\rangle$, are envariant under the phase transformation $\widehat{u}_S^{(\beta_1, \beta_2)}$ given by Eq. (4.5a).

In the spirit of Zurek's interpretation of envariance stated above, this implies that the phases of the Schmidt coefficients are not a property of S alone, so that a local description of S cannot depend on the phases α_1 and α_2 in the composite state $|\psi_{S\mathcal{E}}\rangle$ of Eq. (4.3). This leads Zurek to the conclusion that also the probabilities associated with S must be independent of these phases, and that it thus suffices to show that equal likelihoods arise for the state

$$|\psi_{S\mathcal{E}}\rangle = \frac{1}{\sqrt{2}} \left(|s_1\rangle |e_1\rangle + |s_2\rangle |e_2\rangle \right). \quad (4.7)$$

We shall therefore use this state in the rest of the argument.

(IIb) Another type of envariant transformations relevant to Zurek's derivation are so-called "swaps,"

$$\widehat{u}_S^{(1 \leftrightarrow 2)} = |s_1\rangle \langle s_2| + |s_2\rangle \langle s_1|, \quad (4.8a)$$

$$\widehat{u}_{\mathcal{E}}^{(1 \leftrightarrow 2)} = |e_1\rangle \langle e_2| + |e_2\rangle \langle e_1|. \quad (4.8b)$$

Application of $\widehat{U}_S = \widehat{u}_S^{(1 \leftrightarrow 2)} \otimes \widehat{I}_{\mathcal{E}}$, with $\widehat{u}_S^{(1 \leftrightarrow 2)}$ from Eq. (4.8a), to the state $|\psi_{S\mathcal{E}}\rangle$ in Eq. (4.7) yields

$$\widehat{U}_S |\psi_{S\mathcal{E}}\rangle = \frac{1}{\sqrt{2}} \left(|s_2\rangle |e_1\rangle + |s_1\rangle |e_2\rangle \right), \quad (4.9)$$

i.e., the states of the environment \mathcal{E} correlated with the states of the system S have been interchanged. This swap can obviously be undone by a "counterswap" $\widehat{U}_{\mathcal{E}} = \widehat{I}_S \otimes$

$\hat{u}_{\mathcal{E}}^{(1 \leftrightarrow 2)}$, with $\hat{u}_{\mathcal{E}}^{(1 \leftrightarrow 2)}$ from Eq. (4.8b), applied to the state $\hat{U}_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle$ in Eq. (4.9). Thus, the composite state $|\psi_{\mathcal{SE}}\rangle$, Eq. (4.7), is envariant under swaps. The invariant property is then “ $|s_k\rangle$ is correlated with $|e_l\rangle$.” On the basis of the interpretation of envariance quoted above, this implies that a local description of \mathcal{S} must be independent of which particular environmental state $|e_l\rangle$ is correlated with a given $|s_k\rangle$, i.e., that swapping of the states of the system cannot be detected by a local observation of \mathcal{S} alone.

(IIIa) To make the connection between envariance of $|\psi_{\mathcal{SE}}\rangle$ under swaps with quantum probabilities and Born’s rule, Zurek states [345, p. 120404-2]:

Let us now make a rather general (and a bit pedantic) assumption about the measuring process: When the states are swapped, the corresponding probabilities get relabeled ($i \leftrightarrow j$). This leads us to conclude that the probabilities for any two envariantly swappable $|s_k\rangle$ are equal.

This argument assumes that the swapping transformation (that interchanges the correlations between the states of the system and the environment) also swaps the probabilities associated with the states of the system.

To motivate this assumption, the following line of reasoning has been described to us by Zurek in private communication and has subsequently also appeared in published form in Refs. [33, 346]. Let $p(|s_1\rangle; |\psi_{\mathcal{SE}}\rangle)$ denote the probability for $|s_1\rangle$ when the \mathcal{SE} combination is in the composite state $|\psi_{\mathcal{SE}}\rangle$, and similarly for $|s_2\rangle$, $|e_1\rangle$ and $|e_2\rangle$. Before the first swap, Zurek states that

$$\begin{aligned} p(|s_1\rangle; |\psi_{\mathcal{SE}}\rangle) &= p(|e_1\rangle; |\psi_{\mathcal{SE}}\rangle), \\ p(|s_2\rangle; |\psi_{\mathcal{SE}}\rangle) &= p(|e_2\rangle; |\psi_{\mathcal{SE}}\rangle), \end{aligned} \tag{4.10}$$

by referring to the direct connection between the states of \mathcal{S} and \mathcal{E} in the state vector expansion Eq. (4.3). After the first swap (acting on \mathcal{S}),

$$\begin{aligned} p(|s_1\rangle; \hat{U}_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle) &= p(|e_2\rangle; \hat{U}_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle), \\ p(|s_2\rangle; \hat{U}_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle) &= p(|e_1\rangle; \hat{U}_{\mathcal{S}}|\psi_{\mathcal{SE}}\rangle), \end{aligned} \tag{4.11}$$

where we have used $\hat{U}_S|\psi_{SE}\rangle$ instead of $|\psi_{SE}\rangle$ as the second argument of the probability function to take into account the transformation of the state of SE . Zurek now holds that under a swap, properties of the environment cannot have been affected by the first swap acting on the system S only, so that we must have

$$\begin{aligned} p(|e_1\rangle; \hat{U}_S|\psi_{SE}\rangle) &= p(|e_1\rangle; |\psi_{SE}\rangle), \\ p(|e_2\rangle; \hat{U}_S|\psi_{SE}\rangle) &= p(|e_2\rangle; |\psi_{SE}\rangle). \end{aligned} \quad (4.12)$$

After the application of the counterswap, we get

$$\begin{aligned} p(|s_1\rangle; \hat{U}_E\hat{U}_S|\psi_{SE}\rangle) &= p(|e_1\rangle; \hat{U}_E\hat{U}_S|\psi_{SE}\rangle), \\ p(|s_2\rangle; \hat{U}_E\hat{U}_S|\psi_{SE}\rangle) &= p(|e_2\rangle; \hat{U}_E\hat{U}_S|\psi_{SE}\rangle), \end{aligned} \quad (4.13)$$

where Zurek has that

$$\begin{aligned} p(|s_1\rangle; \hat{U}_E\hat{U}_S|\psi_{SE}\rangle) &= p(|s_1\rangle; \hat{U}_S|\psi_{SE}\rangle), \\ p(|s_2\rangle; \hat{U}_E\hat{U}_S|\psi_{SE}\rangle) &= p(|s_2\rangle; \hat{U}_S|\psi_{SE}\rangle), \end{aligned} \quad (4.14)$$

since the counterswap only acted on E . Moreover, since after the counterswap the final state vector will be identical to the initial state vector, Zurek concludes that

$$\begin{aligned} p(|s_1\rangle; \hat{U}_E\hat{U}_S|\psi_{SE}\rangle) &= p(|s_1\rangle; |\psi_{SE}\rangle), \\ p(|s_2\rangle; \hat{U}_E\hat{U}_S|\psi_{SE}\rangle) &= p(|s_2\rangle; |\psi_{SE}\rangle), \\ p(|e_1\rangle; \hat{U}_E\hat{U}_S|\psi_{SE}\rangle) &= p(|e_1\rangle; |\psi_{SE}\rangle), \\ p(|e_2\rangle; \hat{U}_E\hat{U}_S|\psi_{SE}\rangle) &= p(|e_2\rangle; |\psi_{SE}\rangle). \end{aligned} \quad (4.15)$$

This implies, from the above Eqs. (4.10)–(4.15), that

$$\begin{aligned} p(|s_1\rangle; |\psi_{SE}\rangle) &= p(|s_1\rangle; \hat{U}_E\hat{U}_S|\psi_{SE}\rangle) = p(|s_1\rangle; \hat{U}_S|\psi_{SE}\rangle) \\ &= p(|e_2\rangle; |\psi_{SE}\rangle) = p(|s_2\rangle; |\psi_{SE}\rangle), \end{aligned} \quad (4.16)$$

which establishes the desired result $p(|s_1\rangle; |\psi_{SE}\rangle) = p(|s_2\rangle; |\psi_{SE}\rangle)$.

(IIIb) Since the connection between envariance under swaps and equal probabilities is the crucial step in the derivation, we would like to mention another line of argument found in Zurek's papers [347, 344] that more explicitly connects envariance with ignorance and the information available to a local observer. Here, Zurek considers a von Neumann measurement carried out on the composite state vector $|\psi_{SE}\rangle$ by an observer, described by "memory states" $|\mu_0\rangle$ (the premeasurement memory state) and $|\mu_1\rangle, |\mu_2\rangle$ (the post-measurement memory states corresponding to the perception of the "outcomes" $|s_1\rangle$ and $|s_2\rangle$, respectively):

$$|\mu_0\rangle|\psi_{SE}\rangle \propto |\mu_0\rangle(|s_1\rangle|e_1\rangle + |s_2\rangle|e_2\rangle) \longrightarrow |\mu_1\rangle|s_1\rangle|e_1\rangle + |\mu_2\rangle|s_2\rangle|e_2\rangle. \quad (4.17)$$

Zurek then states [344, p. 755]:

[Envariance of $|\psi_{SE}\rangle$ under swaps] allows the observer (who knows the joint state of SE exactly) to conclude that the probabilities of all the envariantly swappable outcomes must be the same. The observer cannot predict his memory state after the measurement of S because he knows too much: the exact combined state of SE . (...) Probabilities refer to the guess the observer makes on the basis of his information before the measurement about the state of his memory—the future outcome—after the measurement. Since the left-hand side of Eq. (4.17) is envariant under swaps of the system states, the probabilities of all the states must be equal.

In a different paper, Zurek argues [347, p. 12]:

When the state of the observer's memory is not correlated with the system, and the absolute values of the coefficients in the Schmidt decomposition of the entangled state describing SE are all equal, and E cannot be accessed, the resulting state of S is *objectively invariant* under all *local* measure-preserving transformations. Thus, with no need for further excuses, probabilities of events $\{|s_k\rangle\}$ must be—prior to measurement—equal.

Obviously, these arguments appeal to a rather different explanation for the emergence of equal likelihoods from the envariance of $|\psi_{S\mathcal{E}}\rangle$ under swaps than the previously quoted argument. Now, probabilities are introduced from the point of view of the observer to account for his lack of knowledge of the individual state of \mathcal{S} , since he has perfect knowledge of the composite state of $S\mathcal{E}$. Then, so goes Zurek's claim, since the observer cannot detect the swapping of the possible outcome states $|s_1\rangle$ and $|s_2\rangle$ of \mathcal{S} before the measurement, he will regard them as "equivalent" and therefore attach equal likelihoods to them.

4.3 Discussion

(A) First of all, Zurek's derivation intrinsically requires the split of the total Hilbert space \mathcal{H} into at least two subspaces \mathcal{H}_S and $\mathcal{H}_{\mathcal{E}}$ which are identified with a system \mathcal{S} and its environment \mathcal{E} , where \mathcal{S} is presumed to have interacted with \mathcal{E} at some point in the past. The environment is then responsible for the emergence of probabilities within the system, similar to the spirit of decoherence where the environment is responsible for the emergence of subjective classicality within the system [344, 281].

The obvious question is then to what extent the necessity to include the environment constitutes a restriction of generality. Apart from the problem of how to do cosmology, we might take a pragmatic point of view here by stating that any observation of the events to which we wish to assign probabilities will always require a measurement-like context that involves an open system interacting with an external observer, and that therefore the inability of Zurek's approach to derive probabilities for a closed, undivided system should not be considered as a shortcoming of the derivation.

(B) Secondly, we might wonder whether the choice of the entangled pure Schmidt state, Eq. (4.2), implies a lack of generality in the derivation. Any two-system compos-

ite pure state state can be diagonalized in the Schmidt form above, so the particular form of the expansion of $|\psi_{SE}\rangle$ implies no loss of generality. Furthermore, if ρ_{SE} were non-pure, it could be made pure simply by enlarging the space \mathcal{H}_E , which cannot influence probabilities of S since E is assumed to be dynamically decoupled from S after the initial interaction that established the entanglement between S and E [347]. We thus conclude that once the requirement for openness is acknowledged, the consideration of the state $|\psi_{SE}\rangle$, Eq. (4.2), will suffice for a general derivation of the Born rule.

(C) Before introducing any probability concept into quantum theory, we need to define what these probabilities are supposed to be assigned to. Clearly, from the point of view of observations and measurements, we would like to assign probabilities to the occurrence of the specific values of the observable O that has been measured, i.e., to the “outcomes.” The eigenvalue-eigenstate link of quantum mechanics postulates that a system has a value for an observable if and only if the state of the system is an eigenstate characteristic of that value (or a proper mixture of those eigenstates). If we consider only a measurement situation, one way of getting this link is first to assume that the only possible values are outcomes of measurements and that those are restricted to the eigenvalues o_i of an operator \hat{O} that represents the measured observable O . If one then assumes the collapse or projection postulate, that after the measurement the state of the system will be in an eigenstate $|o_i\rangle$ of \hat{O} , it follows that in the non-degenerate case (i.e., when a certain eigenvalue corresponds only to a single eigenvector of the operator observable) an outcome o_i (the value of a physical quantity that appears in a measurement) can be directly related to the eigenstate $|o_i\rangle$ of the measured operator \hat{O} , as the eigenvalue-eigenstate link requires, and we can talk equivalently about the probability for a certain outcome, or eigenvalue or eigenstate.

The basis states $\{|s_1\rangle, |s_2\rangle\}$ and $\{|e_1\rangle, |e_2\rangle\}$ appearing in the composite Schmidt

state $|\psi_{SE}\rangle$ of Eq. (4.3) may then be thought of as the eigenstates of operator observables \hat{O}_S and \hat{O}_E . In this sense, Zurek's derivation tries to establish that for the state $|\psi_{SE}\rangle$ of Eq. (4.3), the outcomes represented by the eigenvalues s_1 and s_2 corresponding to the eigenstates $|s_1\rangle$ and $|s_2\rangle$ of an operator observable $\hat{O}_S = s_1|s_1\rangle\langle s_1| + s_2|s_2\rangle\langle s_2|$ are equally likely. However, in the context of the relative-state view that Zurek promotes, he never explicitly talks about observables and instead directly speaks of determining the “probabilities of events $\{|s_k\rangle\}$ ” [347, p. 12]. This identifies the probability for the occurrence of a specific value of a measured physical quantity with the probability for an eigenstate of the measured observable with an eigenvalue equal to the measured value. That assumption would be justified by the eigenvalue-eigenstate link, although it does not require it.

(D) Zurek furthermore assumes that the probabilities of the outcomes associated with the *individual* states $\{|s_1\rangle, |s_2\rangle\}$ of S and $\{|e_1\rangle, |e_2\rangle\}$ of E are functions of the *composite* state vector $|\psi_{SE}\rangle$ only. (Zurek spells out the assumption that the derivation will be based on the composite state vector but without direct reference to probabilities: “Given the state of the combined SE expressed in the Schmidt form (...) what sort of invariant *quantum facts* can be known about S ?” [345, p. 120404-1].) This assumption about the functional dependence of the probabilities is certainly reasonable, especially since Zurek's aim is clearly to derive Born's rule from within standard quantum mechanics, where the state vector is assumed to provide a complete description of the physical system. The assumption, of course, might well be questioned in a hidden variable or modal interpretation.

But Zurek's argument requires actually a more detailed assumption than stated so far. Obviously, since the SE composition is in an entangled pure state, there is no individual state vector of S alone. But Zurek *infers* the properties of S from the *composite* state vector $|\psi_{SE}\rangle$ by studying its properties under envariance transformations. The idea is to use envariance to deduce statements about S alone. The assumption is

now that probabilities are *local* in the sense that the probabilities that an observer of \mathcal{S} alone can associate with the “events” $|s_k\rangle$ —following Zurek’s identification of outcomes with eigenstates, cf. our discussion in (C)—only depend on the *local* properties of \mathcal{S} (i.e., those properties that cannot be affected by envariant transformations). An analogous assumption must also be invoked with respect to \mathcal{E} for Zurek’s argument to go through: probabilities for the states $|e_k\rangle$ of \mathcal{E} are only dependent on the local properties of \mathcal{E} . This is used to infer the crucial conclusion that probabilities of \mathcal{S} and \mathcal{E} must be independent of the envariant properties of \mathcal{SE} (i.e., properties of \mathcal{SE} that do not belong to \mathcal{S} or \mathcal{E} individually).

This locality of probabilities can be related to the decomposition of the total Hilbert space into the state space of the system and the state space of the environment, together with the focus of any observation on the system alone, on which also the whole definition of envariance relies. But without having explicitly connected the Hilbert state space description with the functional dependence of the probabilities on the state, affirming that probabilities of \mathcal{S} and \mathcal{E} can only depend on the local properties of \mathcal{S} and \mathcal{E} , respectively, must be counted as an important additional assumption.

(E) We would also like to point out that Zurek holds that his argument does not require a causality or locality assumption, but that reference to envariance suffices (see, for example, [345, p. 120404-2]). Zurek suggests that one could alternatively argue for the independence of probabilities from an envariant property of the entangled \mathcal{SE} combination directly if causality and the impossibility of faster-than-light signaling is assumed. He claims that a measurable property associated with \mathcal{S} cannot depend on an envariant property of the entangled \mathcal{SE} state, since otherwise one could influence measurable properties of \mathcal{S} by acting on a “distant” environment \mathcal{E} , and superluminal communication would be possible. We do not find this argument compelling, since influencing measurable properties of a system entangled with a distant

partner by locally acting on the partner does not necessarily require the effect to be instantaneously transmitted to the system. Even if it were, it is not clear that this would entail any violation of relativistic no-signaling requirements (witness the Bohm theory!).

Of course, even if the assumption of causality and the impossibility of faster-than-light signaling indeed justified the conclusion that envariant properties of \mathcal{SE} cannot influence locally measurable physical quantities of \mathcal{S} , any necessity for an appeal to causality to justify Eq. (4.12) would be rather undesired, since the goal is to derive the Born rule from quantum theory alone which, strictly speaking, does not entail the impossibility of superluminal communication. Zurek is clearly aware of this point by stating that causality is “more potent” [344, p. 754] and “more ‘costly’ (and not entirely quantum)” [345, p. 120404-2] than envariance. He consequently holds that his derivation only requires envariance, although he sometimes seems to implicitly refer to causality, for instance in arguing that “only the absolute values of the coefficients can matter since phases [of the coefficients] can be altered by acting on \mathcal{E} alone, and \mathcal{E} is causally disconnected from \mathcal{S} ” [347, p. 10].

(F) Let us now turn to the chain of relations between the probabilities as established in Eqs. (4.10)–(4.16).

(F1) The first relations, Eqs. (4.10), infer equal probabilities for the outcomes represented by $|s_1\rangle$ and $|e_1\rangle$ from their correlation in the direct product $|s_1\rangle|e_1\rangle$ as appearing in the composite state vector $|\psi_{\mathcal{SE}}\rangle$ in the Schmidt decomposition, Eq. (4.7). From a point of view that *presupposes* Born’s rule, this assumption is of course trivially fulfilled, since a simple projection yields

$$\left| \underbrace{\langle e_1 | \langle s_1 | s_1 \rangle | e_1 \rangle}_{=1} + \underbrace{\langle e_2 | \langle s_1 | s_1 \rangle | e_1 \rangle}_{=0} \right|^2 = \left| \underbrace{\langle e_1 | \langle s_1 | s_1 \rangle | e_1 \rangle}_{=1} + \underbrace{\langle e_1 | \langle s_2 | s_1 \rangle | e_1 \rangle}_{=0} \right|^2, \quad (4.18)$$

due to orthonormality of the Schmidt basis states $\{|s_1\rangle, |s_2\rangle\}$ and $\{|e_1\rangle, |e_2\rangle\}$. But without this (obviously undesired) presupposition, the relations in Eqs. (4.10) represent an additional assumption about the connection between state and probabilities which does not follow from the assumption (D) that probabilities are a function of the state vector only.

Of course Eqs. (4.10) may seem innocuous because most of us are accustomed to thinking in terms of state space projections, and we make an intuitive connection to probabilities from such projections. But it seems important in evaluating a derivation of the quantum probability concept and Born's rule to be aware of where such presupposed conceptions enter, as assumptions, into the derivation.

(F2) Yet another important assumption appears to be contained in Eqs. (4.12). We recall that Zurek justified the relations $p(|e_1\rangle; \hat{U}_S|\psi_{SE}\rangle) = p(|e_1\rangle; |\psi_{SE}\rangle)$ and $p(|e_2\rangle; |\psi_{SE}\rangle) = p(|e_2\rangle; \hat{U}_S|\psi_{SE}\rangle)$ by saying that the probabilities associated with the environment \mathcal{E} cannot change as a result of the envariant swap acting on \mathcal{S} since this swap cannot affect properties of \mathcal{E} . An analogous statement is made in justifying the relations of Eqs. (4.14).

But this argument requires the assumption that the probabilities behave similar to the envariant property that the transformation refers to; i.e., , that the behavior of probabilities under envariant transformations, in particular swaps, is somehow known. This knowledge, however, is not established by Zurek's derivation, and we do not see how it could automatically follow from envariance (as suggested by Zurek). To illustrate this point, consider the following two statements regarding the implications derived from envariance in the course of Zurek's argument:

- (i) Phase envariance implies that the probabilities of \mathcal{S} must be independent of the phases of the Schmidt coefficients.
- (ii) Envariance under swaps implies that the probabilities of \mathcal{S} cannot be influenced by a swap acting on \mathcal{E} alone.

The important difference between (i) and (ii) is that (i) aims at demonstrating the independence of the probabilities from an envariant property, whereas (ii) claims invariance of the probabilities under an envariant transformation. Statement (i) only requires assumption (D) to hold: phase envariance implies that a local description of \mathcal{S} cannot depend on the phase factors of the Schmidt coefficients, so if we assume that the probabilities are a function of the local properties ascribed to \mathcal{S} on the basis of the entangled state vector only (and a study of its envariant transformations), (i) follows. But (ii) requires more: Employing a reasoning analogous to (i), envariance of the composite state under swaps solely means that probabilities of \mathcal{S} will not depend on whether $|s_1\rangle$ is entangled with $|e_1\rangle$ or with $|e_2\rangle$, since this “property” of a specific correlation is not property of \mathcal{S} alone; but we have said nothing about whether the application of the swap operation itself to \mathcal{E} might disturb the probabilities associated with \mathcal{S} .

We might reinforce this concern by drawing attention to the physical interpretation of the swap operation. A swap applied to \mathcal{S} implies that the existing correlations $|s_1\rangle|e_1\rangle$ and $|s_2\rangle|e_2\rangle$ between the system \mathcal{S} and the environment \mathcal{E} need to be “undone,” and new correlations of the form $|s_1\rangle|e_2\rangle$ and $|s_2\rangle|e_1\rangle$ between \mathcal{S} and \mathcal{E} have to be created. From the form of the swap transformations, $\hat{U}_{\mathcal{S}} = \hat{u}_{\mathcal{S}} \otimes \hat{I}_{\mathcal{E}}$ and $\hat{U}_{\mathcal{E}} = \hat{I}_{\mathcal{S}} \otimes \hat{u}_{\mathcal{E}}$, it is clear that swaps can be induced by *local* interactions. But we do not see why shifting features of \mathcal{E} , that is, doing something to the environment, should not alter the “guess” (to use Zurek’s expression [344, p. 755]; cf. the quote in (IIIb) above) an observer of \mathcal{S} would make concerning \mathcal{S} -outcomes. Here, if possible, one would like to see some further argument (or motivation) for why the probabilities of one system should be immune to swaps among the basis states of the other system.

(G) Let us finally discuss Zurek’s alternative argument based on the ignorance of an observer of \mathcal{S} with respect to the individual state of the system \mathcal{S} .

In his derivation, Zurek takes the entangled Schmidt state $|\psi_{SE}\rangle$ describing the

correlation between \mathcal{S} and \mathcal{E} as the given starting point and assumes that the observer somehow knows this state exactly already *before* any measurement has taken place. According to Zurek this knowledge seems to imply that the observer is aware of the “menu” of possible outcomes (but cannot attribute a particular outcome state to \mathcal{S} before the measurement). But since the observer has only access to \mathcal{S} , how is this knowledge established in the first place?

In the case of the composite state $|\psi_{SE}\rangle$ with coefficients of equal magnitude, Eq. (4.3), one can choose *any* other orthonormal basis for \mathcal{H}_S and always find a corresponding orthonormal basis of \mathcal{H}_E such that the composite state $|\psi_{SE}\rangle$ has again the diagonal Schmidt form of Eq. (4.3). Therefore, no preferred basis of \mathcal{H}_S or \mathcal{H}_E has been singled out. On one hand, this implies that Zurek’s argument does not require any *a priori* knowledge of the environmental states $|e_k\rangle$ for the observer of \mathcal{S} . On the other hand, however, this also means that there is nothing that would tell the observer of \mathcal{S} which possible “events” $|s_k\rangle$ he is dealing with. (Decoherence provides a mechanism, termed environment-induced superselection, in which the interaction of \mathcal{S} with \mathcal{E} singles out a preferred basis in \mathcal{H}_S [344]; however, a fundamental derivation of the Born rule must of course be independent of decoherence to avoid circularity of the argument.) Even if one holds that a choice of a particular set of basis vectors is irrelevant to the derivation since the aim is solely to demonstrate the emergence of equal likelihoods for *any* orthonormal basis $\{|s_k\rangle\}$, one is still left with the question how the observer of \mathcal{S} establishes the knowledge that the composite state must be described by coefficients of equal magnitude.

Zurek then goes on to claim that (i) because all possible outcome states $|s_k\rangle$ are envariantly swappable, these states appear as “equivalent” to the observer of \mathcal{S} , and (ii) that this “equivalence of outcomes” translates into an attribution of equal likelihoods for each of these outcomes. With respect to part (i) of the argument, perfect knowledge of the pure composite state implies that the observer (before the measurement) cannot know the individual state of \mathcal{S} , which adds in an ignorance-

based probability concept, but without having established equal likelihoods. Now, as mentioned before, envariance under swaps simply means that the question of which $|e_l\rangle$ of \mathcal{E} is correlated with a particular $|s_k\rangle$ of \mathcal{S} is irrelevant to a complete local description of \mathcal{S} . But we do not see how this state of affairs forces the observer of \mathcal{S} to conclude that all the $|s_k\rangle$ are “equivalent.” For part (ii), we note that even if the previous argument did establish an “equivalence of outcomes,” this epistemic indifference about the occurrence of a particular outcome among a set of possible outcomes would not necessarily, from a general point of view of probability theory, force out the implication of equal likelihoods; this conclusion would be particularly questionable when dealing with a set of continuous cardinality.

4.4 Concluding remarks

To summarize, we have pointed out four important assumptions in Zurek’s derivation about the connection between the state vector and probabilities, and about the behavior of probabilities under envariant transformations of the state vector:

- (1) The probability for a particular outcome in a measurement is directly identified with the probability for an eigenstate of the measured observable with an eigenvalue equal to the value of the measured physical quantity, an assumption that would follow from the eigenvalue-eigenstate link.
- (2) For two entangled systems \mathcal{S} and \mathcal{E} described by the Schmidt state $|\psi_{SE}\rangle = \sum_k \lambda_k |s_k\rangle |e_k\rangle$, probabilities associated with the “outcome states” $|s_k\rangle$ and $|e_k\rangle$ of each individual system are a function of the local properties of the systems only; these properties are exclusively determined by the state vector $|\psi_{SE}\rangle$ of the composite system.
- (3) In an entangled Schmidt state of the form $|\psi_{SE}\rangle = \sum_k \lambda_k |s_k\rangle |e_k\rangle$, the “outcome states” $|s_k\rangle$ and $|e_k\rangle$ are equally likely: $p(|s_k\rangle; |\psi_{SE}\rangle) = p(|e_k\rangle; |\psi_{SE}\rangle)$.

- (4) Probabilities associated with the Schmidt states $|s_k\rangle$ of a system \mathcal{S} entangled with another system \mathcal{E} remain unchanged under the application of an envariant transformation $\hat{U}_{\mathcal{E}} = \hat{I}_{\mathcal{S}} \otimes \hat{u}_{\mathcal{E}}$ that only acts on \mathcal{E} (and similarly for \mathcal{S} and \mathcal{E} symmetrically exchanged): $p(|s_k\rangle; \hat{U}_{\mathcal{E}}|\psi_{S\mathcal{E}}\rangle) = p(|s_k\rangle; |\psi_{S\mathcal{E}}\rangle)$ and $p(|s_k\rangle; \hat{U}_{\mathcal{S}}|\psi_{S\mathcal{E}}\rangle) = p(|e_k\rangle; |\psi_{S\mathcal{E}}\rangle)$.

The necessity for an assumption like (3) in the derivation of the Born rule can be traced back to a fundamental statement about any probabilistic theory: We cannot derive probabilities from a theory that does not already contain some probabilistic concept; at some stage, we need to “put probabilities in to get probabilities out.” Our analysis suggests that this has been done via assumption (3) above.

We have pointed out that assumption (4) is necessary to have the argument that is contained in the chain of relations in Eqs. (4.10)–(4.16) between transformed and untransformed probabilities go through, but we claim that this assumption neither follows from envariance alone nor from assumption (2). We have also questioned whether this assumption is physically plausible.

Furthermore we have expressed doubts that Zurek’s alternative approach that appeals to the information available to a local observer is capable of leading to a derivation of the Born rule. It is neither clear to us how exact knowledge of the composite state is established “within” the local observer before the measurement, nor how envariance under swaps leads the observer to conclude that the possible outcomes must be equally likely.

We hope the questions we raise here will not downplay the interest of Zurek’s derivation in the mind of the reader. To the contrary, because we regard it as significant, we aimed at facilitating a balanced and careful evaluation of Zurek’s approach by bringing out central assumptions implicit in his derivation. We note that Zurek uses both “derivation” and “motivation” to describe his treatment of the emergence of the Born rule. Once the critical assumptions are made explicit, however, as they

are here and now in his [346], the former term seems more appropriate. Moreover, any derivation of quantum probabilities and Born's rule will require some set of assumptions that put probabilities into the theory. In the era of the "Copenhagen hegemony," to use Jim Cushing's apt phrase, probabilities were put in by positing an "uncontrollable disturbance" between object and apparatus leading to a brute quantum "individuality" that was taken not to be capable of further analysis. Certainly Zurek's approach improves our understanding of the probabilistic character of quantum theory over that sort of proposal by at least one quantum leap.

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Chapter 5

EXPERIMENTAL MOTIVATION AND EMPIRICAL CONSISTENCY IN MINIMAL NO-COLLAPSE QUANTUM MECHANICS

Summary

We analyze three important experimental domains (SQUIDs, molecular interferometry, and Bose-Einstein condensation) as well as quantum-biophysical studies of the neuronal apparatus to argue that (i) the universal validity of unitary dynamics and the superposition principle has been confirmed far into the mesoscopic and macroscopic realm in all experiments conducted thus far; (ii) all observed “restrictions” can be correctly and completely accounted for by taking into account environmental decoherence effects; and (iii) no positive experimental evidence exists for physical state-vector collapse. We also discuss recent progress in the understanding of the emergence of quantum probabilities and the objectification of observables, and we outline an observable-free formulation of quantum mechanics. We conclude that it is not only viable, but moreover compelling to regard a minimal no-collapse quantum theory as a leading candidate for a physically motivated and empirically consistent interpretation of quantum mechanics.

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5.1 Introduction

Historically, quantum theory was motivated by the need to describe the behavior of microscopic systems not explainable by the laws of classical physics. Not only was

quantum mechanics deemed unnecessary for a description of the macroworld of our experience, it also led to “strange” consequences that seemed to blatantly contradict our experience, as famously illustrated by the Schrödinger-cat *Gedanken* experiment [283] and later generally referred to as the “measurement problem.” Therefore quantum theory was often, as in the Copenhagen interpretation, banned *a priori* from the macroscopic realm.

Over the past decade, however, a rapidly growing number of experiments have demonstrated the existence of quantum superpositions of mesoscopically and macroscopically distinct states on increasingly large scales. Such superpositions are observed as individual quantum states and are perfectly explained by unitarily evolving wave functions. On the other hand, decoherence theory (see Chap. 2 and Refs. [326, 327, 337, 344, 198]) has enabled one to understand the fragility of such superpositions, and thus the extreme difficulty in observing them outside of sophisticated experimental setups, as being due to ubiquitous quantum interactions with environmental degrees of freedom.

These developments have thus extended the domain for an application of quantum theory far into the mesoscopic and macroscopic realm, which lends strong support to assuming a universally exact and applicable Schrödinger equation. To make a physically compelling case for such a purely unitary quantum theory we must pursue two related goals. First, we ought to continue to design experiments which demonstrate the existence of quantum superpositions of macroscopically distinct states — and which, ideally, can explicitly rule out collapse models. Second, since the assumption of a universal Schrödinger dynamics implies that superpositions of (presumably macroscopically) different observer states are both possible and inescapable if we include physical observers into the quantum-mechanical description, we must simultaneously show that environmental decoherence provides the necessary and sufficient mechanism to explain our observation of a “classical” world. The emergence of the latter can then be understood not only in spite of, but precisely *because* of the quantum

formalism — no classical prejudice need to be imposed.

The formal basis for a derivation of a viable interpretation of quantum mechanics from the “bare” unitary formalism alone has been outlined in several papers. The basic idea was introduced in Everett’s proposal of a relative-state view of quantum mechanics [129]. It was later adapted and popularized by deWitt [101, 102, 103] in his “many-worlds” interpretation of quantum mechanics, whose elements go far beyond the abstract sketches of Everett and which must therefore be strictly distinguished from Everett’s proposal [200]. Relative-state interpretations were subsequently fleshed out, by taking into account decoherence effects, in works by Zeh [326, 327, 333], Zurek [341, 344, 346], Wallace [314, 313], and others (see, for example, [97, 302, 113]). Such a theory can be based on the most minimal set of assumptions about the quantum formalism and its interpretation. First, a completely known (pure) state of an isolated quantum system \mathcal{S} is described by a normalized state vector $|\psi\rangle$ in a Hilbert space $H_{\mathcal{S}}$. Second, the time evolution of a state vector $|\psi\rangle$ is given by the Schrödinger equation $i\hbar\frac{\partial}{\partial t}|\psi\rangle = \hat{H}_{\mathcal{S}}$, where $\hat{H}_{\mathcal{S}}$ is the Hamiltonian of the system \mathcal{S} . No mention is made of measurements in this formulation. Instead, measurements are described without special axioms in terms of physical interactions between systems described by state vectors (wave functions) and governed by suitable interaction Hamiltonians. Observables then emerge as a derived concept (see Sec. 5.7 below and Refs. [198, 344]).

In this paper, however, we take a less formal route and focus on an analysis of the experimental and theoretical progress (with an emphasis on the former) towards the two goals mentioned before, namely, the continued acquisition of experimental evidence for superpositions of macroscopically distinct states and an explanation for the emergence of definite perceptions in spite of an assumed universal validity of the superposition principle.

Our goal is to show that there is no experimental evidence for a breakdown of the superposition principle and the related interference effects at any length scale investigated thus far. Whenever a decay of such superpositions is observed, it can be fully

accounted for (both experimentally and theoretically) as resulting from environmental interactions. The absence of any empirical evidence for nonlinear deviations from unitary time evolution, combined with the ability to give an empirically adequate description of the decoherence of superpositions into apparent mixtures, provides good reasons to take the universal validity of the Schrödinger equation as a working assumption and to explore the consequences of this assumption.

The resulting theory will require more attention to a detailed quantum-mechanical description of observers and observations. Such an account is interpretation-neutral, while the question of its relevance for solving the measurement problem may depend on the particular features of an interpretation. This is so because there exist interpretations, for example, Bohmian mechanics or modal interpretations, that claim to solve the measurement problem *without* having to give an explicit account of the physical processes describing observers and observations (see also Sec. 5.3).

This paper is organized as follows. In Sec. 5.2, we shall discuss and analyze three important experimental domains—superconducting quantum interference devices (SQUIDS), matter-wave interferometry, and Bose-Einstein condensation—that have provided evidence for superpositions of macroscopically distinguishable states. Secs. 5.3 and Secs. 5.4 briefly comment on the current status of physical collapse theories and ensemble interpretations in view of the described experiments. In Secs. 5.5 and 5.6, we shall discuss steps towards the resolution of two issues that have often been considered as posing a challenge to relative-state interpretations: The question of the origin of quantum probabilities (see also Chap. 4) and the connection with Born’s rule, and the problem of the “objectification” of observables and thus the emergence of “classical reality.” Sec. 5.7 describes how observables, the collapse postulate, and the usual commutation relations for observables can be understood as a derived (rather than axiomatic) concept once the physical act of measurement and openness of the measured system is properly taken into account. In Sec. 5.8, we shall analyze theoretical models for decoherence in the perceptive and cognitive apparatus,

and the implications of such decoherence processes. Finally, Sec. 5.9 summarizes our main conclusions and discusses possible next steps.

5.2 Superpositions of macroscopically distinct states: Experiments and implications

In the following, we shall describe three recent experimental areas that have led to (or that are very close to achieving) the observation of superpositions of mesoscopically and macroscopically distinguishable states: Coherent quantum tunneling in SQUIDs (Sec. 5.2.2), diffraction of C₇₀ (and larger) molecules in matter-wave interferometers (Sec. 5.2.3), and number-difference superpositions in two-species Bose-Einstein condensates (Sec. 5.2.4). These experiments have achieved the largest such superpositions observed thus far and also represent the most promising experimental domains for achieving even larger superpositions in the future.

For some earlier experiments demonstrating mesoscopic and macroscopic quantum effects, see the setups using superconductors [85, 267, 286, 268, 233], nanoscale magnets [137, 317, 32], laser-cooled trapped ions [231], and photons in a microwave cavity [64, 263]. We would also like to mention Leggett's review article [215] which discusses some experiments that probe the limits of quantum mechanics. Leggett's motivation, however, is somewhat different than that of the present author, as Leggett's main aim is to assess the status of physical collapse theories in view of these experiments.

5.2.1 Measuring the macroscopic distinctness of states in a superposition

Before embarking on an analysis of the experiments, we shall first lend a more precise meaning to the ubiquitous phrase "superposition of macroscopically distinct (or distinguishable) states." If confronted with a superposition of two states |A⟩ and |B⟩ of the form

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|A\rangle + |B\rangle), \quad (5.1)$$

how are we to decide whether this indeed represents a macroscopic Schrödinger-cat state? Clearly, two conditions will need to be fulfilled:

1. The states $|A\rangle$ and $|B\rangle$ must differ macroscopically in some extensive quantity (e.g., spatial separation, total mass, magnetic moment, momentum, charge, current, etc.), relative to a suitable microscopic reference value.
2. The degree of GHZ-type entanglement [161] in the state $|\Psi\rangle$, i.e., the number of correlations that would need to be measured in order to distinguish this state from a mixture, must be sufficiently large. With $|A\rangle$ and $|B\rangle$ usually representing GHZ-like multi-particle states in complex systems such as superconducting currents, molecules, and atomic gases, this measure can typically be well-estimated by the number of microscopic constituents (electrons, protons, neutrons) in the system.

A similar combination of two measures has been suggested by Leggett [214, 215] under the labels “extensive difference” and “disconnectivity.” We shall adopt Leggett’s former term for the first condition, and use the term “degree of entanglement” for the second. A both necessary and sufficient condition for a superposition to be considered a superposition of macroscopically distinct states is then given by the requirement that both the extensive difference between $|A\rangle$ and $|B\rangle$ and the interparticle entanglement in $|\Psi\rangle$ be large relative to an appropriate microscopic unit.

5.2.2 Superconducting quantum interference devices

Experiments using SQUIDs have not only demonstrated that the dynamics of a macroscopic quantity of matter (here $\approx 10^9$ Cooper pairs) can be collectively determined by a single macroscopic coordinate governed by quantum mechanics, but have also achieved the creation and indirect observation of quantum superpositions of two

truly macroscopic states that correspond to currents of several μA running in opposite directions.

SQUID setup and detection of superpositions of macroscopically distinct currents

A SQUID consists of a superconducting loop interrupted by a Josephson junction and immersed into an external magnetic field that creates a flux Φ_{ext} through the loop. This allows for a persistent dissipationless current (“supercurrent”) to flow around the loop, in clockwise or counterclockwise direction, creating an additional flux. Such a current is composed of a very large number of Cooper pairs (i.e., Bose-condensed electron pairs) whose collective center-of-mass motion can be described by a macroscopic wave function around the loop.

Since the wave function must be continuous around the loop, an integer k times its wavelength must equal the circumference of the loop. Since the Josephson junction induces a discontinuous phase drop $\Delta\phi_J$, and since the total change in phase around the superconducting loop is given by $2\pi\Phi/\Phi_0$, where $\Phi_0 = h/2e$ is the flux quantum and Φ is the total trapped flux through the loop, the phase continuity condition implies

$$\Delta\phi_J + 2\pi\Phi/\Phi_0 = 2\pi k, \quad (5.2)$$

with $k = 1, 2, \dots$. This means that the collective quantum dynamics of the SQUID are determined by the single macroscopic variable Φ .

The effective SQUID Hamiltonian can be written as [316]

$$\hat{H} = \frac{\hat{P}_\Phi^2}{2C} + U(\Phi) = -\frac{\hbar^2}{2C} \frac{d^2}{d\Phi^2} + \left[\frac{(\Phi - \Phi_{\text{ext}})^2}{2L} - \frac{I_c\Phi_0}{2\pi} \cos\left(2\pi\frac{\Phi}{\Phi_0}\right) \right], \quad (5.3)$$

where C is the total capacitance (mainly due to the junction), L is the (finite) self-inductance of the loop, and I_c is the critical current of the junction. This Hamiltonian induces dynamics that are analogous to the motion of a particle with effective “mass” C moving in Φ -space in a tilted one-dimensional double-well potential, with the tilt

determined by Φ_{ext} . The role of the canonical variables \hat{X} and \hat{P} is here played by the total trapped flux $\hat{\Phi}$ and the total displacement current $\hat{P}_\Phi = -i\hbar d/d\hat{\Phi}$ (which has units of charge; $Cd\hat{P}_\Phi/dt$ is the charge difference across the junction).

A set of eigenstates $|k\rangle$ of the Hamiltonian of Eq. (5.3), called “ k -fluxoid states,” are localized in one of the wells of the potential below the (classically impenetrable) barrier if the damping induced by the Josephson junction is weak. The corresponding wave functions $\psi_k(\Phi) \equiv \langle \Phi | k \rangle$ are locally s -harmonic, so their amplitudes are peaked around the respective minimum of $U(\Phi)$ with narrow spreads in flux space. Thus these low-lying energy eigenstates have a relatively small range of associated flux values and can therefore (at least for sufficiently small k) also be viewed as “fuzzy” eigenstates of the flux operator. By adjusting Φ_{ext} , the energy levels are shifted, and for certain values of Φ_{ext} , two levels in opposite wells can be made to align, which allows for resonant quantum tunneling between the wells (i.e., between two fluxoid states) [287, 268], leading to a macroscopic change in the magnetic moment of the system.

The most important states for our subsequent treatment are the zero-fluxoid state $|0\rangle$ and the one-fluxoid state $|1\rangle$. Since the states $|0\rangle$ and $|1\rangle$ are localized in, respectively, the left and right well of the potential, let us denote them by $|L\rangle$ and $|R\rangle$ in the following. These states correspond (apart from the quantum zero-point energy [303]) to a classical persistent-current state and thus to macroscopically distinguishable directions of the superconducting current. Since other states are well-separated in energy, the SQUID can thus be effectively modelled as a macroscopic quantum-mechanical two-state system (i.e., as a macroscopic qubit).

At bias $\Phi_{\text{ext}} = \Phi_0/2$, the well becomes symmetric and the corresponding two fluxoid states $|L\rangle$ and $|R\rangle$ would become degenerate (see Fig. 5.1). However, the degeneracy is lifted by the formation of symmetric and antisymmetric superpositions

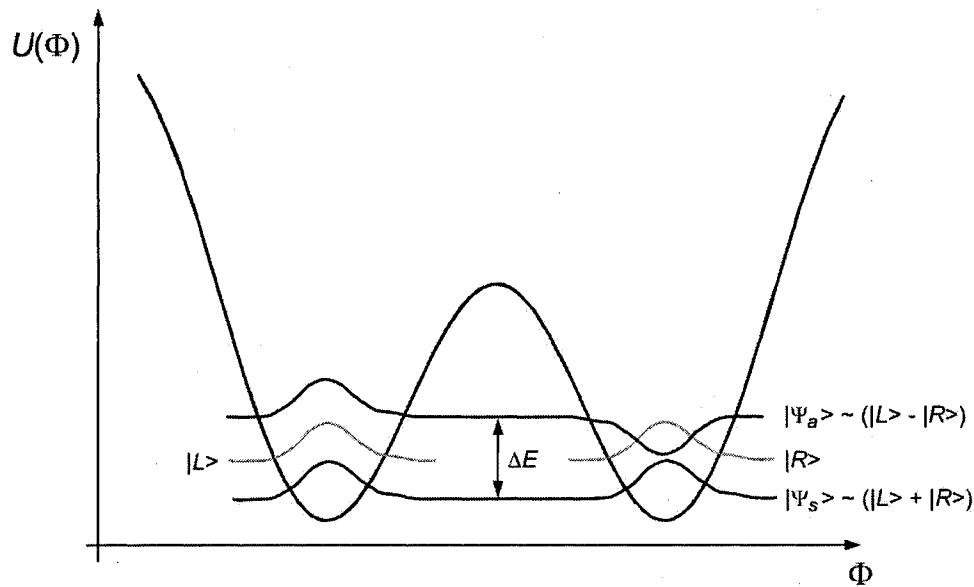


Figure 5.1: Effective SQUID potential at bias $\Phi_{\text{ext}} = \Phi_0/2$. At this point, the double-well potential becomes symmetric. The degeneracy between the two fluxoid states $|L\rangle$ and $|R\rangle$ (which are localized in the left and right well of the potential and correspond to macroscopic currents running in opposite direction around the loop) is lifted by the formation of delocalized coherent superpositions $|\Psi_s\rangle = \frac{1}{\sqrt{2}}(|L\rangle + |R\rangle)$ (the symmetric ground state) and $|\Psi_a\rangle = \frac{1}{\sqrt{2}}(|L\rangle - |R\rangle)$ (the antisymmetric first excited state). The energy difference ΔE between $|\Psi_s\rangle$ and $|\Psi_a\rangle$ has been experimentally measured [136, 303], which confirms the existence of superpositions of the macroscopically distinct states $|L\rangle$ and $|R\rangle$.

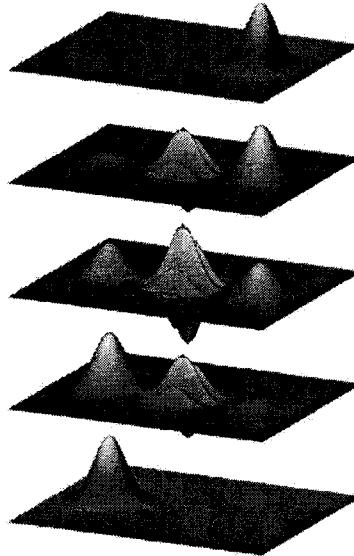


Figure 5.2: Time evolution of the Wigner function corresponding to a superposition $|\Psi(t)\rangle \propto |L\rangle \cos(\Delta Et/2) + i|R\rangle \sin(\Delta Et/2)$ of the two localized opposite-current states $|L\rangle$ and $|R\rangle$ in a SQUID. The state coherently oscillates between the two wells, leading to coherent quantum tunneling. This manifests itself in a macroscopic current oscillating between clockwise and counterclockwise directions. Figure reprinted with permission from Ref. [130]. Copyright 2004 by the American Physical Society.

of $|L\rangle$ and $|R\rangle$ that represent the new energy ground state,

$$|\Psi_s\rangle = \frac{1}{\sqrt{2}}(|L\rangle + |R\rangle) \quad (5.4)$$

with energy E_+ , and the first excited energy eigenstate

$$|\Psi_a\rangle = \frac{1}{\sqrt{2}}(|L\rangle - |R\rangle) \quad (5.5)$$

with energy E_- . Thus these eigenstates are delocalized across the two wells. The (typically very small) energy splitting $\Delta E = E_a - E_s$ is determined by the WKB matrix elements for tunneling between the two wells (and thus between $|L\rangle$ and $|R\rangle$), and is only dependent on the capacitance C of the junction, scaling as $\Delta E \propto e^{-\sqrt{C}}$.

If the system is now more generally described by an arbitrary superposition of $|L\rangle$ and $|R\rangle$, $|\Psi(t)\rangle = \alpha(t)|L\rangle + \beta(t)|R\rangle$, and if we choose the left-localized state $|L\rangle$ as the initial state of the SQUID, i.e., $|\Psi(t=0)\rangle = |L\rangle$, we obtain the time evolution

$$|\Psi(t)\rangle \propto |L\rangle \cos(\Delta Et/2) + i|R\rangle \sin(\Delta Et/2). \quad (5.6)$$

Thus the wave function oscillates coherently between the two localized current states $|L\rangle$ and $|R\rangle$ in each well (see Fig. 5.2) at a rate determined by ΔE , since the probability to find the wave function localized in, say, the left well is oscillatory in time,

$$P_L(t) = |\langle L|\Psi(t)\rangle|^2 = \cos^2(\Delta Et/2). \quad (5.7)$$

This leads to coherent quantum tunneling between the two wells and manifests itself in an oscillation of the current in the SQUID between clockwise and counterclockwise directions. This tunneling effect has been directly observed in superconducting qubit setups similar to the one described here [233, 206, 205, 160, 223, 325, 311].

The indirect route for detecting the presence of superpositions of states corresponding to macroscopic currents running in opposite directions relies on a static spectroscopic measurement of the energy difference ΔE (see Fig. 5.1). Friedman *et al.* [136] have confirmed the existence of such an energy gap (in excellent agreement with theoretical predictions) and, therefore, of superpositions of macroscopically distinct fluxoid states (see also Ref. [303] for a similar experiment and result). In their setup, $|L\rangle$ and $|R\rangle$ (which in this experiment corresponded to $k = 4$ and $k = 10$, respectively) differed in flux by more than $\Phi_0/4$ and in current by 2–3 μA , corresponding to about $10^{10}\mu_B$ in local magnetic moment. Furthermore, the dynamics of the in-unison motion of the approximately 10^9 Cooper pairs represented by $|L\rangle$ and $|R\rangle$ are given by a single unitarily evolving wave function representing the collective flux coordinate Φ .

Scaling

A main advantage of SQUIDs over other experiments (such as those described in the subsequent sections) that probe the limits of quantum mechanics lies in the fact that the relevant macroscopic variable, namely, the trapped flux through the SQUID ring, can be controlled by means of microscopic energy differences in the Josephson junction [215]. As mentioned before, the tunneling matrix element scales as $e^{-\sqrt{C}}$, where C is dominantly determined by the junction rather than by the size of the loop. Thus the difficulty of observing superpositions of macroscopically distinct states scales essentially independently of the degree of macroscopic distinctness between these states (i.e., difference in flux between the opposite currents). This is in stark contrast to the matter-wave diffraction experiments and Bose-Einstein condensates discussed below. In the first case, the grating spacing must decrease as $1/\sqrt{N}$ with the number N of atoms in the molecule, in the second case the decoherence rate increases as N^2 with the number N of atoms in the condensate.

This particular property of SQUIDs has allowed for the creation of superpositions of states that differ by several orders of magnitude more than in other experiments (see Sec. 5.2.5 below).

The interpretation of superpositions

It is well known that quantum-mechanical superpositions must not be interpreted as a simple superposition (addition) of probability distributions. Formally, this conclusion is of course well-reflected in the fact that, in quantum mechanics, we deal with superpositions of probability amplitudes rather than of probabilities, leading to interference terms in the probability distribution.

However, this crucial difference between classical and quantum-mechanical superpositions is sometimes not sufficiently clearly brought out when describing particular experimental situations. In the case of the standard double-slit interference exper-

iment, for example, the state of the diffracted particle is described by a coherent superposition $|\psi\rangle = (|\psi_1\rangle + |\psi_2\rangle)/\sqrt{2}$ of the states $|\psi_1\rangle$ and $|\psi_2\rangle$ corresponding to passage through slit 1 and 2, respectively. This is frequently interpreted as simply representing simultaneous passage of the particle through both slits, i.e., presence of the particle in two distinct spatial regions at the same time, thereby tacitly neglecting the interference terms in the probability distribution.

In the double-slit example, this view will not necessarily be disproven until the stage of the screen is reached at which interference fringes appear. Similarly, and even more drastically, the superpositions of macroscopically distinct current states in a SQUID show that the simplified view of a classical superposition of probability distributions is inadequate. For, if this view were correct, the two contributing opposite currents would mutually cancel out and thus the net “current” described by this superposition would have to be zero, contrary to what is observed. Instead, the SQUID opposite-current superposition represents a novel individually existing physical state that can be described as a coherent “interaction” between simultaneously present states representing currents of opposite direction.

The SQUID example also shows that the “splitting” often referred to in an Everettian framework (for example, in deWitt’s popularization of the “many-worlds view” [101, 102, 103]) should not be taken too literally. The transition, i.e., the “split,” from a single “classical” state—i.e., classically defined definite structures such as particles (defined as having a definite position), currents (defined as a flow of charge into a definite direction), etc.—into a state describing a superposition of such states occurs in a completely unitary and thus reversible manner by changing Φ_{ext} . There is only one single global state vector $|\Psi(t)\rangle$ at all times that corresponds to “physical reality.” The decomposition into a superposition of other states is a primarily formal procedure useful in revealing the physical quantities of our experience contained in the arbitrary state vector $|\Psi(t)\rangle$, since the latter can in general not be related to any “classical” physical structure that would correspond to directly observed objects or properties.

In this sense, the “split” is simply a consequence of trying to trace throughout time a particular (usually “classical”) state that does not coincide with $|\Psi(t)\rangle$. Quantum mechanics shows that this can, in general, only be done in a relative-state sense.

The decomposition obtains also *physical* meaning when the dynamical evolution of the system described by $|\Psi(t)\rangle$ is considered, as the coefficients multiplying the “classical” terms in the superposition will in general be time-dependent. In the example of the SQUID, the coherent-tunneling state does not directly relate to a current in the classical sense (i.e., a current of definite direction), but it can be decomposed into two such currents of opposite direction. The physical relevance of this decomposition and the meaning of the superposition then manifests itself as a current that oscillates between clockwise and counterclockwise directions.

Decoherence and the preferred basis

A particularly interesting feature of the macrocurrent superpositions in SQUIDS is the fact that the interaction with the environment leads to a localization in flux space, rather than to the much more familiar and common localization in position space. In other words, the “preferred basis” (Zurek’s “pointer states” [337, 338]) of the SQUID are flux eigenstates.

This observation is perfectly well accounted for by decoherence theory, which describes the selection of the preferred basis by means of the stability criterion, first formulated by Zurek [337] (see also Chap. 2 and Refs. [338, 340, 341, 344]). According to this criterion, the basis used to represent the possible states of the system must allow for the formation of dynamically stable system-environment correlations. A sufficient (albeit not necessary) requirement for this criterion to be fulfilled is given by the condition that all basis projectors $\hat{P}_n = |s_n\rangle\langle s_n|$ of the system must (at least approximately) commute with the system-environment interaction Hamiltonian \hat{H}_{int} , i.e.,

$$[\hat{H}_{\text{int}}, \hat{P}_n] = 0 \quad \text{for all } n. \quad (5.8)$$

That is, the preferred basis of the system is given by a set of eigenvectors of \hat{H}_{int} .

In the case of the SQUID experiments at bias $\Phi_{\text{ext}} = \Phi_0/2$, if the interaction with the environment is very weak and thus the dynamics of the SQUID system are dominantly governed by the effective SQUID Hamiltonian \hat{H} , Eq. (5.3), the preferred states are predicted to be eigenstates of this Hamiltonian, namely, the dislocalized coherent superpositions $|\Psi_s\rangle = \frac{1}{\sqrt{2}}(|L\rangle + |R\rangle)$ and $|\Psi_a\rangle = \frac{1}{\sqrt{2}}(|L\rangle - |R\rangle)$ of the localized zero-fluxoid and one-fluxoid states $|L\rangle$ and $|R\rangle$. This is in agreement both with the observation of coherent quantum tunneling between the wells and with the measurement of the energy gap $\Delta E = E_a - E_s$ between the states $|\Psi_s\rangle$ and $|\Psi_a\rangle$.

Under realistic circumstances, however, the SQUID is coupled to a dissipative environment \mathcal{E} which can quite generally be modeled as a harmonic heat bath of bosons [316], i.e., as a bath of N harmonic oscillators with generalized coordinates x_α and p_α , natural frequency ω_α , mass m_α , and Hamiltonian

$$\hat{H}_\mathcal{E} = \frac{1}{2} \sum_{\alpha=1}^N \left(\frac{p_\alpha^2}{m_\alpha} + m_\alpha \omega_\alpha^2 x_\alpha^2 \right). \quad (5.9)$$

The reservoir modes x_α couple dynamically to the total flux variable Φ of the SQUID ring. More precisely, they couple to the fluxoid (and essentially opposite-current) states $|L\rangle$ and $|R\rangle$ via the interaction Hamiltonian [316]

$$\hat{H}_{\text{int}} = -\sigma_z \left(\frac{\varphi_0}{2} \sum_\alpha c_\alpha x_\alpha \right), \quad (5.10)$$

where $\sigma_z = (|L\rangle\langle L| - |R\rangle\langle R|)$ is the so-called “pseudospin” operator (owing its name to the fact that the SQUID double-well system can be effectively mapped onto a two-state spin system, with $|L\rangle$ and $|R\rangle$ corresponding to, say, spin “up” and “down,” respectively), and $\pm\varphi_0$ are the flux values associated with the two localized states $|L\rangle$ and $|R\rangle$.

According to the commutativity criterion, Eq. (5.8), the stable states into which the system decoheres are then eigenstates of σ_z , i.e., the preferred basis of the system is given by the two states $|L\rangle$ and $|R\rangle$. This, of course, is in full agreement with

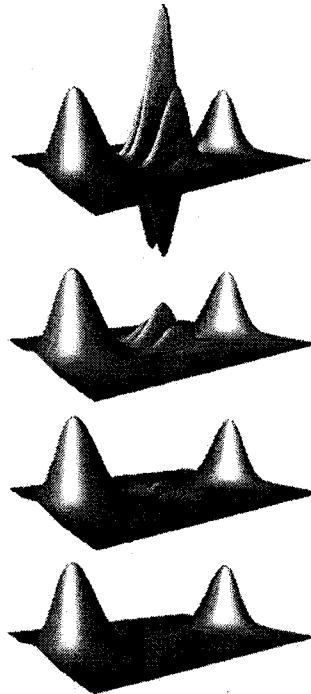


Figure 5.3: Decoherence of the symmetric ground state $|\Psi_s\rangle = \frac{1}{\sqrt{2}}(|L\rangle + |R\rangle)$ at bias $\Phi_{\text{ext}} = \Phi_0/2$ in the Wigner representation. The interaction of the SQUID loop with the environment (here modeled as a monochromatic thermal bath) locally destroys the interference between the two “classical” flux states $|L\rangle$ and $|R\rangle$ represented by the localized peaks on either side. Figure reprinted with permission from Ref. [130]. Copyright 2004 by the American Physical Society.

observations and explains the localization in flux space, i.e., the rapid reduction of the superposition into an apparent ensemble of the macroscopically distinguishable current states $|L\rangle$ and $|R\rangle$.

Fig. 5.3 illustrates this gradual disappearance of interference in the symmetric ground state $|\Psi_s\rangle = \frac{1}{\sqrt{2}}(|L\rangle + |R\rangle)$ due to the interaction of the SQUID ring with a dissipative thermal bath in the Wigner representation of the local density operator of the SQUID [130] (see also Ref. [82]). As predicted by the stability criterion, the robust

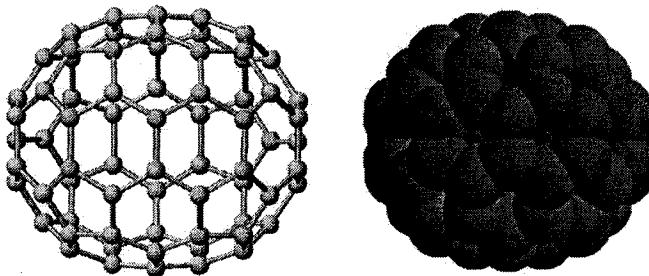


Figure 5.4: Illustration of a C_{70} molecule. The left image shows the “backbone” structure of interlinked carbon atoms. The right image displays the carbon atoms as massive spheres.

states (i.e., the preferred basis) selected by the environment are the macroscopically distinguishable current states $|L\rangle$ and $|R\rangle$. The resulting local loss of coherence—that is, the distribution of coherence, initially associated with the SQUID, over the many degrees of freedom of the SQUID-environment combination—constitutes the main obstacle in the observation of coherent quantum tunneling.

5.2.3 Molecular matter-wave interferometry

Recent experiments by the group of Zeilinger *et al.* [21, 63, 168, 22, 232, 188, 166, 167, 185] have pushed the boundary for the observation of quantum (“wave”) behavior towards larger and larger particles. In the experiment to be described, mesoscopic C_{60} molecules (so-called fullerenes) and C_{70} molecules have been observed to lead to an interference pattern following passage through a diffraction grating (“matter-wave interferometry”). The carbon atoms in the C_{70} molecule are arranged in the shape of an elongated buckyball with a diameter of about 1 nm (see Fig. 5.4). They are complex and massive enough to exhibit properties that position them in the realm of classical solid objects rather than that of atoms. For example, they possess a large number of highly excited internal rotational and vibrational degrees of freedom that

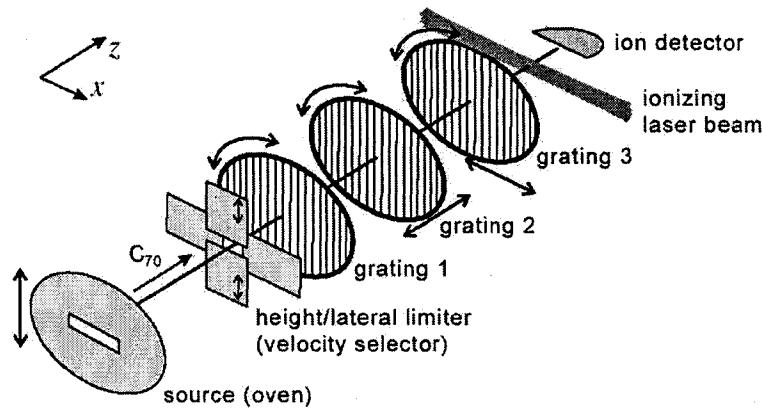


Figure 5.5: Schematic sketch of a Talbot-Lau interferometer used for demonstrating quantum behavior of mesoscopic C₇₀ molecules. The first grating induces a certain degree of coherence in the incident uncollimated beam of molecules. The second grating then acts as the actual diffraction stage. Due to the Talbot-Lau effect, the molecular density pattern at the position of the third grating will be an image of the second grating if the molecules possess a quantum-wave nature. Scanning this pattern by moving the third grating (which acts as a mask) in the x -direction and detecting the transmitted and subsequently ionized molecules will then lead to an oscillatory signal that represents the interference effect. Figure reprinted with permission from Ref. [63]. Copyright 2002 by the American Physical Society.

allow one to attribute a finite temperature to each individual molecule, and heated C₇₀ molecules are observed to emit photons and electrons. The particle aspect seems to be overwhelmingly clear, and yet these molecules have been shown to exhibit quantum interference effects.

Experimental setup and observation of interference

The observation of C₇₀ interference patterns and their controlled disappearance due to environmental decoherence induced by various sources has been made possible by the so-called Talbot-Lau interferometer [63] that has two main advantages over earlier setups used for molecular interferometry [58, 81]. First, the incident beam

of molecules does not need to be collimated, allowing for much higher transmitted intensities. Second, the required period of the gratings used to obtain the interference pattern scales only with the square root of the de Broglie wavelength of the molecules, allowing for the probing of the quantum behavior of, say, sixteen times larger molecules by using an only four times smaller grating spacing.

The Talbot-Lau effect is based on the fact that the transverse part of a plane wave $\psi(z) = e^{ikz}$ incident on a periodic grating located in the xy plane will be identical to the grating pattern at integer multiples of the distance ("Talbot length")

$$L_\lambda = \frac{d^2}{\lambda} \quad (5.11)$$

behind the grating. Since this is a pure interference effect, the presence of the grating pattern at multiples of the Talbot length indicates the wave nature of the incident beam.

The experimental setup that makes use of the Talbot-Lau effect is shown schematically in Fig. 5.5. The main part consists of a set of three gold gratings with a period of about $d = 1 \mu\text{m}$. The first grating acts as a collimator that induces a sufficient degree of coherence in the incident uncollimated beam of C_{70} molecules in order to approximate the plane-wave assumption made above. Each point of the grating can then be viewed as representing a narrow source. The velocity of the molecules can be selected over a range from about 80 m/s to 220 m/s, corresponding to de Broglie wavelengths of approximately 2–6 pm. The second grating is the actual diffraction element, assuming the role of the single grating in the above plane-wave example. The third grating, placed behind the second grating at a distance L equal to the Talbot length $L_{\lambda_{\text{C}70}} = d^2/\lambda_{\text{C}70}$, where $\lambda_{\text{C}70}$ is the de Broglie wavelength of the molecules, can be moved in the x -direction and serves as a scanning detection mask for the molecular density pattern in the transverse plane at this location. The molecules that have passed through the third grating are ionized by a laser beam and then counted by an ion detector.

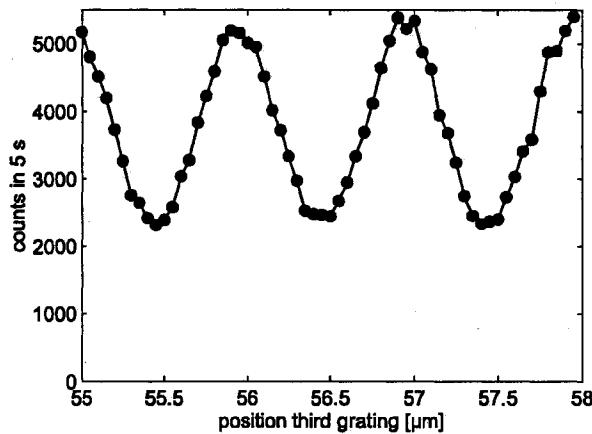


Figure 5.6: Interference fringes for C_{70} molecules measured at the position of the third grating in a Talbot-Lau interferometer. Figure reprinted with permission from Ref. [63]. Copyright 2002 by the American Physical Society.

If the C_{70} molecules indeed possess a quantum-wave nature, the Talbot-Lau effect implies that the molecular density pattern at the position of the third grating should consist of interference fringes with a period equal to the spacing d of the grating pattern. Thus, when the third grating is scanned in the x -direction, we expect an oscillation in the number of transmitted molecules with period d . This is indeed what has been observed experimentally [21, 22, 63, 232, 188] (Fig. 5.6). The possibility that these fringes could result from a classical blocking of rays by the gratings (Moiré fringes) can be excluded, because such patterns would be independent of the de Broglie wavelength, in contrast to what is observed experimentally [63, 166]. This confirms the quantum origin of the measured fringes and thus the wave nature of the C_{70} molecules.

It should be emphasized that the fringes represent *single-particle* interference effects, rather than being due to interference between different molecules [232]. The latter case would require the interfering molecules to be in the same state, which

is practically never the case due to the large number of different excited internal states. Furthermore, the density in the molecular beam is relatively low, such that the average distance between two molecules is much greater than the range of any intermolecular force. Thus, even if the molecules passed at such a slow rate through the apparatus that only a single molecule was present at any time, an interference pattern would emerge. The interference effect is entirely due to the splitting and overlapping of the wave fronts associated with each individual C_{70} molecule. This demonstrates clearly that quantum-mechanical superpositions in configuration space describe individual states that can exhibit interference effects (i.e., phase dependencies) without any statistical aspect.

Disappearance of interference due to controlled decoherence

General numerical estimates for decoherence rates derived from theoretical expressions [197, 141, 294, 186] have clearly demonstrated the extreme efficiency of decoherence on mesoscopic and macroscopic scales. It is therefore usually practically impossible to control the environment in such a way as to explicitly resolve and observe the gradual action of decoherence on larger objects.

The Talbot-Lau interferometer, however, has made such observations possible and has also led to direct confirmations of the predictions of decoherence theory for mesoscopic objects [166, 188, 167, 187, 185]. The main sources of decoherence that have been experimentally investigated are collisions with gas molecules present in the interferometer [188, 166, 187], and thermal emission of radiation when the C_{70} molecules are heated to temperatures beyond 1,000 K [167, 185]. Here, we shall focus on the first case of decoherence, as collisions with environmental particles represent the most natural and ubiquitous source of decoherence in nature.

In the experiments, the vacuum chamber containing the interferometer is filled with gases at different pressures. Each collision between a gas particle and a C_{70} molecule entangles their motional states. Since the C_{70} molecules are much more

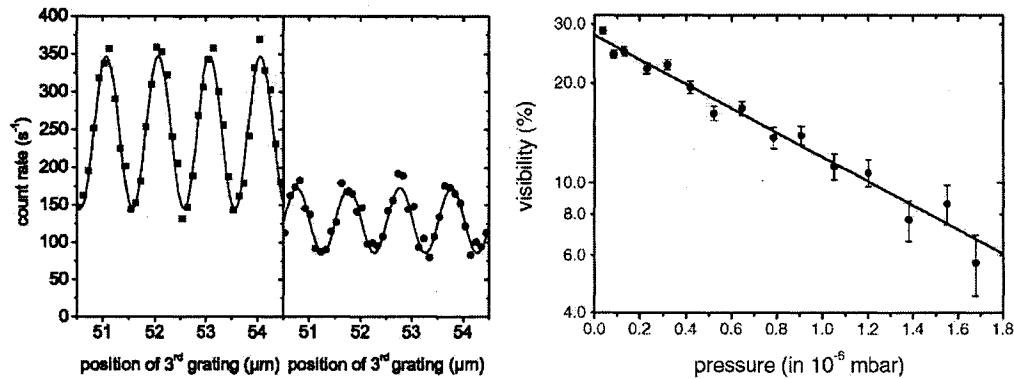


Figure 5.7: Diminished interference effect in C_{70} -molecule interferometry due to decoherence induced by collisions with gas molecules. *Above:* Decreased visibility of the interference fringes when the pressure of the gas is increased from the left to the right panel. *Below:* Dependence of the visibility on the gas pressure. The measured values (circles) are seen to agree well with predictions obtained from decoherence theory (solid line), see Eqs. (5.18) and (5.19). Figures reprinted, with kind permission of Springer Science and Business Media, from Ref. [166].

massive than the gas molecules, the motional state of the gas molecule is distinguishably changed in the collision, while the motion of the C_{70} molecule remains essentially unaffected and can therefore still be detected at the third grating. Thus, each collision encodes which-path information about the trajectory of the C_{70} molecule in the environment (i.e., in the colliding gas particle). This leads to decoherence in the spatial wave function of the C_{70} molecules, since the post-collision environmental states are approximately orthogonal in the position basis due to the significant change of the motional state of the gas molecules in the collisions.

To see this more explicitly, let us denote the state of the C_{70} molecule before and after the scattering by

$$|\psi\rangle_{C_{70}} = \int d\mathbf{x} (\langle \mathbf{x} | \psi \rangle)_{C_{70}} |\mathbf{x}\rangle_{C_{70}} \quad (5.12)$$

and

$$|\psi'\rangle_{C_70} = \int d\mathbf{x} (\langle \mathbf{x} | \psi' \rangle)_{C_70} |\mathbf{x}\rangle_{C_70}, \quad (5.13)$$

respectively, where

$$(\langle \mathbf{x} | \psi \rangle)_{C_70} \approx (\langle \mathbf{x} | \psi' \rangle)_{C_70} \quad (5.14)$$

for all \mathbf{x} . A collision at \mathbf{X} changes the state of the colliding gas molecule from $|\varphi\rangle_{\text{gas}}$ to $|\varphi', \mathbf{X}\rangle_{\text{gas}}$, which encodes which-path information about the C_{70} molecule. Since the $|\varphi', \mathbf{X}\rangle_{\text{gas}}$ represent distinguishable motional states, the environmental states corresponding to scattering events at different locations become approximately orthogonal,

$$(\langle \varphi', \mathbf{X} | \varphi', \mathbf{Y} \rangle)_{\text{gas}} \approx \delta(\mathbf{X} - \mathbf{Y}). \quad (5.15)$$

The collision leads to an entangled state for the combined gas- C_{70} system,

$$|\Psi_0\rangle = |\psi\rangle_{C_70} \otimes |\varphi\rangle_{\text{gas}} \longrightarrow |\Psi\rangle \approx \int d\mathbf{X} (\langle \mathbf{X} | \psi \rangle)_{C_70} |\mathbf{X}\rangle_{C_70} \otimes |\varphi', \mathbf{X}\rangle_{\text{gas}}. \quad (5.16)$$

The reduced density matrix for the C_{70} molecule expressed in the position basis is then obtained by averaging over all possible states $|\varphi', \mathbf{X}\rangle_{\text{gas}}$ of the gas molecule,

$$\begin{aligned} \rho_{C_70} &\approx \int d\mathbf{X} \int d\mathbf{X}' \int d\mathbf{X}'' (\langle \mathbf{X} | \psi \rangle)_{C_70} (\langle \mathbf{X}' | \psi \rangle)_{C_70}^* (\langle \varphi', \mathbf{X}'' | \varphi', \mathbf{X} \rangle)_{\text{gas}} \\ &\quad \times (\langle \varphi', \mathbf{X}' | \varphi', \mathbf{X}'' \rangle)_{\text{gas}} (|\mathbf{X}\rangle \langle \mathbf{X}'|)_{C_70} \\ &\approx \int d\mathbf{X} |\langle \mathbf{X} | \psi \rangle_{C_70}|^2 (|\mathbf{X}\rangle \langle \mathbf{X}|)_{C_70}, \end{aligned} \quad (5.17)$$

where the vanishing of interference terms $(\langle \mathbf{X} | \psi \rangle)_{C_70} (\langle \mathbf{X}' | \psi \rangle)_{C_70}^*$, $\mathbf{X} \neq \mathbf{X}'$, in the last step follows from the approximate orthogonality of the $|\varphi', \mathbf{X}\rangle_{\text{gas}}$. Thus, the gas molecules carry away which-path information, leading to a diffusion of coherence into the environment. Incidentally, in this sense, Bohr's complementarity principle can be understood as a consequence of entanglement: The observability of an interference pattern, and thus the degree of the "wave aspect" of the C_{70} molecules, is directly related to the amount of information, encoded through entanglement with the state of the gas particles, about the path (the "particle aspect") of the molecules.

We expect the visibility V_λ of the interference fringes (defined as $(c_{\max} - c_{\min})/(c_{\max} + c_{\min})$, where c_{\max} and c_{\min} are the maximum and minimum amplitudes of the interference pattern) to decrease as the pressure of the environmental gas is increased. A theoretical analysis [166, 186, 187] predicts that V_λ will decrease exponentially with the pressure $p = nk_B T$ of the colliding gas,

$$V_\lambda(p) = V_\lambda(0)e^{-p/p_0}. \quad (5.18)$$

Here,

$$p_0 = \frac{k_B T}{2L\sigma_{\text{eff}}} \quad (5.19)$$

is the characteristic decoherence constant (“decoherence pressure”), where L denotes the distance between the gratings and σ_{eff} corresponds to the effective cross section [166]. This pressure-dependent decay of the visibility has indeed been confirmed experimentally for C_{70} molecules [166, 188], in excellent agreement with the theoretical predictions (Fig. 5.7).

Studies of collision-induced decoherence in a Talbot-Lau interferometer not only represent an outstanding method to observe the gradual disappearance of quantum-interference effects while having full control over both the source and the strength of decoherence, but also allow one to predict the environmental conditions (in this case, the maximum pressure of the surrounding gas) required to observe quantum effects for even more complex and massive objects than tested thus far. Such experiments are limited by two main factors [188, 166]. First, the velocity of the objects must be quite slow during the passage through the interferometer, in order to keep the de Broglie wavelengths long enough to allow for a sufficient degree of diffraction by practically realizable gratings. Second, the pressure p of the residual gas in the interferometer must be low enough to maintain sufficient visibility of the interference pattern, i.e., we must have $O(p) = p_0$, see Eq. (5.19). Since both limits are purely technical and can be precisely quantified, there is no indication for any fixed quantum-classical boundary in this case other than the observational limit determined by environmental decoherence,

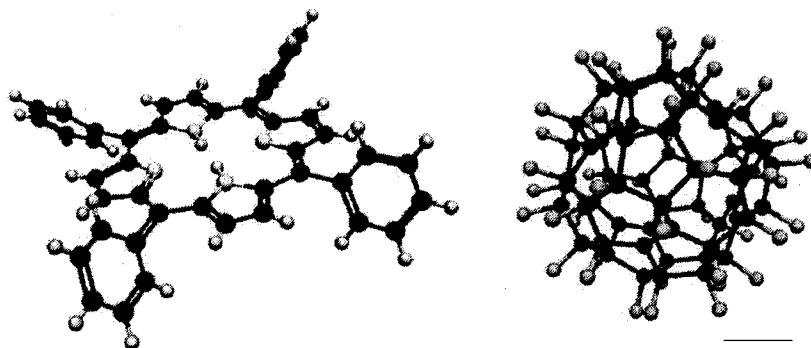


Figure 5.8: Structure of the biomolecule tetraphenylporphyrin C₄₄H₃₀N₄ (left) and of the fluorofullerene C₆₀F₄₈ (right). The wave nature of both molecules has been observed in experiments. Figures reprinted with permission from Ref. [168]. Copyright 2003 by the American Physical Society.

for which rigorous theoretical estimates can be given. Decoherence allows for an exact specification of where the quantum-to-classical transition occurs and what needs to be done to move the boundary.

In fact, the envelope for the observation of the wave nature of mesoscopic molecules has recently been pushed even further in experiments demonstrating quantum interference fringes for the important biomolecule tetraphenylporphyrin C₄₄H₃₀N₄ (with mass $m = 614$ amu and a width over 2 nm) and for the fluorinated fullerene C₆₀F₄₈ (mass $m = 1632$ amu, 108 atoms) [168]. While tetraphenylporphyrin is the first-ever biomolecule whose wave nature has been demonstrated experimentally, fluorofullerenes are the most massive and complex molecules to exhibit quantum behavior thus far. Theoretical estimates for the maximum residual gas pressure that would still allow for the observation of interference fringes for even larger biological objects, up to the size of a rhinovirus, have been given by Hackermüller *et al.* [22, 166] (see Fig. 5.9) and appear to be realizable even with the currently available technology in Talbot-Lau interferometry [188, 166]. One might extrapolate even further and spec-

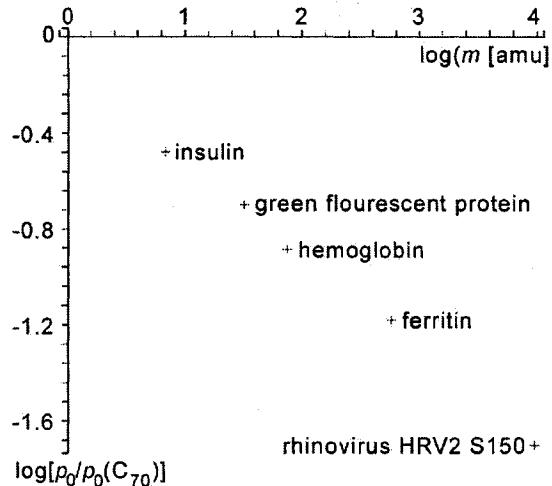


Figure 5.9: Extrapolated maximum residual air pressures p_0 , relative to the value of p_0 for C_{70} molecules, versus molecular weight m (in amu), that would allow for an observation of interference fringes for various biological structures in an elongated ($L = 1$ m) Talbot-Lau interferometer. Data from Ref. [166].

ulate about the feasibility of interference experiments involving human cells, with an average weight and size on the order of 10^{15} amu and 10^4 nm, respectively. While this is certainly beyond the existing technology, there is no reason to assume that such experiments should be impossible.

Implications of the C_{70} interference experiments

The described matter-wave interferometry experiments have led to three crucial results:

1. Interference patterns are observed for particles that clearly reside in the “lump of matter” category.
2. These patterns are due to single-particle (rather than interparticle) interference effects.

3. Any observed disappearance (or absence) of interference patterns can be well understood as resulting from decoherence and can be explicitly controlled and quantified.

Thus there is no theoretical or experimental indication for any fundamental limit on the ability of objects to exhibit quantum behavior (i.e., a wave nature) if these objects are sufficiently shielded from the decohering influence of their environment. Result (2) shows that the initial wave function describing the individual molecule evolves into a spatially extended wave function after passage through the diffraction grating, namely, into a superposition of “classical” localized position states that each correspond to the molecule being in a specific region of space. The gradual disappearance of interference due to controlled interaction with the environment can be understood as entanglement between the different relative states of the environment and the individual components $|x\rangle_{C_70}$ in the superposition. It is important to note that all components $|x\rangle_{C_70}$ are still present regardless of the environmental interaction — decoherence is in principle fully reversible, as experiments on coherent state-vector revival have shown (see, e.g., Ref. [263]).

5.2.4 Bose-Einstein condensation

As a third example, we shall discuss Bose-Einstein condensation (BEC). While this effect had been predicted theoretically already in the 1920s by Einstein [122, 123, 124] based on ideas by Bose [60], explicit experimental verification succeeded only in 1995 [62, 92, 15, 61]. When an atomic bosonic gas confined by a magnetic trap is cooled down to very low temperatures, the de Broglie wavelength $\lambda_{dB} = (2\pi\hbar^2/mk_B T)^{1/2}$ associated with each atom becomes long in comparison with the interparticle separation. At a precise temperature in the ≈ 100 nK range, the collection of atoms can undergo a quantum-mechanical phase transition to a condensate in which the atoms lose their individuality and all occupy the same quantum state. Then a macroscopic number

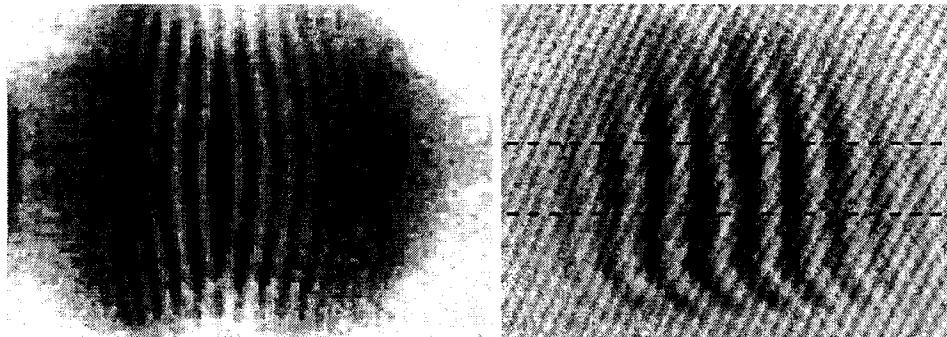


Figure 5.10: Matter-wave interference pattern for the atomic gas density in Bose-Einstein condensates. *Left:* Pattern obtained by letting two independent condensates overlap, demonstrating that the condensate is indeed described by a single wave function with a phase. The fringe period was measured to be $20 \mu\text{m}$. Figure reprinted with permission Ref. [17]. Copyright 1997 by AAAS. *Right:* Fringes due to interference of a single coherently split condensate. This experiment corresponds to a BEC “double-slit” interferometer. Figure reprinted with permission from Ref. [285]. Copyright 2004 by the American Physical Society.

of atoms—large condensates can contain of the order of 10^7 atoms—is described by a single N -particle wave function with a phase,

$$\Psi_N(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) = e^{i\Phi} \prod_{i=1}^N |\psi(\mathbf{r}_i)|, \quad (5.20)$$

i.e., as a product of N identical single-particle wave functions $\psi(\mathbf{r})$. As a consequence, BECs can directly exhibit quantum behavior. For instance, two condensates released from adjacent traps can overlap and form a gas-density interference pattern due to the phase difference between the two wave functions (Fig. 5.10) [194, 17, 265, 193, 272]. Recently, Bose-Einstein “double-slit” interferometers have been experimentally realized [285] and theoretically analyzed [88]. Here, a single condensate is coherently split (corresponding to the diffraction stage in the double-slit experiment) and then allowed to recombine, which leads to the observation of interference fringes (Fig. 5.10).

Macroscopic number-difference superpositions using Bose-Einstein condensates

Various methods have been proposed for the creation of BEC-based Schrödinger cat states in form of a superposition of states with macroscopically distinguishable numbers of particles [83, 270, 158, 118, 69, 218, 228]. BECs are particularly suitable for the generation and the study of Schrödinger cat states, for several reasons. First, as BECs involve up to 10^7 atoms, such superpositions would be the most macroscopic ones ever observed. Second, the condensate is described by a single coherent wave function that pertains to a controllable number of atoms and possesses an extremely long coherence time (up to 10–20 s). Third, the sources of decoherence (mostly loss of particles from the condensate) are fairly well-understood and potentially sufficiently controllable through suitable environmental engineering and trap design [271, 207, 89].

The typically suggested scheme to create superpositions of macroscopically distinguishable states using BECs involves the creation and manipulation of interacting two-species condensates, i.e., BECs in which the atoms possess two different internal states $|A\rangle$ and $|B\rangle$. Experimental realizations of two-species BECs often employ the two hyperfine sublevels $|F, m_F\rangle = |2, 1\rangle$ and $|1, -1\rangle$ of ^{87}Rb . The early proposal by Cirac *et al.* [83] (similar models have been suggested, for example, in Refs. [270, 158, 118, 228, 192]) is based on a Josephson-like coupling between the two species that leads to a number-difference superposition of the form

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|n_A, N - n_A\rangle + e^{i\varphi}|N - n_A, n_A\rangle), \quad (5.21)$$

where $|n_A, n_B\rangle$ is the occupation-number state representing n_A atoms of type A and n_B atoms of type B, and $N = n_A + n_B$ is the total number of atoms. This represents a superposition of two states which differ by a macroscopic number $|N - 2n_A|$ of atoms of a certain type (A or B). Then $n_A = 0$ or $n_A = N$ would correspond to a maximally entangled N -particle GHZ-type state [161] and thus the most “cat-like” state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|N, 0\rangle + e^{i\varphi}|0, N\rangle). \quad (5.22)$$

Another scheme for the creation of macroscopic BEC superpositions that uses a single-component BEC in a double well (with possible generalizations to M wells) has been described in Refs. [219, 220] (see also Refs. [260, 259]). Here, a laser-induced phase shift is imprinted on the condensate in one of the wells, followed by a change of barrier height. This is predicted to lead to a superposition of the form

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|n_L, N - n_L\rangle + e^{i\varphi}|N - n_L, n_L\rangle), \quad (5.23)$$

where $|n_L, n_R\rangle$ is the number state corresponding to n_L (n_R) atoms in the left (right) well. Again, n_L determines the degree of entanglement, with $n_L = 0$ or $n_L = N$ corresponding to maximal “catness.” Even the possibility of creating a coherent superposition of a macroscopic number of atoms with a macroscopic number of molecules using photoassociation in BECs (i.e., the absorption of a photon by two atoms, leading to the formation of a two-atomic bound molecule) has been indicated [69].

To detect a BEC cat state, one might in principle envision experiments similar to those measuring GHZ spin states [225, 89, 273], although this would be very difficult to carry out in practice for the larger values of N relevant to BEC superpositions. Instead, as pointed out in Ref. [89], one could first confirm that measurement statistics indeed give equal likelihoods for the two cat-state terms $|N, 0\rangle$ or $|0, N\rangle$. If the system can also be observed to (approximately) return to its initial state after unitary evolution over a period that is an integer multiple of the time needed for the generation of the cat state, this would provide strong indications for the presence of a cat state.

Decoherence of BEC superpositions

To date, Schrödinger-cat states using BECs have not been realized experimentally, although much progress has been made (see, for example, Ref. [8]). Dissipation and decoherence effects are still too strong to allow for a direct observation of superpositions and will continue to constitute the dominant limit on the size of number-difference Schrödinger cats. These environmental effects are mainly due to elastic and

inelastic scattering between condensate and noncondensate atoms.

Elastic collisions with noncondensate atoms under conservation of the number of condensate atoms lead to phase damping and thus to the destruction of the coherent superposition. The reduced density matrix in the number basis then decoheres according to [218]

$$\langle m | \hat{\rho}(t) | n \rangle = e^{-(m-n)^2 \kappa t} \langle m | \hat{\rho}(0) | n \rangle e^{-i\omega(m-n)t}, \quad (5.24)$$

i.e., the off-diagonal elements $m \neq n$ decay with a decoherence rate that scales with the square of the number difference, $(m - n)^2$.

Furthermore, inelastic collisions with noncondensate atoms lead to a loss of atoms from the condensate, which diminishes coherence. Again, the larger the number difference in the superposition $(|n, N-n\rangle + |N-n, n\rangle)/\sqrt{2}$ is (i.e., the closer n is to 0 or N), the more sensitive the state is to atom loss (see, for example, the detailed analysis in Ref. [118]). In the limit of the maximally entangled state $(|N, 0\rangle + |0, N\rangle)/\sqrt{2}$, already the loss of a single atom of, say, type 1 completely destroys the coherent superposition, since

$$\hat{a}_1(|N, 0\rangle + e^{i\varphi}|0, N\rangle)/\sqrt{2} = \sqrt{N/2}|N-1, 0\rangle, \quad (5.25)$$

where \hat{a}_1 is the destruction operator for particles of type 1.

Thus both decoherence effects will usually limit the size N (i.e., the number difference) of superpositions of the form $(|N, 0\rangle + |0, N\rangle)/\sqrt{2}$. In a detailed analysis that combines the two forms of scattering processes, Dalvit *et al.* [89] have estimated the decoherence rate τ_d^{-1} for an optimal number-difference superposition $(|N, 0\rangle + |0, N\rangle)/\sqrt{2}$ in a standard harmonic trap due to a “thermal cloud” of N_{nc} noncondensate atoms as

$$\tau_d^{-1} \propto a^2 N_{nc} N^2, \quad (5.26)$$

where a is the scattering length. This leads to very short decoherence times even for moderate environment and condensate sizes [89, 218]. For example, for $N_{nc} = 10$ and

$N = 10^3$, τ_d is of the order of milliseconds. For larger Schrödinger cats with $N = 10^7$ and a thermal cloud containing $N = 10^4$ noncondensed atoms, $\tau_d \sim 10^{-13}$ s.

However, several schemes exist to significantly reduce the decoherence rate and to thus render it quite likely that BEC-based number-difference Schrödinger cat states could indeed be observed in future experiments; for example:

1. The construction of modified traps that allow for a faster evaporation of the thermal cloud [89].
2. Generation of number-difference cat states via the creation of macroscopic superpositions of relative-phase states that are not only much less sensitive to atom loss, but might even *require* such loss [118].
3. A “symmetrization” of the environment to reduce the effective size of the thermal cloud [89].
4. Sufficiently fast generation of the cat state [228].

The key lesson to be learned from the example of BEC-based Schrödinger-cat states is that, notwithstanding the fact that such superpositions have not (yet) been explicitly documented in experiments, the physics of these states and the required conditions to create them is very well understood. The failure to experimentally generate these states with currently available setups is well-explained by decoherence models that provide precise numerical estimates for the type of experimental arrangements and parameter ranges that would be required to observe Schrödinger-cat states using BECs. Similar to the case of studying the feasibility of matter-wave interferometry with larger molecules than those investigated thus far (see Sec. 5.2.3), decoherence is the key tool for a precise prediction of the physical conditions required for the experimental observation of superpositions of macroscopically distinct states.

Table 5.1: Estimates for the degree of macroscopic distinctness of the states in superpositions relevant to the three experiments discussed in this paper. \mathcal{S}_{ext} is a measure for the maximum difference in a suitably chosen extensive variable that distinguishes the states in the superposition (here: the total magnetic moment in SQUID experiments; the average separation between two paths in the interferometer in C_{70} molecular diffraction; the difference in angular momentum in two-species Bose-Einstein condensates). \mathcal{S}_{ent} measures the degree of entanglement in multi-particle states and is well-estimated by the number of microscopic constituents involved in the superposition (i.e., the number of Cooper pairs in SQUID, and the number of nucleons and electrons in the C_{70} molecule and the Bose-Einstein condensate). The third column shows the product $\mathcal{S}_{\text{ext}} \times \mathcal{S}_{\text{ent}}$ of the two measures, thus representing the overall degree of macroscopicity of the superpositions. See also Sec. 5.2.1.

Experiment	\mathcal{S}_{ext}	\mathcal{S}_{ent}	$\mathcal{S}_{\text{ext}} \times \mathcal{S}_{\text{ent}}$
SQUID	10^{10}	10^9	10^{19}
C_{70}	10^6	10^3	10^9
Bose-Einstein*	10^7	10^9	10^{16}

*not yet experimentally achieved

5.2.5 Analysis of the degree of macroscopicity of the experimentally achieved superpositions

In the following, let us compare the degree of macroscopic distinctness of the states in the superpositions encountered in the experiments with SQUIDs, diffracted molecules, and BECs. We will use the combination of the two measures introduced in Sec. 5.2.1, namely, the difference \mathcal{S}_{ext} in a relevant extensive quantity between the states in the superposition relative to an appropriate microscopic reference value, and the degree of entanglement \mathcal{S}_{ent} present in the multi-particle superposition.

For the SQUID experiments (Sec. 5.2.2), choosing the total magnetic moment to be the relevant extensive variable, the two states $|L\rangle$ and $|R\rangle$ differ by about $10^{10}\mu_B$ in the experiment by Friedman *et al.* [136]. Taking the Bohr magneton μ_B as the reference unit, the extensive difference \mathcal{S}_{ext} between the two states is thus of the

order of 10^{10} . The degree of entanglement S_{ent} in the multi-Cooper-pair state can be estimated to be of the order of the number of Cooper pairs, i.e., $\sim 10^9$.

In the case of diffraction of C_{70} molecules (Sec. 5.2.3), a suitable extensive quantity would be the center-of-mass displacement between the two paths through the interferometer, which we can estimate to be on the order of 1 mm (corresponding to the lateral width of the molecular beam [166]) relative to the size of the molecule of about 1 nm, which yields a value for S_{ext} on the order of 10^6 . The degree of entanglement S_{ent} is essentially given by the number of microscopic constituents in the molecule, $3 \times 6 \times 70 \sim 10^3$.

For BEC two-species superpositions that use the two hyperfine sublevels $|F, m_F\rangle = |2, 1\rangle$ and $|1, -1\rangle$ of ^{87}Rb atoms (Sec. 5.2.4), a suitable extensive variable would be the total difference in angular momentum due to the hyperfine splitting, in units of \hbar , which is on the order of the number N of atoms in the condensate, which can be as large as 10^7 . Thus the maximum S_{ext} is on the order of 10^7 . The degree of entanglement S_{ent} is again suitably measured by the number of nucleons and electrons in the condensate, which is of the order of $100N$ for ^{87}Rb . Note, however, that such superpositions have not yet been experimentally achieved.

All values are summarized in Table 5.1. We see that the SQUID experiments allow for superpositions that are about ten orders of magnitude “more macroscopic” (in the sense defined above) than those achieved by molecular interferometry. On the other hand, the latter experiments lead to a direct realization of spatial superpositions, which are often considered to be more “counterintuitive” than the superposition of superconducting currents, since position appears to be the dominant definite quantity in our observation of the macroworld. The ubiquitous perception of definiteness in position space has even led some to postulate a fundamentally preferred role to position. For example, Bell [44] stated that “in physics the only observations we must consider are position observations, if only the positions of instrument pointers.” A similar idea underlies the spatial localization mechanism in the GRW theory and is

reflected in the concept of definite particle trajectories in Bohmian mechanics.

Superpositions involving two-species BECs, if experimentally realized, would come close to the degree of macroscopicity achieved in SQUIDs. This result can be understood by noting the striking analogies between the two experiments. In both cases, the multi-particle system (the superconducting material in SQUIDs, or the atomic gas in BECs) is cooled down to extremely low temperatures near absolute zero. The two macroscopically distinguishable states (currents of opposite direction in SQUIDs, or different atom species in BECs) are coupled by a classically impenetrable barrier of the Josephson-junction type. In both experiments, this essentially leads to Schrödinger-cat states of the form

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|N, 0\rangle + e^{i\varphi}|0, N\rangle), \quad (5.27)$$

where the number state $|N, 0\rangle$ denotes N particles (Cooper pairs in SQUIDs, or atoms in BECs) being in the first macroscopically distinguishable state (representing a clockwise current in the SQUID, or the hyperfine sublevel $|F, m_F\rangle = |2, 1\rangle$ in BECs), and no particles being in the second state (corresponding to a counterclockwise current in the SQUID, or the hyperfine sublevel $|F, m_F\rangle = |1, -1\rangle$ in BECs).

5.3 The status of physical collapse models

All existing interpretations of quantum mechanics can be viewed as either adding formal rules¹ or physical elements (as in collapse models) to the axioms of minimal quantum theory stated in the Introduction. With respect to the “formal” category, if the minimal theory can be shown to be sufficient to explain and predict all our observations, there is clearly no empirical reason for introducing purely formal additives. While a similar argument can be made regarding the “physical” category,

¹As, for example, done in the Copenhagen interpretation (that formally postulates a collapse, but regards it merely as an “increase of information,” rather than as a physical process, since it interprets the wave function as representing a probability amplitude), Bohmian mechanics, modal interpretations, and consistent-histories interpretations.

collapse theories might lead to observable deviations from Schrödinger dynamics and could thus be experimentally tested. In both cases, of course, there may be conceptual reasons that motivate the added elements, for example a desire to resolve a felt “weirdness” in the existing quantum theory. While we respect this motivation, we hope to show that in fact the minimal theory is sufficient to resolve the problems without requiring any such additions.

The increasing size of physical systems for which interference effects have been observed imposes bounds on the parameters used in collapse models. However, the current experiments demonstrating mesoscopic and macroscopic interference are still quite far away from disproving the existing collapse theories. For example, even the C₇₀ diffraction experiments described in Sec. 5.2 still fall short of ruling out continuous spontaneous localization models [250, 109, 148] (which lead to the strongest deviations from Schrödinger dynamics among all physical collapse theories) by eleven orders of magnitude [3]. A recently proposed mirror-superposition experiment by Marshall *et al.* [222] that might lead to a superposition involving of the order of 10¹⁴ atoms still fails to rule out continuous spontaneous localization models by about six orders of magnitude [38]. The superpositions observed in coherent quantum tunneling in SQUIDs also appear to be compatible with dynamical reduction models, since the spatial localization mechanism would only result in a small reduction of the supercurrent below the detectable level due to a breaking-up of Cooper pairs, but not in an approximate reduction onto one of the current states [262, 66, 37]. However, given the rapid development of experiments that propose to demonstrate quantum superpositions on increasingly large scales, it appears to be only a matter of time to probe the range relevant to a test of physical reduction models.

It is important to note that no deviations from linear Schrödinger dynamics have ever been observed that could not also be explained (at least in principle) as apparent deviations due to decoherence. In fact, it would be very difficult to distinguish collapse effects from decoherence, since the large number of atoms required for the

collapse mechanism to be effective also leads to strong decoherence [294, 46, 37]. It would therefore be necessary to isolate the system of interest extremely well from its environment, such that decoherence effects can be neglected with respect to the environment-independent localization mechanism. Even in this case it might be difficult to exclude the influence of decoherence due to, for example, thermal emission of radiation, as demonstrated in the case of fullerene and C₇₀ interferometry [167, 185].

This leaves physical collapse theories, at least so far, in the speculative realm, with the added difficulty of obtaining relativistic generalizations [37]. Certainly, such collapse mechanisms might be discovered in the future. However, in the absence of positive experimental evidence for such effects, and given the viable option of constructing a quantum theory consistent with all observations from the minimal formalism alone (a strategy advocated in this paper), the need for a postulated collapse effect, with free parameters tuned such as to avoid inconsistencies with the observation (or nonobservation) of superpositions, appears rather doubtful.

5.4 A comment on ensemble interpretations

Probably the best known ensemble, or “statistical,” interpretation of quantum mechanics is due to Ballentine [30]. Einstein [125] and others [261, 49] seem to have also favored such a view. The key assumption of this type of interpretation is to reject the notion of the quantum state as a *complete* description of an *individual* physical system. Instead, the quantum state is considered to represent only *statistical properties of an ensemble of similarly prepared systems*. The ensemble interpretation thus implies that the entire formal body of quantum mechanics (for example, a probability amplitude) has no direct physical meaning, in the sense of having no direct correspondence to the entities of the physical world (see also the comment in Ref. [215]). This interpretation effectively points towards the need for some hidden-variable theory to fully specify the state of individual systems, but it does not actually specify what this

“complete theory” would be.

A key signature of the ensemble view is its interpretation of superpositions. Ballentine uses the example of the momentum eigenstate of a single electron, which yields a plane wave in configuration space (i.e., a superposition of all spatial positions). In the ensemble interpretation, this quantum state is viewed as representing an (conceptual, infinitely large) ensemble of single electrons with the same momentum, but evenly spread out over all positions. In other words, *superpositions are interpreted as representing the results of an ensemble of measurements*. Note that the ensemble interpretation avoids interpreting superpositions as corresponding to an ensemble of premeasurement values (i.e., of definite but unknown values of position for the electron described by a momentum eigenstate), since this could lead to difficulties with the observed violation of the Bell inequalities [41, 42].

In this way, the ensemble interpretation can be considered instrumentalist, just as the Copenhagen interpretation. That it does not run into obvious empirical conflicts (e.g., with respect to the observation of interference effects in a double-slit experiment) can be explained by referring to the fact that in any measurement of position, we do obtain a single value. That is, although the momentum eigenstate of the electron *formally* is represented by an infinite number of position eigenstates, this superposition of positions is not directly observed, but just the statistical distribution of individual positions of each electron on the screen (which is correctly given by the quantum state). Therefore, in essence, this interpretation is simply based on weaker assumptions regarding the interpretation of quantum states than the standard interpretation of quantum mechanics (that regards quantum states as a complete description of the state of individual systems). It is thus difficult to conceive a way in which the ensemble interpretation could be explicitly refuted.² However, one might legitimately ask whether the restriction to an ensemble view of quantum mechanics is “the best

²It is interesting to note that, conversely, Ballentine has put forward claims that standard collapse quantum mechanics may be empirically invalidated [31].

we can do,” for example, given the recent insights gained from the theoretical and experimental studies outlined in this chapter.

Consider, for instance, the SQUID experiment, discussed in Sec. 5.2.2, that has shown the existence of states corresponding to two macroscopic currents running in opposite direction. Here, the energy difference between the two superpositions $|\Psi_s\rangle = \frac{1}{\sqrt{2}}(|L\rangle + |R\rangle)$ and $|\Psi_a\rangle = \frac{1}{\sqrt{2}}(|L\rangle - |R\rangle)$ has been measured explicitly [136, 303], and coherent oscillations between $|L\rangle$ and $|R\rangle$ have been directly observed in similar superconducting setups [233, 206, 205, 160, 223, 325, 311]. These superpositions are therefore observed as *individually existing*, physical states. Their components $|L\rangle$ and $|R\rangle$ exist simultaneously with oscillatory amplitudes. Nothing in this picture would seem to necessitate (or even suggest) the view that the supercurrent superpositions $|\Psi_s\rangle$ and $|\Psi_a\rangle$ should only represent ensembles of possible current states $|L\rangle$ and $|R\rangle$. In the opinion of this author it therefore appears to be more fruitful to adopt the view that quantum states are a complete description of individual physical states, and to explore the consequences of this view.

We remark that one might try to find some middle ground between the very “nonrealist” (or maybe rather “instrumentalist”) reading of quantum states, as advocated by the ensemble interpretation and other authors (see, e.g., Ref. [138]), and the purely “realist” view of quantum states inherent in many-branches interpretations and physical collapse theories. For example, we have emphasized the importance of quantum entanglement (arising from the superposition principle together with linear time evolution) in many places of this dissertation. Entangled states describe *correlations* between subsystems. In contrast to classical correlations, however, entangled quantum states describe in general a multitude of such correlations, each *relative* to the “state” of the subsystem. The latter state, however, cannot be represented by an individual quantum state that could be ascribed to the subsystems alone. This situation, then, naturally leads to a relative-state view. However, instead of ascribing physical reality to each component of the entangled state (which leads to the many-

branches framework discussed in this chapter), one might advocate the view that, as Mermin [226] suggests,

[t]he only proper subjects of physics are correlations among different parts of the physical world. Correlations are fundamental, irreducible, and objective. They constitute the full content of physical reality. There is no absolute state of being; there are only correlations between subsystems.

Of course, this strategy leaves open the question of the nature of the correlated quantities, i.e., of the subsystems themselves. While Mermin acknowledges this question [226], he suggest that measurable properties (observables) of subsystems attain meaning only from the type of their correlations with observables of other subsystems.

Mermin's approach combines a realist and nonrealist view of quantum states. Namely, quantum states are assumed to correspond directly to physical reality, but this physical reality is identified with correlations between physical systems, rather than with the systems themselves. (See also Refs. [269, 132, 297] for related ideas.)

5.5 *Emergence of probabilities in a relative-state framework*

The question of the origin and meaning of probabilities in a relative state-type interpretation that is based solely on a deterministically evolving global quantum state, and the problem of how to consistently derive Born's rule in such a framework, has been the subject of much discussion and criticism aimed at this type of interpretation (see Chapters 2 and 4). Several decoherence-unrelated proposals have been put forward in the past to elucidate the meaning of probabilities and to arrive at the Born rule in an explicit or implicit relative-state context (see, for instance, Refs. [129, 179, 102, 159, 147, 99]). However, it is highly controversial whether these approaches are successful and represent a noncircular derivation [293, 200, 288]. A derivation that is only based on the non-probabilistic axioms of quantum mechanics and on elements of classical decision theory has been presented by Deutsch [99]. It

was criticized by Barnum *et al.* [34], but was subsequently defended by other authors [151, 315] and embedded into an operational framework by Saunders [279]. It is fair to say that no decisive conclusion appears to have been reached as to the success of these derivations.

Initially, decoherence was thought to provide a natural account of the probability concept in a relative-state framework, by relating the diagonal elements of the decohered reduced density matrix to a collection of possible “events” that can be reidentified over time, and by interpreting the corresponding coefficients as relative frequencies of branches, thus leading to an interpretation of quantum probabilities as empirical frequencies [341, 99]. However, as it has been pointed out before [330, 344, 280] (see also Chap. 2), this argument cannot yield a noncircular derivation of the Born rule, since the formalism (in particular, the trace operation) and interpretation of reduced density matrices presume this rule.

The solution to the problem of understanding the meaning of probabilities and of deriving Born’s rule in a relative-state framework must therefore be sought on a much more fundamental level of quantum mechanics. Since this framework presumes nothing besides the unitarily evolving state vector itself, the solution should preferably be derived solely from properties of this quantum state. However, while we would like to assign probabilities to “outcomes of measurements” on a local system (i.e., probabilities for the system to be found in a certain state), the global quantum state usually contains a high degree of environmental entanglement, i.e., there exists no state vector that could be assigned to the local system alone. Still, we obviously talk regularly of the “state of the system,” and we must therefore distinguish this notion of state from the quantum state vector itself. Following the relative-state viewpoint, the local “events” of the system (or its possible “states of the system”) are then typically identified with the relative-state components of the global state vector in the Hilbert subspace corresponding to the system.

The recent enormous advances in the field of quantum information theory, espe-

cially in the understanding of the properties and implications of quantum entanglement, have shed some light on how one might proceed. Quantum information theory has established the notion that quantum theory can be viewed as a description of what, and how much, “information” Nature is willing to proliferate. For example, a peculiar feature of quantum mechanics is that complete knowledge of the global pure bipartite quantum state $|\Psi\rangle = (|\alpha_1\rangle|\beta_1\rangle + |\alpha_2\rangle|\beta_2\rangle)/\sqrt{2}$ itself does not appear to contain information about the “absolute” state of one of the subsystems. This hints at ways how a concept of “ignorance,” and therefore of probability, may emerge directly from the quantum feature of entanglement without any classical counterpart.

This idea has recently been developed in a series of papers by Zurek [344, 345, 347, 346], leading to a proposal for a derivation of Born’s rule (see also Chapters 2 and 4); cf. Refs. [33, 230]). As pointed out in Chapters 2 and 4), and made more explicit in the most recent of Zurek’s articles on this topic [346], the derivation is still based on certain assumptions that are not contained in the basic measurement-free relative-state framework of quantum mechanics. One might argue how strong these assumptions are. Zurek himself, for example, considers some of them to be “facts” and regards others as “natural” and “modest” [346]; a somewhat more critical position with respect to some of the assumptions has been assumed in Chap. 4. Granted these assumptions, however, we consider Zurek’s proposal a very promising approach towards a deeper understanding of the origin of quantum probabilities, and we shall therefore outline the basic ideas and assumptions in the following (a more detailed description and discussion of the approach can be found in Chapters 2 and 4, and in Refs. [344, 346]).

Zurek’s derivation is based on a particular symmetry property (referred to as “environment-assisted envariance,” or “envariance” for short) of composite quantum states, which is used to infer complete ignorance about the state of the subsystem. The derivation relies on a study of the properties of a composite entangled state and therefore intrinsically requires the decomposition of the Hilbert space into subsystems

and the usual tensor-product structure. The core result to be established is the following. Given a bipartite product Hilbert space $H_{\mathcal{S}_1} \otimes H_{\mathcal{S}_2}$ and a completely known composite pure state in the diagonal Schmidt decomposition

$$|\Psi\rangle = (e^{i\varphi_1} |\alpha_1\rangle_1 |\beta_1\rangle_2 + e^{i\varphi_2} |\alpha_2\rangle_1 |\beta_2\rangle_2) / \sqrt{2}, \quad (5.28)$$

where the $|\alpha_i\rangle_1$ and $|\beta_i\rangle_2$ are orthonormal basis vectors that span the Hilbert spaces $H_{\mathcal{S}_1}$ and $H_{\mathcal{S}_2}$, the probabilities of obtaining either one of the relative states $|\alpha_1\rangle_1$ and $|\alpha_2\rangle_1$ (identified with the “events” of interest to which probabilities are to be assigned [347, p. 12]; see also Chap. 4) are equal. Given this result, generalizations to higher-dimensional Hilbert spaces and to the case of unequal absolute values of the Schmidt coefficients in Eq. (5.28) can be achieved in a rather straightforward way [346].

This result is established in two key steps. First, a few simple assumptions (Zurek’s “facts” [346]) are introduced that connect the global quantum state $|\Psi\rangle$, Eq. (5.28), to the “state of the system” \mathcal{S}_1 . This is necessary because, as mentioned above, the global quantum state is all that the pure state-vector formalism of quantum mechanics provides for the description of a bipartite system containing entanglement. The following assumptions are made about the “state of the system” \mathcal{S}_1 . First, this state is completely determined by the global quantum state, Eq. (5.28); second, it specifies all measurable properties of \mathcal{S}_1 , including probabilities of outcomes of measurements on \mathcal{S}_1 ; and third, unitary transformations can change it only if they act on \mathcal{S}_1 (see Chap. 4 for a discussion of this last assumption).

Granted these three assumptions, one can show that measurable properties of \mathcal{S}_1 can depend neither

1. on the phases φ_i in Eq. (5.28), such that we can assume the simplified form

$$|\Psi\rangle = (|\alpha_1\rangle_1 |\beta_1\rangle_2 + |\alpha_2\rangle_1 |\beta_2\rangle_2) / \sqrt{2} \quad (5.29)$$

for our purpose of discussing probabilities associated with \mathcal{S}_1 ;

2. nor on whether $|\alpha_1\rangle_1$ is paired with $|\beta_1\rangle_2$ or $|\beta_2\rangle_2$, i.e., the unitary transformation acting on \mathcal{S}_1 that changes the quantum state vector

$$|\Psi\rangle = (|\alpha_1\rangle_1|\beta_1\rangle_2 + |\alpha_2\rangle_1|\beta_2\rangle_2) / \sqrt{2} \quad (5.30)$$

into

$$|\Psi'\rangle = (|\alpha_2\rangle_1|\beta_1\rangle_2 + |\alpha_1\rangle_1|\beta_2\rangle_2) / \sqrt{2} \quad (5.31)$$

cannot have altered the state of \mathcal{S}_1 .

In a way, result (2) already indicates a feature of ignorance about the state of \mathcal{S}_1 , since interchanging the potential “outcomes” $|\alpha_i\rangle_1$ through local operations performed on \mathcal{S}_1 does not change any measurable properties of \mathcal{S}_1 and can therefore be viewed as leading to a form of “objective indifference” among these outcomes. It is important to note that this effect is crucially dependent on the feature of entanglement. In a nonentangled pure state of the form $|\Phi\rangle = (|\phi_1\rangle + e^{i\varphi}|\phi_2\rangle) / \sqrt{2}$, the phase φ must of course not be ignored (and would be measurable in a suitable interference experiment), and therefore the system described by the “swapped” state $|\Phi'\rangle = (|\phi_2\rangle + e^{i\varphi}|\phi_1\rangle) / \sqrt{2}$ is clearly physically different from that represented by the original state $|\Phi\rangle$.

To make the above argument more precise, in the second key step of the derivation, the notion of probabilities of the outcomes $|\alpha_i\rangle_1$ in a measurement performed on \mathcal{S}_1 (previously only subsummed under the general heading “measurable properties of \mathcal{S}_1 ”) is now explicitly connected to the global state vector via an additional assumption. In Ref. [346], Zurek offers three possible choices for this assumption, of which we should quote one (see also Ref. [33]). Namely, it is assumed that the form of the Schmidt product states $|\alpha_i\rangle_1|\beta_i\rangle_2$ appearing in Eq. (5.28) implies that the probabilities for $|\alpha_i\rangle_1$ and $|\beta_i\rangle_2$ must be equal. Given this assumption and using result (2) above, it can be readily established (see Chapters 2 and 4, and Ref. [346]) that the probabilities for $|\alpha_1\rangle_1$ and $|\alpha_2\rangle_1$ must be equal, thus completing the derivation.

As we have pointed out in Chap. 4, the need for the final assumption may be considered a reflection of the well-worn phrase that a transition from a nonprobabilistic

theory (such as quantum mechanics solely based on deterministically evolving state vectors) to a probabilistic theory (that refers to “probabilities of outcomes of local measurements”) requires, at some stage, to “put probabilities in to get probabilities out.” However, in the quantum setting, this introduction of a probability concept has a far more objective character than in the classical case. While in the latter setting probabilities refer to subjective ignorance in spite of the existence of a well-defined state (see also Sec. 5.6), in the quantum case all that is available, namely, the global entangled quantum state, is perfectly known. The objectivity of ignorance in quantum mechanics can thus be viewed as a consequence of a form of “complementarity” between local and global observables [346] and could help explain the fundamental need for a probabilistic description in the quantum setting despite the deterministic evolution of the global state vector.

It is the great merit of Zurek’s proposal to have emphasized this objective character of quantum probabilities arising from the feature of quantum entanglement. While the precise role and importance of the assumptions entering the derivation as well as the generality of the approach (given, e.g., the focus on Schmidt decompositions) would benefit from further discussion and analysis, the approach definitely sheds an interesting and new light on the nature of quantum probabilities.

5.6 Objectification of observables in a relative-state framework

A characteristic feature of classical physics is the fact that the state of a system can be found out and agreed upon by many independent observers (with all of them initially completely ignorant about the state) without disturbing this state. In this sense, classical states preexist objectively, resulting in our notion of “classical reality.” In contrast, as is well known, measurements on a closed quantum system will in general alter its state—unless, of course, the observer chooses to measure, by pure luck or prior knowledge, an observable with an eigenstate that coincides with the state of the system. It is therefore impossible to regard quantum states of a closed

system as existing in the way that classical states do. This raises the question of how classical reality emerges from within the quantum substrate, i.e., how observables are “objectified” in the above sense.

In a first step, the decoherence program, in particular the stability criterion and the more general formalism of the “predictability sieve” [337, 338, 340, 341, 344] (see also Sec. 5.2.2), has provided an answer to the question of why only a certain subset of the possible states in the Hilbert space of the system are actually observed. Taking into account the openness of the system and the form of the system-environment interaction is crucial in determining a set of preferred stable states of the system. This supplies an elegant and physically motivated solution to the problem of the preferred basis, an issue that has often been used to challenge the feasibility of relative-state interpretations [200, 292]. Nonetheless, the problem sketched in the previous paragraph remains, as any direct measurement performed on the system would, in general, still alter the state of the system.

The important next step is therefore to realize that in most (if not all) cases observers gather information about the state of a system through indirect observations, namely, by intercepting fragments of environmental degrees of freedom that have interacted with the system in the past and thus carry information about the state of the system [340, 341, 342, 344]. Probably the most common example for such an indirect acquisition of information is the visual registration of photons that have scattered off from the object of interest (see also Sec. 5.8.3). Similar to the case of decoherence, the recognition of the openness of quantum systems is therefore crucial. However, the role of the environment is now broadened, namely, from the selection of preferred states for the system of interest and the delocalization of local phase coherence, to the transmission of information about the state of the system. The idea is then to show how, and which, information is both redundantly and robustly stored in a large number of distinct fragments of the environment in such a way that multiple observers can retrieve this information without disturbing the state of the

system, thereby achieving effective classicality of the state.

This approach has recently been developed under the labels of “environment as a witness” (i.e., the recognition of the role of the environment as a communication channel) and “quantum Darwinism” (namely, the study of what information about the system can be stably stored and proliferated by the environment) [340, 341, 344, 347, 234, 235, 50, 51]. To explicitly quantify the degree of completeness and redundancy of information imprinted on the environment, the measure of (classical [234, 235] or quantum [344, 50, 51]) mutual information has usually been used. Roughly speaking, this quantity represents the amount of information (expressed in terms of Shannon or von Neumann entropies) about the system \mathcal{S} that can be acquired by measuring (a fragment of) the environment \mathcal{E} . Note that the amount of information contained in each fragment is always somewhat less [51] than the maximum information provided by the system itself (as given by the von Neumann entropy of the system).

The measure of classical mutual information is based on the choice of particular observables of \mathcal{S} and \mathcal{E} and quantifies how well one can predict the outcome of a measurement of a given observable of \mathcal{S} by measuring some observable on a fraction of \mathcal{E} [234, 235]. The quantum mutual information $\mathcal{I}_{\mathcal{S},\mathcal{E}}$, used in more recent studies [344, 50, 51], can be viewed as a generalization of classical mutual information and is defined as $\mathcal{I}_{\mathcal{S},\mathcal{E}} = H(\mathcal{S}) + H(\mathcal{E}) - H(\mathcal{SE})$, where $H(\rho) = -\text{Tr}(\rho \log \rho)$ is the von Neumann entropy. Thus $\mathcal{I}_{\mathcal{S},\mathcal{E}}$ measures the amount of entropy produced by destroying all correlations between \mathcal{S} and \mathcal{E} , i.e., it quantifies the degree of correlations between \mathcal{S} and \mathcal{E} . Results derived from these measures have thus far been found to be sufficiently robust with respect to the particular choice of measure [234, 235, 344, 50, 51], although a more detailed analysis of this issue is underway [51].

It has been found that the observable of the system that can be imprinted most completely and redundantly in many distinct subsets of the environment coincides with the “pointer” observable selected by the system-environment interaction (i.e., by the stability criterion of decoherence) [234, 235, 50, 51]. Conversely, most other states

do not seem to be redundantly storable. Thus it suffices to probe a comparably very small fraction of the environment to infer a large amount of the maximum information about the pointer state of the system. On the other hand, if the observer tried to measure other observables on the same fragment, he would learn virtually nothing, as information about the corresponding observables of the system is not redundantly stored. Thus the “pointer” states of the system play a twofold role: They are the states least perturbed by the interaction with the environment, and they are the states that can be most easily found out, without disturbing the system, by probing environmental degrees of freedom. Since the same information about the pointer observable is stored independently in many fragments of the environment, multiple observers can measure this observable on different fragments and will automatically agree on the findings. In this sense, one can ascribe (effective) objective existence to the pointer states.

The research into the objectification of observables along the lines outlined in this section is only in its beginnings. Important aspects, such as the explicit dynamical evolution of the objectification process [235] and the role of the assumptions and definitions in the current treatments of the “objectification through redundancy” idea, are currently still under investigation, as are studies involving more detailed and realistic system-environment models. However, it should have become clear that the approach of departing from the closed-system view and of describing observations as the interception of information that is redundantly and robustly stored in the environment, represents a very promising candidate for a purely quantum-mechanical account of the emergence of classical reality from the quantum domain.

5.7 Observable-free formulation of quantum mechanics

Essentially all standard textbook formulations of the axioms of quantum mechanics are based on the concept of observables associated with Hermitian operators in the respective Hilbert space. In this way, the much-discussed element of “measurement”

is introduced into the theory. As is well known, the concept of measurement seems to play such an important role in quantum mechanics (as opposed to classical physics) because quantum states (of a closed system) do not preexist in the classical sense and can therefore in general not be simply “found out” without invariably altering the state of the system (see also Sec. 5.6). What the system “is” (in terms of its quantum state) and what can be “known” about it (by performing a measurement on it) appears to be intrinsically linked.

On the other hand, the axiomatic subsummation of the complex realm of “measurements” under the abstract and purely formal concept of observables, as well as the seemingly special role assigned to measurements in quantum theory, has often been considered unsatisfactory and problematic. From von Neumann’s early description of quantum measurements in terms of the state-vector formalism itself [312] to Bell’s famous polemic “Against ‘measurement’” [45], it has frequently been attempted to give a more precise (physical) meaning to the term “measurement” and its role in quantum theory.

The main question is therefore as follows. Is it possible to present a formulation of quantum mechanics that is similarly compact as the standard representation but that does not make use of the concept of observables? In other words, can one effectively *derive* observables from a formalism that describes measurements purely in terms of interacting state vectors?

As a cautionary preliminary remark, it should be clear that the concept of observables will always provide the most *compact* formulation, as it encapsulates the entire body of an explicit description of measurements in form of a purely phenomenological representation. For instance, the common statement of “measuring operators-as-observables” simply bypasses the question of how to explicitly describe the registration of measurement outcomes by an apparatus, and instead refers to the “eigenvalues of observables” (actually, eigenvalues of *operators identified with observables* [91]). Therefore, to explain the emergence of observables from the formalism

of interacting state vectors, one will always need to give a somewhat more lengthy (albeit, as we shall see, no less precise!) account. This shall be done in the following; see also the discussion in Sec. 2.4.2 and in Refs. [344, 198].

5.7.1 *The axioms of quantum mechanics*

Let us start with the basic axioms of the “measurement-free” part of quantum mechanics, which we shall state as follows:

- (A1) The (fully known, pure, physical) state of a quantum system \mathcal{S} is (completely) described by a state vector $|\psi\rangle$ in the Hilbert space $H_{\mathcal{S}}$ of \mathcal{S} .
- (A2) The (fully known, pure, physical) state of a composite quantum system is (completely) represented by state vector in a Hilbert space that is the tensor product of the Hilbert spaces of the constituent systems.
- (A3) A quantum state $|\psi\rangle$ of \mathcal{S} evolves in time according to the Schrödinger equation, $i\hbar\frac{\partial}{\partial t}|\psi\rangle = \hat{H}|\psi\rangle$, where \hat{H} is a Hermitian operator in the Hilbert space of \mathcal{S} .

In stating axiom (A3), we have introduced the notion of an operator in a Hilbert space. For our purposes, the Hamiltonian \hat{H} as defined through axiom (A3) is the only operator we will need, and its physical interpretation as describing the time evolution of states vectors [that, according to postulate (A1), directly represent the physical state of the system], follows from (A3) as well. Note that we do not need to make any assumption about the concrete physical interpretation and realization of general operators, or about the connection between operators and observables.

To arrive at an explicit mathematical expression for \hat{H} and its action on state vectors, we also postulate the usual existence of a “dual” space of bras and define the inner product of a bra with a ket. This allows for the definition of orthonormal bases $\{|s_i\rangle\}$ of the Hilbert space $H_{\mathcal{S}}$ via the condition $\langle s_i|s_j\rangle = \delta_{ij}$ (with the familiar

generalizations for the continuous case). With these definitions, we then can, in the usual manner, formally express \hat{H} in terms of the ket-bra notation.

The standard textbook axioms about (idealized) quantum measurement typically read as follows:

- (A4) Every observable O of a system \mathcal{S} is represented by an (Hermitian) operator \hat{O} in the respective Hilbert space $H_{\mathcal{S}}$ of \mathcal{S} .
- (A5) The only possible outcomes of a measurement of O on \mathcal{S} are the eigenvalues o_i of \hat{O} .
- (A6) Immediately following a measurement that yields the value o_i , the system \mathcal{S} is described by the corresponding eigenstate $|\psi_i\rangle$ of \hat{O} .
- (A7) If the system \mathcal{S} is described by the normalized state vector $|\psi\rangle$ before the measurement, the probability $p(o_i)$ for a measurement of O on \mathcal{S} to yield the value o_i is given by $p(o_i) = |\langle\psi_i|\psi\rangle|^2$ (Born's rule).

As pointed out in Ref. [344], it is really the *combination* of axioms (A5) and (A6) that represents the usual “collapse postulate” of orthodox quantum mechanics. We shall now show how axioms (A4)–(A6) can be regarded as naturally emerging from axioms (A1)–(A3) as a derived concept. With respect to the motivation of Born's rule [axiom (A7)], we refer the reader to the discussion in Chap. 4 and in Sec. 5.5 of this chapter.

5.7.2 Observables as Hermitian operators [Axiom (A4)]

A measurement of a quantum system \mathcal{S} requires an apparatus \mathcal{A} , itself a quantum system described by a Hilbert space $H_{\mathcal{A}}$. We then take the following steps.

1. Let us take an arbitrary orthonormal basis $\{|s_k\rangle\}$, $k = 1 \cdots N$, of $H_{\mathcal{S}}$, and let us suppose \mathcal{S} is described by the state vector $|s_i\rangle$.

2. To measure in which state the system is, we have available an apparatus \mathcal{A} that is described by a set of “pointer states” $\{|a_k\rangle\}$ corresponding to the \mathcal{S} -states $\{|s_k\rangle\}$, and interacting with \mathcal{S} such that

$$|s_i\rangle|a_0\rangle \longrightarrow |s_i\rangle|a_i\rangle, \quad (5.32)$$

where $|a_0\rangle$ is the “ready” state of \mathcal{A} .

3. For an arbitrary state vector $|\psi_{\mathcal{S}}\rangle$ of \mathcal{S} , $|\psi_{\mathcal{S}}\rangle = \sum_{k=0}^{N-1} s_k |s_k\rangle$, the linearity of the Schrödinger equation [see axiom (A3)] together with Eq. (5.32) then implies that the measurement interaction for $|\psi_{\mathcal{S}}\rangle$ has the usual “preamble” (von Neumann [312]) form,

$$\left(\sum_{k=0}^{N-1} s_k |s_k\rangle \right) |a_0\rangle \longrightarrow \sum_{k=0}^{N-1} s_k |s_k\rangle |a_k\rangle. \quad (5.33)$$

We will argue below that this process is capable of representing quantum measurement.

4. We will now show that the time evolution of Eq. (5.32), for any state $|s_i\rangle$ of \mathcal{S} , can be accomplished via an interaction Hamiltonian of the form [344]

$$\hat{H}_{\text{int}} = g \hat{O}_{\mathcal{S}} \hat{O}_{\mathcal{A}}. \quad (5.34)$$

Here, g is a coupling-strength parameter, and

$$\hat{O}_{\mathcal{S}} = \sum_{k=0}^{N-1} k |s_k\rangle \langle s_k|. \quad (5.35)$$

We note that in the language of the operator-observable identification established in axiom (A4) (which we of course do not want to presume here), $\hat{O}_{\mathcal{S}}$ would measure in which of the possible states $\{|s_k\rangle\}$ the system “is,” indexed by k .

Furthermore, in Eq. (5.34), we have introduced

$$\hat{O}_{\mathcal{A}} = \sum_{j=0}^{N-1} j |b_j\rangle\langle b_j|, \quad (5.36)$$

which uses the “conjugate” basis [344, 324, 191]

$$|b_j\rangle = \frac{1}{\sqrt{N}} \sum_{k=1}^{N-1} \exp\left(\frac{2\pi i}{N} j k\right) |a_k\rangle. \quad (5.37)$$

Therefore

$$|a_k\rangle = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} \exp\left(-\frac{2\pi i}{N} k j\right) |b_j\rangle. \quad (5.38)$$

It is easy to verify that the time evolution of the composite state $|s_i\rangle|a_0\rangle$ under the action of \hat{H}_{int} yields

$$e^{-i\hat{H}_{\text{int}}t/\hbar} |s_i\rangle|a_0\rangle = |s_i\rangle \left[\frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} \exp\left(-\frac{igt}{\hbar} ij\right) |b_j\rangle \right]. \quad (5.39)$$

If we tune the coupling strength g and/or the interaction time t such that $gt/\hbar = 2\pi/N$, this expression becomes, using Eq. (5.38),

$$e^{-i\hat{H}_{\text{int}}t/\hbar} |s_i\rangle|a_0\rangle = |s_i\rangle \left[\frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} \exp\left(-\frac{2\pi i}{N} ij\right) |b_j\rangle \right] = |s_i\rangle|a_i\rangle, \quad (5.40)$$

as desired.

We note that in the realistic case of nonexact tuning, the measurements will be imperfect, i.e., $|s_i\rangle$ will be correlated not only with $|a_i\rangle$, but with a set of states $\{|a_k\rangle\}$ in the “vicinity” of $|a_i\rangle$. However, this situation can be treated as a straightforward generalization of the formalism and does not have a direct bearing on the general idea of this derivation. Since we know of the existence of well-designed apparatuses that do establish sufficiently exact correlations between \mathcal{S} and \mathcal{A} , we may assume that the tuning of the apparatus is such as to insure that Eq. (5.40) will be fulfilled at least approximately.

Thus we have replaced the formal (but very useful!) concept of “measuring operator-observable \hat{O}_S on \mathcal{S} ” by an explicit description of the time evolution of the combined system-apparatus under the action of a particular interaction Hamiltonian. We also note that, since $[\hat{H}_{\text{int}}, \hat{O}_S] = 0$, the observable \hat{O}_S is a constant of motion and therefore not disturbed by the measurement.

5.7.3 *Eigenvalues of the operator-observable as the only possible outcomes [Axiom (A5)]*

As we have discussed in Sec. 2.2.3, the premeasurement interaction, Eq. (5.33), does not seem to allow one to claim that a measurement (even in the relative-state sense) has been completed, as the final composite state can in general be rewritten in many different bases and therefore correspond to a measurement of several different (in general noncommuting) observables. In Sec. 2.3.5, we described in much detail how to resolve this preferred-basis (and thus preferred-observable) ambiguity via the inclusion of the interaction of the apparatus \mathcal{A} with its environment. In short, we discussed how this interaction selects a set of preferred pointer states $|A_i\rangle$ of \mathcal{A} . These are the only states that remain stable in spite of the immersion of the system-apparatus combination \mathcal{SA} into the environment \mathcal{E} , i.e., they are the only states than can be read off without being perturbed (since they have already been measured by \mathcal{E}). Furthermore, only the pointer states can be measured without decreasing the degree of correlations between \mathcal{A} and \mathcal{S} , while measurements in other bases destroy such correlations on the decoherence timescale [337, 338, 344]. These pointer states can therefore be considered effectively classical (see also Sec. 5.6). In turn, of course, they determine the preferred states of \mathcal{S} , that is, the observable \hat{O}_S that can be measured by \mathcal{A} .

In view of these arguments, axiom (A5) follows immediately. The only reliably measurable apparatus states are the environment-selected, effectively classical pointer states, and these states are one-to-one correlated with the eigenstates (and

thus the eigenvalues) of the corresponding preferred observable of \mathcal{S} [see Eq. (5.33)], i.e., the observable that the apparatus is *designed* to measure (see also the discussion in Sec. 2.4.2).

5.7.4 *Immediate remeasurements yield the same result [Axiom (A6)]*

The “repeatability axiom” (A6) can again be motivated by referring to the environment-induced selection of pointer states [344]. First, outcome repeatability of measurements only needs to apply to these preferred quasiclassical states, because, as we have pointed out in the preceding paragraphs, they are the only states that can be reliably measured by the apparatus. Second, repeated measurements in the pointer basis do not change the density matrix of the system-apparatus combination (since it is approximately diagonal in this basis due to decoherence). Third, in the relative-state view of quantum measurement, observers become correlated with the particular measured pointer states (i.e., they become attached to the corresponding “branch” of the global wave function), such that subsequent measurements made in the pointer basis will confirm the previous outcome (see also Sec. 2.3.5). The combination of these three statements establishes a well-founded motivation for axiom (A6).

5.7.5 *Commutation relations*

An important feature of the concept of observables as used in textbook quantum mechanics is the derivation (or postulation, depending on the book) of commutation relations between observables. The uncertainty principle for the operators \hat{x} and \hat{p} as well as the emerging “ladder” structure of the angular-momentum and spin operators (with quantized eigenvalues) are the most well-known consequences of the commutation relations. We shall now investigate to what extent it is possible to derive these commutation relations from the “observable-free” formalism outlined above.

Position-momentum commutation relations. To derive any commutation relations, we will have to establish a correspondence between the states $|s_i\rangle$ used to describe the system and some physical quantities. In our case, we make the connection by linking position and momentum to continuous position and momentum eigenbases $\{|\mathbf{x}\rangle\}$ and $\{|\mathbf{p}\rangle\}$ of \mathcal{S} . We can then *mathematically* define a “position operator” $\hat{\mathbf{x}} = \int d\mathbf{x}' |\mathbf{x}'\rangle\langle\mathbf{x}'|$ and a “momentum operator” $\hat{\mathbf{p}} = \int d\mathbf{p}' |\mathbf{p}'\rangle\langle\mathbf{p}'|$. Note that this definition is initially purely formal. However, it acquires *physical meaning* in terms of the measurement formalism outlined in Sec. 5.7.2, since $\hat{\mathbf{x}}$ and $\hat{\mathbf{p}}$ define the interaction Hamiltonians that represent a measurement of the states $\{|\mathbf{x}\rangle\}$ and $\{|\mathbf{p}\rangle\}$ of \mathcal{S} [see Eq. (5.34)]. In this sense we can establish the connection between the abstract operators $\hat{\mathbf{x}}$ and $\hat{\mathbf{p}}$ and a measurement of the corresponding physical quantities \mathbf{x} and \mathbf{p} that are represented by the state vectors $|\mathbf{x}\rangle$ and $|\mathbf{p}\rangle$.

Suppose \mathcal{S} is initially described by the state vector $|\mathbf{x}\rangle$, corresponding to the system being localized around \mathbf{x} . If we apply some physical process to \mathcal{S} such that \mathcal{S} changes its position by an infinitesimal amount $d\mathbf{x}$ to $\mathbf{x} + d\mathbf{x}$, the state vector changes from $|\mathbf{x}\rangle$ to $|\mathbf{x} + d\mathbf{x}\rangle$ [see axiom (A1)]. *Formally*, we may introduce an operator $\hat{T}(d\mathbf{x})$ that brings about this change of the state vector,

$$\hat{T}(d\mathbf{x})|\mathbf{x}\rangle = |\mathbf{x} + d\mathbf{x}\rangle. \quad (5.41)$$

Note that we have now generalized the concept of operators from the Hamiltonian only [as introduced in axiom (A3)] to other operators, but still in a strictly formal matter that avoids any identification with physical quantities, observables, etc.

We can then proceed in standard textbook manner (see, for example, Ref. [274, pp. 44–46]) by demanding that $\hat{T}(d\mathbf{x})$ fulfills certain properties needed for it to count as a “infinitesimal-translation operator,” namely: the conservation of probability; the property of additivity; the representation of the inverse operator as an opposite-direction translation; the property $\lim_{d\mathbf{x} \rightarrow 0} \hat{T}(d\mathbf{x}) = 0$; and the requirement that the difference between $\hat{T}(d\mathbf{x})$ and the unit operator $\hat{1}$ should be of first order in $d\mathbf{x}$.

It can then be shown [274, p. 45] that the operator

$$\hat{T}(d\mathbf{x}) = \hat{1} - i\hat{\mathbf{A}} \cdot d\mathbf{x}, \quad (5.42)$$

with $\hat{\mathbf{A}}$ Hermitian, fulfills all of the above requirements, and that the components \hat{x}_i of $\hat{\mathbf{x}}$ and \hat{A}_j of $\hat{\mathbf{A}}$ obey the commutation relation

$$[\hat{x}_i, \hat{A}_j] = i\delta_{ij}. \quad (5.43)$$

The remaining step in the derivation concerns the identification of the operator $\hat{\mathbf{A}}$. Clearly, at this point we do have to make some connection between a mathematical operator and its physical meaning. This seems difficult to avoid, since the commutation relations concern relationships between *physical* (measurable) quantities and therefore cannot simply emerge from within a purely abstract framework.

Since it is well known that in classical physics momentum is the generator of infinitesimal translations [156], it is reasonable to identify $\hat{\mathbf{A}}$ with the momentum operator $\hat{\mathbf{p}}$ defined above (divided by an empirical constant, \hbar , to ensure that $\hat{\mathbf{A}} \cdot d\mathbf{x}$ is dimensionless), which then readily yields the usual commutation relation $[\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij}$.

Angular-momentum commutation relations. According to axiom (A1), a system having angular momentum ℓ along some axis $\hat{\mathbf{n}}$ can be described by a state vector $|L_{\hat{\mathbf{n}}} = \ell\rangle$. Just as in the case of position and momentum, we can then formally define Hilbert-space operators $\hat{L}_{\hat{\mathbf{n}}} = \int d\ell |L_{\hat{\mathbf{n}}} = \ell\rangle \langle L_{\hat{\mathbf{n}}} = \ell|$. Their physical interpretation as representing measurements of the angular momentum component along an axis $\hat{\mathbf{n}}$ follows again from the interaction-Hamiltonian formalism of Sec. 5.7.2. Note that we have not assumed anything specific about these operators, in particular, we have not presumed the existence of quantized eigenvalues.

The commutation relations for the operators $\hat{L}_{\hat{\mathbf{n}}}$ can then be derived in a similar manner as in the case of the position-momentum relations. We shall here follow

the treatment in Ref. [274, pp. 156–158]. We first consider infinitesimal rotations (instead of translations), represented by a “infinitesimal-rotation operator” $\hat{R}(\hat{\mathbf{n}}, d\phi)$, describing a rotation of the system about the axis $\hat{\mathbf{n}}$ by an infinitesimal angle $d\phi$. By analogy with the criteria established for the infinitesimal-translation, we write

$$\hat{R}(\hat{\mathbf{n}}, d\phi) = \hat{1} - i\hat{B}_{\hat{\mathbf{n}}}\varepsilon, \quad (5.44)$$

where $\hat{B}_{\hat{\mathbf{n}}}$ is a Hermitian operator and ε is some real parameter.

The open question is now again how to relate \hat{B} to known physical quantities or operations. From classical mechanics, it is known that angular momentum is the generator of rotations. We are therefore led to the identification $\hat{B}_{\hat{\mathbf{n}}} \equiv \hat{L}_{\hat{\mathbf{n}}}/\hbar$ and $\varepsilon \equiv d\phi$ in Eq. (5.44), i.e.,

$$\hat{R}(\hat{\mathbf{n}}, d\phi) = \hat{1} - \frac{i}{\hbar} \hat{L}_{\hat{\mathbf{n}}} d\phi. \quad (5.45)$$

In this sense, $\hat{L}_{\hat{\mathbf{n}}}$ acquires the additional (but *defined*) meaning of being the generator of an infinitesimal rotation $d\phi$ about $\hat{\mathbf{n}}$.

To finally obtain the commutation relations for the $\hat{L}_{\hat{\mathbf{n}}}$ along different axes $\hat{\mathbf{n}}$, it is also assumed that the operators $\hat{R}(\hat{\mathbf{n}}, d\phi)$, Eq. (5.45), fulfill the same group properties as the standard \mathbb{R}^3 -rotation matrices [274, pp. 157]—a reasonable assumption given that the operators have been established as the quantum-mechanical equivalent of the rotation matrices. Together with the fact that rotations in \mathbb{R}^3 about different axes do not commute, it then follows that [274, pp. 158]

$$[\hat{L}_i, \hat{L}_j] = i\hbar\epsilon_{ijk}\hat{L}_k, \quad (5.46)$$

where the indices i , j , and k label the axes of an orthonormal coordinate system in \mathbb{R}^3 .

We have therefore found that the angular-momentum operators $\hat{L}_{\hat{\mathbf{n}}}$ for different axes $\hat{\mathbf{n}}$ are related in the particular manner given by Eq. (5.46), while the physical meaning of the $\hat{L}_{\hat{\mathbf{n}}}$ is derived from the interaction-Hamiltonian description of measurement (Sec. 5.7.2). The commutation relations Eq. (5.46) then yield the usual

quantization rules for angular momentum [274, pp. 187–191]. Finally, we note that the commutation relations for *spin* angular momentum are usually derived by formal analogy with regular angular momentum (see, for example, Ref. [274, pp. 159]).

5.7.6 Concluding discussion: Do we need observables?

It was the goal of this section to demonstrate that the concept of observables can emerge naturally from the measurement-free axioms of quantum mechanics when the physical act of measurement is properly taken into account (i.e., if the measuring apparatus and its interaction with the system is explicitly included in a purely quantum-mechanical description of measurement). The collapse axioms (A5) and (A6) can be given a similarly natural motivation (at least in a relative-state sense, which of course constitutes the entire scope of this chapter) when the immersion of the system-apparatus combination into the environment is considered. In both cases, therefore, the realization of the *openness* of quantum systems—the pivotal insight of the entire decoherence program—is of crucial importance.

In this sense, observables remain an utterly useful tool for describing quantum measurement theory, as they encapsulate the phenomenological features of measurement without one having to explicitly consider the physical details of the measuring apparatus and its surroundings. They allow one to regard the measured quantum system as *effectively closed* by subsuming the consequences of its openness into axioms that precisely match what is observed. For instance, if one talks about “measuring operator-observable \hat{O} on \mathcal{S} ,” it is already implicitly assumed that one is in possession of an appropriate apparatus that is suitable to measure precisely this observable, i.e., that allows one to reliably find out about the eigenstates of this observable. Decoherence then provides the physical underpinnings of this assumption.

Also, commutation relations for observables allow for a very compact formulation of the connections between different measurements represented by these observables. We have shown how such commutation relations can be derived given the (probably

inevitable) assumption of identifying generators of transformations in classical physics with quantum-mechanical operators that correspond to the physical quantities represented by the generators.

Given the compactness of the observable-based description of quantum measurement, it is unlikely that the concept of observables and the related collapse postulate will soon be eradicated from introductory textbooks of quantum mechanics. However, it is important to realize that their axiomatic role in quantum mechanics may well be dispensable.

5.8 Decoherence in the perceptive and cognitive apparatus

If, motivated by the results of the experiments described in Sec. 5.2, we assume the universal validity of the Schrödinger equation, we immediately face two related consequences:

1. We ought to reconcile this assumption with our perception of definite states in the macroworld, since now there is no underlying stochastic mechanism (of whatever nature) that would select, in an objective manner, a particular “outcome” among the terms in a superposition of, say, spatially localized wave packets. There exists not only a multitude, but also interference effects between them.
2. If Schrödinger dynamics are universal, it is reasonable (at least from a scientifically reasonable functionalist’s standpoint) to also describe observers with their perceptive and cognitive apparatuses—including even what could be grouped together under the rather vague term of “consciousness” [312, 322, 290, 333]—by unitarily evolving wave functions.

Both consequences follow quite naturally from the assumption of universally exact [consequence (1)] and universally applicable [consequence (2)] Schrödinger dynamics. Quite generally, the preferred strategy would be to treat them jointly: Solving the

“measurement problem,” that is, consequence (1), posed by the assumption of a purely unitary quantum theory, by applying this very theory to the observer, i.e., consequence (2). If successful, this would lead to a “subjective” resolution of the measurement problem, i.e., to a quantum-mechanical account of why we, as observers, perceive *definite* states in *specific* bases, rather than superpositions of these states. In the opinion of this author (see Chap. 2) and of others (see, e.g., Refs. [327, 341, 96, 333, 344]), this would also represent a sufficient solution to the problem.

5.8.1 General remarks

First of all, on a rather philosophical sidenote, it is clear that the familiar concepts of the world of our experience are expressed in terms of the observed specific definite states. We do not even *have* a concept available for what a state describing a superposition of an alive and dead cat would represent, because we have never observed such a state. While such a Schrödinger cat might seem exotic, we have seen that quite analogous states are realizable in the laboratory — for example, in terms of superpositions of currents running in opposite directions in SQUIDs. As we have argued in Sec. 5.2.2, the only way we can access such superpositions in terms of our concepts (and not just in mathematical terms) is through the definite current states $|L\rangle$ and $|R\rangle$ that are observable as individual preferred states of the system upon measurement.

Furthermore, it is virtually indisputable that we must describe all observations in terms of physical interactions between the observed system and the observer, i.e., by means of an appropriate interaction Hamiltonian \hat{H}_{int} . Such interactions do not have to be, and usually are not, direct. For example, the probably most common type of observation involves the interception of a number of photons that have interacted with the object of interest in the past and whose state is thus entangled with the state of the object. These photons then contain indirect and redundantly coded information about the object that can be revealed without significantly disturbing the state of the

object (see Sec. 5.6).

If the perception of definiteness is not introduced as an extraneous postulate, but is rather understood as emerging from the unitary quantum formalism itself when observations and observers are described in physical terms, it is inevitable that attempts have to be made to analyze the cognitive apparatus in quantum-mechanical terms. It is clear that giving such an account of subjective definiteness by referring to the physical structure of observers cannot share the mathematical compactness and exactness of axiomatically introduced rules that enforce definiteness on a fundamental level of the theory. However, it is important to note that, given the paramount role of observations in quantum mechanics (mostly owing to the fact that, in general, states do not pre-exist in a classical sense), postulating such “exact” rules is tantamount to simply avoiding a physical analysis of crucial and objective (that is, interpretation-neutral) physical processes (cf. Kent’s objections to “many-worlds” interpretations [200] and Wallace’s defense [314]).

If a purely unitary time evolution is assumed and observations are modeled as physical interactions, the conclusion of the existence of quantum-mechanical superpositions of brain states corresponding to the different “outcomes” of observations is inescapable. Individual perceptions are represented by certain neuronal resting/firing patterns in the brain (see Refs. [112, 114] for more precise definitions of this relationship). As we shall discuss in the next section, superpositions of resting and firing states of a neuron are extremely sensitive to environmental decoherence, with the resting and firing states forming the robust neuronal states. These states can thus be identified with “record states” that are capable of robustly encoding information in spite of environmental interactions [341, 346]. As a consequence of the practically irreversible dislocalization of phase relations between these record states through entanglement with the environment, a dynamical decoupling of these states results. This process represents an objective branching process due to physical interactions between subsystems and with the environment.

The remaining question is then how to relate this objective branching to the perceived subjective “branches of consciousness,” i.e., collective memory states, or “minds” (von Neumann’s principle of the “psycho-physical parallelism” [312]). Of course, the existence (and therefore the locality) of consciousness cannot actually be *derived* from the quantum-mechanical formalism. This has led some authors to conclude that the question of the relationship between subjective experience and its physical correlates can only be fully answered through the introduction of new physical laws [114]. However, in the opinion of this and other authors (see, for example, Refs. [333, 336]), it is an entirely viable (if not compelling) strategy to postulate, within the formalism, the existence of consciousness based on the empirical fact of decohering wavefunction components in neuronal processes, by associating the robust components of the global wave function labelled by the decohered neuronal states with dynamically autonomous observers [326, 327, 217, 341, 333, 344, 336, 346].

Due to the absence of more concrete theoretical and experimental insight into the physical underpinnings of the cognitive apparatus with its associated complex entities such as the “mind,” “consciousness,” and even the comparably basic “record states,” the above brief account of how subjective definiteness may emerge from purely unitary quantum mechanics must (at least for now) remain inherently somewhat vague and nontechnical. Fortunately, however, the main points of the argument are quite independent of, say, the precise details of the structures and dynamics of the information-processing cognitive entities, since the ubiquity and effectiveness of decoherence is likely to lead to very robust results. We shall therefore turn, in the next section, to concrete estimates for decoherence rates in neurons.

5.8.2 Decoherence of neuronal superpositions

The extremely complex network of about 10^{11} interacting neurons in the brain undoubtedly comprises a major part of the cognitive machinery used for processing and storing of information obtained from sensory input. Computer models of such neu-

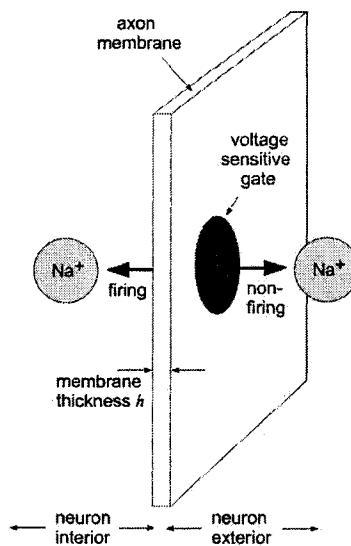


Figure 5.11: Schematic illustration of the axon membrane of a neuron. The firing of the neuron corresponds to a net flow of $N \sim 10^6$ sodium ions into the inside of the axon. Superpositions of firing and non-firing neuronal states (i.e., of N ions being in a spatial superposition of inside and outside the membrane) are decohered on a time scale of about 10^{-20} s [296].

ronal networks (employing a massively parallel interconnected web of “switches” that are turned on and off depending on some, typically nonlinear, activation function) can exhibit rich and complex behavior similar to that encountered in cognitive processes.³ In particular, it is reasonable to identify the “record states” mentioned above with individual neurons or neuronal clusters. One might conjecture that ultimately all cognitive processes (and thus presumably also our perception of consciousness) are due to neuronal activity.

³However, as Donald [114] has pointed out, the brain should not be thought of as a deterministic classical computer with a predictable input/output pattern, since synaptic transmissions have a fairly high failure rate due to the complexity of the underlying biological processes. The large number of about 10^{14} synapses in the human brain, with each neuron firing in average several times per second, inevitably leads to a high degree of unpredictability on the “everyday level” that is much more significant than effects due to pure quantum uncertainties.

Thus the importance of a quantitative investigation of decoherence in neuronal states should be clear. Tegmark [296] has estimated decoherence rates for a superposition of a firing and non-firing neuron in the brain. The firing is represented by a large number $N \sim 10^6$ [296] of Na^+ ions moving across the membrane into the inside of the axon (Fig. 5.11). Thus, a superposition of a firing and nonfiring neuron corresponds to a spatial superposition involving $O(N)$ Na^+ ions.

The extensive difference S_{ext} (see Sec. 5.2.1) can then be estimated to be on the order of 10^2 – 10^3 , given by a small multiple of the thickness $h \sim 10$ nm of the axon membrane separating the inside and outside regions, relative to the size of a Na^+ ion, which is on the order of 0.1 nm. While this value for S_{ext} is comparably small, the degree of entanglement S_{ent} is somewhat closer to the values listed in Table 5.1. Taking it to be equal to the number of microscopic constituents, we obtain $S_{\text{ent}} \sim 3 \times 10^7$. Thus a neuron being in a superposition of firing and resting quite clearly falls into the macroscopic category.

The decoherence rates for this superposition as estimated by Tegmark are, as expected, extremely fast. The three main sources of decoherence in this case, namely, ion-ion scattering, ion-water collisions, and long-range Coulomb interactions due to nearby ions, all result in decoherence times on the order of 10^{-20} s.

One obvious implication of fast neuronal decoherence is that coherent superpositions in neurons could never be sustained long enough to allow for some form of quantum computation. This result appears to be much more clearly established than an answer to the question of whether the relevant decoherence times are long enough to allow for quantum computation in microtubules (dynamically active structures that are a dominant part of the cytoskeleton, i.e., the internal scaffolding of cells). Suggestions in the positive, including the association of such quantum computations with the emergence of consciousness, have been put forward in Refs. [253, 177, 176], criticized in Ref. [296], subsequently defended in Ref. [169], and further evaluated in Ref. [266] (see also Ref. [291]).

However, the question much more relevant to the theme of this paper concerns the implications of neuronal decoherence for a decoherence-based account of subjective definiteness in unitary quantum mechanics — i.e., for a subjective resolution of the “measurement problem.” To this extent, let us in the following discuss a simple step-by-step quantum-mechanical account of the chain of interactions leading to the recording of a visual event in the brain.

5.8.3 *Schematic sketch of the chain of interactions in visual perception and cognition*

Suppose that a small number of photons interact with an object \mathcal{O} described by a pure-state superposition of two macroscopically distinct positions. This step already can be viewed as an environmental decoherence process, where now, however, the environment assumes a crucial role as a carrier of information (see Sec. 5.6). Due to entanglement, the combined object-photon system will be described by a superposition of the form

$$|\Psi_{\mathcal{OP}}\rangle = \frac{1}{\sqrt{2}}(|\omega_1\rangle_{\mathcal{O}}|\phi_1\rangle_{\mathcal{P}} + |\omega_2\rangle_{\mathcal{O}}|\phi_2\rangle_{\mathcal{P}}), \quad (5.47)$$

where ω_i , $i = 1, 2$, are the two distinct (small) spatial regions associated with the object, and $|\phi_i\rangle_{\mathcal{P}}$ denote the corresponding classically distinct collective photon states. A conceptually similar arrangement on the mesoscopic scale has explicitly been studied in experiments involving a single rubidium atom (representing the object) in a superposition of two internal levels and entangled with a cavity radiation mode (corresponding to the collection of photons) [64, 263].

Initial detection of such a collection of photons in the (human) eye is associated with rhodopsin molecules in the retina. Due to their mesoscopic properties, rhodopsin molecules are subject to strong decoherence, such that already at this stage the influence of the environment will have preselected the robust states $|\rho_i\rangle_{\mathcal{R}}$ of the rhodopsin molecule, which correspond to certain photon detection events $|\phi_i\rangle_{\mathcal{P}}$. The total state

$|\Psi_{\mathcal{OPR}}\rangle$ will then be given by

$$|\Psi_{\mathcal{OPR}}\rangle = \frac{1}{\sqrt{2}}(|\omega_1\rangle_{\mathcal{O}}|\phi_1\rangle_{\mathcal{P}}|\rho_1\rangle_{\mathcal{R}} + |\omega_2\rangle_{\mathcal{O}}|\phi_2\rangle_{\mathcal{P}}|\rho_2\rangle_{\mathcal{R}}), \quad (5.48)$$

i.e., the photon-rhodopsin interaction should lead to an (albeit, due to the influence of decoherence, very fragile) superposition of the different biochemically distinct states $|\rho_i\rangle_{\mathcal{R}}$ of the rhodopsin molecule.⁴ These relative states can then be expected to be further correlated with the appropriate states $|\nu_i\rangle_{\mathcal{N}}$ of neuronal arrays that are mainly located in the primary visual area in the occipital lobe of the brain. Suppose that the two “events” represented by the two distinct states $|\rho_i\rangle_{\mathcal{R}}$ of the rhodopsin molecule (corresponding to the different photon states $|\phi_i\rangle_{\mathcal{P}}$ that in turn carry information about the two distinct spatial regions ω_i of the object) are encoded by the states $|\nu_i\rangle_{\mathcal{N}}$, $i = 1, 2$, describing the same collection of N neurons in two different firing/resting patterns.

As a simple example, let us take $N = 3$, and $|\nu_1\rangle_{\mathcal{N}} = |1\rangle_{\mathcal{N}_1}|0\rangle_{\mathcal{N}_2}|1\rangle_{\mathcal{N}_3}$ and $|\nu_2\rangle_{\mathcal{N}} = |0\rangle_{\mathcal{N}_1}|1\rangle_{\mathcal{N}_2}|1\rangle_{\mathcal{N}_3}$, where $|0\rangle_{\mathcal{N}_i}$ and $|1\rangle_{\mathcal{N}_i}$ denote, respectively, the resting and firing state of the i th neuron. Then the combined state $|\Psi_{\mathcal{OPRN}}\rangle$ will be given by

$$|\Psi_{\mathcal{OPRN}}\rangle = \frac{1}{\sqrt{2}}(|\omega_1\rangle_{\mathcal{O}}|\phi_1\rangle_{\mathcal{P}}|\rho_1\rangle_{\mathcal{R}}|1\rangle_{\mathcal{N}_1}|0\rangle_{\mathcal{N}_2}|1\rangle_{\mathcal{N}_3} + |\omega_2\rangle_{\mathcal{O}}|\phi_2\rangle_{\mathcal{P}}|\rho_2\rangle_{\mathcal{R}} \\ \times |0\rangle_{\mathcal{N}_1}|1\rangle_{\mathcal{N}_2}|1\rangle_{\mathcal{N}_3}). \quad (5.49)$$

The extreme fast decoherence rate for the neurons 1 and 2 being in a superposition of firing and resting will lead to a practically irreversible dynamical decoupling of the two branches that now describe two distinct “outcomes” encoded by $|\nu_i\rangle_{\mathcal{N}}$. We may then identify these states with the basic memory states, although, strictly speaking, the physical process of actual information storage in the brain (i.e., learning) occurs only in two subsequent stages [199]. First, in form of short-term memory, believed to

⁴A search for experimental evidence for such superpositions has been suggested in Ref. [284]; for an experimental proposal, see Ref. [183]. Cf. also Ref. [298] for an (unconvincing) suggestion that the visual apparatus itself might trigger a physical collapse.

be due to certain biochemical and electrical interactions between neurons. Second, as long-term memory that is based on actual structural changes in the brain (“neuroplasticity”), most notably, due to the formation of new connections (synapses) between neurons and due to internal changes in the synaptic regions in individual neurons.

However, since all these processes will again be subject to strong decoherence, the essence of our argument is not altered: The states in a superposition of neuronal firing patterns will rapidly entangle with approximately orthogonal (i.e., macroscopically distinguishable) states of the environment and thus lead to the formation of locally noninterfering (that is, dynamically autonomous) branches labelled by these “outcome” states. Regardless of the precise physical, chemical, biological, psychological, etc., details of perceptive and cognitive activity, it is quite clear that decoherence effects are likely to be sufficient to explain the emergence of a subjective perception of single outcomes, represented by stable, “classical,” record states, from a (by all accounts macroscopic) global superposition.

5.9 Discussion and outlook

We have analyzed three important experimental domains—namely, SQUIDs, molecular diffraction, and Bose-Einstein condensation—that have demonstrated (or at least have come very close to demonstrating) the existence of superpositions of states that can be considered macroscopically distinct in comparison with the microscopic states “typically” treated in quantum mechanics. These experiments have provided powerful examples for the validity of unitary Schrödinger dynamics and the superposition principle on increasingly large length scales. They have also shown how the fragility of macroscopic superpositions can be precisely understood and controlled in terms of environmental interactions and the resulting decoherence effects.

Of course, these experiments do not *falsify* the possibility that the Schrödinger equation might not be exact under all circumstances. In fact, no finite number of experiments that show the validity of unitary dynamics could ever do. To do so, a

“positive-test” experiment would be needed that could explicitly demonstrate non-linear deviations from the Schrödinger equation. Leggett [214, 215] has presented a Bell-type inequality that would be obeyed by what he calls the class of “macrorealistic theories,” while it would be violated by the predictions of purely unitary quantum mechanics. The “macrorealistic” class is defined to represent all theories in which macroscopic systems are always in a single definite state among a collection of possible macroscopically distinct states, and in which this definite state can be found out without perturbing the state and dynamics of the system. So one might, at least in principle, through suitable experiments be able to exclude either any such macrorealistic theory or the universal validity of the Schrödinger equation. Such a strategy would be similar in spirit to the tests of Bell’s inequalities, which rule out a large class of, if not all, local realistic theories. (See Sec. 6 of Ref. [215] and references therein for some first ideas in this direction.)

At the current stage, however, it is the opinion of the present author that, in absence of any positive evidence for deviations from unitary dynamics, combined with the continued experimental verification of increasingly large “Schrödinger cats” (whose time evolution, including decoherence effects, is in perfect agreement with unitary dynamics), it appears to be not only reasonable, but moreover compelling, to entertain the possibility of a universally exact Schrödinger equation seriously and to fully explore the consequences of this assumption.

The experiments described in this paper have demonstrated rapid progress in achieving, controlling, and observing superpositions of increasingly distinct states. Experiments involving superpositions of classically distinguishable states of a few photons [64, 263] have been followed by collective superpositions of 10^9 electron pairs in SQUIDs and double slit-type experiments using massive molecules with a large number of degrees of freedom. It is only a matter of time until number-difference superpositions involving on the order of 10^7 rubidium atoms will be experimentally realized in BECs. It is rather unlikely that this progress towards experimental evi-

dence for increasingly large superpositions will encounter any fundamental boundaries in the near future. As we have seen, the main limit seems to be given by the ability to shield the system sufficiently from the decohering influence of the environment. This limit is open to precise quantitative analysis.

In view of this situation, we may now legitimately ask what the next steps in solidifying the empirical support for a purely unitary quantum theory and its consequences might ideally look like. To this extent, we remark that superpositions of macroscopically distinct states that refer to biological (and, even more so, animate) objects seem to have been considered as particular “paradoxical” — after all, Schrödinger chose a cat to illustrate his famous *Gedanken* experiment. This attitude may be traced back to several reasons. For example:

1. The “distinctness” between the states referring to biological objects is usually extremely complex. Not only is the number of physical, chemical, biological, etc., differences between a dead and alive cat overwhelmingly vast, even two functionally different states of a simple biological molecule will be distinct in a large number of features. By contrast, in the examples involving inanimate objects, such as BECs and SQUIDs, the states in the superposition usually differ only in a single physical quantity, such as total angular momentum or magnetic moment.
2. While we might be willing to accept the existence of an “exotic” superposition under extreme physical conditions (such as superconducting currents in a bulk of matter cooled down to temperatures close to absolute zero), biological objects reside in the parameter regime characteristic of the world of our everyday experience.
3. If the superposition principle is applied to human observers (specifically, superpositions of “states of consciousness,” etc.), we feel that our most basic intuition

about possessing a unique identity has been infringed upon.

Especially in light of the first two arguments, the molecular diffraction experiments appear to be the most “natural” realization of superpositions of macroscopically distinct states. In fact, as pointed out in Sec. 5.2.3, interference effects have already been experimentally demonstrated for a biological molecule [168], and larger biological structures are likely to follow [22, 166] (see also Fig. 5.9).

However, another interesting direction could also be taken from here. As suggested for example in Refs. [284, 215], one might try to look for interference effects between (and thus superpositions of) *biologically* distinct states of the same biomolecule, rather than for the spatial superpositions demonstrated in the current molecular-diffraction setups. While such experiments would be considerably more difficult to carry out due to the required near-*in vivo* environmental conditions (room temperatures, presence of a surrounding medium such as an aqueous solution, etc.), which would lead to very strong decoherence effects, there does not seem to exist a fundamental obstacle that would prevent one in principle from the realization of such superpositions in a cleverly designed setup.

Experiments that would find some basic biological structure in a superpositions of distinct states corresponding to different biological “inputs” might in turn indicate the presence of a superposition of input signals originating from the inanimate outside world (e.g., a superposition of photon states entangled with spatially distinct states of a single object — see Sec. 5.8.3). They could also provide direct empirical evidence for consequences of purely unitary quantum mechanics in the regime of more complex structures that are part of conscious (human) observers, and might therefore also ease the discomfort spelled out in item (3) above.

Given that experiments [168] have demonstrated a splitting of the localized state of a biomolecule into “branches”, corresponding to distinct paths, it would also be worth discussing, as Zeh [333] puts it,

the consequences of similar *Gedanken* experiments with objects carrying some primitive form of “core consciousness” — including an elementary awareness of their path through the slits.

In such a situation, after passage through the slits, the state of the object would be described by a superposition of spatially distinct trajectories. However, due to its awareness of the path, it would thus also be in a superposition of multiple (local) “states of consciousness.” Environmental scattering would then lead to entanglement with path-encoding variables (decoherence), which hence would also destroy interference effects between the “branches of consciousness,” and thus the different paths would be “experienced” separately. In the absence of decoherence, it would be possible to coherently recombine the branches into a single localized wavepacket identical to the state before the passage through the slits. It then follows from the standard quantum-mechanical formalism that the associated object then cannot have retained any “memory” of the path taken before the recombination. For related ideas using the example of neutron interferometry, see Ref. [302].

As it is well known, Bohr has repeatedly insisted on the fundamental role of classical concepts (see, for example, Refs. [56, 57]). The experimental evidence for superpositions of macroscopically distinct states on increasingly large length scales counters such a dictum. Superpositions appear to be novel and individually existing states, often without any classical counterparts. Only the physical interactions between systems then determine a particular decomposition into classical states from the view of each particular system. Thus classical concepts are to be understood as locally emergent in a relative-state sense and should no longer claim a fundamental role in the physical theory.

We have already widely acknowledged, based on experimental evidence, the fundamental nonlocality of the quantum world, in spite of the utterly nonclassical implications. We also have obtained direct evidence for the validity of unitary dynamics

and the superposition principle in all experiments conducted so far, although this has forced us to again accept extremely nonclassical situations as physical reality. Why not let these experiences guide us to extend our willingness to entirely give up classical prejudice and instead explore the consequences of a strictly unitary quantum theory embedded into a minimal interpretive framework? After all, exploring the implications of pure quantum features to the largest possible extent can in turn give us back the familiar “classical” notions of the world of our experience. As we have discussed in this paper, consequences of highly nonlocal quantum entanglement lead to the local disappearance of quantum interference effects, may explain the origin of probabilities in quantum mechanics, and are likely capable of accounting for the objectification of observables and therefore the emergence of effective classical reality, thus supplying the missing pieces of the basic Everett theory that have frequently been used to challenge the viability of a relative state-type “minimal interpretation.”

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Chapter 6

SUMMARY AND CONCLUDING DISCUSSION

In this chapter, we will first summarize our results and conclusions from each of the four main chapters of this dissertation (Sec. 6.1). We shall then discuss the larger picture emerging from the “building blocks” of the quantum-to-classical transition as outlined in the previous chapters, and state some general principles and ideas that might guide one in evaluating (and forming) interpretations of quantum mechanics, especially with respect to the different possible stances regarding the meaning of quantum states (Sec. 6.2).

6.1 Summary

We have presented an extensive discussion of the quantum-to-classical transition based on pure quantum features. Each of the four chapters of this dissertation has added a distinct piece to answering the main underlying question, commonly known as the “measurement problem”: How can the perception of a classical world, characterized by a small set of definite and robust physical quantities that appear as stochastic events in measurements, be explained by a quantum formalism that, in principle, should allow for arbitrary, typically flagrantly nonclassical superpositions that are assumed to evolve strictly deterministically in time?

We have identified quantum decoherence, i.e., the dislocalization of local phase relations due to interaction with the environment, as the main ingredient in solutions to the measurement problem. The key contents and results of our study are as follows.

In Chap. 2, we have clarified the main features of the decoherence program. We have shown that, since decoherence is simply a consequence of the application of the

standard quantum formalism of entanglement, and since it is therefore interpretation-neutral, it is not capable of directly solving the measurement problem, if the existence of the measurement problem is acknowledged in the first place. However, decoherence can provide an indispensable basis for a resolution of the problem in interpretive contexts that go beyond the pragmatic view of measurements in the standard interpretation and that instead describe observations and observers in quantum-mechanical terms. By discussing the role of decoherence in a range of interpretations of quantum mechanics, we have demonstrated that decoherence also has far-reaching implications for any existing or future interpretation, by ensuring, or disproving, that its predictions are in agreement with those of decoherence theory.

In Chap. 3, we have critically analyzed an approach that appears to constitute a different, yet similar, viewpoint on quantum decoherence and on the emergence of the classical limit in quantum mechanics. This “self-induced decoherence” approach proposes to identify the collective dynamical suppression of off-diagonal terms in the expression for the expectation value, expanded in the eigenbasis of the total Hamiltonian, with a decoherence effect. If correct, this method would allow for a description of decoherence in closed systems, thereby overcoming the much-discussed requirement for a state-space separation into the “system of interest” and the “observationally irrelevant environment.” We have shown that, however, the self-induced approach does not describe a damping of individual interference terms and simply presupposes the existence of ensembles of measurement outcomes. Therefore it cannot describe a physically relevant decoherence process in individual measurements. In addition, we have demonstrated, through analytical and numerical studies of an explicit spin-bath model, that the “self-induced decoherence” approach will fail to work as suggested in a large class of rather general and realistic model systems. Our analysis has aimed at identifying inappropriate associations of certain formal structures with decoherence, and therefore at clearly delineating the boundary between “true” and “seeming” decoherence.

In Chap. 4, we have slightly departed from the narrow context of decoherence and instead studied the more general feature of quantum entanglement, by means of critically analyzing a proposal for the derivation of quantum probabilities and the Born rule. This proposal relies on properties of entangled states under a specific class of transformations. We have identified several hidden key assumptions on which the derivation is based. We have argued that, if these assumptions are acknowledged, the derivation constitutes a powerful and very general method to understand the emergence of a probability concept from the pure quantum formalism. It might also prove important for a deeper understanding of the role of quantum symmetries in the quantum-to-classical transition, and for the derivation and interpretation of probabilities in non-collapse relative-state quantum mechanics.

In Chap. 5, we have focused on analyzing recent experiments that have allowed, or are very close to allowing, for the creation of superpositions of macroscopically distinguishable states (so-called “Schrödinger cat states”). In particular, we have discussed the examples of superpositions of (i) macroscopic currents running in opposite directions in superconducting quantum interference devices, (ii) matter-wave diffraction experiments with mesoscopic fullerene molecules, and (iii) (albeit not yet achieved) superpositions of number-difference Bose-Einstein condensates. We have demonstrated that these experiments represent realizations of large-scale quantum superpositions that are observed as novel, individually existing states. Another key insight gained from the analysis of these experiments is that the fragility of the macroscopic superpositions can be understood by taking into account environmental decoherence effects. Thus the experiments provide powerful support for the validity of unitary time evolution and the superposition principle on increasingly large scales, and any apparent breakdown of this validity can be attributed to insufficient shielding from the environment. We have also explored how exploring quantum entanglement can motivate explanations for the nature and the origin of probabilities in quantum mechanics (see also Chap. 4), and how observables are “objectified,” i.e., become effectively

classical such that they can be measured by many independent observers without disturbing the state of the system. Furthermore, we have shown how the concept of observables can be understood as emerging from the structure of the Hamiltonians describing the interaction between the measured system and the apparatus, and how the collapse postulate can be motivated, in a relative-state framework, by considering consequences of decoherence and environment-induced selection. Finally, we have investigated how decoherence effects in neuronal structures in the cognitive apparatus may lead to the emergence of noninterfering, dynamically autonomous observer branches that could yield a subjective solution to the measurement problem.

6.2 Concluding Discussion: Does quantum theory need an interpretation?

In prequantum physical theories, the need for an “interpretation” of the theory hardly ever arose, owing mostly to the possibility of a straightforward identification of the mathematical symbols used in the formal body of the theory with the entities of the physical world. To be sure, certain cases, for example, the correspondence between the formal definition of temperature in the theory of thermodynamics and the underlying molecular processes leading to the physical notion of temperature, had indicated that this one-to-one identification could not always be established as easily as in, for example, Newtonian mechanics. However, reference to other physical quantities and phenomena (e.g., by relating a collective thermodynamic variable to the behavior of the microscopic constituents in a bulk of matter) rarely left any doubt about the possibility of resolving this identification problem at least at some level.

In contrast, all that the formalism of quantum theory offers are abstract quantum states and, in textbook quantum mechanics, similarly abstract operators whose action on the quantum states is identified as measurements of the familiar physical quantities (“operators as observables”). The danger of falling into a naive realism about such operators by describing this situation in an oversimplified manner as “measurements

of operators” has been pointed out before [91]. In the absence of a direct and clear correspondence between the abstract quantum states and the physical quantities of our experience, the main interpretive question is therefore concerned with the status (i.e., the meaning) of quantum states.

Curiously, some authors have argued that “quantum theory needs no interpretation” (see, for example, Ref. [138]). But this assertion appears to be circular, as the corresponding arguments implicitly assume some interpretation of the role and ontological status of quantum states. For example, the claim by Fuchs and Peres [138] is based on the view that quantum states are only formal tools to compute the probability distribution of measurements, and do therefore not directly represent physical reality (see also Ballentine’s “ensemble interpretation” [30, 31] and its discussion in Sec. 5.4). In contrast, the “minimal interpretation” of quantum mechanics sketched in Chap. 5 implicitly assumes that quantum states directly represent physical reality on the most fundamental level of the theory. In both cases then, the resulting view of quantum theory is not, and cannot be, completely interpretation-free.

While we have thus argued that one must make some *a priori* assumption about the connection (or the absence of a connection) between the elements of the quantum formalism and those of the physical world, it would be reasonable to let oneself be guided by certain principles in establishing this connection. Of course, the fundamental differences among authors as to what these guiding principles should be have resulted in the large body of “interpretations of quantum mechanics.” Given the important contribution of the interpretation-neutral decoherence program to the description of the quantum-to-classical transition (Chap. 2)—in perfect agreement with experimental observations (Chap. 5)—and the growing number of interference experiments on mesoscopic and macroscopic scales (Chap. 5), we will restrict our main focus to two basic, interrelated principles (for a more detailed list of possible principles, see, for example, Ref. [226]):

1. *Simplicity.* Natural science has always attempted to reduce the great complexity of Nature to a few simple underlying laws, and it has been utterly successful in this endeavor. If an interpretation of quantum mechanics introduces additional rules into the theory whose effect can be shown to emerge directly from the bare quantum formalism, we are given good reason to discard this particular interpretive rule. For example, we have argued (see Sec. 2.4.4) that some types of modal interpretation postulate rules for “property selection,” when the particular selection can be shown to emerge naturally from taking into account decoherence effects. As an additional advantage, derivation of such rules from the formalism itself automatically insures empirical consistency (again, this is an issue sometimes encountered in modal interpretations, see Sec. 2.4.4).
2. *No classical prejudice.* The experiments on mesoscopic and macroscopic superpositions have demonstrated that behavior considered “quantum” can appear on scales typically associated with the realm of classical physics. At the same time, the vanishing of such quantum effects can be described as a consequence of the quantum feature of (environmental) entanglement.

Both principles support the strategy of deriving classicality (i.e., the relationship between quantum states and the classically-appearing entities of our experience) from the quantum formalism itself, instead of trying to tame quantum theory by enforcing classicality through the introduction of extraneous postulates.

In this spirit, we have shown in great detail how the decoherence program, based on a study of the consequences of quantum entanglement and on the assumption of the reality of the wave function, can account for many (if not all) features of classicality. In particular, we have discussed explanations for the local disappearance of interference effects, for the observation of only certain states in the Hilbert space, for the temporal robustness and effective classicality of these states, and for the emergence of quantum probabilities. It is most curious to realize how these features were previously thought

to be in conflict with (rather than be derivable from) the quantum principles of superposition and unitary time evolution, which underlie quantum entanglement.

While nothing *forces* one to adopt the view that quantum states correspond to physical reality, we have argued that decoherence has shown that there are no *empirical obstacles* to upholding such a view. We have also reviewed recent experiments in which quantum superpositions are observed as *individually existing* (physical) states. Therefore, it appears that much insight can be gained by regarding any quantum state as a complete representation of individually existing physical states.

If the universal validity of the Schrödinger equation is assumed, one of the main consequences of such a view appears to be the “existence” of multiple distinct branches present in the global wavefunction. Even though these branches can be shown to be dynamically autonomous for all practical purposes due to decoherence, many authors have felt an (understandable) unease with such a view, as it seems to imply the simultaneous “existence” of a multitude of classical worlds in a single, nonlocal quantum Universe.

In view of this situation, and given a “realistic” interpretation of the state vector, there are at least two main possible interpretations of quantum mechanics. Either, one assumes that the Schrödinger equation is universally valid, which leads to a highly entangled global state vector. Since all branches, besides the one the observer is attached to, are effectively unobservable, one might simply regard their existence as a “heuristic fiction” [333]. Or, one assumes a breakdown of the Schrödinger time evolution on some scale by postulating a new stochastic mechanism. This mechanism would then select one branch and eliminate all other branches, instead of suppressing the latter branches for all practical purposes “only” (cf. Sec. 2.2.2) from the standpoint of (intrinsically local) observers. In this way, we might gain some peace of mind by eradicating all those nebulous other branches, but it can be well argued to what extent such an approach could generate any testable predictions, as decoherence is likely to overshadow any “true” collapse effects. Nonetheless, in Sec. 5.9 we have

hinted at some strategies that could potentially subject classes of collapse models to an experimental test.

In conclusion, the results of our studies suggest that the possibility of universally valid and applicable unitary dynamics should be entertained seriously. The experimental observation of superpositions of macroscopically distinguishable states lend support to this strategy, and theoretical studies on decoherence and entanglement provide strong indications for the ability of such an approach to lead to an empirically adequate quantum theory. Such a theory can be based on a minimal body of interpretive additives, if the whole chain of interactions from the observed system to the registration in the observer is treated in purely quantum-mechanical terms, and if the fundamental role of quantum entanglement is properly recognized.

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