Perception II: Stereo Triangulation

Stereo Tiangulation

(Note that all the answers should be scalar values with two decimal points.)

Consider a camera with the specifications as below:

- Field of view along the horizontal axis (X): 60°
- Field of view along the vertical axis (Y): 45°
- Image plane size: 640 pixels along X axis and 480 pixels along Y axis

Problem 1

Given the information above:

- Calculate the focal length (in pixels) along X axis.
- Calculate the focal length (in pixels) along Y axis.
- And construct the camera intrinsic matrix (K)

Answer

Let α_x and α_y are focal lengths in pixel.

$$\alpha_x \cdot \tan 30^\circ = 640/2 \quad \text{px} \tag{1}$$
$$\alpha_y \cdot \tan 22.5^\circ = 480/2 \quad \text{px} \tag{2}$$

Thus, $\alpha_x = 554.2\text{px}$, $\alpha_y = 579.4\text{px}$

Camera intrinsic matrix is defined as follows:

$$K = \begin{bmatrix} \alpha_x & 0 & u_0 \\ 0 & \alpha_y & v_0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

Assuming u_0 and v_0 are exactly on the middle of image plane,

$$K = \begin{bmatrix} 554.26 & 0 & 320 \\ 0 & 579.41 & 240 \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

Problem 2

The image point p has pixel coordinates (360, 302). If we translate the camera by the vector $T = [0.2, 0.0, 0.0]^T$ meters and take another picture, the point appears at image position (330, 302). Assuming that the first camera frame acts as the world reference frame, estimate the position of point P in 3D coordinates.

Answer

By perspective projection equation:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = K \underbrace{\begin{bmatrix} R_{CW} & {}_C T_{CW} \end{bmatrix}}_{\text{Extrinsic matrix}} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} \tag{5}$$

Where R_{CW} is a rotational matrix of world frame with respect to camera frame and ${}_C T_{cw}$ is a vector from origin of camera frame c to origin of word frame w with respect to camera frame.

For initial pose:

$$\begin{bmatrix} R_{CW} & {}_C T_{CW} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{6}$$

After the translation:

$$\begin{bmatrix} R_{CW} & {}_C T_{CW} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -0.2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{7}$$

Note that the sign of T becomes negative since ${}_C T_{cw} = -R_{CW} \cdot {}_W T_{wc}$ where ${}_W T_{wc} = T$ and $R_{CW} = \mathbb{1}$ in this case.

By solving following equation :

$$\lambda \begin{bmatrix} 360 \\ 302 \\ 1 \end{bmatrix} = \begin{bmatrix} 554.26 & 0 & 320 \\ 0 & 579.41 & 240 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} \tag{8}$$

$$\lambda \begin{bmatrix} 330 \\ 302 \\ 1 \end{bmatrix} = \begin{bmatrix} 554.26 & 0 & 320 \\ 0 & 579.41 & 240 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -0.2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} \tag{9}$$

We get

$$[X_w, Y_w, Z_w]^T = [0.266, 0.395, 3.69]^T \tag{10}$$