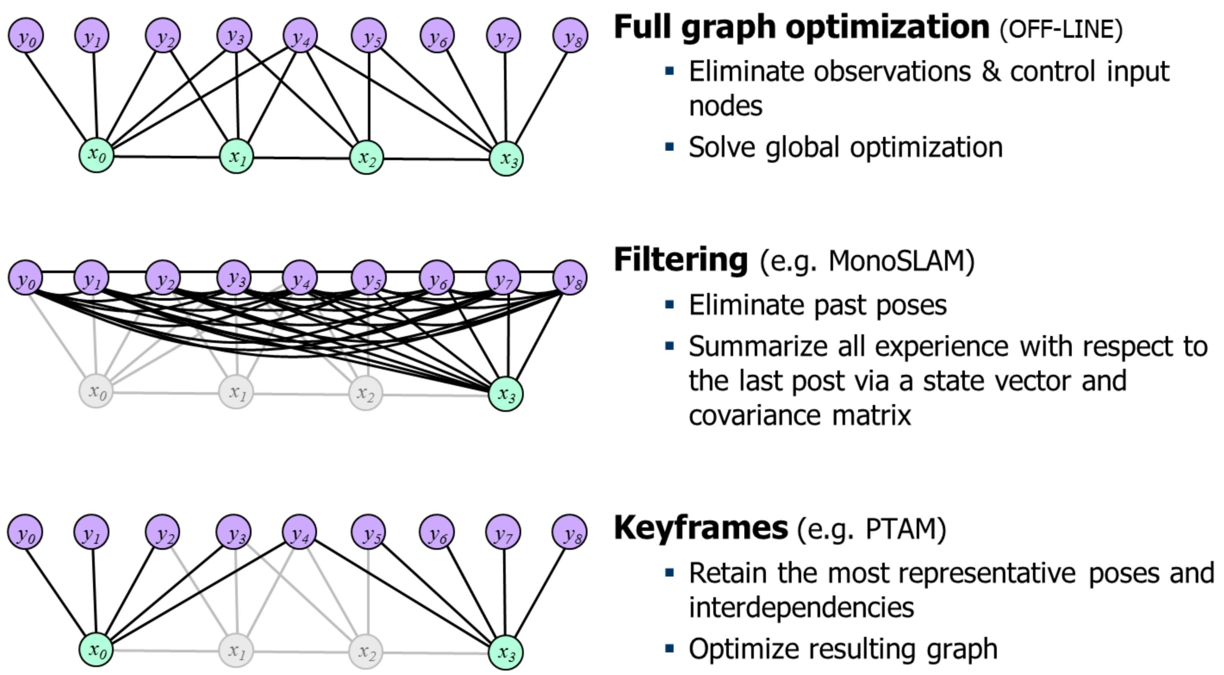
SLAM

The structure of the Covariance Matrix

5/5 points (graded)

In the SLAM lectures we have discussed how the full slam problem is approximated via the keyframes of filtering approaches to enable online estimation, as depicted in the figure below.



SLAM approaches: Approximating the full SLAM problem for online estimation. x_i's denote the robot poses and y_i's denote the landmarks in the scene.

Problem 1. Suppose we have 8 landmarks in the SLAM map, as is the case in the figure above. If we would choose the keyframes approach, no links would be introduced among the landmarks. If we would follow the filtering approach, how many new links *between the landmarks* would have to be introduced? Note: do not consider any new links introduced between the current pose and the landmarks. Enter your answer below:

28

28

Problem 2. If instead the number of landmarks in the SLAM map is now 200, how many new links *between the landmarks* would have to be introduced?:

19900

19900

Problem 3. Consider now the case, where we are conducting SLAM using the Extended Kalman Filter using a wheeled robot that moves in a plane measuring point features on this plane. So the state of this robot is characterized by its location (x,y) and its direction (theta), while the state of each point feature (landmark) is characterized by its estimated loation (in x and y). If we have constructed a SLAM map with 200 features as in Problem 2 above,
how many elements does the EKF covariance matrix contain?:

162409

162409

Problem 4. A covariance matrix is actually symmetric (and positive semi-definite). As a result, there is a lot of repetition in the elements stored, with the unique elements contained in the upper (or lower) triangular space of this matrix (i.e. containing also the elements along the diagonal). Considering the links introduced in the graphical representation of the filtering approach, how many elements does one link *between two point features*(each characterized in x and y) correspond to in
the unique elements of the covariance matrix?:

4

4

Problem 5. How many elements does one link *between a point feature and the robot state* correspond to in
the unique elements of the covariance matrix?:

6

6

Submit

Understanding EKF SLAM

14/14 points (graded)

In the SLAM lecture segments we have studied EKF SLAM in detail. This part of the quiz is aimed at testing your understanding of the different steps during EKF SLAM.

Consider the state x comprising of the robot's state r and the landmarks y_i in the SLAM map, which is described by the probability distribution $p(x)$ representing a multivariate Gaussian. As a result, the SLAM map is described by the mean vector $\hat{x}$ and the covariance matrix P. Note that the notation here differs slightly from any other representations we have seen.

$$p(x) = \frac{1}{\sqrt{(2\pi)^n |P|}} e^{-\frac{1}{2}(x-\hat{x})^T P^{-1}(x-\hat{x})}$$

$$\hat{x} = \begin{pmatrix} \hat{r} \\ \hat{y}_1 \\ \hat{y}_2 \\ \cdots \\ \hat{y}_n \end{pmatrix} \qquad P = \begin{bmatrix} P_{rr} & P_{ry_1} & P_{ry_2} & & P_{ry_n} \\ P_{y_1 r} & P_{y_1 y_1} & P_{y_1 y_2} & \cdots & P_{y_1 y_n} \\ P_{y_2 r} & P_{y_2 y_1} & P_{y_2 y_2} & & P_{y_2 y_n} \\ & \vdots & & \ddots & \vdots \\ P_{y_n r} & P_{y_n y_1} & P_{y_n y_2} & \cdots & P_{y_n y_n} \end{bmatrix}$$

In order to follow the prediction/measurement/update steps that are stated below timing subscripts are introduced where appropriate to aid clarity. For example, the state vector at time t is denoted with $\hat{x}_t$, while the predicted state vector at time t is denoted as $\hat{x}_{t|t-1}$, as the prediction is based on the state at time t-1. While some variables in the equations below are also t-dependent (e.g. the kalman gain K) the subscripts are omitted for the sake of clarity. Study the following equations carefully and proceed with the quiz below.

PREDICTION

$$\hat{x}_{t|t-1} = f(\hat{x}_{t-1}, u_t)$$
$$P_{t|t-1} = F_x P_{t-1} F_x{}^T + F_u Q_t F_u{}^T$$

UPDATE

$$\hat{x}_t = \hat{x}_{t|t-1} + K\left(z - h(\hat{x}_{t|t-1})\right)$$
$$P_t = P_{t|t-1} - KSK^T$$

such that

$$S = HP_{t|t-1}H^T + R$$
$$K = P_{t|t-1}HS^{-1}$$

Complete the statements by selecting the appropriate label from the dropdown menu corresponding to a candidate expression as shown in the table at the bottom of this page. Note that one expression can correspond to multiple statements, while some expressions might remain unused; there is always one correct answer for each statement.

The uncertainty in the state of landmark i in the 3D SLAM map is:

1 ▼ ✔

The correlation of the robot state to the state of a particular landmark i is:

7 ▼ ✔

The vector stacking all the predicted landmark locations in the current image (i.e. captured at time t) is:

10 ▼ ✔

The vector stacking all the observed landmark locations in the current image (i.e. captured at time t) is:

6 ▼ ✔

The vector corresponding to the innovation of landmark i (i.e. capturing the correction of the predicted to the observed landmark location that needs to be applied) corresponds to:

12 ▼ ✔

The uncertainty in the predicted location of landmark i in the current image is described by:

2 ▼ ✔

The measurement function: ...

3 ▼ ✔

... has the jacobian: ...

15 ▼ ✔

..., which in the equations above is denoted as:

5 ▼ ✔

The odometry function: ...

4 ▼ ✔

... has the jacobian: ...

13 ▼ ✔

..., which in the equations above is denoted as:

9 ▼ ✔

When the current image is captured, instead of searching exhaustively across the whole image for each landmark appearing in the state, we can limit the search to more specific locations in the image for efficiency. Namely, for landmark i can search the elliptical region in the current image described by: ...

2 ▼ ✔

... centered at pixel location:

11 ▼ ✔

TABLE OF CANDIDATE EXPRESSIONS

Note that the operator $\{.\}_i$ when used on a column vector, selects the rows corresponding to landmark i (e.g. $\{\hat{x}\}_i = \hat{y}_i$), while when used on a matrix, it selects the sub-matrix on the block diagonal corresponding to landmark i (e.g. $\{P\}_i = P_{y_i y_i}$).