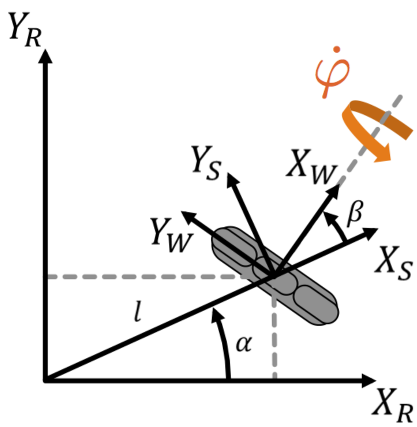
Mobile Robot Kinematics

The 90 Degree Swedish Wheel

Recall that the Swedish wheel has many small wheels around its circumference. In these questions we derive the equations for this wheel. The radius of the wheel is r.



By Rotacaster (Rotacaster Wheel Pty Ltd)
via Wikimedia Commons

Degrees of Freedom

How many degrees of freedom does this wheel have?

Answer

The answer is 3.

- Rotation around the contact point point
- Y axis motion by wheel rotation
- X axis motion by Swedish wheel rotation

The Wheel Equation

Please enter the wheel equation for the 90 degree Swedish wheel pictured above. Pay attention to the turning direction defined in the diagram. This is different than the turning direction defined in the worked example.

In the case of the Swedish wheel, the only actuator/encoder available is the one that spins the large wheel. Movement orthogonal to the large wheel is free due to the rolling of the small wheels. Therefore, the no-sliding-constraint matrix, **C**, is empty and the wheel equation has the form

$$\mathbf{J}\dot{\xi}_R = r\dot{\varphi}$$

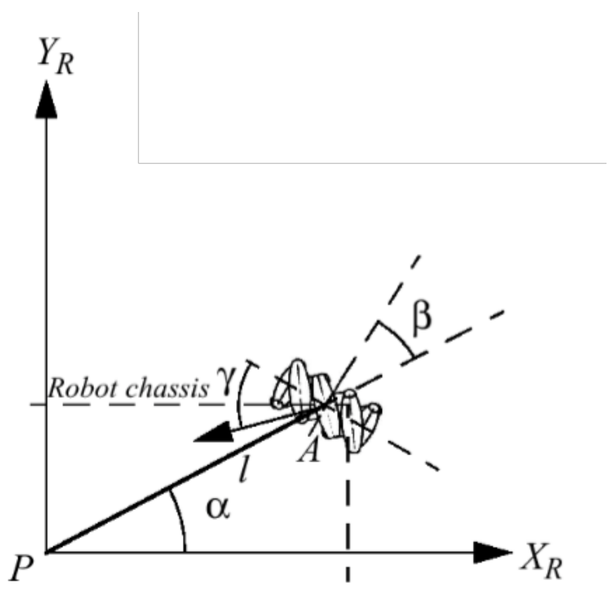
where $\dot{\xi}_R = [\dot{x}\ \dot{y}\ \dot{\theta}]^T$ is the robot's velocity expressed in the robot frame, r is the radius of the big wheel, and $\dot{\varphi}$ is the turning speed (radians per second) of the big wheel. The matrix **J** is 1×3 and has components

$$\mathbf{J} := [j_1 \quad j_2 \quad j_3]$$

Write an expression for each component of **J** in terms of α (alpha), β (beta) and l (l). Use the trigonometric functions sin and cos. Use * for multiplication.

Answer

The constraints of Swedish wheel (with rollers at angle γ):



$$\left[\sin(\alpha+\beta+\gamma) \quad -\cos(\alpha+\beta+\gamma) \quad -l\cos(\beta+\gamma)\right] \mathbf{R}(\theta)\dot{\xi}_I - r\dot{\varphi}\cos\gamma = 0 \tag{1}$$

$$\left[\cos(\alpha+\beta+\gamma) \quad \sin(\alpha+\beta+\gamma) \quad l\sin(\beta+\gamma)\right] \mathbf{R}(\theta)\dot{\xi}_I - r\dot{\varphi}\sin\gamma + r_{sw}\dot{\varphi}_{sw} = 0 \tag{2}$$

For 90 degree Swedish wheel, $\gamma = 0$. Thus the rolling constraint equation (1) is as follows:

$$\begin{bmatrix}\sin(\alpha+\beta) & -\cos(\alpha+\beta) & -l\cos(\beta)\end{bmatrix}\dot{\xi}_R - r\dot{\varphi}\cos\gamma = 0 \tag{3}$$

Note that $\mathbf{R}(\theta)\dot{\xi}_I = \dot{\xi}_R$. Therefore,

$$j_1 = \text{sin (alpha + beta)} \tag{4}$$

$$j_2 = \text{- cos (alpha + beta)} \tag{5}$$

$$j_3 = \text{- l * cos(beta)} \tag{6}$$