TA Session 3: Exercises on Product Differentiation

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Roadmap

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   - Exercise 7.2

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`TA3.m` does all the math.

Product Differentiation

- Product differentiation can be horizontal or vertical (often it's both, but most of the literature studies them separately).
- We call it *horizontal* when neither good is intrinsically better (it's only a matter of taste), while *vertical* differentiation refers to when everyone agrees on the ranking of products (agreeing on the ranking doesn't mean that everyone consumes the same product, all of them have a market, mostly because of the price spread).

Horizontal differentiation

- The following two models are known under the name of spatial horizontal differentiation.
- The most common interpretation is that the competitors sell the same good but at diffferent locations. Buyers live also in different places and, since they incur a transportation cost, they have a preference for buying from the closest shop (as long as the price is the same).
- *Spatial* is just an interpretation. More broadly, those models represent the case of heterogeneous consumers, each having different preferences for one or another seller. In this broader interpretation, goods are similar but each with some characteristics that make the product unique (think of pepsi versus coca-cola).

Hotelling Model: the Linear City

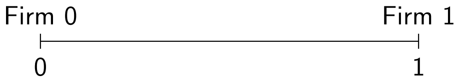
Hotelling Model

- The Hotelling model has the advantage of being very simple and fits very well some real settings, such as political competition.
- Its main drawback is that results, at least when one is interested in the location choice of firms, is very sensible to the assumptions on transportation costs.

Exercise 7.2 of Tirole (1988)

Consider the model of differentiation on the line. The two firms' locations are fixed, and they are the two extremities of the segment. Transportation costs are linear in distance, and the distribution of consumers is uniform along the segment. The firms ($i = 0, 1$) have constant marginal costs, c_0 and c_1, which are not necessarily equal (but, for simplicity, assume that they do not differ too much, so that each firm has a positive market share in equilibrium).

1. Compute the reaction functions $p_i = R_i(p_j)$. Infer the Nash-equilibrium prices $p_i(c_i, c_j)$ and the reduced-form profits $\pi_i(c_i, c_j)$ as functions of the two marginal costs.
2. Show that $\partial^2 \pi_i / \partial c_i \partial c_j < 0$.
3. Suppose that, before competing in price, the firms play a first-period game in which they simultaneously choose their marginal cost. (Think of an investment cost $\phi(c)$ of choosing marginal cost c, with $\phi' < 0$ and $\phi'' > 0$.) Show that this investment game gives rise to a direct effect and a strategic effect.



- Consider a linear city of unitary length, with two firms exogenously located at the two extremes (0 and 1). Firms produce at their marginal costs.
- A mass 1 of consumers is uniformly distributed along the line, a consumer's location describes her ideal point in the product space.
- We assume linear transportation cost. The transportation cost parameter $t > 0$ measures the substitutability between any given pair of products[1].
- The choice-specific utility of an agent located at x is

$$\begin{aligned} u_0(x) &= s - p_0 - tx, & &\text{if buy from 0} \\ u_1(x) &= s - p_1 - t(1-x), & &\text{if buy from 1} \end{aligned}$$

- Let $\tilde{x}$ denote the location of the agent that is indifferent between buying from firm 0 or 1:

$$s - p_0 - t\tilde{x} = s - p_1 - t(1-\tilde{x}) \;\Rightarrow\; \tilde{x} = \frac{p_1 - p_0 + t}{2t}$$

[1] The function t can be understood as an opportunity cost that a consumer incurs if a product in the market does not represent her ideal variety.

Price Competition

Nash Equilibrium

- Assume that the market is covered[2]. All agents that are to the left of the indifferent agent would buy from 0, while those to the right would buy from 1. Therefore, the demands for the two firms are:

$$D_0 = \tilde{x} = \frac{p_1 - p_0 + t}{2t}, \quad D_1 = 1 - \tilde{x} = \frac{p_0 - p_1 + t}{2t}$$

- Given the demand functions, we can write the firms' price competition problems, $i = 0, 1$, $j = 1 - i$:

$$\begin{aligned}
\pi_i(p_i, p_j) &= \max_{p_i} \ (p_i - c_i) D_i = \max_{p_i} \ (p_i - c_i) \frac{p_j - p_i + t}{2t} \\
\Rightarrow & \frac{\partial \pi_i}{\partial p_i} = 0 \ \Leftrightarrow \ p_i = \frac{1}{2}(p_j + t + c_i) \equiv R_i(p_j) \quad \textit{(Best response)} \\
\Rightarrow & p_i^* = t + \frac{1}{3}(2c_i + c_j) \quad \textit{(Bertrand-Nash equilirium prices)}
\end{aligned}$$

[2] This means that the indifferent agent, given prices, obtains a positive utility when buying. This is ensured by a large enough reservation price s.

Price Competition

Nash Equilibrium

- (Symmetric) Equilibrium profits:

$$\begin{aligned}\pi_i &= (p_i^* - c_i) \cdot D_i(p_i^*, p_j^*) \\ &= \frac{1}{2t}\left(t + \frac{1}{3}c_j - \frac{1}{3}c_i\right)^2\end{aligned}$$

- The second-order cross partial derivatives:

$$\frac{\partial^2 \pi_i}{\partial c_i \partial c_j} = -\frac{1}{9t} < 0, \quad i = 1, 2$$

Investment-then-price Game

- Now let us suppose that the firms play a 2-stage game in which they decide their investments endogenously before choosing prices.
  1. Decide costs: (c_0, c_1),
  2. Decide prices: (p_0, p_1)
- Backward induction of finding Subgame Perfect Nash Equilibrium (SPNE):
  1. (stage 2) given costs (c_0, c_1), compute prices $p_0^*(c_0, c_1)$ and $p_1^*(c_0, c_1)$;
  2. (stage 1) plug $p_0^*(c_0, c_1)$ and $p_1^*(c_0, c_1)$ back into profits, maximize the profits over the feasible set $\{(c_1, c_2)\}$.
- The reduced form net-investment profits are given by, $i = 0, 1$, $j = 1 - i$,

$$\begin{aligned}\tilde{\pi}_i(c_0, c_1) &= (p_i - c_i)D_i - \phi(c_i) \\ &= [p_i^*(c_i, c_j) - c_i] \cdot D_i[p_i^*(c_i, c_j), p_j^*(c_i, c_j)] - \phi(c_i)\end{aligned}$$

By the Envelope Theorem,

$$\frac{\mathrm{d}\pi_i}{\mathrm{d}c_i} = \frac{\partial \pi_i}{\partial c_i} + \frac{\partial \pi_i}{\partial p_j^*}\frac{\partial p_j^*}{\partial c_i} = -D_i - \phi'(c_i) + \frac{\partial \pi_i}{\partial p_j^*}\frac{\partial p_j^*}{\partial c_i}$$

  - Direct demand effect of cost adjustment $\mathrm{d}c_i$: $-D_i < 0$.
  - Strategic effect from the competitor's price reaction: $\frac{\partial \pi_i}{\partial p_j^*}\frac{\partial p_j^*}{\partial c_i}$.

Salop Model: the Circular City

Salop Model

- The salop model was mostly introduced to study the entry choice of firms and is often used when the main interest is in computing the equilibrium number of operating firms on a market.

Exercise 7.3 of Tirole (1988)

Show that if transportation costs are td^2, wherer d is the consumer's distance to his selected shop, Salop's model yields

$$p = c + \frac{t}{n^2}$$

and that, with free entry,

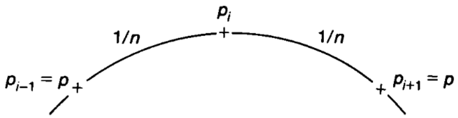
$$n^c = \left(\frac{t}{f}\right)^{\frac{1}{3}} > n^* = \left(\frac{t}{6f}\right)^{\frac{1}{3}}$$

Salop Circle

- The starting point of the circular city (Salop 1979) is almost the same as Hotelling. A mass 1 of buyers with unitary demand, transport cost t, marginal cost of production c.
- However, here we do not assume that the number of firms is fixed. There's an infinite number of potential entrants that would enter the market if profitable.
- With no barriers to entry, this would lead to a perfectly competitive market. However, we assume that firms pay a fixed entry cost f[3].
- By assumption, firms always locate as far as possible from each other, which means the n firms automatically locate at a distance $\frac{1}{n}$ from the next one (maximum differentiation is imposed).
- *Sequential game*: firms choose first if they enter the market and then they choose the price, given the number of firms in equilibrium. The model is solved backward as usual (compute prices given n, then compute n).

[3] We will focus on symmetric equilibria in which f is enough to guarantee that the market is covered

Salop Circle



- Every firm is competing only with the two firms that are one to the left, one to the right. The distance is the same on both side.
- Call p the market price[4]. Call p_i the price of firm i.
- Similar to what we did under Hotelling, we find the indifferent consumer $\tilde{x}$, which must be located where[5]

$$p_i + t\tilde{x}^2 = p + t\left(\frac{1}{n} - \tilde{x}\right)^2 \Rightarrow \tilde{x} = \frac{n}{2t}(p - p_i) + \frac{1}{2n}$$

$$\Rightarrow D_i = 2\tilde{x} = \frac{n}{t}(p - p_i) + \frac{1}{n}$$

[4]Remember that we only focus on symmetric equilibria. Firm i cannot have an incentive to deviate from p, otherwise, that would not be an equilibrium.

[5]Notice that x is interpreted as the distance from firm i, not a specific location on the plane.

Salop Circle

- Profit maximization:

$$
\begin{aligned}
\pi_i &= \max_{p_i} \ (p_i - c)D_i - f \\
&= \max_{p_i} \ (p_i - c)\left(\frac{n}{t}(p - p_i) + \frac{1}{n}\right) - f
\end{aligned}
$$

$$
\frac{\partial \pi_i}{\partial p_i} = 0 \ \& \ p_i = p \Rightarrow p^* - c = \frac{t}{n^2}
$$

  The profit margin $(p^* - c)$ is increasing in t and decreasing in n (increasing in $\frac{1}{n}$, the distance across firms).

- To compute the number of firms that operates in equilibrium, we use the entry condition that firms stop entering the market when $\pi = 0$.

$$
\pi_i = (p^* - c)D_i - f = \frac{t}{n^2} \cdot \frac{1}{n} - f = 0 \ \Rightarrow \ n^c = \left(\frac{t}{f}\right)^{\frac{1}{3}}
$$

Salop Circle

- Let's now check if this number of firms is optimal.
- With n firms, the maximum distance a buyer will travel is $\frac{1}{2n}$, the length of trip d distributes uniformly in $(0, \frac{1}{2n})$, the pdf is $f(d) = \frac{1}{\frac{1}{2n}-0} = 2n$. The average cost is

$$t \cdot \mathbb{E}(d^2) = t \cdot \int_0^{\frac{1}{2n}} x^2 f(x)\mathrm{d}x = 2nt \cdot \int_0^{\frac{1}{2n}} x^2 \mathrm{d}x = 2nt \cdot \left[\frac{1}{3}x^3\right]\Big|_0^{\frac{1}{2n}}$$
$$= \frac{1}{12n^2}$$

- The social planner who wants to minimise the unproductive expenses (fixed and transport costs) would choose

$$n^* = \underset{n}{\arg\min} \quad n \cdot f + t \cdot \mathbb{E}(d^2)$$
$$\Rightarrow n^* = \left(\frac{t}{6f}\right)^{\frac{1}{3}} < \left(\frac{t}{f}\right)^{\frac{1}{3}} = n^c$$

References

- Tirole, J. (1988). *The Theory of Industrial Organization*. MIT press. Chapter 7