

TA Session 1

# Treatment Effects I: RCT, Matching, and IV

Microeconometrics II with Joan Llull  
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# Overview

- 1 Regression Discontinuity
- 2 Differences-in-Differences

# Regression Discontinuity

# Regression Discontinuity

- Regression continuity (RD) research designs exploit precise knowledge of the rules determining treatment (some rules are arbitrary and therefore provide good experiments).
- RD comes in two styles: fuzzy and sharp.
  - The sharp design can be seen as a selection-on-observables story.
  - The fuzzy design leads to an instrumental variables (IV) type of setup.

# Sharp RD

- Sharp RD is used when treatment status is a deterministic and discontinuous function of a covariate,  $x_i$ . Suppose, for example, that

$$D_i = \begin{cases} 1, & \text{if } x_i \geq x_0 \\ 0, & \text{if } x_i < x_0 \end{cases}$$

where  $x_0$  is a known threshold or cutoff.

- *Deterministic*: once we know  $x_i$ , we know  $D_i$ .
- *Discontinuous*: no matter how close  $x_i$  gets to  $x_0$  (from the left), treatment is unchanged until  $x_i = x_0$ .
- **Example**: a standardized test for college entrance, sharp RD compares the post college performance of students with scores just above and just below the threshold.

# Sharp RD

- Consider the following regression that formalizes the RD idea,

$$y_i = f(x_i) + \rho D_i(x_i) + \eta_i$$

where  $\rho$  is the causal effect of interest, and  $D_i(x_i) = \mathbb{1}(x_i \geq x_0)$ .

- As long as  $f(x_i)$  is continuous in a neighborhood of  $x_0$ , it should be possible to estimate this model.

# Sharp RD

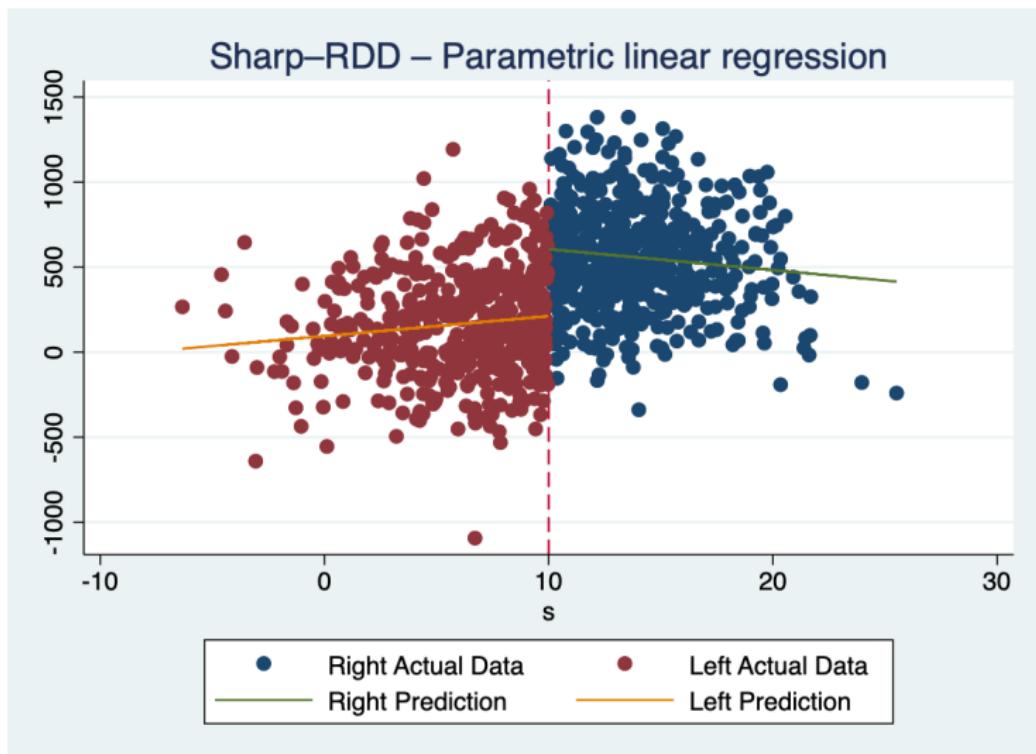
- For example, consider modeling  $f(x_i)$  with a  $p$ th-order polynomial,

$$y_i = \underbrace{\alpha + \beta_1 x_i + \beta_2 x_i^2 + \dots + \beta_p x_i^p}_{=f(x_i)} + \rho D_i + \eta_i$$

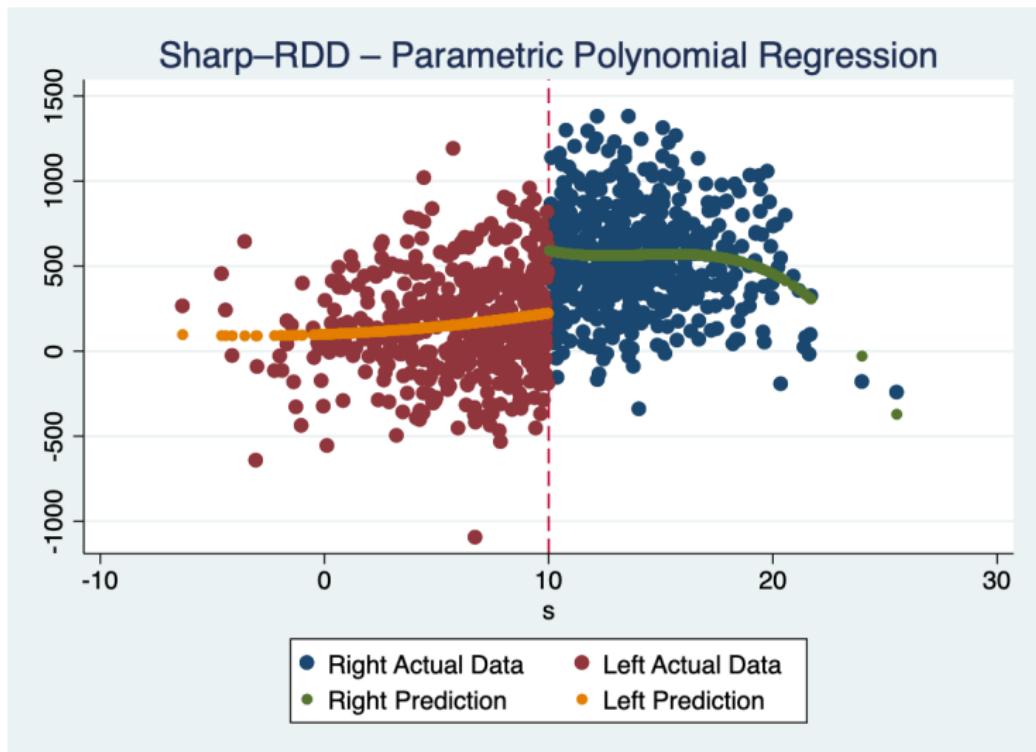
- A generalization of RD model allows different trend functions  $f_0(x_i)$  for  $\mathbb{E}(y_{0i}|x_i)$  and  $f_1(x_i)$  for  $\mathbb{E}(y_{1i}|x_i)$ .
- Calculate the treatment effect:

$$\alpha_{ATE,RD} = \underbrace{\lim_{x \rightarrow x_0^+} \mathbb{E}[y_i|x_i = x]}_{\simeq \mathbb{E}[y_{1i}|x_i]} - \underbrace{\lim_{x \rightarrow x_0^-} \mathbb{E}[y_i|x_i = x]}_{\simeq \mathbb{E}[y_{0i}|x_i]}$$

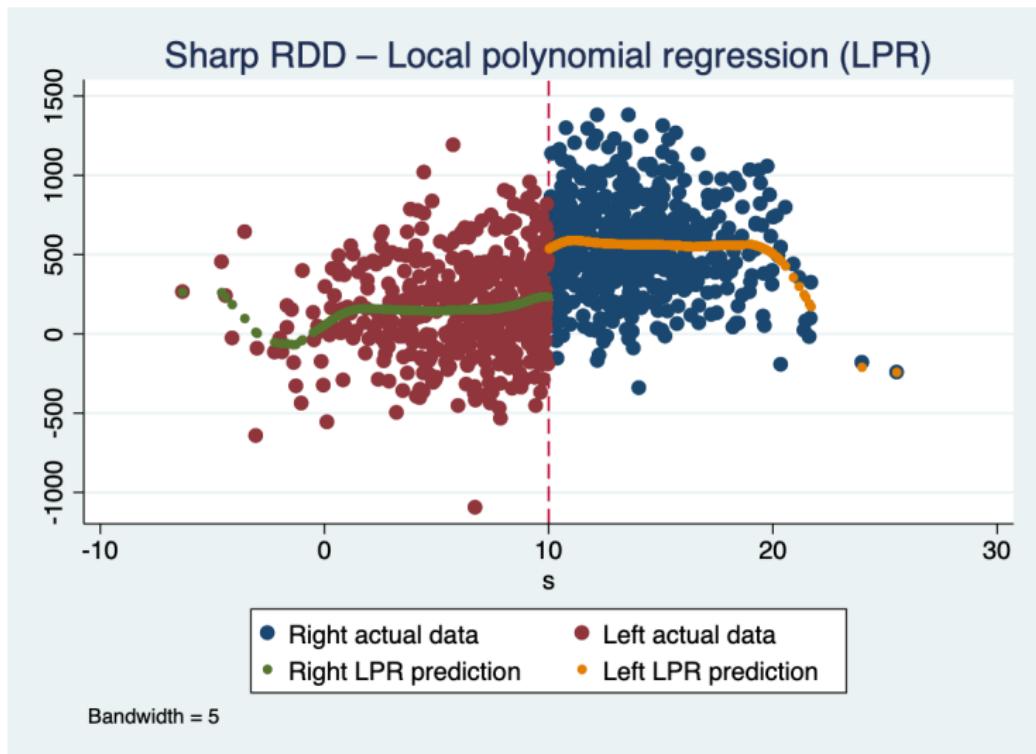
## Sharp RD



## Sharp RD



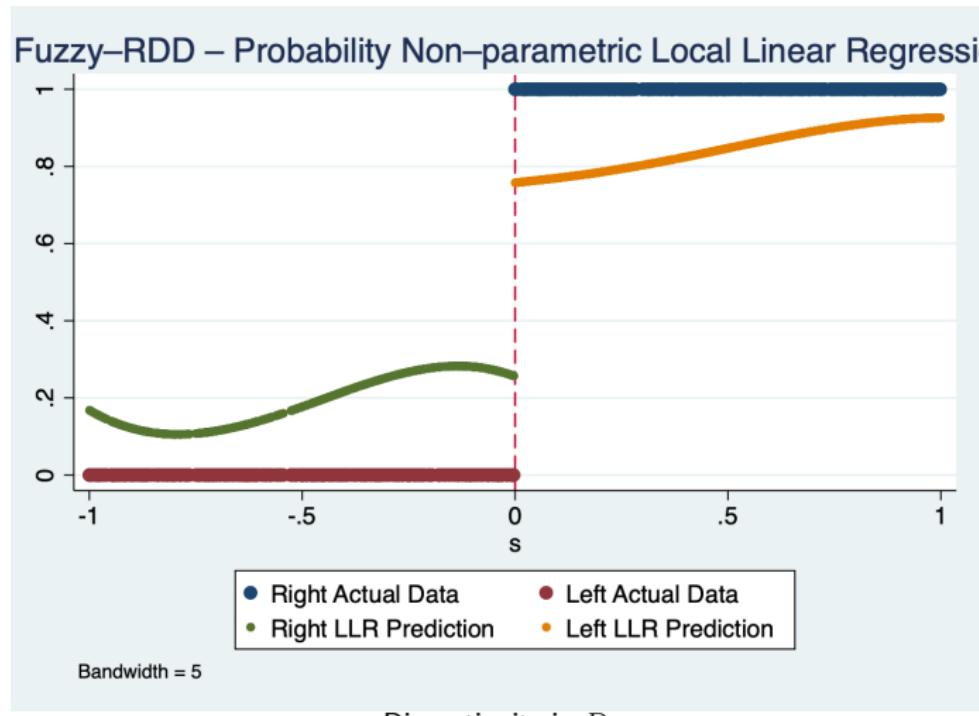
## Sharp RD



# Identification

- The regression discontinuity design identifies the conditional ATE at the treatment cut-off.
- What we need for identification is the continuity of  $f_1(X)$  and  $f_0(X)$ : which means that the conditional expectation of the untreated and treated outcome are continuously affected by the running variable.

# Sharp RD vs. Fuzzy RD



# Fuzzy RD

- Now, there is a jump in the probability in the probability of treatment at  $x_0$ , such that

$$\mathbb{P}(D_i = 1|x_i) = \begin{cases} g_1(x_i), & \text{if } x_i \geq x_0 \\ g_0(x_i), & \text{if } x_i < x_0 \end{cases}$$

where  $g_1(x_0) \neq g_0(x_0)$ . We assume  $g_1(x_0) > g_0(x_0)$  so that  $x_i \geq x_0$  makes treatment more likely.

- Fuzzy RD is IV: the discontinuity becomes an instrumental variable for treatment status instead of deterministically switching treatment on or off.
- The ATE at the cutoff:

$$\alpha_{ATE,RD} = \frac{\mathbb{E}(y|x_0^+) - \mathbb{E}(y|x_0^-)}{\pi(x_0^+) - \pi(x_0^-)}$$

In sharp RD,  $\pi(x_0^+) - \pi(x_0^-) = 1 - 0 = 1$ .

# Fuzzy RD

The Oreopoulos (2006) Example

- The treatment variable  $D_i$  has conditional density:

$$f(D_i|x_i) = \begin{cases} g_1(x_i), & x_i \geq 47 \\ g_0(x_i), & x_i < 47 \end{cases}, \quad g_1(47) > g_0(47)$$

where  $x_i \geq 47$  makes the treatment more likely. Instrument variable:

$$Z_i = \begin{cases} 1, & x_i \geq 47 \\ 0, & x_i < 47 \end{cases}$$

- It follows that

$$\begin{aligned} \mathbb{E}(D_i|x_i) &= \int D_i f(D_i|x_i) dD_i \\ &= \int D_i \left[ g_0(x_i) + (g_1(x_i) - g_0(x_i)) \cdot Z_i \right] dD_i \\ &= \mathbb{E}(D_i) \left[ g_0(x_i) + (g_1(x_i) - g_0(x_i)) \cdot Z_i \right] \end{aligned}$$

# Fuzzy RD is IV

The Oreopoulos (2006) Example

- We repeat the last line here

$$\mathbb{E}(D_i|x_i) = \mathbb{E}(D_i) \left[ g_0(x_i) + (g_1(x_i) - g_0(x_i)) \cdot Z_i \right]$$

The dummy variable  $Z_i$  indicates the point of discontinuity in  $E(D_i|x_i)$ .

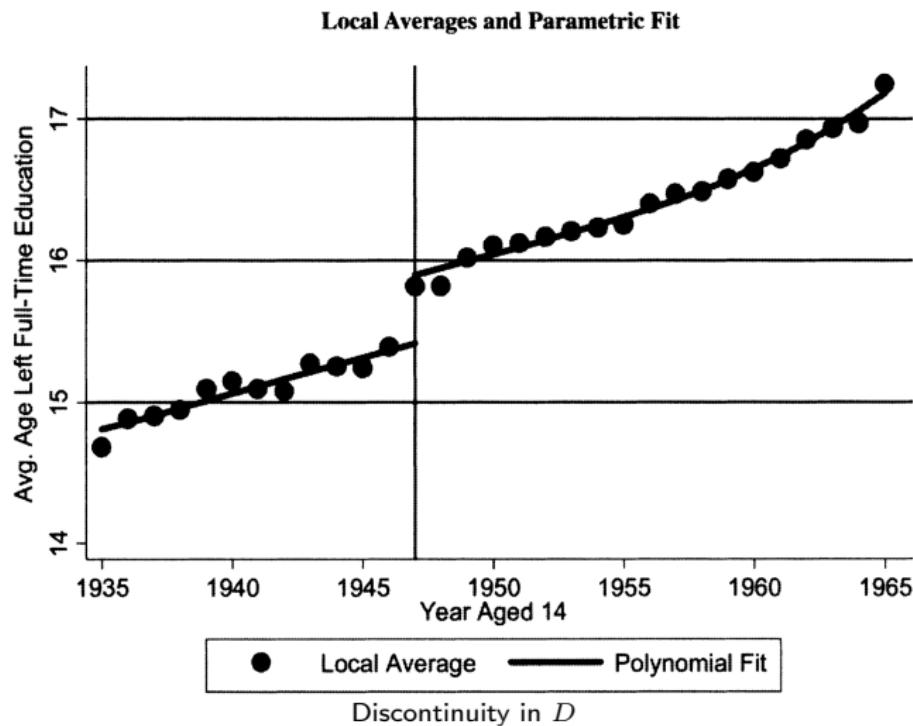
- To capture the non-linearity of the trend, we assume  $g_1(x_i)$  and  $g_0(x_i)$  each be some reasonably smooth function, for example, a  $p$ -th order polynomial:

$$\begin{aligned} E(D_i|x_i) = \mathbb{E}(D_i) & \left[ \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \cdots + \beta_p x_i^p \right. \\ & \left. + (\beta_0^* + \beta_1^* x_i + \beta_2^* x_i^2 + \cdots + \beta_p^* x_i^p) \cdot Z_i \right] \end{aligned}$$

- From this (the relevance condition) we see that  $Z_i$  as well as the interaction terms  $\{x_i Z_i, x_i^2 Z_i, \dots, x_i^p Z_i\}$  can be used as instruments for  $D_i$ .

# Fuzzy RD

The Oreopoulos (2006) Example



# Fuzzy RD is IV

The Oreopoulos (2006) Example

- ① The first stage of the 2SLS: discontinuity in  $D$

$$D_i = \tilde{\beta}_0 + \tilde{\beta}_1 x_i + \tilde{\beta}_2 x_i^2 + \cdots + \tilde{\beta}_p x_i^p + \gamma Z_i + u_i \quad (1)$$

- ② The second stage of the 2SLS: discontinuity in  $y$

Assume  $E(Y_{0i}|x_i) = h(x_i)$ , where  $h(x_i)$  is also a  $p$ -th order polynomial of  $x_i$ .

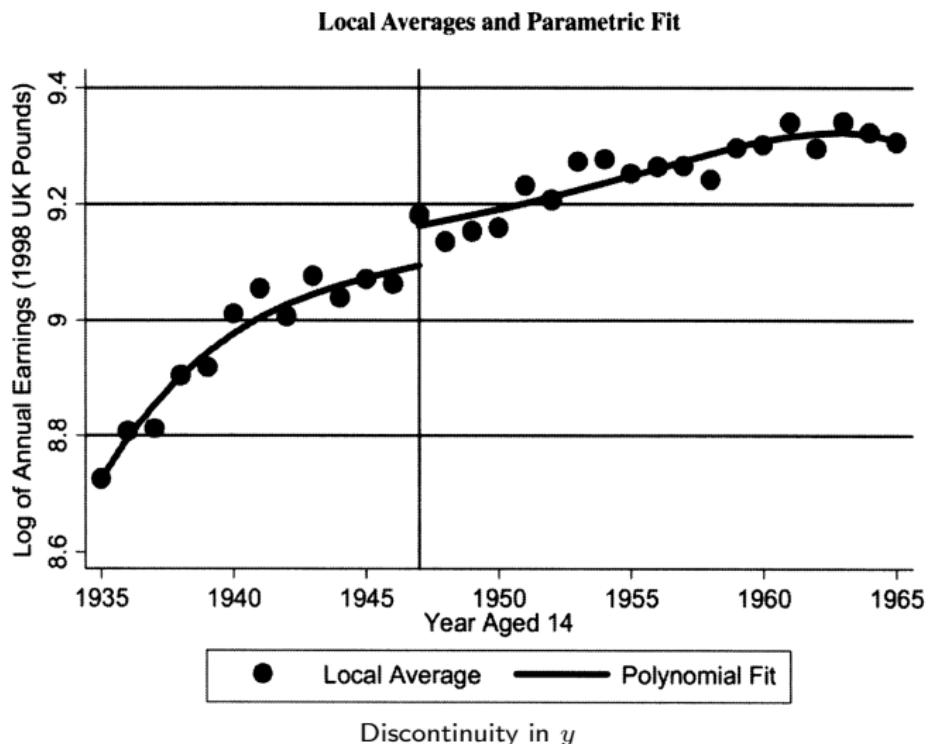
$$\begin{aligned} Y_i &= \alpha_i D_i + h(x_i) + \epsilon_i \\ &= \alpha_i D_i + \rho_0 + \rho_1 x_i + \rho_2 x_i^2 + \cdots + \rho_p x_i^p + \epsilon_i \end{aligned} \quad (2)$$

The fuzzy RD reduced form is obtained by substituting (1) into (2):

$$\begin{aligned} Y_i &= \alpha_i \left[ \tilde{\beta}_0 + \tilde{\beta}_1 x_i + \tilde{\beta}_2 x_i^2 + \cdots + \tilde{\beta}_p x_i^p + \gamma Z_i + u_i \right] \\ &\quad + \rho_0 + \rho_1 x_i + \rho_2 x_i^2 + \cdots + \rho_p x_i^p + \epsilon_i \\ &= \theta_0 + \theta_1 x_i + \theta_2 x_i^2 + \cdots + \theta_p x_i^p + \tilde{\epsilon} \end{aligned}$$

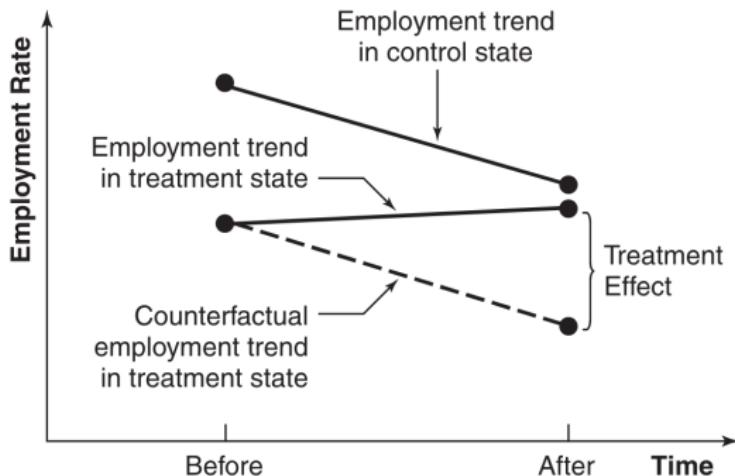
# Fuzzy RD is IV

The Oreopoulos (2006) Example



# Differences-in-Differences

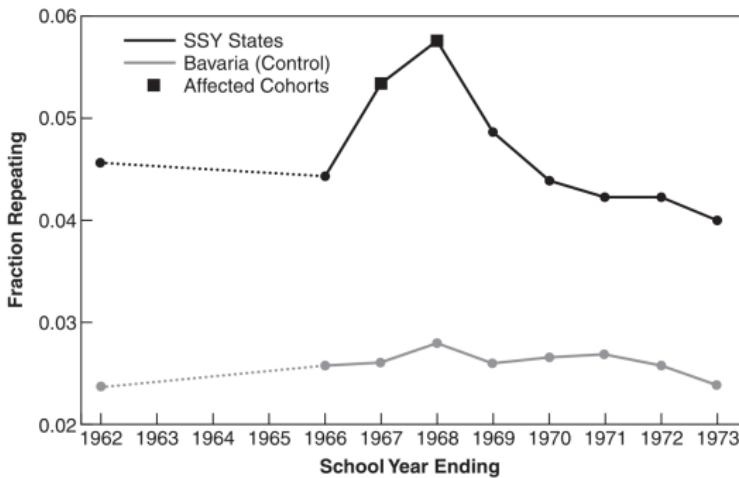
# The Simplest Case: $2 \times 2$



- The basic DiD model is a two-way fixed effects model:

$$y_{it} = \alpha D_{it} + X'_{it}\beta + \nu_i + \gamma_t + \varepsilon_{it}$$

# Trend specification



**Figure 5.2.3** Average grade repetition rates in second grade for treatment and control schools in Germany (from Pischke, 2007). The data span a period before and after a change in term length for students outside Bavaria (SSY states).

- Message from the graph:
  - ① strong visual evidence of treatment and control states with a common underlying trend, and
  - ② a treatment effect that induces a sharp but transitory deviation from this trend.

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