

Ensemble Source Encoding Full-waveform Inversion with Self-Adaptive Regularization

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SUMMARY

We develop an ensemble source encoding full-waveform inversion method with self-adaptive regularization (EnFWI), which approximates the total inversion using the ensemble Kalman filter (EnKF) to alleviate the error caused by inaccurate initial model and data noise with high computational efficiency. It refines an ensemble of velocity models simultaneously by iterative incorporating the waveform observation, and approximate the nonlinear evolution of error covariance by the ensemble covariance. To overcome the problem in EnKF that the updating directions are confined in the low rank space spanned by the model samples which is lack of representation ability in general, we apply the encoded simultaneous-source FWI (ESSFWI) to refine each model sample, so that the gradient directions with perturbation are introduced to improve the representation ability of the low rank updating space. Along with the velocity, we use the EnKF scheme to adjust the anisotropic regularization factors in order to utilize the smoothing constraints in different directions to further reduce the model error. Experiments show that EnFWI achieves larger convergence range and better tolerance to data noise with less computational costs than traditional FWI method.

Key words: Inverse theory; Controlled source seismology; Computational seismology; Wave propagation.

1 INTRODUCTION

Full-waveform inversion (FWI) is a promising data-fitting procedure which aims to extract quantitative information of the earth for high-resolution imaging from seismograms (Tarantola 1984). However, FWI is a non-linear ill-posed problem which suffers from local convergence led by the initial model error and the lack of ultra-low frequency (ULF) data (Virieux & Operto 2009). Data noise further deteriorates the imaging results.

Traditionally, the initial model is provided by some other methods such as traveltime tomography (Luo & Schuster 1991; Pratt & Goult 1991; Shipp & Singh 2002; Priolo & Chiaruttini 2003; Operto et al. 2004; Woodward et al. 2008; Wang et al. 2014) and migration velocity analysis (Al-Yahya 1989; Symes & Carazzone 1991; Sava & Biondi 2004). Other approaches to generate the initial model include first-arrival traveltime tomography (Hole 1992), stereotomography (Lambaré 2008), Laplace-domain inversion and Laplace-Fourier-domain inversion (Shin & Cha 2008; Shin & Cha 2009), seismic envelope inversion (Wu et al. 2014; Luo & Wu 2015) and so on. These methods recover long-wavelength background structure to a certain extent, obtaining initial models with relatively low error. However, it is difficult to meet the demands of conventional gradient-based FWI that the initial model should produce synthetic data that are within a half-cycle, everywhere (Al-Yahya 1989). Consequently, inversion methods with larger convergence domain are required.

Multiscale FWI methods (Bunks et al. 1995; Ravaut et al. 2004; Vigh *et al.* 2011) are developed to reduce the initial model dependence by performing a cascade inversion starting from the lowest frequency available. The lowest frequency band (1.5-2.0 Hz) is crucial for these methods to recover the correct long-wavelength background-velocity structure. However, in most cases, the standard seismic records can go down only to approximately 5 Hz (Wu et al. 2014). Not enough low frequency information is contained in the seismograms to avoid cycle skipping.

The conventional FWI is a simplification of the total inversion (Tarantola & Valette 1982). Instead of minimizing the data misfit directly, the total inversion maximizes the posterior probability of the model parameters under certain initial model and seismograms as measurement. Comparing to FWI, the total inversion is less sensitive to initial model error and data noise (Tarantola 1984). However, in the total inversion, the model and data covariance are required to generate the probability density function, leading to huge computation and memory costs, which make it actually impossible for high dimensional seismic inversion problems even with the state of art supercomputers.

The ensemble Kalman filter (EnKF) incorporates the initial model state and the observation using the same objective function as the total inversion (Evensen 1994; Evensen 2003; Evensen 2009). Instead of generating the covariance explicitly, it keeps an ensemble of model parameters, and uses the ensemble covariance to approximate the real covariance, providing an efficient mean to alleviate the computational costs. It has been applied in seismic inversion (Jin et al. 2008; Quan & Harris 2008). However, in EnKF, the updating increment is confined in the low rank space spanned by the model samples. In high-dimensional problems, e.g., seismic explorations, where model parameters are several order of magnitudes larger than model samples, the low rank updating space is usually difficult to provide significant decreasing of the data misfit when the initial ensemble samples are generated by random perturbation. It is critical to adjust the model samples to improve the representation ability of the low rank space they spanned.

FWI with encoded simultaneous sources (ESSFWI) is an efficient tool to obtain a useful updating direction (Krebs et al. 2009; Ben-Hadj-Ali et al. 2011; Castellanos et al. 2015). It stacks the seismograms to reduce the number of forward modeling, which significantly reduced the computation, and use different source codes in each updating step to suppress the noise resulting from crosstalk between the encoded sources. By the small fluctuations caused by source encoding, ESSFWI may even overcome small local minima (Castellanos et al. 2015). However, ESSFWI is more sensitive to data noise (Krebs et al. 2009) than FWI.

In this paper, we propose an ensemble source encoding full-waveform inversion method with self-adaptive regularization (EnFWI). In this method, we use EnKF to approximate the total inversion to enlarge the convergence domain and promote the resistance to data noise. ESSFWI is applied to refine the model samples to improve the representation for the low rank approximation. We also utilize the EnKF scheme to adjust the anisotropic regularization factor along with the velocity model, in order to utilize the smoothing constrains in different directions to further reduce the model error.

This paper is organized as follows. We firstly derive our EnFWI method. Then, we apply the method in the Marmousi model, to investigate the convergence range, the tolerance of data noise and computational costs of EnFWI.

2 METHOD

In this section, we design the EnFWI scheme. We first apply the EnKF in seismic inversion problems to approximate the total inversion with low computational cost. Then we introduce ESSFWI to refine the model samples in order to improve the representation ability of the low rank updating space. We also use the EnKF scheme to adjust the anisotropic regularization factors in ESSFWI along with the model samples.

2.1 Approximating the total inversion by EnKF

In the seismic inversion problem, we need to obtain the underground model, from an initial model and the seismogram, which serves as an observation of the wave field. We here follow (Evensen 2003) and apply the general EnFK theory to the seismic inversion problem. For simplicity, we consider the acoustic FWI problem. The scalar wave equation is

$$\frac{m(x)\partial^2 u(x, t)}{\partial t^2} - \Delta u(x, t) = f \quad (1)$$

where u is the pressure; the slowness $m = 1/c$ is the model parameter to solve; c is the wave velocity, and f is the source term. The seismogram d is the pressure u recorded in seismic surveys. We define the measurement operator H as $H_m = d_{cal}(m)$, where $d_{cal}(m)$ is the synthetic seismogram.

The total inversion and EnKF solve the inversion problem within the Bayesian framework. Assuming the posterior probability density function (PDF) to be Gaussian, in the k th updating step, by combining the prior knowledge of the initial model $(m_b)_k$, which is called the background field, and the observation d_k , the posterior PDF p satisfies

$$p \propto \exp \left(- \left(((m_b)_k - m)^T B^{-1} ((m_b)_k - m) + (d_k - H_m)^T R_k^{-1} (d_k - H_m) \right) \right) \quad (2)$$

where B is the covariance of m_0 and R_k is the covariance of d_k .

To obtain the optimal model m to maximize the posterior PDF p , we minimize the objective function.

$$2S(m) = ((m_b)_k - m)^T B^{-1} ((m_b)_k - m) + (d_k - Hm)^T R_k^{-1} (d_k - Hm). \quad (3)$$

By assigning the gradient of the objective function $S(m)$ to be 0, we obtain

$$(m_a)_k = (m_b)_k + BH^T (HBH^T + R_k)^{-1} (d_k - H(m_b)_k), \quad (4)$$

where $(m_a)_k$ is an optimal approximation of m by combining the initial model $(m_b)_k$ and the observation d_k , H is the tangent linear approximation of the measurement operator H .

According to eq. (4), the covariance of $(m_a)_k$, which is denoted by P_k , satisfies

$$P_k = B_k - B_k H^T (HB_k H^T + R_k)^{-1} H B_k \quad (5)$$

Eq. (4) and (5) forms the analysis step of the extended Kalman filter (EKF), which is the nonlinear version of the Kalman filter (Kalman, 1960). However, huge computation is required. Denoting the dimension of slowness model m and observation d to be N_m and N_d , the computation to generate $(HB_k H^T + R_k)^{-1}$ is $O(N_m^2 N_d)$, the memory to store B_k is $O(N_m^2)$, which are beyond the ability of

modern computers for high dimension problems. Meanwhile, generating the tangent linear operator H also requires huge computation. Even worse is that the inverting operation may lead to numerical divergence.

EnKF solves these two problems by ensemble approximation. We define the background model ensemble as $\Psi = (\psi_1^k, \psi_2^k, \dots, \psi_N^k)$, where k is the number of the current updating step, N is the number of model samples, $\psi_n^k, n \in \{1, 2, \dots, N\}$ is a model sample. The initial model ensemble Ψ_1 is obtained by adding a spatial low frequency random perturbation field to the initial model $(m_b)_0$. We also generate an observation ensemble $D_k = (d_1^k, d_2^k, \dots, d_N^k)$ by adding Gaussian noise to the observation d_k . To save computation, we stack the sources and seismograms by different source codes in each updating step. Then, the model covariance B and observation covariance R at the k -th updating step are approximated as

$$\begin{cases} B_k = \Psi'_k(\Psi'_k)^T / (N - 1) \\ R_k = D'_k(D'_k)^T / (N - 1), \end{cases} \quad (6)$$

where $\Psi'_k = \Psi_k(I - 1_N)$ and $D'_k = D_k(I - 1_N)$.

By substituting the ensemble approximation eq. (6) into the updating rule eq. (4), we obtain

$$(\Psi_a)_k = \Psi_k + \Psi'_k(H\Psi'_k)^T \left(H\Psi'_k(H\Psi'_k)^T + D'_k(D'_k)^T \right)^{-1} (D_k - H\Psi_k), \quad (7)$$

where $(\Psi_a)_k$ is the analysis model ensemble, $H\Psi'_k$ is computed as $H\Psi_k - \overline{H}\Psi_k$, where $\overline{H}\Psi_k$ is the average of the synthetic records $H\Psi_k$. The tangent linear operator H need not to be generated explicitly. To reduce the computational costs of eq. (7) and eliminate the instability, by assuming $H\Psi'_k$ and D'_k are independent, we obtain

$$\left(H\Psi'_k(H\Psi'_k)^T + D'_k(D'_k)^T \right)^{-1} \approx \left((H\Psi'_k + D'_k)(H\Psi'_k + D'_k)^T \right)^{-1}. \quad (8)$$

By the SVD decomposition, we have

$$H\Psi'_k + D'_k = USV, \quad (9)$$

where U and V are orthogonal matrices, S is a diagonal matrix.

Then, the ensemble updating rule (7) turns to

$$(\Psi_a)_k = \Psi_k + \Psi'_k(H\Psi'_k)^T U(SS^T)^{-1}U^T(D_k - H\Psi_k). \quad (10)$$

For the real slowness model does not change, the analysis model ensemble $(\Psi_a)_k$ is used directly as the background model ensemble Ψ_{k+1} in the next updating step.

In eq. (10), terms of the diagonal matrix S with small absolute values correspond to updating directions which are not effectively represented by the ensemble. Consequently, we omit the small values and zeros of S to enhance the stability. Then, at most N columns of U need to be generated. The dominate computational costs in eq. (10) is N forward modeling to generated $H\Psi_k$, and the memory cost is $O(N(N_m + N_d))$. In most applications, the sample number N is chosen between 10 and 100. The computation is less than traditional FWI for the sources and seismograms are stacked. More memory is needed to store model samples if the method is run on one computing node; however, different model samples can be distributed on different nodes to make full use of the memory of a parallel computing cluster, while duplication of the model is also required on each node when traditional FWI is run in parallel on the same cluster. By the ensemble updating rule eq. (10), we apply the EnKF in the seismic inversion and approximate the total inversion with low computation and memory costs. However, in the eq. (10), the updating direction is confined in the space spanned by the background model ensemble Ψ_k . In general, if the initial background model ensemble Ψ_1 is generated by random perturbation, the low rank updating space cannot provide an effective updating direction which can significantly reduce the data misfit, especially in high dimensional problem where the model parameters are several order of magnitudes more than model samples. Improving the representation ability of the low rank updating space is crucial to provide clear imaging results.

2.2 Refining the model samples using ESSFWI with self-adaptive regularization factors

Bringing in effective updating directions may improve the representation ability of the low rank updating space spanned by the model samples. ESSFWI can provide such directions with low computational costs. It stacks the seismograms using different source codes in each updating step save computation and suppresses the crosstalk noise. The anisotropic Tikhonov regularization term adds smoothing constraints with different weights in the horizontal and vertical direction. Take the 2D problem for example, the objective function of ESSFWI with anisotropic regularization term is

$$\begin{cases} S(m) = S_{data}(m) + S_{regu}(m) \\ 2S_{data}(m) = ||d - Hm||^2 = \int_0^{T_{max}} \sum_{i=1}^{N_{rec}} ||\hat{u}_o(r_{rec_i}, t; f) - u_c(r_{rec_i}, t; f)||^2 dt \\ 2S_{regu}(m) = \lambda_x ||W_x||^2 + \lambda_z ||W_z||^2 \end{cases} \quad (11)$$

where $S_{data}(m)$ is the data misfit, $S_{regu}(m)$ is the anisotropic Tikhonov regularization term, N_{rec} is the number of receivers, r_{rec_i} is the position of the i -th receiver, f is the stacked source using a random

source code, $u_c(r_{rec_i}, t; f)$ is the synthetic seismogram, $\hat{u}_o(r_{rec_i}, t; f)$ is the stacked seismogram using the same source code, λ_x and λ_z are regularization factors. W_x and W_z indicate the discontinuity in the X and Z direction, as

$$\begin{cases} \|W_x\|^2 = \sum_{i=1}^{N_x-1} \sum_{j=1}^{N_z} (m(x_{i+1}, z_j) - m(x_i, z_j))^2 \\ \|W_z\|^2 = \sum_{i=1}^{N_x} \sum_{j=1}^{N_z-1} (m(x_i, z_{j+1}) - m(x_i, z_j))^2 \end{cases} \quad (12)$$

Following Tarantola (1984), the gradient of the data misfit $S_{data}(m)$ can be computed as

$$G_{data}(m) = \int_0^{T_{max}} dt \sum_{i=1}^{N_{rec}} \frac{\partial}{\partial t^2} u^*(r, T_{max} - t; f) u(r, t; f) \quad (13)$$

where r is the position, $u^*(r, T_{max} - t; f)$ is the adjoint wavefield generated by propagate the data residual

$$\delta u(r_{rec_i}, t; f) = \hat{u}_o(r_{rec_i}, t; f) - u_c(r_{rec_i}, t; f) \quad (14)$$

backward in time.

In our realization of ESSFWI, the updating direction is obtained by the conjugate gradient (CG) method, and the step length is obtained by parabolic fitting. When it is used to refine the model samples in EnKF, we choose the negative gradient direction as the updating direction.

The regularization factors have a great influence on the inversing results. It is a challenge problem to find proper regularization factors. The EnKF scheme provides an efficient way to adjust the model samples and the regularization factors simultaneously. We define the new model ensemble combining the slowness model and the regularization factors as

$$\begin{cases} \Psi_k = (\psi_1^k, \psi_2^k, \dots, \psi_N^k) \\ \psi_n^k = ((\psi_n^k)^T, \lambda_x, \lambda_z)^T. \end{cases} \quad (15)$$

We also expand the definitional domain of function H to the new model sample ψ_n^k , as

$$H\psi_n^k = H\psi_n^k \quad (16)$$

We denote an updating step of the ESSFWI as

$$m_{k+1} = Lm_k. \quad (17)$$

Then, we expand the definitional domain of function L to the new model sample as

$$L\Psi_n^k = L \begin{pmatrix} \psi_n^k \\ \lambda_x \\ \lambda_z \end{pmatrix} = \begin{pmatrix} L\psi_n^k \\ \lambda_x \\ \lambda_z \end{pmatrix}. \quad (18)$$

Following the ensemble updating rule eq. (7), we obtain the EnFWI as the analysis step

$$(\Psi_a)_k = \Psi_k + \Psi'_k (\mathbf{H}\Psi'_k)^T (\mathbf{H}\Psi'_k (\mathbf{H}\Psi'_k)^T + D'_k (D'_k)^T)^{-1} (D - \mathbf{H}\Psi_k) \quad (19)$$

and the model step

$$\Psi_{k+1} = L(\Psi_a)_k. \quad (20)$$

In numerical experiments, we find that several updating steps of ESSFWI can be combined to accelerate the convergence, as

$$\Psi_{k+1} = L^{N_{iter}}(\Psi_a)_k, \quad (21)$$

where N^{iter} is typically chosen in [5,10].

3 NUMERICAL RESULTS

3.1 Experiment Setup

In order to examine the ability to tolerate inaccurate initial model and data noise, we applied our EnFWI method in Marmousi model to test results and compare them with FWI and ESSFWI.

The Marmousi model, shown in Figure 1, which is original 2301 grids in X-axis and 751 grids in Z-axis, is subsampled to 461 grids in X-axis and 151 grids in Z-axis, with the grid step of 20m. Equipped with Ricker wavelets with a peak frequency of 10Hz, 47 shots, are placed at the interval of 200m each, at the depth of 120m for a period of 3s. 461 receivers are placed 20m each at the depth of 120m for recording the seismograms.

A relatively large smoothing window size of 1200m in X-axis and 500m in Z-axis is applied on the true Marmousi model in the slowness domain, which is the initial model, shown in Figure 2.

We make a forward modeling on the Marmosi model to acquire the seismograms, which are taken as inputs to three FWI methods, including conventional FWI, ESSFWI and EnFWI.

We set the number of model samples in EnFWI to be 20, which can produce high resolution image at a low computational cost.

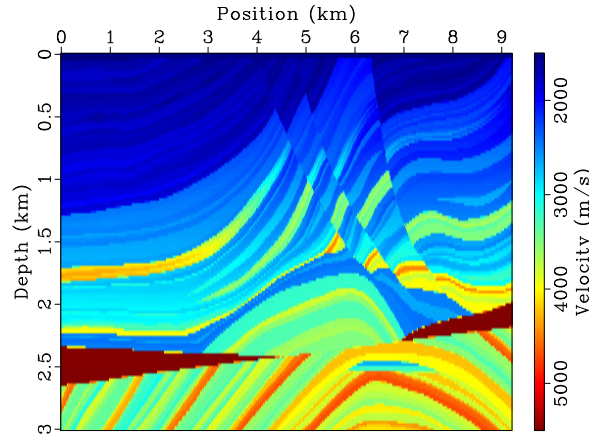


Figure 1. True Marmousi model. The region highlighted is used for comparison with different methods.

3.2 Data Without Noise

In the first experiment, seismograms are observed without data noise. The inversion results are shown in Figure 3-5. We can see that the velocity model inverted by FWI (Figure 3) is in a low quality, which can just describe the very vague outline of the accurate model. Better than FWI, ESSFWI (Figure 4) can produce much clearer image especially in the middle and low-middle region, but ESSFWI is still trapped in local minima in some region. Compared to ESSFWI in the up-middle region of the image, EnFWI (Figure 5) can overcome these minima effectively and the layered structure is much smoother especially in the low-middle region.

Figure 6 shows the variation of model misfit among three methods. Model misfit means the difference between the real model and the inverted model, which is measured by normalized L2 norm of slowness. Figure 6 shows that the model misfit of FWI does not descend but those of ESSFWI

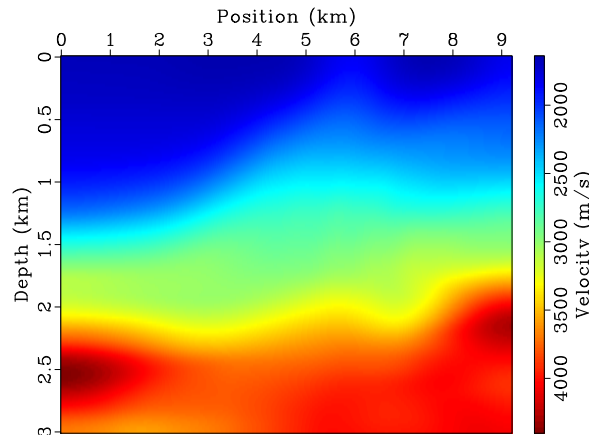


Figure 2. Initial velocity model, smoothed by a 1200m x 500m window.

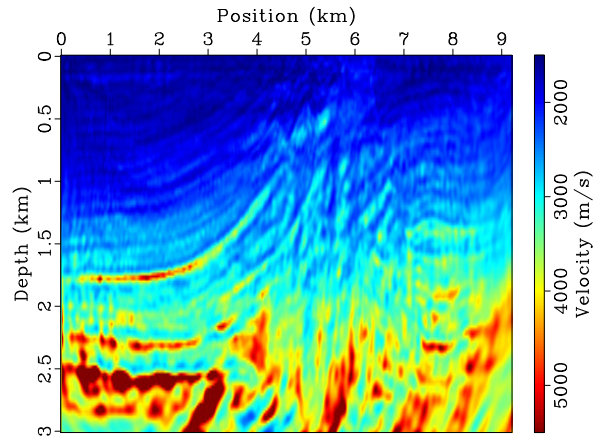


Figure 3. Image inverted by conventional FWI method.

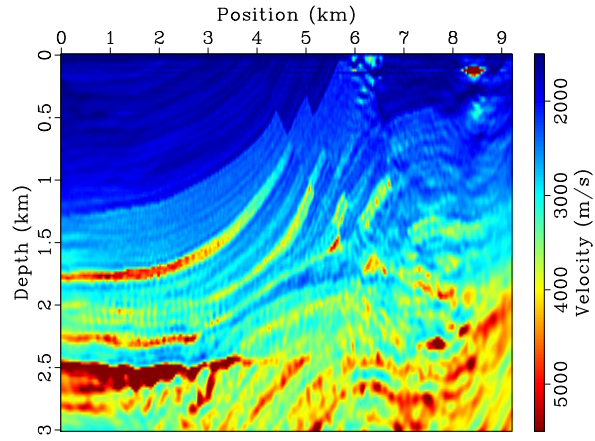


Figure 4. Image inverted by ESSFWI method.

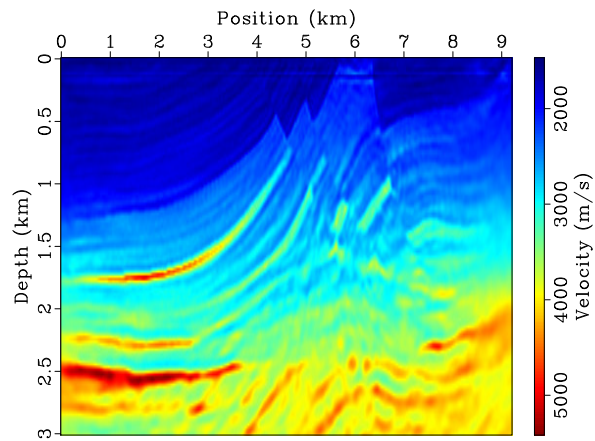


Figure 5. Image inverted by EnFWI.

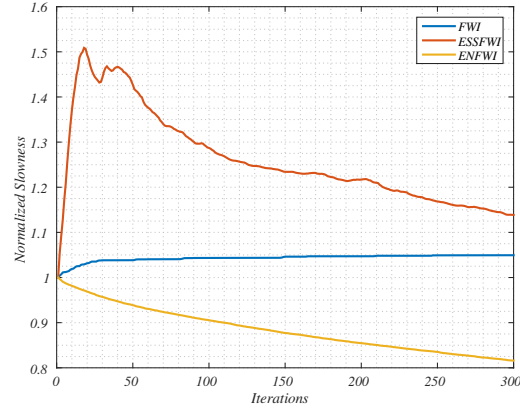


Figure 6. Normalized L2-norm of the slowness error of FWI, ESSFWI and EnFWI without data noise.

and FWI are significantly improved. However, ESSFWI is trapped in local minima in the up-middle region.

3.3 Data With Noise

In the second experiment, we add white noise whose amplitude is 5% of the maximum amplitude of the real seismograms to the data. Because most of the data is much smaller than their maximum, for this reason the noise is quite strong. Figure 7-9 show the inverted results. The imaging of FWI (Figure 7) is so vague and ESSFWI (Figure 8) produce serious artifacts under the challenging strong noise. But our EnFWI (Figure 9) can still reconstruct a good model.

In Figure 10, under the strong noise, the model misfit of ESSFWI and FWI does not descend in a real sense, which means the inverted results are not ameliorated than the original model. The orange curve indicates that our EnFWI method can acquire much better convergence than ESSFWI and FWI.

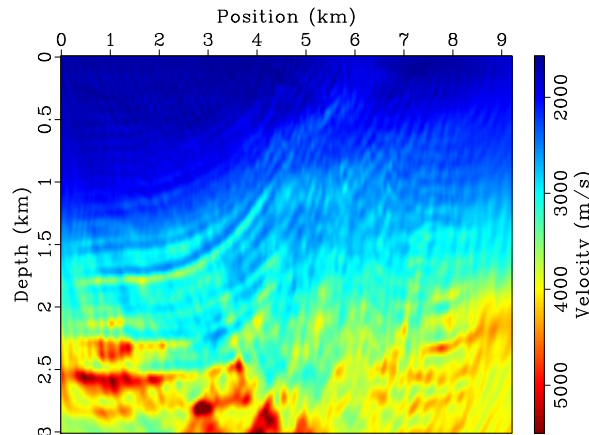


Figure 7. Velocity model inverted by FWI with data noise. The image is so vague.

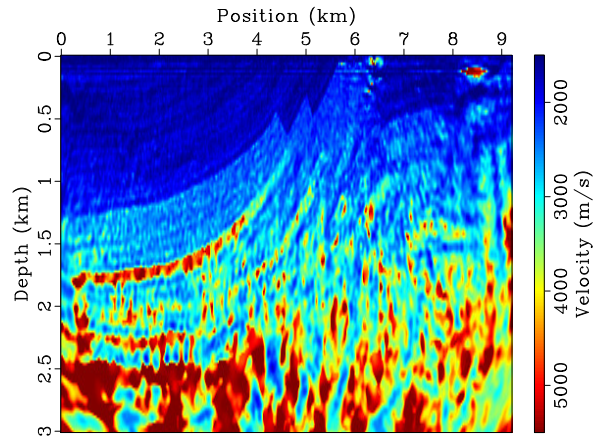


Figure 8. Velocity inverted by ESSFWI with data noise. It falls into severe local minima.

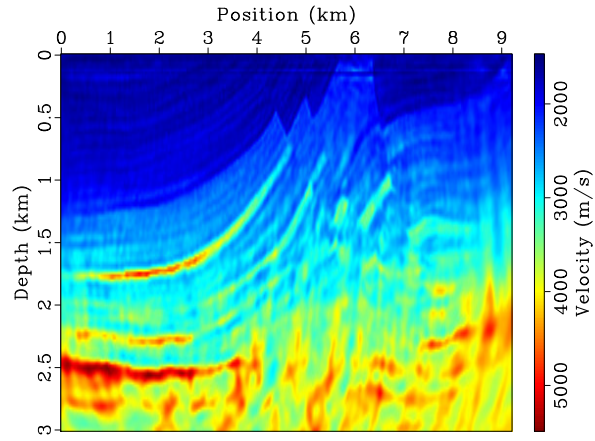


Figure 9. Image inverted by EnFWI. It can still get a better image under strong noise.

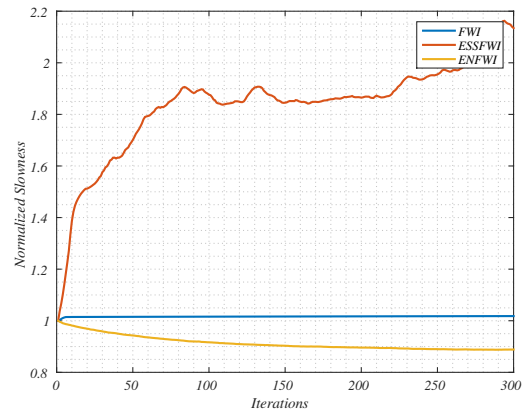


Figure 10. Normalized L2-norm of the slowness error of FWI, ESSFWI and EnFWI **with** data noise.

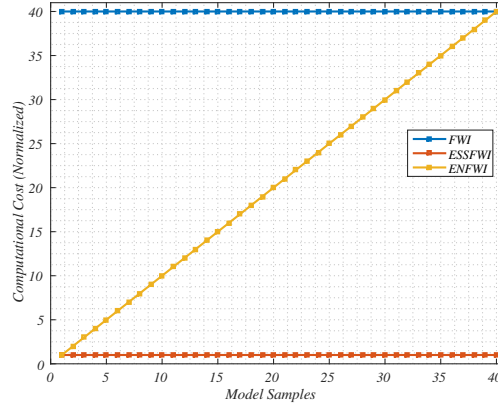


Figure 11. The computational cost of FWI, EssFWI and EnFWI

3.4 Performance

Although the EnFWI method brings significant improvements on the representation of the inverted images, it also comes with additional computational cost. The computational cost is linearly increased with the number of samples in the model ensembles. The EssFWI is employed not only for improving the representation ability but also to reduce the computational cost of EnFWI, which makes EnFWI a practical imaging solution.

The normalized performance of FWI, EssFWI and EnFWI is shown in Figure 11. Compared with FWI, the computational cost of EssFWI is reduced by a factor roughly equal to the number of sources. On the other hand, the computational cost of EnFWI that employs EssFWI as the basic framework increases roughly linearly to the number of model samples. In our case where the number of sources is 40 and the number of model samples is 20, the EnFWI gains a speedup of 2x compared with FWI. Note that in a typical 3D case where the number of sources ranges from thousands to tens of thousands, the EnFWI method can gain a more significant speedup.

4 CONCLUSIONS

We present an EnFWI method with self-adaptive regularization. It approximates the nonlinear evolution of covariance in the total inversion with ensemble covariance to suppress the initial model error and data noise while keeping the computation in a realistic level. ESSFWI is also applied to improve the representation and performance. Anisotropic regularization with factors updated automatically in EnKF analysis steps, is applied to decrease the artifacts utilizing the layered smooth constraints. Numerical experiments show that the proposed method not only has larger convergence range and better tolerance to data noise but also is more efficient than traditional FWI method.

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