2021 第五次数理逻辑作业参考答案

June 12, 2021

P-138 1. 设有如下推理语句:(1)没有无知的教授。(2)所有无知者均爱虚荣。(3)没有爱虚荣的教授。请问(1)和(2)能否推出(3)?

用 $Zhi(x)$ 表示 x 在有知;$Jiao(x)$ 表示 x 是教授;$Love(x)$ 表示 x 爱虚荣。上述三个句子为:

(1)$\neg\exists x(\neg Zhi(x) \wedge Jiao(x))$

(2)$\forall x(\neg Zhi(x) \rightarrow Love(x))$

(3)$\neg\exists x(Love(x) \wedge Jiao(x))$

(1)和(2)不能推出(3)。如果将(2)改成"所有爱虚荣者都无知",那么(1)和(2)才能推出(3)简易证明如下:

(1). $\neg\exists x(\neg Zhi(x) \wedge Jiao(x)), \forall x(Love(x) \rightarrow \neg Zhi(x)), \exists x(Love(x) \wedge Jiao(x)) \vdash \forall x \neg(\neg Zhi(x) \wedge Jiao(x))$

(2). $\neg\exists x(\neg Zhi(x) \wedge Jiao(x)), \forall x(Love(x) \rightarrow \neg Zhi(x)), \exists x(Love(x) \wedge Jiao(x)), Love(x) \wedge Jiao(x) \vdash Love(x)$

(3). $\neg\exists x(\neg Zhi(x) \wedge Jiao(x)), \forall x(Love(x) \rightarrow \neg Zhi(x)), \exists x(Love(x) \wedge Jiao(x)), Love(x) \wedge Jiao(x) \vdash Love(x) \rightarrow \neg Zhi(x)$

(4). $\neg\exists x(\neg Zhi(x) \wedge Jiao(x)), \forall x(Love(x) \rightarrow \neg Zhi(x)), \exists x(Love(x) \wedge Jiao(x)), Love(x) \wedge Jiao(x) \vdash \neg Zhi(x)$

(5). $\neg\exists x(\neg Zhi(x) \wedge Jiao(x)), \forall x(Love(x) \rightarrow \neg Zhi(x)), \exists x(Love(x) \wedge Jiao(x)), Love(x) \wedge Jiao(x) \vdash Jiao(x)$

(6). $\neg\exists x(\neg Zhi(x) \wedge Jiao(x)), \forall x(Love(x) \rightarrow \neg Zhi(x)), \exists x(Love(x) \wedge Jiao(x)), Love(x) \wedge Jiao(x) \vdash \neg Zhi(x) \wedge Jiao(x)$

(7). $\neg\exists x(\neg Zhi(x) \wedge Jiao(x)), \forall x(Love(x) \rightarrow \neg Zhi(x)), \exists x(Love(x) \wedge Jiao(x)), Love(x) \wedge Jiao(x) \vdash \neg(\neg Zhi(x) \wedge Jiao(x))$

(8). $\neg\exists x(\neg Zhi(x) \wedge Jiao(x)), \forall x(Love(x) \rightarrow \neg Zhi(x)), \exists x(Love(x) \wedge Jiao(x)), Love(x) \wedge Jiao(x) \vdash F$

(9). $\neg\exists x(\neg Zhi(x) \wedge Jiao(x)), \forall x(Love(x) \rightarrow \neg Zhi(x)), \exists x(Love(x) \wedge Jiao(x)) \vdash F$

(10). $\neg\exists x(\neg Zhi(x) \wedge Jiao(x)), \forall x(Love(x) \rightarrow \neg Zhi(x)) \vdash \neg\exists x(Love(x) \wedge Jiao(x))$

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P-138 3. 设A, B为FC中的任意公式,变元v在A中无自由出现,试证:

1. $\vdash (A \to \exists vB) \to \exists v(A \to B)$

证明:

(1). $\neg A \to (A \to B)$ PC中的定理6

(2). $(A \to B) \to \exists v(A \to B)$ FC中定理2

(3). $\neg A \to \exists v(A \to B)$ (1)(2)PC中的定理8

(4). $B \to (A \to B)$ 公理1

(5). $\neg(A \to B) \to \neg B$ 对(4)用PC中定理13及分离规则

(6). $\forall v(\neg(A \to B) \to \neg B)$ (5)全称推广定理

(7). $\forall v(\neg(A \to B) \to \neg B) \to ((\forall v)\neg(A \to B) \to \forall v\neg B)$ 公理5

(8). $(\forall v)\neg(A \to B) \to \forall v\neg B$ 对(6)(7)用分离规则

(9). $\neg\forall v\neg B \to \neg\forall v\neg(A \to B)$ 对(8)用PC中的定理13及分离规则

(10). $\exists vB \to \exists v(A \to B)$ 定义

(11). $(A \to \exists vB) \to \exists v(A \to B)$ (3)(10)定理17.5

2. $\vdash \exists v(A \to B) \to (A \to \exists vB)$

证明:

(1). $(A \to B) \to (A \to B)$ PC中定理1

(2). $A \to ((A \to B) \to B)$ 前件互换定理

(3). $(A \to B) \to B) \to (\neg B \to \neg(A \to B))$ PC中定理13

(4). $A \to (\neg B \to \neg(A \to B))$ (2)(3)三段论定理8

(5). $A \vdash \neg B \to \neg(A \to B)$ 演绎定理

(6). $A \vdash \forall v(\neg B \to \neg(A \to B))$ 全程推广定理5

(7). $\forall v(\neg B \to \neg(A \to B)) \to (\forall v\neg B \to \forall v\neg(A \to B))$ 公理5

(8). $A \vdash \forall v\neg B \to \forall v\neg(A \to B)$ (6)(7)分离规则

(9). $(\forall v\neg B \to \forall v\neg(A \to B)) \to (\exists v(A \to B) \to \exists vB)$ 定理13

(10). $A \vdash \exists v(A \to B) \to \exists vB$ (8)(9)分离规则

(11). $\vdash (A \to (\exists v(A \to B) \to \exists vB)$ 演绎定理

(12). $\vdash \exists v(A \to B) \to (A \to \exists vB)$ 前件互换定理

3. $(\forall vB \to A) \to \exists v(B \to A)$

证明：

(1). $A \to (B \to A)$

(2). $(B \to A) \to \exists v(B \to A)$

(3). $A \to \exists v(B \to A)$

(4). $\neg B \to (B \to A)$

(5). $(\neg B \to (B \to A)) \to (\neg(B \to A) \to B)$

(6). $\neg(B \to A) \to B$

(7). $\forall v(\neg(B \to A) \to B)$

(8). $\forall v(\neg(B \to A) \to B) \to (\forall v\neg(B \to A) \to \forall vB)$

(9). $\forall v\neg(B \to A) \to \forall vB$

(10). $(\forall v\neg(B \to A) \to \forall vB) \to (\neg\forall vB \to \neg\forall v\neg(B \to A))$

(11). $\neg\forall vB \to \neg\forall v\neg(B \to A)$

(12). $\neg\forall vB \to \exists v(B \to A)$

(13). $(\forall vB \to A) \to \exists v(B \to A)$

4. $\exists v(B \to A) \to (\forall vB \to A)$

证明：

(1). $(B \to A) \to (B \to A)$ PC中的定理1

(2). $B \to ((B \to A) \to A)$ 对（1）用PC中的前件互换定理2

(3). $((B \to A) \to A) \to (\neg A \to \neg(B \to A))$ PC中的定理13

(4). $B \to (\neg A \to \neg(B \to A))$ （2）和（3）用三段论定理8

(5). $\neg A \to (B \to \neg(B \to A))$ 对（4）用PC中的前件互换定理2

(6). $\neg A \vdash B \to \neg(B \to A)$ 对（5）用演绎定理

(7). $\neg A \vdash \forall v(B \to \neg(B \to A))$ 对（5）用演绎定理

(8). $\neg A \vdash \forall vB \to \forall v\neg(B \to A)$

(9). $\neg A \to (\forall vB \to \forall v\neg(B \to A))$

(10). $\forall vB \to (\neg A \to \forall v\neg(B \to A))$

(11). $\forall vB \to (\exists v(B \to A) \to A)$

(12). $\exists v(B \to A) \to (\forall vB \to A)$

P138-4 在FC中证明：

1. $\forall x(A \to B) \vdash\dashv A \to \forall xB$, x 在 A 中无自由出现

证明：

(1). $\forall x(A \to B), A \vdash A$

(2). $\forall x(A \to B), A \vdash \forall x(A \to B)$

(3). $\forall x(A \to B) \to (A \to B)$

(4). $\forall x(A \to B), A \vdash A \to B$

(5). $\forall x(A \to B), A \vdash B$

(6). $\forall x(A \to B), A \vdash \forall v B$

(7). $\forall x(A \to B) \vdash A \to \forall v B$

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(8). $\neg A \to (A \to B)$

(9). $\neg A \vdash (A \to B)$

(10). $\neg A \vdash \forall x(A \to B)$

(11). $\neg A \to \forall x(A \to B)$

(12). $B \to (A \to B)$

(13). $\forall x(B \to (A \to B))$

(14). $\forall x(B \to (A \to B)) \to (\forall x B \to \forall x(A \to B))$

(15). $\forall x B \to \forall x(A \to B)$

(16). $(A \to \forall x B) \to \forall x(A \to B)$

(17). $A \to \forall x B \vdash \forall x(A \to B)$

2. $\forall x(A \to B) \vdash\dashv \exists x A \to B$, x 在 B 中无自由出现

(1). $\forall x(A \to B), \neg B \vdash \neg B$

(2). $\forall x(A \to B), \neg B \vdash \forall x(A \to B)$

(3). $\forall x(A \to B), \neg B \vdash A \to B$

(4). $(A \to B) \to (\neg B \to \neg A)$

(5). $\forall x(A \to B), \neg B \vdash \neg B \to \neg A$

(6). $\forall x(A \to B), \neg B \vdash \neg A$

(7). $\forall x(A \to B), \neg B \vdash \forall x \neg A$

(8). $\forall x(A \to B) \vdash \neg B \to \forall x \neg A$

(9). $(\neg B \to \forall x \neg A) \to (\exists x A \to B)$

(10). $\forall x(A \to B) \vdash \exists x A \to B$

(11). $B \to (A \to B)$

(12). $B \vdash A \to B$

(13). $B \vdash \forall x(A \to B)$

(14). $\vdash B \to \forall x(A \to B)$

(15). $\vdash \neg A \to (A \to B)$

(16). $\vdash \forall x(\neg A \to (A \to B))$

(17). $\forall x(\neg A \to (A \to B)) \to (\forall x \neg A \to \forall x(A \to B))$

(18). $\vdash \forall x \neg A \to \forall x(A \to B)$

(19). $\vdash \neg \exists x A \to \forall x(A \to B)$

(20). $\vdash (\exists x A \to B) \to \forall x(A \to B)$

(21). $(\exists x A \to B) \vdash \forall x(A \to B)$

3. $\forall x(A \wedge B) \vdash\dashv \forall x A \wedge \forall x B$

证明：

(1). $\vdash \forall x(A \wedge B) \to A \wedge B$

(2). $\forall x(A \wedge B) \vdash A \wedge B$

(3). $\forall x(A \wedge B) \vdash A$

(4). $\forall x(A \wedge B) \vdash B$

(5). $\forall x(A \wedge B) \vdash \forall x A$

(6). $\forall x(A \wedge B) \vdash \forall x B$

(7). $\forall x(A \wedge B) \vdash \forall x A \wedge \forall x B$

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(8). $\vdash \forall x A \wedge \forall x B \to \forall x A$

(9). $\vdash \forall x A \wedge \forall x B \to \forall x B$

(10). $\vdash \forall x A \wedge \forall x B \to A$

(11). $\vdash \forall x A \wedge \forall x B \to B$

(12). $\vdash \forall x A \wedge \forall x B \to A \wedge B$

(13). $\forall x A \wedge \forall x B \vdash A \wedge B$

(13). $\forall x A \wedge \forall x B \vdash \forall x(A \wedge B)$

4. $\exists x(A \vee B) \vdash\dashv \exists x A \vee \exists x B$

证明：

(1). $A \to A \vee B$

(2). $(A \to A \vee B) \to (\neg(A \vee B) \to \neg A)$

(3). $\neg(A \vee B) \to \neg A$

(4). $\forall x(\neg(A \vee B) \to \neg A)$

(5). $\forall x \neg(A \vee B) \to \forall x \neg A$

(6). $\exists x A \to \exists x(A \vee B)$

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(7). $B \to A \vee B$

(8). $(B \to A \vee B) \to (\neg(A \vee B) \to \neg B)$

(9). $\neg(A \vee B) \to \neg B$

(10). $\forall x(\neg(A \vee B) \to \neg B)$

(11). $\forall x \neg(A \vee B) \to \forall x \neg B$

(12). $\exists x B \to \exists x(A \vee B)$

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(13). $(\exists x A \to \exists x(A \vee B)) \to ((\exists x B \to \exists x(A \vee B)) \to (\exists x A \vee \exists x B \to \exists x(A \vee B)))$

(14). $(\exists x B \to \exists x(A \vee B)) \to (\exists x A \vee \exists x B \to \exists x(A \vee B))$

(15). $\exists x A \vee \exists x B \to \exists x(A \vee B)$

(16). $\vdash \exists x A \vee \exists x B \to \exists x(A \vee B)$

(17). $\exists x A \vee \exists x B \vdash \exists x(A \vee B)$

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(18). $\exists x(A \vee B), \exists x A \vdash \exists x A$

(19). $\exists x(A \vee B), \exists x A \vdash \exists x A \vee \exists x B$

(20). $\exists x(A \vee B), \neg\exists x A \vdash \neg\neg\forall x \neg A$

(21). $\exists x(A \vee B), \neg\exists x A \vdash \forall x \neg A$

(22). $\exists x(A \vee B), \neg\exists x A \vdash \neg A$

(23). $\exists x(A \vee B), \neg\exists x A, A \vee B \vdash \neg A$

(24). $\exists x(A \vee B), \neg\exists x A, A \vee B, A \vdash \neg A$

(25). $\exists x(A \vee B), \neg\exists x A, A \vee B, A \vdash A$

(25). $\exists x(A \vee B), \neg\exists x A, A \vee B, A \vdash B$

(26). $\exists x(A \vee B), \neg\exists xA, A \vee B, B \vdash B$

(27). $\exists x(A \vee B), \neg\exists xA, A \vee B \vdash A \vee B$

(28). $\exists x(A \vee B), \neg\exists xA, A \vee B \vdash B$

(29). $\exists x(A \vee B), \neg\exists xA, A \vee B \vdash \exists xB$

(30). $\exists x(A \vee B), \neg\exists xA, A \vee B \vdash \exists xA \vee \exists xB$

(31). $\exists x(A \vee B), \neg\exists xA \vdash \exists x(A \vee B)$

(32). $\exists x(A \vee B), \neg\exists xA \vdash \exists xA \vee \exists xB$

(33). $\exists x(A \vee B) \vdash \exists xA \vee \exists xB$