

Bounded Convergence Theorem

If $X_1, X_2, \dots, X_n \dots$ converge to X in probability,
and $|X_i| < M$ for some finite $M > 0$ for all i
then

$$\lim_{n \rightarrow \infty} E[X_n] = E[X]$$

Monotone Convergence Theorem,

If $X_1, X_2, \dots, X_n \dots$ converge to X in probability,

$$X_1 \leq X_2 \leq X_3 \dots \leq X_n \leq X_{n+1}$$

then

$$\lim_{n \rightarrow \infty} E[X_n] = E[X]$$

Suppose $X(t) = 100 + B(t)$
price of some stock follows

standard Brownian motion.

Someone buy at price 100 and plan to sell
whenever the price get to 120.

If the price never hit 120 by the end of the 2nd year ($t=2$),
then he will sell at $t = 2$ (end of the 2nd year).

$$\min(T_{120}, 2)$$

Q: What is his expected profit?

Use first Wald identity,

$$E(B(\min(T_{120}, 2))) = 0 * E[\min(T_{120}, 2)] = 0$$

since $\mu = 0$