Bài 1 : Tính :

a. $\cos 225^0, \sin 240^0, \cot(-15^0), \tan 75^0$

b. $\sin\frac{7\pi}{2}, \cos\left(-\frac{\pi}{12}\right), \tan\frac{13\pi}{12}$

Lời giải

a. Ta có :

- $\cos 225^0 = \cos(180^0 + 45^0) = -\cos 45^0 = -\frac{\sqrt{2}}{2}$

- $\sin 240^0 = \sin(180^0 + 60^0) = -\sin 60^0 = -\frac{\sqrt{3}}{2}$

- $\cot(-15^0) = -\cot 15^0 = -\tan 75^0 = -\tan(30^0 + 45^0)$

$$= \frac{-\tan 30^0 - \tan 45^0}{1 - \tan 30^0 \tan 45^0} = \frac{-\frac{1}{\sqrt{3}} - 1}{1 - \frac{1}{\sqrt{3}}}$$

$$= -\frac{\sqrt{3}+1}{\sqrt{3}-1} = -\frac{(\sqrt{3}+1)^2}{2} = -2 - \sqrt{3}.$$

b. Ta có :

- $$\sin\frac{7\pi}{12} = \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \sin\frac{\pi}{3}\cos\frac{\pi}{4} + \cos\frac{\pi}{3}\sin\frac{\pi}{4}$$
  $$= \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) = \frac{\sqrt{6}+\sqrt{2}}{4} \approx 0,9659$$

- $$\cos\left(-\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{4} - \frac{\pi}{3}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{4}\sin\frac{\pi}{3}$$
  $$= \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) \approx 0,9659$$

- $$\tan\frac{13\pi}{12} = \tan\left(\pi + \frac{\pi}{12}\right) = \tan\frac{\pi}{12} = \tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$
  $$= \frac{\tan\frac{\pi}{3} - \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{3}\tan\frac{\pi}{4}} = \frac{\sqrt{3}-1}{1+\sqrt{3}} = 2 - \sqrt{3} \approx 0,2679$$

Bài 2 : Tính :

a. $\cos\left(\alpha+\frac{\pi}{3}\right)$, biết $\sin\alpha=\frac{1}{\sqrt{3}}$ và $0<\alpha<\frac{\pi}{2}$

b. $\tan\left(\alpha-\frac{\pi}{4}\right)$, biết $\cos\alpha=-\frac{1}{3}$ và $\frac{\pi}{2}<\alpha<\pi$

c. $\cos(a+b), \sin(a-b)$ biết

$\sin a=\frac{4}{5}, 0<a<90^0, \sin b=\frac{2}{3}, 90^0<b<180^0$

Lời giải

a.Ta có : $\cos\alpha=\sqrt{1-\sin^2\alpha}=\sqrt{1-\frac{1}{3}}=\sqrt{\frac{2}{3}}$

(do $0<\alpha<\frac{\pi}{2}$ nên $\cos\alpha>0$)

vậy $\cos\left(\alpha+\frac{\pi}{3}\right)$

$$=\cos\alpha.\cos\frac{\pi}{3}-\sin\alpha.\sin\frac{\pi}{3}=\sqrt{\frac{2}{3}}.\frac{1}{2}-\frac{1}{\sqrt{3}}.\frac{\sqrt{3}}{2}=\frac{1}{\sqrt{6}}-\frac{1}{2}$$

b. áp dụng $1+\tan^2\alpha=\frac{1}{\cos^2\alpha}$

$$\Rightarrow \tan^2\alpha=\frac{1}{\cos^2\alpha}-1=\frac{1}{\left(-\frac{1}{3}\right)^2}-1=9-1=8$$

$\Rightarrow \tan\alpha=-2\sqrt{2}$ (do $\frac{\pi}{2}<\alpha<\pi$ nên $\tan\alpha<0$)

$$\tan\left(\alpha-\frac{\pi}{4}\right)=\frac{\tan\alpha-\tan\frac{\pi}{4}}{1+\tan\alpha.\tan\frac{\pi}{4}}=\frac{-2\sqrt{2}-1}{1-2\sqrt{2}}=\frac{1+2\sqrt{2}}{2\sqrt{2}-1}$$

$$=\frac{(1+2\sqrt{2})^2}{7}=\frac{9+4\sqrt{2}}{7}$$

c.

* Vì $0^0 < a < 90^0$ nên :

$$\cos a = \sqrt{1-\sin^2 a} = \sqrt{1-\left(\frac{4}{5}\right)^2} = \sqrt{1-\frac{16}{25}} = \frac{3}{5}$$

* Vì $90^0 < b < 180^0$ nên :

$$\cos b = \sqrt{1-\sin^2 b} = \sqrt{1-\frac{4}{9}} = -\sqrt{\frac{5}{9}} = -\frac{\sqrt{5}}{3}$$

* Vậy :

$$\cos(a+b) = \cos a.\cos b - \sin a.\sin b$$

* $$= \frac{3}{5}.\left(-\frac{\sqrt{5}}{3}\right) - \frac{4}{5}.\frac{2}{3} = -\frac{1}{\sqrt{5}} - \frac{8}{15}$$

$$\sin(a-b) = \sin a.\cos b - \cos a.\sin b$$

* $$= \left(\frac{4}{5}\right).\left(-\frac{\sqrt{5}}{3}\right) - \left(\frac{3}{5}\right).\left(\frac{2}{3}\right) = \frac{4}{3\sqrt{5}} - \frac{2}{5}$$

Bài 3 : Rút gọn biểu thức :

a. $\sin(a+b) + \sin\left(\frac{\pi}{2}-a\right)\sin(-b)$

b. $\cos\left(\frac{\pi}{4}+a\right).\cos\left(\frac{\pi}{4}-a\right) + \frac{1}{2}\sin^2 a$

c. $\cos\left(\frac{\pi}{2}-a\right)\sin\left(\frac{\pi}{2}-a\right) - \sin(a-b)$

Lời giải

a. Ta có : $\sin(a+b)+\sin\left(\frac{\pi}{2}-a\right)\sin(-b)$

$=\sin a.\cos b+\sin b.\cos a-\cos a\sin b$

$=\sin a.\cos b$

b . Ta có : $\cos\left(\frac{\pi}{4}+a\right).\cos\left(\frac{\pi}{4}-a\right)+\frac{1}{2}\sin^2 a$

$=\frac{1}{2}\left[\cos 2a+\cos\frac{\pi}{2}\right]+\frac{1}{2}\sin^2 a$

$=\frac{1}{2}\cos 2a+\frac{1}{2}\sin^2 a$

$=\frac{1}{2}(1-2\sin^2 a+\sin^2 a)$

$=\frac{1}{2}\cos^2 a.$

c. Ta có : $\cos\left(\frac{\pi}{2}-a\right)\sin\left(\frac{\pi}{2}-a\right)-\sin(a-b)$

$=\sin a.\cos b-(\sin a.\cos b-\sin b.\cos a)$

$=\cos a.\sin b.$

Bài 4 : Chứng minh các đẳng thức :

a. $\frac{\cos(a-b)}{\cos(a+b)}=\frac{\cot a.\text{cotb}+1}{\cot a.\cot b-1}$

b. $\sin(a+b).\sin(a-b)=\sin^2 a-\sin^2 b=\cos^2 a-\cos^2 b$

c. $\cos(a+b).\cos(a-b)=\cos^2 a-\sin^2 b=\cos^2 b-\sin^2 a$

Lời giải

a. Ta có:

$$\frac{\cos(a-b)}{\cos(a+b)}=\frac{\cos a.\cos b+\sin a.\sin b}{\cos a.\cos b-\text{sina.sinb}}$$

$$=\frac{\dfrac{\cos a.\cos b}{\sin a.\sin b}+1}{\dfrac{\cos a.\cos b}{\sin a.\sin b}-1}$$

$$=\frac{\cot a.\text{cotb}+1}{cota.\cot b-1}$$

b. Ta có : $\sin(a+b).\sin(a-b)$

$=(\sin a.\cos b+\sin b.\cos a)(\sin a.\cos b-\sin b.\cos a)$

$=\cos^2 a.\cos^2 b-\sin^2 a.\sin^2 b=\cos^2 a(1-\sin^2\text{b})-\sin^2 b(1-\sin^2 a)$

$=\sin^2 a-\sin^2 b(dpcm)$

$=(1-\cos^2 a)-(1-\cos^2 b)=\cos^2 b-\cos^2 a\,(dpcm)$

c. Ta có :

$\cos(a+b).\cos(a-b)$

$=(\text{cosa.cosb}-\text{sina.sinb})(\text{cosa.cosb}+\text{sina.sinb})$

$=\cos^2 a.\cos^2 b-\sin^2 a.\sin^2 b$

$=\cos^2 a(1-\sin^2 a)-\sin^2 b(1-\cos^2 a)$

$=\cos^2 a-\sin^2 b(dpcm)=(1-\sin^2 a)-(1-\cos^2\text{b})$

$=\cos^2 b-\sin^2 a$

Bài 5 (trang 154 SGK Đại số 10): Tính sin2a, cos2a, tan2a biết :

a. sina = -0,6 và $\pi < a < \frac{3\pi}{2}$

b. cosa = $-\frac{15}{3}$ và $\frac{\pi}{2} < a < \pi$

c. sina + cosa=$\frac{1}{2}$ và $\frac{3\pi}{4} < a < \pi$

Lời giải

a. Vì sina = -0,6 và $\pi < a < \frac{3\pi}{2}$ nên:

$$\cos a = -\sqrt{1-\sin^2 a} = -\sqrt{1-(-0,6)^2} = \sqrt{1-0,36}$$

$$= -\sqrt{0,64} = -0,8 \text{ và } \tan a = \frac{\sin a}{\cos a} = \frac{-0,6}{-0,8} = \frac{3}{4}$$

Vậy :

* $\sin 2a = 2\sin a.\cos a = 2.(-0,6).(-0,8) = 0,96$

* $\cos 2a = 1 - 2\sin^2 a = 1 - 2.(0.6)^2 = 0,28$

* $\tan 2a = \frac{\sin 2a}{\cos 2a} = \frac{0,96}{0,28} = \frac{96}{28} = \frac{24}{7}$

b. Vì cosa $= -\frac{15}{3}$ và $\frac{\pi}{2} < a < \pi$ nên:

$$\sin a = \sqrt{1 - \sin^2 a} = \sqrt{1 - \frac{25}{169}} = \frac{12}{13}$$

Và $\tan a = \frac{\sin a}{\cos a} = \frac{12}{13} : \left(-\frac{5}{13}\right) = \frac{12}{13} . \left(-\frac{13}{5}\right) = -\frac{12}{5}$

Vậy :

* $\sin 2a = 2\sin a . \cos a = 2.\frac{12}{13} : \left(-\frac{5}{13}\right) = -\frac{120}{169}$

* $\cos 2a = 1 - 2\sin^2 a = 1 - 2.\left(\frac{12}{13}\right)^2 = 1 - \frac{288}{169} = -\frac{119}{169}$

* $\tan 2a = \frac{\sin 2a}{\cos 2a} = \left(-\frac{120}{169}\right) : \left(-\frac{119}{169}\right) = \frac{120}{119}$

c. sina + cosa$=\frac{1}{2} \Rightarrow (\text{sina} + \text{cosa})^2 = \frac{1}{4}$ nên :

$$\sin^2 a + \sin 2a + \cos^2 a = \frac{1}{4} \Rightarrow \sin 2a = \frac{1}{4} - 1 = -\frac{3}{4}$$

Mặt khác $\frac{3\pi}{4} < a < \pi$ nên $\frac{3\pi}{4} < 2a < 2\pi$

Vậy:

$$\cos 2a = \sqrt{1 - \sin^2 a} = \sqrt{1 - \frac{9}{10}} = \frac{\sqrt{7}}{4}$$

$$\tan 2a = \frac{\sin 2a}{\cos 2a} = \left(-\frac{3}{4}\right) : \left(-\frac{\sqrt{7}}{4}\right) = \frac{3}{\sqrt{7}}$$

Bài 6 (trang 154 SGK Đại số 10):

Cho sin2a $= -\frac{5}{9}$ và $\frac{\pi}{2} < a < \pi$. Tính sina và cosa

Lời giải

* Vì $\frac{\pi}{2} < a < \pi$ nên sina > 0 và cos a < 0

* Vì $\frac{\pi}{2} < a < \pi \Leftrightarrow \pi < 2a < 2\pi$

  nên cos2a có thể dương và có thể âm

Vậy cos2a = $\pm\sqrt{1-\sin^2 a} = \pm\sqrt{1-\frac{25}{81}} = \pm\frac{\sqrt{56}}{9} = \pm\frac{2\sqrt{14}}{9}$

Khi $\cos 2a = \frac{2\sqrt{14}}{9}$:

* Cosa = $-\sqrt{\frac{1+\cos 2a}{2}} = -\sqrt{\frac{1+\frac{2\sqrt{14}}{9}}{2}} = -\sqrt{\frac{9+2\sqrt{14}}{18}}$

$$= -\sqrt{\frac{18+4\sqrt{14}}{36}} = -\sqrt{\left(\frac{2+\sqrt{14}}{6}\right)^2} = -\frac{2+\sqrt{14}}{6}$$

* $\sin a = \sqrt{\frac{1-\cos 2a}{2}} = \frac{2-\sqrt{14}}{6}$

Khi $\cos 2a = -\frac{2\sqrt{14}}{9}$:

* $\cos a = -\frac{2-\sqrt{14}}{6}$

* $\sin a = \frac{2+\sqrt{14}}{6}$

Bài 7 (trang 155 SGK Đại số 10): Biến đổi thành tích các biểu thức sau:

a. 1 – sinx

b. 1 + sinx

c. 1 + 2cosx

d. 1 – 2sinx

Lời giải

a. Ta có: $1-\sin x = 1-2\sin\frac{x}{2}.\cos\frac{x}{2}$

$= \sin^2\frac{x}{2} - 2\sin\frac{x}{2}.\cos\frac{x}{2} + \cos^2\frac{x}{2}$

$= \left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)^2$

b. Ta có: $1+\sin x = \left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2$

c. Ta có:

$1+2\cos x = 2\left(\frac{1}{2}+\cos x\right) = 2\left(\cos\frac{\pi}{2}+\cos x\right)$

$= 4\cos\left(\frac{\pi}{6}+\frac{x}{2}\right)\cos\left(\frac{\pi}{6}-\frac{x}{2}\right)$

d. Ta có:

$1-2\sin x = 2\left(\frac{1}{2}-\sin x\right) = 2\left(\sin\frac{\pi}{6}+\sin x\right)$

$= 4\cos\left(\frac{\pi}{12}+\frac{x}{2}\right)\sin\left(\frac{\pi}{12}-\frac{x}{2}\right)$

Bài 8 :

Rút gọn biểu thức A = $\frac{\sin x+\sin 3x+\sin 5x}{\cos x+\cos 3x+\cos 5x}$

Lời giải

Ta có:

+) $\sin x + \sin 3x + \sin 5x$

$= (\sin 5x + \sin x) + \sin 3x$

$= 2\sin 3x.\cos 2x + \sin 3x$

$= \sin 3x(2\cos 2x + 1)$

+) $\cos x + \cos 3x + \cos 5x$

$= (\cos 5x + \cos x) + \cos 3x$

$= 2\cos 3x.\cos 2x + \cos 3x$

$= \cos 3x(2\cos 2x + 1)$

Vậy A = $\frac{\sin 3x(2\cos 2x+1}{\cos 3x(2\cos 2x+1)} = \tan 3x$.