Bài 1 : Tính

a) $9^{\frac{2}{5}}.27^{\frac{2}{5}}$ b) $144^{\frac{3}{4}}:9^{\frac{3}{4}}$

c) $\left(\frac{1}{16}\right)^{-0,75}+0,25^{-\frac{5}{2}}$ d) $(0,04)^{-0,15}-(0,125)^{-\frac{2}{3}}$

Lời giải:

a) $9^{\frac{2}{5}}.27^{\frac{2}{5}}=(9,27)^{\frac{2}{5}}=(3^2.3^3)^{\frac{2}{5}}=\left(3^5\right)^{\frac{2}{5}}=3^2=9.$

b) $(144)^{\frac{3}{4}}:9^{\frac{3}{4}}=\left(\frac{144}{9}\right)^{\frac{3}{4}}=16^{\frac{3}{4}}=\left(2^4\right)^{\frac{3}{4}}=2^3=8.$

c) $\left(\frac{1}{16}\right)^{-0,75}-(0,125)^{\frac{2}{3}}=16^{0,75}+\left(\frac{1}{4}\right)^{\frac{5}{2}}=16^{\frac{3}{4}}+4^{\frac{5}{2}}$

$=\left(2^4\right)^{\frac{3}{4}}+\left(2^2\right)^{\frac{5}{2}}=2^3+2^5=8+32=40.$

d) $(0,04)^{-1,5}-(0,125)^{-\frac{2}{3}}=\left(\frac{1}{25}\right)^{-\frac{3}{2}}-\left(\frac{1}{8}\right)^{-\frac{2}{3}}=(25)^{\frac{3}{2}}-(8)^{\frac{2}{3}}$

$=\left(5^2\right)^{\frac{3}{2}}-\left(2^3\right)^{\frac{2}{3}}=5^3-2^2=125-4=121.$

Bài 2 : Cho a, b là những số thực dương. Viết các biểu thức sau dưới dạng lũy thừa với số mũ hữu tỉ:

a) $a^{\frac{1}{3}}.\sqrt{a}$ b) $b^{\frac{1}{2}}.b^{\frac{1}{3}}.\sqrt[6]{b}$

c) $a^{\frac{4}{3}}:\sqrt[3]{a}$ d) $\sqrt[3]{b}:b^{\frac{1}{6}}$

Lời giải:

a) $a^{\frac{1}{3}}.\sqrt{a} = a^{\frac{1}{3}}.a^{\frac{1}{2}} = a^{\frac{1}{3}+\frac{1}{2}} = a^{\frac{5}{6}}.$

b) $b^{\frac{1}{2}}.b^{\frac{1}{3}}.\sqrt[6]{b} = b^{\frac{1}{2}}.b^{\frac{1}{3}}.b^{\frac{1}{6}} = b^{\frac{1}{2}+\frac{1}{3}+\frac{1}{6}} = b^{1} = b.$

c) $a^{\frac{4}{3}} : \sqrt[3]{a} = a^{\frac{4}{3}} : a^{\frac{1}{3}} = a^{\frac{4}{3}-\frac{1}{3}} = a^{1} = a.$

d) $\sqrt[3]{b} : b^{\frac{1}{6}} = b^{\frac{1}{3}} : b^{\frac{1}{6}} = b^{\frac{1}{3}-\frac{1}{6}} = b^{\frac{1}{6}}.$

Bài 3 : Viết các số sau theo thứ tự tăng dần:

Viết các số sau theo thứ tự tăng dần:

a) $1^{3,75}; 2^{-1}; \left(\frac{1}{2}\right)^{-3}$

b) $98^{0}; \left(\frac{3}{7}\right)^{-1}; 32^{\frac{1}{5}}$

Lời giải:

a) Ta có: $1^{3,75} = 1; 2^{-1} = \frac{1}{2}; \left(\frac{1}{2}\right)^{-3} = 2^{3}$

Vì $\frac{1}{2} < 1 < 2^{3}$ nên thứ tự tăng dần của ba số là:

$2^{-1}; 1^{3,75}; \left(\frac{1}{2}\right)^{-3}$

b) Ta có: $98^{0} = 1; \left(\frac{3}{7}\right)^{-1} = \left(\frac{3}{7}\right); 32^{\frac{1}{5}} = (2^{5})^{\frac{1}{5}} = 2$

Vì $1<3<\frac{7}{3}$ nên thứ tự tăng dần của 3 số là:

$98^{0}; 32^{\frac{1}{5}}; \left(\frac{3}{7}\right)^{-1}$

Bài 4 : Rút gọn các biểu thức sau:

Rút gọn các biểu thức sau:

a) $\dfrac{a^{\frac{4}{3}}\left(a^{-\frac{1}{3}}+a^{\frac{3}{2}}\right)}{a^{\frac{1}{4}}\left(a^{\frac{3}{4}}+a^{-\frac{1}{4}}\right)}$

b) $\dfrac{b^{\frac{1}{5}}\left(\sqrt[5]{b^4}-\sqrt[5]{b^{-1}}\right)}{b^{\frac{3}{2}}\left(\sqrt[3]{b}-\sqrt[3]{b^{-2}}\right)}$

c) $\dfrac{a^{\frac{1}{3}}b^{\frac{1}{3}}-a^{-\frac{1}{3}}b^{\frac{1}{3}}}{\sqrt[3]{a^2}-\sqrt[3]{a^2}}$

d) $\dfrac{a^{\frac{1}{3}}\sqrt{b}+b^{\frac{1}{3}}\sqrt{a}}{\sqrt[6]{a}+\sqrt[6]{b}}$

Lời giải:

a) Ta có: $a^{\frac{4}{3}}\left(a^{-\frac{1}{3}}+a^{\frac{3}{2}}\right)=a^{\frac{4}{3}-\frac{1}{3}}+a^{\frac{4}{3}+\frac{2}{3}}=a+a^2;$

$a^{\frac{1}{4}}\left(a^{\frac{3}{4}}+a^{-\frac{1}{4}}\right)=a^{\frac{1}{4}+\frac{3}{4}}+a^{\frac{1}{4}-\frac{1}{4}}=a+1$

Vậy $\dfrac{a^{\frac{4}{3}}\left(a^{-\frac{1}{3}}+a^{\frac{3}{2}}\right)}{a^{\frac{1}{4}}\left(a^{\frac{3}{4}}+a^{-\frac{1}{4}}\right)}=\dfrac{a+a^2}{a+1}=a\ (a>0).$

b) $b^{\frac{1}{5}}\left(\sqrt[5]{b^4}-\sqrt[5]{b^{-1}}\right)=b^{\frac{1}{5}}.b^{\frac{1}{4}}-b^{\frac{1}{5}}.b^{-\frac{1}{5}}$

$=b^{\frac{1}{5}+\frac{4}{5}}-b^{\frac{1}{5}-\frac{1}{5}}=b-1$

$b^{\frac{3}{2}}\left(\sqrt[3]{b}-\sqrt[3]{b^{-2}}\right)=b^{\frac{2}{3}}.b^{\frac{1}{3}}-b^{\frac{2}{3}}.b^{-\frac{2}{3}}$

$=b^{\frac{2}{3}+\frac{1}{3}}-b^{\frac{2}{3}-\frac{2}{3}}=b-1$

Vậy $\dfrac{b^{\frac{1}{5}}\left(\sqrt[5]{b^4}-\sqrt[5]{b^{-1}}\right)}{b^{\frac{3}{2}}\left(\sqrt[3]{b}-\sqrt[3]{b^{-2}}\right)}=\dfrac{b-1}{b-1}=1.\ (b>0,b\neq 1)$

c) $\frac{a^{\frac{1}{3}}b^{\frac{1}{3}}-a^{-\frac{1}{3}}b^{\frac{1}{3}}}{\sqrt[3]{a^2}-\sqrt[3]{a^2}}=\frac{(a^{\frac{2}{3}-\frac{1}{3}}b^{-\frac{1}{3}}-a^{-\frac{1}{3}}b^{\frac{2}{3}-\frac{1}{3}})}{\left(a^{\frac{1}{6}}+a^{\frac{1}{6}}\right)}=\frac{(a^{\frac{2}{3}}(ab)^{-\frac{1}{3}}-b^{\frac{2}{3}}(ab)^{-\frac{1}{3}})}{\left(a^{\frac{1}{6}}+a^{\frac{1}{6}}\right)}$

$=(ab)^{-\frac{1}{3}}=\frac{1}{\sqrt[3]{ab}}$

$(a>0,b>0,a\neq b)$

d) $\frac{a^{\frac{1}{3}}\sqrt{b}+b^{\frac{1}{3}}\sqrt{a}}{\sqrt[6]{a}+\sqrt[6]{b}}=\frac{a^{\frac{1}{3}}.a^{\frac{1}{2}}+b^{\frac{1}{3}}a^{\frac{1}{2}}}{a^{\frac{1}{6}}+a^{\frac{1}{6}}}=\frac{(a^{\frac{2}{6}}.b^{\frac{2}{6}}b^{\frac{1}{6}}-b^{\frac{2}{6}}.a^{\left(\frac{2}{6}\right)}.a^{\frac{1}{6}})}{a^{\frac{1}{6}}+a^{\frac{1}{6}}}$

$=\frac{(ab)^{\frac{1}{3}}\left[b^{\frac{1}{6}}+a^{\frac{1}{6}}\right]}{a^{\frac{1}{6}}+a^{\frac{1}{6}}}=\sqrt[3]{ab}$

$(a>0,b>0,a.b\neq 0)$

Bài 5 : Chứng minh rằng:

Chứng minh rằng:

a) $\left(\frac{1}{3}\right)^{2\sqrt{5}}<\left(\frac{1}{3}\right)^{3\sqrt{2}}$

b) $7^{6\sqrt{3}}>7^{3\sqrt{6}}$

Lời giải:

a) Ta có: $2\sqrt{5}=\sqrt{2^2.5}=\sqrt{20}$

$3\sqrt{2}=\sqrt{3^2.2}=\sqrt{18}$

vì $\sqrt{20}>\sqrt{18}$nên $2\sqrt{5}>3\sqrt{2}$

mặt khác: $0<\frac{1}{3}<1$ và $2\sqrt{5}>3\sqrt{2}$ nên $\left(\frac{1}{3}\right)^{2\sqrt{5}}<\left(\frac{1}{3}\right)^{3\sqrt{2}}$

b) Ta có: $6\sqrt{3}=\sqrt{6^2.3}=\sqrt{108}$

$3\sqrt{6}=\sqrt{3^3.6}=\sqrt{54}$

Vì $7>1$ và $\sqrt{108}>\sqrt{54}$ nên $7^{\sqrt{108}}>7^{\sqrt{54}}$

Vậy $7^{6\sqrt{3}}>7^{3\sqrt{6}}$.