Bài 1 : Tính các tích phân sau:

a) $\int_{-\frac{1}{2}}^{\frac{1}{2}} \sqrt[3]{(1-x)^2}dx$ b) $\int_0^{\frac{\pi}{2}} \sin(\frac{\pi}{4} - x)dx$

c) $\int_{\frac{1}{2}}^{2} \frac{1}{x(x+1)} dx$ d) $\int_0^2 x(x+1)^2dx$

e) $\int_{\frac{1}{2}}^{2} \frac{1-3x}{(x+1)^2} dx$ g) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin3xcos5xdx$

Lời giải:

a) Đặt 1 – x = u ; du = -dx; $x = -\frac{1}{2} => u = \frac{3}{2}$

$x = \frac{1}{2} => u = \frac{1}{2}$

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} \sqrt[3]{(1-x)^2}\,dx = -\int_{\frac{3}{2}}^{\frac{1}{2}} u^{\frac{2}{3}}du = \left.\frac{u^{\frac{2}{3}+1}}{\frac{2}{3}+1}\right|_{\frac{1}{2}}^{\frac{3}{2}}$$

$$= \frac{3}{5}u^{\frac{5}{3}}\Big|_{\frac{1}{2}}^{\frac{3}{2}} = \frac{3}{5}\left[\sqrt[3]{\left(\frac{3}{2}\right)^5} - \sqrt[3]{\left(\frac{1}{2^5}\right)}\right] = \frac{3}{10}\frac{(3\sqrt[3]{9}-1)}{\sqrt[3]{4}}$$

b) Đặt u(x) = $\frac{\pi}{4} - x$; du(x) = -dx

X = 0 => $\frac{\pi}{4}$; $x = \frac{\pi}{2}$=> u = - $\frac{\pi}{4}$

$$\int_0^{\frac{\pi}{2}} \sin(\frac{\pi}{4} - x)dx = -\int_{\frac{\pi}{4}}^{-\frac{\pi}{4}} \text{sinudu} = cosu\Big|_{\frac{\pi}{4}}^{-\frac{\pi}{4}}$$

$$= \cos\left(-\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{4}\right) = 0$$

c) $\int_{\frac{1}{2}}^{2}\frac{1}{x(x+1)}dx = \int_{\frac{1}{2}}^{2}[\frac{1}{x}-\frac{1}{x+1}]dx$

$$\int_{\frac{1}{2}}^{2}\frac{1}{x}dx-\int_{\frac{1}{2}}^{2}\frac{1}{x+1}dx = lnx|_{\frac{1}{2}}^{2}-\ln(x+1)|_{\frac{1}{2}}^{2}$$

$$= \ln\left(\frac{x}{x+1}\right)\Big|_{\frac{1}{2}}^{2} = \ln\left(\frac{2}{3}\right)-\ln\left(\frac{1}{3}\right) = ln2$$

d) $\int_0^2 x(x+1)^2dx = \int_0^2(x^3+2x^2+x)dx = \frac{x^4}{4}+\frac{2}{3}x^3+\frac{1}{2}x^2\Big|_0^2$

$$= \frac{16}{4}+\frac{16}{3}+\frac{4}{2} = \frac{34}{3}$$

e) $\int_{\frac{1}{2}}^{2}\frac{1-3x}{(x+1)^2}dx = \int_{\frac{1}{2}}^{2}\frac{4-3(x+1)}{(x+1)^2}dx$

$$\int_{\frac{1}{2}}^{2}\frac{4}{(x+1)^2dx}-\int_{\frac{1}{2}}^{2}\frac{3}{x+1}dx = \left(-\frac{4}{x+1}-3\ln|x+1|\right)\Big|_{\frac{1}{2}}^{2}$$

$$= -\frac{4}{3}-3ln3+\frac{8}{3}+3\ln\left(\frac{3}{2}\right) = \frac{4}{3}-3ln2$$

f) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin3xcos5xdx = \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(sin8x-sin2x)dx$

$$= \left(-\frac{1}{16}cos8x+\frac{1}{4}cos2x\right)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= -\frac{1}{16}cos4\pi+\frac{1}{4}\cos\pi+\frac{1}{16}\cos(-4\pi)-\frac{1}{4}\cos(-\pi) = 0$$

Bài 2 : Tính các tích phân sau:

a) $\int_0^2|1-x|dx$

b) $\int_0^{\frac{\pi}{2}}\sin^2 x\,dx$

c) $\int_0^{ln2}\frac{e^{2x+1}+1}{e^x}dx$

d) $\int_0^{\pi} sin2xcos^2xdx$

Lời giải:

a) Ta $|1-x| = \begin{cases} 1-x \text{ với } 0 \le x \le 1 \\ x-1 \text{ với } 1 \le x \le 2 \end{cases}$

$$\Rightarrow \int_0^2 |1-x|dx = \int_0^1 (1-x)dx + \int_1^2 (x-1)dx$$

$$= \left(x - \frac{x^2}{2}\right)\Big|_0^1 + \left(\frac{x^2}{2} - x\right)\Big|_1^2 = \frac{1}{2} + \frac{1}{2} = 1$$

b) Ta có:

$$\int_0^{\frac{\pi}{2}} \sin^2 x dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} (1 - cos2x)dx = \left(\frac{x}{2} - \frac{1}{4}sin2x\right)\Big|_0^{\frac{\pi}{2}} = \frac{\pi}{4}$$

c) Ta có: $\int_0^{ln2} \frac{e^{2x+1}+1}{e^x} dx = \int_0^{ln2} e^{x+1} dx + \int_0^{ln2} e^{-x} dx$

$$= (e^{x+1} - e^{-x})|_0^{ln2} = \left(2e - \frac{1}{2}\right) - (e-1) = e + \frac{1}{2}$$

d) Ta có: $\int_0^{\pi} sin2xcos^2xdx = 2\int_0^{\pi} sinxcos^3xdx$

$$= -2\int_0^{\pi} \cos^3 x d(cosx) = -\frac{\cos^4 x}{2}\Big|_0^{\pi} = -\frac{1}{2} + \frac{1}{2} = 0$$

Bài 3 (): Sử dụng phương pháp đổi biến, hãy tính:

a) $\int_0^3 \frac{x^2}{(x+1)^{\frac{3}{2}}} dx$ (đặt $u = x + 1$)

b) $\int_0^1 \sqrt{1-x^2}\, dx$(đặt x = sint)

c) $\int_0^1 \frac{e^x(1+x)}{1+xe^x} dx$ (đặt $x = 1 + xe^x$)

d) $\int_0^{\frac{a}{2}} \frac{1}{\sqrt{a^2-x^2}} dx$ $(a > 0)$đặt $x = asint$

Lời giải:

a) Đặt u = x + 1 => du = dx, ta có: x= 0 => u= 1

$x = 3 => u = 4$

Ta có: $\int_0^3 \frac{x^2}{(x+1)^{\frac{3}{2}}} dx = \int_1^4 \frac{(u-1)^2}{u^{\frac{3}{2}}} du$

$$\int_1^4 \frac{u^2 - 2u + 1}{u^{\frac{3}{2}}} du = \left(\frac{2}{3} u^{\frac{3}{2}} - 4u^{\frac{1}{2}} - 2u^{-\frac{1}{2}}\right)\Big|_1^4$$

$$= -\frac{11}{3} + \frac{16}{3} = \frac{5}{3}$$

b) Đặt $x = sint, t \in \left[0; \frac{\pi}{2}\right], dx = costdt$

$x = 0 => t = 0;$

$x = 1 => t = \frac{\pi}{2}$

Ta có: $\int_0^1 \sqrt{1 - x^2}\, dx = \int_0^{\frac{\pi}{2}} \sqrt{1 - \sin^2 t}\, costdt$

$$= \int_0^{\frac{\pi}{2}} \cos^2 tdt = \frac{1}{2} \int_0^{\frac{\pi}{2}} (1 + cos2t)dt$$

$$= (\frac{t}{2} + \frac{1}{4} sin2t)\Big|_0^{\frac{\pi}{2}} = \frac{\pi}{4}$$

c) Đặt $u = 1 + xe^x => du = e^x(1 + x)dx$

$x = 0 => u = 1;$

$x = 1 => u = 1 + e;$

$$=> \int_0^1 \frac{e^x(1 + x)}{1 + xe^x} dx = \int_0^{1+e} \frac{du}{u} = lnu\Big|_1^{1+e} = \ln(1 + e)$$

d) Đặt $x = asint => dx = acostdt$

$x = 0 => t = 0;$

$x = \frac{a}{2} => t = \frac{\pi}{6}$

$$=> \int_0^{\frac{a}{2}} \frac{1}{\sqrt{a^2 - x^2}} dx = \int_0^{\frac{\pi}{6}} \frac{acost}{acost} dt = t\Big|_0^{\frac{\pi}{6}} = \frac{\pi}{6}$$

Bài 4 : Sử dụng phương pháp tích phân từng phần, hãy tính:

a) $\int_0^{\frac{\pi}{2}}(x+1)sinxdx$

b) $\int_1^e x^2lnxdx$

c) $\int_0^1 \ln(1+x)\,dx$

d) $\int_0^1(x^2-2x-1)e^{-x}dx$

Lời giải:

a) Đặt $u(x) = x+1, sinxdx = dv(x)$

⇨ $du(x) = dx, v(x) = -cosx$

$$\int_0^{\frac{\pi}{2}}(x+1)sinxdx = -(x+1)cosx|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} cosxdx$$

$$= [sinx - (x+1)cosx]|_0^{\frac{\pi}{2}} = 2$$

b) Đặt $u = lnx, x^2dx = dv$

⇨ $du = \frac{dx}{x}; v = \frac{x^3}{3}$

$$\int_1^e x^2lnxdx = \frac{x^3}{3}lnx\Big|_1^e - \frac{1}{3}\int_1^e \frac{x^3}{x}dx = \frac{1}{3}e^3 - \frac{x^3}{9}\Big|_1^e = \frac{2e^3+1}{9}$$

c) Đặt $u = \ln(1+x)\,, dv = dx$

$=> du = \frac{dx}{1+x}, v = x$

$$\int_0^1 \ln(1+x)\,dx = xln(1+x)|_0^1 - \int_0^1 \frac{x}{1+x}dx$$

$$= ln2 - (x - \ln(1+x))|_0^1 = 2ln2 - 1$$

d) Đặt $u = x^2 - 2x + 1; dv = e^{-x}dx$

⇨ $du = 2x - 2; v = -e^x$

$$\int_0^1(x^2-2x-1)e^{-x}dx$$

$$= (-x^2+2x+1)e^{-x}|_0^1 + 2\int_0^1(x-1)e^{-x}dx$$

$$= 2e^{-1} - 1 + 2[(1-x)e^{-x}|_0^1 + \int_0^1 e^{-x}dx]$$

$$= 2e^{-1} - 1 - 2xe^{-x}|_0^1 = -1$$

Bài 5 : Tính các tích phân sau:

a) $\int_0^1 (1+3x)^{\frac{3}{2}} dx$

b) $\int_0^{\frac{1}{2}} \frac{x^3-1}{x^2-1} dx$

c) $\int_1^2 \frac{\ln(1+x)}{x^2} dx$

Lời giải:

a) Đặt: $u = 1 + 3x => du = 3dx$

$x = 0 => u = 1; x = 1 => u = 4$

$\int_0^1 (1+3x)^{\frac{3}{2}} dx = \frac{1}{3}\int_1^4 u^{\frac{3}{2}} du = \frac{2}{15}(32-1) = \frac{62}{15}$

b) Ta có: $\frac{x^3-1}{x^2-1} = \frac{(x-1)(x^2+x+1)}{(x-1)(x+1)} = x + \frac{1}{x+1}$

$$\int_0^{\frac{1}{2}} \frac{x^3-1}{x^2-1} dx = \int_0^{\frac{1}{2}} x dx + \int_0^{\frac{1}{2}} \frac{1}{x+1} dx$$

$$= \left(\frac{x^2}{2} + \ln(x+1)\right)\Big|_0^{\frac{1}{2}} = \frac{1}{8} + \ln\left(\frac{3}{2}\right)$$

c) Đặt $u = \ln(1+x)\,, dv = \frac{dx}{x^2}$

⇨ $du = \frac{1}{1+x} dx; v = -\frac{1}{x}$

$$\int_1^2 \frac{\ln(1+x)}{x^2} dx = -\frac{\ln(1+x)}{x}\Big|_1^2 + \int_1^2 \frac{1}{x(x+1)} dx \quad (*)$$

Ta có:

$$\int_1^2 \frac{1}{x(1+x)} dx = \int_1^2 \left[\frac{1}{x} - \frac{1}{x+1}\right] dx = \ln\left(\frac{x}{x+1}\right)\Big|_1^2 = 2ln2 - ln3$$

Thay kết quả trên vào (*) ta được: $I = 3\ln\left(\frac{2}{\sqrt{3}}\right)$

Bài 6 : Tính

Tính $\int_0^1 x(1-x)^5 dx$ bằng hai phương pháp:

a) Đổi biến số $u = 1 - x$

b) Tích phân từng phần

Lời giải:

a) Đổi biến sô: $u = 1 - x => du = -dx; x = 1 - u$

Với $x = 0 => u = 1$

$x = 1 => u = 0$

$\int_0^1 x(1-x)^5 dx = -\int_0^1 (1-u)u^5 du$

$= \left(\frac{u^6}{6} - \frac{u^7}{7}\right)\Big|_0^1 = \frac{1}{42}$

b) Tính bằng phương pháp tích phân từng phần.

Đặt $u = x\ ; (1-x)^5 dx = du => du = dx\ ; v - \frac{(1-x)^6}{6}$

$\int_0^1 x(1-x)^5 dx = -\frac{x(1-x)^6}{6}\Big|_0^1 + \frac{1}{6}\int_0^1 (1-x)^6 dx$

$= -\frac{1}{42}(1-x)^7\Big|_0^1 = \frac{1}{42}.$