Low Complexity Resource Allocation for Multiuser Uplink Communication Systems

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***Abstract*—In this paper, we address a practical and large-dimensional problem of joint design of modulation and coding scheme (MCS) and transmit power allocation for an uplink massive multi-antenna system serving multiple users. We propose to jointly optimize the MCS and the transmit power for all the active users in a centralized approach. For this purpose, we formulate a joint MCS and power allocation problem as a multi-dimensional discrete optimization problem and develop a successive coordinate search (SCS) algorithm to obtain optimal solution while yielding low computational complexity. Compared to the brute-force exhaustive search procedure that incurs prohibitively high computational complexity, our proposed approach is shown to solve the problem with much lower computational complexity. Simulation results reveal that our developed SCS algorithm performs very close to the optimal solution for different number of users and antenna configurations.**

***Keywords*—Discrete optimization, modulation and coding scheme (MCS), multi-user multiple input multiple output (MU-MIMO), successive coordinate search (SCS) algorithm.**

I. Introduction

Multi-antenna transmission techniques, being one of the key enablers for the emerging fifth generation (5G) new radio (NR) system, contributed significantly to increase the spectral efficiency and transmission reliability over single-antenna counterparts. When the deployment is constrained by a smaller transmission bandwidth in sub-6GHz frequency bands, and to cover large cells at higher mobility, massive multiple-input and multiple-output (mMIMO) configurations are found to be very attractive as they enable more users spatially multiplexed to improve the network spectral efficiency and overall energy efficiency at the same time [1]–[3]. It is worth pointing out that an efficient allocation of wireless resources for mMIMO system brings additional challenges over conventional MIMO systems and several contributions have been made in the literature to address them while considering performance-complexity trade-off [4], [5]. A low-complexity power allocation algorithm is designed in [6] for an uplink mMIMO system to achieve a target error rate under imperfect channel state information. Two resource allocation schemes are proposed in [7] based on deep learning for a cell free uplink mMIMO system while considering pilot contamination effects. In [8], optimal and multiple sub-optimal dynamic user clustering, antenna selection, and power allocation algorithms are proposed for power-domain non-orthogonal multiple access scheme that demonstrate improved performances over the conventional orthogonal multiple access transmission. In [9], the authors study joint pilot design and power control in cellular massive MIMO systems while proposing a novel pilot design and combining the pilot assignment and uplink power allocation into a max-min fairness based unified optimization framework in terms of spectral efficiency. Moreover, [10] considers a single cell uplink mMIMO with non-orthogonal multiple access (NOMA) system and propose a two step algorithm to perform antenna selection and power allocation sequentially.

In fourth generation (4G) long-term evolution (LTE) and 5G NR cellular communication systems, uplink transmit power for physical uplink channels (physical uplink shared channel (PUSCH), physical uplink control channel (PUCCH), etc.) can possess discrete values depending on the number of allocated resource blocks for data transmission and wireless channel condition (e.g., path loss, etc.) while considering closed-loop system [11], [12]. Furthermore, the modulation and coding scheme (MCS), which selects the baseband modulation class (phase shift keying (PSK), quadrature amplitude modulation (QAM), etc.) and channel coding scheme (polar coding, turbo coding, etc.) while assigning its coding rate, possesses discrete values. Often times, the transmit power and MCS are calculated jointly by maximizing utility (e.g., data throughput, etc.) or minimizing cost (e.g., energy consumption, etc.). Most of the works in the literature consider this joint optimization problem as a continuous or mixed-integer program. However, practical resource (joint power and MCS) allocation algorithms require solving discrete optimization problems that often entails prohibitively high computational complexity for large number of users and antennas.

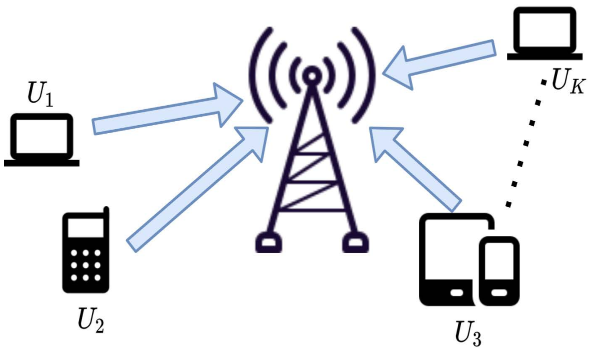

Fig. 1: System model with multi-antenna base station serving multiple users.

In this paper, we address a resource allocation problem for a single-cell uplink multi-user (MU)-MIMO system, where the the access point (AP) operates with a large number of antenna elements. We optimize the transmit power and MCS allocations for the users during uplink transmission by maximizing the network throughput. We incorporate a practical set of constraints to jointly calculate optimal transmit power and MCS, resulting in a discrete optimization problem. The optimal solution approach using brute-force exhaustive search in general yields prohibitively high computational complexity that hinders real-time data transmission. Our objective is to develop a resource allocation scheme that yields close-to-optimal network throughput while assuring lower computational complexity than the exhaustive search algorithm. Following these observations, we leverage the coordinate search approach in solving global optimization problems [13] and thereby develop a computationally efficient successive coordinate search (SCS) algorithm to solve our considered discrete optimization problem.

The rest of the paper is organized as follows. System Model is described in Section II. The proposed joint MCS and power allocation scheme is discussed in Section III, and the simulation results are shown in Section IV. At the end, Section V concludes the paper.

II. System Model

Let us consider a single-cell uplink communication system as shown in Fig. 1, where K users communicate with the AP in MU-MIMO scheme. Each user and AP consist of single antenna and N antennas, respectively. The considered AP can be regarded as the base station (BS) in cellular communication systems (e.g., eNode B (eNB) in LTE or eNode B (gNB) in NR). We assume that all the users are perfectly synchronized in time or frequency or both in time-frequency grid (e.g., using reference signals in LTE/NR). The signal received at the AP during the uplink data transmission can be represented as follows:

$$\boldsymbol{y} = \sum_{k=1}^{K} \sqrt{P_k}\boldsymbol{h}_k x_k + \boldsymbol{n}, \tag{1}$$

where $\boldsymbol{y}$ is $N \times 1$ complex-valued vector containing received signals at the antennas in AP, and $\boldsymbol{h}_k$ represents $N \times 1$ complex-valued channel vector. Each element of $\boldsymbol{h}_k$ represents Rayleigh fading channel h_k with zero mean and unit variance from user $k \in \{1, 2, \cdots, K\}$ to the antennas at AP. Moreover, x_k and P_k denote the transmit signal and its associated (transmit) power, respectively for user $k \in \{1, 2, \cdots, K\}$. Here, $\boldsymbol{n}$ represents an $N \times 1$ complex-valued additive white Gaussian noise (AWGN) vector, which contains (random) samples that follow complex-valued Gaussian distribution with zero mean and variance σ^2. The AP applies linear equalizer, e.g., zero-forcing (ZF), minimum mean square error (MMSE), etc. to detect x_k. In particular, AP applies a finite impulse response (FIR) filter with $1 \times N$ dimensional coefficient vector $\boldsymbol{w}_k$ for user k to equalize the received signal $\boldsymbol{y}$. Therefore, the output (soft estimated) signal $\hat{x}_k$ from the equalizer can be expressed as

$$\hat{x}_k = \boldsymbol{w}_k\boldsymbol{y} = \sqrt{P_k}\boldsymbol{w}_k\boldsymbol{h}_k x_k + \sum_{j \neq k} \sqrt{P_j}\boldsymbol{w}_k\boldsymbol{h}_j x_j + \boldsymbol{w}_k\boldsymbol{n}. \tag{2}$$

The signal-to-interference plus noise ratio (SINR) of user k can be represented as

$$\gamma_k = \frac{P_k G_{k,k}}{\sum\limits_{j \neq k} P_j G_{j,k} + \sigma^2}, \tag{3}$$

where $G_{k,k} = |\boldsymbol{w}_k\boldsymbol{h}_k|^2/(\boldsymbol{w}_k\boldsymbol{w}_k^H)$ and $G_{j,k} = |\boldsymbol{w}_k\boldsymbol{h}_j|^2/(\boldsymbol{w}_k\boldsymbol{w}_k^H)$. Here, $(\cdot)^H$ represents the Hermitian operation.

III. Joint MCS and Power Allocation by SCS Algorithm

In this section, we formulate a joint MCS and power allocation problem and develop a low complexity SCS algorithm to solve the optimization problem. It is worth mentioning that SCS is based on the coordinate descent algorithm [14] that converts a multi-dimensional optimization problem into a multiple single-dimensional search problems.

Problem Formulation: In most of the standardized wireless communication systems, e.g., LTE, NR, etc., MCS index jointly decides the baseband modulation and channel coding schemes according to propagation channel conditions and other factors. In addition to MCS, selecting the baseband transmit power for each user according to propagation path loss plays vital role for successful data transmission. A joint design of MCS and transmit power addresses wireless channel conditions, target received power, offset values to suppress non-linearity, etc. simultaneously and has been regarded as a vital design approach for LTE and NR communication systems [11], [12]. It is worth mentioning that in user equipment (UE), both MCS and transmit power possess discrete values to address the limited energy availability. Hence, the joint allocation of these parameters poses challenges in designing low-complexity solution approaches.

In order to jointly allocate MCS and transmit power, we formulate an optimization problem as follows:

$$\text{P1:} \quad \max_{P_k, a_k, b_k \forall k} \quad \sum_{k=1}^{K} a_k (1 - e^{-b_k \gamma_k}) \tag{4}$$

$$\text{s.t.:} \quad P_k \in \{\mathbb{P}^1, \mathbb{P}^2, \cdots, \mathbb{P}^{L_P}\}, \quad \forall k, \tag{5}$$

$$a_k \in \{\mathbb{A}^1, \mathbb{A}^2, \cdots, \mathbb{A}^{L_a}\}, \quad \forall k, \tag{6}$$

$$b_k \in \{\mathbb{B}^1, \mathbb{B}^2, \cdots, \mathbb{B}^{L_b}\}, \quad \forall k, \tag{7}$$

where (4) represents the MCS-dependent capacity metric with parameters a_k and b_k [15], [16]. In particular, $b_k = f(a_k)$, where function $f(\cdot)$ can be evaluated empirically following [16, Table 1] assuming symmetric channel capacity. Note that the one-to-one mapping between a_k and b_k exists for a wide-range of linear modulation schemes, e.g., 4-QAM, 16-QAM, 64-QAM, etc. [17]. Moreover, (4) represents a tractable format of the uplink network throughput, which assumes that transmit signals present symbols from finite alphabet of linear modulation schemes. Moreover, (4) is a function of the baseband modulation scheme, channel coding scheme, and transmit power. Constraints (5)-(7) represents discrete nature of P_k, a_k, and b_k, respectively for user $k \in \{1, 2, \cdots, K\}$. Here, L_p, L_a, and L_b denote the cardinalities of the sets containing the discrete values of transmit powers, a_k, and b_k, respectively. The solution of P1 represents the maximum data throughput that can be obtained by jointly optimizing P_k, a_k, and b_k. Let us define a vector of optimization variables as $\Theta = [P_1, P_2, \cdots, P_K, a_1, a_2, \cdots, a_K]^T$ of dimension $\mathcal{T} \times 1$. Denoting $C(\Theta) = \sum_{k=1}^{K} a_k (1 - e^{-f(a_k)\gamma_k})$ and exploiting $b_k = f(a_k)$, P1 can be reformulated as

$$\text{P2:} \quad \max_{\Theta} \quad C(\Theta) \tag{8}$$

$$\text{s.t.:} \quad \text{Constraints (5) and (6).} \tag{9}$$

Note that the solution of P2 yields optimal set Θ^*, and the optimal b_k^* can be obtained from $f(a_k^*)$ for $k \in \{1, 2, \cdots, K\}$.

Solution Approach: We observe that P2 is a discrete optimization problem that is computationally challenging to solve. We develop a SCS based joint MCS and transmit power allocation scheme that entails low-computational complexity while yielding close to (global) optimal performance. In particular, we sequentially search the optimal variable along each single dimension of Θ while keeping rest of the (optimization) variables constant in an iterative manner until the convergence is attained. Let us define Θ for a given iteration $i \in \{0, 1, 2, \cdots,\}$ as $\Theta[i] = [P_1[i], P_2[i], \cdots, P_K[i], a_1[i], a_2[i], \cdots, a_K[i]]^T$, where $\Theta[0]$ indicates the initial set of the optimization variables. First, the proposed scheme randomly selects the elements of $\Theta[0]$ from (5) and (6) assuming the elements follow uniform distribution (equally likely outcomes). Then for each successive iteration, we sequentially calculate P_k^* by maximizing $C(\tilde{\Theta}_{P_k}[i]) - C(\Theta[i])$, where $\tilde{\Theta}_{P_k}[i] = [P_1[i], \cdots, \tilde{P}_k, \cdots, P_K[i-1], a_1[i-1], \cdots, a_K[i-1]]^T$ while keeping $P_{j \neq k}$ and a_k constant for $k \in \{1, 2, \cdots, K\}$. Likewise, we calculate a_k^* by maximizing $C(\tilde{\Theta}_{a_k}[i]) - C(\Theta[i])$, where $\tilde{\Theta}_{a_k}[i] = [P_1[i], \cdots, P_K[i], a_1[i], \cdots, \tilde{a}_k, \cdots, a_K[i-1]]^T$ by considering fixed values for $a_{j \neq k}$ and P_k, $k \in \{1, 2, \cdots, K\}$. All these procedures in a given iteration are executed until the convergence is achieved. The step-by-step procedure of the proposed SCS scheme is depicted in Algorithm 1. It is worth pointing out that the developed SCS based scheme entails linear complexity with respect to K.

Exhaustive Search Scheme: A global optimal solution of P2 can be obtained by the brute-force exhaustive search (ES) over all the possible combinations of Θ and finding out the combination (of optimization variables) that yields the maximum network throughput. Note that ES entails prohibitively high

Algorithm 1 Joint Power and MCS Allocation with SCS Scheme

Randomly initialize $\Theta[0]$ following element-wise uniform distribution while satisfying constraints (5) and (6).
Calculate $C(\Theta[0])$ and set $C_{\max} = C(\Theta[0])$.
Set I and ϵ as the maximum number of iterations and minimum absolute error between successive iterations, respectively.
Set $i = 1$ and $e = V$, where V is a large real number.
while $i \leq I$ **or** $e \geq \epsilon$ **do**
  Set $\Theta[i] = [P_1[i-1], \cdots, P_K[i-1], a_1[i-1], \cdots, a_K[i-1]]^T$.
  for $k \in \{1, 2, \cdots, K\}$ **do**
    Set $\tilde{\Theta}_{P_k}[i] = [P_1[i], \cdots, \tilde{P}_k, \cdots, P_K[i-1], a_1[i-1], \cdots, a_K[i-1]]^T$.
    if $C(\tilde{\Theta}_{P_k}[i]) > C(\Theta[i])$ **then**
      Calculate $P_k[i] = \arg\max_{\tilde{P}_k \in \{\mathbb{P}^1, \cdots, \mathbb{P}^{L_P}\}} C(\tilde{\Theta}_{P_k}[i]) - C(\Theta[i])$.
    else
      Set $P_k[i] = P_k[i-1]$.
    end if
    Set $\Theta[i] = [P_1[i], \cdots, P_k[i], \cdots, P_K[i-1], a_1[i-1], \cdots, a_K[i-1]]^T$.
  end for
  for $k \in \{1, 2, \cdots, K\}$ **do**
    Set $\tilde{\Theta}_{a_k}[i] = [P_1[i], \cdots, P_K[i], a_1[i], \cdots, \tilde{a}_k, \cdots, a_K[i-1]]^T$.
    if $C(\tilde{\Theta}_{a_k}[i]) > C(\Theta[i])$ **then**
      Calculate $a_k[i] = \arg\max_{\tilde{a}_k \in \{\mathbb{A}^1, \cdots, \mathbb{A}^{L_a}\}} C(\tilde{\Theta}_{a_k}[i]) - C(\Theta[i])$.
    else
      Set $a_k[i] = a_k[i-1]$.
    end if
    Set $\Theta[i] = [P_1[i], \cdots, P_K[i], a_1[i-1], \cdots, a_k[i], \cdots, a_K[i-1]]^T$.
  end for
  Calculate $e = |C(\Theta[i]) - C_{\max}|$.
  Set $C_{\max} = C(\Theta[i])$.
  Set $i = i + 1$.
end while

computational complexity as the cardinality of search space for Θ grows exponentially with K.

Complexity: The computational complexity of ES scheme for the considered problem is $\mathcal{O}(L_p^K L_a^K)$, i.e., it (complexity) grows exponentially with K. In contrast, the complexity of SCS scheme can be represented as $\mathcal{O}((L_p + L_a)K\tilde{I})$, where $\tilde{I} \leq I$ denotes the number of iterations to converge the algorithm. Here, I represents the maximum number of iterations.

IV. Simulation Results

Simulation Parameters: In this section, we evaluate the performances of the proposed SCS-based joint P_k, a_k, and b_k allocation scheme for the considered MU-MIMO system. In particular, we solve problem P2 while considering that the perfect channel state information is available at AP. To evaluate the performance of the proposed SCS-based scheme and compare it with ES algorithm, we conduct extensive Monte Carlo simulation for 10^5 realizations in MATLAB and show the average uplink network throughput. We consider a wide-range of K (from 2 to 28) and N (from 2 to 64) to observe throughput performance in different use-cases. Moreover, the channel signal-to-noise ratio (SNR), which is defined as $1/\sigma^2$, is considered over a range of -10 dB to 30 dB. The discrete level of transmit power is taken from a set with lower and upper limits of 12 dBm to 23 dBm, respectively. We consider Turbo code with coding rate $1/3$ as channel coding scheme and quadrature PSK (QPSK), 16-QAM, 64-QAM, 256-QAM, and 1024-QAM as baseband modulations schemes. Note that for M-ary modulation scheme with a (n, k) channel coding scheme, a_k can be calculated by $k \log_2 M/n$. Moreover, we calculate b_k empirically for different values of a_k following the method discussed in [16, Table 1]. Throughout the simulations, we set $I = 20$ and $\epsilon = 0.001$.

Comparison Between SCS and ES: Figure 2 depicts the throughput as a function of channel SNR for the proposed SCS-based and ES resource allocation schemes. We set $N = 8$ and consider two scenarios for K. In particular, we assume $K = 2$ and $K = 4$ for Scenarios 1 and 2, respectively. Our objective in Fig. 2 is to show how close the considered resource allocation schemes perform in order to assess the applicability of SCS-based scheme, particularly for large number of users. For both the considered scenarios, the throughput increases linearly in low- and mid-range of SNRs, whereas increase in throughput becomes insignificant for high SNRs. It is worth mentioning that both the proposed and ES schemes perform very close to each other over the entire range of SNR and for both of the considered scenarios. However, our proposed SCS-based scheme exhibits much lower computational complexity in joint allocation of MCS and transmit power in compared to ES scheme. Hence, our proposed technique is a strong contender for a large K and antenna configurations, as observed in Fig. 2.

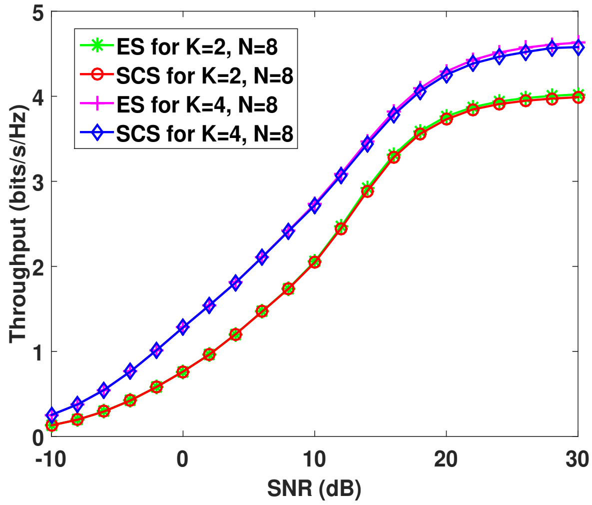

Fig. 2: Throughput as a function of SNR for SCS algorithm compared to ES method.

Throughput for Different Number of Users and Antennas: Furthermore, we analyze the performance of the proposed SCS-based resource allocation scheme for different K (with constant N) as well as for different N (with constant K) as shown in Fig. 3 and Fig. 4, respectively. Figure 3 demonstrates the throughput of the proposed SCS algorithm as a function of SNR for increasing number of users while considering 32 antennas at AP. We observe in Fig. 3 that the throughput increases with increasing number of users. However, the increase in throughput is not linearly proportional with the number of users. If we increase the number of users by the same factor, increase in throughput will not be scaled in similar manner. For instance, at 20 dB channel SNR, increasing K from 4 to 10 helps increasing throughput by 2.6 bits/s/Hz, whereas if K increases from 22 to 28 for the same setup, the throughput increases by 0.3 bits/s/Hz. This finding is observed due to the strong mutual interference among the users, as evident in (3). Similar to Fig. 3, Fig. 4 shows the throughput as a function of channel SNR for increasing number of antennas while considering 10 users in the network. It is evident from Fig. 4 that the throughput improves proportionally with the increasing number of antennas. Thus, the performance improvement for a given number of users can be enhanced with increasing number of antennas at the BS.

Convergence Behavior of SCS Scheme: In Fig. 5, we demonstrate the convergence behavior of the proposed SCS algorithm as a function of computation steps for two different configurations of N and K. In particular, we set $(N, K) = \{(4, 4), (32, 16)\}$, where for each configuration, the proposed scheme is evaluated for 0 dB and 20 dB SNRs. We observe that the proposed algorithm converges quickly within finite number of steps. However, the simulation results show that the lower SNR results in quicker convergence of SCS scheme as compared to the higher SNR for both of the considered configurations.

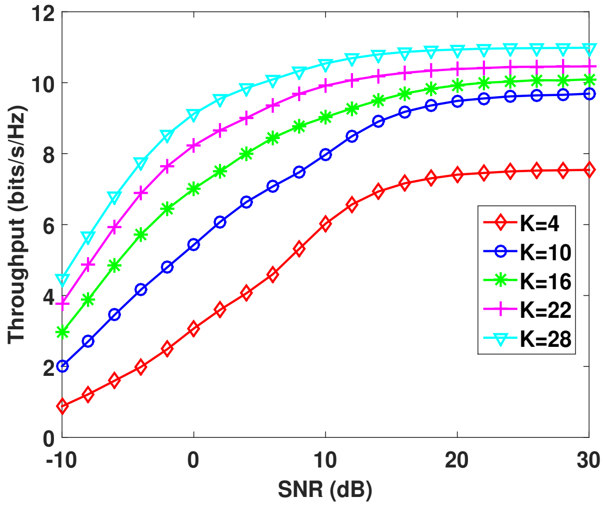

Fig. 3: Throughput of proposed SCS method as a function of SNR for different number of users considering 32 antennas.

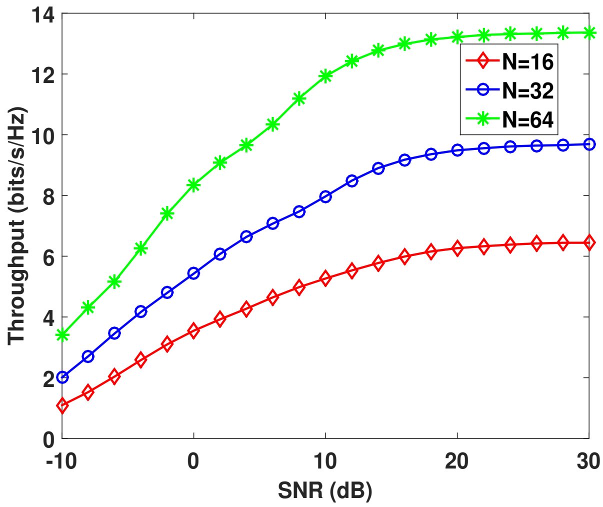

Fig. 4: Throughput of the proposed SCS method as a function of SNR for different number of antennas considering 10 users.

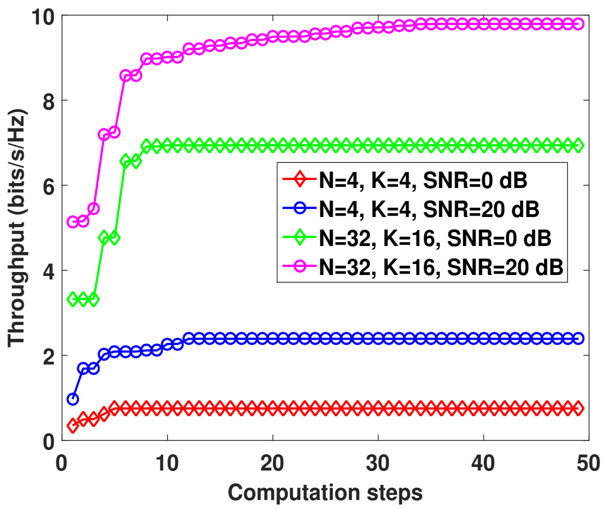

Fig. 5: Convergence plot showing throughput of the proposed SCS method as a function of computation steps.

V. Conclusions

In this paper, we developed SCS-based joint MCS and transmit power allocation scheme for an uplink massive MIMO systems. We maximized MCS dependent capacity metric by joint and optimal allocation of MCS and transmit power of all the users. As practical MCS and power possess discrete values, we formulated a discrete optimization problem that requires ES to find optimal solution, in general. We developed SCS based resource allocation scheme that shows significantly lower computational complexity compared to ES scheme and hence can be implemented efficiently in practical systems. Simulation results reveal that the proposed SCS algorithm performs very close to computationally intensive ES scheme for different number of users and antenna configurations at AP.

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