

Notations:

- p = price
- q = actual quantity
- \hat{q} = predicted quantity

Objective function:

$$\pi(\hat{q}) = \begin{cases} p\hat{q} & , \hat{q} \leq q \\ pq - 0.6p(\hat{q} - q) & , \hat{q} > q \end{cases}$$

Example:

$$\begin{cases} p = 100 \\ q = 10 \end{cases}$$

- $\hat{q} = 0$: $\pi(0) = 0$
- $\hat{q} = 5$: $\pi(5) = 500$
- $\hat{q} = 10$: $\pi(10) = 1000$

Roots:

- $p(q - 0.6\hat{q} + 0.6q) = 0 \Rightarrow \hat{q}_1 = 2.67q$
- $p\hat{q} = 0 \Rightarrow \hat{q}_2 = 0$

- $\hat{q} = 15$: $\pi(15) = 1000 - 300 = 700$
- $\hat{q} = 20$: $\pi(20) = 1000 - 600 = 400$
- $\hat{q} = 26.66$: $\pi(26.66) = 1000 - 1000 = 0$

Plot:

