Construction of a BFP^{2}G

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Definition 1: Given an LPN $G = (PN, M_0, E, \lambda)$, let $\langle \tilde{P}N, \tilde{M}_0 \rangle$ with $\tilde{P}N = (\tilde{P}, \tilde{T}, \tilde{P}re, \tilde{P}ost)$ be its FP^{2}N. The BFP^{2}G of G is a deterministic finite-state automaton $\mathcal{B} = (\tilde{\mathcal{M}}_B, \tilde{T}_{or}, \Delta, \tilde{M}_0)$ computed by using Algorithm 1, where $\tilde{\mathcal{M}}_B \subseteq \tilde{\mathcal{M}}_B^2 \cup \tilde{\mathcal{M}}_{B_{ij}}^1$ for all i and j with $(\tilde{M}_i, B_j) \in \tilde{\mathcal{M}}_{xd}^2$ is the set of states, $\tilde{T}_{or} = \tilde{T}_o \times \mathbb{N}^{|\tilde{T}_u|}$ is the set of events, $\Delta \subseteq \tilde{\mathcal{M}}_B \times \tilde{T}_{or} \times \tilde{\mathcal{M}}_B$ is the transition relation, and $\tilde{M}_0$ is the initial state.

Algorithm 1 Construction of a BFP^{2}G.

Input: An LPN $G = (PN, M_0, E, \lambda)$ with $PN = (P, T, Pre, Post)$ and a fault pattern Σ_F
Output: A BFP^{2}G $\mathcal{B} = (\tilde{\mathcal{M}}_B, \tilde{T}_{or}, \Delta, \tilde{M}_0)$ of G

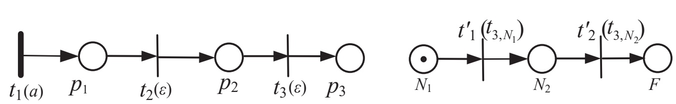
1: Compute the FP^{2}N $\langle \tilde{P}N, \tilde{M}_0 \rangle$;
2: $\tilde{\mathcal{M}}_B := \emptyset$, $\Delta_x := \emptyset$, $\Delta := \emptyset$, $B := \emptyset$, $\tilde{\mathcal{M}}_x := \{(\tilde{M}_0, B)\}$, and $\tilde{\mathcal{M}}_{new} := \{(\tilde{M}_0, B)\}$;
3: **while** $\tilde{\mathcal{M}}_{new} \neq \emptyset$ **do**
4: choose a node $(\tilde{M}, B) \in \tilde{\mathcal{M}}_{new}$;
5: **if** there exists $t \in \tilde{T}_b^F$ such that $Y_{min}^{2reg}(\tilde{M}, t) \neq \emptyset$ **then**
6: assign tag "dangerous" to $\tilde{M}$ and $(\tilde{M}, B)$;
7: call BasisSonNodes$(\tilde{M}, B)$ in Algorithm 2;
8: $\tilde{\mathcal{M}}_x := \tilde{\mathcal{M}}_x \cup \tilde{\mathcal{M}}_x^1$ and $\Delta_x := \Delta_x \cup \Delta_x^1$;
9: **end if**
10: **if** $(\tilde{M}, B)$ has no tag "dangerous" **then**
11: **for** each $t \in \tilde{T}_o^2$ and $Y_{min}^{2reg}(\tilde{M}, t) \neq \emptyset$ **do**
12: **for** each $\pi(\sigma) \in Y_{min}^{2reg}(\tilde{M}, t)$ **do**
13: $\tilde{M}' := \tilde{M} + \tilde{C}_u \cdot \pi(\sigma) + \tilde{C}(\cdot, t)$;
14: $B' := B \cup \{(\tilde{M}, \pi(\sigma))\}$;
15: **if** there exists $(\tilde{M}'', \pi(\sigma')) \in B'$, $\sigma_1 \in (\tilde{T}^2 \setminus \tilde{T}^{FP})^*$, and $\sigma_2 \in (\tilde{T}_u^2 \setminus \tilde{T}^{FP})^*$ satisfying $\tilde{M}[\sigma_1\rangle\tilde{M}_1$ and $\tilde{M}''[\sigma_2\rangle\tilde{M}_2$ with $\tilde{M}_1 \in \mathbb{N}_\omega^{|\tilde{P}|}$, $\tilde{M}_2 \in \mathbb{N}_\omega^{|\tilde{P}|}$, $\pi(\sigma_1) \leq \pi(\sigma t)$, and $\pi(\sigma_2) \leq \pi(\sigma')$ such that $\tilde{M}_1 \gneqq \tilde{M}_2$ **then**
16: find all pairs $(\tilde{M}_1, \tilde{M}_2)$ with $\tilde{M}_1 \gneqq \tilde{M}_2$;
17: for each pair $(\tilde{M}_1, \tilde{M}_2)$ with $\tilde{M}_1 \gneqq \tilde{M}_2$, let $\tilde{M}'(p) = \omega$ for each $p \in \tilde{P}$ such that $\tilde{M}_1(p) > \tilde{M}_2(p)$;
18: **end if**
19: **if** $(\tilde{M}', B') \notin \tilde{\mathcal{M}}_x$ **then**
20: $\tilde{\mathcal{M}}_x := \tilde{\mathcal{M}}_x \cup \{(\tilde{M}', B')\}$;
21: $\tilde{\mathcal{M}}_{new} := \tilde{\mathcal{M}}_{new} \cup \{(\tilde{M}', B')\}$;
22: **end if**
23: $\Delta_x := \Delta_x \cup \{((\tilde{M}, B), (t, \pi(\sigma)), (\tilde{M}', B'))\}$;
24: **end for**
25: **end for**
26: **end if**
27: $\tilde{\mathcal{M}}_{new} := \tilde{\mathcal{M}}_{new} \setminus \{(\tilde{M}, B)\}$;
28: **end while**
29: $\tilde{\mathcal{M}}_B := \{\tilde{M} \in \mathbb{N}_\omega^{|\tilde{P}|} | \exists B \in 2^{\mathbb{N}_\omega^{|\tilde{P}|} \times \mathbb{N}^{|\tilde{T}_u|}} : (\tilde{M}, B) \in \tilde{\mathcal{M}}_x\}$;
30: **for** each $((\tilde{M}, B), (t, \pi(\sigma)), (\tilde{M}', B')) \in \Delta_x$ **do**
31: $\Delta := \Delta \cup \{(\tilde{M}, (t, \pi(\sigma)), \tilde{M}')\}$;
32: **end for**

In Algorithm 1, the set $\tilde{\mathcal{M}}_{new}$ is initialized at $\{(\tilde{M}_0, \emptyset)\}$ in Step 2. For each pair $(\tilde{M}, B) \in \tilde{\mathcal{M}}_{new}$, if it is dangerous, we call function $BasisSonNodes$ in Step 7 to compute a basis son graph for the nonfailure subnet of the FPLN, which describes the normal behaviour of the FP^{2}N from $\tilde{M}$ by firing the transitions in $\tilde{T}_o^1$ and the corresponding minimal explanations. Otherwise, Steps 11 to 25 compute each successor pair $(\tilde{M}', B')$ by firing each transition in $\tilde{T}_o^2$ and its minimal explanation restricted to $\tilde{T}_u^2 \setminus \tilde{T}^{FP}$, and compute the new arc from $(\tilde{M}, B)$ to $(\tilde{M}', B')$. Then, Step 27 removes $(\tilde{M}, B)$ from $\tilde{\mathcal{M}}_{new}$ indicating that $(\tilde{M}, B)$ has been considered. The procedure from Steps 3 to Step 28 is executed iteratively until $\tilde{\mathcal{M}}_{new}$ is empty. Finally, by running Steps 29 to 32, the BFP^{2}G is obtained by removing the second component from each pair $(\tilde{M}, B) \in \tilde{\mathcal{M}}_x$.

Algorithm 2 $Function\ BasisSonNodes(\tilde{M}, B)$.

Input: A dangerous pair $(\tilde{M}, B)$ and $\tilde{P}N^1 = (\tilde{P}, \tilde{T}^1, \tilde{P}re^1, \tilde{P}ost^1)$
Output: A basis son graph $\mathcal{B}^1 = (\tilde{\mathcal{M}}_x^1, \tilde{T}_{or}^1, \Delta_x^1, (\tilde{M}, B))$

1: $\tilde{\mathcal{M}}_x^1 := \{(\tilde{M}, B)\}$, $\Delta_x^1 := \emptyset$, and $\tilde{\mathcal{M}}_{novel}^1 := \{(\tilde{M}, B)\}$;
2: **while** $\tilde{\mathcal{M}}_{novel}^1 \neq \emptyset$ **do**
3: choose a node $(\tilde{M}, B) \in \tilde{\mathcal{M}}_{novel}^1$;
4: **for** each $t \in \tilde{T}_o^1$ and $Y_{min}^1(\tilde{M}, t) \neq \emptyset$ **do**
5: **for** each $\pi(\sigma) \in Y_{min}^1(\tilde{M}, t)$ **do**
6: $\tilde{M}' := \tilde{M} + \tilde{C}_u \cdot \pi(\sigma) + \tilde{C}(\cdot, t)$;
7: $B' := B \cup \{(\tilde{M}, \pi(\sigma))\}$;
8: **if** there exists $(\tilde{M}'', \pi(\sigma')) \in B'$, $\sigma_1 \in (\tilde{T}^1)^*$, and $\sigma_2 \in (\tilde{T}_u^1)^*$satisfying $\tilde{M}[\sigma_1\rangle\tilde{M}_1$ and $\tilde{M}''[\sigma_2\rangle\tilde{M}_2$ with $\tilde{M}_1 \in \mathbb{N}_\omega^{|\tilde{P}|}$, $\tilde{M}_2 \in \mathbb{N}_\omega^{|\tilde{P}|}$, $\pi(\sigma_1) \leq \pi(\sigma t)$, and $\pi(\sigma_2) \leq \pi(\sigma')$ such that $\tilde{M}_1 \gneqq \tilde{M}_2$ **then**
9: find all pairs $(\tilde{M}_1, \tilde{M}_2)$ with $\tilde{M}_1 \gneqq \tilde{M}_2$;
10: for each pair $(\tilde{M}_1, \tilde{M}_2)$ with $\tilde{M}_1 \gneqq \tilde{M}_2$, let $\tilde{M}'(p) = \omega$ for each $p \in \tilde{P}$ such that $\tilde{M}_1(p) > \tilde{M}_2(p)$;
11: **end if**
12: **if** $(\tilde{M}', B') \notin \tilde{\mathcal{M}}_x^1$ **then**
13: $\tilde{\mathcal{M}}_x^1 := \tilde{\mathcal{M}}_x^1 \cup \{(\tilde{M}', B')\}$;
14: $\tilde{\mathcal{M}}_{novel}^1 := \tilde{\mathcal{M}}_{novel}^1 \cup \{(\tilde{M}', B')\}$;
15: **end if**
16: $\Delta_x^1 := \Delta_x^1 \cup \{((\tilde{M}, B), (t, \pi(\sigma)), (\tilde{M}', B'))\}$;
17: **end for**
18: **end for**
19: $\tilde{\mathcal{M}}_{novel}^1 := \tilde{\mathcal{M}}_{novel}^1 \setminus \{(\tilde{M}, B)\}$;
20: **end while**



(a) (b)

Fig. 1: (a) An LPN and (b) a fault pattern net (FPN, SF).

Example 1: Consider the LPN in Fig. 1(a) and its fault pattern Σ_F represented by (FPN, SF) in Fig. 1(b). Let us

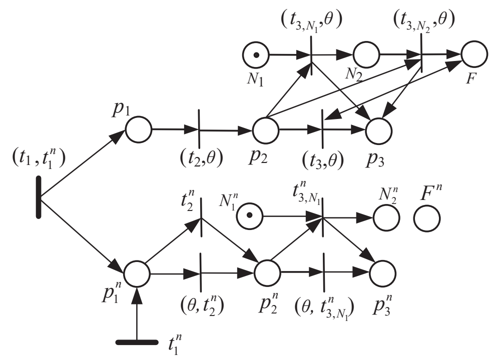


Fig. 2: An FP^{2}N.

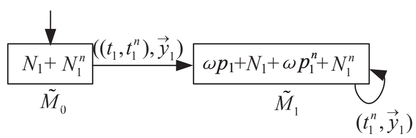


Fig. 3: The BFP^{2}G of the LPN in Fig. 1(a) with its fault pattern in Fig. 1(b).

use Algorithms 1 and 2 to construct the corresponding BFP^{2}G shown in Fig. 3. Now, the detailed application of Algorithms 1 and 2 are illustrated as follows.

As for Algorithm 1, in Step 2, we set $\tilde{\mathcal{M}}_B = \Delta_x = \Delta = \emptyset$ and $\tilde{\mathcal{M}}_x = \tilde{\mathcal{M}}_{new} = \{(\tilde{M}_0, \emptyset)\}$ with $\tilde{M}_0 = N_1 + N_1^n$. Consider the first iteration of Steps 3 to 28, we choose $(\tilde{M}_0, \emptyset) \in \tilde{\mathcal{M}}_{new}$ in Step 4. Since $(\tilde{M}_0, \emptyset)$ has no tag "dangerous", we compute the pair $(\tilde{M}_1, B_1)$ in Steps 13 to 17, where $\tilde{M}_1 = \omega p_1 + N_1 + \omega p_1^n + N_1^n$ and $B_1 = \{(\tilde{M}_0, \vec{0})\}$. In Steps 20 to 23, we have $\tilde{\mathcal{M}}_x = \tilde{\mathcal{M}}_{new} = \{(\tilde{M}_0, \emptyset), (\tilde{M}_1, B_1)\}$ and $\Delta_x = \{((\tilde{M}_0, \emptyset), ((t_1, t_1^n), \vec{0}), (\tilde{M}_1, B_1))\}$. Then, in Step 27, it gives $\tilde{\mathcal{M}}_{new} = \{(\tilde{M}_1, B_1)\}$.

In the second iteration of Steps 3 to 28, we choose $(\tilde{M}_1, B_1) \in \tilde{\mathcal{M}}_{new}$ in Step 4. Since $\tilde{M}_1$ and $(\tilde{M}_1, B_1)$ are dangerous, we call Algorithm 2 in Step 7. Particularly, in Step 1 of Algorithm 2, we set $\tilde{\mathcal{M}}_x^1 = \tilde{\mathcal{M}}_{novel}^1 = \{(\tilde{M}_1, B_1)\}$ and $\Delta_x^1 = \emptyset$. Then, we choose $(\tilde{M}_1, B_1) \in \tilde{\mathcal{M}}_{novel}^1$ in Step 3 of Algorithm 2. In Steps 6 to 10 of Algorithm 2, we compute $(\tilde{M}_1, B_2)$ with $B_2 = \{(\tilde{M}_0, \vec{0}), (\tilde{M}_1, \vec{0})\}$. By executing Steps 13 to 16, we obtain $\tilde{\mathcal{M}}_x^1 = \tilde{\mathcal{M}}_{novel}^1 = \{(\tilde{M}_1, B_1), (\tilde{M}_1, B_2)\}$ and $\Delta_x^1 = \{((\tilde{M}_1, B_1), (t_1^n, \vec{0}), (\tilde{M}_1, B_2))\}$. In Step 19, $\tilde{\mathcal{M}}_{novel}^1$ is updated as $\tilde{\mathcal{M}}_{novel}^1 = \{(\tilde{M}_1, B_2)\}$. Since no new node is generated in the second iteration of Steps 2 to 20 of Algorithm 2 and $\tilde{\mathcal{M}}_{novel}^1$ becomes empty in Step 19, we return Step 8 of Algorithm 1, set $\tilde{\mathcal{M}}_x = \{(\tilde{M}_0, \emptyset), (\tilde{M}_1, B_1), (\tilde{M}_1, B_2)\}$ and $\Delta_x = \{((\tilde{M}_0, \emptyset), ((t_1, t_1^n), \vec{0}), (\tilde{M}_1, B_1)), ((\tilde{M}_1, B_1), (t_1^n, \vec{0}), (\tilde{M}_1, B_2))\}$. In Step 27 of Algorithm 1, $\tilde{\mathcal{M}}_{new}$ is updated as $\tilde{\mathcal{M}}_{new} = \{(\tilde{M}_1, B_2)\}$.

The third iteration of Steps 3 to 28 of Algorithm 1 generates no new node and $\tilde{\mathcal{M}}_{new}$ becomes empty in Step 27. Thus, in Steps 29 to 32, we have $\tilde{\mathcal{M}}_B = \{\tilde{M}_0, \tilde{M}_1\}$ and $\Delta = \{(\tilde{M}_0, ((t_1, t_1^n), \vec{0}), \tilde{M}_1), (\tilde{M}_1, (t_1^n, \vec{0}), \tilde{M}_1)\}$. □