Construction of a BFP^{2}G

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Definition 1: Given an LPN $G = (PN, M_0, E, \lambda)$, let $\langle \tilde{P}N, \tilde{M}_0\rangle$ with $\tilde{P}N = (\tilde{P}, \tilde{T}, \tilde{P}re, \tilde{P}ost)$ be its FP^{2}N. The BFP^{2}G of G is a deterministic finite-state automaton $\mathcal{B} = (\tilde{\mathcal{M}}_B, \tilde{T}_{or}, \Delta, \tilde{M}_0)$ computed by using Algorithm 1, where $\tilde{\mathcal{M}}_B \subseteq \tilde{\mathcal{M}}^2_B \cup \tilde{\mathcal{M}}^1_{B_i}$ for $i = 1, 2, \ldots, q$ with $q = |\tilde{\mathcal{M}}^2_{Bd}|$ is the set of states, $\tilde{T}_{or} = \tilde{T}_o \times \mathbb{N}^{|\tilde{T}_u|}$ is the set of events, $\Delta \subseteq \tilde{\mathcal{M}}_B \times \tilde{T}_{or} \times \tilde{\mathcal{M}}_B$ is the transition relation, and $\tilde{M}_0$ is the initial state.

Algorithm 1 Construction of a BFP^{2}G.

Input: An LPN $G = (PN, M_0, E, \lambda)$ with $PN = (P, T, Pre, Post)$ and a fault pattern Σ_F
Output: A BFP^{2}G $\mathcal{B} = (\tilde{\mathcal{M}}_B, \tilde{T}_{or}, \Delta, \tilde{M}_0)$ of G

1: Compute the FP^{2}N $\langle \tilde{P}N, \tilde{M}_0\rangle$;
2: $\tilde{\mathcal{M}}_B := \emptyset$, $\Delta_x := \emptyset$, $\Delta := \emptyset$, $B := \emptyset$, $\tilde{\mathcal{M}}_x := \{(\tilde{M}_0, B)\}$, and $\tilde{\mathcal{M}}_{new} := \{(\tilde{M}_0, B)\}$;
3: **while** $\tilde{\mathcal{M}}_{new} \neq \emptyset$ **do**
4: choose a node $(\tilde{M}, B) \in \tilde{\mathcal{M}}_{new}$;
5: **if** there exists $t \in \tilde{T}^F_b$ such that $Y^{2reg}_{min}(\tilde{M}, t) \neq \emptyset$ **then**
6: assign tag "dangerous" to $\tilde{M}$ and $(\tilde{M}, B)$;
7: call BasisSonNodes($\tilde{M}, B$) in Algorithm 2;
8: $\tilde{\mathcal{M}}_x := \tilde{\mathcal{M}}_x \cup \tilde{\mathcal{M}}^1_x$ and $\Delta_x := \Delta_x \cup \Delta^1_x$;
9: **end if**
10: **if** $(\tilde{M}, B)$ has no tag "dangerous" **then**
11: **for** each $t \in \tilde{T}^2_o$ and $Y^{2reg}_{min}(\tilde{M}, t) \neq \emptyset$ **do**
12: **for** each $\pi(\sigma) \in Y^{2reg}_{min}(\tilde{M}, t)$ **do**
13: $\tilde{M}' := \tilde{M} + \tilde{C}_u \cdot \pi(\sigma) + \tilde{C}(\cdot, t)$;
14: $B' := B \cup \{(\tilde{M}, \pi(\sigma))\}$;
15: **if** there exists $(\tilde{M}'', \pi(\sigma')) \in B'$, $\sigma_1 \in (\tilde{T}^2 \setminus \tilde{T}^{FP})^*$, and $\sigma_2 \in (\tilde{T}^2_u \setminus \tilde{T}^{FP})^*$ satisfying $\tilde{M}[\sigma_1\rangle\tilde{M}_1$ and $\tilde{M}''[\sigma_2\rangle\tilde{M}_2$ with $\tilde{M}_1 \in \mathbb{N}^{|\tilde{P}|}_\omega$, $\tilde{M}_2 \in \mathbb{N}^{|\tilde{P}|}_\omega$, $\pi(\sigma_1) \leq \pi(\sigma t)$, and $\pi(\sigma_2) \leq \pi(\sigma')$ such that $\tilde{M}_1 \gneqq \tilde{M}_2$ **then**
16: find all pairs $(\tilde{M}_1, \tilde{M}_2)$ with $\tilde{M}_1 \gneqq \tilde{M}_2$;
17: for each pair $(\tilde{M}_1, \tilde{M}_2)$ with $\tilde{M}_1 \gneqq \tilde{M}_2$, let $\tilde{M}'(p) = \omega$ for each $p \in \tilde{P}$ such that $\tilde{M}_1(p) > \tilde{M}_2(p)$;
18: **end if**
19: **if** $(\tilde{M}', B') \notin \tilde{M}_t$ **then**
20: $\tilde{\mathcal{M}}_x := \tilde{\mathcal{M}}_x \cup \{(\tilde{M}', B')\}$;
21: $\tilde{\mathcal{M}}_{new} := \tilde{\mathcal{M}}_{new} \cup \{(\tilde{M}', B')\}$;
22: **end if**
23: $\Delta_x := \Delta_x \cup \{((\tilde{M}, B), (t, \pi(\sigma)), (\tilde{M}', B'))\}$;
24: **end for**
25: **end for**
26: **end if**
27: $\tilde{\mathcal{M}}_{new} := \tilde{\mathcal{M}}_{new} \backslash \{(\tilde{M}, B)\}$;
28: **end while**
29: $\tilde{\mathcal{M}}_B := \{\tilde{M} \in \mathbb{N}^{|\tilde{P}|}_\omega | \exists B \in 2^{\mathbb{N}^{|\tilde{P}|}_\omega \times \mathbb{N}^{|\tilde{T}_u|}} : (\tilde{M}, B) \in \tilde{\mathcal{M}}_x\}$;
30: **for** each $((\tilde{M}, B), (t, \pi(\sigma)), (\tilde{M}', B')) \in \Delta_x$ **do**
31: $\Delta := \Delta \cup \{(\tilde{M}, (t, \pi(\sigma)), \tilde{M}')\}$;
32: **end for**

In Algorithm 1, the set $\tilde{\mathcal{M}}_{new}$ is initialized at $\{(\tilde{M}_0, \emptyset)\}$ in Step 2. For each pair $(\tilde{M}, B) \in \tilde{\mathcal{M}}_{new}$, if it is dangerous, we call function $BasisSonNodes$ in Step 7 to compute a basis son graph for the nonfailure subnet of the FPLN, which describes the normal behaviour of the FP^{2}N from $\tilde{M}$ by firing the transitions in $\tilde{T}^1_o$ and the corresponding minimal explanations. Otherwise, Steps 11 to 25 compute each successor pair $(\tilde{M}', \tilde{B}')$ by firing each transition in $\tilde{T}^2_o$ and its minimal explanation restricted to $\tilde{T}^2_u \setminus \tilde{T}^{FP}$, and compute the new arc from $(\tilde{M}, \tilde{B})$ to $(\tilde{M}', \tilde{B}')$. Then, Step 27 removes $(\tilde{M}, B)$ from $\tilde{\mathcal{M}}_{new}$ indicating that $(\tilde{M}, B)$ has been considered. The procedure from Steps 3 to Step 28 is executed iteratively until $\tilde{\mathcal{M}}_{new}$ is empty. Finally, by running Steps 29 to 32, the BFP^{2}G is obtained by removing the second component from each pair $(\tilde{M}, B) \in \tilde{\mathcal{M}}_x$.

Algorithm 2 $Function\ BasisSonNodes$ $(\tilde{M}, B)$.

Input: A dangerous pair $(\tilde{M}, B)$ and $\tilde{P}N^1 = (\tilde{P}^1, \tilde{T}^1, \tilde{P}re^1, \tilde{P}ost^1)$
Output: A basis son graph $\mathcal{B}^1 = (\tilde{\mathcal{M}}^1_x, \tilde{T}^1_{or}, \Delta^1_x, (\tilde{M}, B))$

$\tilde{\mathcal{M}}^1_x := \{(\tilde{M}, B)\}$, $\Delta^1_x := \emptyset$, and $\tilde{\mathcal{M}}^1_{novel} := \{(\tilde{M}, B)\}$;
while $\tilde{\mathcal{M}}^1_{novel} \neq \emptyset$ **do**
for each $t \in \tilde{T}^1_o$ and $Y^1_{min}(\tilde{M}, t) \neq \emptyset$ **do**
for each $\pi(\sigma) \in Y^1_{min}(\tilde{M}, t)$ **do**
$\tilde{M}' := \tilde{M} + \tilde{C}_u \cdot \pi(\sigma) + \tilde{C}(\cdot, t)$;
$B' := B \cup \{(\tilde{M}, \pi(\sigma))\}$;
if there exists $(\tilde{M}'', \pi(\sigma')) \in B'$, $\sigma_1 \in (\tilde{T}^1)^*$, and $\sigma_2 \in (\tilde{T}^1_u)^*$satisfying $\tilde{M}[\sigma_1\rangle\tilde{M}_1$ and $\tilde{M}''[\sigma_2\rangle\tilde{M}_2$ with $\tilde{M}_1 \in \mathbb{N}^{|\tilde{P}|}_\omega$, $\tilde{M}_2 \in \mathbb{N}^{|\tilde{P}|}_\omega$, $\pi(\sigma_1) \leq \pi(\sigma t)$, and $\pi(\sigma_2) \leq \pi(\sigma')$ such that $\tilde{M}_1 \gneqq \tilde{M}_2$ **then**
find all pairs $(\tilde{M}_1, \tilde{M}_2)$ with $\tilde{M}_1 \gneqq \tilde{M}_2$;
for each pair $(\tilde{M}_1, \tilde{M}_2)$ with $\tilde{M}_1 \gneqq \tilde{M}_2$, let $\tilde{M}'(p) = \omega$ for each $p \in \tilde{P}$ such that $\tilde{M}_1(p) > \tilde{M}_2(p)$;
end if
if $\tilde{M}' \notin \tilde{\mathcal{M}}^1_x$ **then**
$\tilde{\mathcal{M}}^1_x := \tilde{\mathcal{M}}^1_x \cup \{(\tilde{M}', B')\}$;
$\tilde{\mathcal{M}}^1_{novel} := \tilde{\mathcal{M}}^1_{novel} \cup \{(\tilde{M}', B')\}$;
end if
$\Delta^1_x := \Delta^1_x \cup \{((\tilde{M}, B), (t, \pi(\sigma)), (\tilde{M}', B'))\}$;
end for
end for
$\tilde{\mathcal{M}}^1_{novel} := \tilde{\mathcal{M}}^1_{novel} \backslash \{(\tilde{M}, B)\}$;
end while