

# Construction of a BFP<sup>2</sup>G

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**Definition 1:** Given an LPN  $G = (PN, M_0, E, \lambda)$ , let  $\langle \tilde{P}N, \tilde{M}_0 \rangle$  with  $\tilde{P}N = (\tilde{P}, \tilde{T}, \tilde{P}re, \tilde{P}ost)$  be its FP<sup>2</sup>N. The BFP<sup>2</sup>G of  $G$  is a deterministic finite-state automaton  $\mathcal{B} = (\tilde{\mathcal{M}}_B, \tilde{T}_{or}, \Delta, \tilde{M}_0)$  computed by using Algorithm 1, where  $\tilde{\mathcal{M}}_B \subseteq \tilde{\mathcal{M}}_B^2 \cup \tilde{\mathcal{M}}_{Bi}^1$  for  $i = 1, 2, \dots, q$  with  $q = |\tilde{\mathcal{M}}_{Bd}^2|$  is the set of states,  $\tilde{T}_{or} = \tilde{T}_o \times \mathbb{N}^{|\tilde{T}_u|}$  is the set of events,  $\Delta \subseteq \tilde{\mathcal{M}}_B \times \tilde{T}_{or} \times \tilde{\mathcal{M}}_B$  is the transition relation, and  $\tilde{M}_0$  is the initial state.

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## Algorithm 1 Construction of a BFP<sup>2</sup>G.

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**Input:** An LPN  $G = (PN, M_0, E, \lambda)$  with  $PN = (P, T, Pre, Post)$  and a fault pattern  $\Sigma_F$   
**Output:** A BFP<sup>2</sup>G  $\mathcal{B} = (\tilde{\mathcal{M}}_B, \tilde{T}_{or}, \Delta, \tilde{M}_0)$  of  $G$

- 1: Compute the FP<sup>2</sup>N  $\langle \tilde{P}N, \tilde{M}_0 \rangle$ ;
- 2:  $\tilde{\mathcal{M}}_B := \emptyset$ ,  $\Delta_t := \emptyset$ ,  $\Delta := \emptyset$ ,  $B := \emptyset$ ,  $\tilde{\mathcal{M}}_t := \{(\tilde{M}_0, B)\}$ , and  $\tilde{\mathcal{M}}_{new} := \{(\tilde{M}_0, B)\}$ ;
- 3: **while**  $\tilde{\mathcal{M}}_{new} \neq \emptyset$  **do**
- 4:   choose a node  $(\tilde{M}, B) \in \tilde{\mathcal{M}}_{new}$ ;
- 5:   **if** there exists  $t \in \tilde{T}_b^F$  such that  $Y_{min}^{2reg}(\tilde{M}, t) \neq \emptyset$  **then**
- 6:     assign tag “dangerous” to  $\tilde{M}$  and  $(\tilde{M}, B)$ ;
- 7:     call BasisSonNodes( $\tilde{M}, B$ ) in Algorithm 2;
- 8:      $\tilde{\mathcal{M}}_t := \tilde{\mathcal{M}}_t \cup \tilde{\mathcal{M}}_t^1$  and  $\Delta_t := \Delta_t \cup \Delta_t^1$ ;
- 9:   **end if**
- 10:   **if**  $(\tilde{M}, B)$  has no tag “dangerous” **then**
- 11:     **for** each  $t \in \tilde{T}_o^2$  and  $Y_{min}^{2reg}(\tilde{M}, t) \neq \emptyset$  **do**
- 12:       **for** each  $\pi(\sigma) \in Y_{min}^{2reg}(\tilde{M}, t)$  **do**
- 13:          $\tilde{M}' := \tilde{M} + \tilde{C}_u \cdot \pi(\sigma) + \tilde{C}(\cdot, t)$ ;
- 14:          $B' := B \cup \{(\tilde{M}, \pi(\sigma))\}$ ;
- 15:         **if** there exists  $(\tilde{M}'', \pi(\sigma')) \in B'$ ,  $\sigma_1 \in (\tilde{T}^2 \setminus \tilde{T}^{FP})^*$ , and  $\sigma_2 \in (\tilde{T}_u^2 \setminus \tilde{T}^{FP})^*$  satisfying  $\tilde{M}[\sigma_1] \tilde{M}_1$  and  $\tilde{M}''[\sigma_2] \tilde{M}_2$  with  $\tilde{M}_1 \in \mathbb{N}_\omega^{|\tilde{P}|}$ ,  $\tilde{M}_2 \in \mathbb{N}_\omega^{|\tilde{P}|}$ ,  $\pi(\sigma_1) \leq \pi(\sigma t)$ , and  $\pi(\sigma_2) \leq \pi(\sigma')$  such that  $\tilde{M}_1 \not\geq \tilde{M}_2$  **then**
- 16:           find all pairs  $(\tilde{M}_1, \tilde{M}_2)$  such that  $\tilde{M}_1 \not\geq \tilde{M}_2$ ;
- 17:           **for** each pair  $(\tilde{M}_1, \tilde{M}_2)$  such that  $\tilde{M}_1 \not\geq \tilde{M}_2$ , let  $\tilde{M}'(p) = \omega$  for each  $p \in \tilde{P}$  such that  $\tilde{M}_1(p) > \tilde{M}_2(p)$ ;
- 18:         **end if**
- 19:         **if**  $(\tilde{M}', B') \notin \tilde{\mathcal{M}}_t$  **then**
- 20:            $\tilde{\mathcal{M}}_t := \tilde{\mathcal{M}}_t \cup \{(\tilde{M}', B')\}$ ;
- 21:            $\tilde{\mathcal{M}}_{new} := \tilde{\mathcal{M}}_{new} \cup \{(\tilde{M}', B')\}$ ;
- 22:         **end if**
- 23:          $\Delta_t := \Delta_t \cup \{((\tilde{M}, B)(t, \pi(\sigma)), (\tilde{M}', B'))\}$ ;
- 24:       **end for**
- 25:     **end for**
- 26:   **end if**
- 27:    $\tilde{\mathcal{M}}_{new} := \tilde{\mathcal{M}}_{new} \setminus \{(\tilde{M}, B)\}$ ;
- 28: **end while**
- 29:  $\tilde{\mathcal{M}}_B := \{\tilde{M} \in \mathbb{N}_\omega^{|\tilde{P}|} \mid \exists B \in 2^{\mathbb{N}_\omega^{|\tilde{P}|} \times \mathbb{N}^{|\tilde{T}_u|}} : (\tilde{M}, B) \in \tilde{\mathcal{M}}_t\}$ ;
- 30: **for** each  $((\tilde{M}, B), (t, \pi(\sigma)), (\tilde{M}', B')) \in \Delta_t$  **do**
- 31:    $\Delta := \Delta \cup \{(\tilde{M}, (t, \pi(\sigma)), \tilde{M}')\}$ ;
- 32: **end for**

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In Algorithm 1, the set  $\tilde{\mathcal{M}}_{new}$  is initialized at  $\{(\tilde{M}_0, \emptyset)\}$  in Step 2. For each pair  $(\tilde{M}, B) \in \tilde{\mathcal{M}}_{new}$ , if it is dangerous, we call function *BasisSonNodes* in Step 7 to compute

a basis son graph for the nonfailure subnet of the FPLN, which describes the normal behaviour of the FP<sup>2</sup>N from  $\tilde{M}$  by firing the transitions in  $\tilde{T}_o^1$  and the corresponding minimal explanations. Otherwise, Steps 11 to 25 compute each successor pair  $(\tilde{M}', \tilde{B}')$  by firing each transition in  $\tilde{T}_o^2$  and its minimal explanation restricted to  $\tilde{T}_u^2 \setminus \tilde{T}^{FP}$ , and compute the new arc from  $(\tilde{M}, B)$  to  $(\tilde{M}', \tilde{B}')$ . Then, Step 27 removes  $(\tilde{M}, B)$  from  $\tilde{\mathcal{M}}_{new}$  indicating that  $(\tilde{M}, B)$  has been considered. The procedure from Steps 3 to Step 28 is executed iteratively until  $\tilde{\mathcal{M}}_{new}$  is empty. Finally, by running Steps 29 to 32, the BFP<sup>2</sup>G is obtained by removing the second component from each pair  $(\tilde{M}, B) \in \tilde{\mathcal{M}}_t$ .

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## Algorithm 2 Function BasisSonNodes $(\tilde{M}, B)$ .

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**Input:** A dangerous pair  $(\tilde{M}, B)$  and  $\tilde{P}N^1 = (\tilde{P}^1, \tilde{T}^1, \tilde{P}re^1, \tilde{P}ost^1)$   
**Output:** A basis son graph  $\mathcal{B}^1 = (\tilde{\mathcal{M}}_t^1, \tilde{T}_{or}^1, \Delta_t^1, (\tilde{M}, B))$

- 1:  $\tilde{\mathcal{M}}_t^1 := \{(\tilde{M}, B)\}$ ,  $\Delta_t^1 := \emptyset$ , and  $\tilde{\mathcal{M}}_{novel}^1 := \{(\tilde{M}, B)\}$ ;
- 2: **while**  $\tilde{\mathcal{M}}_{novel}^1 \neq \emptyset$  **do**
- 3:   **for** each  $t \in \tilde{T}_o^1$  and  $Y_{min}^1(\tilde{M}, t) \neq \emptyset$  **do**
- 4:     **for** each  $\pi(\sigma) \in Y_{min}^1(\tilde{M}, t)$  **do**
- 5:        $\tilde{M}' := \tilde{M} + \tilde{C}_u \cdot \pi(\sigma) + \tilde{C}(\cdot, t)$ ;
- 6:        $B' := B \cup \{(\tilde{M}, \pi(\sigma))\}$ ;
- 7:       **if** there exists  $(\tilde{M}'', \pi(\sigma')) \in B'$ ,  $\sigma_1 \in (\tilde{T}^1)^*$ , and  $\sigma_2 \in (\tilde{T}_u^1)^*$  satisfying  $\tilde{M}[\sigma_1] \tilde{M}_1$  and  $\tilde{M}''[\sigma_2] \tilde{M}_2$  with  $\tilde{M}_1 \in \mathbb{N}_\omega^{|\tilde{P}^1|}$ ,  $\tilde{M}_2 \in \mathbb{N}_\omega^{|\tilde{P}^1|}$ ,  $\pi(\sigma_1) \leq \pi(\sigma t)$ , and  $\pi(\sigma_2) \leq \pi(\sigma')$  such that  $\tilde{M}_1 \not\geq \tilde{M}_2$  **then**
- 8:         find all pairs  $(\tilde{M}_1, \tilde{M}_2)$  such that  $\tilde{M}_1 \not\geq \tilde{M}_2$ ;
- 9:         **for** each pair  $(\tilde{M}_1, \tilde{M}_2)$  such that  $\tilde{M}_1 \not\geq \tilde{M}_2$ , let  $\tilde{M}'(p) = \omega$  for each  $p \in \tilde{P}$  such that  $\tilde{M}_1(p) > \tilde{M}_2(p)$ ;
- 10:       **end if**
- 11:       **if**  $(\tilde{M}', B') \notin \tilde{\mathcal{M}}_t^1$  **then**
- 12:          $\tilde{\mathcal{M}}_t^1 := \tilde{\mathcal{M}}_t^1 \cup \{(\tilde{M}', B')\}$ ;
- 13:          $\tilde{\mathcal{M}}_{novel}^1 := \tilde{\mathcal{M}}_{novel}^1 \cup \{(\tilde{M}', B')\}$ ;
- 14:       **end if**
- 15:        $\Delta_t^1 := \Delta_t^1 \cup \{((\tilde{M}, B)(t, \pi(\sigma)), (\tilde{M}', B'))\}$ ;
- 16:     **end for**
- 17:   **end for**
- 18:    $\tilde{\mathcal{M}}_{novel}^1 := \tilde{\mathcal{M}}_{novel}^1 \setminus \{(\tilde{M}, B)\}$ ;
- 19: **end while**

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