

Construction of a BFP²G

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Definition 1: Given an LPN $G = (PN, M_0, E, \lambda)$, let $\langle \tilde{P}N, \tilde{M}_0 \rangle$ with $\tilde{P}N = (\tilde{P}, \tilde{T}, \tilde{Pre}, \tilde{Post})$ be its FP²N. The BFP²G of G is a deterministic finite-state automaton $\mathcal{B} = (\tilde{\mathcal{M}}_B, \tilde{T}_{or}, \Delta, \tilde{M}_0)$ computed by using Algorithm 1, where $\tilde{\mathcal{M}}_B \subseteq \tilde{\mathcal{M}}_B^2 \cup \tilde{\mathcal{M}}_{B_{ij}}^1$ for all i and j with $(\tilde{M}_i, \tilde{B}_j) \in \tilde{M}_{xd}^2$ is the set of states, $\tilde{T}_{or} = \tilde{T}_o \times \mathbb{N}^{|\tilde{T}_u|}$ is the set of events, $\Delta \subseteq \tilde{\mathcal{M}}_B \times \tilde{T}_{or} \times \tilde{\mathcal{M}}_B$ is the transition relation, and \tilde{M}_0 is the initial state.

Algorithm 1 Construction of a BFP²G.

Input: An LPN $G = (PN, M_0, E, \lambda)$ with $PN = (P, T, Pre, Post)$ and a fault pattern Σ_F
Output: A BFP²G $\mathcal{B} = (\tilde{\mathcal{M}}_B, \tilde{T}_{or}, \Delta, \tilde{M}_0)$ of G

- 1: Compute the FP²N $\langle \tilde{P}N, \tilde{M}_0 \rangle$;
- 2: $\tilde{\mathcal{M}}_B := \emptyset$, $\Delta_x := \emptyset$, $\Delta := \emptyset$, $B := \emptyset$, $\tilde{\mathcal{M}}_x := \{(\tilde{M}_0, B)\}$, and $\tilde{\mathcal{M}}_{new} := \{(\tilde{M}_0, B)\}$;
- 3: **while** $\tilde{\mathcal{M}}_{new} \neq \emptyset$ **do**
- 4: choose a node $(\tilde{M}, B) \in \tilde{\mathcal{M}}_{new}$;
- 5: **if** there exists $t \in \tilde{T}_b^F$ such that $Y_{min}^{2reg}(\tilde{M}, t) \neq \emptyset$ **then**
- 6: assign tag “dangerous” to \tilde{M} and (\tilde{M}, B) ;
- 7: call BasisSonNodes(\tilde{M}, B) in Algorithm 2;
- 8: $\tilde{\mathcal{M}}_x := \tilde{\mathcal{M}}_x \cup \tilde{\mathcal{M}}_x^1$ and $\Delta_x := \Delta_x \cup \Delta_x^1$;
- 9: **end if**
- 10: **if** (\tilde{M}, B) has no tag “dangerous” **then**
- 11: **for** each $t \in \tilde{T}_o^2$ and $Y_{min}^{2reg}(\tilde{M}, t) \neq \emptyset$ **do**
- 12: **for** each $\pi(\sigma) \in Y_{min}^{2reg}(\tilde{M}, t)$ **do**
- 13: $\tilde{M}' := \tilde{M} + \tilde{C}_u \cdot \pi(\sigma) + \tilde{C}(\cdot, t)$;
- 14: $B' := B \cup \{(\tilde{M}, \pi(\sigma))\}$;
- 15: **if** there exists $(\tilde{M}'', \pi(\sigma')) \in B'$, $\sigma_1 \in (\tilde{T}^2 \setminus \tilde{T}^{FP})^*$, and $\sigma_2 \in (\tilde{T}_u^F \setminus \tilde{T}^{FP})^*$ satisfying $\tilde{M}[\sigma_1] \tilde{M}_1$ and $\tilde{M}''[\sigma_2] \tilde{M}_2$ with $\tilde{M}_1 \in \mathbb{N}_\omega^{|\tilde{P}|}$, $\tilde{M}_2 \in \mathbb{N}_\omega^{|\tilde{P}|}$, $\pi(\sigma_1) \leq \pi(\sigma t)$, and $\pi(\sigma_2) \leq \pi(\sigma')$ such that $\tilde{M}_1 \not\geq \tilde{M}_2$ **then**
- 16: find all pairs $(\tilde{M}_1, \tilde{M}_2)$ with $\tilde{M}_1 \not\geq \tilde{M}_2$;
- 17: **for** each pair $(\tilde{M}_1, \tilde{M}_2)$ with $\tilde{M}_1 \not\geq \tilde{M}_2$, let $\tilde{M}'(p) = \omega$ for each $p \in \tilde{P}$ such that $\tilde{M}_1(p) > \tilde{M}_2(p)$;
- 18: **end if**
- 19: **if** $(\tilde{M}', B') \notin \tilde{\mathcal{M}}_x$ **then**
- 20: $\tilde{\mathcal{M}}_x := \tilde{\mathcal{M}}_x \cup \{(\tilde{M}', B')\}$;
- 21: $\tilde{\mathcal{M}}_{new} := \tilde{\mathcal{M}}_{new} \cup \{(\tilde{M}', B')\}$;
- 22: **end if**
- 23: $\Delta_x := \Delta_x \cup \{((\tilde{M}, B), (t, \pi(\sigma)), (\tilde{M}', B'))\}$;
- 24: **end for**
- 25: **end for**
- 26: **end if**
- 27: $\tilde{\mathcal{M}}_{new} := \tilde{\mathcal{M}}_{new} \setminus \{(\tilde{M}, B)\}$;
- 28: **end while**
- 29: $\tilde{\mathcal{M}}_B := \{\tilde{M} \in \mathbb{N}_\omega^{|\tilde{P}|} \mid \exists B \in 2^{\mathbb{N}_\omega^{|\tilde{P}|} \times \mathbb{N}^{|\tilde{T}_u|}} : (\tilde{M}, B) \in \tilde{\mathcal{M}}_x\}$;
- 30: **for** each $((\tilde{M}, B), (t, \pi(\sigma)), (\tilde{M}', B')) \in \Delta_x$ **do**
- 31: $\Delta := \Delta \cup \{(\tilde{M}, (t, \pi(\sigma)), \tilde{M}')\}$;
- 32: **end for**

In Algorithm 1, the set $\tilde{\mathcal{M}}_{new}$ is initialized at $\{(\tilde{M}_0, \emptyset)\}$ in Step 2. For each pair $(\tilde{M}, B) \in \tilde{\mathcal{M}}_{new}$, if it is dangerous, we call function *BasisSonNodes* in Step 7 to compute

a basis son graph for the nonfailure subnet of the FPLN, which describes the normal behaviour of the FP²N from \tilde{M} by firing the transitions in \tilde{T}_o^1 and the corresponding minimal explanations. Otherwise, Steps 11 to 25 compute each successor pair (\tilde{M}', \tilde{B}') by firing each transition in \tilde{T}_o^2 and its minimal explanation restricted to $\tilde{T}_u^2 \setminus \tilde{T}^{FP}$, and compute the new arc from (\tilde{M}, B) to (\tilde{M}', \tilde{B}') . Then, Step 27 removes (\tilde{M}, B) from $\tilde{\mathcal{M}}_{new}$ indicating that (\tilde{M}, B) has been considered. The procedure from Steps 3 to Step 28 is executed iteratively until $\tilde{\mathcal{M}}_{new}$ is empty. Finally, by running Steps 29 to 32, the BFP²G is obtained by removing the second component from each pair $(\tilde{M}, B) \in \tilde{\mathcal{M}}_x$.

Algorithm 2 Function BasisSonNodes(\tilde{M}, B).

Input: A dangerous pair (\tilde{M}, B) and $\tilde{P}N^1 = (\tilde{P}^1, \tilde{T}^1, \tilde{Pre}^1, \tilde{Post}^1)$
Output: A basis son graph $\mathcal{B}^1 = (\tilde{\mathcal{M}}_x^1, \tilde{T}_{or}^1, \Delta_x^1, (\tilde{M}, B))$

- 1: $\tilde{\mathcal{M}}_x^1 := \{(\tilde{M}, B)\}$, $\Delta_x^1 := \emptyset$, and $\tilde{\mathcal{M}}_{novel}^1 := \{(\tilde{M}, B)\}$;
- 2: **while** $\tilde{\mathcal{M}}_{novel}^1 \neq \emptyset$ **do**
- 3: **for** each $t \in \tilde{T}_o^1$ and $Y_{min}^1(\tilde{M}, t) \neq \emptyset$ **do**
- 4: **for** each $\pi(\sigma) \in Y_{min}^1(\tilde{M}, t)$ **do**
- 5: $\tilde{M}' := \tilde{M} + \tilde{C}_u \cdot \pi(\sigma) + \tilde{C}(\cdot, t)$;
- 6: $B' := B \cup \{(\tilde{M}, \pi(\sigma))\}$;
- 7: **if** there exists $(\tilde{M}'', \pi(\sigma')) \in B'$, $\sigma_1 \in (\tilde{T}^1)^*$, and $\sigma_2 \in (\tilde{T}_u^1)^*$ satisfying $\tilde{M}[\sigma_1] \tilde{M}_1$ and $\tilde{M}''[\sigma_2] \tilde{M}_2$ with $\tilde{M}_1 \in \mathbb{N}_\omega^{|\tilde{P}^1|}$, $\tilde{M}_2 \in \mathbb{N}_\omega^{|\tilde{P}^1|}$, $\pi(\sigma_1) \leq \pi(\sigma t)$, and $\pi(\sigma_2) \leq \pi(\sigma')$ such that $\tilde{M}_1 \not\geq \tilde{M}_2$ **then**
- 8: find all pairs $(\tilde{M}_1, \tilde{M}_2)$ with $\tilde{M}_1 \not\geq \tilde{M}_2$;
- 9: **for** each pair $(\tilde{M}_1, \tilde{M}_2)$ with $\tilde{M}_1 \not\geq \tilde{M}_2$, let $\tilde{M}'(p) = \omega$ for each $p \in \tilde{P}$ such that $\tilde{M}_1(p) > \tilde{M}_2(p)$;
- 10: **end if**
- 11: **if** $\tilde{M}' \notin \tilde{\mathcal{M}}_x^1$ **then**
- 12: $\tilde{\mathcal{M}}_x^1 := \tilde{\mathcal{M}}_x^1 \cup \{(\tilde{M}', B')\}$;
- 13: $\tilde{\mathcal{M}}_{novel}^1 := \tilde{\mathcal{M}}_{novel}^1 \cup \{(\tilde{M}', B')\}$;
- 14: **end if**
- 15: $\Delta_x^1 := \Delta_x^1 \cup \{((\tilde{M}, B), (t, \pi(\sigma)), (\tilde{M}', B'))\}$;
- 16: **end for**
- 17: **end for**
- 18: $\tilde{\mathcal{M}}_{novel}^1 := \tilde{\mathcal{M}}_{novel}^1 \setminus \{(\tilde{M}, B)\}$;
- 19: **end while**
