Construction of a BFP^{2}G

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Definition 1: Given an LPN $G = (PN, M_0, E, \lambda)$, let $\langle \tilde{P}N, \tilde{M}_0 \rangle$ with $\tilde{P}N = (\tilde{P}, \tilde{T}, \tilde{Pre}, \tilde{Post})$ be its FP^{2}N. The BFP^{2}G of G is a deterministic finite-state automaton $\mathcal{B} = (\tilde{\mathcal{M}}_B, \tilde{T}_{or}, \Delta, \tilde{M}_0)$ computed by using Algorithm 1, where $\tilde{\mathcal{M}}_B \subseteq \tilde{\mathcal{M}}_B^2 \cup \tilde{\mathcal{M}}_{Bi}^1$ for $i = 1, 2, \ldots, q$ with $q = |\tilde{\mathcal{M}}_{Bd}^2|$ is the set of states, $\tilde{T}_{or} = \tilde{T}_o \times \mathbb{N}^{|\tilde{T}_u|}$ is the set of events, $\Delta \subseteq \tilde{\mathcal{M}}_B \times \tilde{T}_{or} \times \tilde{\mathcal{M}}_B$ is the transition relation, and $\tilde{M}_0$ is the initial state.

Algorithm 1 Construction of a BFP^{2}G.

Input: An LPN $G = (PN, M_0, E, \lambda)$ with $PN = (P, T, Pre, Post)$ and a fault pattern Σ_F
Output: A BFP^{2}G $\mathcal{B} = (\tilde{\mathcal{M}}_B, \tilde{T}_{or}, \Delta, \tilde{M}_0)$ of G
1: Compute the FP^{2}N $\langle \tilde{P}N, \tilde{M}_0 \rangle$;
2: $\tilde{\mathcal{M}}_B := \emptyset$, $\Delta_t := \emptyset$, $\Delta := \emptyset$, $B := \emptyset$, $\tilde{\mathcal{M}}_t := \{(\tilde{M}_0, B)\}$, and $\tilde{\mathcal{M}}_{new} := \{(\tilde{M}_0, B)\}$;
3: **while** $\tilde{\mathcal{M}}_{new} \neq \emptyset$ **do**
4:     choose a node $(\tilde{M}, B) \in \tilde{\mathcal{M}}_{new}$;
5:     **if** there exists $t \in \tilde{T}_b^F$ such that $Y_{min}^{2reg}(\tilde{M}, t) \neq \emptyset$ **then**
6:         assign tag "dangerous" to $\tilde{M}$ and $(\tilde{M}, B)$;
7:         call BasisSonNodes($\tilde{M}, B$) in Algorithm 2;
8:         $\tilde{\mathcal{M}}_t := \tilde{\mathcal{M}}_t \cup \tilde{\mathcal{M}}_t^1$ and $\Delta_t := \Delta_t \cup \Delta_t^1$;
9:     **end if**
10:    **if** $(\tilde{M}, B)$ has no tag "dangerous" **then**
11:        **for** each $t \in \tilde{T}_o^2$ and $Y_{min}^{2reg}(\tilde{M}, t) \neq \emptyset$ **do**
12:            **for** each $\pi(\sigma) \in Y_{min}^{2reg}(\tilde{M}, t)$ **do**
13:                $\tilde{M}' := \tilde{M} + \tilde{C}_u \cdot \pi(\sigma) + \tilde{C}(\cdot, t)$;
14:                $B' := B \cup \{(\tilde{M}, \pi(\sigma))\}$;
15:                **if** there exists $(\tilde{M}'', \pi(\sigma')) \in B'$, $\sigma_1 \in (\tilde{T}^2 \setminus \tilde{T}^{FP})^*$, and $\sigma_2 \in (\tilde{T}_u^2 \setminus \tilde{T}^{FP})^*$ satisfying $\tilde{M}[\sigma_1\rangle\tilde{M}_1$ and $\tilde{M}''[\sigma_2\rangle\tilde{M}_2$ with $\tilde{M}_1 \in \mathbb{N}_\omega^{|\tilde{P}|}$, $\tilde{M}_2 \in \mathbb{N}_\omega^{|\tilde{P}|}$, $\pi(\sigma_1) \leq \pi(\sigma t)$, and $\pi(\sigma_2) \leq \pi(\sigma')$ such that $\tilde{M}_1 \gneqq \tilde{M}_2$ **then**
16:                    find all pairs $(\tilde{M}_1, \tilde{M}_2)$ with $\tilde{M}_1 \gneqq \tilde{M}_2$;
17:                    for each pair $(\tilde{M}_1, \tilde{M}_2)$ with $\tilde{M}_1 \gneqq \tilde{M}_2$, let $\tilde{M}'(p) = \omega$ for each $p \in \tilde{P}$ such that $\tilde{M}_1(p) > \tilde{M}_2(p)$;
18:                **end if**
19:                **if** $(\tilde{M}', B') \notin \tilde{\mathcal{M}}_t$ **then**
20:                    $\tilde{\mathcal{M}}_t := \tilde{\mathcal{M}}_t \cup \{(\tilde{M}', B')\}$;
21:                    $\tilde{\mathcal{M}}_{new} := \tilde{\mathcal{M}}_{new} \cup \{(\tilde{M}', B')\}$;
22:                **end if**
23:                $\Delta_t := \Delta_t \cup \{((\tilde{M}, B), (t, \pi(\sigma)), (\tilde{M}', B'))\}$;
24:            **end for**
25:        **end for**
26:    **end if**
27:    $\tilde{\mathcal{M}}_{new} := \tilde{\mathcal{M}}_{new} \setminus \{(\tilde{M}, B)\}$;
28: **end while**
29: $\tilde{\mathcal{M}}_B := \{\tilde{M} \in \mathbb{N}_\omega^{|\tilde{P}|} | \exists B \in 2^{\mathbb{N}_\omega^{|\tilde{P}|} \times \mathbb{N}^{|\tilde{T}_u|}} : (\tilde{M}, B) \in \tilde{\mathcal{M}}_t\}$;
30: **for** each $((\tilde{M}, B), (t, \pi(\sigma)), (\tilde{M}', B')) \in \Delta_t$ **do**
31:    $\Delta := \Delta \cup \{(\tilde{M}, (t, \pi(\sigma)), \tilde{M}')\}$;
32: **end for**

In Algorithm 1, the set $\tilde{\mathcal{M}}_{new}$ is initialized at $\{(\tilde{M}_0, \emptyset)\}$ in Step 2. For each pair $(\tilde{M}, B) \in \tilde{\mathcal{M}}_{new}$, if it is dangerous, we call function *BasisSonNodes* in Step 7 to compute

a basis son graph for the nonfailure subnet of the FPLN, which describes the normal behaviour of the FP^{2}N from $\tilde{M}$ by firing the transitions in $\tilde{T}_o^1$ and the corresponding minimal explanations. Otherwise, Steps 11 to 25 compute each successor pair $(\tilde{M}', \tilde{B}')$ by firing each transition in $\tilde{T}_o^2$ and its minimal explanation restricted to $\tilde{T}_o^2 \setminus \tilde{T}^{FP}$, and compute the new arc from $(\tilde{M}, \tilde{B})$ to $(\tilde{M}', \tilde{B}')$. Then, Step 27 removes $(\tilde{M}, B)$ from $\tilde{\mathcal{M}}_{new}$ indicating that $(\tilde{M}, B)$ has been considered. The procedure from Steps 3 to Step 28 is executed iteratively until $\tilde{\mathcal{M}}_{new}$ is empty. Finally, by running Steps 29 to 32, the BFP^{2}G is obtained by removing the second component from each pair $(\tilde{M}, B) \in \tilde{\mathcal{M}}_t$.

Algorithm 2 *Function BasisSonNodes $(\tilde{M}, B)$.*

Input: A dangerous pair $(\tilde{M}, B)$ and $\tilde{P}N^1 = (\tilde{P}^1, \tilde{T}^1, \tilde{Pre}^1, \tilde{Post}^1)$
Output: A basis son graph $\mathcal{B}^1 = (\tilde{\mathcal{M}}_t^1, \tilde{T}_{or}^1, \Delta_t^1, (\tilde{M}, B))$
1: $\tilde{\mathcal{M}}_t^1 := \{(\tilde{M}, B)\}$, $\Delta_t^1 := \emptyset$, and $\tilde{\mathcal{M}}_{novel}^1 := \{(\tilde{M}, B)\}$;
2: **while** $\tilde{\mathcal{M}}_{novel}^1 \neq \emptyset$ **do**
3:     **for** each $t \in \tilde{T}_o^1$ and $Y_{min}^1(\tilde{M}, t) \neq \emptyset$ **do**
4:         **for** each $\pi(\sigma) \in Y_{min}^1(\tilde{M}, t)$ **do**
5:             $\tilde{M}' := \tilde{M} + \tilde{C}_u \cdot \pi(\sigma) + \tilde{C}(\cdot, t)$;
6:             $B' := B \cup \{(\tilde{M}, \pi(\sigma))\}$;
7:             **if** there exists $(\tilde{M}'', \pi(\sigma')) \in B'$, $\sigma_1 \in (\tilde{T}^1)^*$, and $\sigma_2 \in (\tilde{T}_u^1)^*$ satisfying $\tilde{M}[\sigma_1\rangle\tilde{M}_1$ and $\tilde{M}''[\sigma_2\rangle\tilde{M}_2$ with $\tilde{M}_1 \in \mathbb{N}_\omega^{|\tilde{P}|}$, $\tilde{M}_2 \in \mathbb{N}_\omega^{|\tilde{P}|}$, $\pi(\sigma_1) \leq \pi(\sigma t)$, and $\pi(\sigma_2) \leq \pi(\sigma')$ such that $\tilde{M}_1 \gneqq \tilde{M}_2$ **then**
8:                 find all pairs $(\tilde{M}_1, \tilde{M}_2)$ with $\tilde{M}_1 \gneqq \tilde{M}_2$;
9:                 for each pair (M_1, M_2) with $\tilde{M}_1 \gneqq \tilde{M}_2$, let $\tilde{M}'(p) = \omega$ for each $p \in \tilde{P}$ such that $\tilde{M}_1(p) > \tilde{M}_2(p)$;
10:            **end if**
11:            **if** $\tilde{M}' \notin \tilde{\mathcal{M}}_t^1$ **then**
12:                $\tilde{\mathcal{M}}_t^1 := \tilde{\mathcal{M}}_t^1 \cup \{(\tilde{M}', B')\}$;
13:                $\tilde{\mathcal{M}}_{novel}^1 := \tilde{\mathcal{M}}_{novel}^1 \cup \{(\tilde{M}', B')\}$;
14:            **end if**
15:            $\Delta_t^1 := \Delta_t^1 \cup \{((\tilde{M}, B), (t, \pi(\sigma)), (\tilde{M}', B'))\}$;
16:        **end for**
17:    **end for**
18:    $\tilde{\mathcal{M}}_{novel}^1 := \tilde{\mathcal{M}}_{novel}^1 \setminus \{(\tilde{M}, B)\}$;
19: **end while**