Quiz 1 Solution

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10/13/2020

- i. With $\Psi = 1$,

$$logf(x, \mu) = -(x - \mu) - exp\{-(x - \mu)\}$$

  So

$$l(\mu; x_1, \cdots, x_n = n\mu - \sum_{i=1}^{n} x_i - exp(\mu) \sum_{i=1}^{n} exp(-x_i))$$

```
river <- c(50.4, 49.7, 49.9, 51.1, 49.2, 53.4, 51.1, 49.9, 51.3, 49.6)
ll_river <- function(theta, datavec) {
  n=length(datavec)
  s1=sum(datavec)
  s2=sum(exp(-datavec))
  return(n*theta-s1-exp(theta)*s2)
}
ll_river(48, river)
```

```
## [1] -26.80713
```

- ii. The score is- ii. The score is u(μ) =

$$\frac{\partial}{\partial \mu} l(\mu; x_1, \cdots, x_n) = n - exp(\mu) \sum_{i=1}^{n} exp(-x_i).$$

$u(\hat{\mu}) = 0 \Leftrightarrow$

$$\hat{\mu} = logn - log(\sum_{i=1}^{n} exp(-x_i))$$
$$= -log(\frac{1}{n} \sum_{i=1}^{n} exp(-x_i))$$

```
n <- length(river)
s2 <- sum(exp(-river))
muhat = -log(s2 / n)
print(muhat)
```

```
## [1] 50.11434
```

- iii.

$$l_O(\mu) = -\frac{\partial}{\partial \mu^2} - \frac{\partial}{\partial \mu} = exp(\mu) \sum_{i=1}^{n} exp(-x_i).$$

```
IObs <- exp(muhat) * s2
print(IObs)
```

```
## [1] 10
```

This is n, which is also obvious from part ii

- iv. $Var[\hat{\mu}] \approx 1/I_O = 0.1$
- v. Test is (a) one-tailed (asking wheter μ **exceeds** 50).
- vi. The Wald test statistic is

$$W = \frac{\hat{\mu} - 50}{se(\hat{\mu})}$$

```
W = (muhat - 50) / sqrt(0.1)
print(W)
```

```
## [1] 0.3615794
```

- vii. The p-value is $Pr[Z > W] = Pr[Z > 0.3615794]$

```
p = 1 - pnorm(W)
print(p)
```

```
## [1] 0.3588332
```

- viii. Since $p > 0.05$ we (a) fail to reject the null hypothesis.
- ix. Evaluate the score at $\mu_0 = 50$

```
u_river <- function(theta, datavec) {
  n = length(datavec)
  s2 = sum(exp(-datavec))
  return(n - exp(theta) * s2)
}
u <- u_river(50, river)
u
```

```
## [1] 1.080466
```

- x. The χ^2-distributed function of the score statistic using the information at μ_0 is $U^2/I_O(\mu_0)$

```
u^2/10
```

```
## [1] 0.1167408
```

For 20-21 only, because of an ambiguity in the original version of the question, I am also allowing:

```
u / sqrt(10)
```

```
## [1] 0.3416735
```

- xi. The p-value for the test is

```
1 - pchisq(u^2/10, df=1)
```

```
## [1] 0.7325966
```

- xii The alternative hypothesis is that (b) $\mu \neq \mu_0$