

1

$$E = \frac{1}{2} \sum_k (t_k - a_k)^2$$

$$\Delta W \propto -\frac{\partial E}{\partial W}$$

$$\Delta w_{kj} \propto -\frac{\partial E}{\partial w_{kj}}$$

$$\Delta w_{kj} = -\varepsilon \frac{\partial E}{\partial a_k} \frac{\partial a_k}{\partial net_k} \frac{\partial net_k}{\partial w_{kj}}$$

$$\frac{\partial E}{\partial a_k} = \frac{\partial(\frac{1}{2}(t_k - a_k)^2)}{\partial a_k} = -(t_k - a_k)$$

$$\frac{\partial a_k}{\partial net_k} = \frac{\partial(1 + e^{-net_k})^{-1}}{\partial net_k} = \frac{e^{-net_k}}{(1 + e^{-net_k})^2}$$

$$1 - \frac{1}{1 + e^{-net_k}} = \frac{e^{-net_k}}{1 + e^{-net_k}}$$

$$a_k(1 - a_k)$$

$$\frac{\partial net_k}{\partial w_{kj}} = \frac{\partial(w_{kj}a_j)}{\partial w_{kj}} = a_j$$

For output layer

$$\Delta w_{kj} = \varepsilon \overbrace{(t_k - a_k)a_k(1 - a_k)}^{\delta_k} a_j$$

$$\Delta w_{kj} = \varepsilon \delta_k a_j$$

For the input layer

$$\Delta w_{ji} \propto -\left[\sum_k \frac{\partial E}{\partial a_k} \frac{\partial a_k}{\partial net_k} \frac{\partial net_k}{\partial a_j}\right] \frac{\partial a_j}{\partial net_j} \frac{\partial net_j}{\partial w_{ji}}$$

$$\begin{aligned}
&= \varepsilon \left[\sum_k \overbrace{(t_k - a_k) a_k (1 - a_k)}^{\delta_k} w_{kj} \right] a_j (1 - a_j) a_i \\
&= \varepsilon \left[\sum_k \overbrace{\delta_k w_{kj}}^{\delta_j} \right] a_j (1 - a_j) a_i \\
\Delta w_{ji} &= \varepsilon \delta_j a_i
\end{aligned}$$

2 1) the log-likelihood of the complete data

$$Z_{im} = \begin{cases} 1, & \text{if } (X_i, Y_i) \text{ is in the } m\text{th group} \\ 0, & \text{otherwise.} \end{cases}$$

$$\sum_{i=1}^n \sum_{m=1}^M Z_{im} [\log p_m(X_i) + \log \lambda_m(X_i) - \lambda_m(X_i) Y_i].$$

2) E step

$$r_{im}^{(l+1)} = \frac{p_m^{(l)}(X_i) \lambda_m^{(l)}(X_i) \exp\{-\lambda_m^{(l)}(X_i) Y_i\}}{\sum_{m=1}^M p_m^{(l)}(X_i) \lambda_m^{(l)}(X_i) \exp\{-\lambda_m^{(l)}(X_i) Y_i\}}.$$

3) in the M step we need to maximize

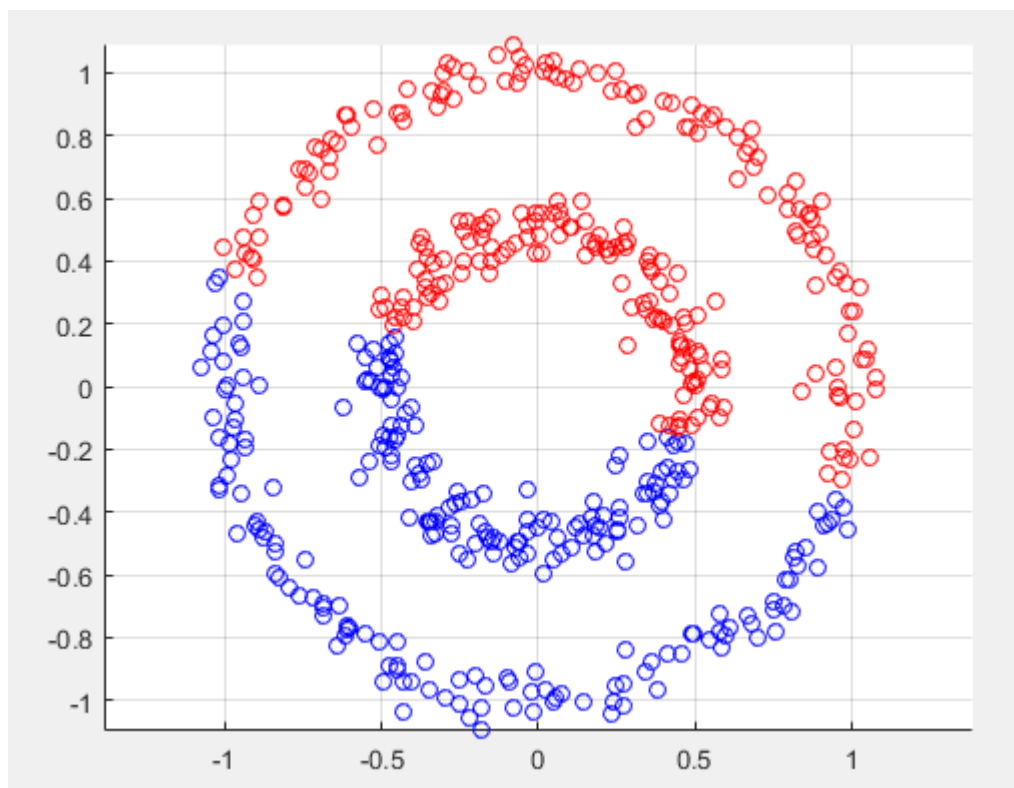
$$\sum_{i=1}^n \sum_{m=1}^M r_{im}^{(l+1)} [\log p_m + \log \lambda_m - \lambda_m Y_i]$$

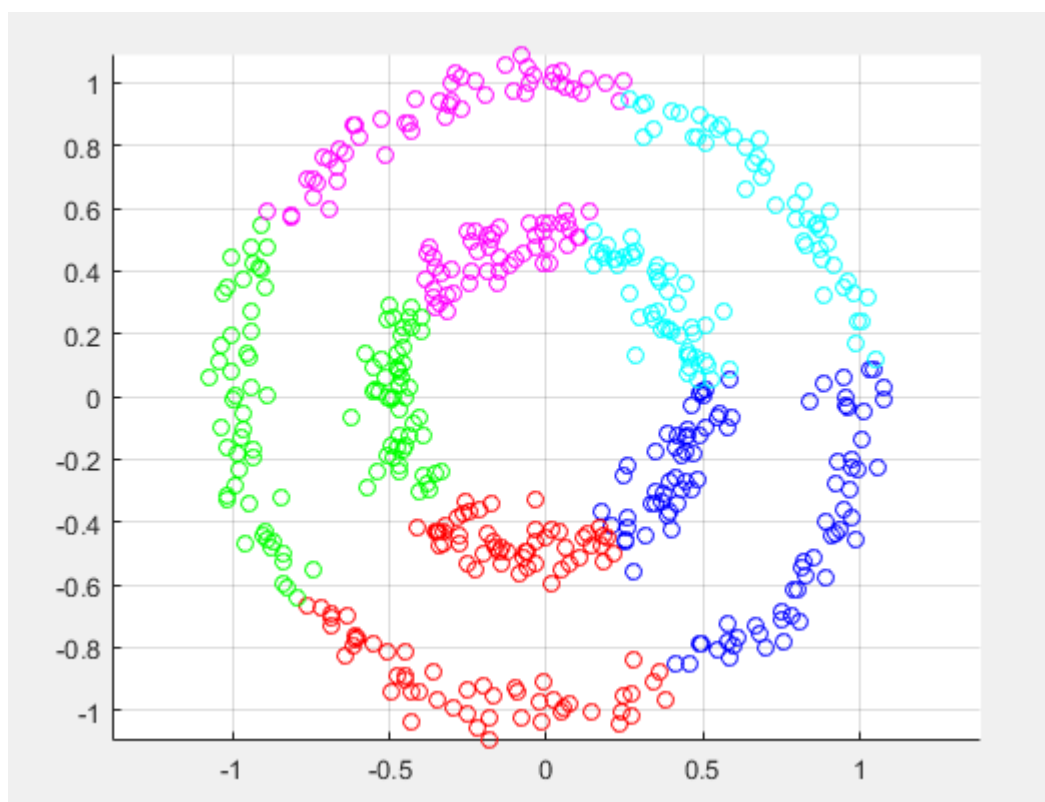
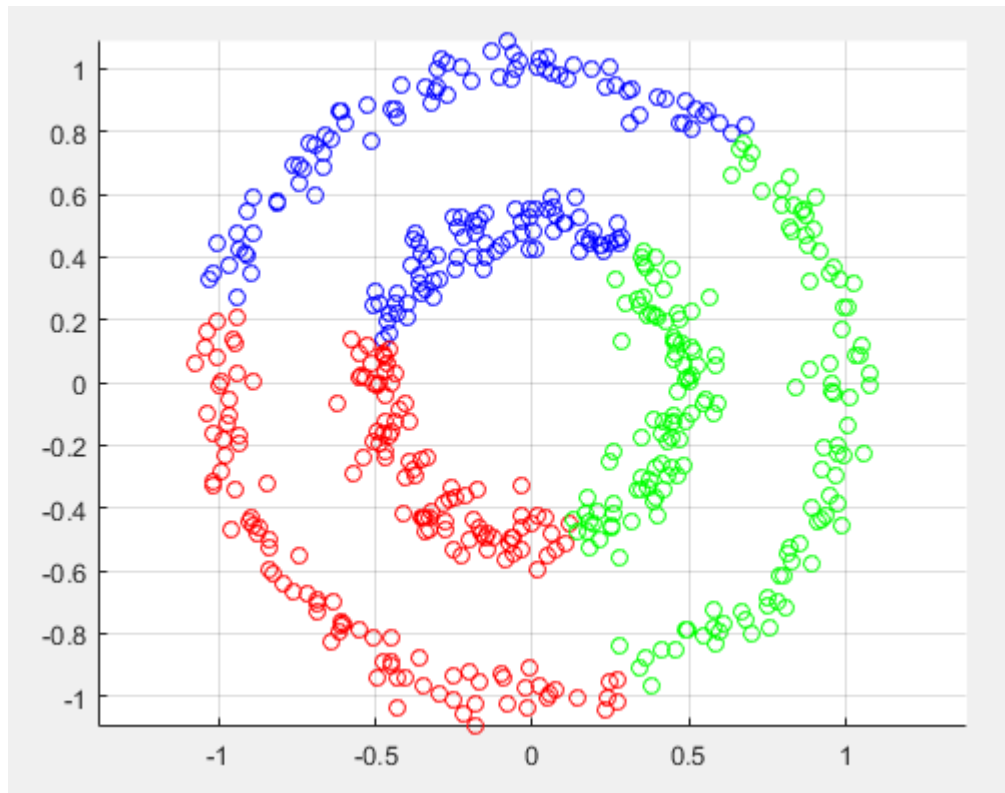
$$\lambda_m^{(l+1)}(x) = \frac{\sum_{i=1}^n r_{im}^{(l+1)} F}{\sum_{i=1}^n r_{im}^{(l+1)} Y_i}$$

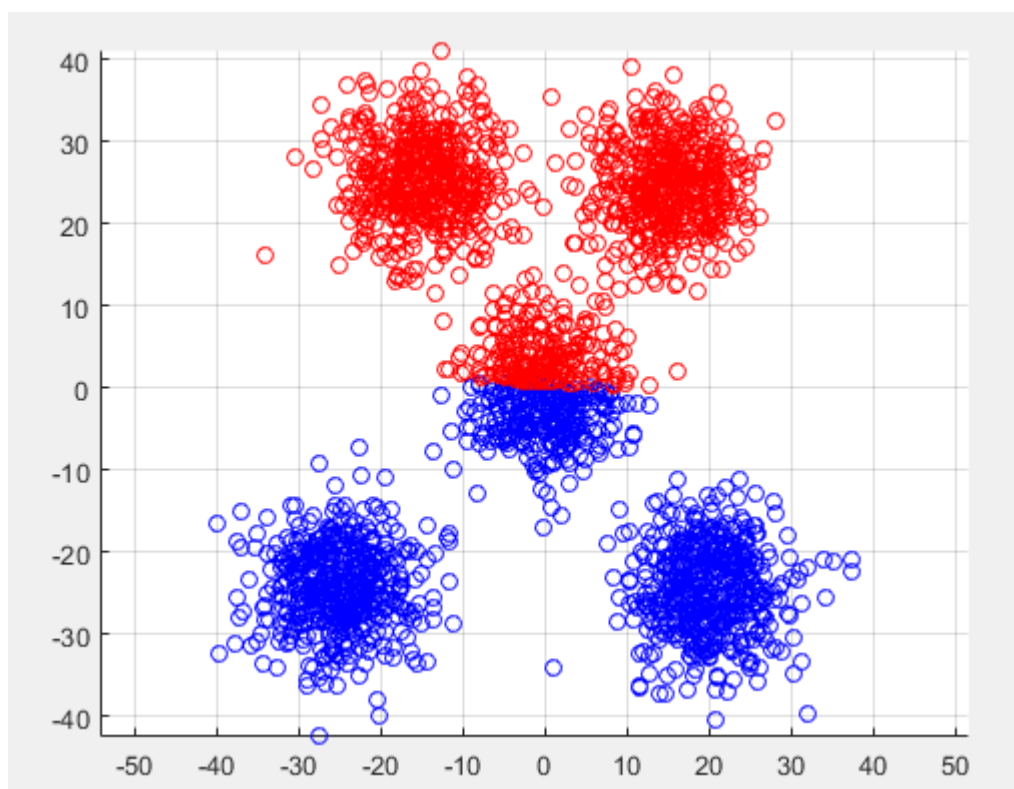
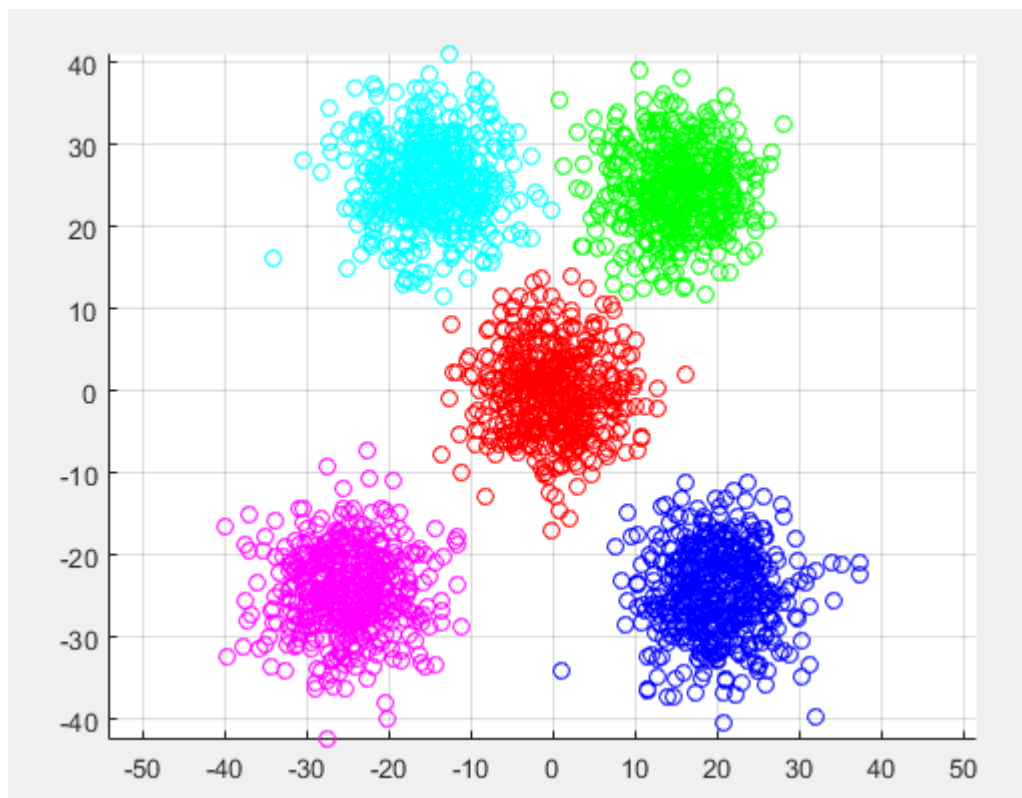
3

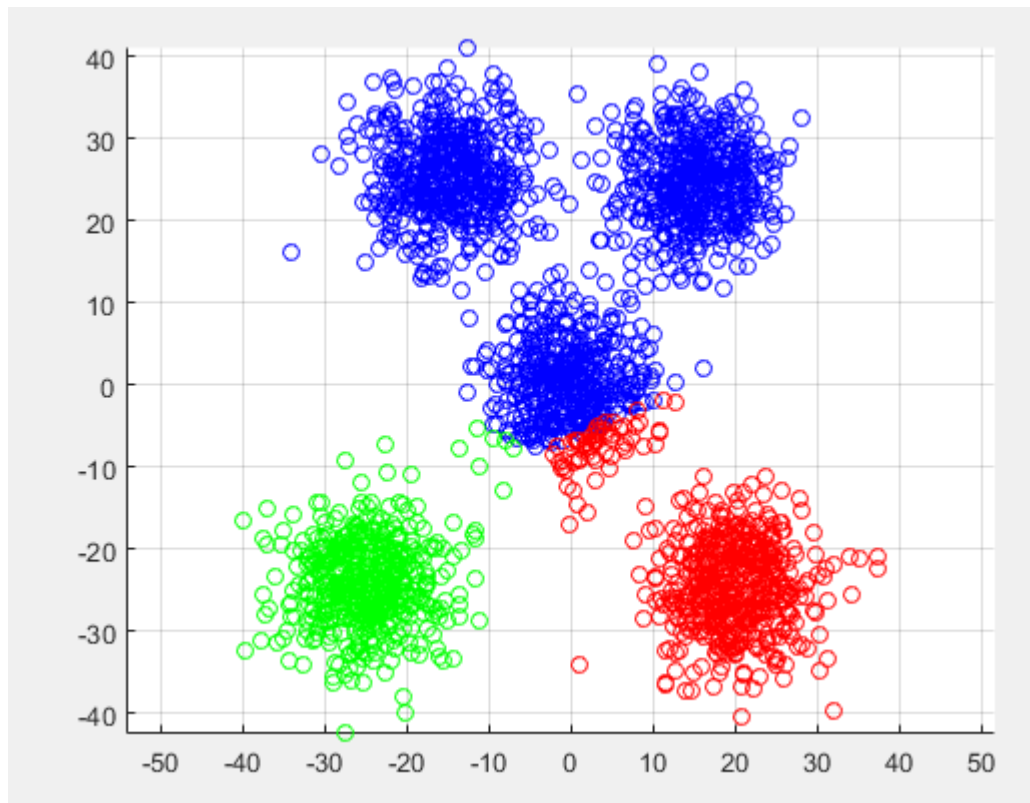
2) K-Means algorithm is efficient for smaller data sets and K-Medoids algorithm seems to perform better for large data sets.

k means sensitive to outlier and objective function.









For the circle it is failed to separate it since the distance to the center is the same.

the NO. 1 model.

initial mu

2.8791	-0.4866
-0.0374	1.2096
5.4871	0.7125

initial sigma (,:,1)= 0.5309 0

0 0.0915

(,:,2)= 0.5309 0

0 0.0915

(,:,3)= 0.5309 0

0 0.0915

initial prior probs

0.3333	0.3333	0.3333
--------	--------	--------

initial log likelihood is:

-1.7388e+04

final iteration is

112

final likelihood is:

-1.7381e+03

difference

1.6629e-05

my gmm:

my mu

2.3399 0.8565

2.3950 3.4136

5.7088 4.1591

my Sigma

(:,:,1) =

2.0459 0.3604

0.3604 0.4099

(:,:,2) =

1.0018 1.2935

1.2935 2.7074

(:,:,3) =

0.4280 0.0466

0.0466 0.4718

my prop

0.4268 0.3726 0.2007

the NO. 2 model.

initial mu

2.1850 5.5090

-0.4494 1.1877

-0.3842 3.9230

initial sigma

(:,:,1)=0.9644 0

0 0.4325

(:,:,2)=0.9644 0

0 0.4325

(:,:,3)=0.9644 0

0 0.4325

inital prior probs

0.3333 0.3333 0.3333

initial log likelihood is:

-3.8687e+03

final iteration is

120

final likelihood is:

-1.7293e+03

difference

1.0683e-05

my gmm:

my mu 4.3903 4.4395

3.6040 0.9653

1.3894 1.2819

my Sigma (:,:,1) = 2.2263 -0.2868

-0.2868 0.6176

(:,:,2) = 0.2684 0.0864

0.0864 0.3318

(:,:,3) = 0.7881 0.6014

0.6014 1.0386

my prop

0.3974 0.2052 0.3974

the NO. 3 model.

initial mu 4.7227 4.3786

5.2782 -0.4064

2.4246 6.8183

initial sigma

(:,:,1) = 0.1875 0

0 0.2662

(:,:,2) = 0.1875 0

0 0.2662

(:,:,3) = 0.1875 0

0 0.2662

inital prior probs

0.3333 0.3333 0.3333

initial log likelihood is:

-1.1767e+04

final iteration is

164

final likelihood is:

-1.7381e+03

difference

1.6050e-05

my gmm:

my mu 5.7089 4.1592

2.3337 0.8560

2.4026 3.4266

my Sigma

(:,:,1) = 0.4279 0.0466

0.0466 0.4718


```

(:,2)=2.0488    0.3614
    0.3614    0.4108
(:,3)=0.9927    1.2777
    1.2777    2.6824
my prop 0.2006    0.4286    0.3708
the NO. 4  model.
initial mu 5.6265    2.1034
    2.9065    1.0244
    5.3734    -1.0422
initial sigma (:,1) = 0.6733    0
    0    0.4296
(:,2)=0.6733    0
    0    0.4296
(:,3)=0.6733    0
    0    0.4296
inital prior probs
    0.3333    0.3333    0.3333
initial log likelihood is:
-3.8414e+03
final iteration is
41
final likelihood is:
-1.7293e+03
difference
1.5497e-05
my gmm:
my mu 4.3901    4.4395
    1.3897    1.2816
    3.6046    0.9653
my Sigma
(:,1) = 2.2264    -0.2866
    -0.2866    0.6176
(:,2)=0.7883    0.6010
    0.6010    1.0380
(:,3)=0.2679    0.0864
    0.0864    0.3317
my prop
    0.3975    0.3975    0.2050
the NO. 5  model.
initial mu 2.5921    1.4003
    3.9784    5.4064
    -0.8479    4.7375
initial sigma (:,1) =0.1253    0
    0    0.1302

```

```

(:,2)=0.1253      0
      0      0.1302
(:,3)=0.1253      0
      0      0.1302
initial prior probs 0.3333      0.3333      0.3333
initial log likelihood is: -6.6951e+03
final iteration is 153
final likelihood is: -1.7381e+03
difference 1.7286e-05
my gmm:
my mu 2.3336      0.8560
      5.7089      4.1592
      2.4027      3.4268
my Sigma(:,1)=2.0488      0.3614
              0.3614      0.4108
(:,2)= 0.4279      0.0466
      0.0466      0.4718
(:,3)= 0.9925      1.2774
      1.2774      2.6819
my prop 0.4286      0.2006      0.3708

```

the NO. 1 model for holdout data.

```

initial mu
      2.3399      0.8565
      2.3950      3.4136
      5.7088      4.1591
initial sigma
(:,1)=2.0459      0.3604
      0.3604      0.4099
(:,2)=1.0018      1.2935
      1.2935      2.7074
(:,3)=0.4280      0.0466
      0.0466      0.4718
initial prior probs 0.4268      0.3726      0.2007
initial log likelihood is: -4.6437e+03
final iteration is 62
final likelihood is: -4.4798e+03
difference 4.3214e-05
my gmm: 2.2692      0.9090
      2.4711      3.6114
      5.7262      4.1480
(:,1)=2.1199      0.2742
      0.2742      0.3626
(:,2)=

```

```

0.7666    0.9497
    0.9497    2.3070
(:,3) = 0.3804    0.0370
    0.0370    0.3942
    0.3409    0.2497    0.4094
the NO. 2  model for holdout data.
initial mu  4.3903    4.4395
    3.6040    0.9653
    1.3894    1.2819
initial sigma
(:,1) = 2.2263    -0.2868
    -0.2868    0.6176
(:,2) = 0.2684    0.0864
    0.0864    0.3318
(:,3) = 0.7881    0.6014
    0.6014    1.0386
inital prior probs 0.3974    0.2052    0.3974
initial log likelihood is: -4.7798e+03
final iterat    35
final likelihood is: -4.6074e+03
differenc    4.0285e-05
my gmm:
    5.0506    4.3018
    3.5109    0.9705
    1.3605    1.3854
(:,1) = 1.7368    -0.2836
    -0.2836    0.5095
(:,2) = 0.5404    0.0810
    0.0810    0.2898
(:,3) = 0.7403    0.5667
    0.5667    1.0368
0.5464    0.1665    0.2871
the NO. 3  model for holdout data.
initial mu
    5.7089    4.1592
    2.3337    0.8560
    2.4026    3.4266

initial sigma

(:,1) =

    0.4279    0.0466
    0.0466    0.4718

```

```

(:,2)=2.0488    0.3614
    0.3614    0.4108
(:,3)=0.9927    1.2777
    1.2777    2.6824
inital prior probs    0.2006    0.4286    0.3708
initial log likelihood is: -4.6436e+03
final iteration is    61
final likelihood is: -4.4798e+03
difference    4.4752e-05
my gmm:    5.7262    4.1480
    2.2692    0.9089
    2.4710    3.6113
(:,1)=0.3804    0.0370
    0.0370    0.3942
(:,2)=2.1199    0.2742
    0.2742    0.3626
(:,3)=0.7666    0.9498
    0.9498    2.3072
0.4094    0.3409    0.2497
the NO. 4 model for holdout data.
Initial mu    4.3901    4.4395
    1.3897    1.2816
    3.6046    0.9653
initial sigma(:,1)=2.2264    -0.2866
    -0.2866    0.6176
(:,2)=0.7883    0.6010
    0.6010    1.0380
(:,3)=0.2679    0.0864
    0.0864    0.3317
inital prior probs
    0.3975    0.3975    0.2050
initial log likelihood is: -4.7799e+03
final iteration is    35
final likelihood is: -4.6074e+03
difference    4.0657e-05
my gmm:
    5.0506    4.3018
    1.3605    1.3854
    3.5109    0.9705
(:,1)=1.7368    -0.2836
    -0.2836    0.5095
(:,2)=0.7403    0.5667

```

```

0.5667    1.0368
(:,3)=0.5403    0.0810
0.0810    0.2898
0.5464    0.2871    0.1665
the NO. 5 model for holdout data.
initial mu 2.3336    0.8560
5.7089    4.1592
2.4027    3.4268
initial sigma(:,1)=2.0488    0.3614
0.3614    0.4108
(:,2)=0.4279    0.0466
0.0466    0.4718
(:,3)=0.9925    1.2774
1.2774    2.6819
inital prior probs 0.4286    0.2006    0.3708
initial log likelihood is: -4.6436e+03
final iteration is 61
final likelihood is: -4.4798e+03
difference 4.4683e-05
my gmm: 2.2692    0.9089
5.7262    4.1480
2.4710    3.6113
(:,1)=2.1199    0.2742
0.2742    0.3626
(:,2)=0.3804    0.0370
0.0370    0.3942
(:,3)=0.7666    0.9498
0.9498    2.3072
0.3409    0.4094    0.2497

```

So we can see that the best likelihood of holdout data is the one with high log
The iteration is 61 likelyhood is -4.4798e+03
The likelihood of the training data is -1.7381e+03

K=5

```

the NO. 1 model.
initial mu 5.2182    7.0422
0.8679    3.4292
3.0674    -0.1533
4.7607    -0.0597
6.4426    0.8891
initial sigma
(:,1)=3.9726    0
0    3.5544

```

```

(:,,2) = 3.9726      0
                0    3.5544
(:,,3) = 3.9726      0
                0    3.5544
(:,,4) = 3.9726      0
                0    3.5544
(:,,5) = 3.9726      0
                0    3.5544
                initial prior probs
                0.2000    0.2000    0.2000    0.2000    0.2000
initial log likelihood is -2.3548e+03
final iteration is 174
final likelihood is: -1.6764e+03
difference 1.3680e-05
my gmm:
my mu  3.7502    5.2922
        2.9310    4.5226
        1.3475    1.1861
        3.6098    0.9643
        5.7450    4.1179
my Sigma
(:,,1) = 0.8827    0.0338
        0.0338    0.0066
(:,,2) = 0.4563    0.2301
        0.2301    0.7889
(:,,3) = 0.7736    0.5374
        0.5374    0.8824
(:,,4) = 0.2653    0.0896
        0.0896    0.3340
(:,,5) = 0.3990    0.0814
        0.0814    0.4506
my prop 0.0253    0.1972    0.3791    0.2042    0.1942
the NO. 2 model. initial mu
        5.7708    0.8328
        0.7665    4.0326
        6.7789    2.7810
        1.6999    1.7146
        0.3550    5.9158
initial sigma  (:,1) = 3.7172      0
                0    3.8498
(:,,2) = 3.7172      0
                0    3.8498
(:,,3) = 3.7172      0
                0    3.8498

```

```
(:,:,4)=3.7172      0
              0      3.8498
(:,:,5) = 3.7172      0
              0      3.8498
initial prior probs 0.2000    0.2000    0.2000    0.2000    0.2000
```

initial log likelihood is: -2.3125e+03

final iteration is 269

final likelihood is: -1.6764e+03

difference 1.6680e-05

my gmm: my mu

```
3.7029    1.0112
1.3879    1.3957
5.7264    4.1436
2.0216    0.8306
2.9957    4.6978
```

```
my Sigma (:,:,1) = 0.1715    0.0738
              0.0738    0.3096
```

```
(:,:,2)=0.8170    0.7705
              0.7705    1.0892
```

```
(:,:,3)=0.4093    0.0588
              0.0588    0.4720
```

```
(:,:,4)=1.4790    0.1807
              0.1807    0.4694
```

```
(:,:,5)=0.5347    0.2379
              0.2379    0.6326
```

```
my prop 0.1595    0.2947    0.1983    0.1435    0.2040
```

the NO. 3 model. initial mu

```
6.6731    3.6098
1.1375    -0.7036
5.2703    -0.8957
5.2398    3.2852
1.9670    5.4629
```

initial sigma

```
(:,:,1)=4.0659      0
              0      3.4300
```

```
(:,:,2)=4.0659      0
              0      3.4300
```

```
(:,:,3)=4.0659      0
              0      3.4300
```

```
(:,:,4)=4.0659      0
              0      3.4300
```

```
(:,:,5)=4.0659      0
              0      3.4300
```

initial prior probs 0.2000 0.2000 0.2000 0.2000 0.2000
 initial log likelihood is -2.3066e+03
 final iteration is 263
 final likelihood is -1.6764e+03
 difference 1.6564e-05

my gmm: my mu 5.7264 4.1436
 1.3878 1.3956
 3.7029 1.0112
 2.0218 0.8306
 2.9957 4.6977

my Sigma (:,:,1) = 0.4093 0.0588
 0.0588 0.4720

(:,:,2) = 0.8170 0.7704
 0.7704 1.0891

(:,:,3) = 0.1715 0.0738
 0.0738 0.3096

(:,:,4) = 1.4791 0.1806
 0.1806 0.4694

(:,:,5) = 0.5347 0.2379
 0.2379 0.6326

my prop 0.1983 0.2947 0.1595 0.1435 0.2040

the NO. 4 model. initial mu

3.6186 5.5954
 2.7468 1.3600
 -1.2643 3.2654
 1.5871 0.0837
 0.0533 3.9093

initial sigma(:,:,1) = 3.3949 0
 0 3.9542

(:,:,2) = 3.3949 0
 0 3.9542

(:,:,3) = 3.3949 0
 0 3.9542

(:,:,4) = 3.3949 0
 0 3.9542

(:,:,5) = 3.3949 0
 0 3.9542

initial prior probs 0.2000 0.2000 0.2000 0.2000 0.2000

initial log likelihood is: -2.2490e+03

final iteration is 212

final likelihood is: -1.6764e+03

difference 1.6514e-05

my gmm my mu 5.7263 4.1437

	3.7046	1.0096			
	1.3832	1.3912			
	2.0256	0.8508			
	2.9949	4.6963			
my Sigma(:,1)	=0.4093	0.0588			
	0.0588	0.4719			
(:,:,2)	=0.1702	0.0743			
	0.0743	0.3087			
(:,:,3)	=0.8182	0.7770			
	0.7770	1.0947			
(:,:,4)	=1.4657	0.1748			
	0.1748	0.4809			
(:,:,5)	=0.5351	0.2388			
	0.2388	0.6344			
my prop	0.1983	0.1586	0.2916	0.1473	0.2042

the NO. 5 model. initial mu

	4.6742	6.6392			
	5.1909	-0.0327			
	2.5816	5.8719			
	-0.6338	3.3514			
	0.6389	7.3652			

initial sigma (:,1) = 3.2101 0

		0	3.7428		
(:,:,2)	= 3.2101	0			
	0	3.7428			
(:,:,3)	= 3.2101	0			
	0	3.7428			
(:,:,4)	= 3.2101	0			
	0	3.7428			
(:,:,5)	= 3.2101	0			
	0	3.7428			

inital prior probs 0.2000 0.2000 0.2000 0.2000 0.2000

initial log likelihood is -2.5780e+03

final iteration is 135

final likelihood is: -1.6734e+03

difference 1.6561e-05

my gmm: my mu

	5.7278	4.1427
	3.6300	0.9660
	2.0401	2.4891
	1.2415	0.9107
	2.9876	4.6476

```

my Sigma (:,:,1)=0.4082    0.0595
                0.0595    0.4717
(:,:,2)=0.2484    0.0865
                0.0865    0.3337
(:,:,3)=0.4741    0.2318
                0.2318    0.1553
(:,:,4)=0.7661    0.3921
                0.3921    0.5776
(:,:,5)=0.5466    0.2519
                0.2519    0.6689
my prop  0.1981    0.1996    0.0697    0.3185    0.2141
the NO. 1  model. initial mu
                5.2182    7.0422
                0.8679    3.4292
                3.0674   -0.1533
                4.7607   -0.0597
                6.4426    0.8891
initial sigma
(:,:,1)=3.9726    0
                0    3.5544
(:,:,2)=3.9726    0
                0    3.5544
(:,:,3)=3.9726    0
                0    3.5544
(:,:,4)=3.9726    0
                0    3.5544
(:,:,5)=3.9726    0
                0    3.5544
inital prior probs 0.2000    0.2000    0.2000    0.2000    0.2000
initial log likelihood is: -2.3548e+03
final iteration is 174
final likelihood is: -1.6764e+03
difference 1.3680e-05
my gmm: my mu
                3.7502    5.2922
                2.9310    4.5226
                1.3475    1.1861
                3.6098    0.9643
                5.7450    4.1179
my Sigma (:,:,1)=0.8827    0.0338
                0.0338    0.0066
(:,:,2)=0.4563    0.2301
                0.2301    0.7889
(:,:,3)=0.7736    0.5374

```

```

0.5374    0.8824
(:,4)=0.2653    0.0896
0.0896    0.3340
(:,5)=0.3990    0.0814
0.0814    0.4506
my prop  0.0253    0.1972    0.3791    0.2042    0.1942
the NO. 2  model.initial mu
5.7708    0.8328
0.7665    4.0326
6.7789    2.7810
1.6999    1.7146
0.3550    5.9158
initial sigma(:,1)=3.7172    0
0    3.8498
(:,2)=3.7172    0
0    3.8498
(:,3)=3.7172    0
0    3.8498
(:,4)=3.7172    0
0    3.8498
(:,5)=3.7172    0
0    3.8498
inital prior probs 0.2000    0.2000    0.2000    0.2000    0.2000
initial log likelihood is: -2.3125e+03
final iteration is 269
final likelihood is: -1.6764e+03
difference 1.6680e-05
my gmm: my mu
3.7029    1.0112
1.3879    1.3957
5.7264    4.1436
2.0216    0.8306
2.9957    4.6978
my Sigma(:,1)=0.1715    0.0738
0.0738    0.3096
(:,2)=0.8170    0.7705
0.7705    1.0892
(:,3)=0.4093    0.0588
0.588    0.4720
(:,4)=1.4790    0.1807
0.1807    0.4694
(:,5)=0.5347    0.2379
0.2379    0.6326
my prop  0.1595    0.2947    0.1983    0.1435    0.2040

```

```

the NO. 3  model. initial mu
      6.6731      3.6098
      1.1375     -0.7036
      5.2703     -0.8957
      5.2398      3.2852
      1.9670      5.4629
initial sigma (:,:,1)=4.0659      0
      0      3.4300
(:,:,2)=4.0659      0
      0      3.4300
(:,:,3)=4.0659      0
      0      3.4300
(:,:,4)=4.0659      0
      0      3.4300
(:,:,5)=4.0659      0
      0      3.4300
inital prior probs 0.2000      0.2000      0.2000      0.2000      0.2000
initial log likelihood is: -2.3066e+03
final iteration is 263
final likelihood is: -1.6764e+03
difference 1.6564e-05
my gmm: my mu
      5.7264      4.1436
      1.3878      1.3956
      3.7029      1.0112
      2.0218      0.8306
      2.9957      4.6977
my Sigma(:,:,1)=0.4093      0.0588
      0.0588      0.4720
(:,:,2)=0.8170      0.7704
      0.7704      1.0891
(:,:,3)=0.1715      0.0738
      0.0738      0.3096
(:,:,4)=1.4791      0.1806
      0.1806      0.4694
(:,:,5)=0.5347      0.2379
      0.2379      0.6326
my prop 0.1983      0.2947      0.1595      0.1435      0.2040
the NO. 4  model. initial mu
      3.6186      5.5954
      2.7468      1.3600
      -1.2643      3.2654
      1.5871      0.0837
      0.0533      3.9093

```

```

initial sigma (:,:,1)=3.3949      0
                0      3.9542
(:, :,2)=3.3949      0
                0      3.9542
(:, :,3)=3.3949      0
                0      3.9542
(:, :,4)=3.3949      0
                0      3.9542
(:, :,5)=3.3949      0
                0      3.9542
initial prior probs 0.2000      0.2000      0.2000      0.2000      0.2000
initial log likelihood is: -2.2490e+03
final iteration is 212
final likelihood is: -1.6764e+03
difference 1.6514e-05
my gmm: my mu
        5.7263      4.1437
        3.7046      1.0096
        1.3832      1.3912
        2.0256      0.8508
        2.9949      4.6963
my Sigma(:,:,1)=0.4093      0.0588
                0.0588      0.4719
(:, :,2)=0.1702      0.0743
                0.0743      0.3087
(:, :,3)=0.8182      0.7770
                0.7770      1.0947
(:, :,4)=1.4657      0.1748
                0.1748      0.4809
(:, :,5)=0.5351      0.2388
                0.2388      0.6344
my prop 0.1983      0.1586      0.2916      0.1473      0.2042
the NO. 5 model initial mu
        4.6742      6.6392
        5.1909     -0.0327
        2.5816      5.8719
        -0.6338      3.3514
        0.6389      7.3652
initial sigma(:,:,1)=3.2101      0
                0      3.7428
(:, :,2)=3.2101      0
                0      3.7428
(:, :,3)=3.2101      0
                0      3.7428

```

```
(:,:,4)=3.2101      0
              0      3.7428
(:,:,5)=3.2101      0
              0      3.7428
inital prior probs 0.2000      0.2000      0.2000      0.2000      0.2000
initial log likelihood is: -2.5780e+03
final iteration is 135
final likelihood is: -1.6734e+03
difference  1.6561e-05
```

my gmm: my mu

```
5.7278      4.1427
3.6300      0.9660
2.0401      2.4891
1.2415      0.9107
2.9876      4.6476
```

```
my Sigma(:,:,1)=0.4082      0.0595
                0.0595      0.4717
```

```
(:,:,2)=0.2484      0.0865
              0.0865      0.3337
```

```
(:,:,3)=0.4741      0.2318
              0.2318      0.1553
```

```
(:,:,4)=0.7661      0.3921
              0.3921      0.5776
```

```
(:,:,5)=0.5466      0.2519
              0.2519      0.6689
```

```
my prop 0.1981      0.1996      0.0697      0.3185      0.2141
```

the NO. 1 model for holdout data. initial mu

```
3.7502      5.2922
2.9310      4.5226
1.3475      1.1861
3.6098      0.9643
5.7450      4.1179
```

```
initial sigma(:,:,1)=0.8827      0.0338
                0.0338      0.0066
```

```
(:,:,2)=0.4563      0.2301
              0.2301      0.7889
```

```
(:,:,3)=0.7736      0.5374
              0.5374      0.8824
```

```
(:,:,4)=0.2653      0.0896
              0.0896      0.3340
```

```
(:,:,5)=0.3990      0.0814
              0.0814      0.4506
```

```
inital prior probs 0.0253      0.1972      0.3791      0.2042      0.1942
```

initial log likelihood is: -4.5966e+03

final iteration is 60

final likelihood is: -4.3929e+03

difference 4.0136e-05

my gmm:4.8229 5.4811

2.9591 4.6370

1.3236 1.2822

3.5364 0.9659

5.7322 4.1409

(:,:,1) =0.4330 0.1081

0.1081 0.0271

(:,:,2) =0.3382 0.0722

0.0722 0.6726

(:,:,3) =0.7182 0.4759

0.4759 0.8494

(:,:,4) =0.5150 0.0825

0.0825 0.2871

(:,:,5) =0.3727 0.0391

0.0391 0.3935

0.0030 0.1496 0.2773 0.1627 0.4074

the NO. 2 model for holdout data.

initial mu

3.7029 1.0112

1.3879 1.3957

5.7264 4.1436

2.0216 0.8306

2.9957 4.6978

initial sigma(:,:,1) =0.1715 0.0738

0.0738 0.3096

(:,:,2) =0.8170 0.7705

0.7705 1.0892

(:,:,3) =0.4093 0.0588

0.0588 0.4720

(:,:,4) =1.4790 0.1807

0.1807 0.4694

(:,:,5) =0.5347 0.2379

0.2379 0.6326

inital prior probs 0.1595 0.2947 0.1983 0.1435 0.2040

initial log likelihood is: -4.5942e+03

final iteration is 141

final likelihood is: -4.3799e+03

difference 4.1152e-05

my gmm:3.6227 0.9938

1.7994 1.6964

```

5.7288    4.1483
0.6287    0.6027
3.0187    4.6903
(:,,1) =0.4330    0.0497
0.0497    0.2644
(:,,2) =0.4182    0.1368
0.1368    0.8853
(:,,3) =0.3739    0.0347
0.0347    0.4032
(:,,4) =0.3942    0.1353
0.1353    0.2654
(:,,5) =0.2915    0.0140
0.0140    0.6241
0.1508    0.1881    0.4097    0.1071    0.1443
the NO. 3 model for holdout data.
initial mu 5.7264    4.1436
1.3878    1.3956
3.7029    1.0112
2.0218    0.8306
2.9957    4.6977
initial sigma (:,1) =0.4093    0.0588
0.0588    0.4720
(:,,2) =0.8170    0.7704
0.7704    1.0891
(:,,3) =0.1715    0.0738
0.0738    0.3096
(:,,4) =1.4791    0.1806
0.1806    0.4694
(:,,5) =0.5347    0.2379
0.2379    0.6326
inital prior probs 0.1983    0.2947    0.1595    0.1435    0.2040
initial log likelihood is: -4.5942e+03
final iteration is 141
final likelihood is: -4.3799e+03
difference 4.1212e-05
my gmm:5.7288    4.1483
1.7994    1.6964
3.6227    0.9938
0.6287    0.6027
3.0187    4.6903
(:,,1) =0.3739    0.0347
0.0347    0.4032
(:,,2) =0.4182    0.1368
0.1368    0.8853

```



```

(:,3)=0.4330    0.0497
          0.0497    0.2644
(:,4)=0.3942    0.1353
          0.1353    0.2654
(:,5)=0.2915    0.0140
          0.0140    0.6241
0.4097    0.1881    0.1508    0.1071    0.1443
the NO. 4 model for holdout data. initial mu
          5.7263    4.1437
          3.7046    1.0096
          1.3832    1.3912
          2.0256    0.8508
          2.9949    4.6963
initial sigma(:,1)=0.4093    0.0588
                  0.0588    0.4719
(:,2)=0.1702    0.0743
          0.0743    0.3087
(:,3)=0.8182    0.7770
          0.7770    1.0947
(:,4)=1.4657    0.1748
          0.1748    0.4809
(:,5)=0.5351    0.2388
          0.2388    0.6344
inital prior probs
          0.1983    0.1586    0.2916    0.1473    0.2042
initial log likelihood is:-4.5948e+03
final iteration is 150
final likelihood is: -4.3799e+03
difference 4.2264e-05
my gmm:
          5.7288    4.1483
          3.6248    0.9939
          1.8050    1.6998
          0.6339    0.6057
          3.0188    4.6902
(:,1)=0.3739    0.0347
          0.0347    0.4032
(:,2)=0.4315    0.0494
          0.0494    0.2636
(:,3)=0.4174    0.1324
          0.1324    0.8849
(:,4)=0.3957    0.1361
          0.1361    0.2678
(:,5)=0.2913    0.0139

```

0.0139 0.6242
0.4097 0.1504 0.1873 0.1082 0.1443

the NO. 5 model for holdout data.

initial mu5.7278 4.1427
3.6300 0.9660
2.0401 2.4891
1.2415 0.9107
2.9876 4.6476

initial sigma

(:,:,1)=0.4082 0.0595
0.0595 0.4717
(:,:,2)=0.2484 0.0865
0.0865 0.3337
(:,:,3)=0.4741 0.2318
0.2318 0.1553
(:,:,4)=0.7661 0.3921
0.3921 0.5776
(:,:,5)=0.5466 0.2519
0.2519 0.6689

inital prior probs 0.1981 0.1996 0.0697 0.3185 0.2141

initial log likelihood is -4.5858e+03

final iteration is 347

final likelihood is: -4.3799e+03

difference 4.3540e-05

my gmm:

5.7288 4.1482
3.6263 0.9939
1.8071 1.7000
0.6350 0.6063
3.0189 4.6903
(:,:,1)=0.3739 0.0347
0.0347 0.4032
(:,:,2)=0.4305 0.0493
0.0493 0.2631
(:,:,3)=0.4179 0.1309
0.1309 0.8849
(:,:,4)=0.3959 0.1362
0.1362 0.2683
(:,:,5)=0.2912 0.0139
0.0139 0.6241
0.4097 0.1502 0.1874 0.1085 0.1443

The best log for the holdout data is -4.3799e+03 iteration is 141 the log of the training data is -1.6764e+03

K=7

the NO. 1 model.

initial mu

4.6528	5.4326
0.2403	2.8994
1.8621	2.4528
4.1166	2.5486
5.4722	1.3173
-0.6574	3.0898
6.7800	3.1096

initial sigma (:,:,1) = 3.9495 0

0 4.0950

(:,:,2) = 3.9495 0

0 4.0950

(:,:,3) = 3.9495 0

0 4.0950

(:,:,4) = 3.9495 0

0 4.0950

(:,:,5) = 3.9495 0

0 4.0950

(:,:,6) = 3.9495 0

0 4.0950

(:,:,7) = 3.9495 0

0 4.0950

initial prior probs 0.1429 0.1429 0.1429 0.1429 0.1429 0.1429 0.1429

initial log likelihood is: -2.2069e+03

final iteration is 411

final likelihood is: -1.6629e+03

difference 1.6351e-05

my gmm: my mu

2.9712	4.6530
1.9171	1.8283
1.8589	0.3343
3.4058	4.6263
3.6813	1.0276
0.6519	0.4775
5.7191	4.1512

my Sigma(:,:,1) = 0.4926 0.1475

0.1475 0.5928

```

(:,2)=0.4226    0.0966
          0.0966    0.5506
(:,3)=2.0794   -0.1032
          -0.1032    0.2545
(:,4)=0.0846    0.3151
          0.3151    1.2351
(:,5)=0.1854    0.0833
          0.0833    0.2944
(:,6)=0.2778    0.1631
          0.1631    0.2969
(:,7)= 0.4151    0.0507
          0.0507    0.4781
my prop
          0.1807    0.2183    0.0522    0.0294    0.1760    0.1436    0.1997

```

the NO. 2 model.

initial mu

```

4.2805    3.4550
1.9509    4.0890
5.7471    3.7784
3.3030    0.4527
1.7064    1.2726
6.8643    2.7603
6.3114    0.6522

```

initial sigma(:,1) = 3.9762 0

```

          0    3.4949
(:,2)=3.9762          0
          0    3.4949
(:,3)=3.9762          0
          0    3.4949
(:,4)=3.9762          0
          0    3.4949
(:,5)=3.9762          0
          0    3.4949
(:,6)=3.9762          0
          0    3.4949
(:,7)=3.9762          0
          0    3.4949

```

inital prior probs

```

0.1429    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429

```

initial log likelihood is: -2.1583e+03

final iteration is 217

final likelihood is: -1.6567e+03

```

difference 6.6306e-06
my gmm: my mu
    1.9325    2.4329
    2.9929    4.6759
    5.1591    5.0393
    2.4258    1.2866
    0.8006    0.5797
    5.7237    4.1303
    3.7764    1.0401
my Sigma (:,:,1) = 1.0626    0.5658
                    0.5658    0.3424
(:, :, 2) = 0.4999    0.2227
                    0.2227    0.6105
(:, :, 3) = 2.5019   -1.0183
                    -1.0183    0.4145
(:, :, 4) = 0.7207   -0.1799
                    -0.1799    0.6485
(:, :, 5) = 0.3785    0.2331
                    0.2331    0.3532
(:, :, 6) = 0.3695    0.0687
                    0.0687    0.4732
(:, :, 7) = 0.1430    0.0860
                    0.0860    0.2978
my prop
    0.0738    0.2038    0.0091    0.1925    0.1916    0.1917    0.1375
the NO. 3 model.
    initial mu
        0.6121    0.2517
        0.1280    6.5651
        0.6274    7.2215
        2.4514    2.4792
        1.3590   -0.3944
        6.7273    0.8940
        2.4033    2.2149
    initial sigma(:,:,1) = 3.7268    0
                            0    3.5623
(:, :, 2) = 3.7268    0
                    0    3.5623
(:, :, 3) = .7268    0
                    0    3.5623
(:, :, 4) = 3.7268    0
                    0    3.5623
(:, :, 5) = 3.7268    0
                    0    3.5623

```

```

(:,6)=3.7268      0
      0      3.5623
(:,7)=3.7268      0
      0      3.5623
initial prior probs 0.1429    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429
initial log likelihood is: -2.3160e+03
final iteration is    371
final likelihood is:  -1.6575e+03
difference    1.5138e-05
my gmm: my mu
      0.7107    0.5197
      3.3600    4.6263
      2.9972    4.7567
      1.9876    2.4971
      3.6824    0.9945
      5.7290    4.1469
      2.0167    1.5761
my Sigma(:,1) = 0.3102    0.1800
      0.1800    0.3116
(:,2)=0.0976    0.3559
      0.3559    1.3090
(:,3)=0.5018    0.1676
      0.1676    0.4908
(:,4)=1.4013    0.7742
      0.7742    0.4670
(:,5)=0.2042    0.0630
      0.0630    0.3236
(:,6)=0.4083    0.0549
      0.0549    0.4772
(:,7) = 0.3201   -0.0111
      -0.0111    0.7746
my prop 0.1758    0.0206    0.1775    0.0613    0.1896    0.1977    0.1776

```

the NO. 4 model.

```

initial mu
      3.9169    3.0841
      4.8671   -0.6189
      0.5755    0.9327
      -0.3392    5.6544
      1.2325   -1.1125
      1.4263    6.7753
      2.3503    5.0347
initial sigm(:,1) =3.6205    0
      0      3.8786

```

```

(:,2) = 3.6205      0
      0      3.8786
(:,3) = 3.6205      0
      0      3.8786
(:,4) = 3.6205      0
      0      3.8786
(:,5) = 3.6205      0
      0      3.8786
(:,6) = 3.6205      0
      0      3.8786
(:,7) = 3.6205      0
      0      3.8786
initial prior probs
      0.1429      0.1429      0.1429      0.1429      0.1429      0.1429      0.1429
initial log likelihood is: -2.3174e+03
final iteration is 447
final likelihood is: -1.6630e+03
difference 1.5841e-05
my gmm: my mu
      5.7162      4.1281
      3.6887      0.9697
      1.7462      1.8067
      1.2176      2.1300
      1.5208      0.8451
      3.0338      4.8251
      5.1630      5.0378
my Sigma(:,1) = 0.3752      0.0712
      0.0712      0.4724
(:,2) = 0.2045      0.0688
      0.0688      0.3216
(:,3) = 1.1791      1.2711
      1.2711      1.4950
(:,4) = 0.3647      0.5433
      0.5433      1.3543
(:,5) = 1.0281      0.3479
      0.3479      0.4981
(:,6) = 0.4148      0.1736
      0.1736      0.4984
(:,7) = 2.5010      -1.0179
      -1.0179      0.4143

my prop
      0.1930      0.1806      0.1613      0.0642      0.2171      0.1745      0.0091

```

the NO. 5 model.

initial mu

0.7118	4.1029
2.6544	4.5858
7.0755	2.0991
3.4256	1.8529
3.2005	7.2937
0.6619	-1.0378
2.9179	6.3923

initial sigma (:,:,1) = 4.0452 0

0 4.0963

(:,:,2) = 4.0452 0

0 4.0963

(:,:,3) = 4.0452 0

0 4.0963

(:,:,4) = 4.0452 0

0 4.0963

(:,:,5) = 4.0452 0

0 4.0963

(:,:,6) = 4.0452 0

0 4.0963

(:,:,7) = 4.0452 0

0 4.0963

nital prior probs

0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429
--------	--------	--------	--------	--------	--------	--------

initial log likelihood is -2.3047e+03

final iteration is 169

final likelihood is: -1.6664e+03

difference 1.4350e-05

my gmm: my mu

1.9265	1.7425
3.0064	4.6339
5.7461	3.9540
3.6716	0.9803
5.2111	5.0268
0.6352	0.4964
5.4826	4.5715

my Sigma (:,:,1) = 0.4035 0.0971

0.0971 0.8236

(:,:,2) = 0.3844 0.1099

0.1099 0.6336

(:,:,3) = 0.3330 0.1070

0.1070 0.4696

(:,:,4) = 0.2123 0.0703


```

        0.0703    0.3219
(:,5) = 1.3343   -0.5845
        -0.5845    0.2887
(:,6) = 0.3492    0.1421
        0.1421    0.3100
(:,7) = 0.5673    0.2016
        0.2016    0.0931
my prop
        0.2415    0.1911    0.1452    0.1910    0.0367    0.1643    0.0302

```

the NO. 1 model for holdout data.

initial mu

```

        2.9712    4.6530
        1.9171    1.8283
        1.8589    0.3343
        3.4058    4.6263
        3.6813    1.0276
        0.6519    0.4775
        5.7191    4.1512

```

initial sigma

```

(:,1) = 0.4926    0.1475
        0.1475    0.5928
(:,2) = 0.4226    0.0966
        0.0966    0.5506
(:,3) = 2.0794   -0.1032
        -0.1032    0.2545
(:,4) = 0.0846    0.3151
        0.3151    1.2351
(:,5) = 0.1854    0.0833
        0.0833    0.2944
(:,6) = 0.2778    0.1631
        0.1631    0.2969
(:,7) = 0.4151    0.0507
        0.0507    0.4781

```

inital prior probs

```

        0.1807    0.2183    0.0522    0.0294    0.1760    0.1436    0.1997

```

initial log likelihood is -4.5919e+03

final iteration is 243

final likelihood is: -4.3721e+03

difference 4.3580e-05

my gmm:

```

        3.0029    4.7038
        1.7306    1.7829
        1.7072   -0.1397

```

	3.1470	3.4856				
	3.5775	1.0160				
	0.6493	0.6105				
	5.7281	4.1481				
(:,:,1) =	0.2962	0.0109				
	0.0109	0.6073				
(:,:,2) =	0.3916	0.1814				
	0.1814	0.7938				
(:,:,3) =	0.5534	-0.1681				
	-0.1681	0.1080				
(:,:,4) =	0.1901	0.6590				
	0.6590	2.2859				
(:,:,5) =	0.4641	0.0319				
	0.0319	0.2570				
(:,:,6) =	0.4261	0.1473				
	0.1473	0.2604				
(:,:,7) =	0.3749	0.0348				
	0.0348	0.4034				
0.1396	0.1705	0.0088	0.0070	0.1584	0.1058	0.4099

the NO. 2 model for holdout data.

initial mu

	1.9325	2.4329				
	2.9929	4.6759				
	5.1591	5.0393				
	2.4258	1.2866				
	0.8006	0.5797				
	5.7237	4.1303				
	3.7764	1.0401				

initial sigma(:,1) = 1.0626 0.5658

			0.5658	0.3424		
(:,:,2) =	0.4999	0.2227				
	0.2227	0.6105				
(:,:,3) =	2.5019	-1.0183				
	-1.0183	0.4145				
(:,:,4) =	0.7207	-0.1799				
	-0.1799	0.6485				
(:,:,5) =	0.3785	0.2331				
	0.2331	0.3532				
(:,:,6) =	0.3695	0.0687				
	0.0687	0.4732				
(:,:,7) =	0.1430	0.0860				
	0.0860	0.2978				

inital prior probs

0.0738	0.2038	0.0091	0.1925	0.1916	0.1917	0.1375
--------	--------	--------	--------	--------	--------	--------

initial log likelihood is: -4.6189e+03

final iteration is 166

final likelihood is: -4.3713e+03

difference 4.1926e-05

my gmm:

	1.5299	2.0535				
	3.0267	4.6801				
	5.2091	5.0204				
	2.1971	1.8678				
	0.8969	0.7023				
	5.7334	4.1403				
	3.6888	1.0132				
(:,:,1)	=0.2911	0.0720				
	0.0720	0.1375				
(:,:,2)	=0.2814	0.0127				
	0.0127	0.6389				
(:,:,3)	=0.0246	-0.0089				
	-0.0089	0.0033				
(:,:,4)	=0.3459	-0.1399				
	-0.1399	1.1252				
(:,:,5)	=0.5096	0.1726				
	0.1726	0.3186				
(:,:,6)	=0.3743	0.0394				
	0.0394	0.4004				
(:,:,7)	=0.3965	0.0322				
	0.0322	0.2427				
0.0405	0.1446	0.0035	0.1053	0.1611	0.4062	0.1388

the NO. 3 model for holdout data.

initial mu

	0.7107	0.5197				
	3.3600	4.6263				
	2.9972	4.7567				
	1.9876	2.4971				
	3.6824	0.9945				
	5.7290	4.1469				
	2.0167	1.5761				
initial sigma(:,1)	=0.3102	0.1800				
	0.1800	0.3116				
(:,:,2)	=0.0976	0.3559				
	0.3559	1.3090				
(:,:,3)	=0.5018	0.1676				
	0.1676	0.4908				
(:,:,4)	=1.4013	0.7742				
	0.7742	0.4670				

```

(:,5)=0.2042    0.0630
          0.0630    0.3236
(:,6)=  0.4083    0.0549
          0.0549    0.4772
(:,7)= 0.3201   -0.0111
          -0.0111    0.7746
initial prior probs
          0.1758    0.0206    0.1775    0.0613    0.1896    0.1977    0.1776
initial log likelihood is: -4.6004e+03
final iteration is      279
final likelihood is: -4.3716e+03
difference    4.3088e-05
my gmm:
          0.9027    0.7046
          3.1379    3.4542
          3.0128    4.6912
          1.5350    2.0597
          3.6731    1.0195
          5.7275    4.1483
          2.1657    1.8930
(:,1)= 0.5132    0.1753
          0.1753    0.3196
(:,2)= 0.2038    0.7059
          0.7059    2.4476
(:,3)= 0.2782    0.0083
          0.0083    0.6265
(:,4)= 0.2958    0.0804
          0.0804    0.1462
(:,5)= 0.4017    0.0288
          0.0288    0.2486
(:,6)= 0.3756    0.0345
          0.0345    0.4037
(:,7)= 0.3212   -0.1487
          -0.1487    1.1580
          0.1620    0.0069    0.1397    0.0423    0.1423    0.4101    0.0967
the NO. 4  model for holdout data.
initial mu
          5.7162    4.1281
          3.6887    0.9697
          1.7462    1.8067
          1.2176    2.1300
          1.5208    0.8451
          3.0338    4.8251
          5.1630    5.0378

```

```

initial sigma  (:,:,1) =  0.3752    0.0712
                  0.0712    0.4724
(:,:,2) =  0.2045    0.0688
            0.0688    0.3216
(:,:,3) =  1.1791    1.2711
            1.2711    1.4950
(:,:,4) =  0.3647    0.5433
            0.5433    1.3543
(:,:,5) =  1.0281    0.3479
            0.3479    0.4981
(:,:,6) =  0.4148    0.1736
            0.1736    0.4984
(:,:,7) =  2.5010   -1.0179
            -1.0179    0.4143

inital prior probs
    0.1930    0.1806    0.1613    0.0642    0.2171    0.1745    0.0091
initial log likelihood is: -4.6027e+03
final iteration is      724
final likelihood is: -4.3705e+03
difference      4.2298e-05
my gmm:
    5.7341    4.1407
    3.5449    0.9693
    1.8647    2.0846
    1.9777    3.1676
    0.9870    0.7445
    3.0015    4.6488
    5.2091    5.0204
(:,:,1) =  0.3737    0.0393
            0.0393    0.3992
(:,:,2) =  0.5041    0.0799
            0.0799    0.2893
(:,:,3) =  0.3438    0.0187
            0.0187    0.1784
(:,:,4) =  0.3502   -0.1581
            -0.1581    0.1010
(:,:,5) =  0.5738    0.1877
            0.1877    0.3402
(:,:,6) =  0.3027    0.0300
            0.0300    0.6552
(:,:,7) =  0.0246   -0.0089
            -0.0089    0.0033
    0.4059    0.1619    0.0862    0.0137    0.1786    0.1503    0.0035

```

the NO. 5 model for holdout data. initial mu

1.9265	1.7425
3.0064	4.6339
5.7461	3.9540
3.6716	0.9803
5.2111	5.0268
0.6352	0.4964
5.4826	4.5715

initial sigma (:,:,1) = 0.4035 0.0971
 0.0971 0.8236

(:,:,2) = 0.3844 0.1099
 0.1099 0.6336

(:,:,3) = 0.3330 0.1070
 0.1070 0.4696

(:,:,4) = 0.2123 0.0703
 0.0703 0.3219

(:,:,5) = 1.3343 -0.5845
 -0.5845 0.2887

(:,:,6) = 0.3492 0.1421
 0.1421 0.3100

(:,:,7) = 0.5673 0.2016
 0.2016 0.0931

inital prior probs

0.2415	0.1911	0.1452	0.1910	0.0367	0.1643	0.0302
--------	--------	--------	--------	--------	--------	--------

initial log likelihood is: -4.5925e+03

final iteration is 995

final likelihood is: -4.3682e+03

difference 4.3230e-05

my gmm:

1.8017	1.6978
3.0236	4.6879
5.5166	3.7719
3.6242	0.9939
5.8993	4.5024
0.6301	0.6036
6.9322	4.9705

(:,:,1) = 0.4183 0.1358
 0.1358 0.8858

(:,:,2) = 0.2948 0.0099
 0.0099 0.6219

(:,:,3) = 0.2972 -0.0081
 -0.0081 0.2862

(:,:,4) = 0.4326 0.0501
 0.0501 0.2642

```

(:,5) =    0.3115   -0.1085
          -0.1085    0.2386
(:,6) =    0.3945    0.1355
          0.1355    0.2661
(:,7) =    0.1568   -0.0332
          -0.0332    0.0292
          0.1881    0.1448    0.2038    0.1506    0.1961    0.1074    0.0091

```

The high log is -4.3713e+03 with iteration 166 the log of training data is -1.6567e+03

K=9

the NO. 1 model.

initial mu

```

-0.5032    5.4619
  0.9273    4.9006
  1.5716    6.5549
  4.5910    6.4427
 -0.1714    1.5612
  4.9549    4.7578
 -0.4326    0.3657
  4.3633   -1.1009
  2.9641    5.1552

```

initial sigma

```

(:,1)=3.6319    0
           0    3.7800
(:,2)=3.6319    0
           0    3.7800
(:,3)=3.6319    0
           0    3.7800
(:,4)=3.6319    0
           0    3.7800
(:,5)=3.6319    0
           0    3.7800
(:,6)=3.6319    0
           0    3.7800
(:,7)=3.6319    0
           0    3.7800
(:,8)=3.6319    0
           0    3.7800
(:,9)=3.6319    0
           0    3.7800

```

initial prior probs

```

          0.1111    0.1111    0.1111    0.1111    0.1111    0.1111    0.1111
0.1111    0.1111

```

initial log likelihood is: -2.3895e+03

final iteration is 176

final likelihood is: -1.6497e+03

difference 1.5074e-05

my gmm:

my mu

3.4406	5.6017
2.1381	2.5273
3.0362	4.6241
4.9587	5.1235
1.4989	1.0662
5.7127	4.0696
0.4471	0.3736
3.6779	0.9728
1.6977	3.6612

my Sigma

(:,:,1) =0.1077	0.2487
0.2487	0.5830
(:,:,2) =0.4264	0.2259
0.2259	0.1586
(:,:,3) =0.2741	0.0045
0.0045	0.4394
(:,:,4) =1.4091	-0.6039
-0.6039	0.2910
(:,:,5) =0.7789	0.2773
0.2773	0.5892
(:,:,6) =0.3785	0.0913
0.0913	0.4658
(:,:,7) = 0.2017	0.2235
0.2235	0.2898
(:,:,8) =0.2104	0.0719
0.0719	0.3215
(:,:,9) =0.0582	-0.0331
-0.0331	0.5054

my prop

0.0126	0.0638	0.1562	0.0409	0.2756	0.1766	0.0625
0.1870	0.0248					

the NO. 2 model.

initial mu

6.5637	-1.1173
3.9785	2.9267
4.0469	0.1036
6.1667	7.2122
5.6937	4.8801

3.6879	3.0193
0.2351	2.7617
0.7350	-0.8460
6.4040	4.6107

initial sigma

(:,:,1) = 3.1743	0
0	3.3716
(:,:,2) = 3.1743	0
0	3.3716
(:,:,3) = 3.1743	0
0	3.371
(:,:,4) = 3.1743	0
0	3.3716
(:,:,5) = 3.1743	0
0	3.3716
(:,:,6) = 3.1743	0
0	3.3716
(:,:,7) = 3.1743	0
0	3.3716
(:,:,8) = 3.1743	0
0	3.3716
(:,:,9) = 3.1743	0
0	3.3716

initial prior probs

0.1111	0.1111	0.1111	0.1111	0.1111	0.1111	0.1111
0.1111	0.1111					

initial log likelihood is: -2.2363e+03

final iteration is 324

final likelihood is: -1.6458e+03

Difference 1.6390e-05

my gmm:

my mu

3.9165	1.1828
1.8343	1.9623
3.1945	1.0762
3.9363	5.5601
2.9316	4.5980
3.4819	4.9295
1.8819	2.4190
0.8134	0.6027
5.7437	4.1220

```

my Sigma
(:,,1) = 0.0909    0.1309
           0.1309    0.2860
(:,,2) = 0.0684   -0.1394
           -0.1394    1.9205
(:,,3) = 0.4591   -0.1575
           -0.1575    0.3945
(:,,4) = 0.5321   -0.1471
           -0.1471    0.0656
(:,,5) = 0.2570   -0.0241
           -0.0241    0.3877
(:,,6) = 0.0638    0.2275
           0.2275    0.8633
(:,,7) = 1.0674    0.5410
           0.5410    0.3387
(:,,8) = 0.3678    0.2299
           0.2299    0.3598
(:,,9) = 0.4022    0.0755
           0.0755    0.4505
my prop 0.0695     0.0810    0.1802    0.0314    0.1300    0.0287    0.0834
0.2018    0.1940
the NO. 3 model.initial mu
           3.2050    4.3215
           -0.5206    5.6484
           5.8047    2.6101
           5.7994    2.4224
           4.9655    5.8675
           -0.0547   -0.6369
           4.4146   -0.2011
           3.1782    0.1515
           7.1621    2.0590
initial sigma(:,,1) = 3.9633    0
           0    4.1035
(:,,2) = 3.9633    0
           0    4.1035
(:,,3) = 3.9633    0
           0    4.1035
(:,,4) = 3.9633    0
           0    4.1035
(:,,5) = 3.9633    0
           0    4.1035
(:,,6) = 3.9633    0
           0    4.1035
(:,,7) = 3.9633    0

```

```

0      4.1035
(:,8)=3.9633      0
0      4.1035
(:,9)=3.9633      0
0      4.1035
initial prior probs
0.1111      0.1111      0.1111      0.1111      0.1111      0.1111      0.1111
0.1111      0.1111
initial log likelihood is:-2.3014e+03
final iteration is 325
final likelihood is: -1.6494e+03
difference 1.4906e-05
my gmm:my mu 3.4246      4.7506
2.7561      4.6564
1.6841      2.3344
2.0227      2.1315
4.5706      5.2760
0.7497      0.4910
3.7826      1.0475
2.4321      1.2601
5.7225      4.1090
my Sigma(:,1)=0.0685      0.2345
0.2345      0.8460
(:,2)=0.4348      0.2282
0.2282      0.5064
(:,3)=1.0796      0.5632
0.5632      0.3299
(:,4)=1.5943      1.6934
1.6934      1.8463
(:,5)=1.5731      -0.6426
-0.6426      0.2685
(:,6)=0.3233      0.1451
0.1451      0.2833
(:,7)=0.1397      0.0871
0.0871      0.2982
(:,8)=0.7455      -0.1869
-0.1869      0.5973
(:,9)=0.3892      0.0755
0.0755      0.4668
my prop
0.0324      0.1455      0.0501      0.0712      0.0250      0.1563      0.1340
0.1970      0.1885
the NO. 4 model initial mu
-0.8385      7.2594

```

```

2.1319    0.0970
3.2508   -0.4374
2.2857    1.8965
4.3905    0.3684
4.1372    2.9248
1.1914    1.6079
2.4159    6.9750
-1.2329    6.7006
initial sigma(:,1)=3.1846      0
                                0    4.0380
(:,2)=3.1846      0
                                0    4.0380
(:,3)=3.1846      0
                                0    4.0380
(:,4)=3.1846      0
                                0    4.0380
(:,5)=3.1846      0
                                0    4.0380
(:,6)=3.1846      0
                                0    4.0380
(:,7)=3.1846      0
                                0    4.0380
(:,8)=3.1846      0
                                0    4.0380
(:,9)=3.1846      0
                                0    4.0380
inital prior probs
0.1111    0.1111    0.1111    0.1111    0.1111    0.1111    0.1111
0.1111    0.1111
initial log likelihood is -2.2697e+03
final iteration is 366
final likelihood is: -1.6491e+03
difference 1.5444e-05
my gmm:my mu
3.6438    5.4206
0.4814    0.0039
3.2427    0.9529
1.9552    2.4271
3.9254    1.1821
5.6810    4.1730
1.0859    0.8338
3.0178    4.3934
2.1353    4.3088
my Sigma(:,1)=0.0897    0.0147

```

```

0.0147    0.0945
(:,,2)=0.9896  -0.4058
      -0.4058    0.1735
(:,,3)=0.4591  -0.0205
      -0.0205    0.3716
(:,,4)=0.5363    0.2790
      0.2790    0.1897
(:,,5)=0.0769    0.1184
      0.1184    0.2770
(:,,6)=0.4534    0.0256
      0.0256    0.4839
(:,,7)=0.5576    0.3847
      0.3847    0.4839
(:,,8)=0.1673  -0.0783
      -0.0783    0.3316
(:,,9)=0.5493    0.6919
      0.6919    1.4201
my prop
      0.0447    0.0162    0.1794    0.0826    0.0629    0.2060    0.2427
0.1004    0.0652
the NO. 5 model.
initial mu
      0.9909    3.3582
      2.3386    4.7521
      3.4349    4.4753
      6.8970    0.1931
      2.2940   -0.2463
      7.2505    7.3910
      1.2744    0.1317
      4.7784   -1.0829
      4.4736    3.5518
initial sigma(:,,1)=4.0138    0
      0    3.9693
(:,,2)=4.0138    0
      0    3.9693
(:,,3)=4.0138    0
      0    3.9693
(:,,4)=4.0138    0
      0    3.9693
(:,,5)=4.0138    0
      0    3.9693
(:,,6)=4.0138    0
      0    3.9693
(:,,7)=4.0138    0

```

```

0      3.9693
(:,8)=4.0138      0
0      3.9693
(:,9)=4.0138      0
0      3.9693
initial prior probs
0.1111      0.1111      0.1111      0.1111      0.1111      0.1111      0.1111
0.1111      0.1111
initial log likelihood is: -2.2472e+03
final iteration is 218
final likelihood is -1.6473e+03
difference 1.6228e-05
my gmm: my mu
1.8637      2.3800
2.8958      4.6876
2.9424      4.6708
3.3428      0.9270
1.8237      1.2690
5.7441      4.1310
0.6252      0.3743
3.7076      0.8551
2.8911      2.6864
my Sigma(:,1)=0.3556      0.1679
0.1679      0.1339
(:,2)=0.5994      0.4093
0.4093      0.6762
(:,3)=0.1038      -0.1802
-0.1802      0.5791
(:,4)=0.7690      0.6918
0.6918      0.6892
(:,5)=1.0147      0.2047
0.2047      0.2438
(:,6)=0.3968      0.0783
0.0783      0.4490
(:,7)=0.2881      0.1612
0.1612      0.2576
(:,8)=0.1727      0.0282
0.0282      0.2767
(:,9)=1.4320      2.0041
2.0041      2.8105
my prop
0.0803      0.1629      0.0364      0.0520      0.1662      0.1943      0.1264
0.1363      0.0453
the NO. 1 model for holdout data.initial mu

```

3.4406	5.6017						
2.1381	2.5273						
3.0362	4.6241						
4.9587	5.1235						
1.4989	1.0662						
5.7127	4.0696						
0.4471	0.3736						
3.6779	0.9728						
1.6977	3.6612						
initial sigma(:,1) = 0.1077		0.2487					
0.2487	0.5830						
(:,2) = 0.4264		0.2259					
0.2259	0.1586						
(:,3) = 0.2741		0.0045					
0.0045	0.4394						
(:,4) = 1.4091		-0.6039					
-0.6039	0.2910						
(:,5) = 0.7789		0.2773					
0.2773	0.5892						
(:,6) = 0.3785		0.0913					
0.0913	0.4658						
(:,7) = 0.2017		0.2235					
0.2235	0.2898						
(:,8) = 0.2104		0.0719					
0.0719	0.3215						
(:,9) = 0.0582		-0.0331					
-0.0331	0.5054						
inital prior probs							
0.0126	0.0638	0.1562	0.0409	0.2756	0.1766	0.0625	
0.1870	0.0248						
initial log likelihood is: -4.6050e+03							
final iteration is 284							
final likelihood is: -4.3588e+03							
difference 4.2308e-05							
my gmm:							
3.0231	4.4368						
1.8596	2.0589						
3.0077	4.6666						
5.9271	4.5820						
0.9813	0.6623						
5.7102	4.1060						
0.5408	1.1169						
3.5551	0.9720						
1.7004	3.3855						

```

(:,,1) = 0.0111    0.0318
          0.0318    0.0908
(:,,2) =0.3634    0.0690
          0.0690    0.2548
(:,,3) =0.3098    0.0206
          0.0206    0.6645
(:,,4) =0.3687   -0.1841
          -0.1841    0.1324
(:,,5) =0.6059    0.1833
          0.1833    0.3029
(:,,6) =0.3694    0.0468
          0.0468    0.4099
(:,,7) =0.0103    0.0373
          0.0373    0.1646
(:,,8) =0.4924    0.0763
          0.0763    0.2900
(:,,9) =0.1854   -0.0379
          -0.0379    0.0291
0.0049    0.1047    0.1440    0.0364    0.1578    0.3731    0.0095    0.1605
0.0091

```

the NO. 2 model for holdout data.

initial mu

```

3.9165    1.1828
1.8343    1.9623
3.1945    1.0762
3.9363    5.5601
2.9316    4.5980
3.4819    4.9295
1.8819    2.4190
0.8134    0.6027
5.7437    4.1220

```

initial sigma(:,,1) =0.0909 0.1309

```

0.1309    0.2860

```

```

(:,,2) =0.0684   -0.1394
          -0.1394    1.9205
(:,,3) =0.4591   -0.1575
          -0.1575    0.3945
(:,,4) =0.5321   -0.1471
          -0.1471    0.0656
(:,,5) =0.2570   -0.0241
          -0.0241    0.3877
(:,,6) =0.0638    0.2275
          0.2275    0.8633
(:,,7) =1.0674    0.5410

```



```

0.5410    0.3387
(:,8)=0.3678    0.2299
0.2299    0.3598
(:,9)=0.4022    0.0755
0.0755    0.4505
inital prior probs
0.0695    0.0810    0.1802    0.0314    0.1300    0.0287    0.0834
0.2018    0.1940
initial log likelihood is: -4.6310e+03
final iteration is 470
final likelihood is: -4.3566e+03
difference 4.0722e-05
my gmm:
3.2690    0.9762
2.2242    1.8409
3.7750    1.0228
4.5320    5.6155
3.0154    4.6615
3.1404    3.4631
1.4869    2.0527
0.9102    0.7053
5.7228    4.1386
(:,1)=0.0093    -0.0039
-0.0039    0.0035
(:,2)=0.3373    -0.1761
-0.1761    1.1063
(:,3)=0.3716    0.0315
0.0315    0.2637

(:,4)=3.7667    -0.9643
-0.9643    0.2754
(:,5)=0.2608    0.0224
0.0224    0.5800
(:,6)=0.2074    0.7190
0.7190    2.4947
(:,7)=0.2748    0.0758
0.0758    0.1387
(:,8)=0.5186    0.1747
0.1747    0.3193
(:,9)=0.3676    0.0333
0.0333    0.3959
0.0111    0.1079    0.1222    0.0073    0.1358    0.0068    0.0390    0.1633
0.4065
the NO. 3 model for holdout data. initial mu

```

```

3.4246    4.7506
2.7561    4.6564
1.6841    2.3344
2.0227    2.1315
4.5706    5.2760
0.7497    0.4910
3.7826    1.0475
2.4321    1.2601
5.7225    4.1090
initial sigma(:,1) =0.0685    0.2345
0.2345    0.8460
(:,2) =0.4348    0.2282
0.2282    0.5064
(:,3) =1.0796    0.5632
0.5632    0.3299
(:,4) =1.5943    1.6934
1.6934    1.8463
(:,5) =1.5731   -0.6426
-0.6426    0.2685
(:,6) =0.3233    0.1451
0.1451    0.2833
(:,7) =0.1397    0.0871
0.0871    0.2982
(:,8) =0.7455   -0.1869
-0.1869    0.5973
(:,9) =0.3892    0.0755
0.0755    0.4668
inital prior probs
0.0324    0.1455    0.0501    0.0712    0.0250    0.1563    0.1340
0.1970    0.1885
initial log likelihood is  -4.6218e+03
final iteration is  483
final likelihood is: -4.3604e+03
difference 4.3488e-05
my gmm:
3.3152    4.4391
2.9983    4.6657
1.4411    2.3619
1.7352    1.7534
3.2033    6.2065
0.7324    0.6548
3.6704    0.9996
1.9376    1.7229
5.7279    4.1451

```

```

(:,1)=0.0095    0.0719
      0.0719    0.5748
(:,2)=0.2858    0.0227
      0.0227    0.5251
(:,3)=0.2429    0.2044
      0.2044    0.1726
(:,4)= 0.9956    1.1951
      1.1951    1.4363
(:,5)=0.8594   -0.0825
      -0.0825    0.0090
(:,6)=0.4346    0.1440
      0.1440    0.3009
(:,7)=0.4043    0.0415
      0.0415    0.2486
(:,8)=0.4238    0.0301
      0.0301    0.8891
(:,9)=0.3762    0.0369
      0.0369    0.3988
0.0063    0.1304    0.0067    0.0118    0.0058    0.1250    0.1424    0.1621
0.4096

```

the NO. 4 model for holdout data initial mu

```

      3.6438    5.4206
      0.4814    0.0039
      3.2427    0.9529
      1.9552    2.4271
      3.9254    1.1821
      5.6810    4.1730
      1.0859    0.8338
      3.0178    4.3934
      2.1353    4.3088
initial sigma (,1)=0.0897    0.0147
      0.0147    0.0945
(:,2)=0.9896   -0.4058
      -0.4058    0.1735
(:,3)=0.4591   -0.0205
      -0.0205    0.3716
(:,4)=0.5363    0.2790
      0.2790    0.1897
(:,5)=0.0769    0.1184
      0.1184    0.2770
(:,6)= 0.4534    0.0256
      0.0256    0.4839
(:,7)=0.5576    0.3847
      0.3847    0.4839

```

```

(:,8)=0.1673   -0.0783
          -0.0783   0.3316
(:,9)=0.5493   0.6919
          0.6919   1.4201
initial prior probs
          0.0447   0.0162   0.1794   0.0826   0.0629   0.2060   0.2427
0.1004   0.0652
initial log likelihood is: -4.6035e+03
final iteration is 435
final likelihood is: -4.3592e+03
difference 4.0478e-05
my gmm:
          3.1150   4.9151
          0.9772  -0.0842
          3.6179   0.9725
          1.5076   2.0605
          4.5003   1.0311
          5.7286   4.1484
          0.9471   0.7467
          2.8129   4.1969
          2.3046   2.0585
(:,1)= 0.2586  -0.0244
          -0.0244   0.5552
(:,2)= 0.8045  -0.3144
          -0.3144   0.1241
(:,3)=0.3763   0.0540
          0.0540   0.2109
(:,4)=0.2594   0.0553
          0.0553   0.1212
(:,5)=0.0118   0.0122
          0.122    0.3669
(:,6)=0.3740   0.0340
          0.0340   0.4038
(:,7)=0.5368   0.1931
          0.1931   0.3298
(:,8)=0.2531  -0.1552
          -0.1552   0.1243
(:,9)=0.3965  -0.1226
          -0.1226   1.1130
0.1065   0.0051   0.1224   0.0380   0.0119   0.4099   0.1651   0.0295
0.1115
the NO. 5 model for holdout data. initial mu
          1.8637   2.3800
          2.8958   4.6876

```

```

2.9424    4.6708
3.3428    0.9270
1.8237    1.2690
5.7441    4.1310
0.6252    0.3743
3.7076    0.8551
2.8911    2.6864
initial sigma (:,:,1) =0.3556    0.1679
                        0.1679    0.1339
(:,:,2) =0.5994    0.4093
                0.4093    0.6762
(:,:,3) =0.1038   -0.1802
                -0.1802    0.5791
(:,:,4) =0.7690    0.6918
                0.6918    0.6892
(:,:,5) =1.0147    0.2047
                0.2047    0.2438
(:,:,6) =0.3968    0.0783
                0.0783    0.4490
(:,:,7) =0.2881    0.1612
                0.1612    0.2576
(:,:,8) =0.1727    0.0282
                0.0282    0.2767
(:,:,9) =1.4320    2.0041
                2.0041    2.8105
inital prior probs
0.0803    0.1629    0.0364    0.0520    0.1662    0.1943    0.1264    0.1363
0.0453
initial log likelihood is:-4.6344e+03
final iteration is 880
final likelihood is: -4.3635e+03
difference 4.3505e-05
my gmm:
1.7337    2.1309
1.7013    3.3810
3.0058    4.6724
3.5486    0.8614
2.4054    1.0871
5.7308    4.1517
0.8865    0.7457
3.6288    0.9636
2.0504    1.5080
(:,:,1) =0.3421    0.1091
                0.1091    0.2257

```

```

(:,2)=0.1808   -0.0369
          -0.0369    0.0297
(:,3)=0.2978    0.0228
          0.0228    0.6304
(:,4)=0.6046    0.4152
          0.4152    0.4724
(:,5)=0.6110    0.6057
          0.6057    0.7056
(:,6)=0.3725    0.0328
          0.0328    0.3992
(:,7)=0.5365    0.2144
          0.2144    0.2936
(:,8)=0.4326   -0.0341
          -0.0341    0.1876
(:,9)=0.4233    0.6174
          0.6174    0.9024
0.0869    0.0094    0.1476    0.0495    0.0333    0.4086    0.1516    0.1019
0.0112

```

The high log is -4.3588e+03 with iteration 284 the log of training data is -1.6497e+03.

K=11

the NO. 1 model. initial mu

```

          0.3010   -0.3112
          1.8659    3.8000
          2.6708    0.6145
          7.2381    2.0036
          0.0026    3.7428
          6.1323    0.8391
          4.2845    1.1778
          1.9304    4.0418
          0.3053    0.9572
          2.3861    5.8593
          2.8576    7.2471
initial sigma(:,1)=3.8621    0
          0    3.6440
(:,2)=3.8621    0
          0    3.6440
(:,3)=3.8621    0
          0    3.6440
(:,4)=3.8621    0
          0    3.6440
(:,5)=3.8621    0

```

	0	3.6440
(:,:,6) =3.8621		0
	0	3.6440
(:,:,7) =3.8621		0
	0	3.6440
(:,:,8) =3.8621		0
	0	3.6440
(:,:,9) =3.8621		0
	0	3.6440
(:,:,10) =3.8621		0
	0	3.6440
(:,:,11) =3.8621		0
	0	3.6440

inital prior probs

0.0909	0.0909	0.0909	0.0909	0.0909	0.0909	0.0909	0.0909
0.0909	0.0909	0.0909					

initial log likelihood is: -2.2577e+03

final iteration is 282

final likelihood is: -1.6418e+03

difference 1.6068e-05

my gmm: my mu

0.9107	0.4360
1.7588	2.3449
3.4718	0.9198
5.7114	4.0743
1.4210	2.8673
3.9927	1.3230
2.1253	1.4194
3.4489	4.8362
1.5620	1.5761
2.8896	4.7078
4.8966	5.1436

my Sigma

(:,:,1) =0.5497	0.0023
	0.0023
(:,:,2) =0.3906	0.2021
	0.2021
(:,:,3) =0.2709	-0.0214
	-0.0214
(:,:,4) =0.3817	0.0889
	0.0889
(:,:,5) =0.3593	0.6186
	0.6186

```

(:,6)=0.0732    0.1127
              0.1127    0.2530
(:,7)=0.1564    0.0776
              0.0776    0.1240
(:,8)=0.0675    0.2364
              0.2364    0.8742
(:,9)=1.3294    1.4094
              1.4094    1.5554
(:,10)=0.3034   0.0634
              0.0634    0.4037
(:,11)=1.4785   -0.6253
              -0.6253    0.2940
my prop 0.1383    0.0532    0.1632    0.1781    0.0414    0.0477    0.0581
0.0307    0.1217    0.1278    0.0399
the NO. 2  model. initial mu
              3.7523    0.1987
              -0.4238    2.3391
              6.5776   -0.5425
              6.3439    3.8790
              5.8013    2.7603
              0.9173    4.7332
              3.8425    4.7678
              -1.1713    4.2298
              2.3599   -1.0741
              1.3732   -0.7654
              0.0472    1.4335
initial sigma(:,1) = 3.6628    0
                      0    3.9546
(:,2)=3.6628    0
              0    3.9546
(:,3)=3.6628    0
              0    3.9546
(:,4)=3.6628    0
              0    3.9546
(:,5)=3.6628    0
              0    3.9546
(:,6)=3.6628    0
              0    3.9546
(:,7)=3.6628    0
              0    3.9546
(:,8)=3.6628    0
              0    3.9546
(:,9)=3.6628    0
              0    3.9546

```



```

(:,10) = 3.6628      0
           0      3.9546
(:,11) = 3.6628      0
           0      3.9546
initial prior probs 0.0909      0.0909      0.0909      0.0909      0.0909      0.0909      0.0909
0.0909      0.0909      0.0909      0.0909
initial log likelihood is -2.2936e+03
final iteration is 257
final likelihood is: -1.6397e+03
difference 1.6340e-05
my gmm: my mu
      3.4865      0.9329
      1.9612      2.4278
      3.9950      1.3277
      5.7125      4.1860
      5.8055      3.8058
      3.1252      4.6550
      3.7471      5.2973
      1.7815      3.9914
      2.1104      1.4591
      0.4695      0.0162
      0.8296      0.6347
my Sigma(:,1) = 0.2542      -0.0361
      -0.0361      0.3197
(:,2) = 0.6432      0.3518
           0.3518      0.2344
(:,3) = 0.0775      0.1177
           0.1177      0.2587
(:,4) = 0.4998      0.0630
           0.0630      0.3369
(:,5) = 0.0564      0.1716
           0.1716      0.8297
(:,6) = 0.2683      0.1461
           0.1461      0.6152
(:,7) = 1.0408      0.0405
           0.0405      0.0070
(:,8) = 0.3947      0.6783
           0.6783      1.5021
(:,9) = 0.1413      -0.0132
           -0.0132      0.6086
(:,10) = 1.0778      -0.4455
           -0.4455      0.1904
(:,11) = 0.3553      0.2389
           0.2389      0.3664

```

my prop	0.1600	0.0729	0.0487	0.1617	0.0342	0.1550	0.0253
	0.0388	0.0967	0.0142	0.1925			

the NO. 3 model. initial mu

2.2052	0.9677
5.8207	-0.0215
4.9297	1.0951
7.1242	2.4899
3.2899	3.2532
1.4821	2.6419
-0.4426	6.3064
3.9881	3.1735
5.4597	6.9048
2.3440	4.2226
-0.5724	7.0282

initial sigma (:,:,1) = 3.3726	0
	0 3.9762

(:,:,2) = 3.3726	0
	0 3.9762

(:,:,3) = 3.3726	0
	0 3.9762

(:,:,4) = 3.3726	0
	0 3.9762

(:,:,5) = 3.3726	0
	0 3.9762

(:,:,6) = 3.3726	0
	0 3.9762

(:,:,7) = 3.3726	0
	0 3.9762

(:,:,8) = 3.3726	0
	0 3.9762

(:,:,9) = 3.3726	0
	0 3.9762

(:,:,10) = 3.3726	0
	0 3.9762

(:,:,11) = 3.3726	0
	0 3.9762

inital prior probs	0.0909	0.0909	0.0909	0.0909	0.0909	0.0909	0.0909
	0.0909	0.0909	0.0909				

initial log likelihood is -2.2579e+03

final iteration is 755

final likelihood is: -1.6399e+03

difference 1.5018e-05

my gmm: my mu

0.4718	0.0098					
3.7989	1.0811					
2.5649	1.2083					
5.7020	4.0797					
2.0666	2.1153					
0.7954	0.5696					
2.6038	4.7578					
1.9133	2.4142					
5.1072	5.0422					
1.7346	3.7776					
3.3662	4.7936					
my Sigma(:,1) = 1.1084 -0.4604						
-0.4604	0.1971					
(:,:,2) = 0.1320	0.0953					
0.0953	0.2990					
(:,:,3) = 0.7817	-0.2387					
-0.2387	0.4905					
(:,:,4) = 0.3843	0.0848					
0.0848	0.4678					
(:,:,5) = 1.9290	1.9960					
1.9960	2.0792					
(:,:,6) = 0.3259	0.1772					
0.1772	0.3125					
(:,:,7) = 0.0510	-0.0154					
-0.0154	0.4053					
(:,:,8) = 0.5017	0.2575					
0.2575	0.1730					
(:,:,9) = 1.4097	-0.6099					
-0.6099	0.2880					
(:,:,10) = 0.0650	-0.0204					
-0.0204	0.4979					
(:,:,11) = 0.1137	0.1502					
0.1502	0.5980					
my prop	0.0140	0.1219	0.1931	0.1790	0.0405	0.1657
0.0633	0.0343	0.0287	0.0969			0.0625
the NO. 4 model. initial mu						
1.1658	-1.3098					
4.5216	3.9110					
4.7261	2.0224					
-0.7726	6.6625					
0.8652	-1.3586					
0.5956	2.6860					
4.4867	2.3519					
6.0348	2.6725					

```

1.6515    5.3839
5.4747    1.4587
4.5525    5.5117
initial sigma(:,1) = 3.6033    0
0    3.3359
(:,2) = 3.6033    0
0    3.3359
(:,3) = 3.6033    0
0    3.3359
(:,4) = 3.6033    0
0    3.3359
(:,5) = 3.6033    0
0    3.3359
(:,6) = 3.6033    0
0    3.3359
(:,7) = 3.6033    0
0    3.3359
(:,8) = 3.6033    0
0    3.3359
(:,9) = 3.6033    0
0    3.3359
(:,10) = 3.6033    0
0    3.3359
(:,11) = 3.6033    0
0    3.3359
inital prior probs
0.0909    0.0909    0.0909    0.0909    0.0909    0.0909    0.0909
0.0909    0.0909    0.0909    0.0909
nitial log likelihood is: -2.2398e+03
final iteration is 286
final likelihood is -1.6356e+03
difference 1.5523e-05

```

my gmm:

my mu

```

1.4625    0.7652
3.1597    4.0583
2.7204    1.2723
2.4861    4.3075
1.0213    0.4509
0.6539    0.6041
2.0065    2.4586
5.6926    3.6308
3.2022    5.3064

```

	3.7919	1.0940					
	5.7104	4.2480					
my Sigma(:, :, 1) =	0.1945	0.2902					
		0.2902	0.4484				
(:, :, 2) =	0.1337	-0.0207					
	-0.0207	0.1205					
(:, :, 3) =	0.7261	-0.3290					
	-0.3290	0.4439					
(:, :, 4) =	0.5214	0.4123					
	0.4123	0.4505					
(:, :, 5) =	0.8383	-0.1101					
	-0.1101	0.2596					
(:, :, 6) =	0.2869	0.3119					
	0.3119	0.4026					
(:, :, 7) =	0.5906	0.3183					
	0.3183	0.2203					
(:, :, 8) =	0.0086	0.0148					
	0.0148	0.4091					
(:, :, 9) =	0.5378	0.0821					
	0.0821	0.2545					
(:, :, 10) =	0.1388	0.1047					
	0.1047	0.3137					
(:, :, 11) =	0.5179	0.0564					
	0.0564	0.4160					
my prop							
	0.0430	0.0509	0.1588	0.0594	0.0794	0.1065	0.0841
0.0325	0.0943	0.1228	0.1682				

the NO. 5 model. initial mu

	0.1733	5.3430
	4.9595	0.2854
	2.7827	1.1520
	-0.0297	-0.5698
	1.6222	3.6834
	3.9568	4.6229
	0.3125	3.4237
	5.1057	2.3640
	0.7606	4.2816
	6.6751	4.3095
	0.9904	4.5848

initial sigma

(:, :, 1) =	3.7677	0
	0	4.2453
(:, :, 2) =	3.7677	0

```

0      4.2453
(:,,3) = 3.7677      0
0      4.2453
(:,,4) = 3.7677      0
0      4.2453
(:,,5) = 3.7677      0
0      4.2453
(:,,6) = 3.7677      0
0      4.2453
(:,,7) = 3.7677      0
0      4.2453
(:,,8) = 3.7677      0
0      4.2453
(:,,9) = 3.7677      0
0      4.2453
(:,,10) = 3.7677      0
0      4.2453
(:,,11) = 3.7677      0
0      4.2453
initial prior probs
0.0909      0.0909      0.0909      0.0909      0.0909      0.0909      0.0909
0.0909      0.0909      0.0909      0.0909
initial log likelihood is: -2.2398e+03
final iteration is      310
final likelihood is: -1.6382e+03
differenc      1.5696e-05
my gmm: my mu
3.0248      4.2165
3.7067      1.0340
1.8875      0.3216
0.7509      0.5870
1.7947      1.8965
3.3727      5.3497
2.0231      2.4747
5.7348      4.8759
2.3257      1.5368
5.7324      4.1084
2.1878      4.3014
my Sigma(:,,1) = 0.1874      -0.0399
-0.0399      0.1603
(:,,2) = 0.1720      0.0870
0.0870      0.2990
(:,,3) = 2.0462      -0.0948
-0.0948      0.2135

```

```

(:,4)=0.3073    0.2080
          0.2080    0.3502
(:,5)=0.0025   -0.0041
          -0.0041    1.3567
(:,6)=0.5599   -0.0058
          -0.0058    0.1241
(:,7)=0.5978    0.3129
          0.3129    0.2055
(:,8)=0.3410    0.7014
          0.7014    1.4426
(:,9)=0.2734   -0.0192
          -0.0192    0.1980
(:,10)=0.4123   0.0561
          0.0561    0.4157
(:,11)=0.8117   0.9680
          0.9680    1.3888
my prop
          0.0832    0.1687    0.0566    0.1702    0.0296    0.0852    0.0817
0.0049    0.0898    0.1915    0.0386

```

the NO. 1 model for holdout data.

initial mu

```

0.9107    0.4360
1.7588    2.3449
3.4718    0.9198
5.7114    4.0743
1.4210    2.8673
3.9927    1.3230
2.1253    1.4194
3.4489    4.8362
1.5620    1.5761
2.8896    4.7078
4.8966    5.1436

```

initial sigma(:,1)=0.5497 0.0023

```

          0.0023    0.2499

```

```

(:,2)=0.3906    0.2021
          0.2021    0.1442
(:,3)=0.2709   -0.0214
          -0.0214    0.3103
(:,4)=0.3817    0.0889
          0.0889    0.4636
(:,5)=0.3593    0.6186
          0.6186    1.6533
(:,6)=0.0732    0.1127

```

```

0.1127    0.2530
(:,,7) = 0.1564    0.0776
0.0776    0.1240
(:,,8) = 0.0675    0.2364
0.2364    0.8742
(:,,9) = 1.3294    1.4094
1.4094    1.5554
(:,,10) = 0.3034    0.0634
0.0634    0.4037
(:,,11) = 1.4785    -0.6253
-0.6253    0.2940
initial prior probs
0.1383    0.0532    0.1632    0.1781    0.0414    0.0477    0.0581
0.0307    0.1217    0.1278    0.0399
initial log likelihood is: -4.6104e+03
final iteration is 712
final likelihood is: -4.3519e+03
difference 4.2569e-05
my gmm:
0.9008    0.6888
1.6187    2.2433
3.5622    0.9707
5.7072    4.1056
1.9993    1.8794
4.5011    1.0495
2.7968    1.8413
3.6136    5.3847
1.7814    1.7409
2.9822    4.5978
5.9180    4.5827
(:,,1) = 0.5108    0.1670
0.1670    0.3064
(:,,2) = 0.4095    0.1632
0.1632    0.1364
(:,,3) = 0.3885    0.0434
0.0434    0.2048
(:,,4) = 0.3730    0.0464
0.0464    0.4131
(:,,5) = 0.2186    -0.2149
-0.2149    1.3461
(:,,6) = 0.0116    0.0119
0.0119    0.3665
(:,,7) = 0.2377    0.0735
0.0735    0.0276

```



```

(:,8)=0.0499    0.0866
          0.0866    0.2332
(:,9)=0.2806    0.1504
          0.1504    0.0834
(:,10)=0.2636   -0.0147
          -0.0147    0.6821
(:,11)=0.3755   -0.1854
          -0.1854    0.1318
0.1609    0.0427    0.1335    0.3739    0.0647    0.0122    0.0145    0.0077
0.0128    0.1404    0.0368
the NO. 2  model for holdout data. initial mu
          3.4865    0.9329
          1.9612    2.4278
          3.9950    1.3277
          5.7125    4.1860
          5.8055    3.8058
          3.1252    4.6550
          3.7471    5.2973
          1.7815    3.9914
          2.1104    1.4591
          0.4695    0.0162
          0.8296    0.6347
initial sigma (,:,1) = 0.2542   -0.0361
                        -0.0361   0.3197
(:,2) = 0.6432    0.3518
          0.3518    0.2344
(:,3) = 0.0775    0.1177
          0.1177    0.2587
(:,4) = 0.4998    0.0630
          0.0630    0.3369
(:,5) = 0.0564    0.1716
          0.1716    0.8297

(:,6) = 0.2683    0.1461
          0.1461    0.6152
(:,7) = 1.0408    0.0405
          0.0405    0.0070
(:,8) = 0.3947    0.6783
          0.6783    1.5021
(:,9) = 0.1413   -0.0132
          -0.0132    0.6086
(:,10) = 1.0778   -0.4455
          -0.4455    0.1904
(:,11) = 0.3553    0.2389

```

```

0.2389    0.3664
inital prior probs
0.1600    0.0729    0.0487    0.1617    0.0342    0.1550    0.0253
0.0388    0.0967    0.0142    0.1925
initial log likelihood is: -4.6433e+03
final iteration is 220
final likelihood is: -4.3518e+03
difference 4.2001e-05
my gmm:
3.6323    0.9766
1.5074    2.0613
4.5006    1.0320
5.7268    4.2061
5.7314    3.1765
3.0410    4.6643
4.8222    5.4809
2.0811    2.8765
2.3784    1.9399
0.6766    0.0375
0.9799    0.7511
(:,1)=0.3753    0.0576
0.0576    0.2116
(:,2)=0.2537    0.0602
0.0602    0.1330
(:,3)= 0.0118    0.0122
0.0122    0.3694
(:,4)= 0.3978    0.0386
0.0386    0.3436
(:,5)= 0.1062    0.0372
0.0372    0.1481
(:,6)=0.2272    0.0362
0.0362    0.6203
(:,7)= 0.4371    0.1092
0.1092    0.0273
(:,8)=0.0474    -0.4192
-0.4192    3.9969
(:,9)=0.4554    -0.2661
-0.2661    0.9591
(:,10)=0.6934    -0.2591
-0.2591    0.0973
(:,11)=0.5490    0.1929
0.1929    0.3343
0.1183    0.0408    0.0118    0.3830    0.0258    0.1389    0.0031    0.0128
0.0901    0.0045    0.1708

```

the NO. 3 model for holdout data.

initial mu

0.4718	0.0098
3.7989	1.0811
2.5649	1.2083
5.7020	4.0797
2.0666	2.1153
0.7954	0.5696
2.6038	4.7578
1.9133	2.4142
5.1072	5.0422
1.7346	3.7776
3.3662	4.7936

initial sigma(:,1) = 1.1084 -0.4604

-0.4604 0.1971

(:,:,2) = 0.1320 0.0953

0.0953 0.2990

(:,:,3) = 0.7817 -0.2387

-0.2387 0.4905

(:,:,4) = 0.3843 0.0848

0.0848 0.4678

(:,:,5) = 1.9290 1.9960

1.9960 2.0792

(:,:,6) = 0.3259 0.1772

0.1772 0.3125

(:,:,7) = 0.0510 -0.0154

-0.0154 0.4053

(:,:,8) = 0.5017 0.2575

0.2575 0.1730

(:,:,9) = 1.4097 -0.6099

-0.6099 0.2880

(:,:,10) = 0.0650 -0.0204

-0.0204 0.4979

(:,:,11) = 0.1137 0.1502

0.1502 0.5980

initial prior probs 0.0140 0.1219 0.1931 0.1790 0.0405 0.1657 0.0625

0.0633 0.0343 0.0287 0.0969

initial log likelihood is: -4.6298e+03

final iteration is 407

final likelihood is: -4.3449e+03

difference 4.2210e-05

my gmm:

1.0632 -0.1141

	3.7108	0.9984					
	2.2529	1.7136					
	5.7859	3.9965					
	5.1864	4.8442					
	0.8577	0.7038					
	1.9685	2.8455					
	1.5122	2.0610					
	5.6032	4.7433					
	1.3580	2.9617					
	3.0602	4.6583					
(:,:,1) =	0.6135	-0.2414					
	-0.2414	0.0969					
(:,:,2) =	0.3928	0.0423					
	0.0423	0.2351					
(:,:,3) =	0.4010	-0.1124					
	-0.1124	0.8733					
(:,:,4) =	0.3977	0.0968					
	0.0968	0.3415					
(:,:,5) =	0.1693	0.1287					
	0.1287	0.1092					
(:,:,6) =	0.5096	0.1818					
	0.1818	0.2780					
(:,:,7) =	0.1835	0.8325					
	0.8325	3.7788					
(:,:,8) =	0.3436	0.1236					
	0.1236	0.1623					
(:,:,9) =	0.1604	-0.0686					
	-0.0686	0.2054					
(:,:,10) =	0.1065	0.6004					
	0.6004	4.5566					
(:,:,11) =	0.2573	0.0414					
	0.0414	0.5929					
	0.0056	0.1332	0.1064	0.3302	0.0198	0.1434	0.0113
	0.0594	0.0093	0.1399				0.0414
the NO. 4 model for holdout data.initial mu							
	1.4625	0.7652					
	3.1597	4.0583					
	2.7204	1.2723					
	2.4861	4.3075					
	1.0213	0.4509					
	0.6539	0.6041					
	2.0065	2.4586					
	5.6926	3.6308					
	3.2022	5.3064					

```

        3.7919    1.0940
        5.7104    4.2480
initial sigma(:,1) = 0.1945    0.2902
                0.2902    0.4484
(:,2) = 0.1337   -0.0207
        -0.0207    0.1205
(:,3) = 0.7261   -0.3290
        -0.3290    0.4439
(:,4) = 0.5214    0.4123
        0.4123    0.4505
(:,5) = 0.8383   -0.1101
        -0.1101    0.2596
(:,6) = 0.2869    0.3119
        0.3119    0.4026
(:,7) = 0.5906    0.3183
        0.3183    0.2203
(:,8) = 0.0086    0.0148
        0.0148    0.4091
(:,9) = 0.5378    0.0821
        0.0821    0.2545
(:,10) = 0.1388   0.1047
        0.1047    0.3137
(:,11) = 0.5179   0.0564
        0.0564    0.4160
initial prior probs
        0.0430    0.0509    0.1588    0.0594    0.0794    0.1065    0.0841
0.0325    0.0943    0.1228    0.1682
initial log likelihood is: -4.6396e+03
final iteration      824
final likelihood is: -4.3454e+03
difference  4.2701e-05
my gmm:
        1.8016    1.5490
        2.8451    4.1708
        3.0547    0.9948
        1.9903    3.4180
        0.5899    0.4873
        0.5335    1.0633
        1.5690    2.3177
        5.6723    4.1006
        3.0744    4.8966
        3.8772    0.9862
        5.7307    4.1506
(:,1) = 0.3773    0.1699

```

```

0.1699    0.7415
(:,2) = 0.2607    -0.1590
        -0.1590    0.1261
(:,3) = 0.0769    0.1137
        0.1137    0.4821
(:,4) = 0.4179    -0.0562
        -0.0562    0.0180
(:,5) = 0.4369    0.1263
        0.1263    0.2126
(:,6) = 0.0143    0.0315
        0.0315    0.1375
(:,7) = 0.5678    0.2711
        0.2711    0.1628
(:,8) = 0.0028    0.0363
        0.0363    0.7606
(:,9) = 0.2928    0.0012
        0.0012    0.5485
(:,10) = 0.3046    0.0459
        0.0459    0.2249
(:,11) = 0.3918    0.0339
        0.0339    0.3862
0.1750    0.0290    0.0397    0.0093    0.0865    0.0138    0.0164    0.0176
0.1110    0.1094    0.3922

```

the NO. 5 model for holdout data.

initial mu

```

3.0248    4.2165
3.7067    1.0340
1.8875    0.3216
0.7509    0.5870
1.7947    1.8965
3.3727    5.3497
2.0231    2.4747
5.7348    4.8759
2.3257    1.5368
5.7324    4.1084
2.1878    4.3014

```

```

initial sigma(:,1) = 0.1874    -0.0399
                    -0.0399    0.1603
(:,2) = 0.1720    0.0870
        0.0870    0.2990
(:,3) = 2.0462    -0.0948
        -0.0948    0.2135
(:,4) = 0.3073    0.2080

```

```

0.2080    0.3502
(:,5)=0.0025 -0.0041
-0.0041    1.3567
(:,6)=0.5599 -0.0058
-0.0058    0.1241
(:,7)= 0.5978    0.3129
0.3129    0.2055
(:,8)=0.3410    0.7014
0.7014    1.4426
(:,9)=0.2734 -0.0192
-0.0192    0.1980
(:,10)=0.4123    0.0561
0.0561    0.4157
(:,11)=0.8117    0.9680
0.9680    1.3888
initial prior probs
0.0832    0.1687    0.0566    0.1702    0.0296    0.0852    0.0817
0.0049    0.0898    0.1915    0.0386

```

initial log likelihood is:
-4.6415e+03

The high log is -4.3518e+03 with iteration 220 the log of training is -1.6397e+03

So from the log we can see that it is better to choose k=3 with similar log but less iteration.